# Conformal Field Theories

**Tutorial 3**

*Higgs School 2022*

**Exercise 3.1.** *Compute the correlation function of two scalar fields on the cylinder. In order to to this, recall the relation between the flat metric and the cylinder metric*

$$ds^2 = dr^2 + r^2 d\Omega_{d-1} = e^{2\tau}\left(d\tau^2 + d\Omega_{d-1}\right) \tag{3.1}$$

*and the fact that, in absence of Weyl anomalies, the correlation functions of a theory with* $T^\mu_\mu = 0$ *satisfy the nice relation*

$$\langle \mathcal{O}_1(x)\dots\mathcal{O}_n(x)\rangle_{g_{\mu\nu}} = \left(\prod_{i=1}^{n}\Omega(x_i)^{\Delta_i}\right)\langle \mathcal{O}_1(x)\dots\mathcal{O}_n(x)\rangle_{\Omega^2 g_{\mu\nu}}\,. \tag{3.2}$$

*Using the above relation with* $g_{\mu\nu}$ *the cylinder metric, it allows to compute the correlation function of operators on the cylinder. Show that*

$$\langle \mathcal{O}(\tau_1,\vec{n}_1)\mathcal{O}(\tau_2,\vec{n}_2)\rangle_{cyl} = \sum_n c_n e^{-(\Delta+n)\tau_{21}} \tag{3.3}$$

*where* $\tau$ *is the time on the cylinder and* $,\vec{n}$ *is a unit vector on the sphere* $S_{d-1}$*. Interpret the coefficients* $c_n$ *and the exponent* $\Delta + n$*.*

**Exercise 3.2.** *Consider the function* $C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}(x_{12},\partial_2)$ *describing the OPE of two scalars and a third scalar*

$$\mathcal{O}_1(x_1)\times\mathcal{O}_2(x_2) \sim C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}(x_{12},\partial_2)\mathcal{O}_3(x_2) + \dots \tag{3.4}$$

*Find the form of* $C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}$ *(stop at oder* $O(x_{12}^2)$ *) by comparing with the three point function of scalars* $\langle\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\rangle$

**Exercise 3.3.** *Compute the unitary bound for scalars* $\Delta \geq (d-2)/2$ *starting from the positivity of the second descendant of a scalar* $O_\Delta$ *with dimension* $\Delta$*:* $||P^2|O_\Delta\rangle||^2 = 0$*.*