

Conformal Field Theories

Tutorial 4

Higgs School 2022

Exercise 4.1. *A Generalized Free Theory (GFT) is the simplest example of solvable CFT that is not a free theory. In AdS/CFT framework it corresponds to a free field propagating in the bulk of Anti deSitter in $d + 1$ dimensions.*

As a CFT, it is defined by a single fundamental field (scalar in this case) $\mathcal{O}(x)$ with scaling dimension $\Delta \geq (d - 2)/2$. All correlation functions of the theory are defined using the Wick theorem through the simple contraction

$$\underbrace{\mathcal{O}(x_i)\mathcal{O}(x_j)} = \frac{1}{x_{ij}^{2\Delta}} \quad (4.1)$$

Compute the four point function $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4) \rangle$ and show that it satisfies the crossing equation, i.e. it is invariant under exchange $1 \leftrightarrow 3$.

Consider the OPE

$$\mathcal{O}(0) \times \mathcal{O}(x) = \underbrace{\mathcal{O}(0)\mathcal{O}(x)} + : \mathcal{O}(0)\mathcal{O}(x) : = \mathbb{1} \frac{1}{x_{ij}^{2\Delta}} + : \mathcal{O}(0)\mathcal{O}(0) : + \dots \quad (4.2)$$

where the symbol $::$ indicates the normal ordering. By Taylor expanding in x identify the operators appearing in the rhs of the above OPE.