Status of BMWc's work on hadronic vacuum polarization

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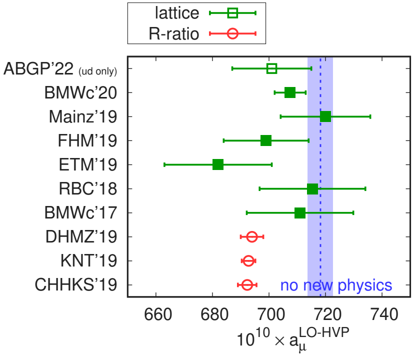
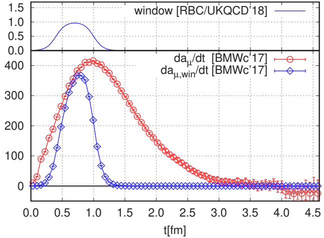
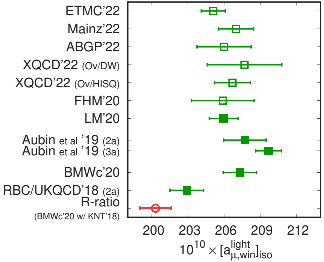
Current situation



- 4.2σ: WP'20 vs FNAL'21 & BNL'04
- 1.5σ: BMWc'20 vs FNAL'21 & BNL'04
- 2.1σ: BMWc'20 vs WP'20





- 3.7σ: BMWc'20 vs data-driven
- Recent lattice results confirm our high value: RBC/UKQCD? FNAL/HPQCD/MILC?

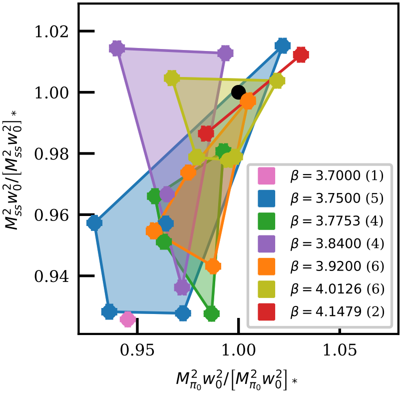
Reduce dominant, continuum-extrapolation source of error:
New finer lattice w/ $a = 0.048\,\mathrm{fm}$

Simulations

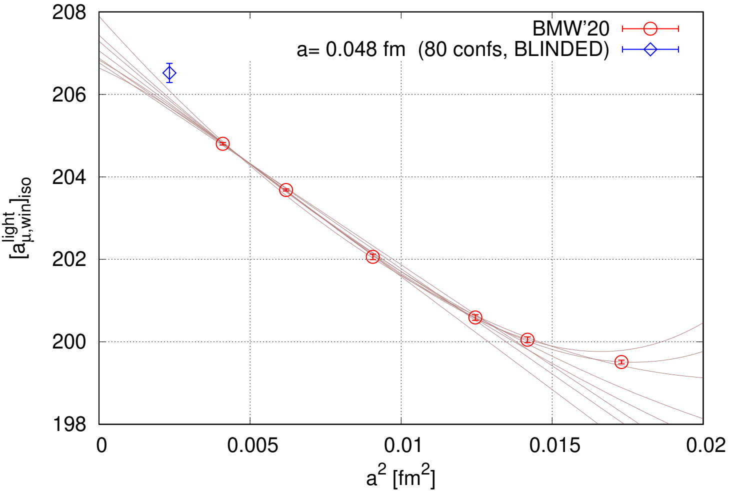
- Tree-level Symanzik-improved gauge action
- $N_f = 2+1+1$ staggered fermions
- Stout smearing
  4 steps, $\varrho = 0.125$

β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980
4.1479	0.0483	128×192	4439

- $L \sim 6$ fm, $T \sim 9$ fm
- m_{ud}, m_s and m_c around physical point



Intermediate window (blinded)



- If $a = 0.048\,\mathrm{fm}$ compatible with current $a \to 0$ extrapolation
  $\Rightarrow$ significantly increased tension w/ R-ratio
- If not, . . .

Reduce dominant, continuum-extrapolation source of error:
Vary discretization of quark EM current operator

Tastes of staggered EM current operator

Vector meson operators [Golterman '85]

OP	spin $\otimes$ taste	mult.	# hops	# paths	id*
1-timeslice operators					
Li00	$\gamma_i \otimes \xi_i$ local	3	0	1×1	4
L100	$\gamma_i \otimes 1$ conserved	3	1	2×1	5
Lij0	$\gamma_i \otimes \xi_i\xi_j$	6	1	2×1	10
Lijk	$\gamma_i \otimes \xi_i\xi_j\xi_k$	3	2	4×2	11
Lj00	$\gamma_i \otimes \xi_j$	6	2	4×2	15
Ljk0	$\gamma_i \otimes \xi_j\xi_k$	3	3	8×6	20
2-timeslice operators					
Ni00	$\gamma_i \otimes \xi_i\xi_4$	3	1	1×1	24
N100	$\gamma_i \otimes \xi_4$	3	2	2×2	25
Nij0	$\gamma_i \otimes \xi_i\xi_j\xi_4$	6	2	2×2	30
Nijk	$\gamma_i \otimes \xi_i\xi_j\xi_k\xi_4$	3	3	4×6	31
Nj00	$\gamma_i \otimes \xi_j\xi_4$	6	3	4×6	35
Njk0	$\gamma_i \otimes \xi_j\xi_k\xi_4$	3	4	8×24	40
Total	12	48			

* [Altmeyer et al '93], [Ishizuka et al '94]

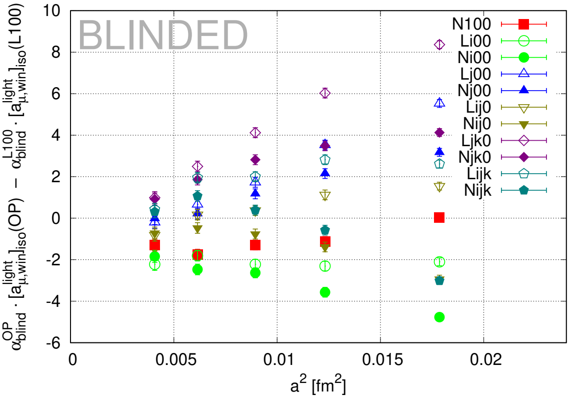
Must all give same continuum limit, but approach it differently

$\Rightarrow$ can use to constrain continuum limit

Continuum extrapolation of 12 taste current correlators

- LMA for all 12 operators
- 5 lattice spacings
- 1 ensemble at each β
- 48 configs on each ensemble

- Do they extrapolate to the same point?
- How best to use the information to constrain continuum limit?



Check universality of continuum limit: Intermediate window with overlap on staggered

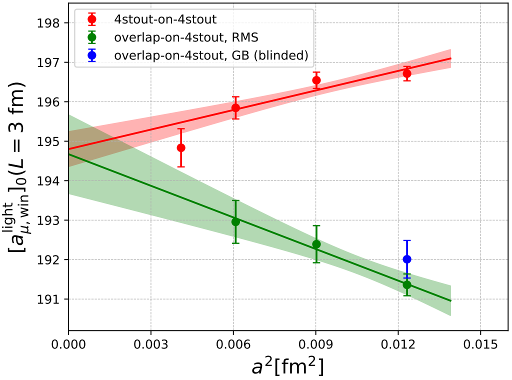
Intermediate window: overlap on staggered

Universality: all valid discretizations of QCD should give the same results in the continuum

Check staggered continuum limit for $a_{\mu,\text{win}}^{\text{light}}$ w/ overlap on staggered in $L = 3\,\text{fm}$ boxes [BMWc'20]:

- Increasing statistics of finest staggered point (not yet analyzed)
- In BMWc'20, overlap with RMS pion-mass matching to make computation possible
- New: implemented LMA for overlap to allow for Goldstone pion-mass matching (blinded and in progress)

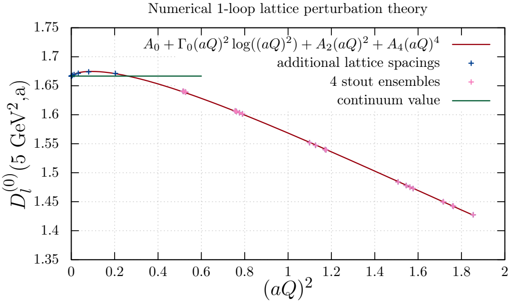
  $\Rightarrow$ expect smaller discretization errors



Understanding short-distance discretization effects: Adler function and logarithmically-enhanced discretization errors

Logarithmically-enhanced discretization errors

- For on-shell quantities, discretization error $\sim a^m \alpha_s(1/a)^n \sim a^m/\ln(1/a)^n$
- $a_\mu^{\text{LO-HVP}}$, $\hat{\Pi}(Q^2)$, $D(Q^2)$, ... are not on-shell QCD quantities
  $\rightarrow$ more sensitive to short distances
  $\rightarrow$ suffer from log-enhanced discretization error $\sim a^2 \ln a$ [Cè et al '21]
- Study effect on $D(Q^2)$ which depends on single scale Q^2 (for $\pi/a \gg Q \gg \Lambda_{\text{QCD}},\ m_q$)
  $\rightarrow$ by tuning Q^2 can determine at what scales these discretization errors modify continuum limit significantly

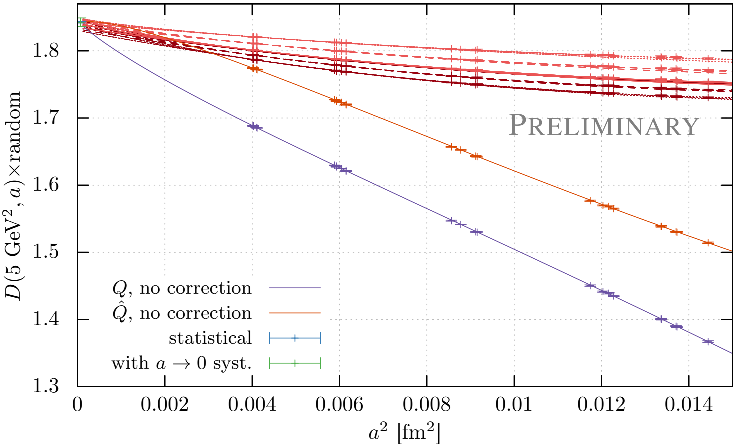


- Here: 1-loop lattice pQCD
- Naive polynomial extrapolation from simulation lattice spacings overshoots known continuum limit
- Log starts taking over for $a \lesssim 0.04\,\text{fm}$ in lattice pQCD for $Q^2 = 5\,\text{GeV}^2$
- Coefficient of log term (Γ_0) computed analytically
- Use 1-loop lattice pQCD to correct simulation results (see also ETMc '22)

$D_l(5\,\mathrm{GeV}^2)$: preliminary fits w/ $a \to 0$ systematic

Strong IB and QED corrections included to $O(\delta m, e^2)$ as in BMWc '20

$$D(Q^2,a) = D(Q^2,0) + \underbrace{A_2[a\alpha_s^n(1/a)]^2 + A_{2l}a^2\log(a^2)(+A_4[a^2\alpha_s^n(1/a)]^2)}_{\text{continuum extrapolation}} + \cdots$$



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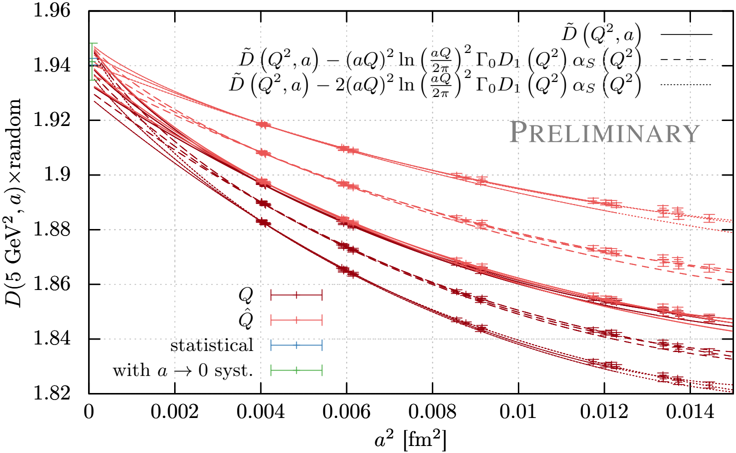


QED and SIB corrections w/out chiral extrapolation

QED valence contribution

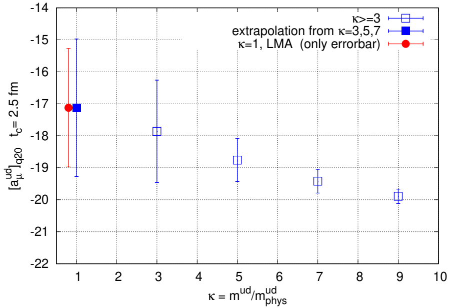
- Valence sector of quark-connected HVP

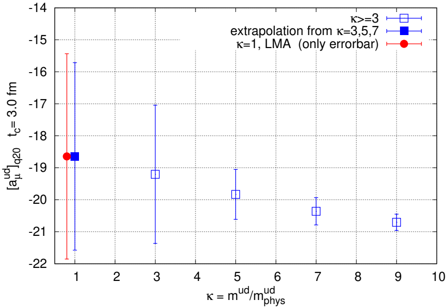
$$\left\langle \langle \mathcal{C}_2''(U,A)\rangle_{A,\mathrm{q.}} \right\rangle_U \quad = $$  $+$ 

- Compute as finite difference: measure $\mathcal{C}(0)$, $\mathcal{C}(+\frac{1}{3}e_*)$, $\mathcal{C}(-\frac{1}{3}e_*)$

- Calculated at $\kappa = \frac{m^{ud}_{\mathrm{val}}}{m^{ud}_{\mathrm{phys}}} = 3, 5, 7 \quad \longrightarrow \quad \kappa = 1$

- New strategy: implement LMA to calculate directly at $\kappa = 1$ (blinded)





($\beta = 3.7000$, 300 configs)

Benchmarking of our very precise fixing of the QCD scale and quark masses and general analysis procedure w/ f_π and f_K

BMWc's physical points

In full $N_f = 4 \times 1$ QCD+QED, fix scale and quark masses w/ M_{Ω^-}, M_{π^0}, $M_{K_\chi} = \sqrt{[M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}]/2}$, $\Delta M^2_K = M^2_{K^0} - M^2_{K^+}, \ldots$

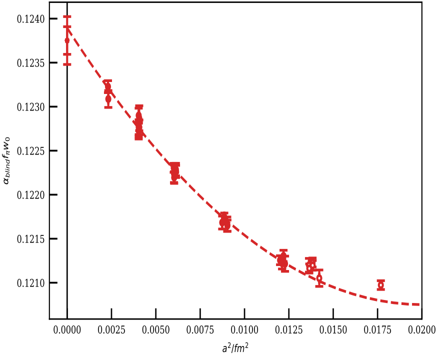
In isospin symmetric QCD:

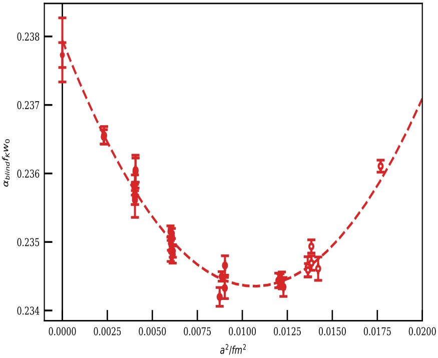
- Want quantities that do not depend strongly on $O(\alpha, m_d - m_u)$ corrections
- Fix scale w/ w_0 (purely gluonic observable)
- Mass m_q of q fixed via mass of corresponding quark-connected $\bar{q}q$, PS meson, M_{qq} for $q = u, d, s$ (neutral and no magnetic moment)
- Fix $m_{ud} = (m_u + m_d)/2$ w/ $M^2_{uu} + M^2_{qq} = M_{\pi^0}|_{\text{PDG}}$ (good up to NLO IB corrections [Bijnens et al '07])
- Compute $w_0|_{\text{ph}}$ and $M_{ss}|_{\text{ph}}$ using full, $N_f = 4 \times 1$ QCD+QED calculations [BMWc '20]:

$$w_0|_{\text{ph}} = 0.17236(70)\,\text{fm} \qquad M_{ss}|_{\text{ph}} = 689.89(49)\,\text{MeV}$$

Use to compute $w_0 f_\pi$ & $w_0 f_K$ in isospin limit, including new $a = 0.048\,\text{fm}$ lattices

Continuum extrapolation of $w_0 f_\pi$ and $w_0 f_K$: blinded





- Gives $\sigma^{\rm rel}_{w_0 f_\pi} = 0.22\%$
- Compare to $\sigma^{\rm rel}_{w_0 M_{\Omega^-}} = 0.40\%$ ...
- ... and $\sigma^{\rm rel}_{f_\pi}|_{\rm FLAG} = 0.6\%$ ...
- ... and $\sigma^{\rm rel}_{|V_{ud}|f_\pi}|_{\rm PDG} = 0.10\%$

- Gives $\sigma^{\rm rel}_{w_0 f_K} = 0.20\%$
- Compare to $\sigma^{\rm rel}_{w_0 M_{\Omega^-}} = 0.40\%$ ...
- ... and $\sigma^{\rm rel}_{f_K}|_{\rm FLAG} = 0.2\%$ (dependent on older PDG f_π) ...
- ... and $\sigma^{\rm rel}_{|V_{us}|f_K}|_{\rm PDG} = 0.16\%$

Conclusion

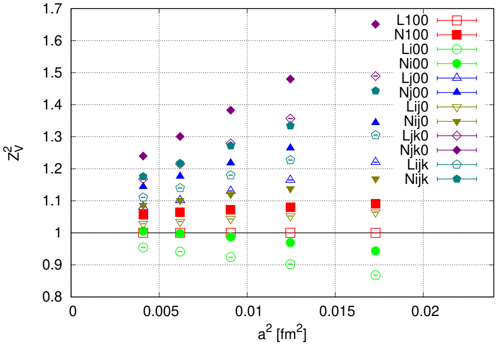
- New simulations on finer $a = 0.048\,\mathrm{fm}$ lattices
- Check of $a \to 0$ w/ 12 taste variations of quark, EM current
- Improve check of universality w/ overlap fermions
- Adler fn and investigations of impact of logarithmically-enhanced discretization errors on $a \to 0$ extrapolations for observables sensitive to different physical scales
- QED and SIB corrections w/out chiral extrapolations
- f_π and f_K as benchmarks of our physical-point-fixing and analysis procedures
- Other windows, ...

BACKUP

Renormalization

- $L100$ $(\gamma_i \otimes 1)$ is conserved $\longrightarrow$ no renormalization ($Z_V(\text{L100}) = 1$)
- Other tastes: $J^\mu(\text{L100}) = Z_V(\text{OP}) \cdot J^\mu(\text{OP}) + \mathcal{O}(a^2)$
- Fix Z_V at each a, via:

$$Z_V^2(\text{OP}) \stackrel{\text{def}}{=} \frac{a_{\mu,\text{win}}^{\text{strange}}(\text{L100})}{a_{\mu,\text{win}}^{\text{strange}}(\text{OP})}$$



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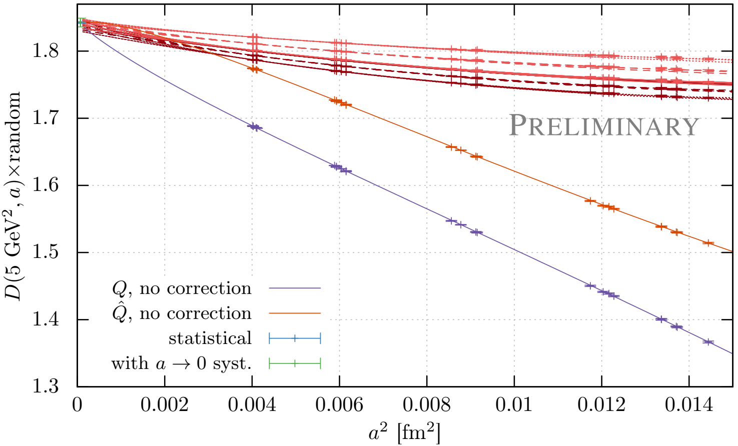

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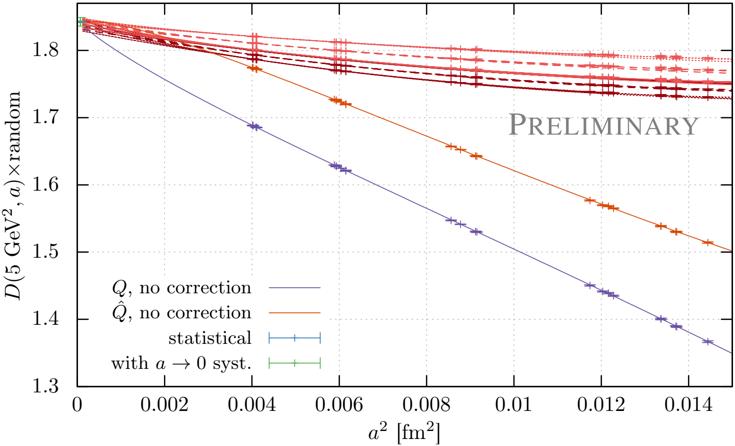

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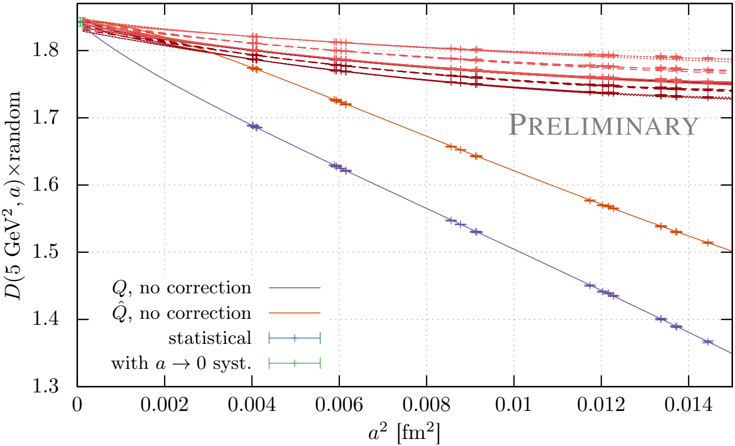

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