HVP in the intermediate window(s)

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for the FNAL/HPQCD/MILC Collaborations
09/05/22

*connected HVP analysis subgroup

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(CU Boulder)

Outline

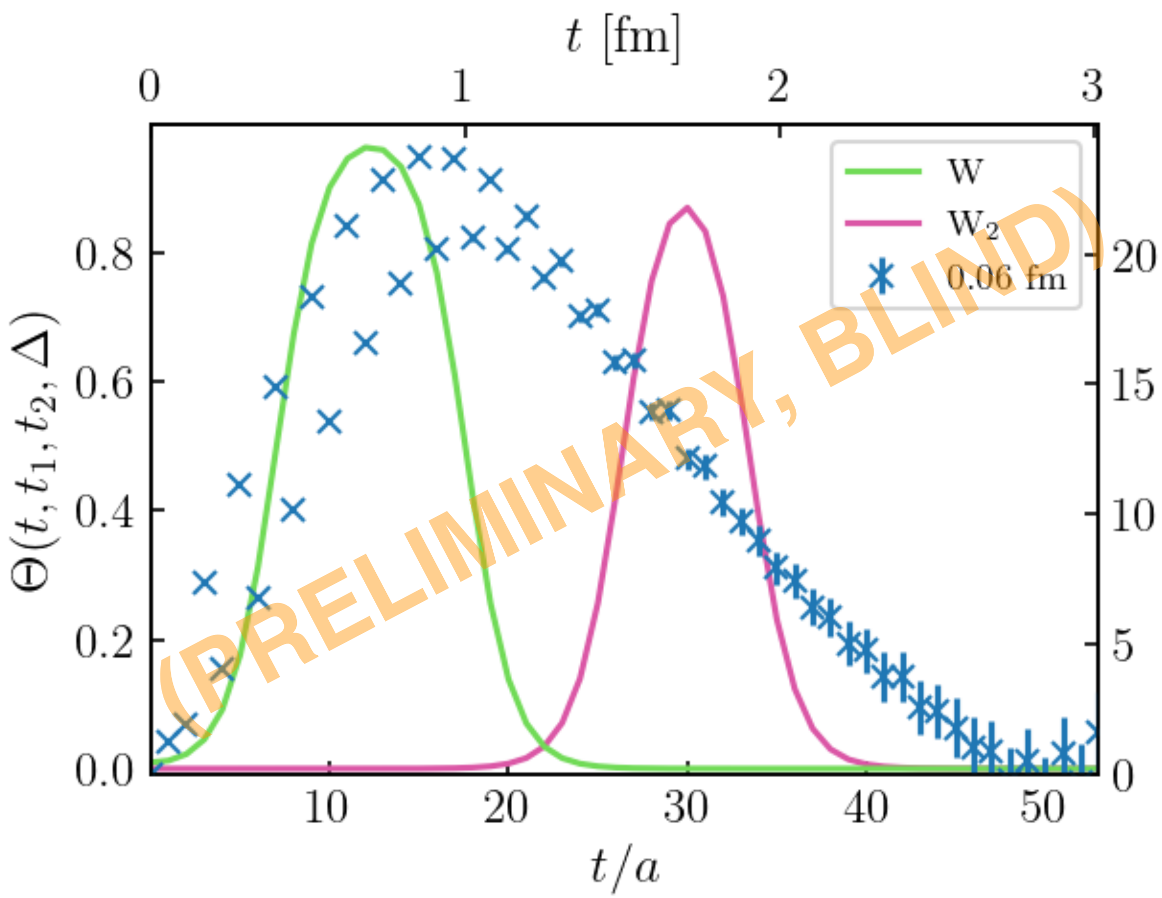
- Intermediate time windows: W and W2
- Results **(still blinded)**
- Ensembles used
- Corrections and models
- Continuum extrapolation and model averaging

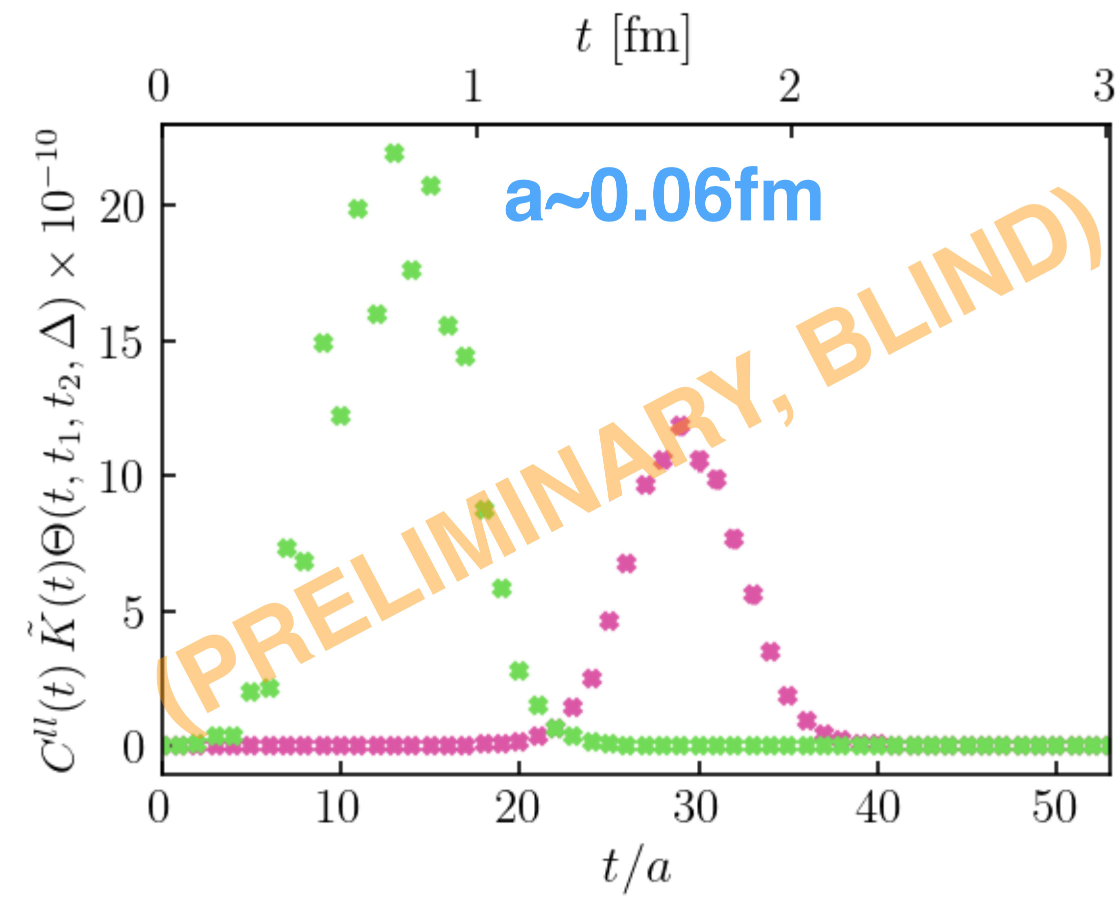
*Bernecker and Meyer, Eur.Phys.J. A47 (2011) 148

Setting the stage

- This is a Theory Initiative workshop, so I don't need to introduce or motivate muon (g-2)! But to be explicit:
  - We calculate the connected HVP contribution to $a_{\mu,\ell\ell}$ from degenerate light quarks ($m_\ell=(m_u+m_d)/2$), using the standard time-momentum representation*.
  - Not included: QED, strong isospin breaking, disconnected HVP, strange or charm
  - We focus on two intermediate windows - so not the whole connected HVP.

- a_μ can be split into pieces*, with the middle part (no short-distance or long-distance) relatively clean - high precision, lower systematics.





- Window function:

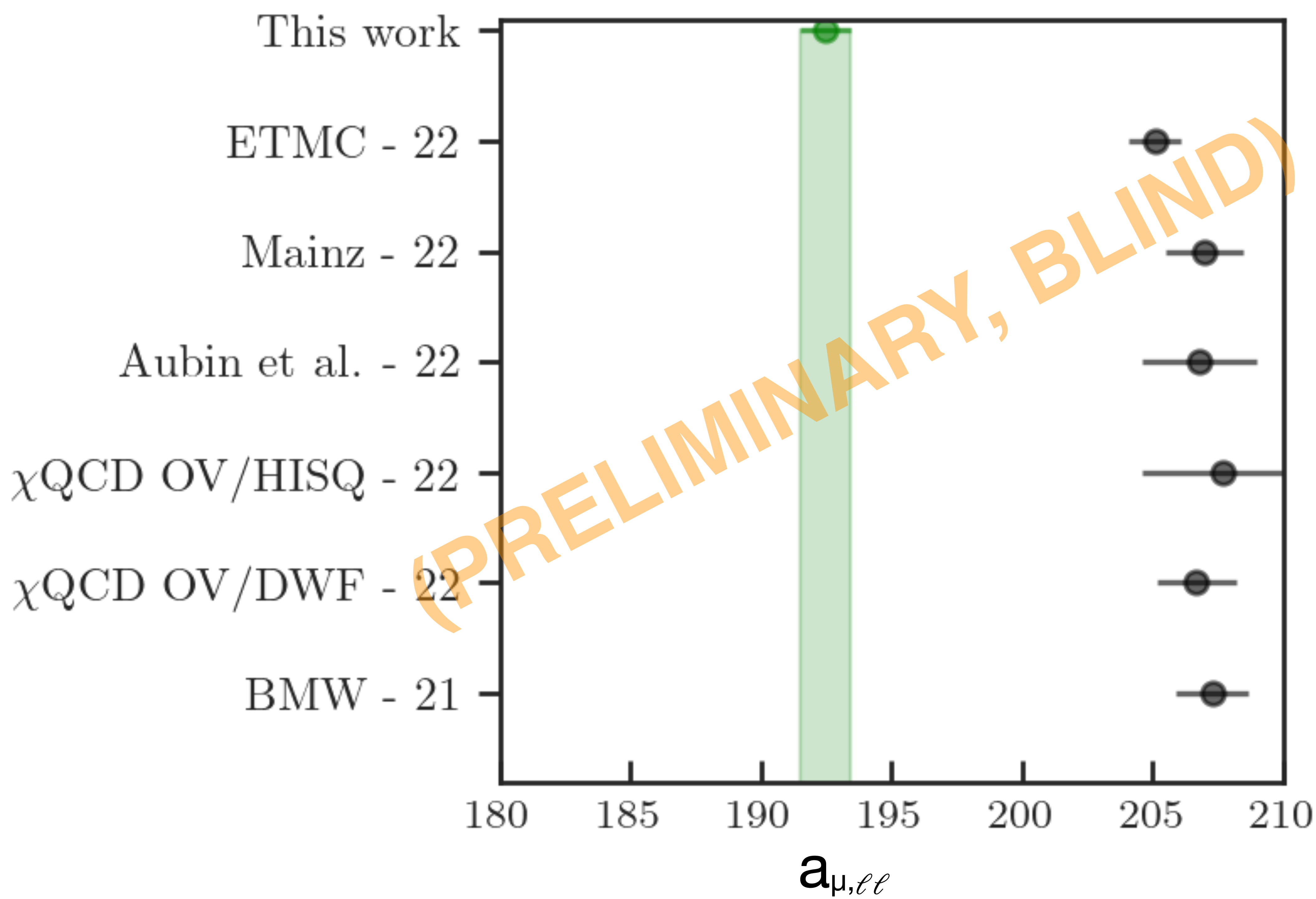
$$\Theta\left(t, t_0, t_1, \Delta\right) = \frac{1}{2}\left[\tanh\left(\frac{t-t_0}{\Delta}\right) - \tanh\left(\frac{t-t_1}{\Delta}\right)\right] + (t \to -t)$$

- W is the standard "intermediate window*" **[0.4, 1] fm**; W_2 is a longer-distance proposal* **[1.5, 1.9] fm** which is intended to be more amenable to EFT study. **Δ=0.15/fm** for both.

*Blum et al. (RBC/UKQCD), Phys. Rev. Lett 121 (2018) 2, 022003

*Aubin, Blum, Golterman, Peris, arXiv:2204.12256

Results


This work
ETMC - 22
Mainz - 22
Aubin et al. - 22
χQCD OV/HISQ - 22
χQCD OV/DWF - 22
BMW - 21
(PRELIMINARY, BLIND)
180
185
190
195
200
205
210
$a_{\mu,\ell\ell}$

*C.T.H. Davies et al (FNAL/HPQCD/MILC), *Phys.Rev.D* 101 (2020) 3, 034512

Lattice Ensembles

$\approx a$[fm]	$N_s^3 \times N_t$	$am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$	w_0/a	M_{π_5}(MeV)	$N_{\rm conf.}$	$N_{\rm wall}$
0.15	$32^3 \times 48$	0.002426 / 0.0673 / 0.8447	1.13215(35)	134.73(71)	9362	48
0.12	$48^3 \times 64$	0.001907 / 0.05252 / 0.6382	1.41110(59)	134.86(71)	9011	64
0.09	$64^3 \times 96$	0.00120 / 0.0363 / 0.432	1.95180(70)	128.34(68)	5384	48
0.06	$96^3 \times 128$	0.0008 / 0.022 / 0.260	3.0170(23)	134.95(72)	2621	24

- **HISQ action**. 2+1+1 flavor. Four lattice spacings for continuum extrapolation. Non-perturbative current renormalization - Z_V from η_s correlation functions. Scale setting with w_0. *Note pion mass mistuning at 0.09fm.*

- Compared to previous publication*, statistics greatly increased at 0.12fm (x9), 0.09fm (x3.5), 0.06fm (x2)

- Data also available at two other valence quark masses at 0.15fm, 0.12fm - estimate SIB (future work) and data-driven pion mass correction.

- Before continuum extrapolation, we include **three types of corrections** to our data:

$$a_\mu(L_\infty, m_{\pi,\mathrm{phys}}) = a_\mu(L_{\mathrm{lat}}, m_{\pi,\mathrm{lat},\xi_1}, \dots, m_{\pi,\mathrm{lat},\xi_{16}}) + \Delta_{\mathrm{FV}} + \Delta_{m_\pi} + \Delta_{TB}$$

- **FV = finite volume.** Calculated in several model variants:

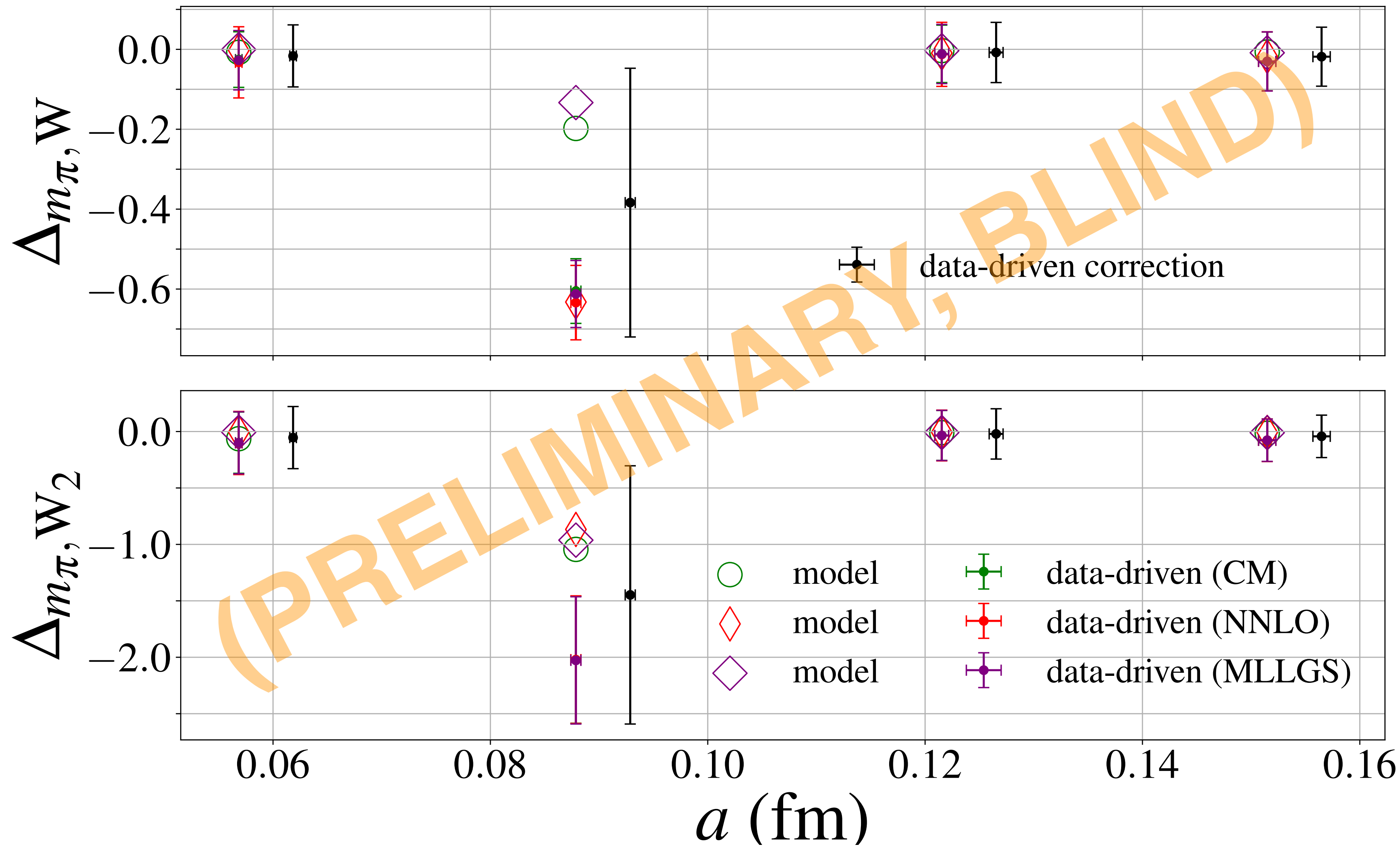
*B. Chakraborty et al (HPQCD), PRD 96 (2017), 034516
*A. Francis, B. Jäger, H. Meyer and H. Wittig, PRD 88 (2013), 054502
*M.T. Hansen and A. Patella, JHEP 10, 029 (2020)

  - ChiPT (NLO and NNLO)
  - Meyer-Lellouch-Lüscher-Gounaris-Sakurai (MLLGS)*
  - Chiral Model (CM)*, now including additional pion-loop taste-breaking effects
  - Hansen-Patella (HP)*

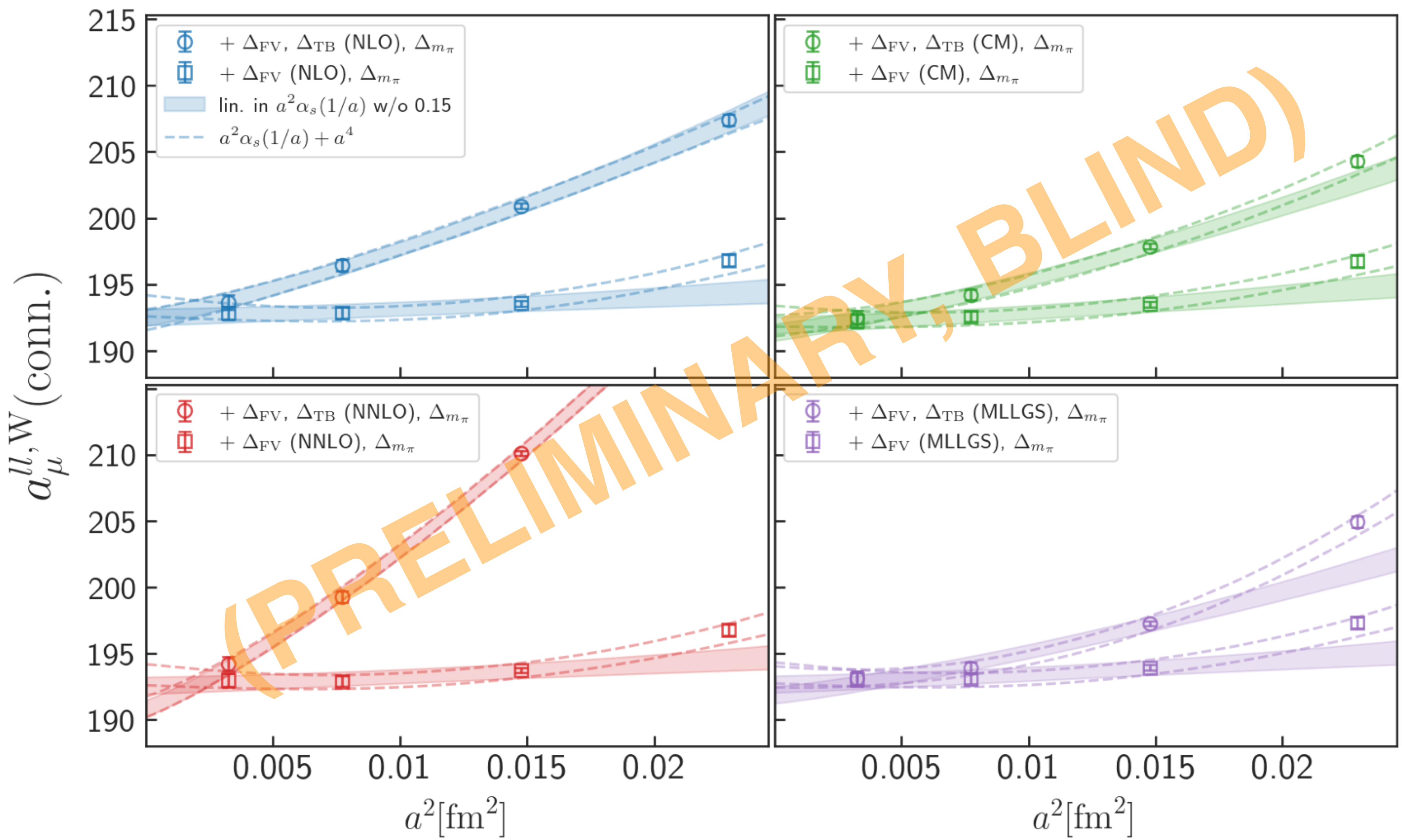
- **mistune = pion mass tuning correction.** Calculated in all models but HP, then compared to data-driven analysis using partially quenched measurements (next slide.)

- **TB = taste breaking.** Calculated in all models but HP. We also include variant fits where we set $\Delta_{TB} = 0$ and let continuum extrapolation deal with it.

- We allow *mixing models* (i.e. one model for FV, another for TB.)



- Data-driven pion mass correction: fit to a_μ data at different valence masses at a~0.12, 0.15fm (models used for FV.)
- Indicates potential for model underestimation of correction, especially in W2!
- We adopt a $\Delta_{\mathrm{mistune}}$ equal to the average maximum correction between models and data-driven, with error equal to |max_model - max_DD| / 2 + σ_{DD}.



- Continuum extrapolations with and without Δ_{TB} included generally consistent with each other

- Account for possible systematic error by including both variations in Bayesian model average

Continuum extrapolation

- Continuum extrapolations, most general functional form:

$$a_\mu^{\ell\ell}(a) = a_\mu^{\ell\ell}\left(1 + C_{a^2}^n\left[(a\Lambda)^2\alpha_s(1/a)^n\right] + C_{a^4}(a\Lambda)^4 + C_{a^6}(a\Lambda)^6 + C_s \sum_{f=\ell,\ell,s} \delta m_f/\Lambda\right)$$

- $a^2\alpha_s$ is the leading expected behavior for HISQ, so $O(a^2)$ is not included - but higher-order a^4, a^6 can be tree-level. We take Λ=500 MeV.
- We consider variant fits where C_s=0, with no C_{a6} ("quadratic", and with neither C_{a6} nor C_{a4} ("linear").
- We also consider variations where we drop the coarsest ensemble (a~0.15fm) from our analysis.

Other details/variations

- At finite a, oscillations occur in the integrand for a_μ, due to unphysical “parity partner” contributions. We fit to remove the oscillating terms and reconstruct.

- We also test not removing the oscillations at all - effects vanish in continuum limit.

- We consider a variation where the corrections in W are only computed over the longer-time half of the window [0.7, 1] fm.

*W.I. Jay and EN, Phys.Rev.D 103 (2021), 114502 *EN and J. Sitison, arXiv:2208.14983

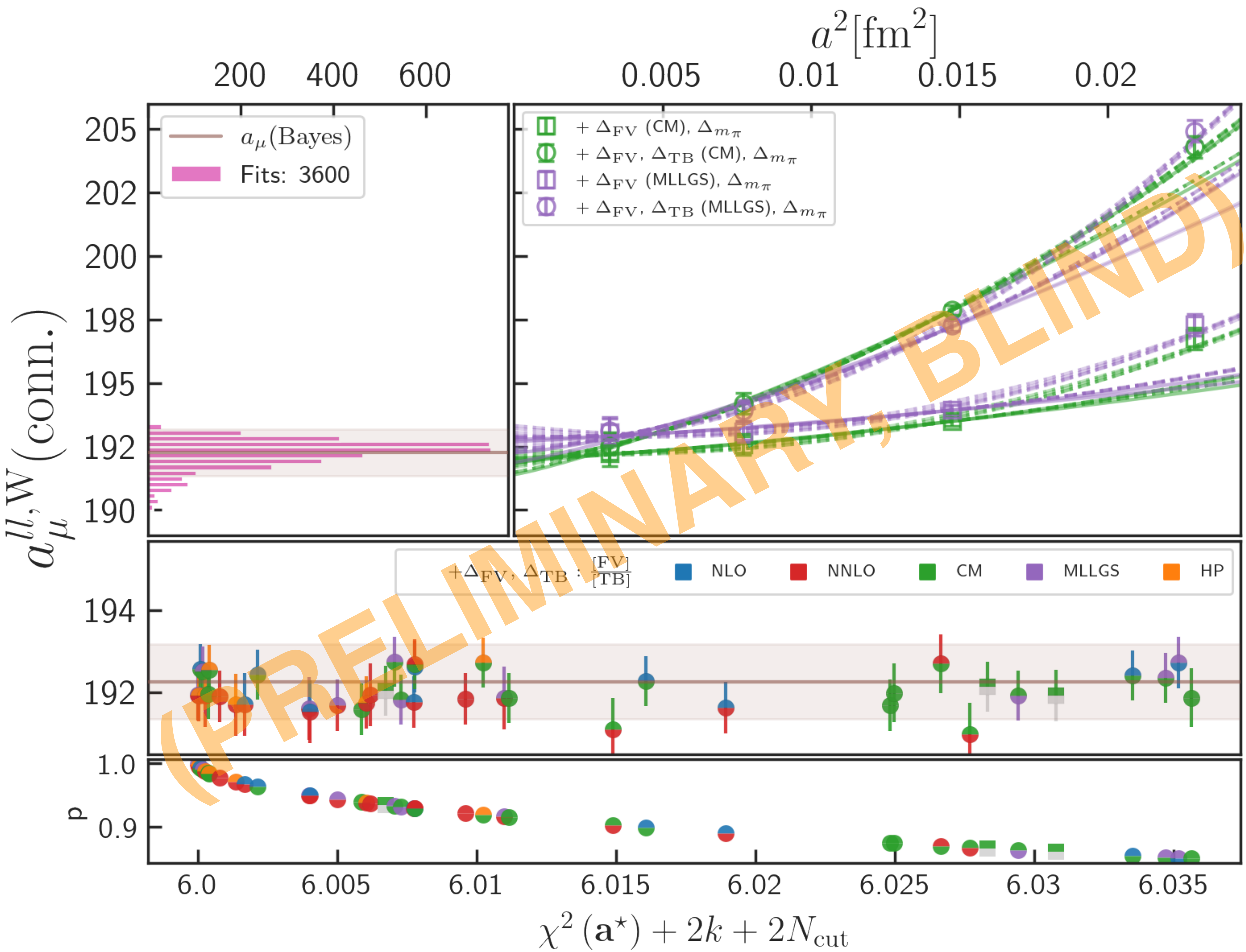
Bayesian model averaging

- To account for all of the variations in our analysis discussed above, we use Bayesian model averaging; fits to all variations are carried out, and then a weighted average is taken.

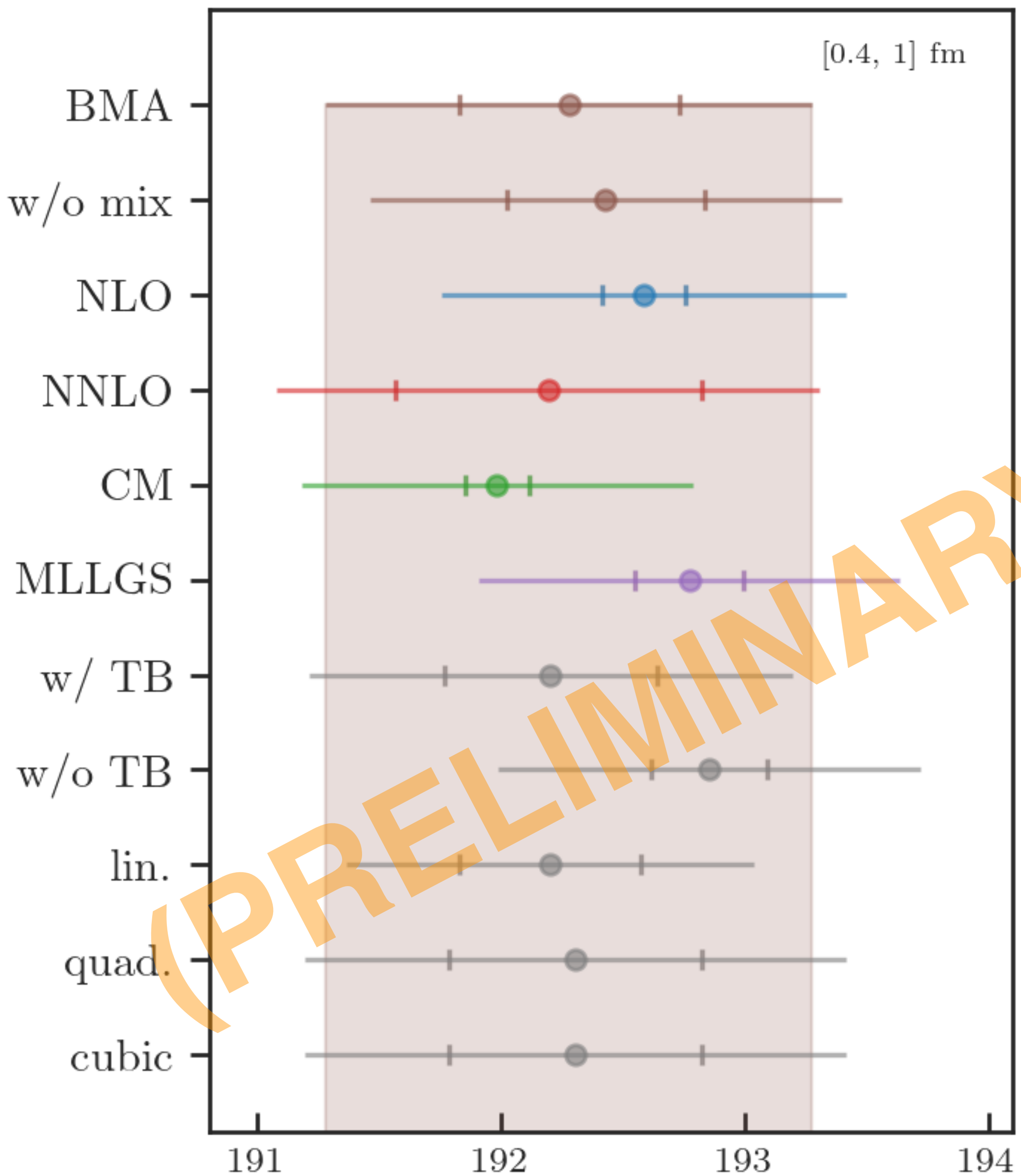
- Weights are computed using "Bayesian AIC":

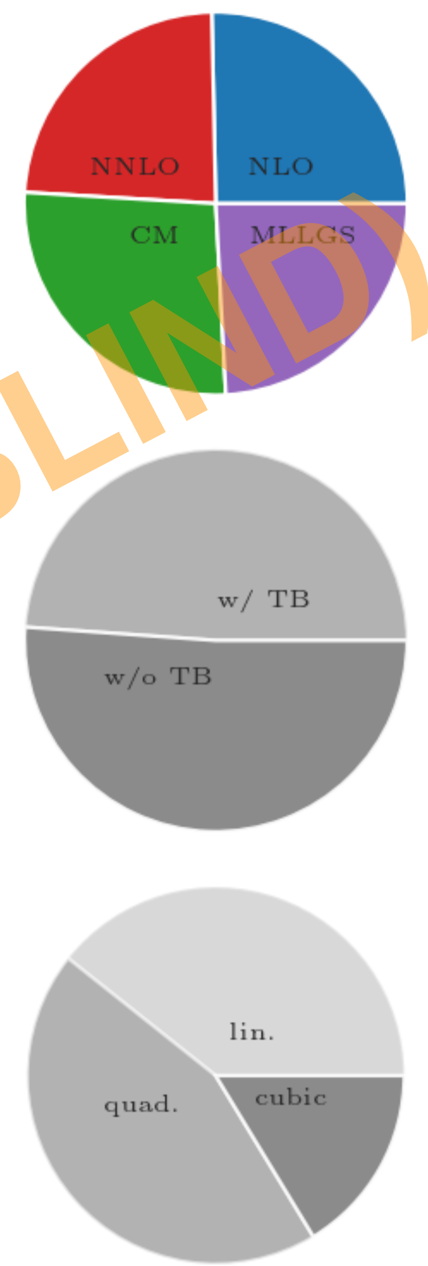
$$\mathrm{pr}(M \mid D) \equiv \exp\left[-\frac{1}{2}\left(\hat{\chi}^2\left(\mathbf{a}^{\star}\right)+2k+2N_{\mathrm{cut}}\right)\right]$$

- k = # of fit parameters. N_{cut} penalizes removal of data points - applicable in variants where ensembles are dropped.



$a^2[\mathrm{fm}^2]$
200
400
600
0.005
0.01
0.015
0.02
205
202
200
198
195
192
190
$a_\mu(\mathrm{Bayes})$
Fits: 3600
$+\Delta_{\mathrm{FV}}$ (CM), Δ_{m_π}
$+\Delta_{\mathrm{FV}}, \Delta_{\mathrm{TB}}$ (CM), Δ_{m_π}
$+\Delta_{\mathrm{FV}}$ (MLLGS), Δ_{m_π}
$+\Delta_{\mathrm{FV}}, \Delta_{\mathrm{TB}}$ (MLLGS), Δ_{m_π}
$a_\mu^{ll,\mathrm{W}}(\mathrm{conn.})$
$+\Delta_{\mathrm{FV}}, \Delta_{\mathrm{TB}} : \frac{[\mathrm{FV}]}{[\mathrm{TB}]}$
NLO
NNLO
CM
MLLGS
HP
194
192
1.0
0.9
p
6.0
6.005
6.01
6.015
6.02
6.025
6.03
6.035
$\chi^2(\mathbf{a}^\star) + 2k + 2N_{\mathrm{cut}}$
(PRELIMINARY, BLIND)





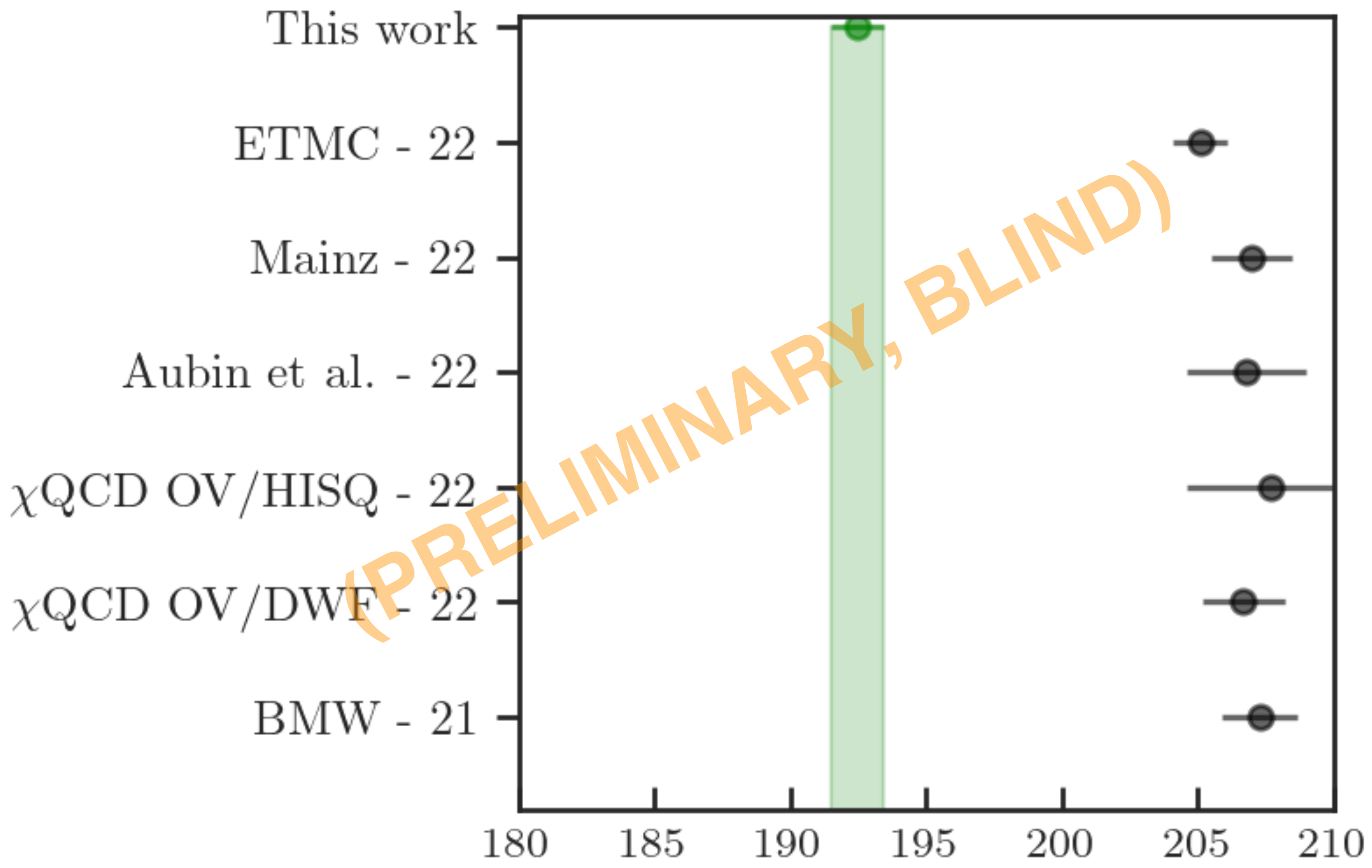
- **<u>Left:</u>** comparison of results with restricted model subsets.
- **<u>Right:</u>** posterior probability (sum of model weights over given subsets.)

Error breakdown

Source	$a_\mu^{ll,\mathrm{W}}$(conn.) %	$a_\mu^{ll,\mathrm{W2}}$(conn.) %
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.29	0.89
Scale setting (w_0 (fm), w_0/a)	0.26	1.37
Current renormalization (Z_V)	0.24	0.21
Monte Carlo statistics	0.16	2.33
Pion-mass adjustment (Δ_{m_π})	0.13	1.04
Finite-volume correction (Δ_{FV})	0.11	0.50
Total	0.52%	3.11%

(PRELIMINARY)

*C. Lehner, arXiv:1710.06874
*C.T.H. Davies et al (FNAL/HPQCD/MILC), arXiv:2207.04765



$a_{\mu,\ell\ell}$

- **Next steps:**
  - Finalize analysis and unblind!
  - One-sided windows and short-distance windows
  - Disconnected, SIB, heavy-flavor contribution analysis all pending
  - More statistics and more masses at 0.09fm to improve mass tuning

Backup

Bayesian model averaging

Model average for single parameter a_0

$$\langle a_0 \rangle = \sum_M \langle a_0 \rangle_M \,\mathrm{pr}(M|D)$$ Central value

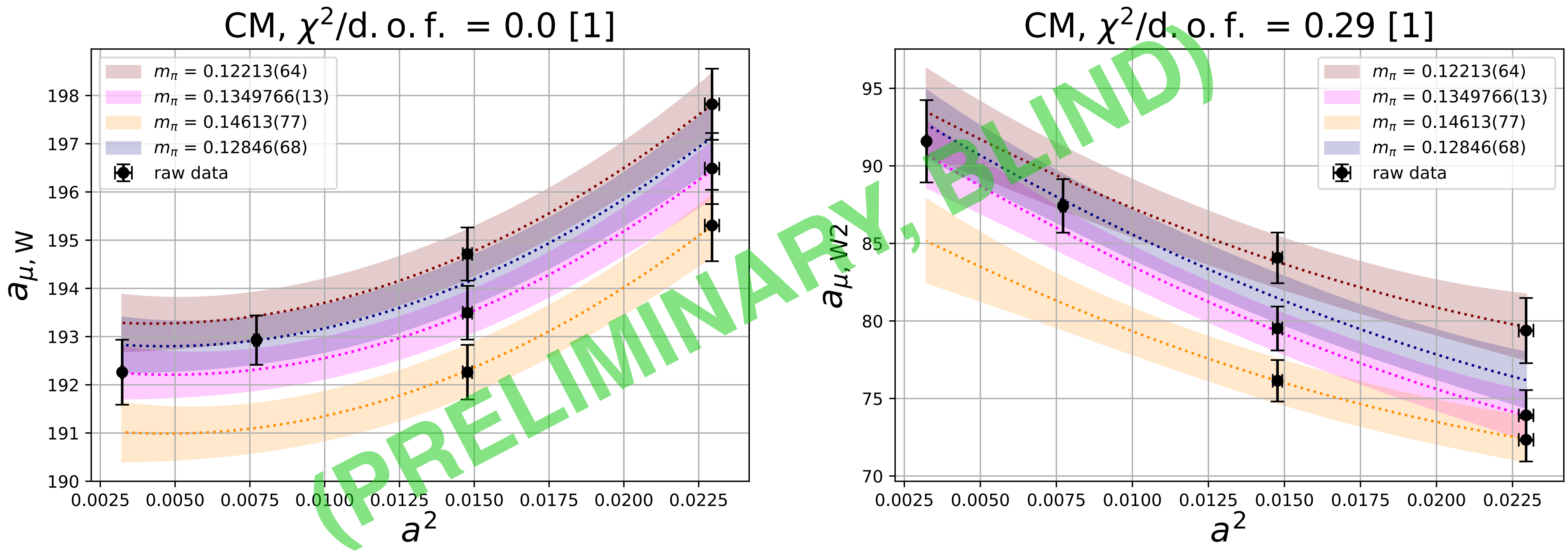
$$\sigma_{a_0}^2 = \langle a_0^2 \rangle - \langle a_0 \rangle^2$$ Combined statistical+systematic error

$$= \sum_{i=1}^N \langle a_0^2 \rangle_i \,\mathrm{pr}(M_i|D) - \left(\sum_{i=1}^N \langle a_0 \rangle_i \,\mathrm{pr}(M_i|D) \right)^2$$

$$= \sum_{i=1}^N \sigma_{a_0,i}^2 \mathrm{pr}(M_i|D) + \sum_{i=1}^N \langle a_0 \rangle_i^2 \,\mathrm{pr}(M_i|D) - \left(\sum_{i=1}^N \langle a_0 \rangle_i \;\mathrm{pr}(M_i|D) \right)^2$$

avg. stat error

model variation systematic error



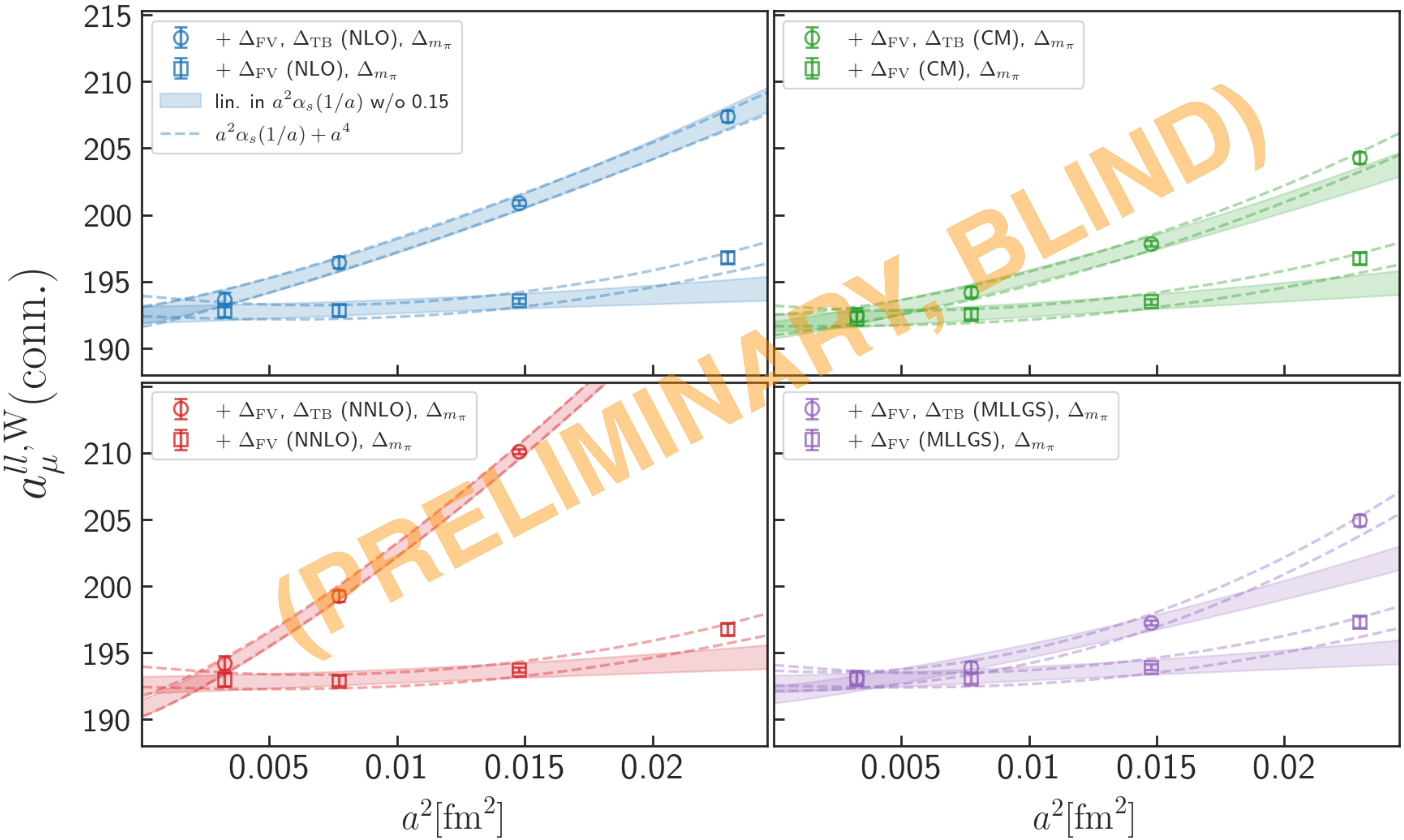
- Interpolating fits for data-driven analysis of mass corrections shown above.
- "CM" = chiral model used for finite-volume corrections before these fits.

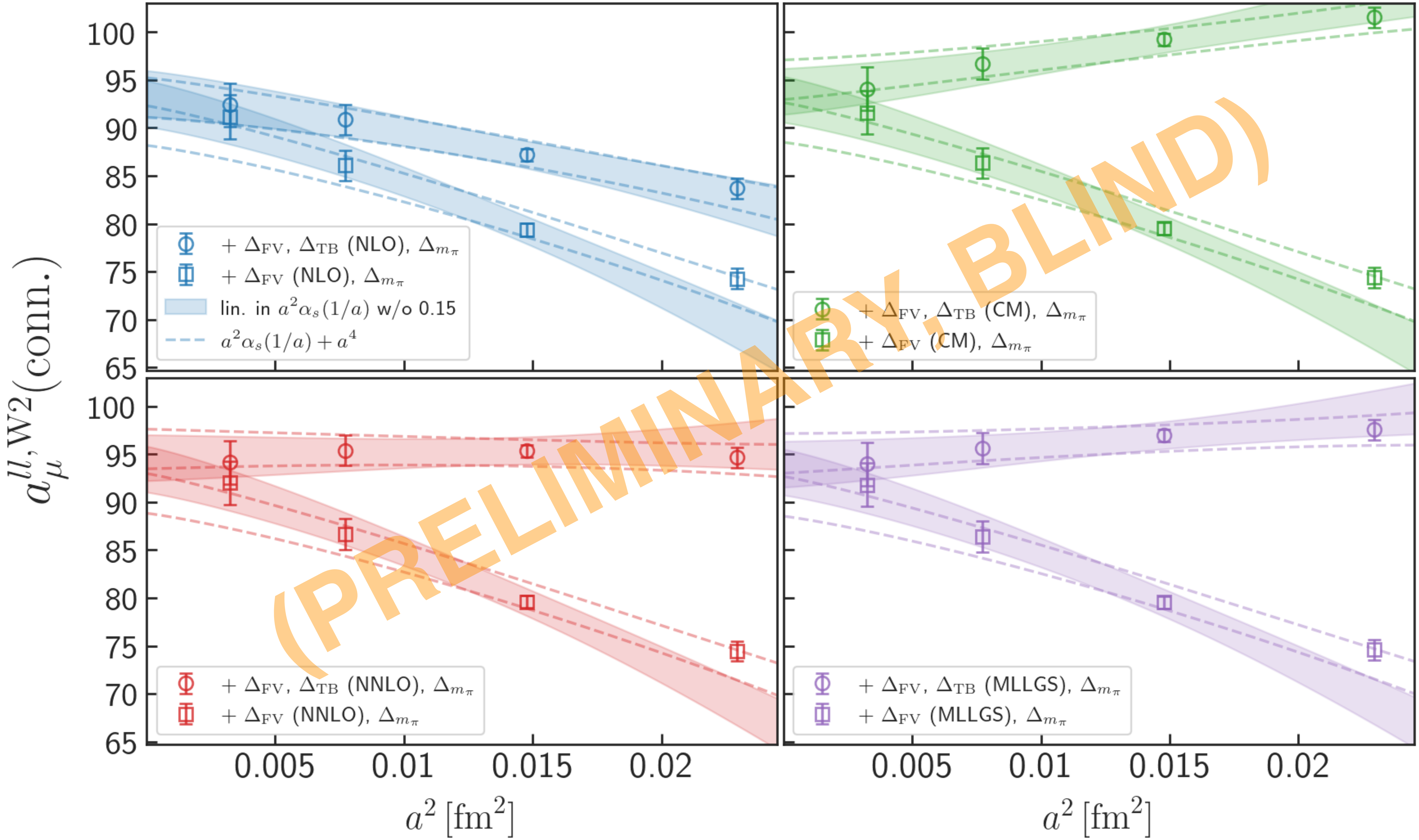
<u>Fit model:</u>

$$a_\mu = c_0(a) + c_1(a)m_\pi^2 + c_{-1}(a)m_\pi^{-2}$$

(all c_i are polynomials in a^2. $\log(m_\pi^2)$ tested instead of m_π^{-2}, results don't change noticeably.)

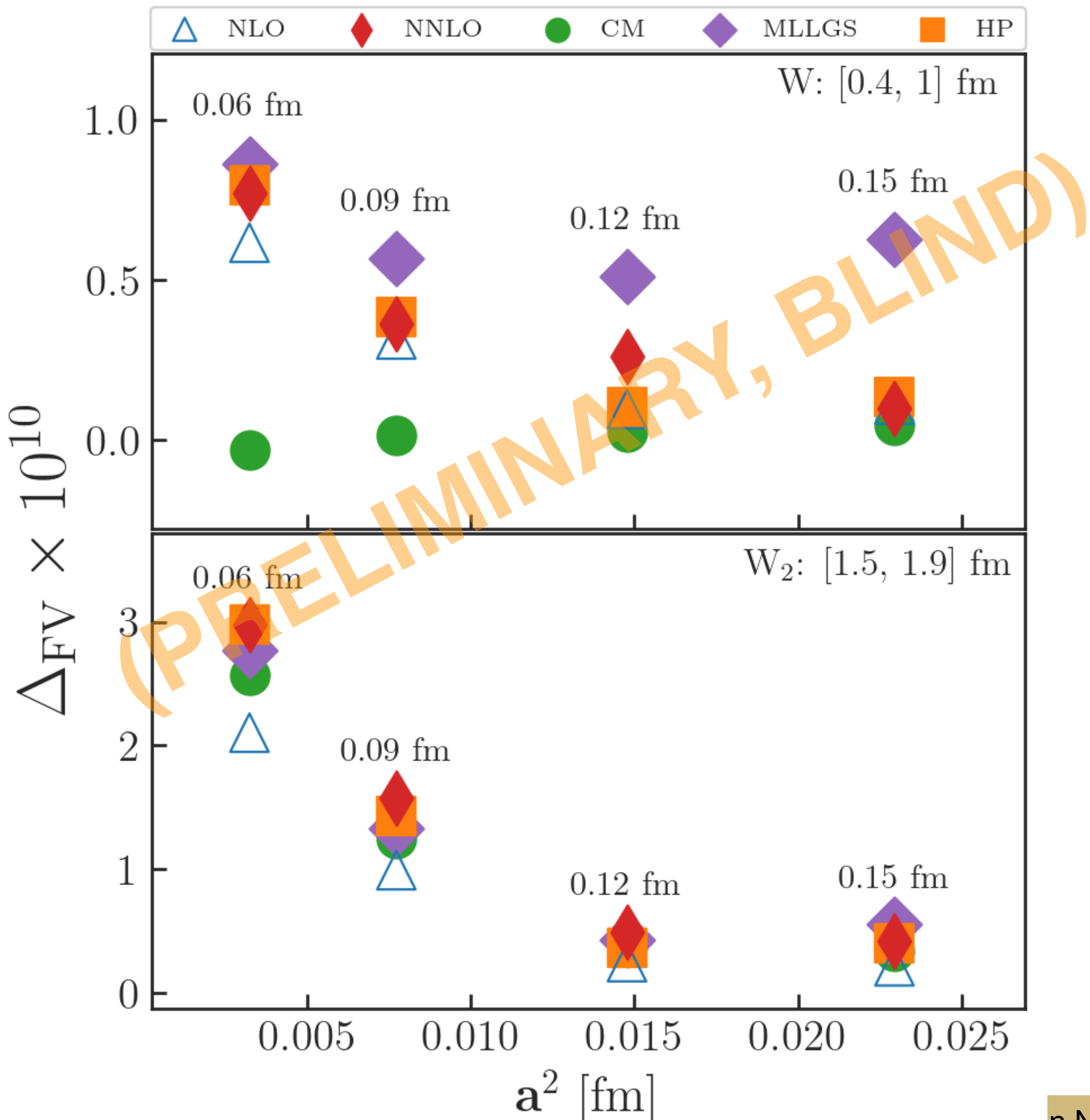
TB correction, or ignore it?





$a_\mu^{ll,\mathrm{W2}}$(conn.)
100
95
90
85
80
75
70
65
$+\Delta_{\mathrm{FV}}, \Delta_{\mathrm{TB}}$ (NLO), Δ_{m_π}
$+\Delta_{\mathrm{FV}}$ (NLO), Δ_{m_π}
lin. in $a^2\alpha_s(1/a)$ w/o 0.15
$a^2\alpha_s(1/a) + a^4$
$+\Delta_{\mathrm{FV}}, \Delta_{\mathrm{TB}}$ (CM), Δ_{m_π}
$+\Delta_{\mathrm{FV}}$ (CM), Δ_{m_π}
$+\Delta_{\mathrm{FV}}, \Delta_{\mathrm{TB}}$ (NNLO), Δ_{m_π}
$+\Delta_{\mathrm{FV}}$ (NNLO), Δ_{m_π}
$+\Delta_{\mathrm{FV}}, \Delta_{\mathrm{TB}}$ (MLLGS), Δ_{m_π}
$+\Delta_{\mathrm{FV}}$ (MLLGS), Δ_{m_π}
0.005
0.01
0.015
0.02
a^2 [fm^2]
(PRELIMINARY, BLIND)

FV corrections:



Corrections - FV, mass, TB in that order:

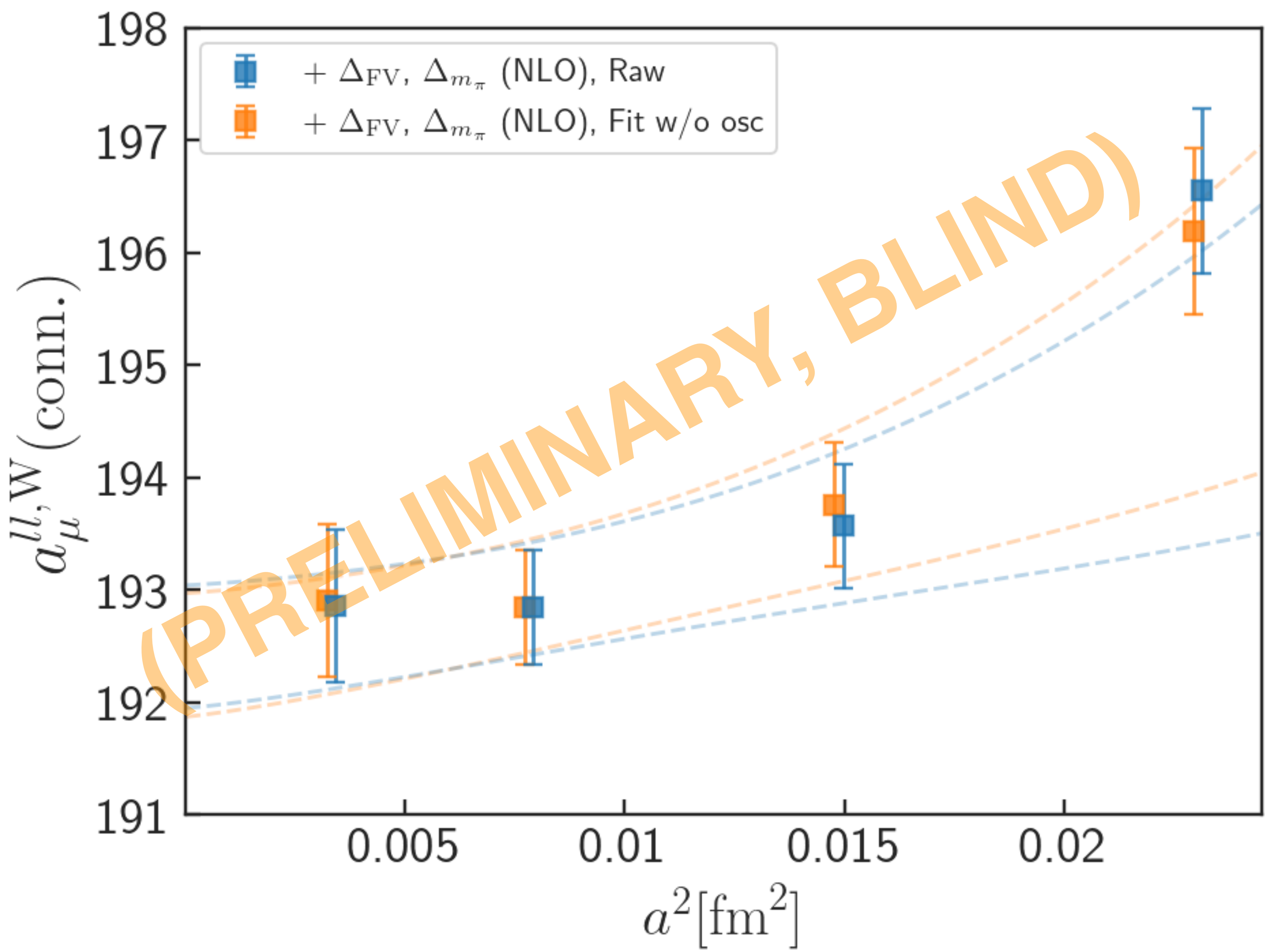
$$a_\mu(L_\infty, m_{\pi_{phys}}) = a_\mu(L_{latt}, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}}) + \Delta_{\mathrm{FV}} + \Delta_{\mathrm{mistune}} + \Delta_{\mathrm{TB}}$$

$$\Delta_{\mathrm{FV}} = a_\mu(L_\infty, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}}) - a_\mu(L_{latt}, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}})$$

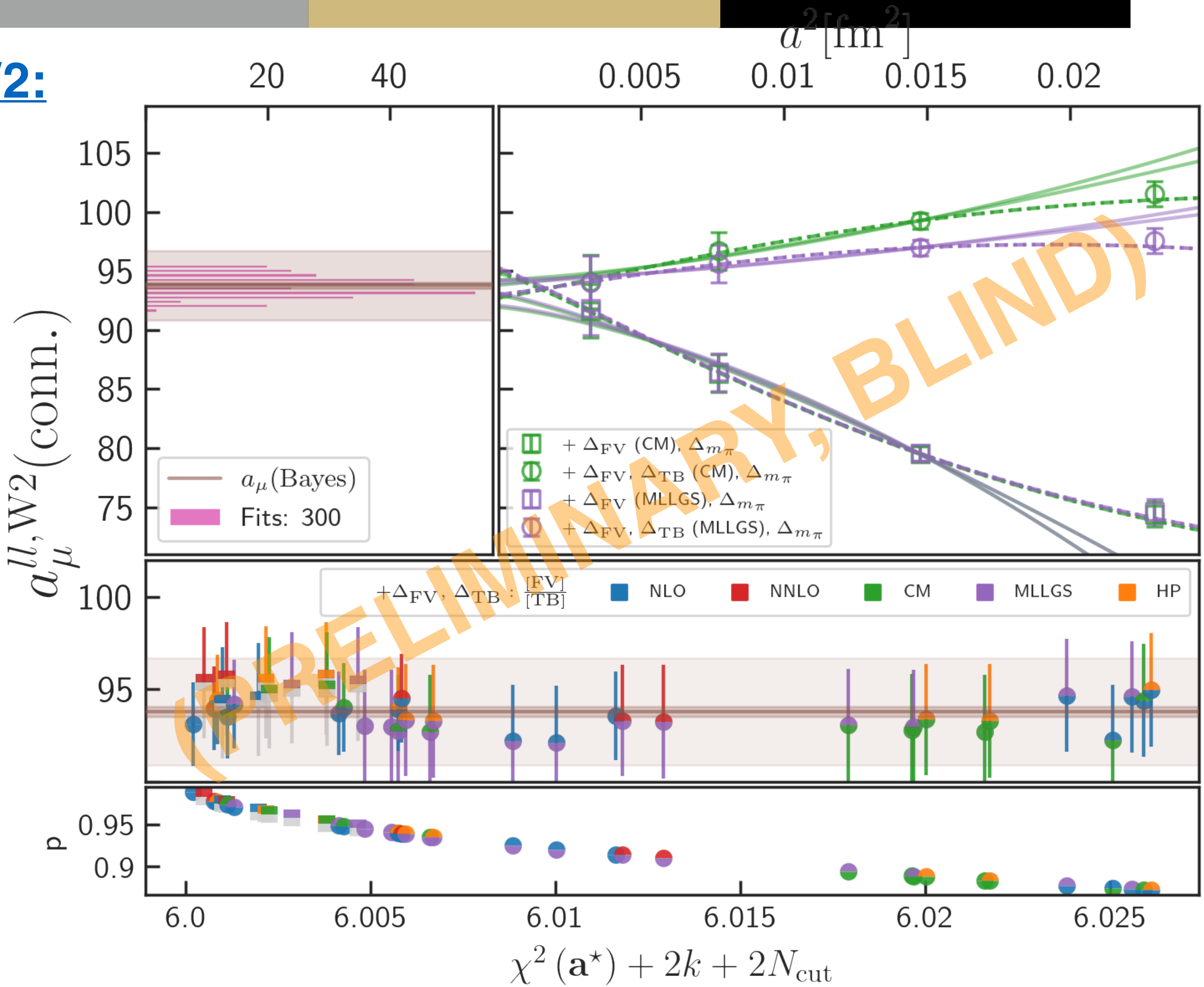
$$\Delta_{\mathrm{mistune}} = a_\mu(L_\infty, m_{\pi_{phys,\xi_1}}, \cdots, m_{\pi_{phys,\xi_{16}}}) - a_\mu(L_\infty, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}})$$

$$\Delta_{\mathrm{TB}} = a_\mu(L_\infty, m_{\pi_{phys}}, \cdots, m_{\pi_{phys}}) - a_\mu(L_\infty, m_{\pi_{phys,\xi_1}}, \cdots, m_{\pi_{phys,\xi_{16}}})$$

Oscillation removal

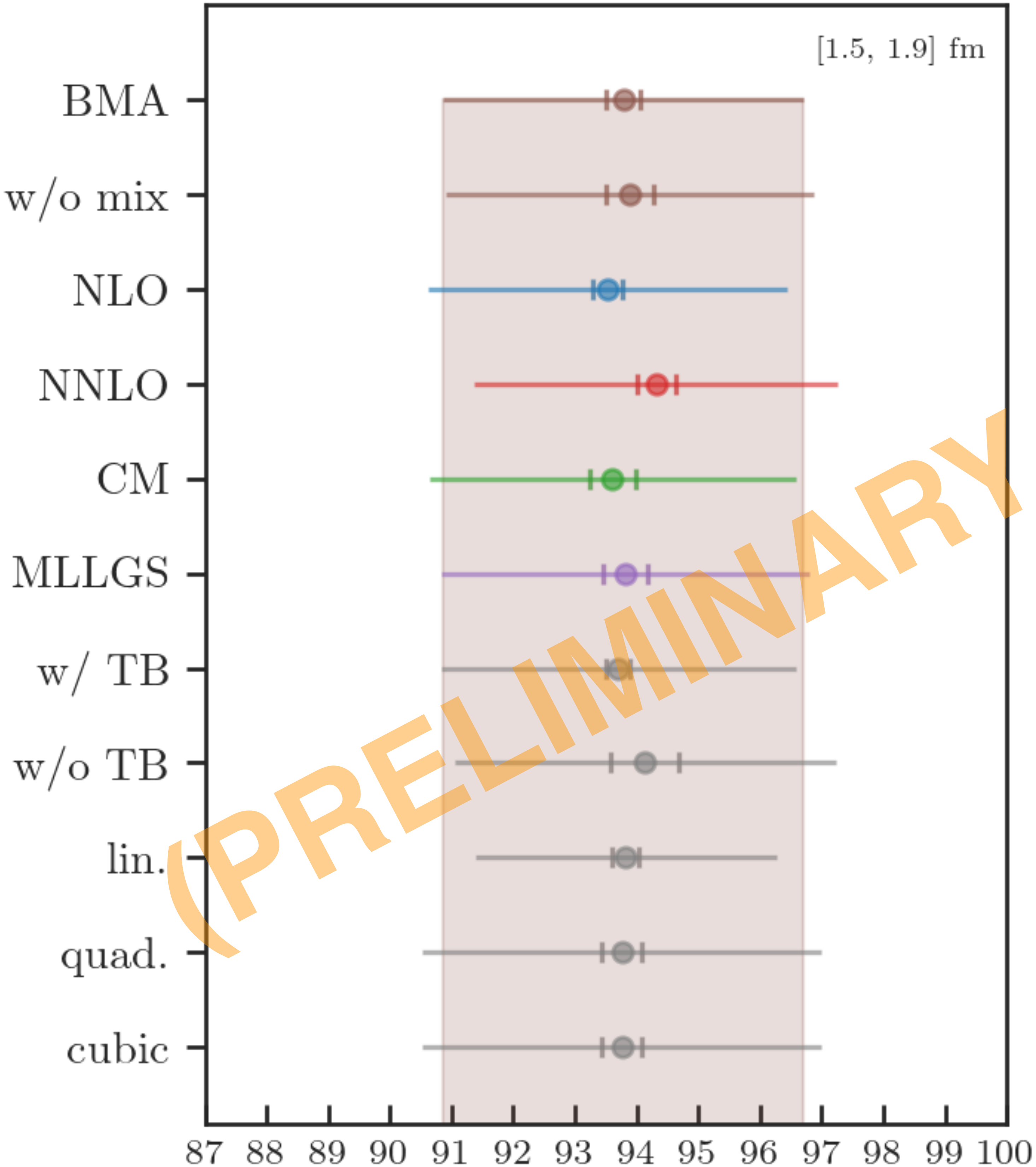


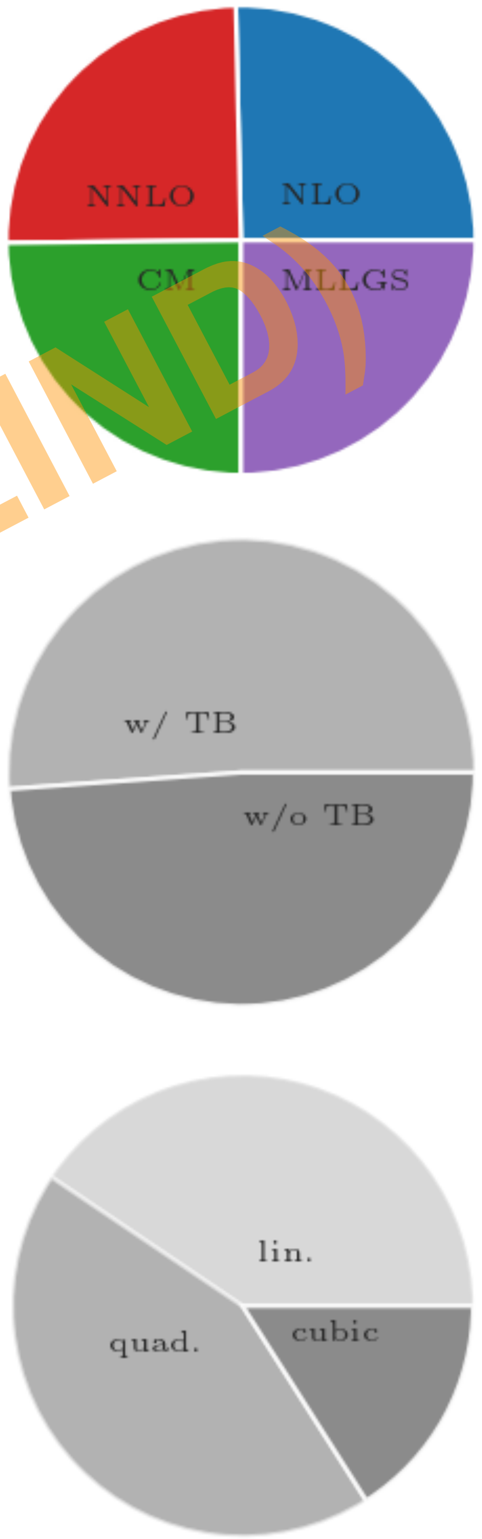
W2:



a^2[fm^2]
20
40
0.005
0.01
0.015
0.02
105
100
95
90
85
80
75
$a_\mu^{ll,\mathrm{W2}}$(conn.)
a_μ(Bayes)
Fits: 300
+ Δ_{FV} (CM), Δ_{m_π}
+ Δ_{FV}, Δ_{TB} (CM), Δ_{m_π}
+ Δ_{FV} (MLLGS), Δ_{m_π}
+ Δ_{FV}, Δ_{TB} (MLLGS), Δ_{m_π}
(PRELIMINARY, BLIND)
+Δ_{FV}, Δ_{TB} : $\frac{[\mathrm{FV}]}{[\mathrm{TB}]}$
NLO
NNLO
CM
MLLGS
HP
p
0.95
0.9
6.0
6.005
6.01
6.015
6.02
6.025
$\chi^2(\mathbf{a}^\star) + 2k + 2N_{\mathrm{cut}}$

W2:


[1.5, 1.9] fm
BMA
w/o mix
NLO
NNLO
CM
MLLGS
w/ TB
w/o TB
lin.
quad.
cubic
87 88 89 90 91 92 93 94 95 96 97 98 99 100
(PRELIMINARY, BLIND)


NNLO
NLO
CM
MLLGS
w/ TB
w/o TB
lin.
quad.
cubic