Isospin-breaking effects in HVP from e^+e^- data

b UNIVERSITÄT BERN

AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

Martin Hoferichter

Albert Einstein Center for Fundamental Physics,

Institute for Theoretical Physics, University of Bern

Sep 05, 2022

Fifth Plenary Workshop of the Muon $g-2$ Theory Initiative

Higgs Centre, Edinburgh

Colangelo, MH, Stoffer JHEP 02 (2019) 006, PLB 814 (2021) 136073 [2π disp]

Colangelo, MH, Kubis, Niehus, Ruiz de Elvira PLB 825 (2022) 136852 [2π IAM+disp]

Stamen, Hariharan, MH, Kubis, Stoffer EPJC 82 (2022) 432 [$\bar{K}K$ disp]

Colangelo, MH, Kubis, Stoffer 2208.08993 [IB in 2π]

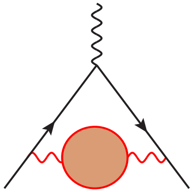
thanks to B.-L. Hoid [$\pi^0\gamma$], A. Keshavarzi [$\eta\gamma$], and L. Lellouch [IB in BMWc 2020]

Motivation

effect	$\pi^0\gamma$	$\eta\gamma$	ρ–ω mixing	FSR	M_{π^0} vs. $M_{\pi^\pm}$	total
size **in units of 10^{-10}**	4.64(4)	0.65(1)	2.71(1.36)	4.22(2.11)	−4.47(4.47)	7.8(5.1)

BMWc 2017, Jegerlehner

- Detailed comparison between e^+e^- data and lattice QCD
- Well-defined for total and windows, here: **what about isospin breaking?**
- Can do much better than previous estimates, but still caveats:
  - Cannot cover all channels in exclusive approach
  - Scheme dependence
- Dominant effects:
  - **Radiative channels $\pi^0\gamma$, $\eta\gamma$**: data
  - **ρ–ω mixing**: residue in dispersive representation
  - **FSR**: scalar QED + dispersive corrections
  - **M_{π^0} vs. $M_{\pi^\pm}$ for 2π channel**: IAM + Omnès
  - **$\bar{K}K$**: resonance/threshold enhancement



Dispersive representation of 2π contribution

- Most relevant effects in 2π channel
- ρ–ω mixing, FSR from dispersive representation talk by P. Stoffer
- Pion-mass dependence more subtle, main idea:

$$\Omega_1^1(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\delta_1^1(s')}{s'(s'-s)}\right\}$$

$\hookrightarrow$ can get **pion-mass dependence from IAM** Guo et al. 2009, Colangelo et al. 2022

- **Isospin breaking** due to pion mass difference:

$$a_\mu^{\text{HVP}}[\pi\pi]\big|_{M_{\pi^\pm}} - a_\mu^{\text{HVP}}[\pi\pi]\big|_{M_{\pi^0}} = -7.67(4)_{\text{ChPT}}(3)_{\text{polynomial}}(4)_{\langle r_\pi^2\rangle}(21)_{r_{V1}^r}[22]_{\text{total}}$$

$\hookrightarrow$ **threshold effect**, almost exclusively in LD window

- Biggest change compared to previous estimate $-4.47(4.47)$

ρ–ω mixing and FSR

- FSR dominated by **infrared enhanced effects**
  $\hookrightarrow$ scalar QED, corrections small Moussallam 2013

$$a_\mu^{\text{HVP}}[\pi\pi\gamma, \text{non-Born}] = 0.15_{\pi^+\pi^-\gamma} + 0.03_{\pi^0\pi^0\gamma} = 0.18(4)$$

- Higher-order terms $\mathcal{O}(e^2\epsilon_{\rho\omega})$ small, $\lesssim 0.1$, pure $\epsilon_{\rho\omega}$ part gives

$$a_\mu^{\text{HVP}}[\pi\pi, \text{FSR}, \text{Born}] = 4.24(2) \qquad a_\mu^{\text{HVP}}[\pi\pi, \rho\text{–}\omega] = 3.68(17)$$

ρ–ω mixing and FSR

- FSR dominated by **infrared enhanced effects**

  $\hookrightarrow$ scalar QED, corrections small Moussallam 2013

$$a_\mu^{\text{HVP}}[\pi\pi\gamma, \text{non-Born}] = 0.15_{\pi^+\pi^-\gamma} + 0.03_{\pi^0\pi^0\gamma} = 0.18(4)$$

- Higher-order terms $\mathcal{O}(e^2\epsilon_{\rho\omega})$ small, $\lesssim 0.1$, pure $\epsilon_{\rho\omega}$ part gives

$$a_\mu^{\text{HVP}}[\pi\pi, \text{FSR}, \text{Born}] = 4.24(2) \qquad a_\mu^{\text{HVP}}[\pi\pi, \rho\text{–}\omega] = 3.68(17)$$

- Separation of $\epsilon_{\rho\omega}$ into $\mathcal{O}(e^2)$ and $\mathcal{O}(\delta) = \mathcal{O}(m_u - m_d)$ Urech 1995

$$\Theta_{\rho\omega} \simeq -3\epsilon_{\rho\omega} M_V^2 \qquad \Theta_{\rho\omega}\big|_{e^2} = e^2 F_\rho F_\omega \qquad \Gamma(V \to e^+e^-) = \frac{e^4 F_V^2}{12\pi M_V}$$

- Correction actually relative to $\rho(770)$, so

$$\epsilon_{\rho\omega}\big|_{e^2} \simeq -e^2\left(\frac{F_\omega}{M_\omega}\right)^2 \simeq -0.34(1)\times 10^{-3}$$

- However: $\mathcal{O}(e^2)$ goes away after VP removal!

  $\hookrightarrow$ $a_\mu^{\text{HVP}}[\pi\pi, \rho\text{–}\omega]$ purely $\mathcal{O}(\delta)$

Isospin breaking in $\bar{K}K$ channel

- Why $\bar{K}K$? talk by D. Stamen
  - ϕ resonance close to $\bar{K}K$ threshold
  - **Isospin breaking from masses enhanced**
- Threshold region dominated by isoscalar form factor via ϕ

  $\hookrightarrow$ analyzed in terms of ϕ resonance parameters
- Significant isospin breaking in residues

$$c_\phi^{K^+K^-} = 0.977(6) \qquad c_\phi^{\bar{K}^0K^0} = 1.001(6)$$

$\hookrightarrow$ dominant source of uncertainty

- Definition of isospin limit via self energy $(M^2_{K^\pm})_{\text{EM}} = 2.12(18)\times 10^{-3}\,\text{GeV}^2$ from Cottingham formula

$$M_{K^\pm} = (494.58 - 3.05_\delta + 2.14_{e^2})\,\text{MeV} \qquad M_{K^0} = (494.58 + 3.03_\delta)\,\text{MeV}$$

very close to scheme just introduced by Antonin

Isospin breaking in $\bar{K}K$ channel

- Results

$$a_\mu^{\text{HVP}}[K^+K^-, \leq 1.05\,\text{GeV}] = 18.45(20) \qquad a_\mu^{\text{HVP}}[K^0\bar{K}^0, \leq 1.05\,\text{GeV}] = 11.83(15)$$

$$a_\mu^{\text{HVP}}[K^+K^-, \text{FSR}] = 0.75(4)$$

$$a_\mu^{\text{HVP}}[K^+K^-, e^2] = -3.24(17) \qquad a_\mu^{\text{HVP}}[K^0\bar{K}^0, e^2] = -0.02(0)$$

$$a_\mu^{\text{HVP}}[K^+K^-, \delta] = 4.98(26) \qquad a_\mu^{\text{HVP}}[K^0\bar{K}^0, \delta] = -4.62(23)$$

$$a_\mu^{\text{HVP}}[K^+K^-, e^2\delta] = -0.33(1)$$

- Note:
  - K^0 self energy negligible, but indirect $\mathcal{O}(e^2)$ effect from the $K^\pm$ contribution to the ϕ spectral function
  - Remaining differences between "isospin limit" K^+K^- (16.29) and $\bar{K}^0K^0$ (16.47) due to c_ϕ and isovector form factor

- Isospin-breaking effects huge due to **threshold/resonance enhancement**

  $\hookrightarrow$ higher-order terms $\mathcal{O}(e^2\delta)$ in K^+K^- larger than in 2π!

Summing everything up

	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	–	0.05(0)	–	0.83(6)	–	2.79(11)	–	3.68(17)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	$-0.09(7)$	–	$-7.62(14)$	–	$-7.67(22)$	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	$-0.29(1)$	0.44(2)	$-1.71(9)$	2.63(14)	$-1.24(6)$	1.91(10)	$-3.24(17)$	4.98(26)
kaon mass ($\bar{K}^0K^0$)	0.00(0)	$-0.41(2)$	$-0.01(0)$	$-2.44(12)$	$-0.01(0)$	$-1.78(9)$	$-0.02(0)$	$-4.62(23)$
total	0.14(1)	0.08(3)	1.61(12)	1.02(20)	$-2.44(16)$	2.92(17)	$-0.68(29)$	4.04(39)
BMWc 2020	–	–	$-0.09(6)$	0.52(4)	–	–	$-1.5(6)$	1.9(1.2)
JLM 2021	–	–	–	–	–	–	–	3.32(89)

- Note: error estimates only refer to the effects included

  $\hookrightarrow$ **additional channels missing** (most relevant for SD and int window)

- Systematic error $\simeq 0.8$ from c_ϕ due to ambiguity how to define the isospin limit

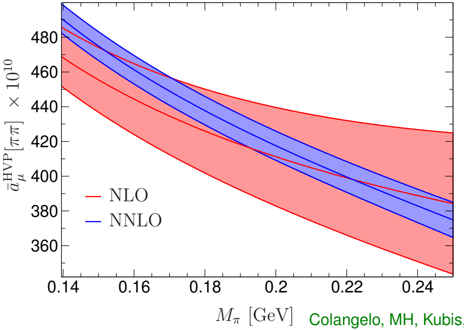
- Reasonable agreement with BMWc 2020 and James, Lewis, Maltman 2021

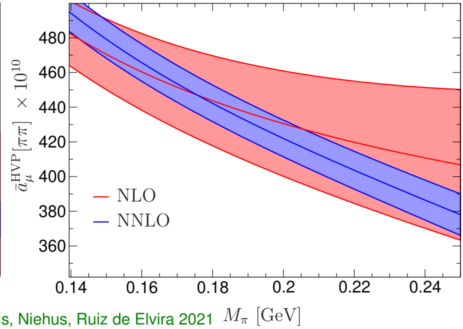
  $\hookrightarrow$ if anything, the result would become even larger with pheno estimates

Other channels

- **3π channel**:
  - Naive estimate for FSR by comparing to 2π: $\simeq 0.4$ Kubis, Prabhu, Schuh, work in progress
  - BaBar 2021 quotes $\rho \to 3\pi$ contribution in VMD fit ($\simeq -0.6$ in a_μ^HVP Boito et al. 2022), but hard to extract beyond the model context
  - ω peak scales with $1/\Gamma_\omega$ in narrow-width approximation, dependence of Γ_ω on pion mass could play a role Dax, Isken, Kubis 2018, but effect cancels out in the integral
  - Threshold effects suppressed by $(s - 9M_\pi^2)^3$, far away from ω peak
  - Main uncertainty again from residue $c_\omega^{3\pi}$, is there any isospin breaking as in $c_\phi^{\bar{K}K}$?
- ***R*-ratio from perturbative QCD**
  - QED corrections included in `rhad` Harlander, Steinhauser 2003, but $\mathcal{O}(10^{-3})$ compared to leading-order result, and $10^{-3} a_\mu^\text{HVP}[\geq 1.8\,\text{GeV}] \lesssim 0.05$

Predicting the pion-mass dependence from the IAM





Colangelo, MH, Kubis, Niehus, Ruiz de Elvira 2021

- $\bar{a}_\mu^{\mathrm{HVP}}[\pi\pi]$ only $I = 1$ correlator (with $\epsilon_{\rho\omega} = 0$)
- Free parameters:
  - LECs in $\delta_1^1(s)$: combined fit to data Colangelo, MH, Stoffer 2019 and lattice Andersen et al. 2019
  - r_{V1}^r: resonance saturation $r_{V1}^r = 2.0 \times 10^{-5}$ in concord with lattice Feng, Fu, Jin 2020
- Validated at physical point

Hadronic vacuum polarization from lattice QCD

- **Chiral extrapolation** part of systematic error budget

  $\hookrightarrow$ extrapolation to (or interpolation around) physical quark masses

- Biggest contribution from ***$I=1$ ud* isospin-symmetric correlator**

  $\hookrightarrow$ phenomenologically dominated by 2π channel, first correction from 4π

- ChPT not enough Golterman, Maltman, Peris 2017

$$a_\mu^{I=1} = \frac{\alpha^2}{24\pi^2}\bigg(-\log\frac{M_\pi^2}{m_\mu^2} - \frac{31}{6} + 3\pi^2\sqrt{\frac{M_\pi^2}{m_\mu^2}} + \mathcal{O}\bigg(\frac{M_\pi^2}{m_\mu^2}\log^2\frac{M_\pi^2}{m_\mu^2}\bigg)\bigg)$$

  $\hookrightarrow$ "convergence" in M_π/m_μ

- Need to provide information on the $\rho(770)$ resonance

  $\hookrightarrow$ **inverse-amplitude method at two-loop order**

Possible application to lattice QCD

1. **Chiral LECs as fit parameters**:
   - Describes $\pi\pi$ physics
   - Need to add $a_\mu^{\text{HVP}}[ud, I=1, \text{non-}\pi\pi] = \zeta + M_\pi^2 \xi$
     $\hookrightarrow$ **infrared singularities** will be totally dominated by 2π
   - Can provide **independent constraints from other lattice calculations**: δ_1^1, F_π, $\langle r_\pi^2 \rangle$
2. **Simple parameterizations**:
   - Only possible for integrated HVP or space-like integrand $\frac{\bar{\Pi}(-Q^2)}{Q^2} = \frac{a+bQ^2}{1+cQ^2+dQ^4}$
   - Test infrared singularities Golterman, Maltman, Peris 2017, e.g., M_π^{-2}, $\log M_\pi^2$
   - Fits to $\{a, b, c, d\}$ indicate singularity as strong as M_π^{-2} in $[0.14, 0.25]$ GeV
   - **Purely empirical finding**, no analytic approximation to full IAM nor true chiral behavior
   - Could help inform lattice fits

Possible application to lattice QCD