Muon g-2 with overlap valence fermion

The Fifth Plenary Workshop of the Muon g-2 Theory Initiative at the University of Edinburgh

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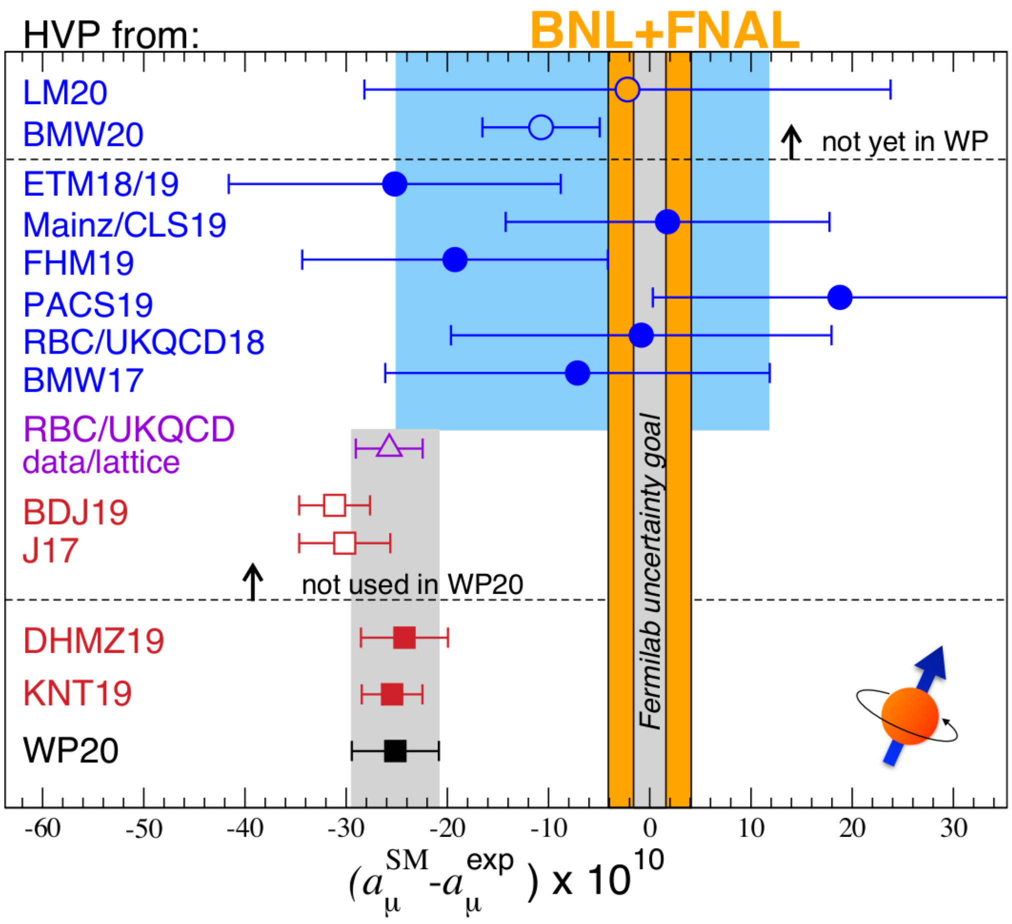
arXiv:2204.01280

- Motivations
- Simulation strategies with Overlap fermions
- Lattices
- Intermediate windows
- Short-distance windows
- Total contributions (light and strange preliminary)
- Conclusions

Muon anomalous magnetic moments



Colangelo et al., arXiv:2203.15810

- Experimental result is 4.2σ SM predictions (WP Aoyama et al., 2020)
- BMW20 result is 2.1σ higher than R-ratio and consistent with experiment value at 1.5σ level
- Comparison among lattice groups on window quantities [Blum et al (2018)]
  - No signal-to-noise problem
  - Small t cutoff effects suppressed
  - Long distance volume effects suppressed

Euclidean time windows

- Time-momentum representation :

D. Bernecker and H. B. Meyer, Eur. Phys. J. A 47, 148 (2011)

$$a_\mu^{\mathrm{LO-HVP}} = \int \mathrm{d}t\, \omega(t) C(t)$$

- Intermediate window quantities:

T. Blum, et al., (RBC/UKQCD), Phys. Rev. Lett. 121, 022003 (2018)

$$a_\mu^{\mathrm{win}} = \sum_t w_t C(t) \times w^W(t)$$

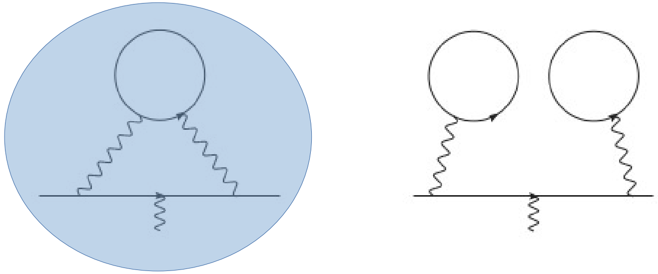
$$w^W(t) = \theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)$$

$$\theta(t, t_0, \Delta) = \tfrac{1}{2}(1 + \tanh(\tfrac{t-t'}{\Delta}))$$

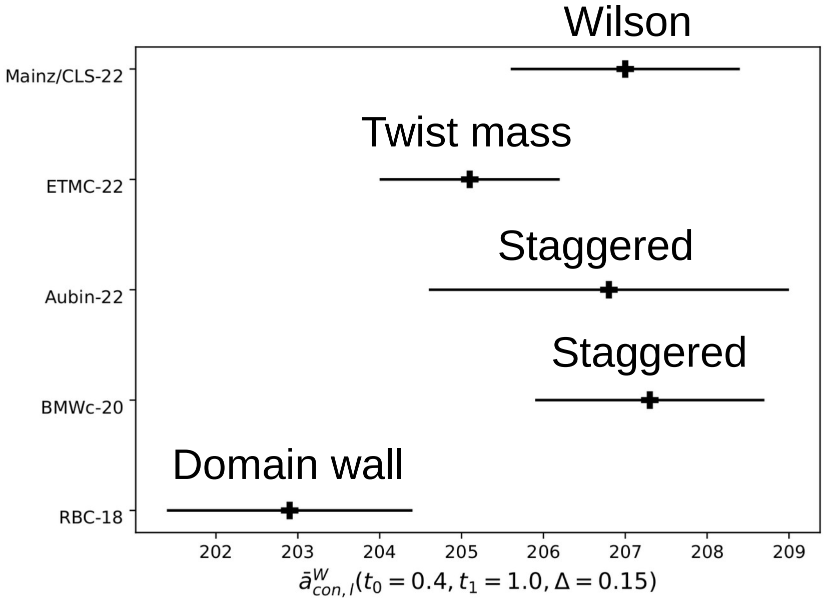
$$t_0 = 0.4,\ t_1 = 1.0,\ \Delta = 0.15$$

$$w^S(t) = 1 - \theta(t, t_0, \Delta)$$

$$w^L(t) = \theta(t, t_1, \Delta)$$



Connected light/strange under Iso-spin symmetric



- Motivations
- ***Simulation strategies with Overlap fermions***
  - ***Overlap fermions***
  - ***Renormalization constants***
- Lattices
- Intermediate windows
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Overlap fermions and two-point functions

- Overlap fermions (Neuberger, 1998)

$$D_{ov}(\rho) = \rho(1 + \gamma_5 \epsilon(\gamma_5 D_w(-\rho)))$$

Ginsparg-Wilson relationship

$$D_{ov}\gamma_5 + \gamma_5 D_{ov} = \frac{1}{\rho} D_{ov}\gamma_5 D_{ov}$$

- Connected piece of the vector meson correlation functions

$$C_{2pt} = \langle \sum_y \mathrm{Tr}[\gamma_i D^{-1}(y|x_0)\gamma_i D^{-1}(x_0|y)]\rangle$$

$$D^{-1}(y|x) = D_L^{-1}(y|x) + D_H^{-1}(y|x)$$

$$D_L^{-1}(y|x) = \sum_i \frac{1}{\lambda_i + m} v_i(y) v^\dagger(x)$$

~2000 paris

- Measure each parts separately

sum over all x_0 with v_i

Grid sources (~8) with ~1 fm separation in spatial direction

$$C_{2pt} = \langle C(P_L, P_L) + C(P_L, P_H) + C(P_H, P_L) + C(P_H, P_H)\rangle$$

Renormalization constants

- Chiral symmetry of Overlap fermions guarantees

$$Z_V = Z_A$$

- Partially conserved axial current (PCAC)

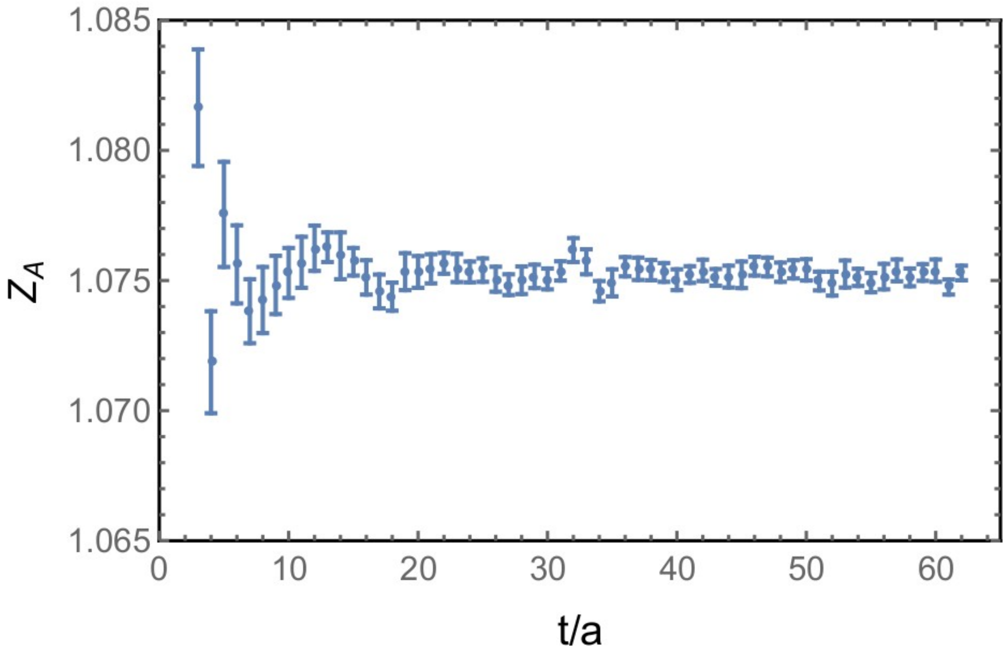
$$Z_A \partial_\mu A_\mu = 2 Z_m Z_p m_q P$$

- Correlated ratio from two-pt

$$Z_A = \frac{2m_q \langle\Omega| P(t)P^\dagger(0) |\Omega\rangle}{\langle\Omega| \partial_\mu A_\mu(t) P^\dagger(0) |\Omega\rangle}$$

- **Reaches less than 0.02% error**

$Z_A(64\mathrm{I})$ from PCAC



F.-C. He, Y.-J. Bi, T. Draper, K.-F. Liu, Z.F. Liu, Y.-B. Yang, arXiv:2204.09246

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Lattices

	Symbol	$L^3 \times T$	a (fm)	m_π	N_{cfg}	N_{src}	N_g	λ_c
2+1 DWF (RBC/UKQCD)	48I	$48^3 \times 96$	0.11406(26)	139	100	12	4	234
	64I	$64^3 \times 128$	0.08365(25)	139	92	8	4	187
	24D	$24^3 \times 64$	0.1940(19)	141	232	8	2	263
	32D	$32^3 \times 64$	0.1940(19)	141	134	8	4	230
	48D	$48^3 \times 64$	0.1940(19)	141	47	8	6	116
2+1+1 HISQ (MILC)	a12m130	$48^3 \times 64$	0.12121(64)	131	23	8	4	180
	a09m130	$64^3 \times 96$	0.08786(47)	128	22	8	4	200
	a12m310	$24^3 \times 64$	0.12129(89)	305	54	16	1	224
	a09m310	$32^3 \times 96$	0.08821(71)	313	39	16	1	195
	a06m310	$48^3 \times 144$	0.05740(50)	319	32	8	1	243
	a04m310	$64^3 \times 192$	0.04250(40)	310	54	2	1	167

- Motivations
- Simulation strategies with Overlap fermions
- Lattices
- ***Intermediate windows***
  - ***Light quark results***
  - ***Strange quark results***
  - ***Charm contributions (preliminary)***
  - ***Disconnect contributions (preliminary)***

Short-distance windows

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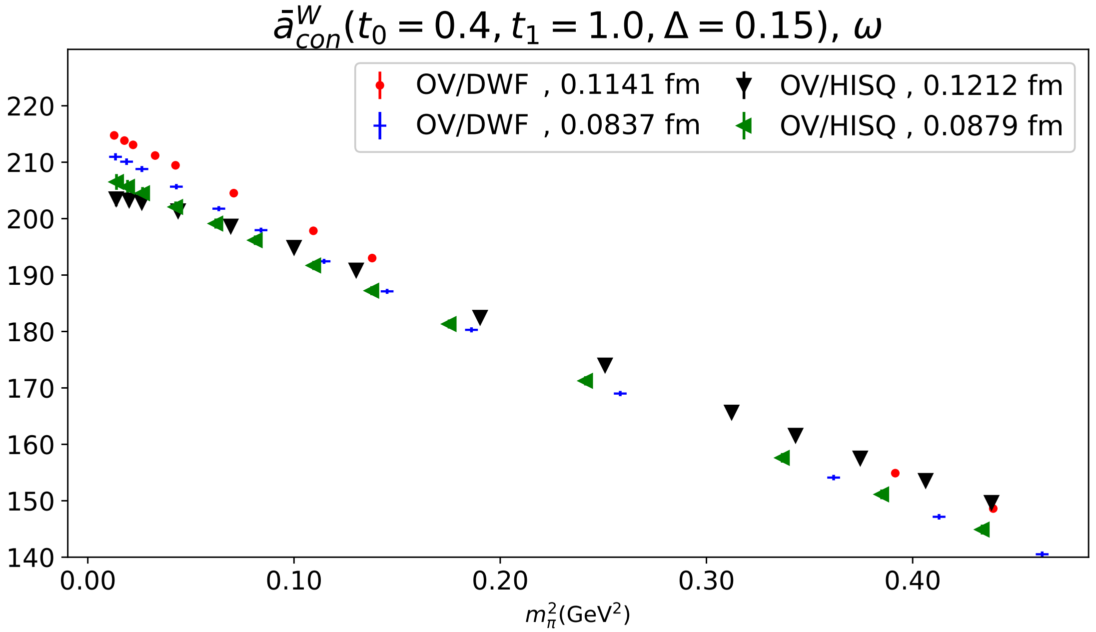
Valence quark mass dependence



- Overlap multi-mass inverter
  valence pion mass [120, 700] MeV

- Light quarks :
  valence pion mass = 135 MeV

- Strange quark :
  valence quark mass =
  light quark mass * 27.42(12)

Light quark connect contributions

- Connected light window contributions

$$a_\mu^{\text{win}} = \sum_t w_t C(t) \times [\theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)]$$

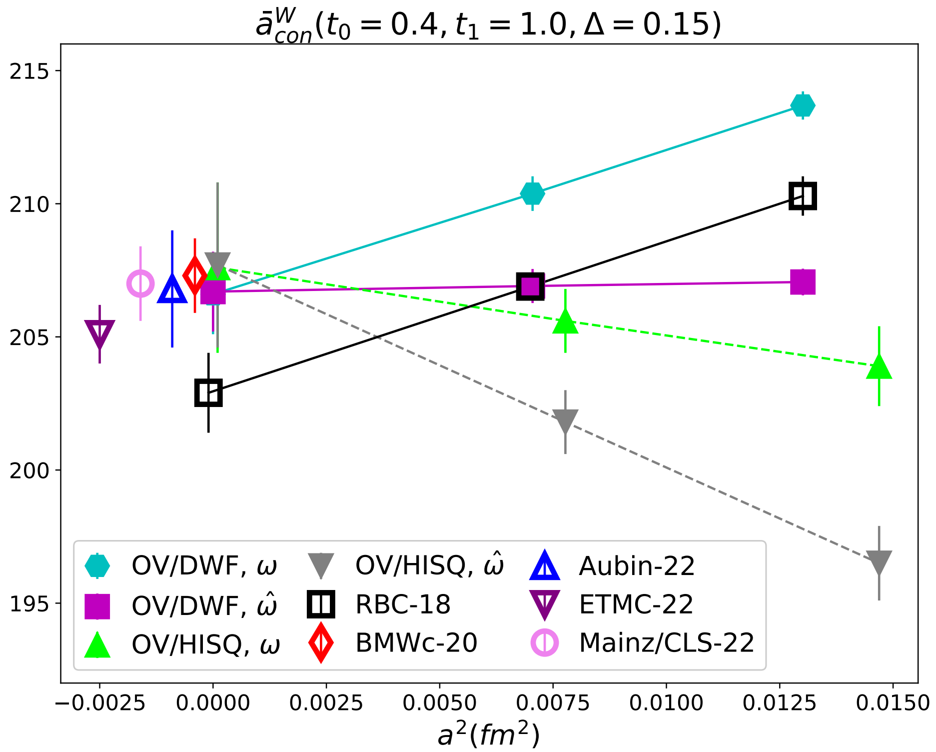
- Two difference weighting functions

$$\omega(t) = 4\alpha^2 \int_0^\infty \frac{\mathrm{d}q^2}{m_\mu^2} f\left(\frac{q^2}{m_\mu^2}\right) \left[\frac{\cos(tq) - 1}{q^2} + \frac{1}{2}t^2\right]$$

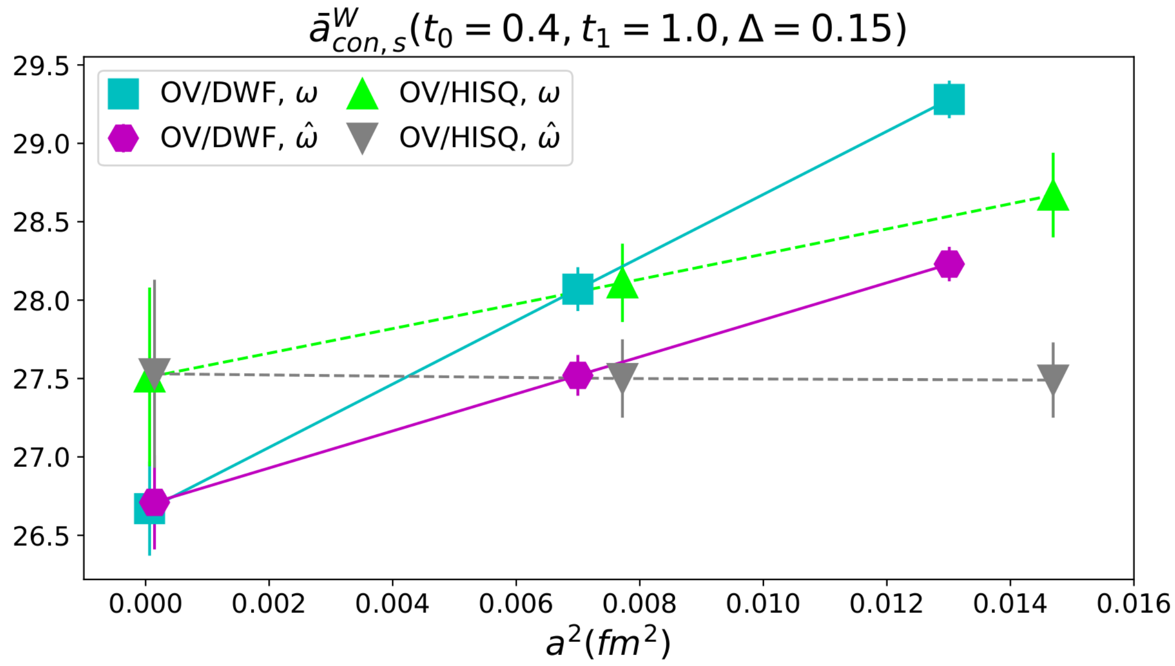
$$\hat{\omega}(t) = 4\alpha^2 \int_0^\infty \frac{\mathrm{d}q^2}{m_\mu^2} f\left(\frac{q^2}{m_\mu^2}\right) \left[\frac{\cos(tq) - 1}{[\frac{2}{a}\sin(\frac{qa}{2})]^2} + \frac{1}{2}t^2\right]$$

- OV/DWF and OV/HISQ are consistent at continuum limit

- OV/DWF result is higher than unitary DWF



Strange quark connect contributions



- All cases have obvious slope
- OV/DWF and OV/HISQ are consistent at continuum limit
- Consistent with unitary DWF [1] with value 27.0(2)

[1] T. Blum, et al., (RBC/UKQCD), Phys. Rev. Lett. 121, 022003 (2018)

Continuum extrapolations

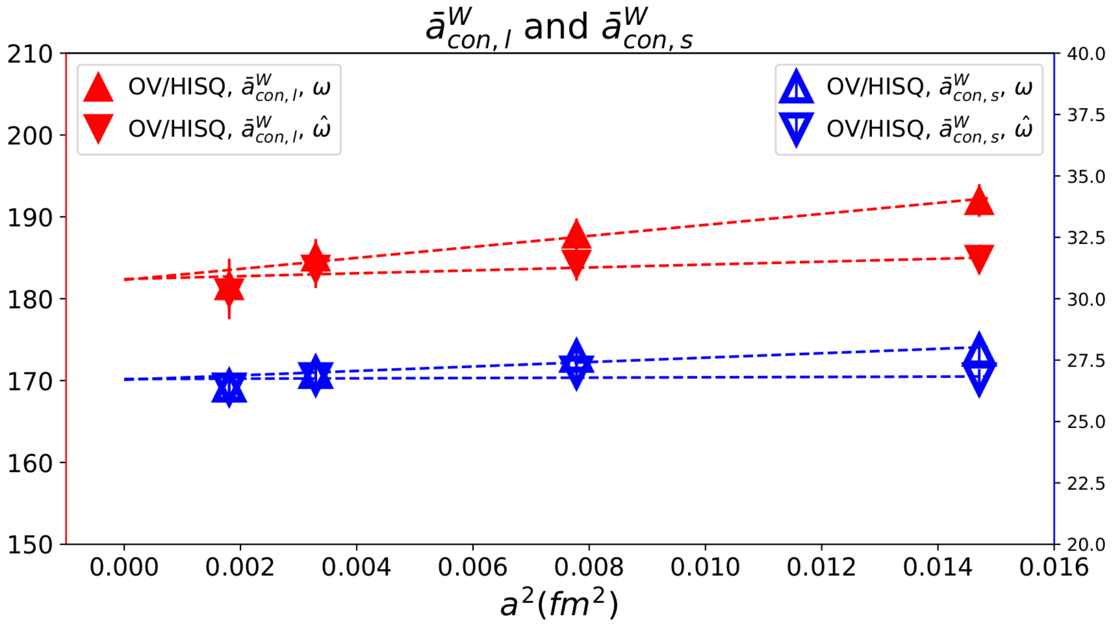
- HISQ ensembles (MILC) with a ~ [0.04, 0.12] fm at 310 MeV pion mass

- Simple linear extrapolations (conf ~#50)

$$\text{light quark}\ \chi^2 \sim 0.22$$

$$\text{strange quark}\ \chi^2 \sim 1.20$$

- Situation may be different at
  - Physical pion mass
  - OV/DWF cases



Finite Volume effects

- DWF ensembles (RBC/UKQCD) with L ~ [4.66, 9.31] fm at physical pion mass

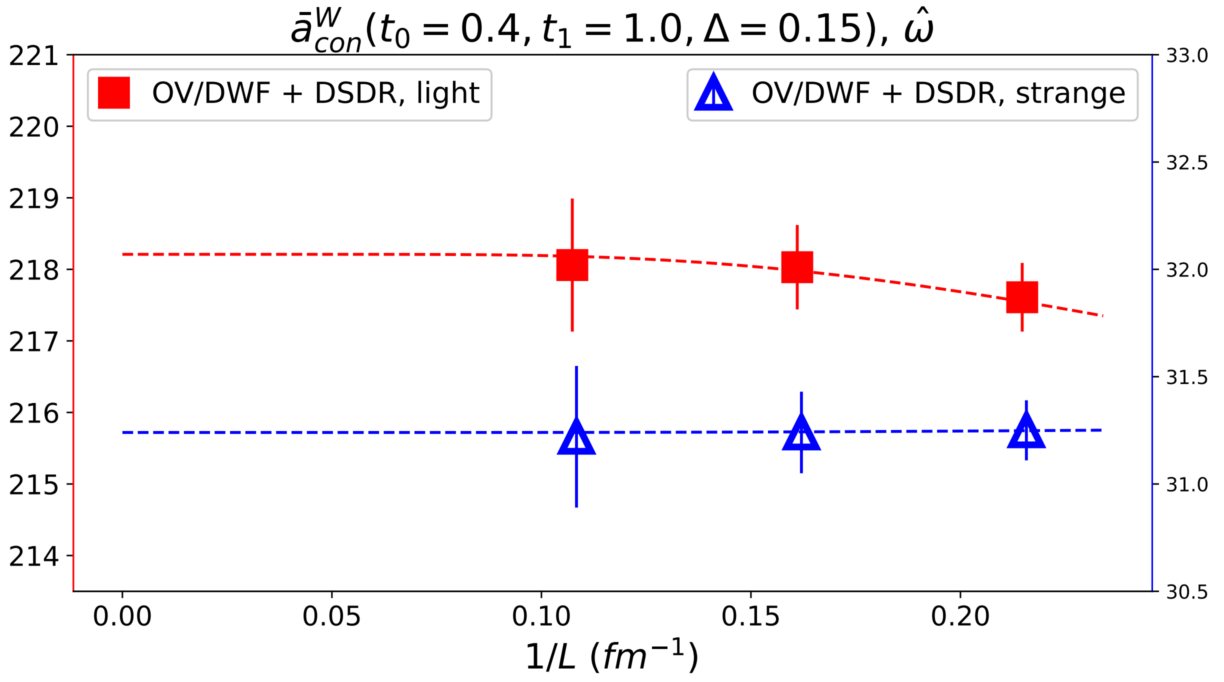
- Fit with simple exponential

$$a + be^{(-m_\pi L)}$$

- Finite volume effects

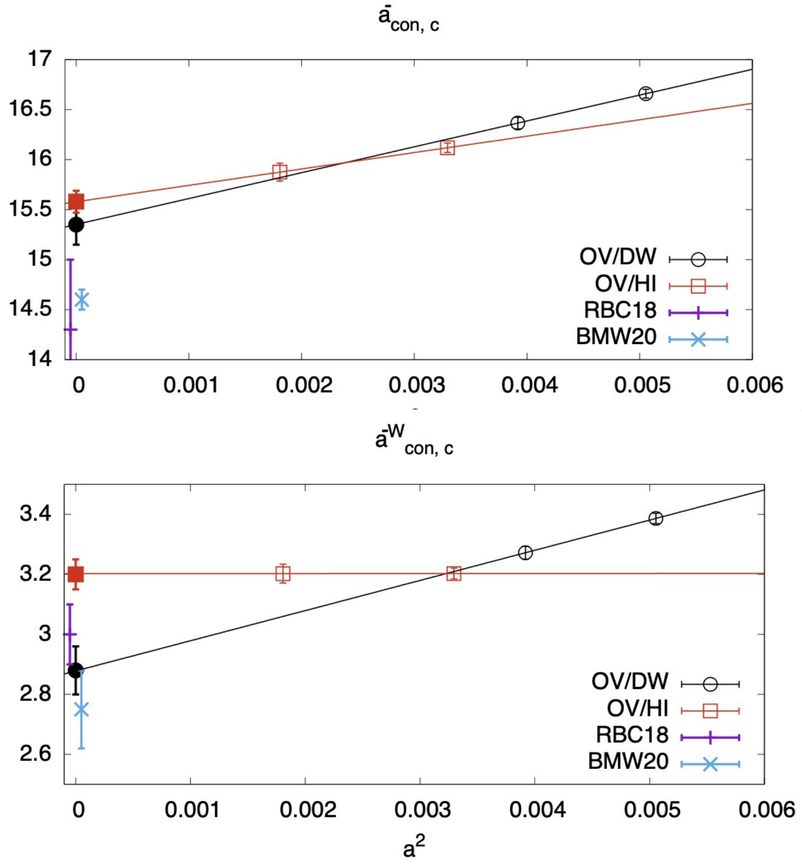
  Light quark : −0.36(56)

  Strange quark : 0.01(18)



Charm contributions (preliminary)

Total contribution and the standard window contribution



- Tuning the charm quark mass by requiring

$$m_{J/\Psi} = 3.0969\,\mathrm{GeV}$$

- Overlap valence fermion on:
  - RBC ensembles at

$$0.071\ \mathrm{fm},\ 267\ \mathrm{MeV}\ \text{and}\ \ 0.063\ \mathrm{fm},\ 370\ \mathrm{MeV}$$

  - MILC ensembles at

$$0.057\ \mathrm{fm},\ 319\ \mathrm{MeV}\ \text{and}\ \ 0.043\ \mathrm{fm},\ 310\ \mathrm{MeV}$$

- Tension between two setups suggest a^4 effects, while would not be crucial given the precision required.

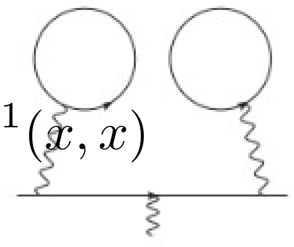
- Statistical errors only

Disconnect contribution (preliminary)

$$D^{-1}_{u/d,s}(x,x) = D^{L,-1}_{u/d,s}(x,x) + D^{H,-1}_{u/d,s}(x,x)$$

Exact from low modes

Z_4-noise grid with even-odd dilution



- Reusing previous quark loops

Y.B. Yang, et al., (χQCD), Phys. Rev. Lett. 121, 212001 (2018)

  - 48I (DWF), 0.114 fm, -0.868(63) (preliminary and statistical errors only)

- Further improvements
  - Cluster-decomposition error (CDER) [1] technique
  - Difference of two quark loops with different masses[2]

$$\mathrm{Tr}\left[\Gamma(D_l^{-1} - D_s^{-1})\right] = (m_s - m_l)\mathrm{Tr}\left[\Gamma(D_l^{-1} D_s^{-1})\right]$$

[1] K.-F. Liu, J. Liang, and Y.-B. Yang, Phys. Rev. D 97, 034507 (2018)
[2] L. Giusti, et al., Eur. Phys. J. C 79, 586 (2019)

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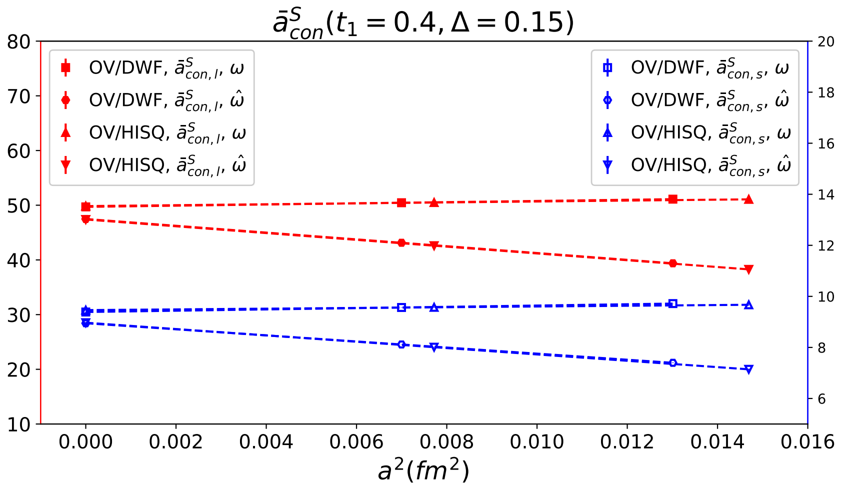
Short-distance contributions

$$w^S(t) = 1 - \theta(t, t_0, \Delta)$$

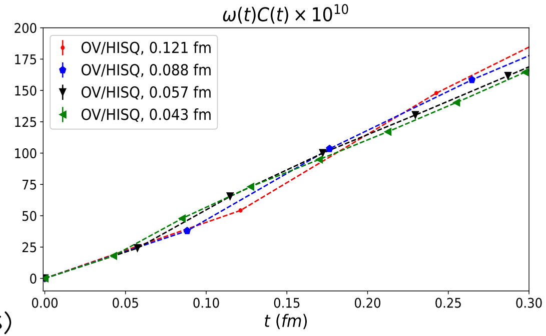


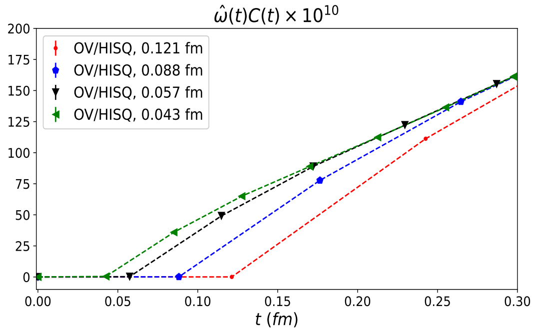
- Sea effects are small (DWF and HISQ agree at current two lattice spacings)
- The disagreement between two weighting definitions hits a^4 effects

$$\hat{\omega}(t) = 4\alpha^2 \int_0^\infty \frac{dq^2}{m_\mu^2} f\left(\frac{q^2}{m_\mu^2}\right)\left[\frac{\cos(tq)-1}{[\frac{2}{a}\sin(\frac{qa}{2})]^2} + \frac{1}{2}t^2\right]$$

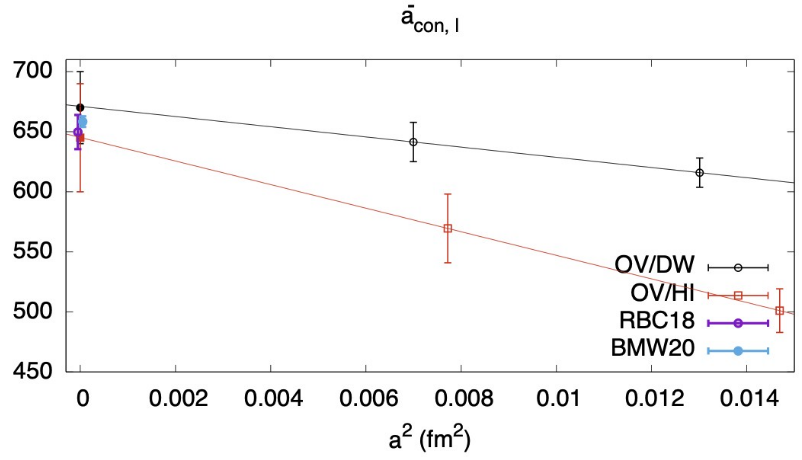
- Singular near t = 0

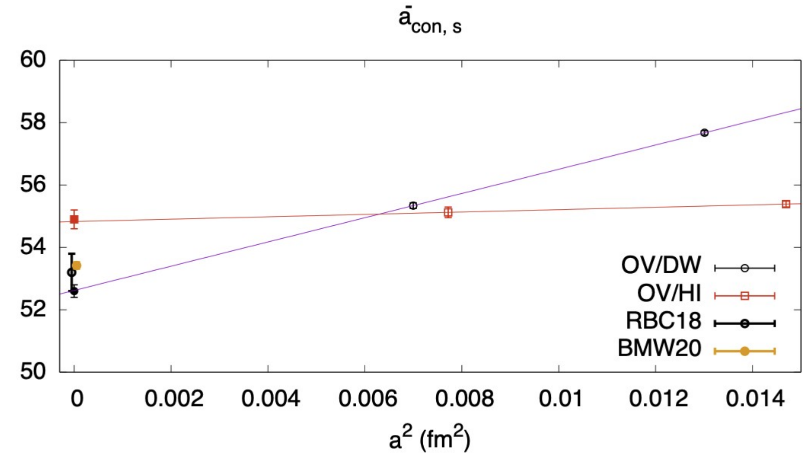
[1] T. Blum, et al., (RBC/UKQCD), Phys. Rev. Lett. 121, 022003 (2018)





Total contributions (light and strange preliminary)





- Bounding method is applied to the light quark case (~2.5 fm)

- Overlap valence fermion on:
  - Physical point RBC ensembles at
    $$0.112 \text{ fm and } \ 0.083 \text{ fm}$$
  - Physical point MILC ensembles at
    $$0.121 \text{ fm and } \ 0.088 \text{ fm}$$

- Tension between two setups suggest a^4 effects, requires further study at finer lattice spacing

- Statistical errors only

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Conclusions

- High statistical measurements with Overlap fermions are feasible
- Gauge dependence is obvious for intermediate windows but not short-distance windows

- On-going disconnect and charm contributions
- Smaller lattice spacing results are needed
- **Challenging** : full contributions using Overlap fermions with high statistics

Thank You