Status and Progress for HVP with C* boundary conditions

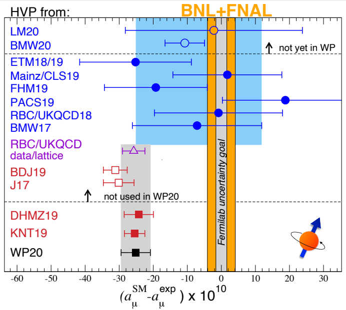
Javad Komijani

ETH zürich

RC*: Anian Altherr, Lucius Bushnaq, Isabel Campos-Plasencia, Marco Catillo, Alessandro Cotellucci, Madeleine Dale, Roman Gruber, Patrick Fritzsch, JK, Jens Luecke, **Marina Marinkovic**, Sofie Martins, Agostino Patella, Joao Pinto Barros, Nazario Tantalo & Paola Tavella

September 5, 2022

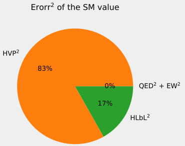
Motivation



Contribution	value $\times 10^{11}$
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP	6845(40)
HLbL	92(18)
Total SM value	116 591 810(43)
Difference: $a_\mu^{exp} - a_\mu^{SM}$	251(59)

[arXiv:{2104.03281, 2006.04822, 2203.15810}]

- HVP is the main contributor to the total theory uncertainty
- Lots of activities in lattice QCD to improve the precision
- ...
- Strong isospin breaking & QED:
  - direct effects in HVP of order 1%, yet need to be measured precisely...
  - indirect effects through scale setting quantities



Motivation

- To include QED effects (& deal with Gauss's law):
  - Removal of the zero mode of the photon (with non-local prescriptions) [Hayakawa & Uno, 2008]
    QED_L prescription is often used, where $\sum_{\vec{x}} A_\mu(x_0, \vec{x}) = 0$ for all μ and x_0
  - C* boundary conditions [Kronfeld & Wiese, 1991, Wiese, 1992; Polley, 1993]
    RC* collaboration: full simulation of QCD + QED with C* boundary conditions, which is a local solution to the zero-mode of the photon field
  - Massive photon [Endres et al., 2015]

Outline

1. Intro to C* boundary conditions
2. Ensembles with C* boundary conditions
3. HVP in QCD and QCD+QED simulations with C* boundary conditions

Related talks/posters presented in the lattice conference

- Anian Altherr [slides]: HVP with C* boundary conditions
- Roman Grover [slides]: HVP with C* boundary conditions
- Sofie Martins [slides]: Finite-Size Effects of the Hadronic Vacuum Polarization Contribution to the Muon $(g-2)$ with C* Boundary Conditions
- Jens Lücke [slides]: An update on QCD+QED simulations with C* boundary conditions
- Paola Tavella [poster]: Strange and charm contribution to the HVP from C* boundary conditions
- Alessandro Cotellucci [poster]: Tuning of QCD+QED simulations with C* boundary conditions

Intro to C* boundary conditions

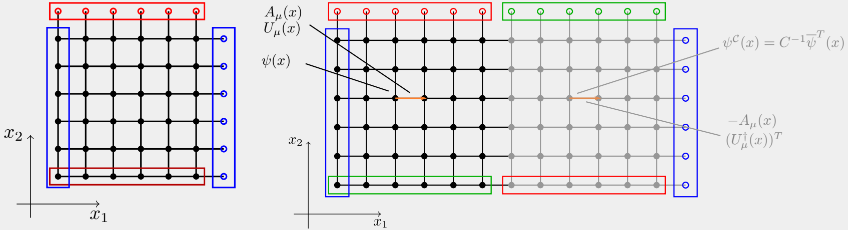


Figure: Left: periodic boundary conditions & Right: C* boundary conditions

- physical lattice: $x_1 < L_1$
- mirror lattice: $L_1 \leq x_1 \leq 2L_1$
- periodic boundaries on extended lattice, i.e. $\psi(x + 2L_1\hat{1}) = \psi(x)$

C* BOUNDARY CONDITIONS: PROS & CONS[1]

- [+] $A_\mu(x + L_1\hat{x}_1) = -A_\mu(x)$ (C-odd) $\implies$ allowed momenta: $\frac{\pi}{L}(2\mathbb{Z}+1)$
  $\rightarrow$ no need for zero-mode dropping
- [+] propagation of electrically charged states can be simulated on the lattice from first principles
- [+] suppressed finite volume effects
- [-] lattice volume doubled by introducing a mirror lattice
- [-] flavour mixing $\rightarrow$ exponentially suppressed by volume
- [-] global U(1) broken down to $\mathbb{Z}_2$ $\rightarrow$ charge conservation is partially violated

[1]For details see Lucini et al. [arXiv:1509.01636].

C* BOUNDARY CONDITIONS[3]

The spinor fields ψ and $\bar{\psi}$ are NOT independent anymore

	periodic boundary		C* boundary
Action:	$\sum_{x\in\Lambda_{\text{phys}}} \bar{\psi}(x)D\psi(x)$	$\longrightarrow$	$\sum_{x\in\Lambda_{\text{phys+mirror}}} -\frac{1}{2}\psi^T(x)CTD\psi(x)$
Integration measure:	$[\mathcal{D}\psi]_{\Lambda_{\text{phys}}}\,[\mathcal{D}\bar{\psi}]_{\Lambda_{\text{phys}}}$	$\longrightarrow$	$[\mathcal{D}\psi]_{\Lambda_{\text{phys+mirror}}}$
Determinant:[2]	$\det(D)$	$\longrightarrow$	$\text{Pf}(CTD)$
Wick-contraction :	$\overbrace{\psi(x)\bar{\psi}}(y) = D^{-1}(x\vert y)$	$\longrightarrow$	$\overbrace{\psi(x)\psi^T}(y) = -D^{-1}(x\vert y)TC^{-1}$

- C: charge-conjugation matrix
- T: translation operator flips physical $\leftrightarrow$ mirror lattice

$$T\psi(x_{\text{phys}}) = \psi(x_{\text{phys}} + L_1\hat{1})$$
$$T\psi(x_{\text{mirr}}) = \psi(x_{\text{mirr}} - L_1\hat{1})$$

[3]For details see Lucini et al. [arXiv:1509.01636]

Vector correlator turns into the usual expression (with modified Dirac operator)

$$\langle j_\mu(x) j_\nu(y) \rangle = \mathrm{tr}_{CD} \left[\gamma_\mu D^{-1}(x|y) \gamma_\nu D^{-1}(y|x) \right]$$

Ensembles with C* boundary conditions

OPENQ*D CODE

Publicly available under `https://gitlab.com/rcstar/openQxD`

- available solvers:
  - conjugate gradient on the normal equations (CGNE)
  - generalized conjugate residual using Schwarz alternating procedure (SAP+GCR)
  - (inexact) deflation-accelerated solver (DFL+SAP+GCR)

RC* ensembles with C* boundary conditions[4]

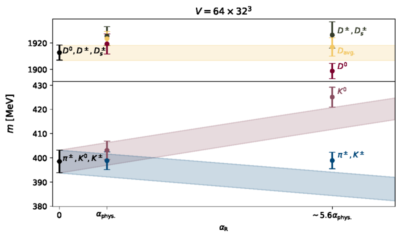
ensemble	lattice	flavor	α	a [fm]	$m_{\pi^\pm}$ [MeV]
A400a00	64×32^3	3 + 1	0	0.054	400
B400a00	80×48^3	3 + 1	0	0.054	400
A450a07	64×32^3	1 + 2 + 1	0.007299	0.055	450
A380a07	64×32^3	1 + 2 + 1	0.007299	0.053	380
A500a50	64×32^3	1 + 2 + 1	0.05	0.053	495
A360a50	64×32^3	1 + 2 + 1	0.05	0.051	360
C380a50	96×48^3	1 + 2 + 1	0.05	0.051	390

- Full simulation of QCD + QED:
  - Lüscher-Weisz action for SU(3) gauge
  - Wilson action for Compact U(1) gauge
  - Four flavours of $\mathcal{O}(a)$-improved Wilson fermions
- C*/periodic boundary conditions in three spatial/time directions
- $\beta = 3.24$ and $\alpha \in [0, 1/137, 0.05]$

[4]See Jens Lücke [lattice 2022 slides]: An update on QCD+QED simulations with C* boundary conditions

RC* ENSEMBLES: LINE OF CONSTANT PHYSICS

ensemble	lattice	flavor	α	a [fm]	$m_{\pi^\pm}$ [MeV]
A400a00	64×32^3	3 + 1	0	0.054	400
B400a00	80×48^3	3 + 1	0	0.054	400
A450a07	64×32^3	1 + 2 + 1	0.007299	0.055	450
A380a07	64×32^3	1 + 2 + 1	0.007299	0.053	380
A500a50	64×32^3	1 + 2 + 1	0.05	0.053	495
A360a50	64×32^3	1 + 2 + 1	0.05	0.051	360
C380a50	96×48^3	1 + 2 + 1	0.05	0.051	390



J. Lücke [lattice 2022 slides]

- Tuning 4 flavors of quark masses:

$$\phi_0 = m^2_{K^\pm} - m^2_{\pi^\pm} \quad \to 0 \qquad \text{fixes } m_d/m_s \qquad \text{(non-physical, degenerate } d \text{ \& } s\text{)}$$

$$\phi_1 = m^2_{K^\pm} + m^2_{\pi^\pm} + m^2_{K^0} \quad \to \phi_1^{\text{phys}} \qquad \text{fixes } m_u + m_d + m_s$$

$$\phi_2 = \frac{1}{\alpha_{\text{ren}}}\left(m^2_{K^0} - m^2_{K^\pm}\right) \quad \to \phi_2^{\text{phys}} \qquad \text{fixes } \delta m_{strong}/\delta_{EM} \qquad (\Rightarrow \text{isospin limit when } \alpha_{\text{ren}} = 0)$$

$$\phi_3 = m^2_{D_s^\pm} + m^2_{D^\pm} + m^2_{D^0} \quad \to \phi_3^{\text{phys}} \qquad \text{fixes } m_c$$

- $\alpha = 0.05$ amplifies both QED & strong isospin breaking effects

HVP in QCD and QCD+QED simulations with C* boundary conditions

- QCD with C* ($\alpha = 0$) + Pert. QED: [Divitiis et al, arXiv:1303.4896]
  - leading light contribution to HVP [partially performed for 2 ens.]
  - Pert. QED (requires $\sim$ 16 more sets of inversions per source)
  - strong isospin breaking effects
- QCD + QED with C*:
  - no extra inversions for QED effects
  - main cost is hidden in generation of gauge configurations & related tuning [Bushnaq et al (RC* collaboration), to be published]
- Ensembles we have partially analyzed:

ensemble	lattice	flavor	α	a [fm]	$m_{\pi^\pm}$ [MeV]
A400a00	64×32^3	3 + 1	0	0.054	400
B400a00	80×48^3	3 + 1	0	0.054	400
A450a07	64×32^3	1 + 2 + 1	0.007299	0.055	450
A380a07	64×32^3	1 + 2 + 1	0.007299	0.053	380
A500a50	64×32^3	1 + 2 + 1	0.05	0.053	495
A360a50	64×32^3	1 + 2 + 1	0.05	0.051	360
C380a50	96×48^3	1 + 2 + 1	0.05	0.051	390

- New ensembles with lighter $m_{\pi^\pm}$ & different a will be generated and used

- Time-momentum representation
- local & point-split vector currents; renormalization is required
- QCD: the renormalization constant is evaluated from $\langle j_k^{ps}(x) j_k^{loc}(0)\rangle \;/\; \langle j_k^{loc}(x) j_k^{loc}(0)\rangle$
- QCD+QED: (conserved) vector current requires renormalization [Collins et al, arXiv:hep-th/0512187]

$$j_\mu^{\text{ren}}(x) = j_\mu^{\text{ps}}(x) + \mathcal{O}\left(\partial_\nu F^{\nu\mu}\right)$$

- Single-exponential fit (for now) where signal fades

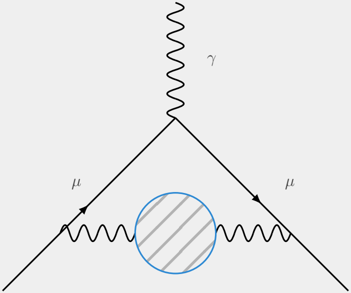
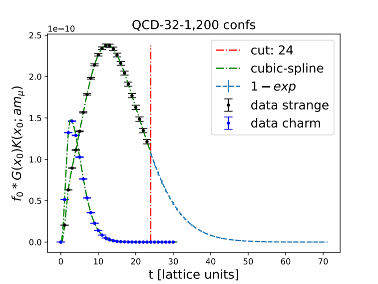


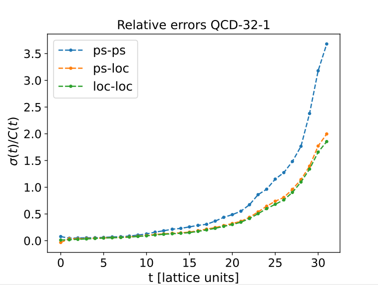
Figure: Hadronic vacuum polarization

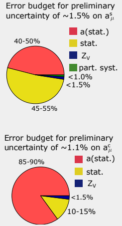
$$\int_0^\infty \mathrm{d}t \underbrace{\int_{\mathbb{R}^3} \mathrm{d}^3x \langle j_k^{ps}(t,\vec{x}) j_k^{loc}(0,\vec{0})\rangle}_{G_k(t)} \tilde{K}(t;m_\mu)$$

- **QCD with C* ($\alpha = 0$) + Pert. QED:**
  - leading light contribution to HVP [partially performed for 2 ens.]
  - Pert. QED (requires $\sim$ 16 more sets of inversions per source)
  - strong isospin breaking effects

ensemble	lattice	flavor	α	a [fm]	$m_{\pi^\pm}$ [MeV]	N_{cfg}
A400a00	64×32^3	3 + 1	0	0.054	400	200
B400a00	80×48^3	3 + 1	0	0.054	400	108
A450a07	64×32^3	1 + 2 + 1	0.007299	0.055	450	
A380a07	64×32^3	1 + 2 + 1	0.007299	0.053	380	
A500a50	64×32^3	1 + 2 + 1	0.05	0.053	495	
A360a50	64×32^3	1 + 2 + 1	0.05	0.051	360	
C380a50	96×48^3	1 + 2 + 1	0.05	0.051	390	



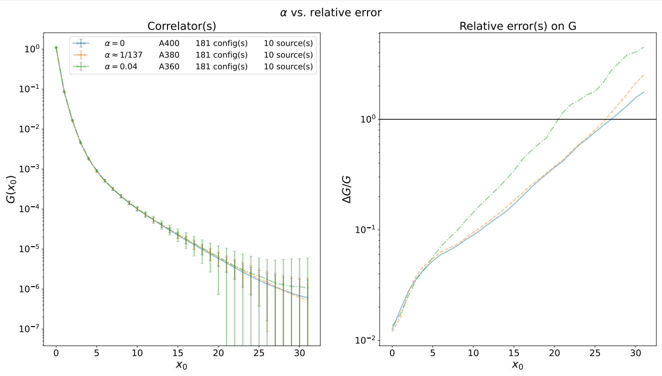


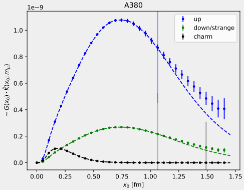


- The PIE chart does not include any extrapolations

Second study: Including QED with C* & relative error

ensemble	lattice	flavor	α	a [fm]	$m_{\pi^\pm}$ [MeV]	N_{cfg}
A400a00	64×32^3	3 + 1	0	0.054	400	181
B400a00	80×48^3	3 + 1	0	0.054	400	
A450a07	64×32^3	1 + 2 + 1	0.007299	0.055	450	
A380a07	64×32^3	1 + 2 + 1	0.007299	0.053	380	181
A500a50	64×32^3	1 + 2 + 1	0.05	0.053	495	
A360a50	64×32^3	1 + 2 + 1	0.05	0.051	360	181
C380a50	96×48^3	1 + 2 + 1	0.05	0.051	390	





- Current-current correlator for up quark; slightly different masses

$\Rightarrow$ Comparable noise levels for $\alpha \in [0, 1/137]$

Third study: variant sources

The propagator $D^{-1}(x|y)$ needs inversions of the Dirac operator using source fields located at source point y:

1. Point sources
   - a point source η at point (y, β, b) obeys $\eta(x)_{\alpha a} = \delta_{xy}\delta_{\alpha\beta}\delta_{ab}$
   - **12 inversions per point source** y
2. Spin-diluted stochastic wall sources
   - located at a time-slice y_0
   - diluted in the 4 spin-indices $\implies$ **4 inversions per wall source**
   - $(\mathbb{Z}_2 + i\mathbb{Z}_2)$-noise
   - gives an average over spatial lattice: $\Omega = \Lambda_3$
3. Smeared sources for mass extraction

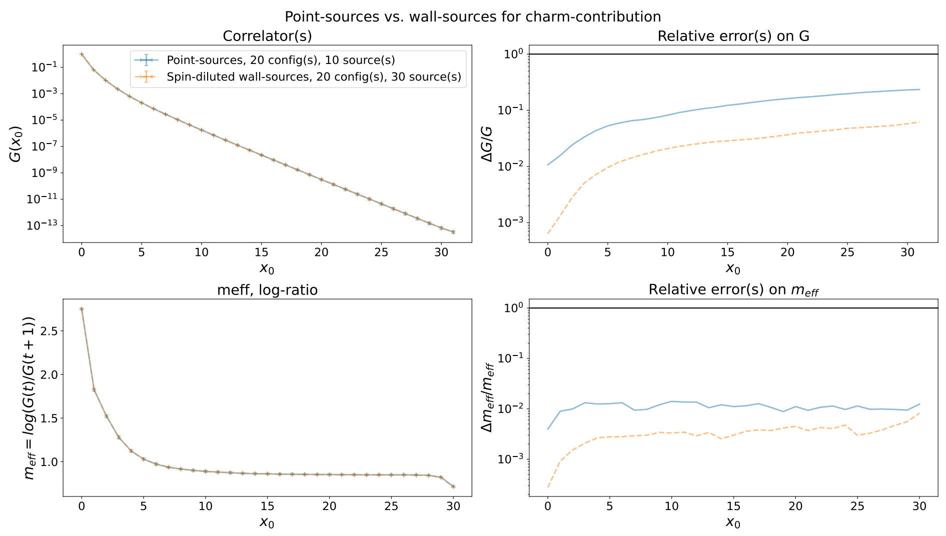


Figure: Choosing a type of stochastic source for the charm contribution ($q_c = 2/3$)

Up: Point sources $\leftrightarrow$ spin-diluted wall sources

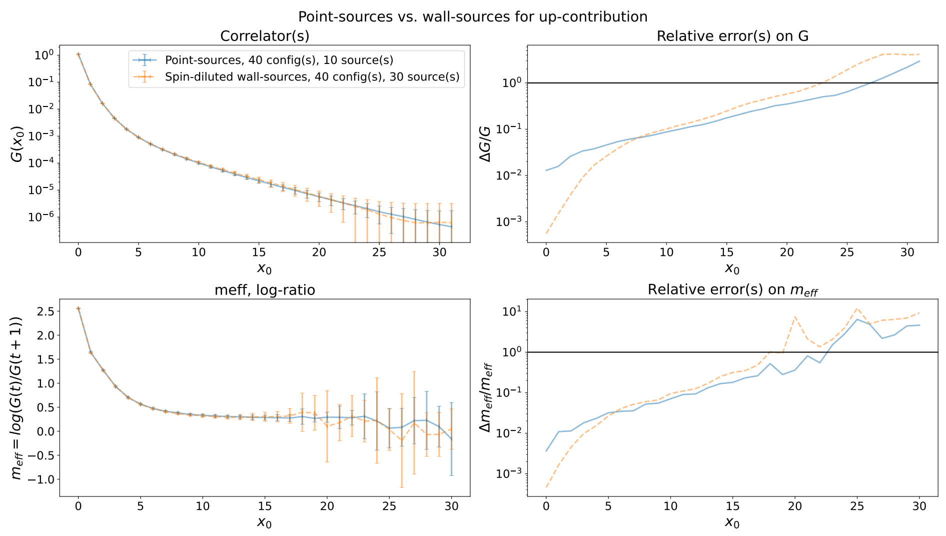


Figure: Low-statistics study: Sources for **up**-contribution ($q_u = 2/3$)

Down/strange: Point sources ↔ spin-diluted wall sources

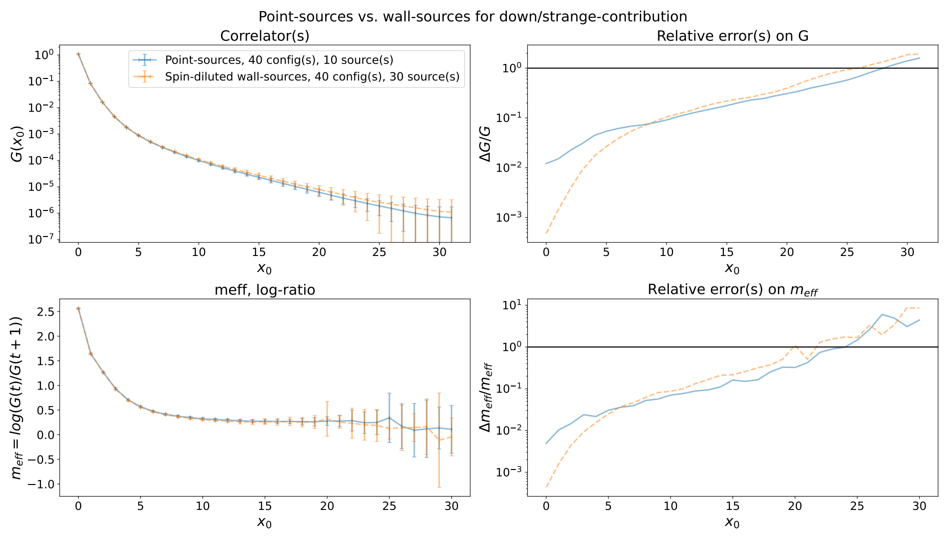


Figure: Low-statistics study: Sources for **down/strange**-contribution ($q_{d/s} = -1/3$)

Point sources $\leftrightarrow$ spin-diluted wall sources

Different measurement setup for different quark flavours:

- **heavy quarks** $\longrightarrow$ spin-diluted stochastic wall-sources
- **light quarks** $\longrightarrow$ point-sources
- **mass spectra** $\longrightarrow$ smeared sources

Outlook: a long but promising to-do list

- Short-term:
  - Model part: include excited states
  - Variance reduction: low mode averaging
  - Finite size effects due to QED
- Medium-term:
  - Disconnected contributions
  - Renormalization factor due to QED
  - Comparing Pert. QED method with our dynamical QCD+QED results
- Long-term:
  - Chiral extrapolation
  - Continuum extrapolation

Thank you!