A DATA-BASED ROUTE TO THE DISCONNECTED CONTRIBUTON TO $a_\mu^{LO,HVP}$

and to the isospin-limit light-quark-connected contribution, if time permits

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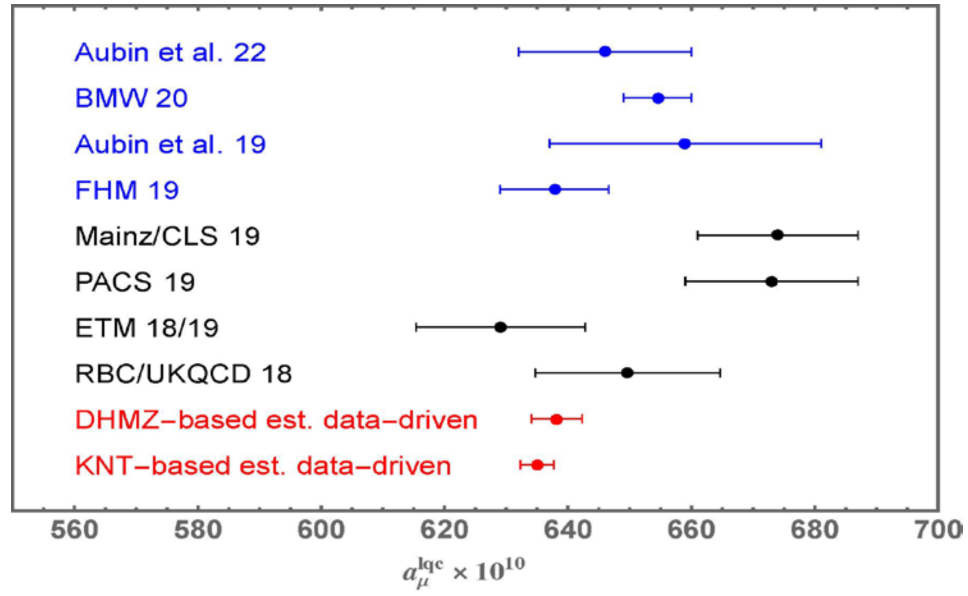
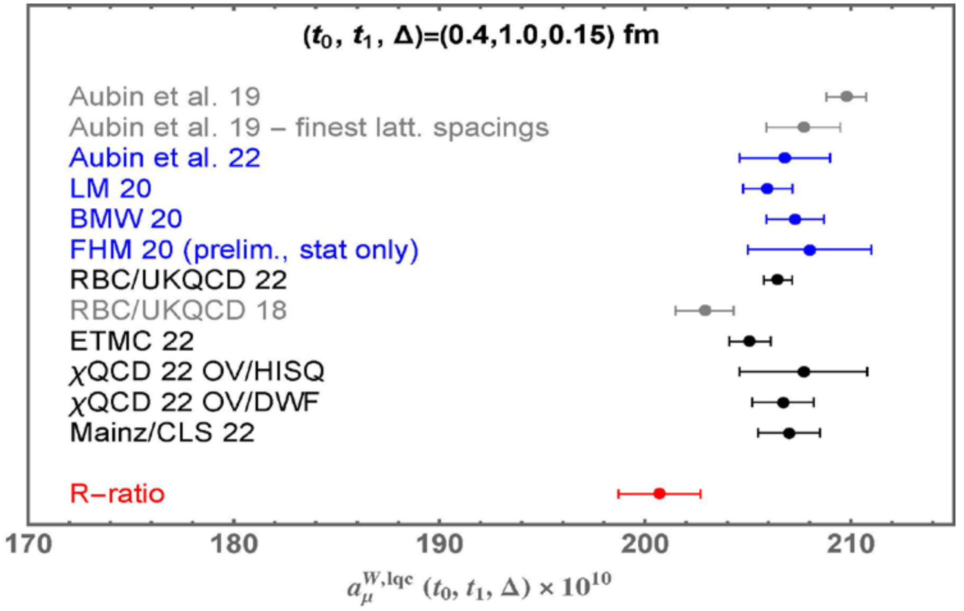
CONTEXT: LATTICE-DISPERSIVE $a_\mu^{LO,HVP}$ TENSION

$a_\mu^{LO,HVP}$ THEORY

- $a_\mu^{LO,HVP}$ (dispersive) x 10^{10}
  - DHMZ 2020: 694.0(4.0)
  - KNT 2019: 692.8(2.4)
- $a_\mu^{LO,HVP}$ (lattice) x 10^{10}
  - ETM 2019: 692.1(16.3)
  - FHM 2019: 699(15)
  - Mainz 2019: 720.0(15.9)
  - PACS 2019: 737(20)
  - **BMW 2020: 707.5(5.5)**

- **Components amenable to both lattice and dispersive determinations of interest for further exploration**





THE STRANGE CONNECTED + FULL DISCONNECTED CONTRIBUTION FROM DATA

- **Notation: $SU(3)_F$ decompositions**

$$J_\mu^{EM} = V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8 \equiv J_\mu^{EM,3} + J_\mu^{EM,8}$$

$$= \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s)$$

$$\begin{aligned}\hat{\Pi}_{EM}(Q^2) &= \hat{\Pi}^{33}_{EM}(Q^2) + \frac{2}{\sqrt{3}}\hat{\Pi}^{38}_{EM}(Q^2) + \frac{1}{3}\hat{\Pi}^{88}_{EM}(Q^2)\\ &\equiv \hat{\Pi}^{I=1}_{EM}(Q^2) + \hat{\Pi}^{MI}_{EM}(Q^2) + \hat{\Pi}^{I=0}_{EM}(Q^2)\\ \rho_{EM}(s) &= \rho^{33}(s) + \frac{2}{\sqrt{3}}\rho^{38}(s) + \frac{1}{3}\rho^{88}(s)\\ &\equiv \rho^{I=1}_{EM}(s) + \rho^{MI}_{EM}(s) + \rho^{I=0}_{EM}(s)\end{aligned}$$

$$a_\mu^{LO,HVP} = a_\mu^{33} + \frac{2}{\sqrt{3}} a_\mu^{38} + \frac{1}{3} a_\mu^{88} \equiv a_\mu^{I=1} + a_\mu^{MI} + a_\mu^{I=0}$$

- **Isospin limit (IL) content**
  - 38 (MI) component 0
  - 33 (I=1) pure light-quark-connected (lqc)
  - 88 (I=0) lqc+ strange-quark-connected (sconn) + full disconnected (disc)

- **⇒ appropriate 33, 88 combination gives sconn+disc sums in IL:**

$$\begin{aligned}\hat{\Pi}^{sconn+disc}_{EM} &\equiv \hat{\Pi}^{I=0}_{EM} - \frac{1}{9}\hat{\Pi}^{I=1}_{EM}\\ \rho^{sconn+disc}_{EM}(s) &= \rho^{I=0}_{EM}(s) - \frac{1}{9}\rho^{I=1}_{EM}(s)\\ a_\mu^{sconn+disc} &= a_\mu^{I=0} - \frac{1}{9} a_\mu^{I=1}\end{aligned}$$

PRACTICAL IMPLEMENTATION (I: neglecting IB correction)

- **identifying I=0, 1 spectral contributions**
  - G-parity +/- states: I=1/0
  - ⇒ I=0, 1 separation for states with only narrow π, η, ω, ϕ (>93% of total)
  - G-parity ambiguous states X (e.g. with $K\bar{K}$ pairs): from maximally conservative 50±50% I=1/50∓50% I=0 separation if not available from other sources

$$[a_\mu^{\rm scorn+disc}]_X = \left(\frac{4}{9} \pm \frac{5}{9}\right) [a_\mu^{\rm LO,HVP}]_X$$

  - pQCD above exclusive-mode region

E.g. G-parity-unambiguous modes, KNT19 data, to KNT exclusive-mode-region endpoint s=(1.937 GeV)^2

$I=1$ modes X	$[a_\mu^{\rm LO,HVP}]_X \times 10^{10}$	$I=0$ modes X	$[a_\mu^{\rm LO,HVP}]_X \times 10^{10}$
Low-s $\pi^+\pi^-$	0.87(02)	Low-s 3π	0.01(00)
$\pi^+\pi^-$	503.46(1.91)	$\pi^0\gamma$ (ω, ϕ dominated)	4.46(10)
$2\pi^+2\pi^-$	14.87(20)	3π	46.73(94)
$\pi^+\pi^-2\pi^0$	19.39(78)	$2\pi^+2\pi^-\pi^0$ (no ω, η)	0.98(09)
$3\pi^+3\pi^-$ (no ω)	0.23(01)	$\pi^+\pi^-3\pi^0$ (no η)	0.62(11)
$2\pi^+2\pi^-2\pi^0$ (no η)	1.35(17)	$3\pi^+3\pi^-\pi^0$ (no ω, η)	0.00(01)
$\pi^+\pi^-4\pi^0$ (no η)	0.21(21)	$\eta\gamma$ (ω, ϕ dominated)	0.70(02)
$\eta\pi^+\pi^-$	1.34(05)	$\eta\pi^+\pi^-\pi^0$ (no ω)	0.71(08)
$\eta 2\pi^+2\pi^-$	0.08(01)	$\eta\omega$	0.30(02)
$\eta\pi^+\pi^-2\pi^0$	0.12(02)	$\omega(\to npp)2\pi$	0.13(01)
$\omega(\to\pi^0\gamma)\pi^0$	0.88(02)	$\omega 2\pi^+2\pi^-$	0.01(00)
$\omega(\to npp)3\pi$	0.17(03)	$\eta\phi$	0.41(02)
$\omega\eta\pi^0$	0.24(05)	$\phi\to$ unaccounted	0.04(04)
Total:	543.21(2.09)	Total:	55.10(96)

G-PARITY-AMBIGUOUS CONTRIBUTIONS (KNT19 exclusive-mode region)

- **$K^+K^- + K_SK_L$:**
  - I=0+1 $a_\mu^{LO,HVP}$, 36.07(29)x10^{-10}: too large for useful maximally conservative separation
  - BaBar PRD98 (2018) 032010 $\tau\to\bar{K}K\upsilon_\tau$ data (I=1, V) + CVC: 0.764(28)x10^{-10} for I=1 up to 2.76 GeV^2; maximally conservative I=0/1 separation: 0.089(89) x10^{-10} for I=1 from 2.76 GeV^2 to KNT19 exclusive-mode endpoint (1.937 GeV)^2 ⇒

$$[a_\mu^{\rm sconn+disc}]_{K\bar{K}} \equiv [a_\mu^{I=0}]_{K\bar{K}} - \frac{1}{9}[a_\mu^{I=1}]_{K\bar{K}} = [a_\mu^{\rm LO,HVP}]_{K\bar{K}} - \frac{10}{9}[a_\mu^{I=1}]_{K\bar{K}} = 35.12(31)\mathrm{x}10^{-10}$$

- **$K\bar{K}\pi$:** KNT19 I=0+1 contribution 2.71(12)x10^{-10}, I=1 contribution 0.74(12)x10^{-10} from BaBar PRD77 (2008) 092002 Dalitz plot I=0/1 separation analysis ⇒

$$[a_\mu^{\rm sconn+disc}]_{K\bar{K}\pi} = [a_\mu^{\rm LO,HVP}]_{K\bar{K}\pi} - \frac{10}{9}[a_\mu^{I=1}]_{K\bar{K}\pi} = 1.89(18)\times 10^{-10}$$

G-PARITY-AMBIGUOUS CONTRIBUTIONS (KNT19 exclusive-mode region) (II)

- **$K\bar{K}\pi\pi$:**
  - G-parity-unambiguous G = -, I=0 $\varphi[\rightarrow K\bar{K}]\ \pi\pi$ contribution: $0.16(1)\times10^{-10}$
  - maximally conservative assessment for the remaining G-parity-ambiguous part (KNT19 data and correlations): $0.79(98)\times10^{-10}$
  - $\Rightarrow$ $\left[a_\mu^{\mathrm{sconn+disc}}\right]_{K\bar{K}2\pi} = 0.95(98)\times10^{-10}$ **(dominant G-parity-ambiguous uncertainty)**

- **All other KNT19 G-parity-ambiguous modes:**
  - $K\bar{K}3\pi$, $\omega(\rightarrow npp)K\bar{K}$ (no φ), $\eta(\rightarrow npp)K\bar{K}$ (no φ), $p\bar{p}$, $n\bar{n}$, low-s $\pi\gamma$, $\eta\gamma$
  - total contribution: $0.10(8)\times10^{-10}$

RESULTS (I): NEGLECTING ISOSPIN-BREAKING CORRECTIONS

- **With KNT19 exclusive-mode input**
  - $a_\mu^{sconn+disc} = 39.08(1.44)\mathrm{x}10^{-10}$
  - Lattice results, a_μ^{sconn}:

    2020 WP average: $53.2(3)\mathrm{x}10^{-10}$

    2020 BMW: $53.39(11)\mathrm{x}10^{-10}$
  - ⇒ disconnected contribution (with naïve WP/BMW a_μ^{sconn} average)

    $[a_\mu^{disc}]_{IL} = -14.3(1.4)\ \mathrm{x}10^{-10}$

- **With DHMZ20 exclusive-mode input**
  - $a_\mu^{sconn+disc} = 37.76(1.39)(1.33)_{lin}\mathrm{x}10^{-10}$
  - Lattice results, a_μ^{sconn}:

    2020 WP average: $53.2(3)\mathrm{x}10^{-10}$

    2020 BMW: $53.39(11)\mathrm{x}10^{-10}$
  - ⇒ disconnected contribution (with naïve WP/BMW a_μ^{sconn} average)

    $[a_\mu^{disc}]_{IL} = -15.6(1.4)(1.3)_{lin}\ \mathrm{x}10^{-10}$

ISOSPIN-BREAKING CORRECTIONS

➢ To first order in IB

- ❖ strong IB (SIB) only in a_μ^{MI}, EM in all of a_μ^{MI}, $a_\mu^{I=1}$, $a_\mu^{I=0}$
- ❖ EM+SIB a_μ^{MI} contribution "contaminates" nominal I=1/0 exclusive mode assignments

➢ Experimental input on a_μ^{MI}

- ❖ Expect dominant contributions from low-s resonance region (2π, 3π, $K\bar{K}$)
- ❖ **2π exclusive mode:** ρ-ω interference region IB contribution from Colangelo, Hoferichter, Stoffer dispersive analysis: $[a_\mu^{MI}]_{2\pi} = 3.65(67)\text{x}10^{-10}$

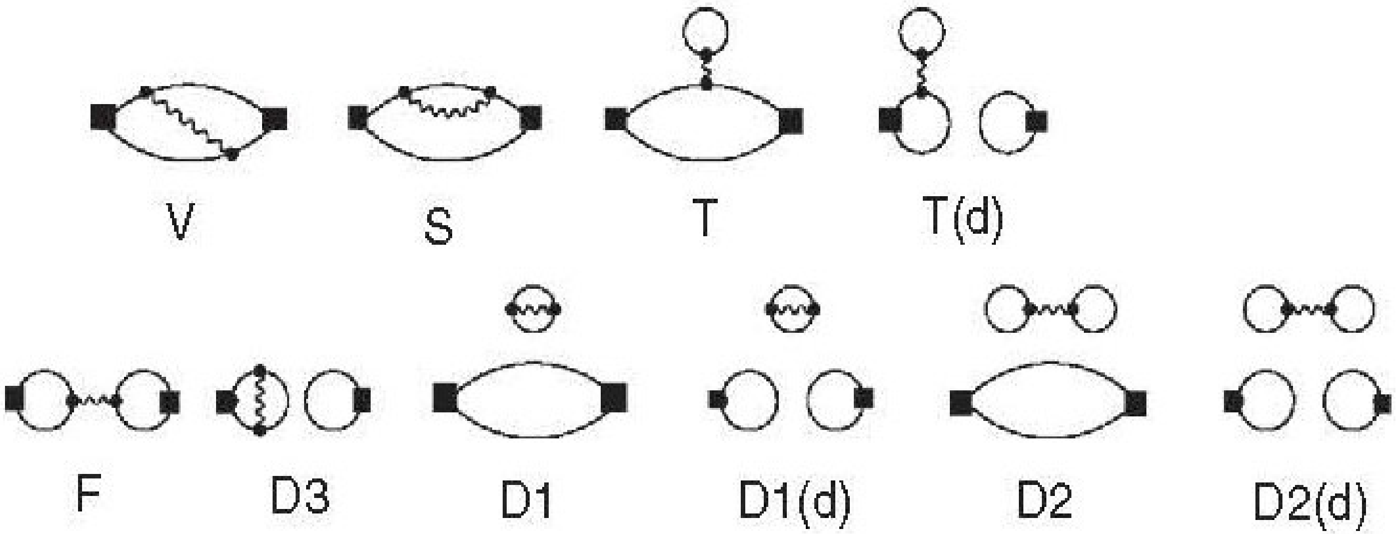
  [Colangelo, Hoferichter, Kubis, Stoffer 2208.08993 update: 3.68(17) $\text{x}10^{-10}$] **[P. Stoffer talk]**
- ❖ **3π exclusive mode:** IB ρ→3π contribution from BaBar VMD model fit to PRD104 (2021) 112003 BaBar 3π cross sections: $[a_\mu^{MI}]_{3\pi} = -0.56(12)\text{x}10^{-10}$ rough estimate
- ❖ **other (higher-s) exclusive modes:** with narrow-resonance-enhanced 2π, 3π IB ~1% of corresponding IC contributions, assign ~1% of IC sums ($\pm 0.40\text{x}10^{-10}$ in total) as uncertainty from missing MI contributions of other exclusive modes
- ❖ **Resulting estimated MI $a_\mu^{sconn+disc}$ correction:**

$$-[-0.56(12) - (1/9)\, 3.65(67) \pm 0.40]\,\text{x}10^{-10} = 0.97(14)(40)\text{x}10^{-10}$$

MORE ON IB CORRECTIONS: EM CORRECTIONS TO $a_\mu^{I=1}$, $a_\mu^{I=0}$

- **EM valence-valence (vv), valence-sea (vs), sea-sea (ss) connected (c) and disconnected (d) contributions**

(vv,c) (vv,c) (vs,c) (vs,d)



(vv,d) (vv,d) (ss,c) (ss,d) (ss,c) (ss,d)

BMW 2020 results

$$[a_\mu^{\rm LO,HVP}]_{(vv,c)} = -1.23(40)(31)\times 10^{-10}\ (V+S),$$

$$[a_\mu^{\rm LO,HVP}]_{(vv,d)} = -0.55(15)(10)\times 10^{-10}\ (F+D3),$$

$$[a_\mu^{\rm LO,HVP}]_{(vs,c)} = -0.0093(86)(95)\times 10^{-10}\ (T),$$

$$[a_\mu^{\rm LO,HVP}]_{(vs,d)} = 0.011(24)(14)\times 10^{-10}\ (T(d)),$$

$$[a_\mu^{\rm LO,HVP}]_{(ss,c)} = 0.37(21)(24)\times 10^{-10}\ (D1+D2),$$

$$[a_\mu^{\rm LO,HVP}]_{(ss,d)} = -0.040(33)(21)\times 10^{-10}\ (D1(d)+D2(d))$$

NOTE smallness of (vs,c), (vs,d), (ss,d) contributions (vanish in $SU(3)_F$ limit)

CANCELLATIONS IN THE EM $a_\mu^{\rm sconn+disc}$ CONTRIBUTIONS

EM (vv,c) contributions

- with all charge factors made explicit, everything else in "loop factors":

$$\left[a_\mu^{\rm LO,HVP}\right]_{(vv,c)} = \left[17\bar{L}_\ell^{(vv,c)} + \bar{L}_s^{(vv,c)}\right]/81$$

$$\left[a_\mu^{I=1}\right]_{(vv,c)} = 5\bar{L}_\ell^{(vv,c)}/36$$

$$\left[a_\mu^{I=0}\right]_{(vv,c)} = \left[5\bar{L}_\ell^{(vv,c)} + 4\bar{L}_s^{(vv,c)}\right]/324$$

- $SU(3)_F$-breaking factor: $x_{(vv,c)} \equiv \bar{L}_s^{(vv,c)}/\bar{L}_\ell^{(vv,c)}$

- Strong cancellation in *sconn+disc* combination c.f. LO, HVP

$$[a_\mu^{\rm sconn+disc}]_{(vv,c)}/[a_\mu^{\rm LO,HVP}]_{(vv,c)} = \frac{1}{17}\frac{x_{(vv,c)}}{1+(x_{(vv,c)}/17)}$$

- E.g., for $0 \leq x_{(vv,c)} \leq 1$: $-0.068(22)(17)\times 10^{-10} \leq [a_\mu^{\rm sconn+disc}]_{(vv,c)} \leq 0$

CANCELLATIONS IN THE EM $a_\mu^{\text{sconn+disc}}$ CONTRIBUTIONS (II)

- **EM (vv,d), unsuppressed diagram F contributions**

$$\left[a_\mu^{\text{LO,HVP}}\right]_{(vv,d)} = \left[25\overline{LL}^{(F)}_{\ell,\ell} + 10\overline{LL}^{(F)}_{\ell,s} + \overline{LL}^{(F)}_{s,s}\right]/81$$

$$\left[a_\mu^{I=1}\right]_{(vv,d)} = \overline{LL}^{(F)}_{\ell,\ell}/4$$

$$\left[a_\mu^{I=0}\right]_{(vv,d)} = \left[\overline{LL}^{(F)}_{\ell,\ell} + 4\overline{LL}^{(F)}_{\ell,s} + 4\overline{LL}_{s,s}\right]/324$$

$$x_F \equiv \overline{LL}^{(F)}_{\ell,s}/\overline{LL}^{(F)}_{\ell,\ell} \qquad y_F \equiv \overline{LL}^{(F)}_{s,s}/\overline{LL}^{(F)}_{\ell,\ell}$$

$$\left[a_\mu^{\text{sconn+disc}}\right]_{(vv,d)}/\left[a_\mu^{\text{LO,HVP}}\right]_{(vv,d)} = -\left[\frac{2-x_F-y_F}{25+10x_F+y_F}\right]$$

$$0 \leq \left[a_\mu^{\text{sconn+disc}}\right]_{(vv,d)} \leq 0.044(12)(8)\times 10^{-10}$$ for $0 \leq x_F, y_F \leq 1$

CANCELLATIONS IN THE EM $a_\mu^{\text{sconn+disc}}$ CONTRIBUTIONS (III)

- **EM (ss,c), unsuppressed diagram D1 contributions**

$$\left[a_\mu^{LO,HVP}\right]_{(ss,c)} = \left[5\overline{LL}_\ell^{(D1)} + \overline{LL}_s^{(D1)}\right]/9$$

$$\left[a_\mu^{I=1}\right]_{(ss,c)} = \overline{LL}_\ell^{(D1)}/2$$

$$\left[a_\mu^{I=0}\right]_{(ss,c)} = \left[\overline{LL}_\ell^{(D1)} + 2\overline{LL}_s^{(D1)}\right]/18$$

$$x_{D1} \equiv \overline{LL}_s^{(D1)}/\overline{LL}_\ell^{(D1)}$$

$$\left[a_\mu^{\text{sconn+disc}}\right]_{(ss,c)}/\left[a_\mu^{\text{LO,HVP}}\right]_{(ss,c)} = \frac{1}{5}\frac{x_{D1}}{1+(x_{D1}/5)}$$

$\Rightarrow$ $0 \leq \left[a_\mu^{\text{sconn+disc}}\right]_{(ss,c)} \leq 0.062(35)(40)\times 10^{-10}$ for $0 \leq x_{D1} \leq 1$

IB-CORRECTED RESULTS

(applying MI correction, neglecting tiny $a_\mu^{sconn+disc} = a_\mu^{I=0} - a_\mu^{I=1}/9$ EM corrections)

With KNT19 exclusive-mode input

- $a_\mu^{sconn+disc} = 40.1(1.4)(0.4)\times10^{-10}$
- $\Rightarrow$ disconnected contribution (with naïve WP/BMW a_μ^{sconn} average)

  $a_\mu^{disc} = -13.3(1.4)(0.4)\ \times10^{-10}$

With DHMZ20 exclusive-mode input

- $a_\mu^{sconn+disc} = 38.7(1.4)(1.3)_{lin}(0.4)\times10^{-10}$
- $\Rightarrow$ disconnected contribution (with naïve WP/BMW a_μ^{sconn} average)

  $[a_\mu^{disc}]_{IL} = -14.6(1.4)(1.3)_{lin}\ (0.4)\times10^{-10}$

Source	n_f	$a_\mu^{disc}\times10^{10}$	$a_\mu^{sconn+disc}\times10^{10}$
RBC/UKQCD [4,11]	$2+1$	$-11.2(3.3)(2.3)$	$42.0(3.3)(2.3)$
BMW [9]	$2+1+1$	$-12.8(1.1)(1.6)$	$40.9(1.2)(1.7)$
Mainz [18]	$2+1$	$-23.2(2.2)(4.5)$	$31.3(3.3)(4.5)$
BMW [20]	$2+1+1$	$-13.36(1.18)(1.36)$	$40.03(1.18)(1.36)$
This work, Eq. (5.30)	$2+1$	$-13.3(1.4)(0.4)$	$40.1(1.4)(0.4)$
This work, Eq. (6.2)	$2+1$	$-14.6(1.4)(1.3)_{lin}(0.4)$	$38.7(1.4)(1.3)_{lin}(0.4)$

SUMMARY/CONCLUSIONS

- $a_\mu^{sconn+disc}$, a_μ^{disc} in agreement with most precise lattice results, comparable precision
- Contrast: tension with Mainz result (m_π extrapolation required)
- Confirms light-quark-connected component as source of lattice-dispersive tension
- Back-up slide for analogous data-based light-quark-connected determination
- Analogous data-based *sconn+disc, lqc* contributions for window quantities also feasible (with s-dependent exclusive-mode distributions)

BACK-UP: ANALOGOUS LIGHT-QUARK-CONNECTED DETERMINATION

- **PRELIMINARY ASSESSMENT**
- **In IL, $a_\mu^{lqc} = \frac{10}{9}\, a_\mu^{I=1}$**
- ***KNT19/DHMZ nominal IL $a_\mu^{I=1}$ = $574.3(2.3)(0.1)_{DV} \times 10^{-10}$ / $577.2(3.4)(1.1)_{lin}(0.2)_{DV} \times 10^{-10}$***
- ***IB-corrected $a_\mu^{I=1}$ as per treatment above, with***
  - *removal of MI ππ contamination from nominal I=1 contribution*
    [$3.68(17) \times 10^{-10}$] ***[P. Stoffer talk]***
  - *analogous EM $a_\mu^{I=1}$ correction estimate, here using BMW lattice numerics*
    [estimated EM $a_\mu^{I=1}$ contribution $-0.85(56) \times 10^{-10}$]

  - ***⇒ (mildly-lattice-EM-dependent) estimates:***
    KNT19: a_μ^{lqc} = $635.0(2.7)(0.1)_{DV} \times 10^{-10}$
    DHMZ: a_μ^{lqc} = $638.2(3.9)(1.1)_{lin}(0.2)_{DV} \times 10^{-10}$