Kaon electromagnetic form factors in dispersion theory

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HISKP (Theorie)
Bonn University

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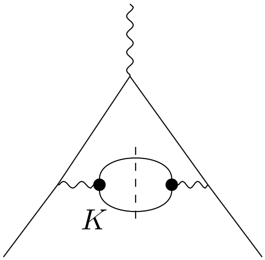
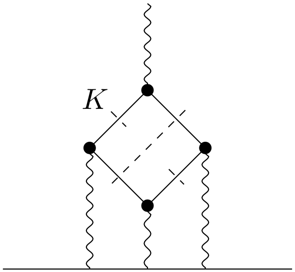
[DS, Hariharan, Hoferichter, Kubis, Stoffer; Eur. Phys. J. C **82** (2022) 432]

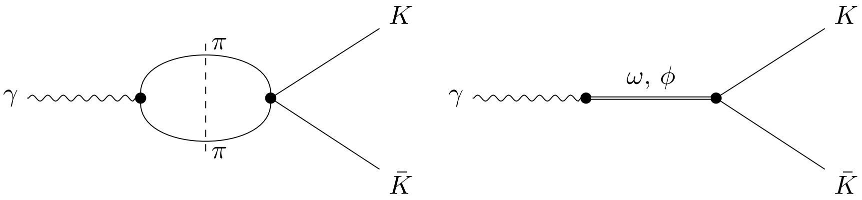
Motivation

- kaon HVP and HLbL contribution to $g-2$ of the muon
- kaon form factor
- global analysis of time- and spacelike data





Formalism



- dispersive representation
- $\pi\pi \to \bar{K}K$ partial-wave amplitude as input
- decompose in isovector (v) and isoscalar (s) part

$$F_{K^\pm} = F_K^s(s) + F_K^v(s)$$
$$F_{K^0} = F_K^s(s) - F_K^v(s)$$

Pion vector form factor

- unitarity relation: $\pi\pi$ intermediate states

$$\text{Im} F_\pi^V(s) = F_\pi^V(s) \sin \delta(s) e^{-i\delta(s)} \theta(s - s_{\text{th}})$$

- solution: Omnès function [Omnès (1958)]

$$\Omega(s) = \exp\left(\frac{s}{\pi} \int_{s_{\text{th}}}^{\infty} \mathrm{d}s' \frac{\delta(s')}{s'(s'-s)} \right)$$
$$F_\pi^V(s) = P(s)\Omega(s)$$

Isovector part

- unitarity relation for isovector part

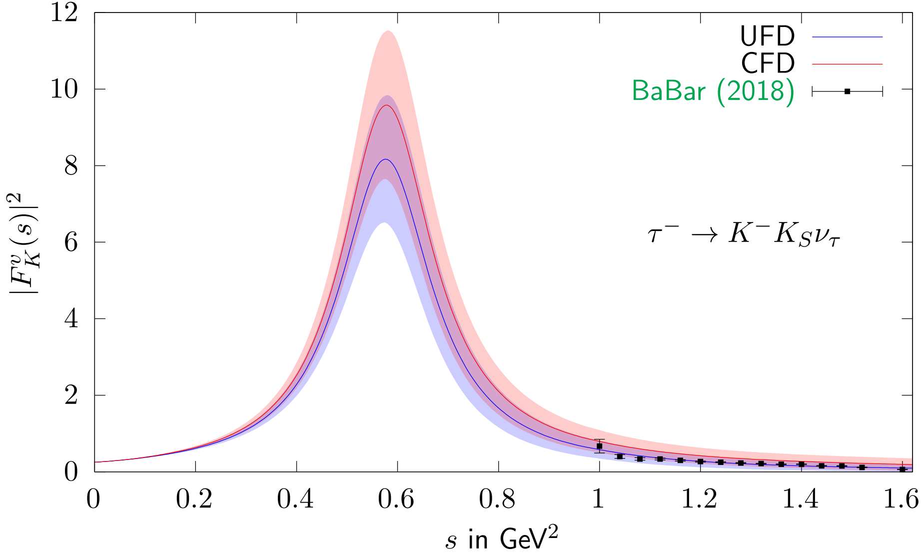
$$\mathrm{Im} F_K^v(s) = \frac{s}{4\sqrt{2}} \sigma_\pi^3(s) (g_1^1(s))^* F_\pi^V(s)$$

- $g_1^1(s)$: $\pi\pi \to \bar{K}K$ P-wave amplitude [Peláez and Rodas (2018)]
- $F_\pi^V(s)$: pion vector form factor
- compute pion vector form factor with phase from $\pi\pi \to \bar{K}K$ to render imaginary part real by construction
- fix polynomial with BaBar (2009) data
- dispersion integral

$$F_K^v(s) = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \mathrm{d}s' \frac{\mathrm{Im} F_K^v(s')}{s' - s}$$

- add BW ρ' to fix normalization to $F_K^v(0) = 1/2$

Isovector part



Isoscalar part

- dominant contributions from 3π and $\bar{K}K$
- VMD ansatz with isoscalar vector resonances
- small widths of $\omega(782)$ and $\phi(1020)$
- ω' pole to guarantee correct normalization

$$F_K^s(s) = \frac{c_\phi}{3}\frac{M_\phi^2}{M_\phi^2 - s - i\sqrt{s}\Gamma_\phi(s)} + \frac{c_\omega}{6}\frac{M_\omega^2}{M_\omega^2 - s - iM_\omega\Gamma_\omega} + \left(\frac{1}{2} - \frac{c_\phi}{3} - \frac{c_\omega}{6}\right)\frac{M_{\omega'}^2}{M_{\omega'}^2 - s - iM_{\omega'}\Gamma_{\omega'}}$$

- energy-dependent width for ϕ resonance
- SU(3)-symmetric limit corresponds to $c_\phi = c_\omega = 1$

Data

- timelike data:
  - charged: (CMD-2 (2008)), CMD-3 (2018), SND (2001), BaBar (2013)
  - neutral: CMD-2 (2004), CMD-3 (2016), SND (2001)
- spacelike data (charged): Dally et al. (1980), Amendolia et al. (1986)
- remove vacuum polarization effects [KNT (2018)]
- Born cross section

$$\sigma^{(0)}(s) = \frac{\pi\alpha^2}{3s}\sigma_K^3(s)|F_K(s)|^2$$

- final-state radiation by $1 + \frac{\alpha}{\pi}\eta(s)$
- $\eta(s)$ known from pion processes

Fits

- include full covariance matrices (systematic + statistical)
- chi-squared minimization

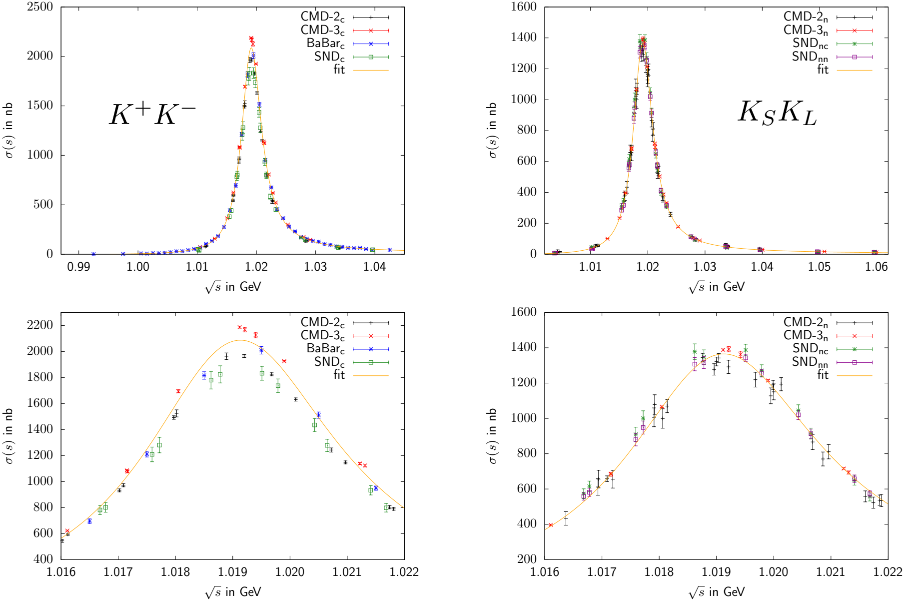
$$\chi^2 = \sum_{i,j} (f(x_i) - y_i) V(i,j)^{-1} (f(x_j) - y_j)$$

- remove d'Agostini bias [d'Agostini (1994)] by iterative method [NNPDF (2010)]

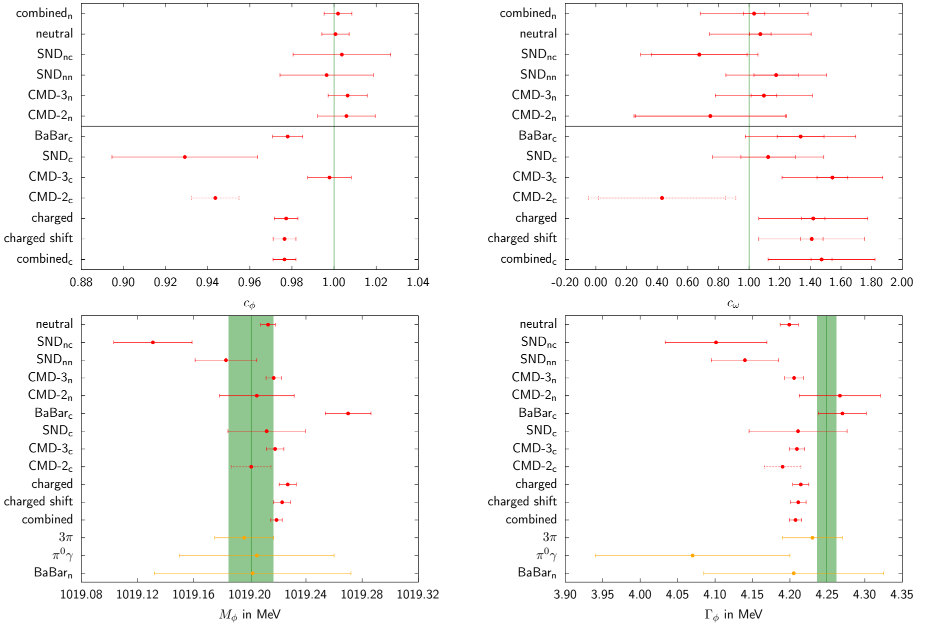
$$V_{n+1}(i,j) = V^{\text{stat}}(i,j) + \frac{V^{\text{syst}}(i,j)}{y_i y_j} f_n(x_i) f_n(x_j)$$

- inflate uncertainty of parameters by scale factor $S = \sqrt{\chi^2/\text{dof}}$ [PDG (2020)]

Fits


K^+K^-
K_SK_L
$\sigma(s)$ in nb
$\sqrt{s}$ in GeV
CMD-2$_c$
CMD-3$_c$
BaBar$_c$
SND$_c$
fit
CMD-2$_n$
CMD-3$_n$
SND$_{nc}$
SND$_{nn}$
fit
2500
2000
1500
1000
500
0
0.99
1.00
1.01
1.02
1.03
1.04
1600
1400
1200
1000
800
600
400
200
0
1.01
1.02
1.03
1.04
1.05
1.06
2200
2000
1800
1600
1400
1200
1000
800
600
400
1.016
1.017
1.018
1.019
1.020
1.021
1.022
1600
1400
1200
1000
800
600
400
200
1.016
1.017
1.018
1.019
1.020
1.021
1.022

Fits


combined$_n$
neutral
SND$_{nc}$
SND$_{nn}$
CMD-3$_n$
CMD-2$_n$
BaBar$_c$
SND$_c$
CMD-3$_c$
CMD-2$_c$
charged
charged shift
combined$_c$
0.88 0.90 0.92 0.94 0.96 0.98 1.00 1.02 1.04
c_ϕ
-0.20 0.00 0.20 0.40 0.60 0.80 1.00 1.20 1.40 1.60 1.80 2.00
c_ω
combined
3π
$\pi^0\gamma$
BaBar$_n$
1019.08 1019.12 1019.16 1019.20 1019.24 1019.28 1019.32
M_ϕ in MeV
3.90 3.95 4.00 4.05 4.10 4.15 4.20 4.25 4.30 4.35
Γ_ϕ in MeV

HLbL

- HLbL contribution [Aoyama et al. (2020)]

$$a_\mu^{\text{HLbL}} = 90(17) \times 10^{-11}$$

- $\bar{K}K$ appreciably suppressed compared to $\pi\pi$
- fully determined by kaon electromagnetic form factors
- minor importance for $(g-2)_\mu$

	$a_\mu^{K^\pm\text{-box}} \times 10^{11}$	$a_\mu^{K^0\text{-box}} \times 10^{15}$
VMD [Aoyama et al. (2020)]	-0.50	-1.2
DS [Eichmann et al. (2020)]	$-0.48(2)(4)$	
DS [Miramontes et al. (2022)]	$-0.48(4)$	
This work	$-0.484(5)(10)$	$-0.5(2)(4)$

HVP

- master formula [Bouchiat and Michel (1961); Brodsky and de Rafael (1968)]

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{M_{\pi^0}^2}^{\infty} \mathrm{d}s \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s)$$

$$R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons})$$

$\sqrt{s} \leq 1.05$ GeV	$K^+K^- \times 10^{-11}$	$K_S K_L \times 10^{-11}$
[KNT (2018, 2020)]	181.2(1.7)	119.7(1.8)
combined	184.5(2.0)	118.3(1.5)
BaBar	182.5(2.2)	
CMD-2		117.2(2.4)
CMD-3	192.6(3.9)	119.5(2.2)
SND	166.7(11.9)	119.0(5.1) 121.1(5.0)

HVP

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combined	$184.5(2.0)$	$118.3(1.5)$

- for neutral channel less than 1σ
- for charged channel discrepancy greater
- manifestation of tension between BaBar and CMD-3 (2.3σ)

HVP

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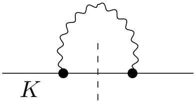
- for neutral channel less than 1σ
- for charged channel discrepancy greater
- manifestation of tension between BaBar and CMD-3 (2.3σ)
- our approach: constrained to follow shape of ϕ resonance
- local averages can obfuscate global shape if inconsistencies among the data sets
- fit of our representation to [KNT (2018, 2020)] gives $\chi^2/\text{dof} > 2$

Corrections to Dashen's theorem

- precise electromagnetic mass difference for kaons
  $\rightarrow$ extraction of quark mass difference
- large corrections to Dashen's theorem



$$(\Delta M_K^2)_{\text{EM}} = (\Delta M_\pi^2)_{\text{EM}} + \mathcal{O}(e^2 m_q)$$

$$\epsilon = \frac{(\Delta M_K^2)_{\text{EM}}}{(\Delta M_\pi^2)_{\text{EM}}} - 1$$

- use Cottingham formula with kaon form factor and pion form factor
  [Colangelo et al. (2019)]

$$(\Delta M_K^2)_{\text{EM}} = 2.12(18) \times 10^{-3}\text{GeV}^2$$
$$(\Delta M_\pi^2)_{\text{EM}} = 1.3(3) \times 10^{-3}\text{GeV}^2$$

- comparable with lattice results

$$\epsilon = 0.63(40)$$

lattice: $\epsilon = 0.79(6), 0.73(17)$ [FLAG Review 2021]

Isospin breaking in HVP

- define isospin limit with previous result
  [discussion by Hoferichter on Monday]

$$M_{K^+} = (494.58 - 3.05_\delta + 2.14_{e^2})\text{MeV}$$
$$M_{K^0} = (494.58 + 3.03_\delta)\text{MeV}$$

- significant isospin breaking in residues

$$c_\phi^{K^+K^-} = 0.976(5)\,, \quad c_\phi^{\bar{K}^0K^0} = 1.002(7)$$

- corresponding shift:

in 10^{-11}	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
kaon mass (K^+K^-)	$-32.4(1.7)$	$49.8(2.6)$
kaon mass $(\bar{K}^0K^0)$	$-0.2(0)$	$-46.2(2.3)$

Conclusion

- kaon form factor fit to available space- and timelike data
- isovector part constrained by $\pi\pi \to \bar{K}K$ scattering and pion VFF
- applications:
  - charge radii

$$\langle r^2 \rangle_\mathsf{n} = -0.060(4)\mathsf{fm}^2 \qquad \langle r^2 \rangle_\mathsf{c} = 0.359(3)\mathsf{fm}^2$$

  - corrections to Dashen's theorem
  - small HLbL contribution in agreement with previous calculations
  - $> 1\sigma$ shift in HVP due to tension in underlying charged kaon data

Spares | Spacelike region