The interplay of axial and pseudoscalar mesons in the HLbL

Pablo Sanchez-Puertas
psanchez@ifae.es

IFAE & BIST (UAB, Barcelona), University of Granada (Granada), Spain

Based on J. Phys. G49 (2022) 015002, in Coll. with P. Masjuan & P. Roig

5^{th} plenary workshop of the Muon g-2 Theory initiative, Edinburgh

Section 1

Motivation

Current HLbL status

- Dispersive foundations for analytic approaches
  - The HLbL $(g-2)$ kernel enhances light degrees of freedom
  - $\{\pi, \eta, \eta'\}$-poles: main contribution $\Rightarrow$ good control
  - $\pi\pi, KK$: boxes + S-wave rescattering $\Rightarrow$ good control
- Above $2h$ states less known: a resonance approach has been suggested
  - Axial-vector mesons numerically relevant (+ th. *funny* things)
- Well above, SD constraints cannot be satisified with finite meson exchanges
  - Could we use SD to better understand multihadron states?

Our Focus: Melnikov-Vainshtein OPE constraint (MV)[1]

[1] K. Melnikov and A. Vainshtein, PRD 70 (2004) 11300

Section 2

MV constraint and $\langle VVA \rangle$

Melnikov-Vainshtein SD constraint

- The HLbL tensor is defined as

$$\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3)=\int d^4x d^4y d^4z e^{i(q_1\cdot x+q_2\cdot y+q_3\cdot z)}\times\langle 0|\,T\{j^\mu(x)j^\nu(y)j^\lambda(z)j^\sigma(0)\}\,|0\rangle$$

- What happens if $q_1^2\sim q_2^2\sim(\frac{q_1-q_2}{2})^2\equiv\hat{q}^2\gg\{(q_1+q_2)^2,q_{3,4}^2,\Lambda_{\mathrm{QCD}}^2\}$?

Melnikov-Vainshtein SD constraint

- The HLbL tensor is defined as

$$\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3)=\int d^4x d^4y d^4z e^{i(q_1\cdot x+q_2\cdot y+q_3\cdot z)}\times\langle 0|\,T\{j^\mu(x)j^\nu(y)j^\lambda(z)j^\sigma(0)\}\,|0\rangle$$

- What happens if $q_1^2\sim q_2^2\sim(\frac{q_1-q_2}{2})^2\equiv\hat{q}^2\gg\{(q_1+q_2)^2,q_{3,4}^2,\Lambda_{\rm QCD}^2\}$?
- Use the LO OPE $\Rightarrow$ insert it in the HLbL above ($j_5^\mu=\bar{q}\gamma^\mu\gamma^5\mathcal{Q}^2q$)

$$\Pi^{\mu\nu\lambda\sigma}_{\rm OPE}(q_1,q_2,q_3)=\frac{2i\epsilon^{\mu\nu\alpha\hat{q}}}{\hat{q}^2}i\int d^4z d^4w e^{i(q_{12}\cdot w+q_3\cdot z)}\langle 0|\,T\{j_{5\alpha}(w)j^\lambda(z)j^\sigma(0)\}\,|0\rangle$$

Melnikov-Vainshtein SD constraint

- The HLbL tensor is defined as

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \int d^4x d^4y d^4z e^{i(q_1\cdot x + q_2 \cdot y + q_3 \cdot z)} \times \langle 0 | T\{j^\mu(x) j^\nu(y) j^\lambda(z) j^\sigma(0)\} |0\rangle$$

- What happens if $q_1^2 \sim q_2^2 \sim (\frac{q_1 - q_2}{2})^2 \equiv \hat{q}^2 \gg \{(q_1+q_2)^2, q_{3,4}^2, \Lambda^2_{\mathrm{QCD}}\}$?
- Use the LO OPE $\Rightarrow$ insert it in the HLbL above ($j_5^\mu = \bar{q}\gamma^\mu\gamma^5 \mathcal{Q}^2 q$)

$$\Pi^{\mu\nu\lambda\sigma}_{\mathrm{OPE}}(q_1, q_2, q_3) = \frac{2i\epsilon^{\mu\nu\alpha\hat{q}}}{\hat{q}^2} i \int d^4z d^4w e^{i(q_{12}\cdot w + q_3\cdot z)} \langle 0 | T\{j_{5\alpha}(w) j^\lambda(z) j^\sigma(0)\} |0\rangle$$

- Relates the HLbL to the $\langle VVA \rangle$ Green's function

$$i \int d^4x d^4y e^{i(q_1\cdot x + q_2\cdot y)} \langle 0 | T\{V^a_\mu(x) V^b_\nu(y) A^c_\rho(0)\} |0\rangle \equiv \mathcal{W}^{abc}_{\mu\nu\rho}(q_1, q_2) = \mathcal{W}_{\mu\nu\rho}(q_1, q_2)\, \mathrm{tr}(t^c\{t^a, t^b\})$$

Melnikov-Vainshtein SD constraint

- The HLbL tensor is defined as

$$\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3) = \int d^4x d^4y d^4z e^{i(q_1\cdot x+q_2\cdot y+q_3\cdot z)} \times \langle 0|\, T\{j^\mu(x) j^\nu(y) j^\lambda(z) j^\sigma(0)\}\, |0\rangle$$

- What happens if $q_1^2 \sim q_2^2 \sim (\frac{q_1-q_2}{2})^2 \equiv \hat{q}^2 \gg \{(q_1+q_2)^2, q_{3,4}^2, \Lambda_{\rm QCD}^2\}$?
- Use the LO OPE $\Rightarrow$ insert it in the HLbL above ($j_5^\mu = \bar{q}\gamma^\mu\gamma^5\mathcal{Q}^2 q$)

$$\Pi^{\mu\nu\lambda\sigma}_{\rm OPE}(q_1,q_2,q_3) = \frac{2i\epsilon^{\mu\nu\alpha\hat{q}}}{\hat{q}^2} i \int d^4z d^4w e^{i(q_{12}\cdot w+q_3\cdot z)} \langle 0|\, T\{j_{5\alpha}(w) j^\lambda(z) j^\sigma(0)\}\, |0\rangle$$

- Relates the HLbL to the $\langle VVA \rangle$ Green's function

$$i\int d^4x d^4y e^{i(q_1\cdot x+q_2\cdot y)} \langle 0|\, T\{V_\mu^a(x) V_\nu^b(y) A_\rho^c(0)\}\, |0\rangle \equiv \mathcal{W}^{abc}_{\mu\nu\rho}(q_1,q_2) = \mathcal{W}_{\mu\nu\rho}(q_1,q_2)\,\mathrm{tr}(t^c\{t^a,t^b\})$$

In chiral (large-N_c) limit, longitudinal part known exactly $\Rightarrow$ MV remarks
Explore (hadronic) implications in the remainder of the talk

Transverse dof in $\langle VVA \rangle$ and relation with anomaly

- Choosing a basis for $\langle VVA \rangle$ (free of kin. singularities)[2]

$$\mathcal{W}^{\mu\nu\rho}(q_1,q_2) = \epsilon^{\mu\nu q_1 q_2} q_{12}^{\rho} C_S + \epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha} q_2^{\nu} - g_{\alpha}^{\nu} q_2^2) B_2 + \epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha} q_1^{\mu} - g_{\alpha}^{\mu} q_1^2) \bar{B}_2 + \epsilon^{\mu\nu q_1 q_2} \bar{q}_{12}^{\rho} C_A$$

- Since anomaly known exactly, sensible to isolate it $[B_2(\bar{B}_2) = B_{2S} \pm B_{2A}]$

$$q_{12}^{\rho} \mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \epsilon_{\mu\nu q_1 q_2}\left[q_{12}^2 C_S + (q_1^2+q_2^2) B_{2S} + (q_1^2 - q_2^2)(C_A - B_{2A})\right] = \epsilon_{\mu\nu q_1 q_2} \frac{N_c}{4\pi^2}$$

[2] P. Roig, PSP PRD101 (2020) 074019 and P. Masjuan, P. Roig, PSP JPG49 (2022) 015002

Transverse dof in $\langle VVA \rangle$ and relation with anomaly

- Choosing a basis for $\langle VVA \rangle$ (free of kin. singularities)[2]

$$\mathcal{W}^{\mu\nu\rho}(q_1,q_2) = \epsilon^{\mu\nu q_1 q_2} q_{12}^{\rho} C_S + \epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha} q_2^{\nu} - g_{\alpha}^{\nu} q_2^2) B_2 + \epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha} q_1^{\mu} - g_{\alpha}^{\mu} q_1^2)\bar{B}_2 + \epsilon^{\mu\nu q_1 q_2} \bar{q}_{12}^{\rho} C_A$$

- Since anomaly known exactly, sensible to isolate it $[B_2(\bar{B}_2) = B_{2S} \pm B_{2A}]$

$$q_{12}^{\rho}\mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \epsilon_{\mu\nu q_1 q_2}\left[q_{12}^2 C_S + (q_1^2+q_2^2)B_{2S} + (q_1^2-q_2^2)(C_A - B_{2A})\right] = \epsilon_{\mu\nu q_1 q_2}\frac{N_c}{4\pi^2}$$

- Note the $q_{12}^2 \to 0$ limit (real photons $\leftrightarrow \pi^0 \to \gamma\gamma$) ($X^0 = \lim_{q_{12}^2 \to 0} X$)

$$\lim_{q_{12}^2\to 0} \mathrm{Res}(C_S, q_{12}^2 = 0) + (q_1^2+q_2^2)B_{2S} + (q_1^2-q_2^2)(C_A - B_{2A}) = \frac{N_c}{4\pi^2}$$

[2] P. Roig, PSP PRD101 (2020) 074019 and P. Masjuan, P. Roig, PSP JPG49 (2022) 015002

Transverse dof in $\langle VVA\rangle$ and relation with anomaly

- Choosing a basis for $\langle VVA\rangle$ (free of kin. singularities)[2]

$$\mathcal{W}^{\mu\nu\rho}(q_1,q_2)=\epsilon^{\mu\nu q_1 q_2}q_{12}^{\rho}C_S+\epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha}q_2^{\nu}-g_{\alpha}^{\nu}q_2^2)B_2+\epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha}q_1^{\mu}-g_{\alpha}^{\mu}q_1^2)\bar{B}_2+\epsilon^{\mu\nu q_1 q_2}\bar{q}_{12}^{\rho}C_A$$

- Since anomaly known exactly, sensible to isolate it $[B_2(\bar{B}_2)=B_{2S}\pm B_{2A}]$

$$q_{12}^{\rho}\mathcal{W}_{\mu\nu\rho}(q_1,q_2)=\epsilon_{\mu\nu q_1 q_2}\left[q_{12}^2C_S+(q_1^2+q_2^2)B_{2S}+(q_1^2-q_2^2)(C_A-B_{2A})\right]=\epsilon_{\mu\nu q_1 q_2}\frac{N_c}{4\pi^2}$$

- Note the $q_{12}^2\to 0$ limit (real photons $\leftrightarrow \pi^0\to\gamma\gamma$) ($X^0=\lim_{q_{12}^2\to 0}X$)

$$q_{12}^2\frac{\frac{N_c}{4\pi^2}\tilde{F}_{\mathrm{pGB}VV}(q_1^2,q_2^2)}{q_{12}^2-m_{\mathrm{pGB}}^2}+(q_1^2+q_2^2)B_{2S}^0+(q_1^2-q_2^2)(C_A^0-B_{2A}^0)=\frac{N_c}{4\pi^2}$$

[2] P. Roig, PSP PRD101 (2020) 074019 and P. Masjuan, P. Roig, PSP JPG49 (2022) 015002

Transverse dof in $\langle VVA \rangle$ and relation with anomaly

- Choosing a basis for $\langle VVA \rangle$ (free of kin. singularities)[2]

$$\mathcal{W}^{\mu\nu\rho}(q_1,q_2) = \epsilon^{\mu\nu q_1 q_2} q_{12}^{\rho} C_S + \epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha} q_2^{\nu} - g_{\alpha}^{\nu} q_2^2) B_2 + \epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha} q_1^{\mu} - g_{\alpha}^{\mu} q_1^2)\bar{B}_2 + \epsilon^{\mu\nu q_1 q_2} \bar{q}_{12}^{\rho} C_A$$

- Since anomaly known exactly, sensible to isolate it $[B_2(\bar{B}_2) = B_{2S} \pm B_{2A}]$

$$q_{12}^{\rho} \mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \epsilon_{\mu\nu q_1 q_2}\left[q_{12}^2 C_S + (q_1^2+q_2^2) B_{2S} + (q_1^2-q_2^2)(C_A - B_{2A})\right] = \epsilon_{\mu\nu q_1 q_2} \frac{N_c}{4\pi^2}$$

- Note the $q_{12}^2 \to 0$ limit (real photons $\leftrightarrow \pi^0 \to \gamma\gamma$) ($X^0 = \lim_{q_{12}^2 \to 0} X$)

$$\frac{\frac{N_c}{4\pi^2}\tilde{F}_{\mathrm{pGB}VV}(q_1^2,q_2^2)}{q_{12}^2 - m_{\mathrm{pGB}}^2} + \frac{q_1^2+q_2^2}{q_{12}^2} B_{2S}^0 + \frac{q_1^2-q_2^2}{q_{12}^2}(C_A^0 - B_{2A}^0) = \frac{N_c}{4\pi^2 q_{12}^2} = \frac{w_L}{8\pi^2}$$

[2] P. Roig, PSP PRD101 (2020) 074019 and P. Masjuan, P. Roig, PSP JPG49 (2022) 015002

Transverse dof in $\langle VVA\rangle$ and relation with anomaly

- Choosing a basis for $\langle VVA\rangle$ (free of kin. singularities)[2]

$$\mathcal{W}^{\mu\nu\rho}(q_1,q_2)=\epsilon^{\mu\nu q_1 q_2}q_{12}^{\rho}C_S+\epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha}q_2^{\nu}-g_{\alpha}^{\nu}q_2^2)B_2+\epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha}q_1^{\mu}-g_{\alpha}^{\mu}q_1^2)\bar{B}_2+\epsilon^{\mu\nu q_1 q_2}\bar{q}_{12}^{\rho}C_A$$

- Since anomaly known exactly, sensible to isolate it $[B_2(\bar{B}_2)=B_{2S}\pm B_{2A}]$

$$q_{12}^{\rho}\mathcal{W}_{\mu\nu\rho}(q_1,q_2)=\epsilon_{\mu\nu q_1 q_2}\left[q_{12}^2C_S+(q_1^2+q_2^2)B_{2S}+(q_1^2-q_2^2)(C_A-B_{2A})\right]=\epsilon_{\mu\nu q_1 q_2}\frac{N_c}{4\pi^2}$$

- Note the $q_{12}^2\to 0$ limit (real photons $\leftrightarrow \pi^0\to\gamma\gamma$) ($X^0=\lim_{q_{12}^2\to 0}X$)

$$\frac{\frac{N_c}{4\pi^2}\tilde{F}_{\mathrm{pGB}VV}(q_1^2,q_2^2)}{q_{12}^2-m_{\mathrm{pGB}}^2}+\frac{q_1^2+q_2^2}{q_{12}^2}B_{2S}^0+\frac{q_1^2-q_2^2}{q_{12}^2}(C_A^0-B_{2A}^0)=\frac{N_c}{4\pi^2q_{12}^2}=\frac{w_L}{8\pi^2}$$

- If longitudinal part is separated (later replace C_S in favor of the anomaly)

$$\mathcal{W}^{\mu\nu\rho}=\left\{t_{\mu\nu\rho}^{(+)}(-B_{2S})+t_{\mu\nu\rho}^{(-)}(C_A-B_{2A})+\tilde{t}_{\mu\nu\rho}^{(-)}(B_{2A})-\epsilon_{\mu\nu q_1 q_2}q_{12\rho}\left[-C_S-\frac{q_1^2+q_2^2}{q_{12}^2}B_{2S}-\frac{q_1^2-q_2^2}{q_{12}^2}(C_A-B_{2A})\right]\right\}$$

$$t_{\mu\nu\rho}^{(+)}=\epsilon_{q_1 q_2\mu\rho}q_{1\,\nu}+\epsilon_{q_2 q_1\nu\rho}q_{2\,\mu}-(q_1\cdot q_2)\epsilon_{\mu\nu\rho\bar{q}_{12}}+\frac{q_1^2+q_2^2-q_{12}^2}{q_{12}^2}\epsilon_{\mu\nu q_1 q_2}q_{12\,\rho},\qquad t_{\mu\nu\rho}^{(-)}=\epsilon_{\mu\nu q_1 q_2}\left[\bar{q}_{12\,\rho}-\frac{q_1^2-q_2^2}{q_{12}^2}q_{12\,\rho}\right]$$

[2] P. Roig, PSP PRD101 (2020) 074019 and P. Masjuan, P. Roig, PSP JPG49 (2022) 015002

Transverse dof in $\langle VVA\rangle$ and relation with anomaly

- Choosing a basis for $\langle VVA\rangle$ (free of kin. singularities)[2]

$$\mathcal{W}^{\mu\nu\rho}(q_1,q_2)=\epsilon^{\mu\nu q_1 q_2}q_{12}^{\rho}C_S+\epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha}q_2^{\nu}-g_{\alpha}^{\nu}q_2^2)B_2+\epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha}q_1^{\mu}-g_{\alpha}^{\mu}q_1^2)\bar{B}_2+\epsilon^{\mu\nu q_1 q_2}\bar{q}_{12}^{\rho}C_A$$

- Since anomaly known exactly, sensible to isolate it $[B_2(\bar{B}_2)=B_{2S}\pm B_{2A}]$

$$q_{12}^{\rho}\mathcal{W}_{\mu\nu\rho}(q_1,q_2)=\epsilon_{\mu\nu q_1 q_2}\left[q_{12}^2C_S+(q_1^2+q_2^2)B_{2S}+(q_1^2-q_2^2)(C_A-B_{2A})\right]=\epsilon_{\mu\nu q_1 q_2}\frac{N_c}{4\pi^2}$$

- Note the $q_{12}^2\to 0$ limit (real photons $\leftrightarrow \pi^0\to\gamma\gamma$) $(X^0=\lim_{q_{12}^2\to 0}X)$

$$\frac{\frac{N_c}{4\pi^2}\tilde{F}_{\mathrm{pGB}VV}(q_1^2,q_2^2)}{q_{12}^2-m_{\mathrm{pGB}}^2}+\frac{q_1^2+q_2^2}{q_{12}^2}B_{2S}^0+\frac{q_1^2-q_2^2}{q_{12}^2}(C_A^0-B_{2A}^0)=\frac{N_c}{4\pi^2q_{12}^2}=\frac{w_L}{8\pi^2}$$

- If longitudinal part is separated (later replace C_S in favor of the anomaly)

$$\mathcal{W}^{\mu\nu\rho}=\left\{t_{\mu\nu\rho}^{(+)}(-B_{2S})+t_{\mu\nu\rho}^{(-)}(C_A-B_{2A})+\tilde{t}_{\mu\nu\rho}^{(-)}(B_{2A})+\epsilon_{\mu\nu q_1 q_2}q_{12\rho}\frac{w_L}{8\pi^2}\right\}$$

$$t_{\mu\nu\rho}^{(+)}=\epsilon_{q_1 q_2\mu\rho}q_{1\,\nu}+\epsilon_{q_2 q_1\nu\rho}q_{2\,\mu}-(q_1\cdot q_2)\epsilon_{\mu\nu\rho\bar{q}_{12}}+\frac{q_1^2+q_2^2-q_{12}^2}{q_{12}^2}\epsilon_{\mu\nu q_1 q_2}q_{12\,\rho},\qquad t_{\mu\nu\rho}^{(-)}=\epsilon_{\mu\nu q_1 q_2}\left[\bar{q}_{12\,\rho}-\frac{q_1^2-q_2^2}{q_{12}^2}q_{12\,\rho}\right]$$

[2] P. Roig, PSP PRD101 (2020) 074019 and P. Masjuan, P. Roig, PSP JPG49 (2022) 015002

Transverse dof in $\langle VVA \rangle$ and relation with anomaly

- Choosing a basis for $\langle VVA \rangle$ (free of kin. singularities)[2]

$$\mathcal{W}^{\mu\nu\rho}(q_1,q_2) = \epsilon^{\mu\nu q_1 q_2} q_{12}^{\rho} C_S + \epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha} q_2^{\nu} - g_{\alpha}^{\nu} q_2^2) B_2 + \epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha} q_1^{\mu} - g_{\alpha}^{\mu} q_1^2)\bar{B}_2 + \epsilon^{\mu\nu q_1 q_2}\bar{q}_{12}^{\rho} C_A$$

- Since anomaly known exactly, sensible to isolate it $[B_2(\bar{B}_2) = B_{2S} \pm B_{2A}]$

$$q_{12}^{\rho}\mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \epsilon_{\mu\nu q_1 q_2}\left[q_{12}^2 C_S + (q_1^2+q_2^2)B_{2S} + (q_1^2-q_2^2)(C_A - B_{2A})\right] = \epsilon_{\mu\nu q_1 q_2}\frac{N_c}{4\pi^2}$$

- Note the $q_{12}^2 \to 0$ limit (real photons $\leftrightarrow \pi^0 \to \gamma\gamma$) ($X^0 = \lim_{q_{12}^2\to 0} X$)

$$\frac{\frac{N_c}{4\pi^2}\tilde{F}_{\mathrm{pGB}VV}(q_1^2,q_2^2)}{q_{12}^2 - m_{\mathrm{pGB}}^2} + \frac{q_1^2+q_2^2}{q_{12}^2}B_{2S}^0 + \frac{q_1^2-q_2^2}{q_{12}^2}(C_A^0 - B_{2A}^0) = \frac{N_c}{4\pi^2 q_{12}^2} = \frac{w_L}{8\pi^2}$$

- If longitudinal part is separated (later replace C_S in favor of the anomaly)

$$\mathcal{W}^{\mu\nu\rho} = \left\{ t_{\mu\nu\rho}^{(+)}(-B_{2S}) + t_{\mu\nu\rho}^{(-)}(C_A - B_{2A}) + \tilde{t}_{\mu\nu\rho}^{(-)}(B_{2A}) + \epsilon_{\mu\nu q_1 q_2} q_{12\rho}\left[\frac{\frac{N_c}{4\pi^2}\tilde{F}_{\mathrm{pGB}VV}(q_1^2,q_2^2)}{q_{12}^2 - m_{\mathrm{pGB}}^2} + \frac{q_1^2+q_2^2}{q_{12}^2}B_{2S}^0 + \frac{q_1^2-q_2^2}{q_{12}^2}(C_A^0 - B_{2A}^0)\right]\right\}$$

$$t_{\mu\nu\rho}^{(+)} = \epsilon_{q_1 q_2 \mu\rho} q_{1\,\nu} + \epsilon_{q_2 q_1 \nu\rho} q_{2\,\mu} - (q_1\cdot q_2)\epsilon_{\mu\nu\rho\bar{q}_{12}} + \frac{q_1^2+q_2^2-q_{12}^2}{q_{12}^2}\epsilon_{\mu\nu q_1 q_2} q_{12\,\rho}, \qquad t_{\mu\nu\rho}^{(-)} = \epsilon_{\mu\nu q_1 q_2}\left[\bar{q}_{12\,\rho} - \frac{q_1^2-q_2^2}{q_{12}^2} q_{12\,\rho}\right]$$

[2] P. Roig, PSP PRD101 (2020) 074019 and P. Masjuan, P. Roig, PSP JPG49 (2022) 015002

Transverse dof in $\langle VVA \rangle$ and relation with anomaly

- From analytic properties, the standard $\langle VVA \rangle$ basis can be expressed as

$$\mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \frac{-1}{8\pi^2}\Bigg[-\epsilon_{\mu\nu \mathbf{q_1} \mathbf{q_2}}\, q_{12\rho}\left(\frac{2N_c\tilde{F}_{PVV}(q_1^2,q_2^2)}{q_{12}^2 - m_{\rm pGB}^2} + \frac{(q_1^2+q_2^2)w_{T0}^{(+)} - (q_1^2-q_2^2)w_{T0}^{(-)}}{q_{12}^2}\right)$$
$$+t_{\mu\nu\rho}^{(+)}[w_{TS}^{(+)}+w_{T0}^{(+)}]+t_{\mu\nu\rho}^{(-)}[w_{TS}^{(-)}+w_{T0}^{(-)}]+\tilde{t}_{\mu\nu\rho}^{(-)}\tilde{w}_T^{(-)}\Bigg], \qquad w_T^{(\pm)}(q_{12}^2,\ldots) = w_{TS}^{(\pm)}(q_{12}^2,\ldots)+w_{T0}^{(\pm)}(0,\ldots)$$

- Analiticity implies that anomaly fulfillment not possible using only pseudoscalar mesons or modified F_{PVV} models as in former literature

- Interesting consequences

$$(q_1^2+q_2^2)w_{T0}^{(+)}(q_1^2,q_2^2) - (q_1^2-q_2^2)w_{T0}^{(-)}(q_1^2,q_2^2) = 2N_c[1-\tilde{F}_{P\gamma\gamma}(q_1^2,q_2^2)]$$

Transverse dof in $\langle VVA \rangle$ and relation with anomaly

- From analytic properties, the standard $\langle VVA \rangle$ basis can be expressed as

$$\mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \frac{-1}{8\pi^2}\left[-\epsilon_{\mu\nu q_1 q_2} q_{12\rho}\left(\frac{2N_c\tilde{F}_{PVV}(q_1^2,q_2^2)}{q_{12}^2-m^2_{\mathrm{pGB}}} + \frac{(q_1^2+q_2^2)w_{T0}^{(+)}-(q_1^2-q_2^2)w_{T0}^{(-)}}{q_{12}^2}\right)\right.$$

$$\left.+t^{(+)}_{\mu\nu\rho}[w^{(+)}_{TS}+w^{(+)}_{T0}]+t^{(-)}_{\mu\nu\rho}[w^{(-)}_{TS}+w^{(-)}_{T0}]+\tilde{t}^{(-)}_{\mu\nu\rho}\tilde{w}^{(-)}_{T}\right], \qquad w_T^{(\pm)}(q_{12}^2,\ldots) = w_{TS}^{(\pm)}(q_{12}^2,\ldots)+w_{T0}^{(\pm)}(0,\ldots)$$

- Analiticity implies that anomaly fulfillment not possible using only pseudoscalar mesons or modified F_{PVV} models as in former literature

- Interesting consequences

$$(q_1^2+q_2^2)w_{T0}^{(+)}(q_1^2,q_2^2)-(q_1^2-q_2^2)w_{T0}^{(-)}(q_1^2,q_2^2) = 2N_c[1-\tilde{F}_{P\gamma\gamma}(q_1^2,q_2^2)]$$

$$\lim_{q^2\to\infty} w_{T0}^{(+)}(q^2,q^2) = \frac{N_c}{q^2}\left[1+\frac{8\pi^2}{N_c}\frac{F_P^2}{q^2}\right]+\mathcal{O}(q^{-6}).$$

Transverse dof in $\langle VVA \rangle$ and relation with anomaly

- From analytic properties, the standard $\langle VVA \rangle$ basis can be expressed as

$$\mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \frac{-1}{8\pi^2}\left[-\epsilon_{\mu\nu q_1 q_2} q_{12\rho}\left(\frac{2N_c \tilde{F}_{PVV}(q_1^2,q_2^2)}{q_{12}^2 - m_{\rm pGB}^2} + \frac{(q_1^2+q_2^2)w_{T0}^{(+)} - (q_1^2-q_2^2)w_{T0}^{(-)}}{q_{12}^2}\right)\right.$$

$$\left. + t_{\mu\nu\rho}^{(+)}[w_{TS}^{(+)} + w_{T0}^{(+)}] + t_{\mu\nu\rho}^{(-)}[w_{TS}^{(-)} + w_{T0}^{(-)}] + \tilde{t}_{\mu\nu\rho}^{(-)}\tilde{w}_T^{(-)}\right], \qquad w_T^{(\pm)}(q_{12}^2,\ldots) = w_{TS}^{(\pm)}(q_{12}^2,\ldots) + w_{T0}^{(\pm)}(0,\ldots)$$

- Analiticity implies that anomaly fulfillment not possible using only pseudoscalar mesons or modified F_{PVV} models as in former literature
- Interesting consequences

$$(q_1^2+q_2^2)w_{T0}^{(+)}(q_1^2,q_2^2) - (q_1^2-q_2^2)w_{T0}^{(-)}(q_1^2,q_2^2) = 2N_c[1-\tilde{F}_{P\gamma\gamma}(q_1^2,q_2^2)]$$

$$w_{T0}^{(+)}(q^2,0) - w_{T0}^{(-)}(q^2,0) = \frac{2N_c}{q^2}[1-\tilde{F}_{P\gamma\gamma}(q^2,0)]$$

$$w_{T0}^{(+)}(0,0) = -2N_c b_\pi$$

$$(w_{T0}^{(+)} - w_{T0}^{(-)})(q^2,0) = \frac{2N_c}{q^2}\left[1 + \frac{24\pi^2 F_P^2}{N_c q^2}\right] + \ldots$$

Section 3

Consequences for hadronic models

The role of pseudoscalar mesons

- The key element is the TFF

$$i\int d^4x e^{iq_1\cdot x}\langle 0|\,T\{j^\mu(x)j^\nu(0)\}\,|P(q_{12})\rangle = \epsilon^{\mu\nu q_1 q_2}F_{P\gamma\gamma}(q_1^2,q_2^2),$$

- The correct SD HLbL$\Rightarrow$ $\langle VVA\rangle$ transition guaranteed if correct OPE

$$\lim_{q^2\to\infty} F_{P\gamma\gamma}(q^2,q^2) = -\frac{1}{\hat{q}^2}\sum_c 2F_P^c\,\mathrm{tr}(\lambda^c\mathcal{Q}^2),$$

- Its contribution to the $\langle VVA\rangle$ Green's function (chiral/large-N_c limit)

$$\mathcal{W}^{abc}_{\mu\nu\rho} = \epsilon_{\mu\nu q_1 q_2}q_{12\rho}\sum_P \frac{F_P^c F_{PV^aV^b}(q_1^2,q_2^2)}{q_{12}^2-m_P^2} = \frac{\mathrm{tr}(\{t^a,t^b\}t^c)}{8\pi^2}\sum_P \frac{2N_c\tilde{F}_{PV^aV^b}(q_1^2,q_2^2)}{q_{12}^2-m_P^2}$$

- Which is the quoted contribution to w_L

$$\frac{2N_c\tilde{F}_{PV^aV^b}(q_1^2,q_2^2)}{q_{12}^2-m_P^2} \neq \frac{2N_c}{q_{12}^2}\ \text{if}\ q_{1,2}^2\neq 0$$

- This was the point of discussion in the past

The role of axial-vector mesons I

- The key element is

$$i\int d^4x e^{iq_1\cdot x}\langle 0|\,T\{j^\mu(x)j^\nu(0)\}\,|A(q_{12})\rangle = \mathcal{M}_A^{\mu\nu\rho}(q_1,q_2)\varepsilon_{A\rho}$$

- Can use the previous basis; other basis related via Schouten ids.

$$\mathcal{M}_A^{\mu\nu\rho}(q_1,q_2) = i\Big[\epsilon^{\mu\alpha\rho q_1}(q_{2\,\alpha}q_2^\nu - g_\alpha^\nu q_2^2)B_2 + \epsilon^{\nu\alpha\rho q_2}(q_{1\,\alpha}q_1^\mu - g_\alpha^\mu q_1^2)\bar{B}_2 + \epsilon^{\mu\nu q_1 q_2}\bar{q}_{12}^\rho C_A + \epsilon^{\mu\nu q_1 q_2} q_{12}^\rho C_S\Big]$$

- The correct SD HLbL$\Rightarrow\langle VVA\rangle$ transition guaranteed if correct OPE

$$\lim_{\hat{q}^2\to\infty} B_{2S}(\hat{q}^2,\hat{q}^2)\varepsilon_A^\rho = \frac{1}{\hat{q}^4}\langle 0|\,j_5^\rho\,|A\rangle \equiv \sum_a \frac{m_A F_A^a}{\hat{q}^4}\varepsilon_A^\rho\,\mathrm{tr}\,\mathcal{Q}^2\lambda^a,$$

- Can drop $q\cdot\varepsilon$ terms (C_S) onshell, but then other basis inequivalent ... basis-dependent axial-vector meson contributions?

The role of axial-vector mesons II

- Their $\langle VVA\rangle$ contribution ($D^{\rho\tau}$ non transverse!)

$$\mathcal{W}^{\mathcal{Q}\mathcal{Q}a}_{\mu\nu\rho}|_A = i\sum_{A,\mathrm{pol.}}\mathcal{M}^A_{\mu\nu\tau} m_A F^a_A \frac{\varepsilon^\tau_A \varepsilon^*_{A\rho}}{q^2_{12}-m^2_A} = -i\sum_{A,\mathrm{pol.}}\mathcal{M}^A_{\mu\nu\tau} m_A F^a_A \frac{g^{\tau\rho}-\frac{q^\tau_{12}q^\rho_{12}}{m^2_A}}{q^2_{12}-m^2_A}$$

- Using the std. basis that separates w_L from w_T (w_L no pole, see also[3])

$$\frac{\{w_T^{(+)}, w_T^{(-)}, \tilde{w}_T^{(-)}\}}{8\pi^2} = \frac{\{B_{2S}, B_{2A}-C_A, -B_{2A}\}}{q^2_{12}-m^2_A} m_A F^a_A,$$

$$\frac{w_L}{8\pi^2} = \left[C_S + \frac{q_1^2+q_2^2}{q^2_{12}} B_{2S} - \frac{q_1^2-q_2^2}{q^2_{12}}(B_{2A}-C_A)\right]\left[1-\frac{q^2_{12}}{m^2_A}\right]\frac{m_A F^a_A}{q^2_{12}-m^2_A},$$

- If added to pGBs (clearly, in our basis $C_S \to 0 \Rightarrow$ basis-ambiguity fixed!)

$$\frac{N_c\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2 q^2_{12}} = \frac{N_c\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2}\frac{\tilde{F}_{P\gamma\gamma}(q_1^2,q_2^2)}{q^2_{12}-m^2_{\mathrm{pGB}}} - \sum_n \frac{F^a_{A_n}}{m^a_{A_n}}\Big[C_S + \frac{q_1^2+q_2^2}{q^2_{12}}B_{2S} - \frac{q_1^2-q_2^2}{q^2_{12}}(B_{2A}-C_A)\Big],$$

[3] K. Melnikov and A. Vainshtein, 2112.05770

The role of axial-vector mesons III

- Anomaly constraint can be implemented with axial-vector meson model
- Such (longitudinal) contribution cannot show an axial pole

$$D^{\rho\tau} = -\frac{g^{\rho\tau} - \frac{q^\rho q^\tau}{m_A^2}}{q^2 - m_A^2} = -\frac{q^2 g^{\rho\tau} - q^\rho q^\tau}{m_A^2(q^2 - m_A^2)} + \frac{g^{\rho\tau}}{m_A^2} \equiv D_T^{\rho\tau} + \frac{g^{\rho\tau}}{m_A^2}$$

- In terms of the $\langle VVA \rangle$, $D_T^{\rho\tau}$ leads to w_{TS} and $g^{\rho\tau}$ to w_{T0}

Explore consequences in HLbL
(i) Basis dependence (ii) Separate the pole (iii) Model AVM tower

Section 4

Applications to HLbL

Illustrating basis dependence and pole separation

- The most relevant contribution from symmetric form factor:

$$B_{2S}^{OPE} = \frac{B_{2S}(0,0)\Lambda^4}{(q_1^2+q_2^2-\Lambda^2)^2}, \quad \{B_{2S}(0,0),\Lambda\} \sim \{\Gamma^{\rm exp}_{A\to\gamma\gamma},\Lambda_{\rm exp}\}$$

- Fulfills the OPE for $B_{2S} \sim Q^{-4}$ unlike $B_{2S}^{\rm fact}(q_1^2,q_2^2) \sim B_{2S}(q_1^2,0)B_{2S}(0,q_2^2)$

	f_1	f_1'	a_1	Total
Fact	$4.3(^{+1.8}_{-1.5})$	$1.2(^{+0.6}_{-0.5})$	$2.8(^{+1.9}_{-1.7})$	$8.3(^{+2.7}_{-3.4})$
OPE	$8.3(^{+3.4}_{-2.9})$	$2.3(^{+1.1}_{-0.9})$	$5.4(^{+3.7}_{-3.3})$	$16.0(^{+5.1}_{-4.5})$

Illustrating basis dependence and pole separation

- The most relevant contribution from symmetric form factor:

$$B_{2S}^{OPE} = \frac{B_{2S}(0,0)\Lambda^4}{(q_1^2+q_2^2-\Lambda^2)^2}, \quad \{B_{2S}(0,0),\Lambda\} \sim \{\Gamma^{\rm exp}_{A\to\gamma\gamma},\Lambda_{\rm exp}\}$$

- Fulfills the OPE for $B_{2S} \sim Q^{-4}$ unlike $B_{2S}^{\rm fact}(q_1^2,q_2^2) \sim B_{2S}(q_1^2,0)B_{2S}(0,q_2^2)$

	f_1	f_1'	a_1	Total
Fact	$4.3(^{+1.8}_{-1.5})$	$1.2(^{+0.6}_{-0.5})$	$2.8(^{+1.9}_{-1.7})$	$8.3(^{+2.7}_{-3.4})$
OPE	$8.3(^{+3.4}_{-2.9})$	$2.3(^{+1.1}_{-0.9})$	$5.4(^{+3.7}_{-3.3})$	$16.0(^{+5.1}_{-4.5})$

- To illustrate basis-dependence, take alt. basis[a]

$$w_0 = C_S + B_{2S} \qquad w_1 = C_A \qquad w_2 = -B_{2S} \qquad w_3 = B_{2A}$$
$$C_S = w_0 + w_2 \qquad C_A = w_1 \qquad B_{2S} = -w_2 \qquad B_{2A} = w_3$$

	f_1	f_1'	a_1	Total
OPE	8.3	2.3	5.4	16.0
OPE'	3.5	1.0	2.3	6.8

- The 2nd choice in violence with anomaly if nothing else cancels

[a] M. Knecht, JHEP 08 (2020) 056

Illustrating basis dependence and pole separation

- The most relevant contribution from symmetric form factor:

$$B_{2S}^{OPE} = \frac{B_{2S}(0,0)\Lambda^4}{(q_1^2+q_2^2-\Lambda^2)^2}, \quad \{B_{2S}(0,0),\Lambda\} \sim \{\Gamma^{\rm exp}_{A\to\gamma\gamma},\Lambda_{\rm exp}\}$$

- Fulfills the OPE for $B_{2S}\sim Q^{-4}$ unlike $B_{2S}^{\rm fact}(q_1^2,q_2^2)\sim B_{2S}(q_1^2,0)B_{2S}(0,q_2^2)$

	f_1	f_1'	a_1	Total
Fact	$4.3(^{+1.8}_{-1.5})$	$1.2(^{+0.6}_{-0.5})$	$2.8(^{+1.9}_{-1.7})$	$8.3(^{+2.7}_{-3.4})$
OPE	$8.3(^{+3.4}_{-2.9})$	$2.3(^{+1.1}_{-0.9})$	$5.4(^{+3.7}_{-3.3})$	$16.0(^{+5.1}_{-4.5})$

- Separate $D_T^{\mu\nu}$ (subtracted) from $g^{\mu\nu}$ (contact)

	f_1	f_1'	a_1	Total
Subt	$-1.3(^{+0.5}_{-0.7})$	$-0.27(^{+0.12}_{-0.17})$	$-0.85(^{+0.55}_{-0.68})$	$-2.42(^{+0.75}_{-1.0})$
Cont	$9.6(^{+4.1}_{-3.5})$	$2.6(^{+1.3}_{-1.0})$	$6.2(^{+4.3}_{-3.8})$	$18.4(^{+6.1}_{-5.3})$

- Conctact term is the leading part: suffices to have a model for (part of) this?

A Regge-like model for the anomaly

- For simplicity, consider the $q_1^2 = q_2^2 = q^2$ regime (only B_{2S}) and take

$$m_{A_n^a}^2 = m_{A^a}^2 + n\mu_a^2, \quad B_{2S}^{A_n^a}(q_1^2, q_2^2) = \frac{4F_A \,\mathrm{tr}(\mathcal{Q}^2\lambda^a) m_{A_n^a}}{[q_1^2 + q_2^2 - (M_a^2 + n\Lambda^2)]^2},$$

- Match to the anomaly and resum

$$\frac{N_c \,\mathrm{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2}\left[1 - \tilde{F}_{P\gamma\gamma}(Q^2, Q^2)\right] = \lim_{Q_{1,2}^2 \to Q^2} \sum_A \frac{F_A}{m_{A_n^a}}(Q_1^2 + Q_2^2) B_{2S}(Q_1^2, Q_2^2) + \ldots$$

A Regge-like model for the anomaly

- For simplicity, consider the $q_1^2 = q_2^2 = q^2$ regime (only B_{2S}) and take

$$m^2_{A^a_n} = m^2_{A^a} + n\mu_a^2, \quad B_{2S}^{A^a_n}(q_1^2, q_2^2) = \frac{4F_A\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)m_{A^a_n}}{[q_1^2 + q_2^2 - (M_a^2 + n\Lambda^2)]^2},$$

- Match to the anomaly and resum

$$\frac{N_c}{4\pi^2}\left[1 - \tilde{F}_{P\gamma\gamma}(Q^2, Q^2)\right] = \lim_{Q^2_{1,2}\to Q^2} \frac{4F_A^2(Q_1^2 + Q_2^2)}{\Lambda^4}\psi^{(1)}\left(\frac{M_a^2 + Q_1^2 + Q_2^2}{\Lambda^2}\right)$$

A Regge-like model for the anomaly

- For simplicity, consider the $q_1^2 = q_2^2 = q^2$ regime (only B_{2S}) and take

$$m_{A_n^a}^2 = m_{A^a}^2 + n\mu_a^2, \quad B_{2S}^{A_n^a}(q_1^2, q_2^2) = \frac{4F_A\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)m_{A_n^a}}{[q_1^2 + q_2^2 - (M_a^2 + n\Lambda^2)]^2},$$

- Match to the anomaly and resum

$$\frac{N_c}{4\pi^2}\left[1 - \tilde{F}_{P\gamma\gamma}(Q^2, Q^2)\right] = \lim_{Q_{1,2}^2 \to Q^2} \frac{4F_A^2(Q_1^2 + Q_2^2)}{\Lambda^4}\psi^{(1)}\left(\frac{M_a^2 + Q_1^2 + Q_2^2}{\Lambda^2}\right)$$

- High energies (for $a \neq 3$, M_a from $B_{2S}(0,0)$)

$$1 - \frac{8\pi^2F_P^2}{3Q^2} = \left(\frac{4\pi F_A}{\Lambda\sqrt{N_c}}\right)^2\left[1 + \frac{\Lambda^2 - 2M_a^2}{4Q^2}\right] \Rightarrow \begin{cases} \Lambda = \frac{4\pi F_A}{\sqrt{N_c}} \\ M_a^2 = \frac{8\pi^2}{3}[F_A^2 + 2F_P^2] \\ b_\pi = \frac{3\psi^{(1)}\left(\frac{F_A^2 + 2F_\pi^2}{2F_A^2}\right)}{16\pi^2 F_A^2} \sim 1.8(1)\ \mathrm{GeV}^{-2} \end{cases}$$

A Regge-like model for the anomaly

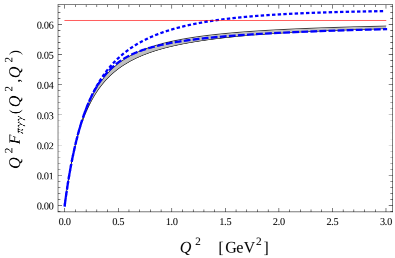
- For simplicity, consider the $q_1^2 = q_2^2 = q^2$ regime (only B_{2S}) and take

$$m^2_{A^a_n} = m^2_{A^a} + n\mu_a^2, \quad B_{2S}^{A^a_n}(q_1^2, q_2^2) = \frac{4F_A \,\mathrm{tr}(\mathcal{Q}^2\lambda^a) m_{A^a_n}}{[q_1^2 + q_2^2 - (M_a^2 + n\Lambda^2)]^2},$$

- Match to the anomaly and resum

$$\frac{N_c}{4\pi^2}\left[1 - \tilde{F}_{P\gamma\gamma}(Q^2, Q^2)\right] = \lim_{Q_{1,2}^2 \to Q^2} \frac{4F_A^2(Q_1^2 + Q_2^2)}{\Lambda^4}\psi^{(1)}\left(\frac{M_a^2 + Q_1^2 + Q_2^2}{\Lambda^2}\right)$$

- High energies (for $a \neq 3$, M_a from $B_{2S}(0,0)$)



A Regge-like model for the anomaly

- For simplicity, consider the $q_1^2 = q_2^2 = q^2$ regime (only B_{2S}) and take

$$m^2_{A_n^a} = m^2_{A^a} + n\mu_a^2, \quad B_{2S}^{A_n^a}(q_1^2, q_2^2) = \frac{4F_A \,\mathrm{tr}(\mathcal{Q}^2\lambda^a) m_{A_n^a}}{[q_1^2 + q_2^2 - (M_a^2 + n\Lambda^2)]^2},$$

- Match to the anomaly and resum

$$\frac{N_c}{4\pi^2}\left[1 - \tilde{F}_{P\gamma\gamma}(Q^2, Q^2)\right] = \lim_{Q_{1,2}^2 \to Q^2} \frac{4F_A^2(Q_1^2 + Q_2^2)}{\Lambda^4}\psi^{(1)}\left(\frac{M_a^2 + Q_1^2 + Q_2^2}{\Lambda^2}\right)$$

- Their HLbL contribution

n	0	1	5	10	40	99	$(1-99)$
a_1	5.89	7.35	8.73	9.12	9.48	9.56	3.67
f_1	10.52	13.55	16.84	17.83	18.77	18.98	8.46
f_1'	1.97	2.35	2.69	2.77	2.85	2.87	0.90
Total	18.38	23.25	28.26	29.71	31.10	31.41	13.03

A Regge-like model for the anomaly

- For simplicity, consider the $q_1^2 = q_2^2 = q^2$ regime (only B_{2S}) and take

$$m^2_{A^a_n} = m^2_{A^a} + n\mu_a^2, \quad B_{2S}^{A^a_n}(q_1^2, q_2^2) = \frac{4F_A\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)m_{A^a_n}}{[q_1^2+q_2^2-(M_a^2+n\Lambda^2)]^2},$$

- Match to the anomaly and resum

$$\frac{N_c}{4\pi^2}\left[1-\tilde{F}_{P\gamma\gamma}(Q^2,Q^2)\right] = \lim_{Q^2_{1,2}\to Q^2} \frac{4F_A^2(Q_1^2+Q_2^2)}{\Lambda^4}\psi^{(1)}\left(\frac{M_a^2+Q_1^2+Q_2^2}{\Lambda^2}\right)$$

- Their HLbL contribution[a]

n	0	1	5	10	40	99	$(1-99)$
Std	18.4	23.3	28.3	29.7	31.1	31.4	13.0
Subt	-2.9	-3.7	-4.5	-4.8	-5.0	-5.1	-2.2
Cont	21.3	27.0	32.9	34.6	36.2	36.6	15.3

- Again, Contact-like term dominates

[a] Also PRD101 (2020) 114015 and PRD102 (2020) 016009: holographic

Outlook

- Interesting to work out chiral limit corrections (heavy P)

$$\langle V^\mu V^\nu \partial^\rho A^a_\rho \rangle = \text{anomaly} + \langle V^\mu V^\nu \{M, P^a\}\rangle \left(+\delta^{a0}\langle V^\mu V^\nu G\tilde{G}\rangle\right)$$

$$\epsilon^{\mu\nu q_1 q_2} q_{12}^\rho \frac{F_\pi F_{\pi\gamma\gamma}}{q_{12}^2} \to \epsilon^{\mu\nu q_1 q_2} q_{12}^\rho F'_\pi F'_{\pi\gamma\gamma} \left[\frac{1}{q_{12}^2 - m_\pi^2} + \ldots \Rightarrow \frac{1}{q^2} + \frac{m_\pi^2}{q^2(q_{12}^2 - m_\pi^2)} + \ldots\right]$$

- When taking divergence

$$\epsilon^{\mu\nu q_1 q_2} F'_\pi F'_{\pi\gamma\gamma}(q_1^2, q_2^2)\left[1 + \frac{m_\pi^2}{(q_{12}^2 - m_\pi^2)} + \ldots\right]$$

- That is roughly anomaly + $2m\langle PVV\rangle$ from anomaly equation
- Possibly use different constraints to infer heavy P properties

Conclusions

- OPE provides valuable information about the interplay of pGB and axials
- High-energy behavior of axial TFF ($\Rightarrow$ larger HLbL contribution)
- Proper basis for axial TFF via anomaly matching
- Anomaly-matching should (also) involve transverse dof (full model!)
- Might be interesting to separate non-pole part
- Simple Regge-like model illustrating assertions
- Procedure might be useful to understand deviations beyond chiral limit

The role of axial-vector mesons III

- Anomaly constraint can be implemented with axial-vector meson model
- Such (longitudinal) contribution cannot show an axial pole

$$D^{\rho\tau} = -\frac{g^{\rho\tau} - \frac{q^\rho q^\tau}{m_A^2}}{q^2 - m_A^2} = -\frac{q^2 g^{\rho\tau} - q^\rho q^\tau}{m_A^2(q^2 - m_A^2)} + \frac{g^{\rho\tau}}{m_A^2} \equiv D_T^{\rho\tau} + \frac{g^{\rho\tau}}{m_A^2}$$

- In terms of the $\langle VVA \rangle$, $D_T^{\rho\tau}$ leads to w_{TS} and $g^{\rho\tau}$ to w_{T0}

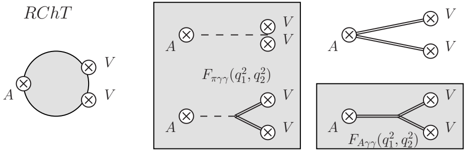
[4] P. Roig, PSP PRD101 (2020) 074019

The role of axial-vector mesons III

- Anomaly constraint can be implemented with axial-vector meson model
- Such (longitudinal) contribution cannot show an axial pole

$$D^{\rho\tau} = -\frac{g^{\rho\tau} - \frac{q^\rho q^\tau}{m_A^2}}{q^2 - m_A^2} = -\frac{q^2 g^{\rho\tau} - q^\rho q^\tau}{m_A^2(q^2 - m_A^2)} + \frac{g^{\rho\tau}}{m_A^2} \equiv D_T^{\rho\tau} + \frac{g^{\rho\tau}}{m_A^2}$$

- In terms of the $\langle VVA \rangle$, $D_T^{\rho\tau}$ leads to w_{TS} and $g^{\rho\tau}$ to w_{T0}
- In purely transverse models (i.e. RChT) axials do not fulfill anomaly, but require extra terms, explaining current results pasar a apendice![4]



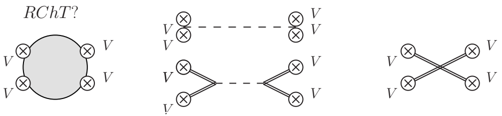
[4] P. Roig, PSP PRD101 (2020) 074019

The role of axial-vector mesons III

- Anomaly constraint can be implemented with axial-vector meson model
- Such (longitudinal) contribution cannot show an axial pole

$$D^{\rho\tau} = -\frac{g^{\rho\tau} - \frac{q^\rho q^\tau}{m_A^2}}{q^2 - m_A^2} = -\frac{q^2 g^{\rho\tau} - q^\rho q^\tau}{m_A^2(q^2 - m_A^2)} + \frac{g^{\rho\tau}}{m_A^2} \equiv D_T^{\rho\tau} + \frac{g^{\rho\tau}}{m_A^2}$$

- In terms of the $\langle VVA \rangle$, $D_T^{\rho\tau}$ leads to w_{TS} and $g^{\rho\tau}$ to w_{T0}
- In purely transverse models (i.e. RChT) axials do not fulfill anomaly, but require extra terms, explaining current results pasar a apendice![4]



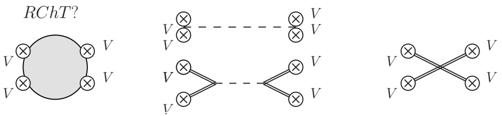
[4] P. Roig, PSP PRD101 (2020) 074019

The role of axial-vector mesons III

- Anomaly constraint can be implemented with axial-vector meson model
- Such (longitudinal) contribution cannot show an axial pole

$$D^{\rho\tau} = -\frac{g^{\rho\tau} - \frac{q^\rho q^\tau}{m_A^2}}{q^2 - m_A^2} = -\frac{q^2 g^{\rho\tau} - q^\rho q^\tau}{m_A^2(q^2 - m_A^2)} + \frac{g^{\rho\tau}}{m_A^2} \equiv D_T^{\rho\tau} + \frac{g^{\rho\tau}}{m_A^2}$$

- In terms of the $\langle VVA \rangle$, $D_T^{\rho\tau}$ leads to w_{TS} and $g^{\rho\tau}$ to w_{T0}
- In purely transverse models (i.e. RChT) axials do not fulfill anomaly, but require extra terms, explaining current results pasar a apendice![4]



- Interesting? Model (part of) the $g^{\rho\tau}$ term + data-based for $D_T^{\rho\tau}$ term

Explore consequences in HLbL
(i) Basis dependence (ii) Separate the pole (iii) Model AVM tower

[4] P. Roig, PSP PRD101 (2020) 074019