Hadronic Light-by-Light scattering contribution with Wilson-Clover quarks

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Fifth Plenary Workshop of the Muon g-2 Theory Initiative

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ

PRISMA

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Aix Marseille université

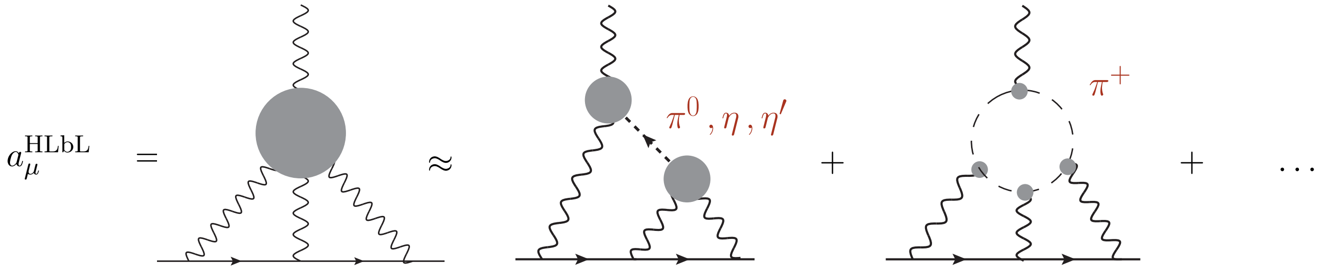
- White Paper : status in 2020

  The anomalous magnetic moment of the muon in the Standard Model [Phys.Rept. 887 (2020) 1-166]

Contribution	$a_\mu \times 10^{11}$	
- QED (leptons, 10^{th} order)	$116\ 584\ 718.931 \pm 0.104$	[Aoyama et al. '12 '19]
- Electroweak	153.6 ± 1.0	[Gnendiger et al. '13]
- Strong contributions		
HVP (LO)	$6\ 931 \pm 40$	[DHMZ '19, KNT '20]
HVP (NLO)	-98.3 ± 0.7	[Hagiwara et al. '11]
HVP (NNLO)	12.4 ± 0.1	[Kurtz et al. ''14]
HLbL	92 ± 18	[See WP]
Total (theory)	$116\ 591\ 810 \pm 43$	

$\rightarrow$ Error budget dominated by hadronic contributions

$\rightarrow$ Goal : 10% on the HLbL contribution



Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
HLbL (dispersive)	92 ± 19
HLbL (lattice)	79 ± 35
LO HVP	6931 ± 40

- results from the 2020 White Paper
- $\Delta a_\mu^{\rm exp/SM} = 251(59) \times 10^{-11} \approx 3 \times a_\mu^{\rm hlbl}$
- π^0, η, η' : accessible on the Lattice
- Complete lattice calculation
  $\rightarrow$ fully-independent

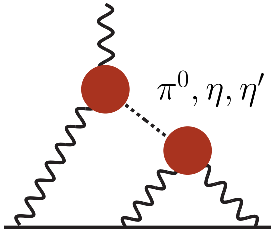
- **Pion-pole contribution on the lattice**

  ↪ Dominant contribution to the HLbL scattering in $(g-2)_\mu$

  ↪ Can be used to estimate systematic error in the full HLbL calculation

  [A. G et al, Phys.Rev. D 94 (2016)]

  [A. G et al, Phys.Rev. D100 (2019)]

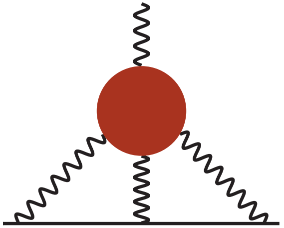


- **Direct lattice QCD calculation**

  ↪ Compute the 4-point correlation function directly on the lattice

  [Eur.Phys.J.C 80 (2020) 9, 869]

  [Eur.Phys.J.C 81 (2021) 7, 651]



- **HLbL forward scattering amplitudes**

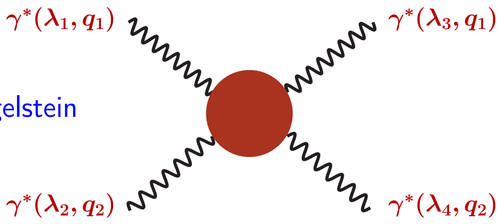
  ↪ Full HLbL amplitudes contain more info than just a_μ

  ↪ Extract information about single-meson transition form factor

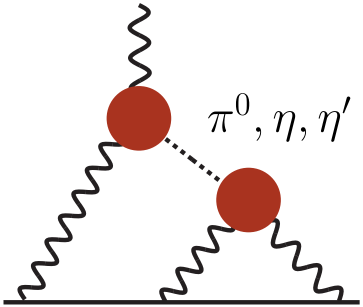
  ↪ Also related to e.m corrections to LO-HVP (Talk by Franziska Hagelstein yesterday)

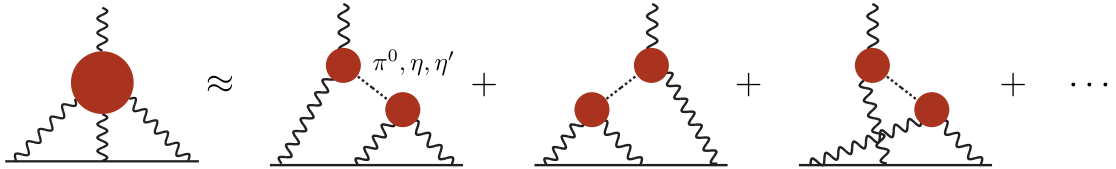
  [Phys.Rev.Lett. 115 (2015) 22, 222003]

  [Phys.Rev. D 98 (2018)]



Pseudoscalar poles contribution





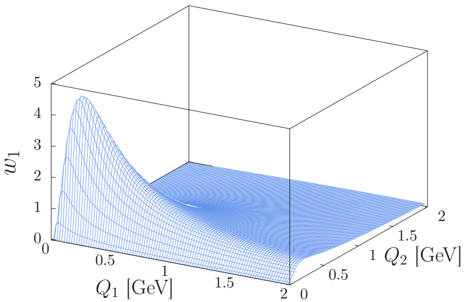
[Jegerlehner & Nyffeler '09]

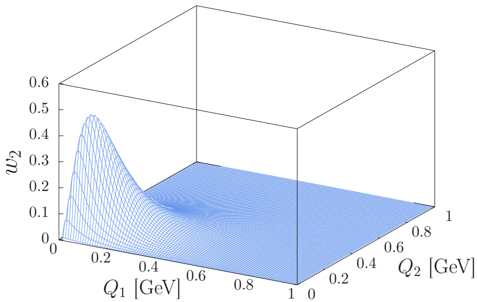
$$a_\mu^{\mathrm{HLbL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \; w_1(Q_1,Q_2,\tau)\; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0) \; + \; w_2(Q_1,Q_2,\tau)\; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0)$$

→ Product of one single-virtual and one double-virtual transition form factors

→ $w_{1,2}(Q_1,Q_2,\tau)$ are model-independent weight functions

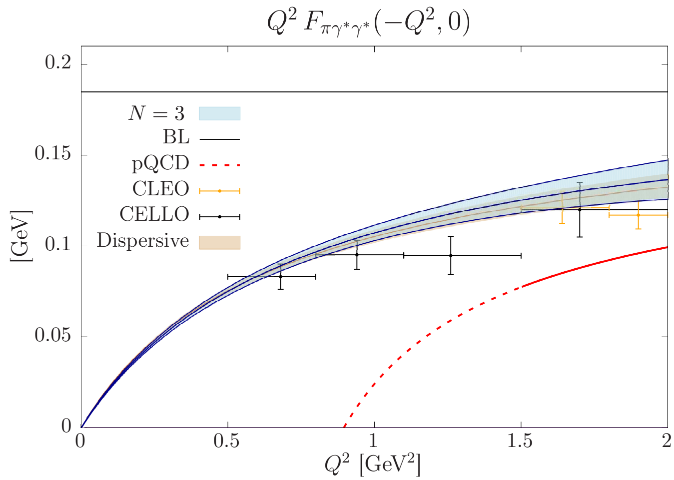
→ The weight functions are **concentrated at small momenta below 1** GeV

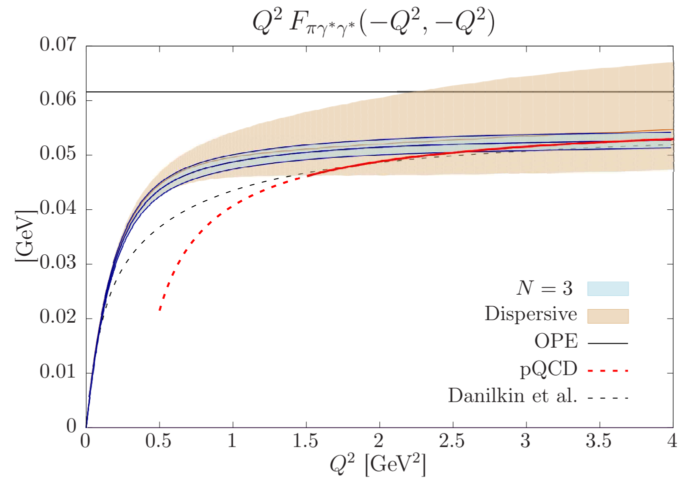




↪ Need the pion TFF for arbitrary spacelike virtualities in the momentum range $[0-3]$ GeV^2

- Pion transition form factor at the physical point (model independent parametrization)





$$a_\mu^{\mathrm{HLbL};\pi^0} = (59.9 \pm 3.6) \times 10^{-11}$$

[A. G et al, Phys.Rev. D100 (2019)]

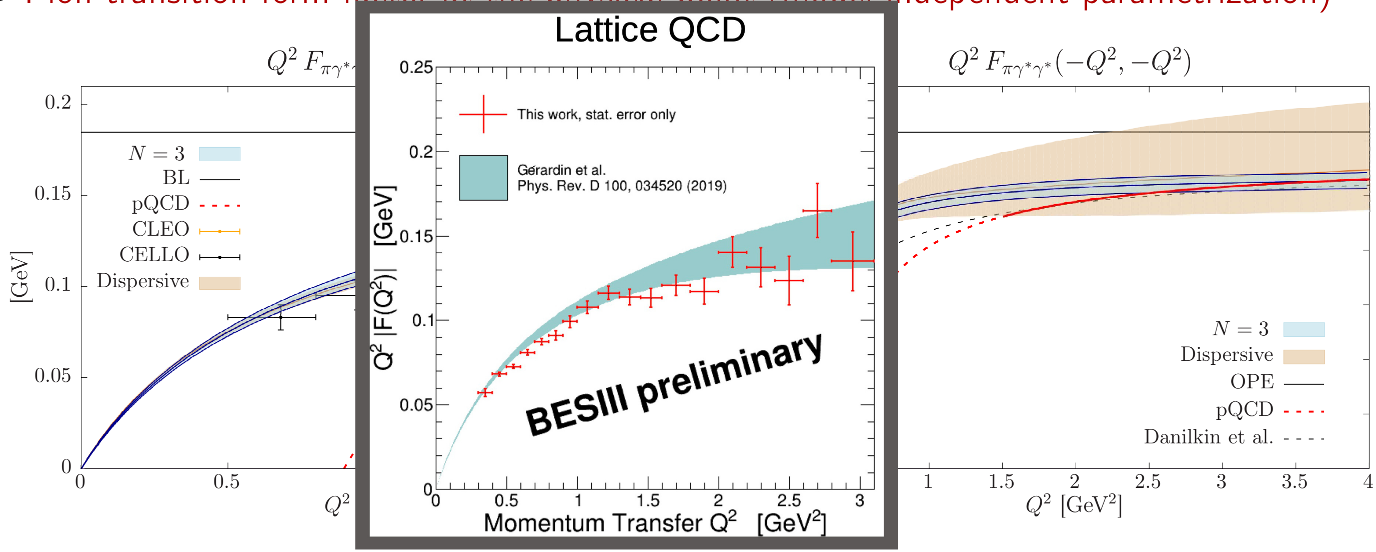
→ Compatible with data-driven result

$$a_\mu^{\mathrm{HLbL};\pi^0} = 62.6^{+3.0}_{-2.5} \times 10^{-11}$$

[Hoferichter et al. '18]

- Also useful for the direct lattice calculation of HLbL
  - → study of finite-size effects
  - → chiral extrapolation ($m_\pi \to m_\pi^{\mathrm{phys}}$)

- Pion transition form factor at the physical point (model independent parametrization)



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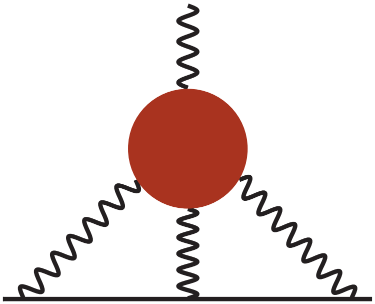
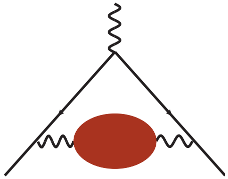
Direct lattice calculation of the HLbL contribution from Mainz



- HVP contribution : time momentum representation [Bernecker, Meyer '12]

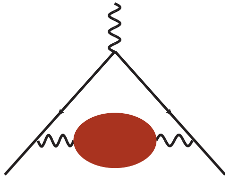
$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \mathrm{d}x_0 \; K(x_0) G(x_0)$$

$$G(x_0) = -\frac{1}{3} \sum_{k=1}^{3} \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$



- HVP contribution : Lorentz-covariant coordinate-space representation [Meyer '17]

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \mathrm{d}^4x \; H_{\mu\nu}(x) G_{\mu\nu}(x)$$



$$G_{\mu\nu}(x) = \langle J_\mu(x) J_\nu(0) \rangle$$

[Talk by Julian Parrino at Lattice 22]

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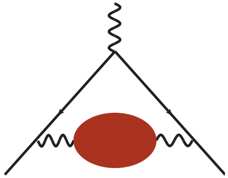
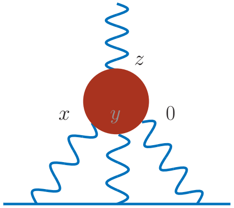


- Compute the QED part perturbatively (continuum + infinite volume + position space)

[J. Green et al. '16] [N. Asmussen et al. '16 '17]

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int \mathrm{d}^4y \int \mathrm{d}^4x \; \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

$$i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y) = -\int \mathrm{d}^4z \, z_\rho \, \langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \rangle$$

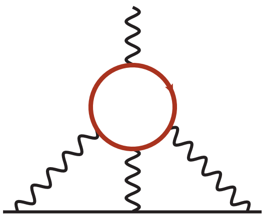


→ $\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$ is the four-point correlation function computed on the lattice

→ $\mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ is the QED kernel, computed semi-analytically (infra-red finite)

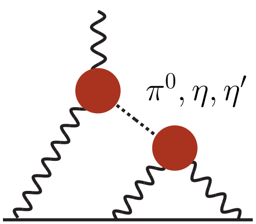
→ Avoid $1/L^2$ finite-volume effects from the massless photons $\Rightarrow \sim e^{-m_\pi L}$

- The kernel has been computed in the **continuum** and **infinite volume**
- **Semi-analytical calculation** : the kernel is expressed in term of a few scalar functions
- Scalar functions are pre-computed on a fine grid
- Computing the kernel for a couple (x, y) is very cheap
- Several tests of the kernel function have been done (in the continuum) :



Lepton-loop contribution to LbL

- 8d $\rightarrow$ 3d integral $(x^2,\ y^2$ and $x \cdot y)$
- we reproduce the exact result at the sub-percent level



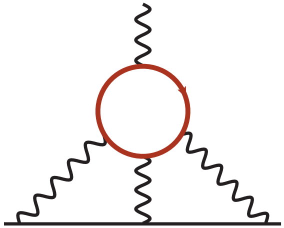
π^0-pole contribution to HLbL

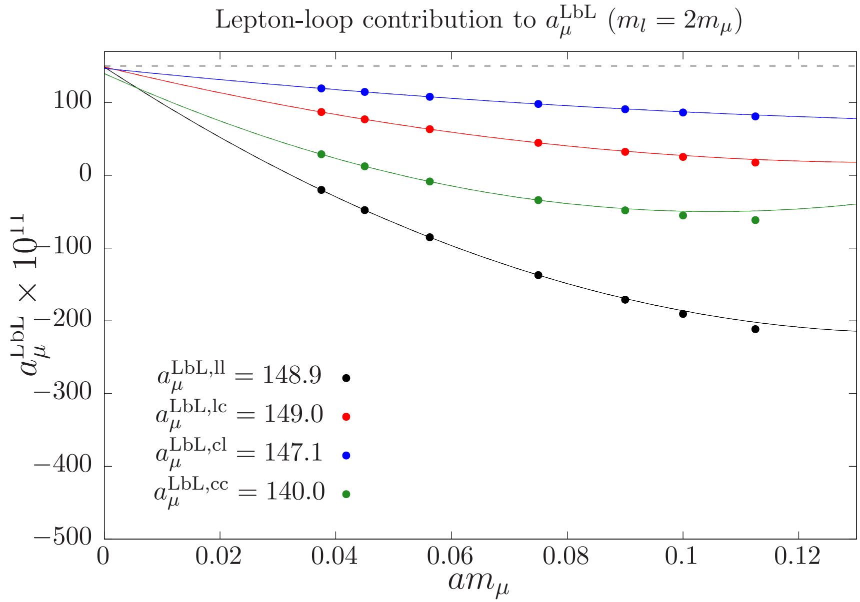
- assume a vector-meson dominance (VMD) model for $\widehat{\Pi}$
- again, we reproduce the exact result at the sub-percent level

[F. Jegerlehner and A. Nyffeler, '09)]

- **The QED kernel will be made publicly available (soon !)**

- Perform a lattice QCD calculation with unit gauge links
  - $\rightarrow$ it corresponds to the well-known lepton-loop contribution
  - $\rightarrow$ check of the QED kernel (and of the lattice implementation)





different colors
=
different discretizations of the correlation function

$$J_\mu^l(x) = \overline{\psi}(x)\gamma_\mu\psi(x)$$

$$J_\mu^c(x) = \overline{\psi}(x+a\hat{\mu})\frac{1+\gamma_\mu}{2}U_\mu^\dagger(x)\psi(x) - \overline{\psi}(x)\frac{1-\gamma_\mu}{2}U_\mu(x)\psi(x+a\hat{\mu})$$

$\rightarrow$ The QED kernel $\mathcal{L}^{(0)}(x,y)$ suffers from very large lattice artifacts !

$\rightarrow$ QCD : the situation is even worse (very noisy estimator)

$$a_\mu^{\rm HLbL} = \frac{me^6}{3} \int \mathrm{d}^4 y \int \mathrm{d}^4 x \; \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

- Conservation of the vector current : $\partial_\mu J_\mu(x) = 0$:

$$0 = \sum_x \partial_\mu^{(x)} \left(x_\alpha \widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y) \right) = \sum_x \widehat{\Pi}_{\rho,\alpha\nu\lambda\sigma}(x,y) + \sum_x x_\alpha \, \partial_\mu^{(x)} \widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

→ we can add any fonction $f(y)$ to the standard QED kernel [RBC/UKQCD '17]

- Idea : subtract short distance contributions

$$\mathcal{L}^{(2)}(x,y) = \mathcal{L}(x,y) - \mathcal{L}(0,y) - \mathcal{L}(x,0) \quad \Rightarrow \quad \mathcal{L}^{(2)}(x,0) = \mathcal{L}^{(2)}(0,y) = \mathcal{L}^{(2)}(0,0) = 0$$

- Our preferred subtracted kernel :

$$\bar{\mathcal{L}}^{(\Lambda)}_{[\rho,\sigma];\mu\nu\lambda}(x,y) = \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) - \partial_\mu^{(x)}(x_\alpha e^{-\Lambda m_\mu^2 x^2/2}) \bar{\mathcal{L}}_{[\rho,\sigma];\alpha\nu\lambda}(0,y) - \partial_\nu^{(y)}(y_\alpha e^{-\Lambda m_\mu^2 y^2/2}) \bar{\mathcal{L}}_{[\rho,\sigma];\mu\alpha\lambda}(x,0)$$

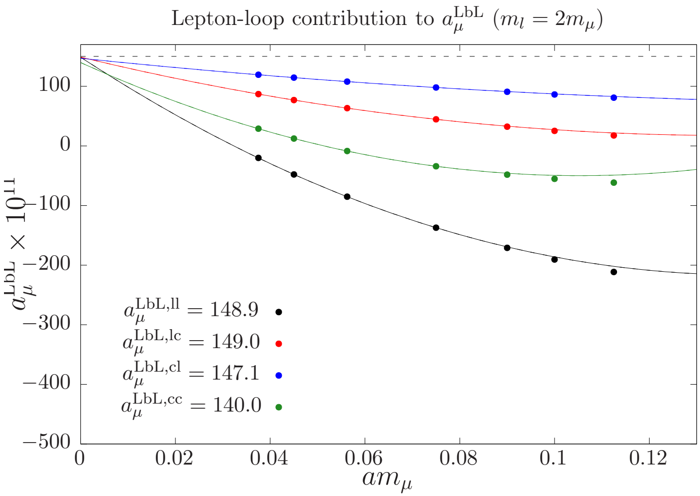
If $\Lambda \to 0$: $\mathcal{L}^{(2)}$

If $\Lambda \to \infty$: $\mathcal{L}^{(0)}$

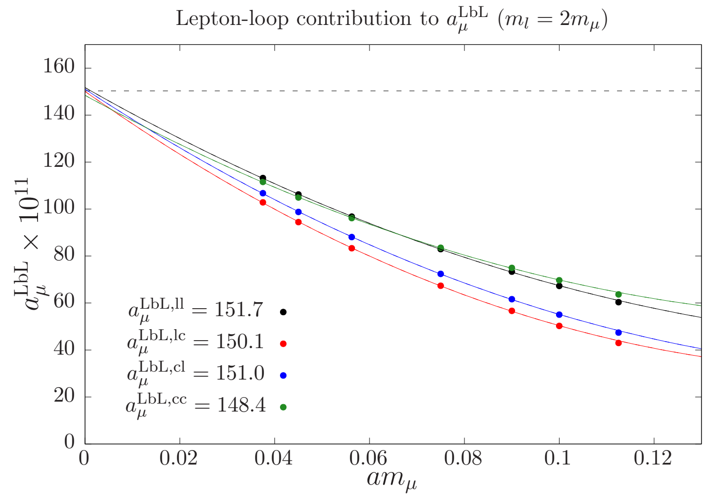
- Different definitions may affect :

→ Discretization effects / Finite-size effects / Statistical precision of the estimator

$\mathcal{L}^{(0)}(x,0)$



$\mathcal{L}^{(2)}(x,0)$



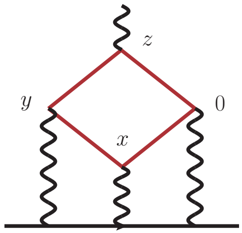
→ $\mathcal{L}^{(2)}(x,y)$ has much smaller discretization effects

→ we can reproduce the known result ($a_\mu^{\rm LbL} = 150.31 \times 10^{-11}$) with a very good precision

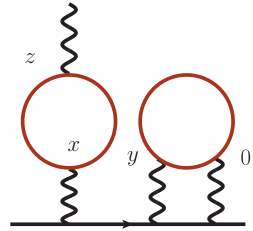
→ Wilson-Clover : local vector current has smaller discretization effects (also observed in QCD !)

$\mathcal{L}^{(\Lambda)}(x,y)$ with $\Lambda = 0.4$ is our preferred choice

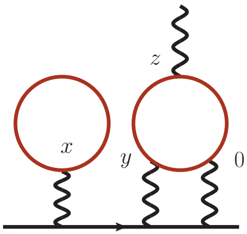
- Fully connected contribution

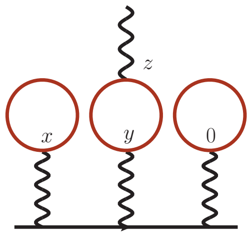


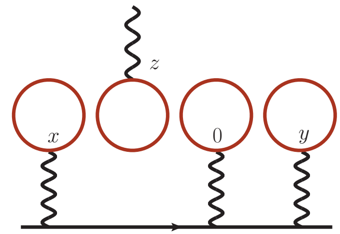
- Leading 2+2 (quark) disconnected contribution



- Sub-dominant disconnected contributions (3+1, 2+1+1, 1+1+1+1)







- 2+2 disconnected diagrams are not negligible !

  → Large-N_c prediction : 2+2 disc ≈ - 50 % × connected [Bijnens '16] [RBC-UKQCD '16]

  → Cancellation ⇒ more difficult (correlations does not seem to help in practice ...)

- Second set of diagrams vanish in the SU(3) limit (at least one quark loop which couple to a single photon)

Three recent publications

- Focus on systematics at the SU(3)$_f$ point ($m_\pi = m_K \approx 440$ MeV) [Eur.Phys.J.C 80 (2020) 9, 869]
- Complete calculation [Eur.Phys.J.C 81 (2021) 7, 651] [Eur.Phys.J.C 82 (2022) 8, 664]

Contribution	Value $\times 10^{11}$
Light-quark connected and $(2+2)$	107.4(11.3)(9.2)
Strange-quark connected and $(2+2)$	$-0.6(2.0)$
Charm-quark connected and $(2+2)$	2.8(0.5)
$(3+1)$	0.0(0.6)
$(2+1+1)$	0.0(0.3)
$(1+1+1+1)$	0.0(0.1)
Total	109.6(15.9)

$\rightarrow$ Sub-leading disconnected contributions : irrelevant at the 10% level

$\rightarrow$ Light quark contribution is the main challenge

Master formula :

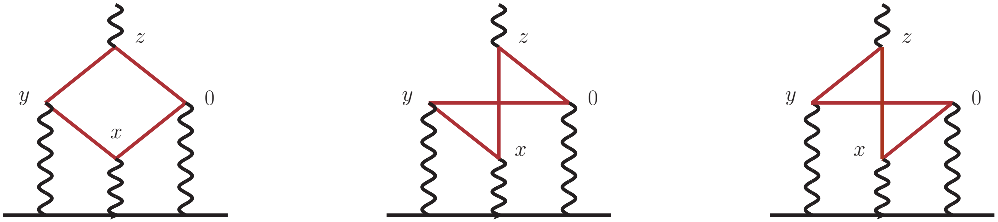
$$a_\mu^{\rm HLbL} = \frac{me^6}{3} \int \, \mathrm{d}^4y \int \, \mathrm{d}^4x \; \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

$$i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y) = - \int \, \mathrm{d}^4z \, z_\rho \, \langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \rangle$$

- 8 of the 12 integrals (over x and z) are done exactly (sums over lattice points) $\rightarrow f(y)$
- Once all indices have been contracted : $f(y) = f(|y|)$
- I will ofter show the partially integrated sum :

$$a_\mu^{\rm hlbl}(|y|_{\rm max}) = \int_0^{|y|_{\rm max}} d|y| \; f(|y|)$$

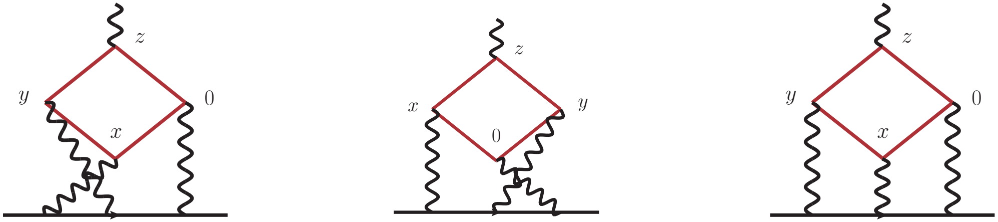
- Method 1



  - Start with point sources at 0 and y
  - Naively, it requires sequential inversions (2$^{\rm nd}$ and 3$^{\rm rd}$ diagrams)
  - For N values of y : 7(N + 1) quark propagator inversions

- Method 2

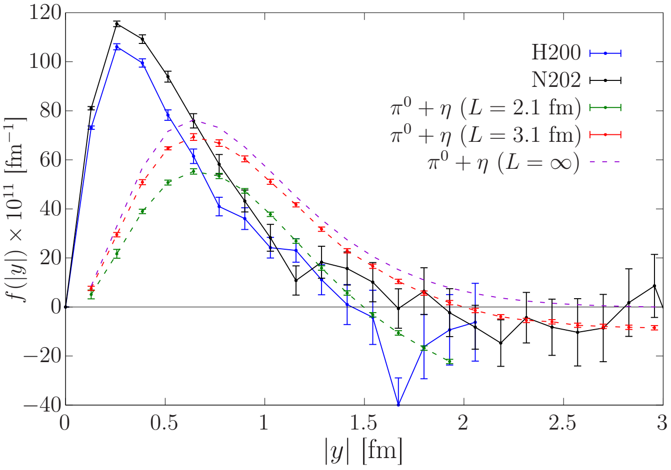
  - Trick : reordering of the vertices at le level of the muon line

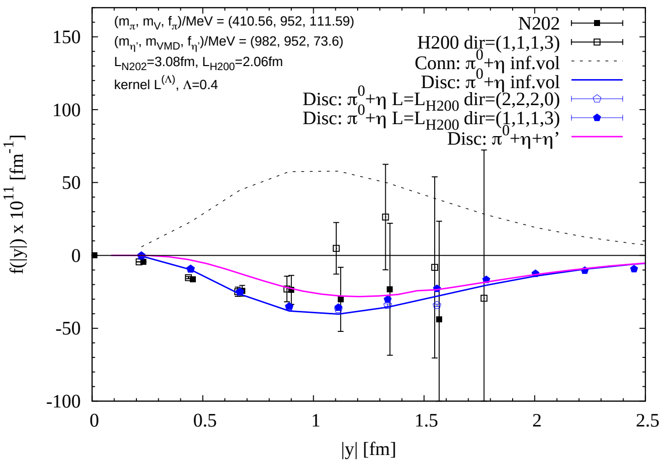


  - Replace inversions of quark propagators by kernel evaluations (much cheaper !)

$\rightarrow$ Reduces the cost by an order of magnitude

- Two ensembles with $m_\pi L = 4.4$ (blue) and $m_\pi L = 6.4$ (balck)





- Reasonably good understanding of finite-size effect
  - → pion does not saturate the integrand until very large values of $|y|$
  - → pion-pole transition form factor from dedicated lattice calculation

Comment about $O(a)$ improvement with Wilson-Clover quarks

- one needs to improve the action (this is done at the level of gauge ensembles generation)
- but one also needs to use improved currents (simplified version here ...)

$$J_\mu^{\mathrm{improved}}(x) = J_\mu(x) + a c_{\mathrm{V}}(g_0)\, \partial_\nu \Sigma_{\mu\nu}(x)$$

→ This has been fully implemented in the HVP (see talk by Simon Kuberski)

→ This step was also done for the pion Transition Form Factor

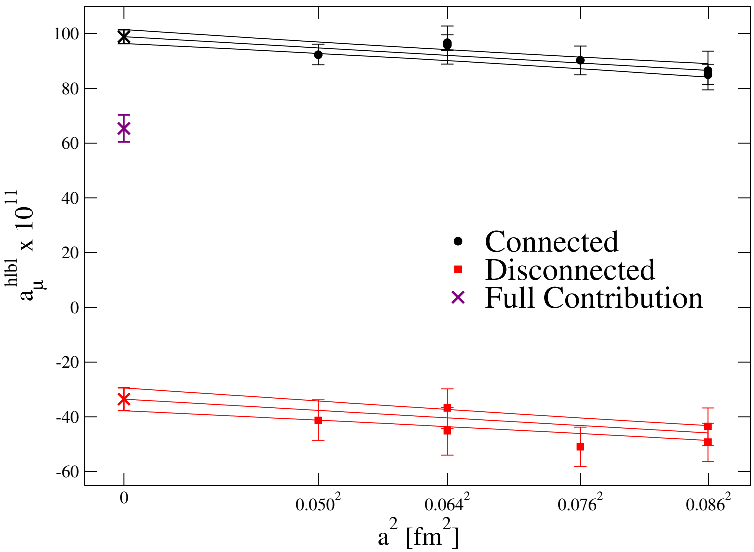
→ ... but no for the HLbL : would significantly increase the cost (more inversions)

- What we learned from the HVP and TFF calculations :

→ most of the $O(a)$ contribution is removed by simply using the improved action

→ residual $O(a)$ effects (due to the use of un-improved currents) are found to be small

- $m_\pi = m_K \approx 440$ MeV : much cheaper calculation [Eur.Phys.J.C 80 (2020) 9, 869]



- Continuum value

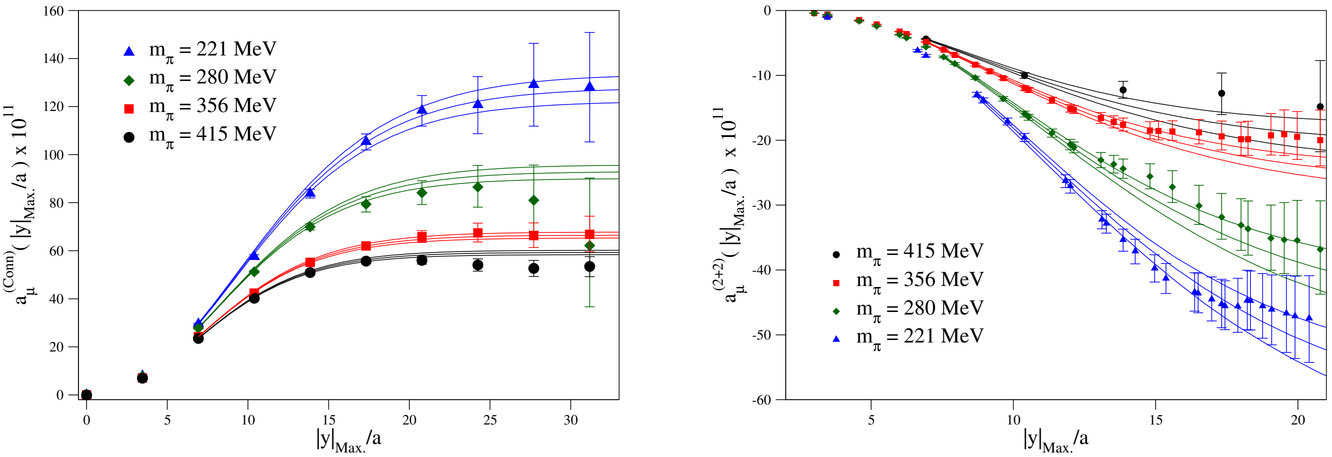
$$a_\mu^{\mathrm{hlbl,SU(3)_f}} = (65.4 \pm 4.9) \times 10^{-11}.$$

- Guess the value at the physical point by subtracting and adding back the pion-pole contribution

$$a_\mu^{\mathrm{hlbl,SU(3)_f}} + \left(a_\mu^{\mathrm{hlbl},\pi^0,\mathrm{phys}} - a_\mu^{\mathrm{hlbl},\pi^0,\mathrm{SU(3)_f}} \right) = (104.1 \pm 9.1) \times 10^{-11}.$$

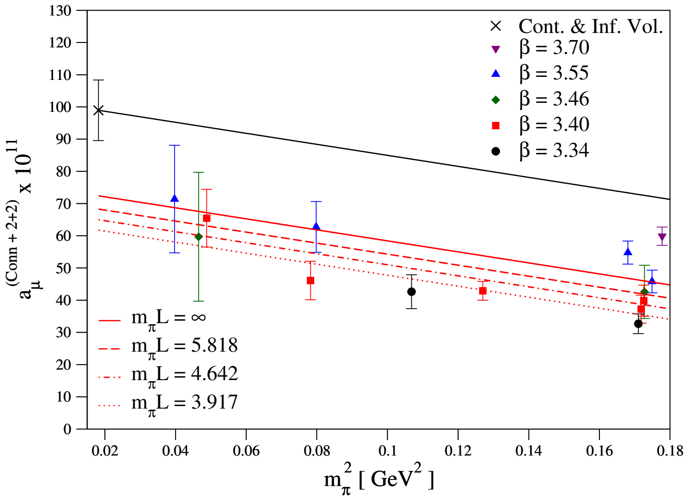
[Eur.Phys.J.C 81 (2021) 7, 651]

- Connected and disconnected contributions for $m_\pi \to m_\pi^{\text{phys}}$



- Noise increases as m_π decreases

  $\rightarrow$ extend the data using the model $f(|y|) = A|y|^3 \exp(-M|y|)$ (works very well for the π^0)
- Strong pion mass observed, but largely suppress in the sum of the two contribution

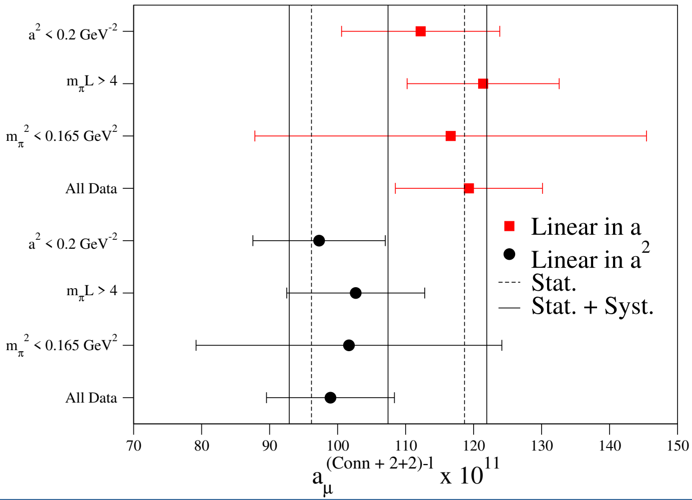


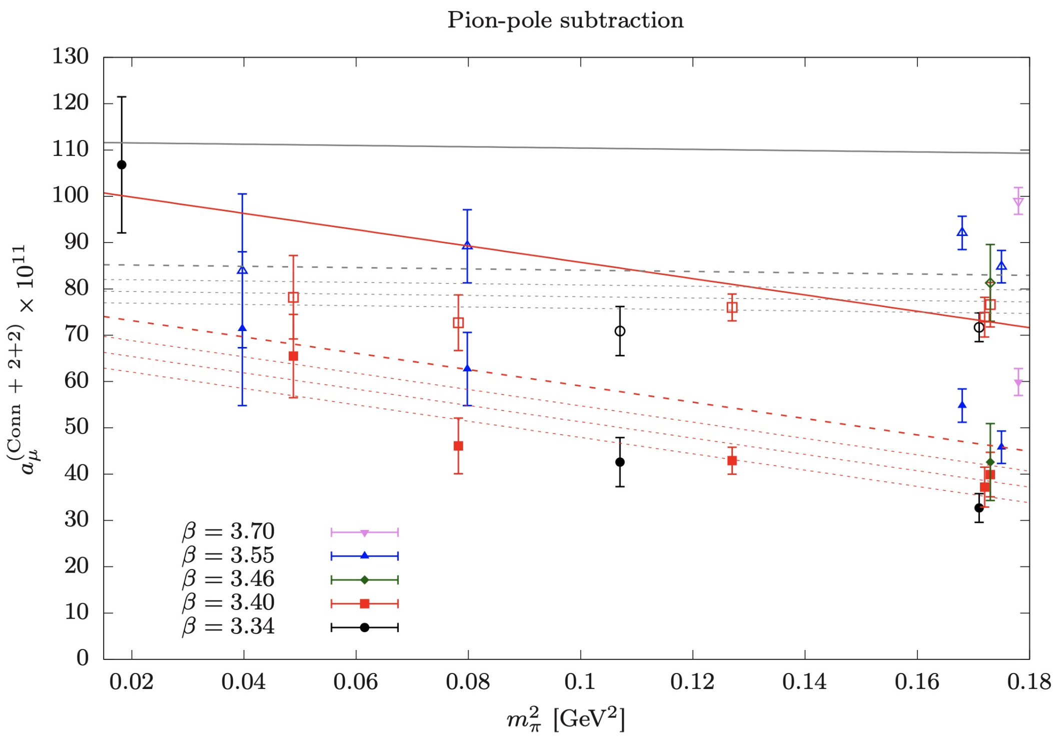
Main sources of error :

- Statistical error increases as $m_\pi \to m_\pi^{\text{phys}}$
- Finite-volume effects $\propto \exp(-m\pi L/2)$
- Continuum extrapolation

- Light quark contribution :

$$a_\mu^{\text{hlbl,light}} = (107.4 \pm 11.3 \pm 9.2) \times 10^{-11}$$

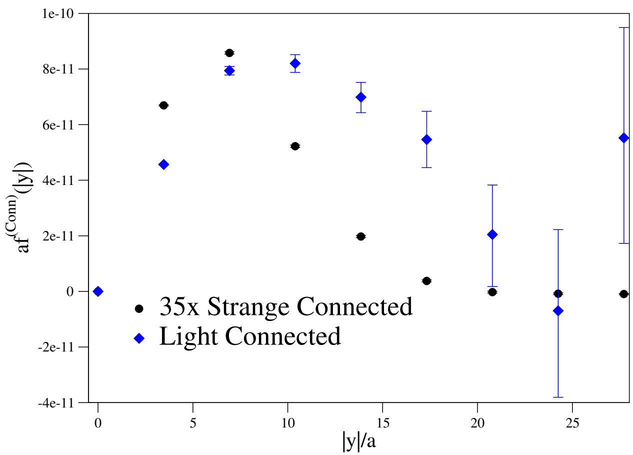




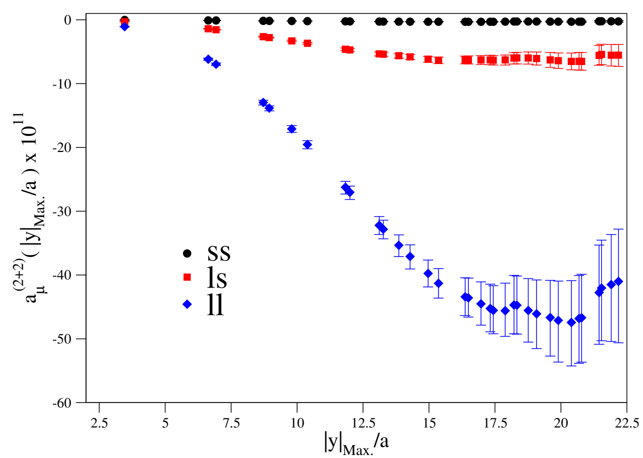
- Filled symbols : original data
- Open symbols : add $a_\mu^{\mathrm{hlbl},\pi^0,\mathrm{phys}}$ and subtract $a_\mu^{\mathrm{hlbl},\pi^0}(a, m_\pi)$

Chiral extrapolation very flat : improving the continuum extrapolation (away from the physical pion mass) might be a better strategy !

Integrand (Conn)



Integrated (2+2)

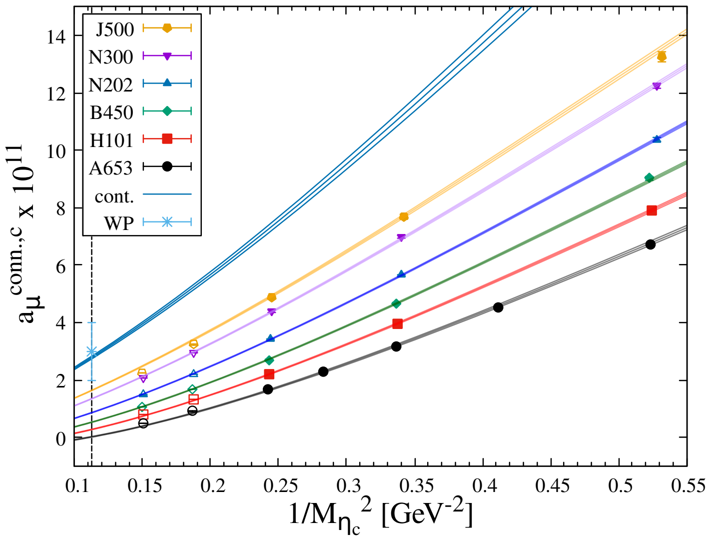


- Highly suppress as compared to light quark (charge factor)
- Strange quark contribution at the physical point :

$$a_\mu^{\mathrm{hlbl,strange}} = (-0.6 \pm 2.0) \times 10^{-11}$$

[Chao, Hudspith, Gérardin, Green, HM arXiv :2204.08844]

- Large lattice artefacts : challenging calculation
- Strategy : unphysical charm quark masses + extrapolation
- Error estimate : model averaging procedure

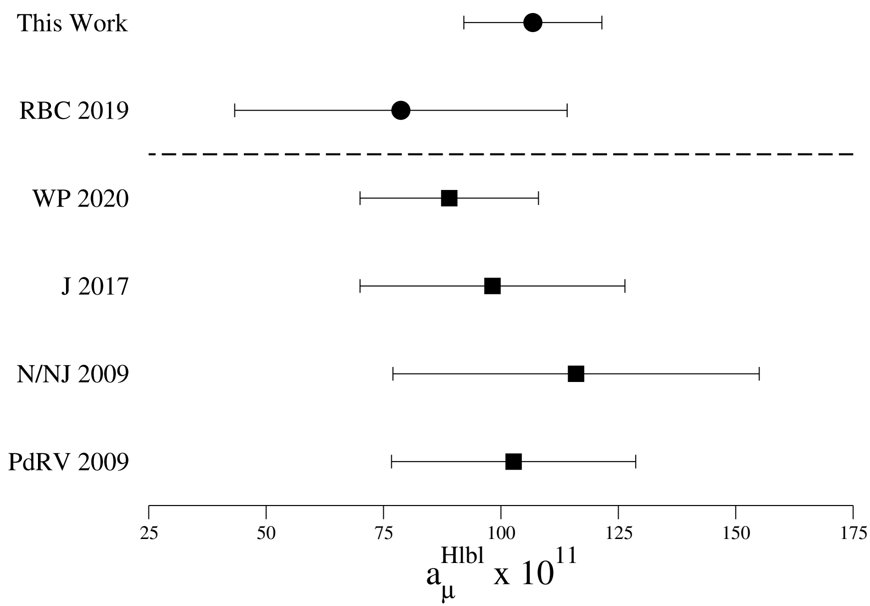


- Charm quark contribution (dominated by the systematic error) :

$$a_\mu^{\mathrm{hlbl,charm}} = (2.8 \pm 0.5) \times 10^{-11}$$

Contribution	Value $\times 10^{11}$
Light-quark connected and $(2+2)$	107.4(11.3)(9.2)
Strange-quark connected and $(2+2)$	$-0.6(2.0)$
Charm-quark connected and $(2+2)$	2.8(0.5)
$(3+1)$	0.0(0.6)
$(2+1+1)$	0.0(0.3)
$(1+1+1+1)$	0.0(0.1)
Total	109.6(15.9)

- Not discussed in details here : sub-leading contributions have been computed and are small $\rightarrow$ challenging to compute ! (lot's of work done by En-Hung Chao)
- Precision of 15%



- Update since WP : complete lattice QCD results.
- Many challenges (continuum extrapolation, finite-volume effects, noisy correlators)
  $\rightarrow$ lattice calculation of pseudoscalar-pole contributions (π^0, η, η') can help there
- Close, but not yet at the target precision ($< 10\%$)

$$\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) = \sum_{A=\mathrm{I,II,III}} \mathcal{G}^{A}_{\delta[\rho\sigma]\mu\alpha\nu\beta\lambda}\, T^{(A)}_{\alpha\beta\delta}(x,y)$$

- $\mathcal{G}^{A}_{\delta[\rho\sigma]\mu\alpha\nu\beta\lambda}$ = traces of gamma matrices $\rightarrow$ sums of products of Kronecker deltas
- The tensors $T^{(A)}_{\alpha\beta\delta}$ are decomposed into a scalar S, vector V and tensor T part

$$T^{(\mathrm{I})}_{\alpha\beta\delta}(x,y) = \partial^{(x)}_{\alpha}(\partial^{(x)}_{\beta} + \partial^{(y)}_{\beta}) V_{\delta}(x,y)$$

$$T^{(\mathrm{II})}_{\alpha\beta\delta}(x,y) = m\partial^{(x)}_{\alpha}\left(T_{\beta\delta}(x,y) + \frac{1}{4}\delta_{\beta\delta}S(x,y)\right)$$

$$T^{(\mathrm{III})}_{\alpha\beta\delta}(x,y) = m(\partial^{(x)}_{\beta} + \partial^{(y)}_{\beta})\left(T_{\alpha\delta}(x,y) + \frac{1}{4}\delta_{\alpha\delta}S(x,y)\right)$$

They are parametrized by six weight functions

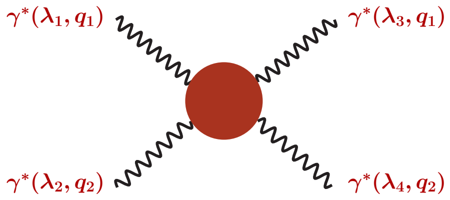
$$S(x,y) = \bar{\mathfrak{g}}^{(0)}$$

$$V_{\delta}(x,y) = x_{\delta}\,\bar{\mathfrak{g}}^{(1)} + y_{\delta}\,\bar{\mathfrak{g}}^{(2)}$$

$$T_{\alpha\beta}(x,y) = (x_{\alpha}x_{\beta} - \frac{x^2}{4}\delta_{\alpha\beta})\,\bar{\mathfrak{l}}^{(1)} + (y_{\alpha}y_{\beta} - \frac{y^2}{4}\delta_{\alpha\beta})\,\bar{\mathfrak{l}}^{(2)} + (x_{\alpha}y_{\beta} + y_{\alpha}x_{\beta} - \frac{x\cdot y}{2}\delta_{\alpha\beta})\,\bar{\mathfrak{l}}^{(3)}$$

- the weight functions depend on the three variables x^2, $x \cdot y = |x||y|\cos\beta$ and y^2
- Semi-analytical expressions for the weight functions have been computed to about 5 digits precision

- Forward scattering amplitudes $\mathcal{M}_{\lambda_3\lambda_4\lambda_1\lambda_2}$



  - 81 helicity amplitudes ($\lambda_i = 0, \pm 1$)

$$\mathcal{M}_{\lambda_1'\lambda_2'\lambda_1\lambda_2} = \mathcal{M}_{\mu\nu\rho\sigma}\ \epsilon^{*\mu}(\lambda_1')\ \epsilon^{*\nu}(\lambda_2')\ \epsilon^{\rho}(\lambda_1)\ \epsilon^{\sigma}(\lambda_2)$$

  - Photons virtualities : $Q_1^2 = -q_1^2 > 0$ and $Q_2^2 = -q_2^2 > 0$
  - Crossing-symmetric variable : $\nu = q_1 \cdot q_2$

- Using parity and time invariance : only 8 independent amplitudes

$$(\mathcal{M}_{++,++} + \mathcal{M}_{+-,+-}),\ \mathcal{M}_{++,--},\ \mathcal{M}_{00,00},\ \mathcal{M}_{+0,+0},\ \mathcal{M}_{0+,0+},\ (\mathcal{M}_{++,00} + \mathcal{M}_{0+,-0}),$$
$$(\mathcal{M}_{++,++} - \mathcal{M}_{+-,+-}),\ (\mathcal{M}_{++,00} - \mathcal{M}_{0+,-0})$$

↪ Either even or odd with respect to ν

↪ The eight amplitudes have been computed on the lattice for different values of ν, Q_1^2, Q_2^2

Strategy :

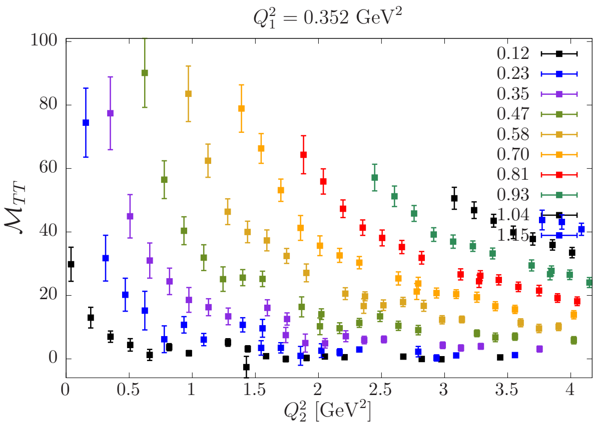
1) Compute all the forward LbL scattering amplitudes on the lattice [Green et. al '15]
2) Use a simple model to describe the lattice data (input : TFFs)
3) Extract information about TFFs by fitting the model parameters to lattice data

→ For each of the eight amplitudes, we have a dispersion relation :

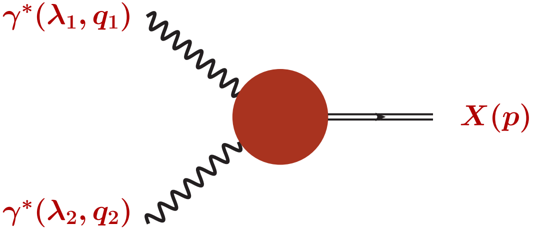
$$\overline{\mathcal{M}}_\alpha(\nu) = \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{X'} \, \sigma_\alpha/\tau_\alpha(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$

Lattice calculation

↪ 4-pt correlation function



$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \mathrm{X}(p_\mathrm{X})$ fusion cross sections



↪ Main contribution is expected from mesons :

Pseudoscalars (0^{-+}) Axial-vectors (1^{++})
Scalar (0^{++}) Tensors (2^{++})

↪ Input : transition form factors

↪ Assume monopole/dipole masses (fit parameters)