MUonE: theory

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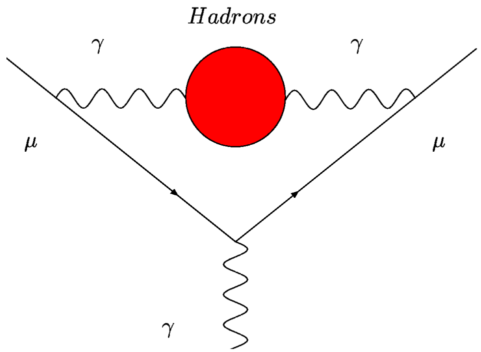
Fifth Plenary Workshop of the Muon g-2 Theory Initiative
University of Edinburgh
September 5-9 2022

- MUonE: basic ideas
- Theory progress
- Experimental progress
- Outlook

} This talk (MUonE: basic ideas; Theory progress)

} Pilato’s talk (Experimental progress; Outlook)

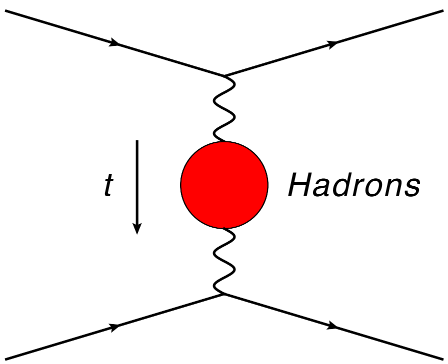
- **Leading hadronic contribution computed via the usual dispersive (timelike) formula:**



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}_{\rm had}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,\left(s/m_\mu^2\right)}$$

- **Alternatively, simply exchanging the x and s integrations:**



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

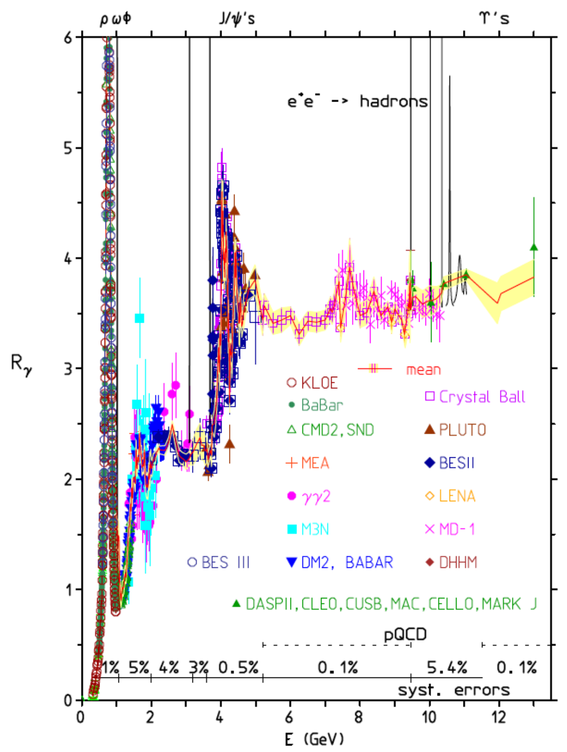
Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{had}(t)$ is the hadronic contribution to the space-like running of α: proposal to measure a_μ^{HLO} via scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

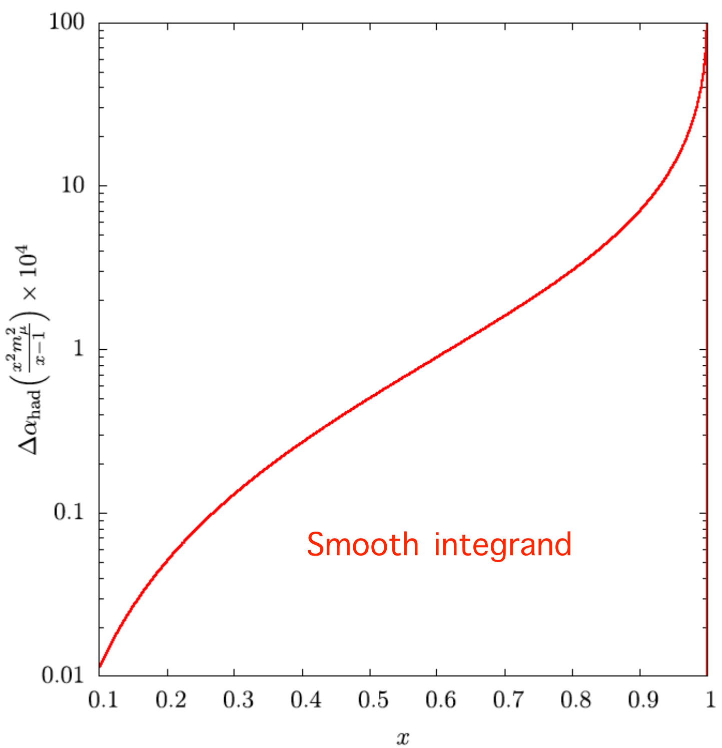
a_μ^{HLO} : time-like vs space-like method

Time-like



F. Jegerlehner, arXiv:1511.04473

Space-like

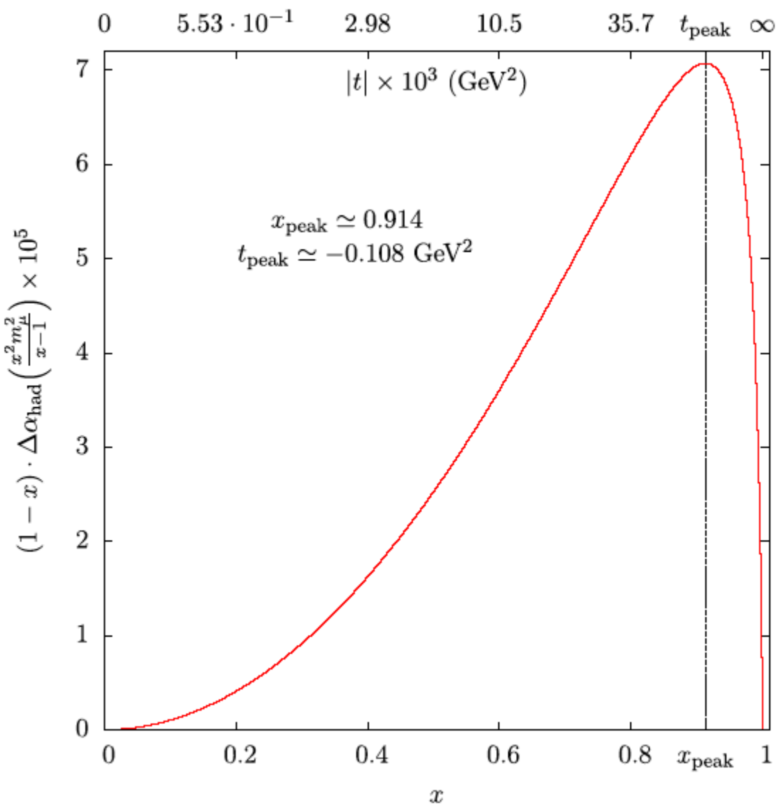


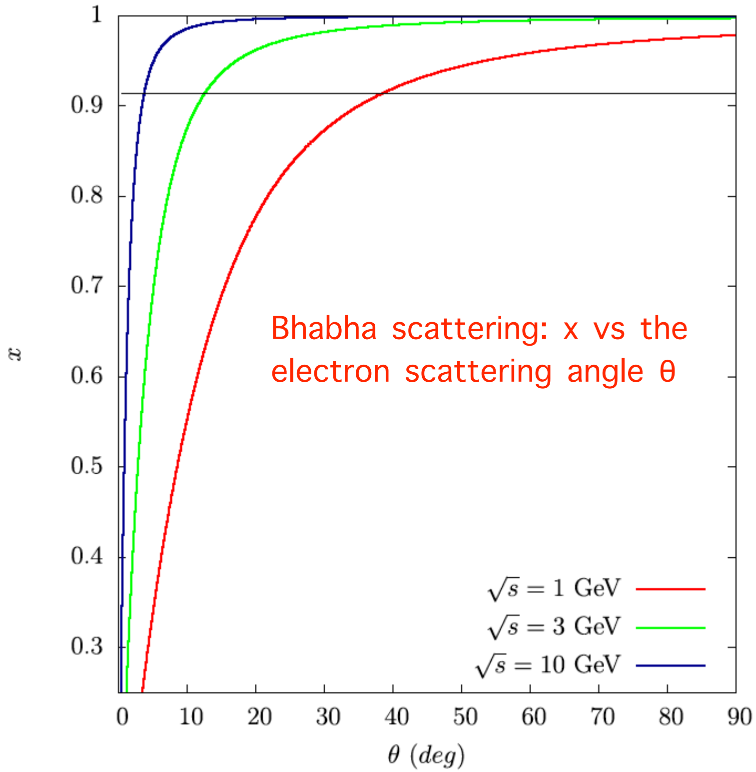
Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

- ☑ **Smooth integrand**
- ☑ **Inclusive measurement**
- ☑ **Direct interplay with lattice QCD**

Space-like method for a_μ^{HLO}: which experiment?

- **Can $\Delta\alpha_{had}(t)$ be precisely measured via Bhabha scattering?**



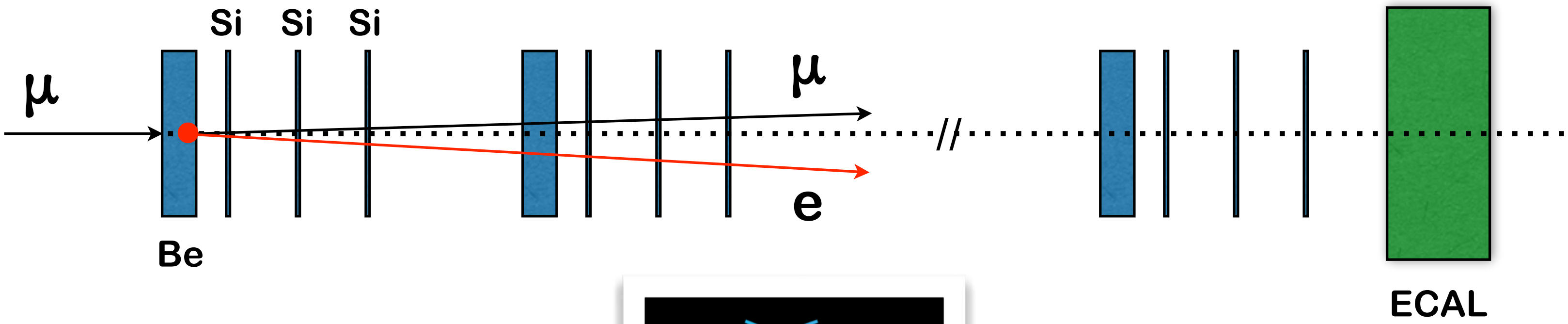


- **The peak occurs at $x_{peak} = 0.914$, $t_{peak} = -0.108$ GeV^2 $\simeq -(330$ MeV$)^2$**

MUonE: muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the **elastic scattering** $\mu\, e \to \mu\, e$.

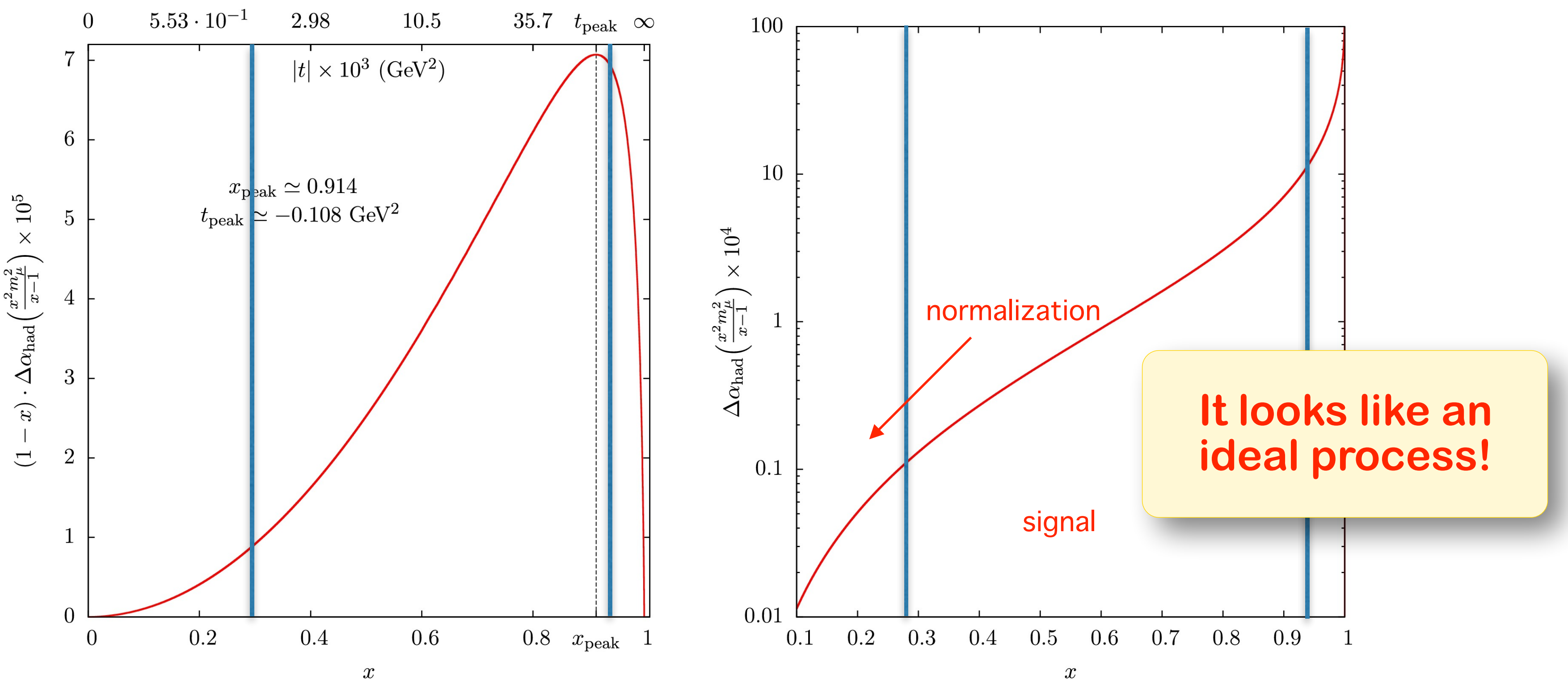
- We proposed to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by three thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

See Pilato's talk

- For a 150 GeV muon beam (√s~400 MeV), MUonE's scan region extends up to x=0.932, ie beyond the x=0.914 peak!



- HVP parametrisations under investigation in full integration domain

Greynat & de Rafael 2022; Colangelo & Jenny 2022; MUonE collaboration

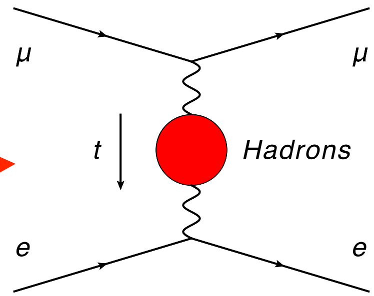
- **Statistics**: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total Be thickness: 60cm), 2-3 years of data taking (2×10^7 s/yr) → $\mathscr{L}_{int} \sim 1.5 \times 10^7\,nb^{-1}$.

- With this $\mathscr{L}_{int}$ we estimate that measuring the shape of $d\sigma/dt$ we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie $\sim 20 \times 10^{-11}$.

- **Systematic** effects must be known at

$$\lesssim 10\text{ppm}$$

- Theory: To extract $\Delta\alpha_{had}(t)$ from this measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at $\lesssim$ 10ppm!



Theory progress

C.M. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni, PLB 746 (2015) 325 [1504.02228 [hep-ph]]
P. Mastrolia, M. Passera, A. Primo, U. Schubert, JHEP 1711 (2017) 198 [1709.07435 [hep-ph]]
S. Di Vita, S. Laporta, P. Mastrolia, A. Primo, U. Schubert, JHEP 1809 (2018) 016 [1806.08241 [hep-ph]]
M. Alacevich et al, JHEP 02 (2019) 155 [1811.06743 [hep-ph]]
M. Fael, JHEP 1902 (2019) 027 [1808.08233 [hep-ph]]
M. Fael and M. Passera, PRL 122 (2019) 192001 [1901.03106 [hep-ph]]
P. Banerjee et al. [The MUonE TI], EPJC 80 (2020) 591 [2004.13663 [hep-ph]]
C. M. Carloni Calame et al, JHEP 11 (2020) 028 [2007.01586 [hep-ph]]
P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027 [2007.01654 [hep-ph]]
R. Bonciani et al, PRL 128 (2022) 022002 [2106.13179 [hep-ph]]
E. Budassi et al, JHEP 11 (2021) 098 [2109.14606 [hep-ph]]
A.V. Nesterenko, JPG 49 (2022) 055001 [2112.05009 [hep-ph]]
E. Balzani, S. Laporta, M. Passera, arXiv:2112.05704 [hep-ph]
M. Fael, F. Lange, K. Schönwald, M. Steinhauser, PRL128 (2022) 172003 [2202.05276 [hep-ph]]
M. Fael, F. Lange, K. Schönwald, M. Steinhauser, PRD 106 (2022) 034029 [2207.00027 [hep-ph]]
D. Greynat and E. de Rafael, JHEP 05 (2022) 084 [2202.10810 [hep-ph]]
E. Budassi, C.M. Carloni Calame, C.L. Del Pio, F. Piccinini, PLB 829 (2022) 137138 [2203.01639 [hep-ph]]

NP:

P.S.B. Dev, W. Rodejohann, X. J. Xu, Y. Zhang, JHEP05 (2020), 053 [2002.04822 [hep-ph]]
A. Masiero, P. Paradisi, M. Passera, PRD102 (2020) 7, 075013 [2002.05418 [hep-ph]]
U. Schubert, C. Williams, PRD100 (2019) 3, 035030
K. Asai, K. Hamaguchi, N. Nagata, S.Y. Tseng, J. Wada, arXiv:2109.10093 [hep-ph]
I. Galon, D. Shih, I.R. Wang, arXiv:2202.08843 [hep-ph]
G. Grilli di Cortona, E. Nardi, PRD 105 (2022) 11, L111701 [2204.04227 [hep-ph]]

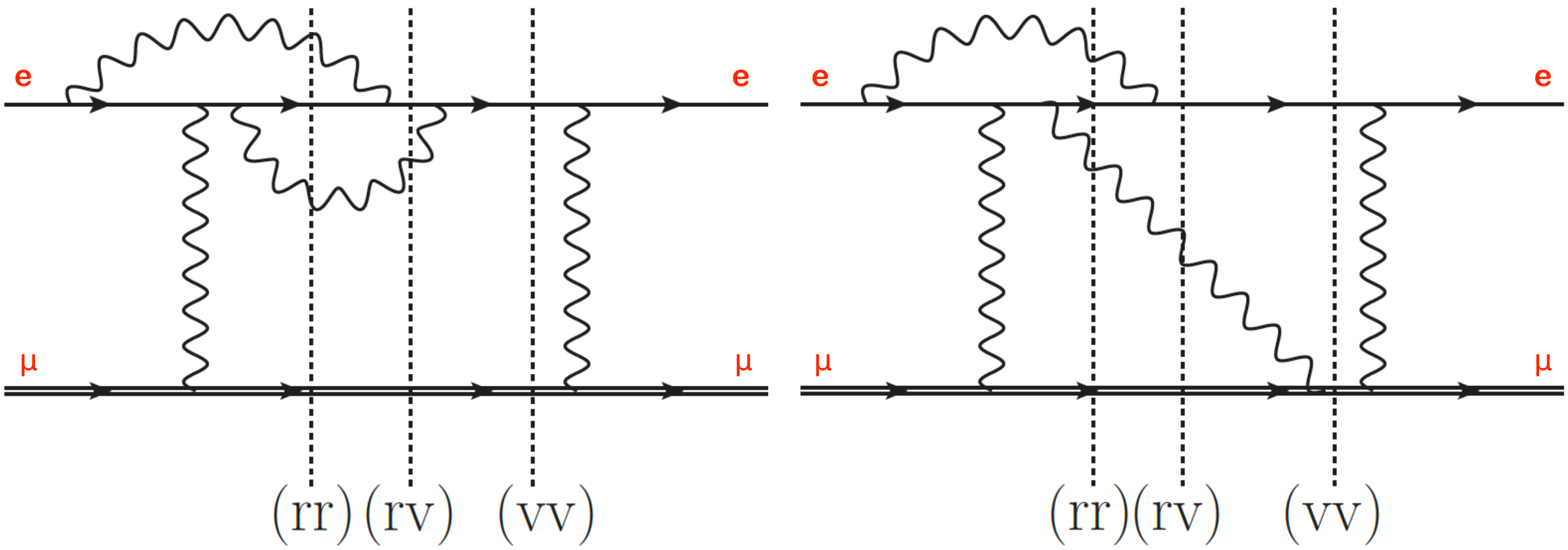
Interplay with NNLO MC for e^+e^- data. See G. Venanzoni's talk

Theory of μe scattering: NLO & NNLO corrections

- **Full set of QED NLO corrections computed & checked**
- **Fully differential fixed-order MC @ NLO ready**
- **EW NLO corrections known but not required at 10ppm level**

Pavia & PSI groups 2018-19

- **QED corrections at NNLO. Examples of contributions:**



MUonE Theory Initiative, 2004.13663

Complete NNLO results not yet available, but large TH effort under way.

Theory of μe scattering: NNLO corrections (II)

Mesmer	Pavia: Budassi, Carloni Calame, Chiesa, Del Pio, Hasan, Montagna, Nicrosini, Piccinini
McMule	PSI: Banerjee, Coutinho, Engel, Gurgone, Hagelstein, Kollatzsch, Naterop, Rocco, Schalch, Sharkovska, Signer, Ulrich

- **Two Monte Carlo event generators built including:**
  - **Exact NLO corrections**
  - **Exact NNLO virtual leptonic and hadronic corrections**
  - **Exact NNLO real leptonic pairs emission**
  - **Single π^0 production (real hadron production doesn't affect $\Delta\alpha_h$ extraction)**
  - **Almost exact NNLO photonic corrections (approximated virtual corrections due to diagrams with at least two photons connecting μ and e lines)**
- **The typical size of these RC is a few 10^{-4} (basic acceptance cuts) but it can grow in corners of the phase space.**
- **Missing (at least): full NNLO & resummation of higher-order corrections matched to exact NNLO.**

Theory of μe scattering: NNLO corrections (III)

- All missing master integrals for 2-loop box diagrams computed (m_e=0, full muon mass dependence)

Padova: Di Vita, Laporta, Mastrolia, MP, Primo, Schubert, 2017-18

- Fully analytic evaluation of the 2-loop QED amplitude completed (m_e=0)!

Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano, PRL (2022) 128

- "Massification" in progress, ie obtaining the leading electron mass terms from the corresponding massless amplitude and virtual soft contribution.

PSI: Engel, Gnendiger, Rocco, Signer, Ulrich, Zoller

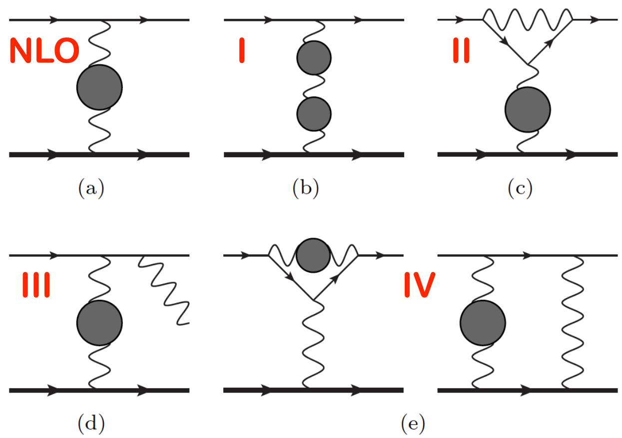
- Full NNLO MC should be ready very soon.

Padova, Pavia, PSI, …

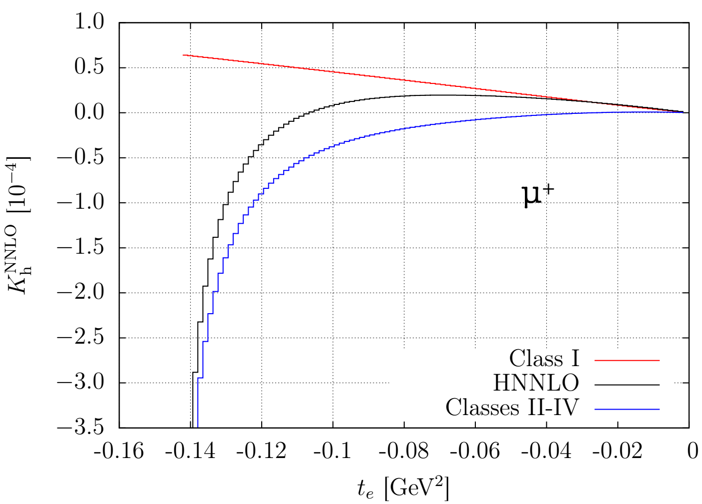
- Resummation: match the NNLO calculations with resummation of the log contributions. Resummation & experimental cuts: Log(of what?)?

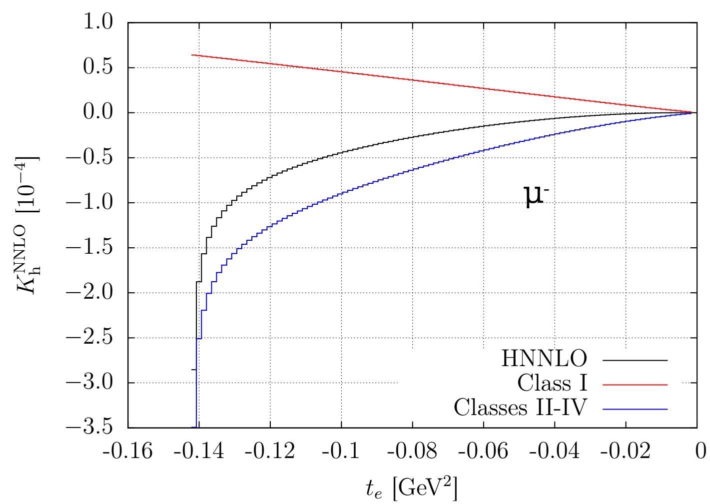
Theory of μe scattering: NNLO hadronic

- **The NNLO hadronic RC are of order $10^{-4} - 10^{-5}$ ⇒ they are crucial**



NB: there are no LbL corrections @ NNLO!

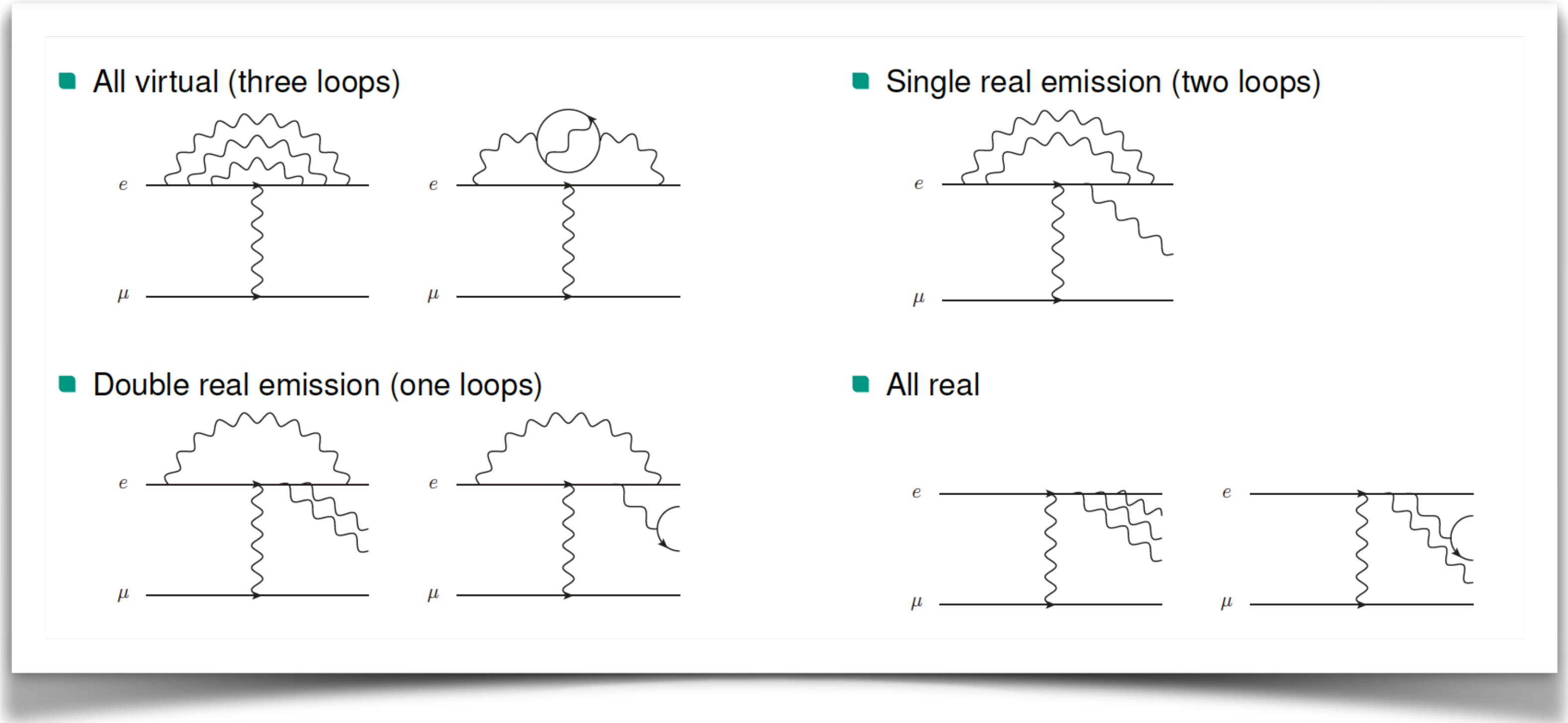




Fael & MP, PRL 2019

- **Can also be computed with MUonE data via the hyperspherical integration method.** Fael JHEP 2019

- **Towards <u>dominant</u> N^3LO corrections for $\mu e \to \mu e$:**



M. Fael, 3rd MUonE Collaboration Meeting, CERN, 11.05.2022

- **Massive vector form factors to three loops computed very recently**

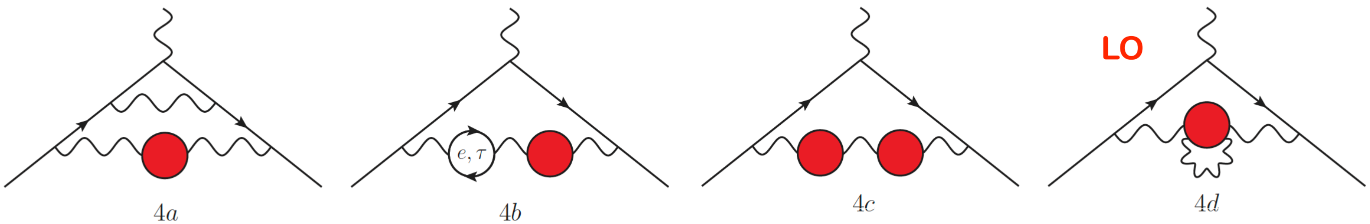
Fael, Lange, Schönwald, Steinhauser, PRL 2022 & PRD 2022

- **Dedicated Durham (IPPP) N^3LO kick-off workstop/thinkstart, Aug 2022**

Y. Ulrich, https://conference.ippp.dur.ac.uk/event/1104

- **NLO HVP contribution:** $a_\mu^{HVP}(NLO) = -98.3\,(7) \times 10^{-11}$

Krause '96; Keshavarzi, Nomura, Teubner 2019; WP20



Normally incorporated in $a_\mu^{HVP}(LO)$. MUonE will naturally include it!

- **Just like at LO,**

$$a_\mu^{\rm HVP}({\rm LO}) = -\frac{\alpha}{\pi^2}\int_{-\infty}^{0}\frac{{\rm d}t}{t}\,\Pi_{\rm h}(t)\,{\rm Im}K^{(2)}(t/m^2) = \frac{\alpha}{\pi}\int_0^1 {\rm d}x\,(1-x)\,\Delta\alpha_{\rm h}(t(x))$$

the NLO space-like kernel is ready for MUonE:

$$a_\mu^{(4a)} = \left(\frac{\alpha}{\pi}\right)^2\int_0^1 {\rm d}x\,\kappa^{(4)}(x)\,\Delta\alpha_{\rm h}(t(x)), \qquad \kappa^{(4)}(x) = \frac{2-x}{x\,(x-1)}\,2F^{(4)}(x-1)$$

$$F^{(4)}(u) = R_1(u) + R_2(u)\ln(-u) + R_3(u)\ln(1+u) + R_4(u)\ln(1-u) + \\ + R_5(u)\left[4{\rm Li}_2(u) + 2{\rm Li}_2(-u) + \ln(-u)\ln\left((1-u)^2(1+u)\right)\right]$$

$R_i(u)$ are rational functions

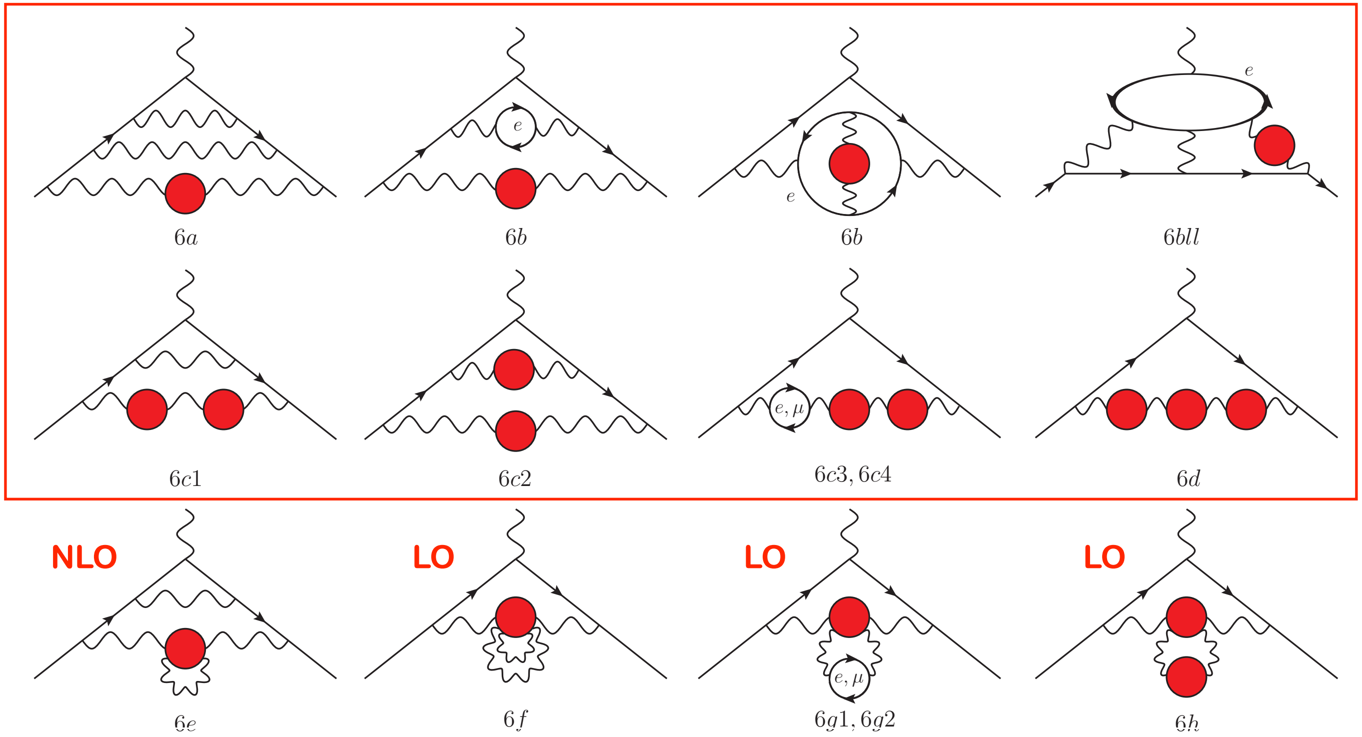
Nesterenko 2112.05009. Balzani, Laporta, MP 2112.05704

See Balzani & Nesterenko's talks

- **NNLO HVP contribution:**

$$a_\mu^{HVP}(NNLO) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014
Comparable to the final uncertainty expected at Fermilab



- **Also the NNLO space-like kernels are ready for MUonE!**
- **The NLO & NNLO space-like kernels can also be employed in lattice QCD.**

Balzani, Laporta, MP, 2112.05704

New physics in MUonE's $\Delta\alpha_h$ extraction?

- MUonE will be very precise → could NP affect its $\Delta\alpha_h$ extraction? Using existing experimental bounds we showed that this is unlikely:

- Consider "light" or "heavy" mediators [ie, mass lower or higher than MUonE's energy scale of O(1 GeV)]:

  - Heavy NP — EFT formalism:

    S & T effects suppressed by electron mass and, for T, also by $(g\text{-}2)_e$

    P doesn't interfere with QED

    V & A effects excluded by $e^+e^- \to \mu^+\mu^-$ data (mainly)

    LFV effects excluded by muonium-antimuonium oscillation limits

  - Light NP — spin 0 and 1 mediators:

    ALPs, Dark Photons and light Z' bosons effects excluded by direct searches and dipole moments. LVF excluded by muonium oscillation

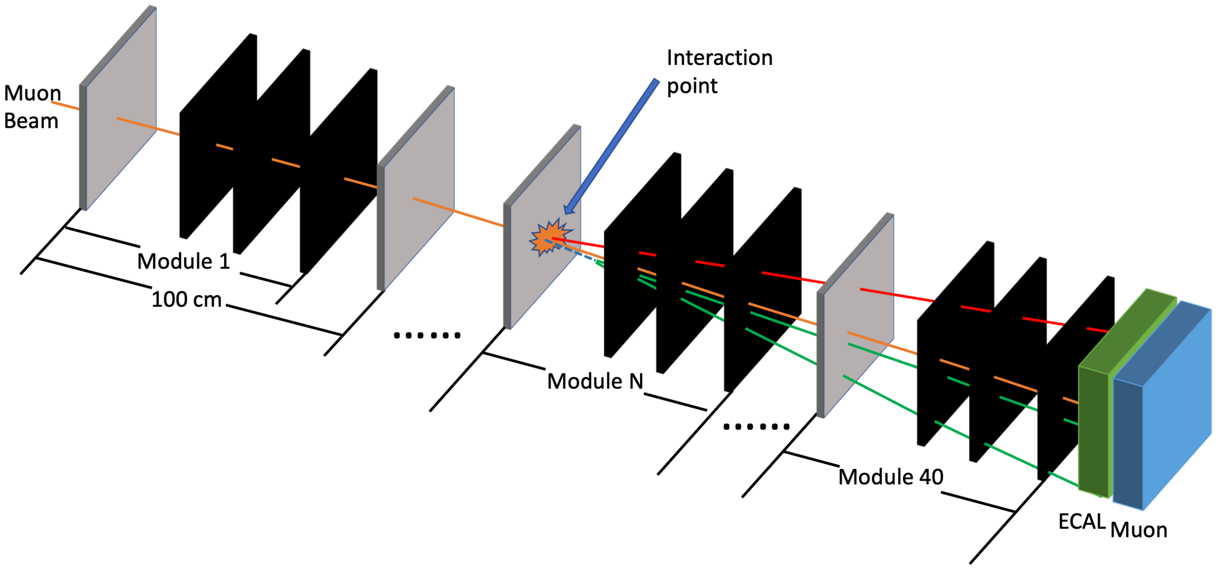
- NB: MUonE's extraction of $\Delta\alpha_h$ will not be sensitive to NP signals which are constant in t relative to the LO QED one!

Dev, Rodejohann, Xu, Zhang, JHEP 2020
Masiero, Paradisi, MP, PRD 2020

- Probing new light mediators at MUonE via the $\mu e \to \mu e X$ process?
  - Invisibly decaying Z' production in $\mu e \to \mu e Z'$

    Asai, Hamaguchi, Nagata, Tseng, Wada, 2109.10093
  - Long-lived mediators with displaced vertex signatures

    Galon, Shih, Wang, 2202.08843



- … or via μ scattering off the target nuclei $\mu N \to \mu N X$?

  Grilli di Cortona, Nardi, 2204.04227

MUonE: theory workshops

Muon-electron scattering:
Theory kickoff workshop

4-5 September 2017

Padova
Europe/Rome timezone

- **Overview**
- Venue
- Timetable
- Logistic
- Map

Support

MUonE theory workshops: Padova 2017, MITP Mainz 2018, Zurich 2019

Next MUonE theory workshop: MITP Mainz Nov 14-18 2022

Conclusions

- MUonE will provide a new independent determination of the HVP contribution to a_μ alternative to the dispersive and lattice ones.

- Lots of theory progress for MUonE: two independent MC in progress, fully analytic evaluation of the two-loop QED amplitude completed, studies towards dominant N3LO corrections, new NLO and NNLO space-like kernels ready, possible new physics searches, etc ...

- ... but lots of work still needed!