QED corrections in $\bar{B} \to \bar{K}\ell^+\ell^-$ at the fully differential level: An EFT approach

QED in Weak Decays Workshop

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Lepton Flavour Universality (LFU) predicted by SM.

One can thus define *lepton flavour universality* ratios, such as R_K:

$$R_K\left[q^2_{\min}, q^2_{\max}\right] = \frac{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \frac{d\Gamma\left(B\to K\mu^+\mu^-\right)}{dq^2}}{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \frac{d\Gamma(B\to Ke^+e^-)}{dq^2}},$$

where $q^2 = \left(\ell^+ + \ell^-\right)^2$.

Naively expect $R_K = 1 + \mathcal{O}(\frac{\alpha}{\pi})$, whereas LHCb [2103.11769] reports

$$R_K\left[1.1\mathrm{GeV}^2, 6\mathrm{GeV}^2\right] = 0.846^{+0.042+0.013}_{-0.039-0.012}$$

This represents a *3.1 σ deviation* from the SM.

Motivation

Why are QED corrections to $\bar{B} \to \bar{K}\ell^+\ell^-$ important?

QED corrections are expected to be small, since $\frac{\alpha}{\pi} \approx 2{\cdot}10^{-3}$.

Due to kinematic effects however, QED corrections are enhanced to $\mathcal{O}(\frac{\alpha}{\pi}) \ln \hat{m}_\ell \gtrsim 2 - 3\,\%$ [*Note:* $\hat{m}_\ell \equiv \frac{m_\ell}{m_B}$].

Moreover, R_K is a theoretically *clean observable*.

Therefore, need to make sure QED corrections properly accounted for in experiments (PHOTOS).

Also, precise determination of CKM matrix elements.

Based on 2009:00929 [G. Isidori, SN, R. Zwicky] and 2205.08635 [G. Isidori, D. Lancierini, SN, R. Zwicky]

and future work to come...

Motivation

Bordone et al. [1605.07633] already performed a calculation to estimate QED corrections in $\bar{B} \to \bar{K}\ell^+\ell^-$ and R_K, working in single differential in q^2.

In our work,

- Results at the *full (double)* differential level are given, and hence they can be used for angular analysis (moments). Moreover, knowledge of the lepton angles are necessary for *applying cuts* on the photon energy.
- We work with *full matrix elements* (real and virtual), starting from an *EFT Lagrangian description*. Hence, we can capture effects beyond collinear $\ln \hat{m}_\ell$ terms, such as $\ln \hat{m}_K$ which are not necessarily so small.
- We present a *detailed discussion on IR divergences*, and demonstrate explicitly the conditions under which they cancel.

We use an *EFT*, for $\bar{B}(p_B) \to \bar{K}(p_K)\,\ell^+(\ell_2)\,\ell^-(\ell_1)$.

$$\mathcal{L}_{\rm int}^{\rm EFT} = g_{\rm eff}\, L^\mu V_\mu^{\rm EFT} + \text{h.c.}\ ,$$

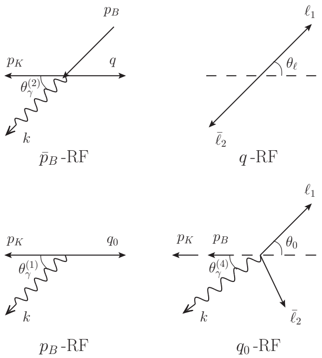
$$V_\mu^{\rm EFT} = \sum_{n\geq 0} \frac{f_\pm^{(n)}(0)}{n!}(-D^2)^n[(D_\mu B^\dagger)K \mp B^\dagger(D_\mu K)]\ ,$$

where D_μ is the covariant derivative and $f_\pm^{(n)}(0)$ denotes the $n^{\rm th}$ derivative of the $B \to K$ *form factor* $f_\pm(q^2)$.

$$\begin{aligned} H_0^\mu(q_0^2) &\equiv \langle \bar{K}|V_\mu|\bar{B}\rangle = f_+(q_0^2)(p_B+p_K)^\mu + f_-(q_0^2)(p_B-p_K)^\mu \\ &= \langle \bar{K}|V_\mu^{\rm EFT}|\bar{B}\rangle + \mathcal{O}(e), \end{aligned}$$

$$L_\mu \equiv \bar{\ell}_1\Gamma^\mu \ell_2\,, \quad V_\mu \equiv \bar{s}\gamma_\mu(1-\gamma_5)b\,,$$

$$g_{\rm eff} \equiv \frac{G_F}{\sqrt{2}}\lambda_{\rm CKM}, \qquad \Gamma^\mu \equiv \gamma^\mu(C_V + C_A\gamma_5) \qquad C_{V(A)} = \alpha\frac{C_{9(10)}}{2\pi}$$



$$\{q_a^2, c_a\} = \begin{cases} q_\ell^2 = (\ell_1 + \ell_2)^2, & c_\ell = -\left(\frac{\vec{\ell_1} \cdot \vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q-\mathrm{RF}} \quad [\text{"Hadron collider"}] , \\ q_0^2 = (p_B - p_K)^2 , & c_0 = -\left(\frac{\vec{\ell_1} \cdot \vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q_0-\mathrm{RF}} \quad [\text{"B-factory"}] , \end{cases}$$

where $q - \mathrm{RF}$ and $q_0 - \mathrm{RF}$ denotes the rest frames of $q \equiv \ell_1 + \ell_2$ and $q_0 \equiv p_B - p_K = q + k$ respectively.

For the *real contribution* to the differential rate, we implement a *physical cut-off on the photon energy* (based on the visible kinematics),

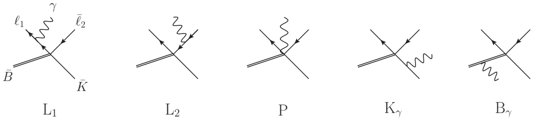
$$\bar{p}_B^2 \equiv m_{B_{\rm rec}}^{\,2} = (p_B - k)^2 = (\ell_1 + \ell_2 + p_K)^2.$$

with

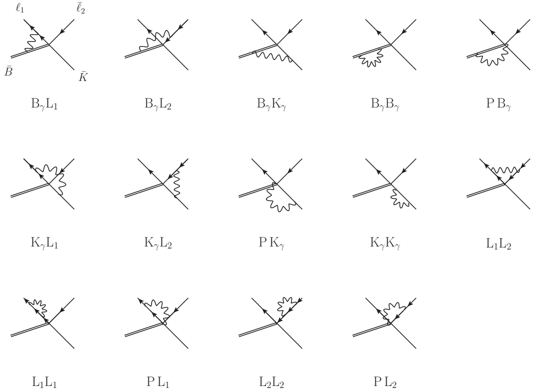
$$\bar{p}_B^2 \ \geq \ m_B^2\,(1 - \delta_{\rm ex})\,,$$

For the *virtual contribution*, since there is *no photon-emission*, there is no difference between the $\{q^2, c_\ell\}$- and $\{q_0^2, c_0\}$-variables.

Real Amplitudes:



Virtual Amplitudes:



$\Longrightarrow$ *Explicit gauge invariance*

The real integrals are split into *IR sensitive parts* which can be done *analytically* and a necessarily regular part which is dealt with numerically.

$$\mathcal{F}_{ij}^{(a)}(\delta_{\rm ex}) = \frac{d^2\Gamma^{\rm LO}}{dq^2 dc_\ell}\tilde{\mathcal{F}}_{ij}^{(s)}(\omega_s) + \tilde{\mathcal{F}}_{ij}^{(hc)(a)}(\underline{\delta}) + \Delta\mathcal{F}_{ij}^{(a)}(\underline{\delta}) \; ,$$

with $\tilde{\mathcal{F}}_{ij}^{(s)}$ ($\tilde{\mathcal{F}}_{ij}^{(hc)(a)}$) containing all *soft* (*hard-collinear*) singularities, whereas $\Delta\mathcal{F}$ is regular.

We adopt the *phase space slicing method*, which requires the introduction of two auxiliary (unphysical) cut-offs $\omega_{s,c}$,

$$\omega_s \ll 1 \; , \quad \frac{\omega_c}{\omega_s} \ll 1 \; .$$

[*Note: Hard-collinear* $\equiv \ln \hat{m}_\ell$ *sensitive terms.*]

Phase Space slicing conditions

$$\bar{p}_B^2 \geq m_B^2 (1 - \omega_s) \iff E_\gamma^{p_B-\mathrm{RF}} \leq \frac{\omega_s m_B}{2},$$

$$k \cdot \ell_{1,2} \leq \omega_c m_B^2$$

All soft divergences cancel between real and virtual, independent of the choice of differential variables.

In the collinear limit ($k||\ell_1$), the matrix element squared factorises:

$$|\mathcal{A}^{(1)}_{\ell_1||\gamma}|^2 = \frac{e^2}{(k\cdot\ell_1)}\hat{Q}^2_{\ell_1}\tilde{P}_{f\to f\gamma}(z)|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 + \mathcal{O}(m^2_{\ell_1})\ ,$$

where $|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 = |\mathcal{A}^{(0)}_{\bar{B}\to\bar{K}\ell_{1\gamma}\bar{\ell}_2}|^2$ and $\tilde{P}_{f\to f\gamma}(z)$ is the collinear part of the splitting function for a fermion to a photon

$$\tilde{P}_{f\to f\gamma}(z) \equiv \left(\frac{1+z^2}{1-z}\right)\ .$$

z gives the momentum fraction of the photon and lepton.

$$\ell_1 = z\ell_{1\gamma}, \quad k = (1-z)\,\ell_{1\gamma}$$

which then implies

$$q^2 = zq_0^2$$

Lower limit on z integration: Depends on the cut-off δ_{ex}.

In $\{q_0^2, c_0\}$ variables, *when fully photon inclusive*,

$$\left.\frac{d^2\Gamma}{dq_0^2 dc_0}\right|_{\ln \hat{m}_{\ell_1}} = \frac{d^2\Gamma^{\,\mathrm{LO}}}{dq_0^2 dc_0} \left(\frac{\alpha}{\pi}\right) \hat{Q}_{\ell_1}^2 \ln \hat{m}_{\ell_1} \times C_{\ell_1}^{(0)} \,,$$

where

$$C_{\ell_1}^{(0)} = \left[\frac{3}{2} + 2\ln \bar{z}(\omega_s)\right]_{\tilde{\mathcal{F}}^{(hc)}} + \left[-1 - 2\ln \bar{z}(\omega_s)\right]_{\tilde{\mathcal{F}}^{(s)}} + \left[\frac{3}{2} - 2\right]_{\tilde{\mathcal{H}}} = 0$$

On the other hand, in $\{q^2, c_\ell\}$ variables,

$$\left.\frac{d^2\Gamma}{dq^2 dc_\ell}\right|_{\mathrm{hc}} = \frac{\alpha}{\pi}\left(\hat{Q}_{\ell_1}^2 K_{\mathrm{hc}}(q^2, c_\ell) \ln \hat{m}_{\ell_1} + \hat{Q}_{\ell_2}^2 K_{\mathrm{hc}}(q^2, -c_\ell) \ln \hat{m}_{\ell_2}\right) \,,$$

where $K_{\mathrm{hc}}(q^2, c_\ell)$ is a non-vanishing function.

After integration over q^2 and c_ℓ, the above vanishes.

However, with a cut-off δ_{ex}, collinear logs survive in both differential variables!

Q: Do we miss any $\ln \hat{m}_\ell$ contributions due to structure dependence, by doing an EFT calculation?

A: No, gauge invariance ensures that there are no such additional contributions. [Sec. 3.4, Isidori, SN, Zwicky '20]

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$\implies$ *Structure Dependent Contributions:* See Roman's talk! [Ongoing].

Results

We consider *relative* corrections. For a single differential in $\frac{d}{dq_a^2}$,

$$\Delta^{(a)}(q_a^2; \delta_{\mathrm{ex}}) = \left(\frac{d\Gamma^{\mathrm{LO}}}{dq_a^2}\right)^{-1} \frac{d\Gamma(\delta_{\mathrm{ex}})}{dq_a^2}\bigg|_{\alpha},$$

where the numerator and denominator are integrated separately over $\int_{-1}^{1} dc_a$ respectively.

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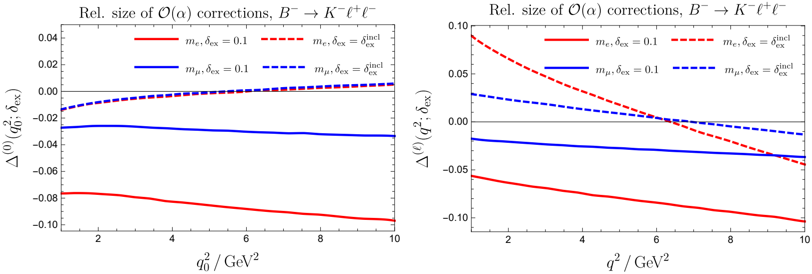
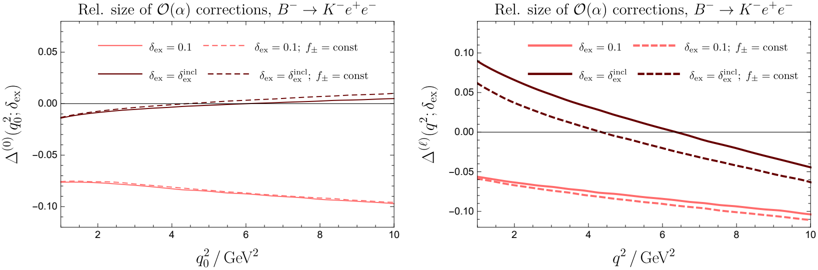
QED corrections are taken into account in the experimental analysis. $\Longrightarrow$ See Davide's talk!

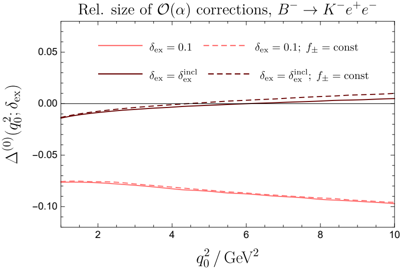


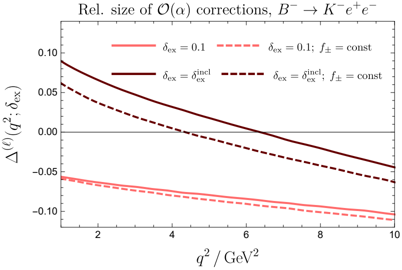
- In photon-inclusive case ($\delta_{ex} = \delta_{ex}^{inc}$, dashed lines), all IR sensitive terms cancel in the q_0^2 variable locally.
- (Approximate) lepton universality on the plots on the left.
- δ_{ex} effects are sizeable since hard-collinear logs do not cancel in that case. More pronounced for electrons.
- In charged case, we see finite effects of the $\mathcal{O}(2\%)$ due to $\ln \hat{m}_K$ effects which do not cancel.

Results
Distortion of the $\bar{B} \to \bar{K}\ell^+\ell^-$ spectrum



Effects are more prominent in the photon-inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$) since there is more phase space for the q^2- and q_0^2-variables to differ. *In fact, a fixed q^2 probes the full range of q_0^2 in that case!!*





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Could be problematic for probing R_K in $q^2 \in [1.1, 6]$ GeV^2 range, due to charmonium resonances!

ℓ	$m_B^{\rm rec}$[GeV]	$\delta_{\rm ex}$	$(q_0^2)_{\rm max}$
μ	5.175	0.0486	$q^2 + 1.36$ GeV^2
e	4.88	0.146	$q^2 + 4.07$ GeV^2

- $(q_0^2)_{\rm max} = q^2 + \delta_{\rm ex} m_B^2$ for zero angle between the photon and the radiating particle.
- Photon energy cut-off on the muon is tighter, so the migration of radiation effect is smaller.

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Thus for $q^2 = 6\,\text{GeV}^2$, in the electron case, the system probes the pole location of the first charmonium resonance, but not the second one:

$$m_{\Psi(2S)}^2 \approx 13.6\,\text{GeV}^2 > (q_0^2)_{\rm max} > m_{J/\Psi}^2 \approx 9.58\,\text{GeV}^2.$$

The net QED correction that should be applied to R_K according to our analysis amounts to

$$\Delta_{\mathrm{QED}} R_K \approx \left.\frac{\Delta\Gamma_{K\mu\mu}}{\Gamma_{K\mu\mu}}\right|^{m_B^{\mathrm{rec}}=5.175\ \mathrm{GeV}}_{q_0^2\in[1.1,6]\ \mathrm{GeV}^2} - \left.\frac{\Delta\Gamma_{Kee}}{\Gamma_{Kee}}\right|^{m_B^{\mathrm{rec}}=4.88\ \mathrm{GeV}}_{q_0^2\in[1.1,6]\ \mathrm{GeV}^2} \approx +1.7\%$$

$\Longrightarrow$ *Well below the current experimental error reported by LHCb.*

However, effect of cuts can be significant. In Bordone et al. '16, in addition to the above energy cuts, a tight angle cut was also used, and a correction to R_K of

$$\Delta_{\mathrm{QED}} R_K \approx +3.0\% \quad ,$$

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Results
R_K

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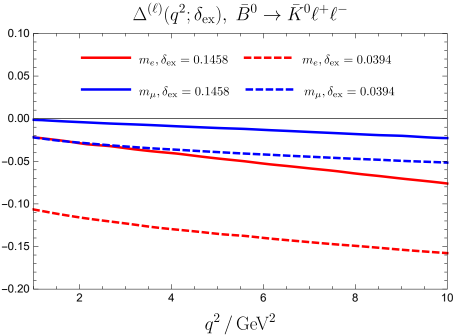
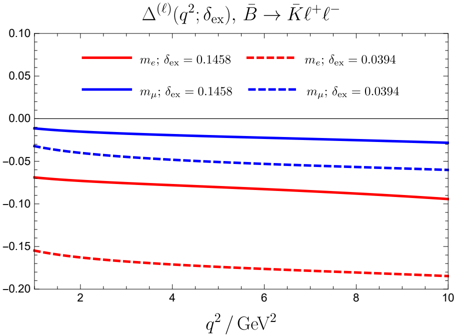
$\Longrightarrow$ *Highlights the importance of building a MC to cross-check the experimental analysis: PHOTOS.*

Results
R_K

Our work



Bordone et al. '16



- The different photon energy cuts for the electron and the muon cases causes the shift in R_K due to QED corrections to be relatively low.

Effect of charmonium resonances
Implementation

2205.08635 [G. Isidori, D. Lancierini, SN, R. Zwicky]

Charmonium resonances implemented through

$$C_9^{\text{eff}}(q^2) = C_9 + \Delta C_9(q^2) \,,$$

$$\Delta C_9(q^2) = \Delta C_9(0) + \sum_{\psi} \eta_\psi e^{i\delta_\psi} \frac{q^2}{m_\psi^2} \frac{m_\psi \Gamma_\psi}{\left(m_\psi^2 - q^2\right) - i m_\psi \Gamma_{J/\psi}} \,,$$

using single-subtracted dispersion relation (at $q^2 = 0$).

The parameter $\eta_{J/\psi}$ is fixed by using the measured values of the branching fractions $\mathcal{B}(\bar{B} \to \bar{K} J/\psi)$ and $\mathcal{B}(J/\psi \to \mu^+\mu^-)$.

Master equation for collinear divergences ($k||\ell_1$)

$$\Delta^{(\ell)}_{\rm hc}(\hat{q}_0^2, c_0) = \frac{\alpha}{\pi}\hat{Q}^2_{\ell_1}\left(\frac{d^2\Gamma^{\rm LO}}{d\hat{q}_0{}^2 dc_0}\right)^{-1}\left(\int_{z_{\ell_1}^{\delta_{\rm ex}}}^1 dz P_{f\to f\gamma}(z)\frac{d^2\Gamma^{\rm LO}}{d\hat{q}_0{}^2 dc_0}\right)\ln\frac{\mu_{\rm hc}}{m_\ell}\;,$$

where $\mu^2_{\rm hc} = \mathcal{O}(m_B^2) \approx 6q_0^2$, and

$$P_{f\to f\gamma}(z) \;=\; \lim_{z^*\to 0}\left[\frac{1+z^2}{(1-z)}\theta((1-z^*)-z) + (\frac{3}{2} + 2\ln z^*)\delta(1-z)\right]\;,$$

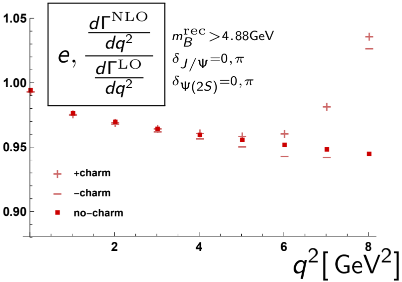
is the splitting function of a fermion to a photon.

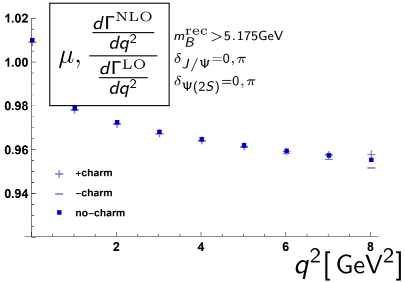
Recall: z is the momentum fraction of the photon-lepton system carries by the lepton ($q^2 = zq_0^2$).

The differential rate factorises from the z-integration in the above variables.

Effect of charmonium resonances:
Results: With window

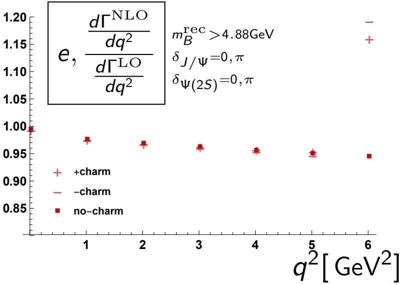
Include contributions from J/ψ and $\psi(2S)$ resonances.

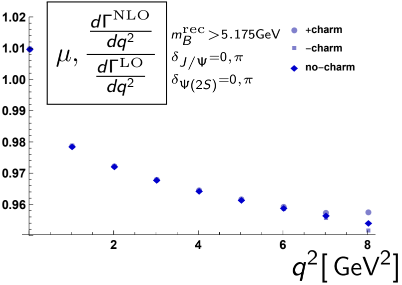




- Peak of the resonance (only modulus squared part) eliminated through a window $\Delta\omega^2 = 0.1\,\mathrm{GeV}^2$ around it.
- For $q^2 < 6\,\mathrm{GeV}^2$, the interference effects are small, even in the electron case, and do not indicate any contamination to R_K in particular.

Effect of charmonium resonances:
Results: Without window





- Without window.
- With an electron-like photon energy cut-off, the peak of the J/ψ is probed at $q^2 = 6\,\mathrm{GeV}^2$, due to migration of radiation effects.

Experimental analysis takes charmonium resonances into account in principle *(incoherently!)*, but...

1506.03970 [Gratrex et al.]

We can expand

$$\frac{d^2\Gamma(B \to K\ell^+\ell^-)}{dq^2\, dc_\ell} = \sum_{l_\ell \geq 0} G^{(l_\ell)} P_{l_\ell}(c_\ell)\,.$$

where $P_{l_\ell}(c_\ell)$ are *Legendre polynomials*.

By restricting to *dimension-6 operators* in the effective Hamiltonian, as well as imposing the *lepton-pair factorisation approximation (LFA)*, we only have S and P waves (operators of spin$=0,1$).

$$\implies l_\ell \leq 2$$

In fact, in our EFT description, the $m_\ell \to 0$ limit gives

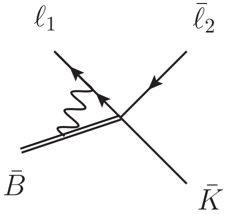
$$\frac{d^2\Gamma_{\mathrm{LO}}(B \to K\ell^+\ell^-)}{dq^2\, dc_\ell} \propto \left(1 - c_\ell^2\right).$$

QED Moments in $\bar{B} \to \bar{K}\ell^+\ell^-$
Generalities

Higher dimensional operators (in the LFA) lead to higher moments, but are suppressed by powers of (m_b/m_W).

On the other hand, QED corrections break LFA - Hence, they can give rise to any $l_\ell \geq 0$ moments.

e.g. in the EFT description, $\ell_1 \cdot p_B$ occurs in logs and dilogs.



Important question: How big are they??

Naively, we can expect them to be relatively large, especially for electrons, due to collinear logs, $\ln(m_\ell/m_B)$.

$$\implies G_e^{(l_\ell>2)} \gg G_\mu^{(l_\ell>2)} \neq 0$$

Motivates experimental measurement of higher moments in $\bar{B} \to \bar{K}\ell^+\ell^-$, to compare with theoretical prediction of QED corrections.

Recall master equation in splitting function formalism $(k||\ell_1)$,

$$\Delta_{\rm hc}^{(\ell)}(\hat{q}_0^2, c_0) = \frac{\alpha}{\pi}\hat{Q}_{\ell_1}^2 \left(\frac{d^2\Gamma^{\rm LO}}{d\hat{q}_0{}^2 dc_0}\right)^{-1} \left(\int_{z_{\ell_1}^{\delta_{\rm ex}}}^{1} dz P_{f\to f\gamma}(z) \frac{d^2\Gamma^{\rm LO}}{d\hat{q}_0{}^2 dc_0}\right) \ln\frac{\mu_{\rm hc}}{m_\ell}$$

The z-integration can in principle introduce logs involving c_ℓ/c_0, making higher moments sensitive to collinear $\ln(m_\ell/m_B)$.

In going from $\{q_0^2, c_0\}$ to $\{q^2, c_\ell\}$ variables, the above equation is further complicated by a *Jacobian*, and the fact that $\frac{d^2\Gamma^{\rm LO}}{d\hat{q}_0{}^2 dc_0}$ does *not factorise* from the integral.

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Still, no odd moments present, since collinear contributions are even in c_ℓ. ($c_\ell \to -c_\ell$ swaps ℓ^+ and ℓ^- in the collinear limit)

- EFT analysis shows that hard collinear logs ($\ln \hat{m}_\ell$) cancel when differential in $\left\{q_0^2, c_0\right\}$ variables when fully photon inclusive.
- Using gauge invariance, it can be shown that there are no further collinear logs from structure-dependent contributions.
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$\Longrightarrow$ *Perform refined q^2-binning for R_K!*

- EFT analysis shows that hard collinear logs ($\ln \hat{m}_\ell$) cancel when differential in $\left\{q_0^2, c_0\right\}$ variables when fully photon inclusive.
- Using gauge invariance, it can be shown that there are no further collinear logs from structure-dependent contributions.
- Charmonium resonances could potentially affect the q^2 bin relevant for R_K.

  $\Longrightarrow$ *Perform refined q^2-binning for R_K!*
- QED corrections enhance higher moments, which are typically suppressed by powers of (m_b/m_W).

Future Work

- Fixing ambiguities in the UV counterterms, and structure-dependent corrections (including $\ln \hat{m}_K$ contributions) [Ongoing].
- Analysis of moments of the angular distribution. Also differential in m_B^{rec}. [Ongoing].
- Charged-current semileptonic decays ($\bar{B} \to D\ell\nu$). Unidentified neutrino in final state makes it hard to reconstruct B meson and to apply a cut-off on photon energy.

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The END

BACKUP SLIDES

The real amplitude can be decomposed,

$$\mathcal{A}^{(1)} = \hat{Q}_{\ell_1} a^{(1)}_{\ell_1} + \delta\mathcal{A}^{(1)} \, ,$$

into a term $\hat{Q}_{\ell_1} a^{(1)}_{\ell_1}$ with all terms proportional to $\hat{Q}_{\ell_1}$, and the remainder $\delta\mathcal{A}^{(1)}$.

$$a^{(1)}_{\ell_1} = -e g_{\text{eff}} \bar{u}(\ell_1) \left[\frac{2\epsilon^* \cdot \ell_1 + \not{\epsilon}^* \not{k}}{2k \cdot \ell_1} \Gamma \cdot H_0(q_0^2) \right] v(\ell_2) \, ,$$

which contains all $1/(k \cdot \ell_1)$-terms.

The structure-dependence of this term is encoded in the form factor H_0.

The amplitude square is given by

$$\sum_{\rm pol} |\mathcal{A}^{(1)}|^2 = \sum_{\rm pol} |\delta\mathcal{A}^{(1)}|^2 - \hat{Q}_{\ell_1}^2 \sum_{\rm pol} |a_{\ell_1}^{(1)}|^2 + 2\hat{Q}_{\ell_1} \mathrm{Re}[\sum_{\rm pol} \mathcal{A}^{(1)} a_{\ell_1}^{(1)*}] \,,$$

where it will be important that $\mathcal{A}^{(1)}$ is gauge invariant.

The *first term* is manifestly free from hard-collinear logs $\ln m_{\ell_1}$.

We use *gauge invariance* and set $\xi = 1$ under which the polarisation sum

$$\sum_{\rm pol} \epsilon_\mu^* \epsilon_\nu = (-g_{\mu\nu} + (1-\xi) k_\mu k_\nu / k^2) \to -g_{\mu\nu}$$

collapses to the metric term only.

The *second term* evaluates to

$$\int d\Phi_\gamma \, \hat{Q}_{\ell_1}^2 \sum_{\text{pol}} |a_{\ell_1}^{(1)}|^2 = \int d\Phi_\gamma \, \hat{Q}_{\ell_1}^2 \frac{\mathcal{O}(m_{\ell_1}^2) + \mathcal{O}(k\cdot\ell_1)}{(k\cdot\ell_1)^2} = \mathcal{O}(1)\, \hat{Q}_{\ell_1}^2 \ln m_{\ell_1}$$

where we used $k - \ell_1 = \mathcal{O}(m_{\ell_1}^2)$, valid in the collinear region.

We now turn to the *third term*.

Using anticommutation relations, $k - \ell_1 = \mathcal{O}(m_{\ell_1}^2)$ in the collinear limit, and the EoMs, we rewrite $a_{\ell_1}^{(1)}$ as

$$a_{\ell_1}^{(1)} = -e g_{\rm eff} \bar{u}(\ell_1) \left[\frac{4\epsilon^* \cdot \ell_1 + m_{\ell_1} \not{\epsilon}^*}{2k \cdot \ell_1} \Gamma \cdot H_0(q_0^2) \right] v(\ell_2) \, ,$$

Gauge invariance $k \cdot \mathcal{A}^{(1)} = 0$ implies $\ell_1 \cdot \mathcal{A}^{(1)} = \mathcal{O}(m_{\ell_1}^2)$ in the collinear region

Therefore, the first part of $a_{\ell_1}^{(1)}$ contributes to

$$\hat{Q}_{\ell_1}\mathrm{Re}[\sum_{\mathrm{pol}} \mathcal{A}^{(1)} a_{\ell_1}^{(1)*}] \to c_1 \hat{Q}_{\ell_1}^2 \frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)^2} + c_2 \hat{Q}_{\ell_1}\hat{Q}_X \frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)}$$

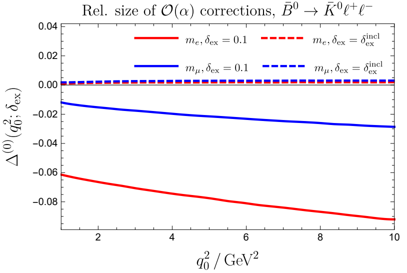
where $X \in \{\bar{B}, \bar{K}, \bar{\ell}_2\}$.

The second part of $a_{\ell_1}^{(1)}$ contributes to

$$\hat{Q}_{\ell_1}\mathrm{Re}[\sum_{\mathrm{pol}} \mathcal{A}^{(1)} a_{\ell_1}^{(1)*}] \to c_1' \hat{Q}_{\ell_1}^2 \frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)^2} + c_2' \hat{Q}_{\ell_1}\hat{Q}_X \frac{\mathcal{O}(m_{\ell_1})}{(k\cdot\ell_1)}$$

Thus, using gauge invariance, one concludes that $\delta\mathcal{A}^{(1)}$ (indicated by terms $\propto \hat{Q}_X$ in the above) does not lead to collinear logs.

Results
$\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$ in q_a^2



- In photon-inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$, dashed lines), all IR sensitive terms cancel in the q_0^2 variable locally.
- (Approximate) lepton universality on the plots on the left.
- Effects due to the photon energy cuts are sizeable since hard-collinear logs do not cancel in that case. More pronounced for electrons.

We consider *relative* QED corrections. For a single differential in $\frac{d}{dq_a^2}$,

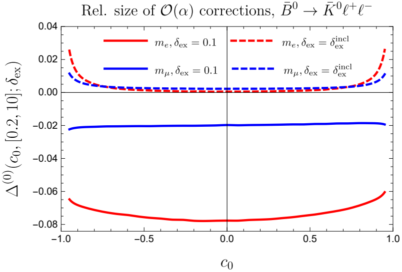
$$\Delta^{(a)}(q_a^2;\delta_{\rm ex}) = \left(\frac{d\Gamma^{\rm LO}}{dq_a^2}\right)^{-1} \frac{d\Gamma(\delta_{\rm ex})}{dq_a^2}\Big|_\alpha ,$$

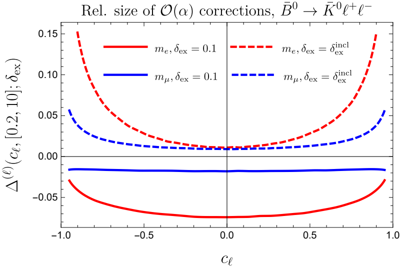
where the numerator and denominator are integrated separately over $\int_{-1}^{1} dc_a$ respectively. In addition, we define the single differential in $\frac{d}{dc_a}$

$$\Delta^{(a)}(c_a,[q_1^2,q_2^2];\delta_{\rm ex}) = \left(\int_{q_1^2}^{q_2^2} \frac{d^2\Gamma^{\rm LO}}{dq_a^2 dc_a} dq_a^2\right)^{-1} \int_{q_1^2}^{q_2^2} \frac{d^2\Gamma(\delta_{\rm ex})}{dq_a^2 dc_a} dq_a^2\Big|_\alpha ,$$

where the non-angular variable is binned.

It is important to integrate the QED correction and the LO separately as this corresponds to the experimental situation.



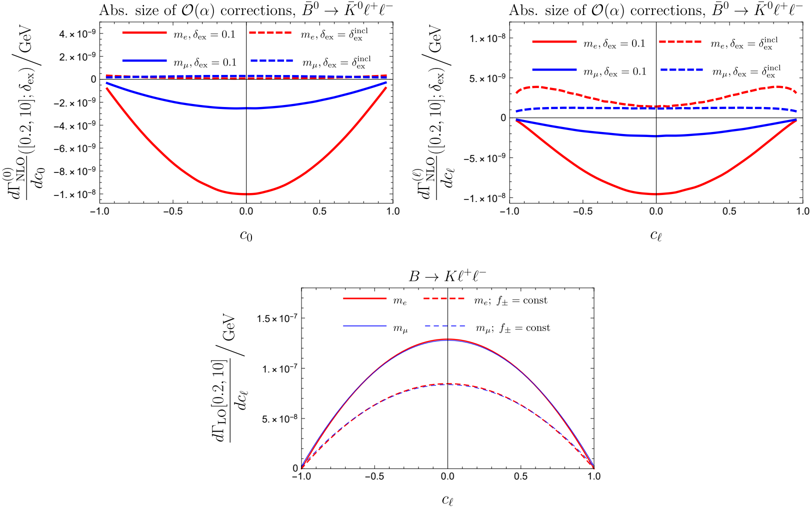


Enhanced effect towards the endpoints $\{-1, 1\}$ is partly due to the special behaviour of the LO differential rate which behaves like $\propto (1 - c_\ell^2) + \mathcal{O}(m_\ell^2)$ and explains why the effect is less pronounced for muons.

Even in c_ℓ. Almost even in c_0 (up to non-collinear effects).

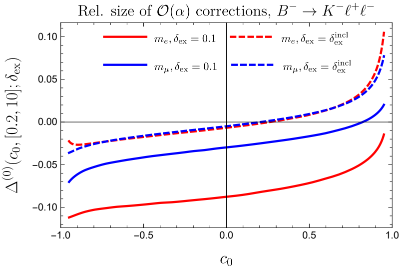
Results

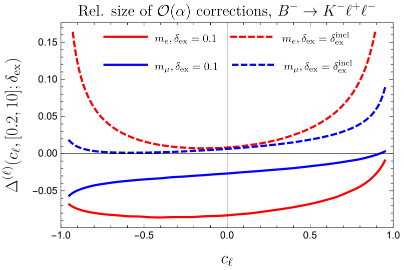
c_a distribution



Results

c_a distribution in charged meson mode

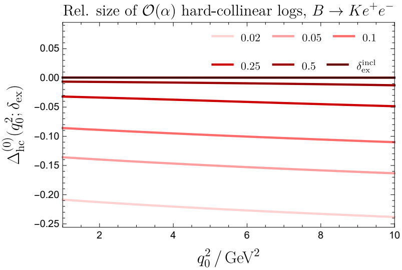


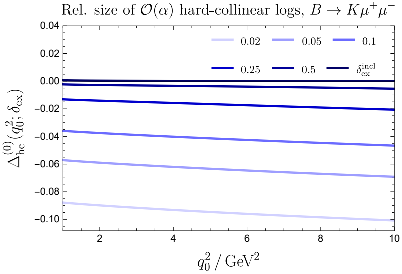


- Same comments as before apply.
- More enhanced than the neutral meson case.
- ‘Collinear’ ln m_K odd in c_0/c_ℓ.

Results

Hard collinear $\ln \hat{m}_\ell$ contributions in q_a^2





- Cancellation of hc $\ln \hat{m}_\ell$ in fully inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$).
- Tighter cut $\implies$ larger corrections
- Electron and muon cases are scaled by a factor $\approx \frac{\ln \hat{m}_e}{\ln \hat{m}_\mu} \approx 2.36$

Tighter cut on electrons than muons $\implies$ Partial compensation $\implies$ QED corrections to R_K 'relatively' small.

To understand the distortion better, consider the following analysis in the collinear region:

$$|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 \propto f_+(q_0^2)^2 = f_+(q^2/z)^2.$$

Since $z < 1$ in general, it is clear that momentum transfers of a higher range are probed.

For example, when $c_\ell = -1$, maximising the effect, one gets

$$z_{\delta_{\rm ex}}(q^2)\Big|_{c_\ell=-1} = \frac{q^2}{q^2 + \delta_{\rm ex} m_B^2} \,, \quad (q_0^2)_{\rm max} = q^2 + \delta_{\rm ex} m_B^2 \,,$$

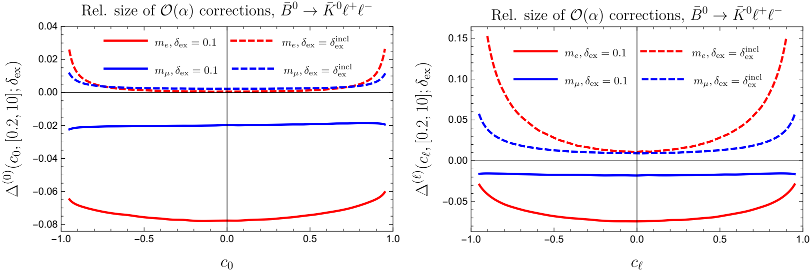
For $\delta_{\rm ex} = 0.15$, $q^2 = 6\,\text{GeV}^2$ one has $(q_0^2)_{\rm max} = 10.18\,\text{GeV}^2$

$\Longrightarrow$ *Problematic for probing R_K in $q^2 \in [1.1, 6]$ GeV^2 range, due to charmonium resonances!*

Furthermore, in photon-inclusive case, the lower boundary for z becomes $z_{\rm inc}(c_\ell)|_{m_K \to 0} = \hat{q}^2$ such that $(q_0^2)_{\rm max} = m_B^2$.

$\Longrightarrow$ *Entire spectrum is probed for any fixed value of q^2*

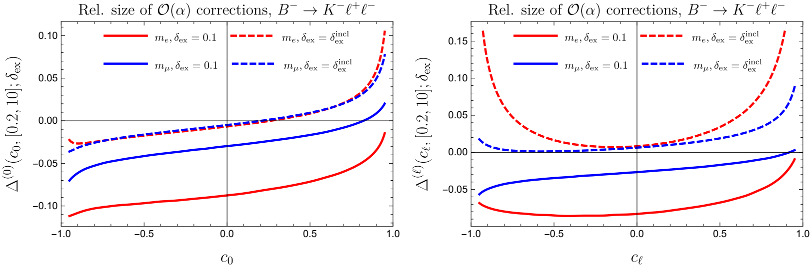
c_0 distribution



Features:

1. Approximate LFU in c_0 with $\delta_{\rm ex}^{\rm inc}$.
2. Even in c_ℓ.
3. (almost) even in c_0, since c_0 measured wrt to ℓ_1 in q_0-RF.

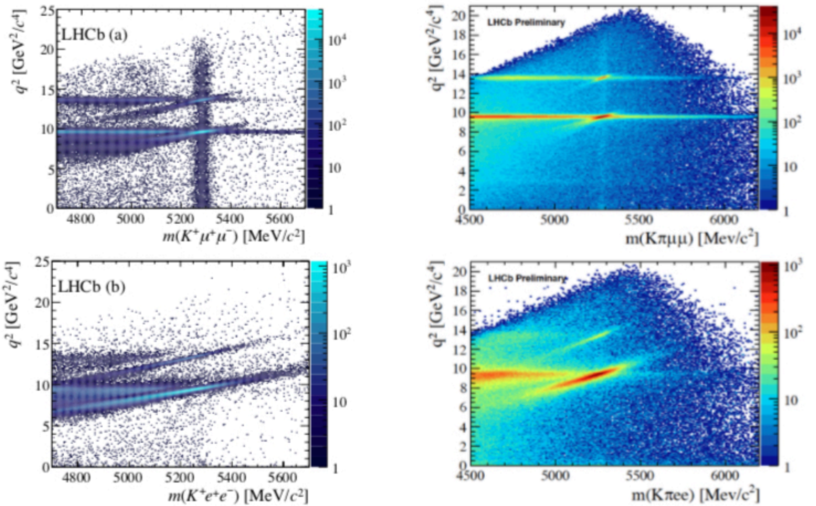
QED Moments in $\bar{B} \to \bar{K}\ell^+\ell^-$
c_ℓ distribution



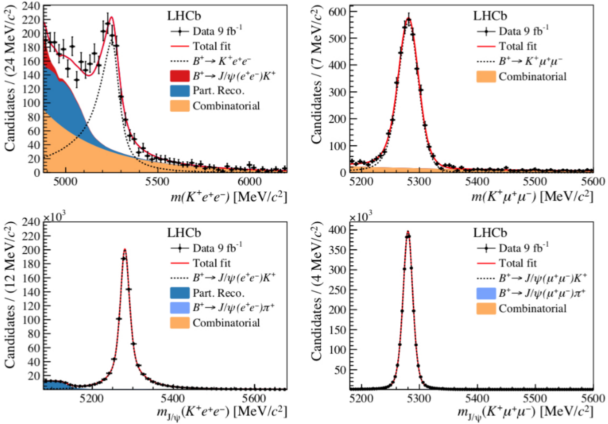
Features:

1. As before, approximate LFU in c_0 with δ_{ex}^{inc}.
2. Effects of odd moments are smallest in c_ℓ, with δ_{ex}^{inc} and $m_\ell = m_e$, as collinear $\ln(m_\ell/m_B)$ (which are even in c_ℓ) dominate.

Heat plots from Borsato's talk:



LHCb plot



- Resonant mode has 10^3 more events than non-resonant mode.
- For the electron case, the non-resonant mode has contributions from $\bar{B} \to J/\psi(e^+e^-)\bar{K}$ due to QED, and loose photon energy cut.