Radiative corrections in chiral perturbation theory

Marc Knecht

Centre de Physique Théorique UMR7332,
CNRS Luminy Case 907, 13288 Marseille cedex 09 - France
knecht@cpt.univ-mrs.fr

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OUTLINE

- Introduction
- QED corrections in pion and kaon decays: the low-energy EFT point of view
- Two case studies or radiative corrections in real life
- Conclusion

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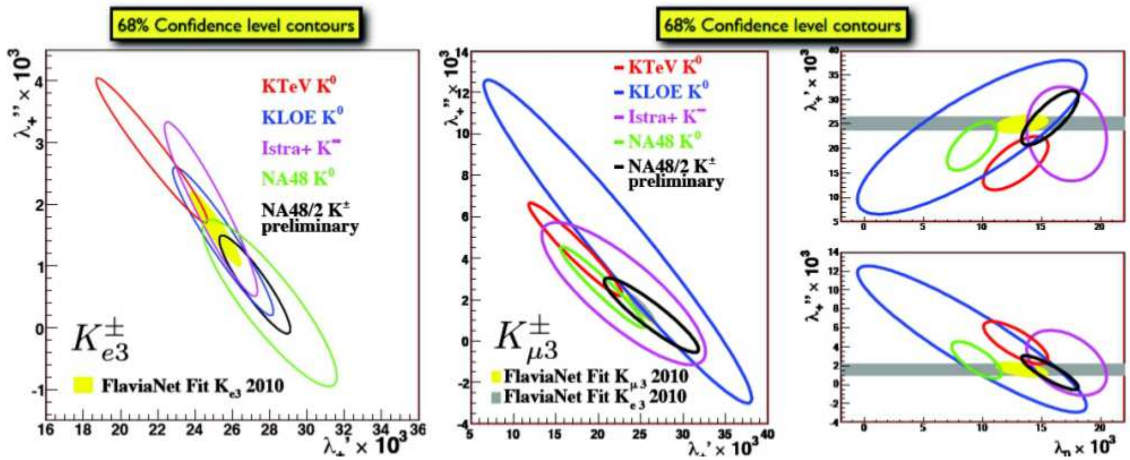
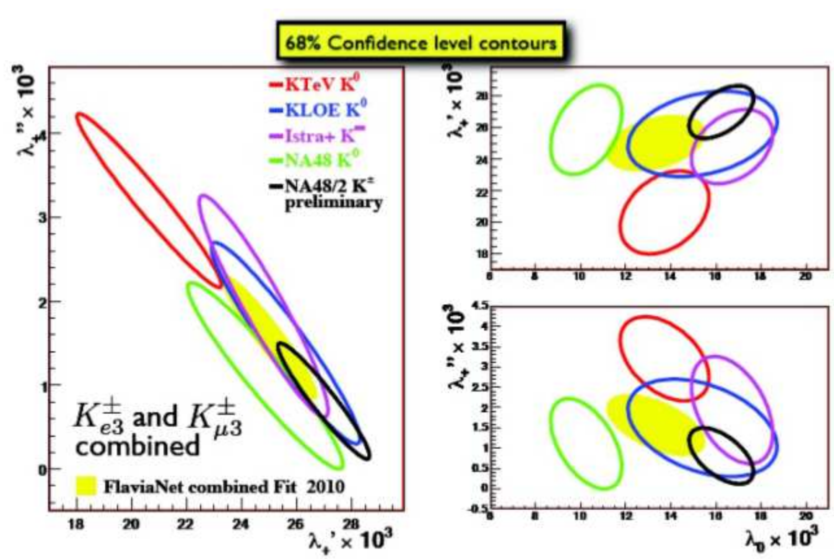
Illustration with structure of $K_{\ell 3}$ form factors...

M. Antonelli et al., Eur Phys. J. C 69, 399 (2010)





D. Madigozhin [NA48/2 Coll.], PoS DIS2013, 135 (2013)

... or with $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$

- Geneva-Saclay high-statistics experiment: $3 \cdot 10^4$ events, a_0 at 20%

L. Rosselet et al., Phys. Rev. D 15, 574 (1977)

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S. Pislak et al., Phys. Rev. 67, 072004 (2003) [Phys. Rev. 81, 119903 (2010)] [hep-ex/0301040]

- NA48/2: $1.1 \cdot 10^6$ events, a_0 at 6%

J. R. Batley et al., Eur. Phys. J. C 70, 635 (2010)

The experimental values of the two S-wave scattering lengths

$$a_0 = 0.222(14) \qquad a_2 = -0.0432(97)$$

compare quite well with the prediction from two-loop chiral perturbation theory

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But taking isospin corrections ($m_u \neq m_d$ and $M_\pi \neq M_{\pi^0}$) into account turns out to be crucial in order to reach this agreement

J. Gasser, PoS KAON , 033 (2008), arXiv:0710.3048 [hep-ph]

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note: $M_{\pi^0} \neq M_{\pi^\pm}$ is an electromagnetic effect!

Some general remarks

Radiative corrections to total decay rates are typically at the level of a few %

$$\Gamma = \Gamma_0 \left[1 + \alpha \frac{\Delta\Gamma}{\Gamma_0} \right] \qquad \alpha \frac{\Delta\Gamma}{\Gamma_0} \sim \pm(1-3)\%$$

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Emission of soft photons can sometimes lift the helicity suppression:
for instance in $B \to \mu\nu_\mu$

$$\left(\frac{M_B}{m_\mu} \right)^2 \times \alpha \qquad \text{is not small...}$$

D. Bećirević, B. Haas, E. Kou, Phys. Lett. B 681, 257 (2009)

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R. Zwicky, Symmetry 13, 2036 (2021)

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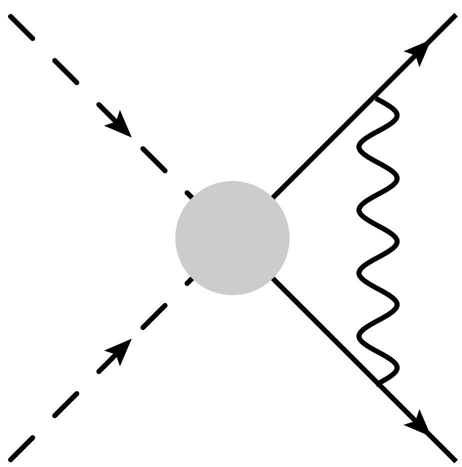

$$\sim \mathcal{M}_0 \times \frac{(-1)}{2} e_1 e_2 \frac{\pi\alpha}{v_{12}((p_1+p_2)^2)}$$

$$v_{12}((p_1+p_2)^2) \equiv \frac{\lambda^{1/2}((p_1+p_2)^2, m_1^2, m_2^2)}{(p_1+p_2)^2 - m_1^2 - m_2^2}$$

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$$|\mathcal{M}_0|^2 \left[1 + \frac{(-1)}{2}\, e_1 e_2 \, \frac{2\pi\alpha}{v_{12}((p_1+p_2)^2)} \right]$$

$\longrightarrow$

$$|\mathcal{M}_0|^2 T(\eta_{12}), \quad T(\eta) = \frac{\eta}{1-e^{-\eta}} = 1 + \frac{\eta}{2} + \cdots, \quad \eta_{12} \equiv -e_1 e_2 \frac{2\pi\alpha}{v_{12}}$$

Most of the time, radiative corrections are small, knowing them at 10% or even 20% relative precision is usually sufficient

G. Martinelli, talk at KAON2016

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Following discussion restricted to the meson sector

For radiative corrections in the baryon sector, see

U.-G. Meißner, S. Steininger, Phys. Lett. B 419, 403 (1998); B 451, 233 (1999)

G. Müller, U.-G. Meißner, Nucl. Phys. B 556, 265 (1999)

QED corrections in pion and kaon decays from the low-energy EFT perspective

For $\mu \ll \Lambda_{\rm had} \sim 1{\rm GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

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$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion, taking into account all the constraints that follow from the spontaneously broken chiral $SU(3)_L \times SU(3)_R$ chiral symmetry

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

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$\longrightarrow$ systematic expansion in powers of $p/\Lambda_{\rm had}$ (with $m_q \sim p^2$, i.e. $M_\pi^2 \sim p^2$)

$\longrightarrow$ starts at $\mathcal{O}(p^2)$, power counting consistent with loop expansion

$$\mathcal{L}^{\rm str}(2) = \frac{F_0^2}{4}\langle \partial^\mu U^\dagger \partial_\mu U\rangle - \frac{<\bar{q}q>}{2}\langle \mathcal{M}(U+U^\dagger)\rangle + \cdots$$

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- strong interactions among mesons at low-energies

$$\mathcal{L}^{\rm str} = \mathcal{L}_2^{\rm str}(2) + \mathcal{L}_4^{\rm str}(10+0) + \mathcal{L}_6^{\rm str}(90+23) + \mathcal{L}_8^{\rm str}(1233{+}??) + \cdots$$

J. Gasser, H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

J. Bijnens, G. Colangelo, G. Ecker, JHEP 02, 020 (1999); Annals Phys. 280, 100 (2000)

J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23, 539 (2002)

T. Ebertshaüser, H. W. Fearing, S. Scherer, Phys. Rev. D 65, 054033 (2002)

J. Bijnens, N. Hermansson-Truedsson, S. Wang, JHEP 01 (2019)

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$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_{\rm F}}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

- $Q_i(\mu)$ $\longrightarrow$ four-quark operators (current-current and QCD penguin operators
- $C_i(\mu)$ $\longrightarrow$ perturbative QCD corrections from M_W down to $\mu \lesssim m_c$
- contains both $SU(3)_L$ octet and 27-plet components

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$$\mathcal{L}_{\rm eff}^{\Delta S=1} \longrightarrow \mathcal{L}_2^{\Delta S=1}(1+1) + \mathcal{L}_4^{\Delta S=1}(22+28) + \cdots$$

J. A. Cronin, Phys. Rev. 161, 1483 (1967)

J. Kambor, J. H. Missimer, D. Wyler, Nucl. Phys. B 346, 17 (1990)

G. Esposito-Farese, Z. Phys. C 50, 255 (1991)

G. Ecker, J. Kambor, D. Wyler, Nucl. Phys. B 394, 101 (1993)

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- factorized form, the description of semi-leptonic decays amounts to the evaluation of the relevant form factors

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Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedom

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q,U] \quad Q = \mathrm{diag}(2/3,-1/3,-1/3) \quad eQ \sim p$$

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G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

R. Urech, Nucl. Phys. B 433, 234 (1995)

H. Neufeld, H. Rupertsberger, Z. Phys. C 71, 131 (1996)

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The construction extends in a straightforward manner to the weak sector

$$\mathcal{L}^{\Delta S=1;\mathrm{EM}} = \mathcal{L}_2^{\Delta S=1;EM}(1) + \mathcal{L}_4^{\Delta S=1;EM}(14+?) + \cdots$$

$$\mathcal{L}_2^{\Delta S=1;EM} = e^2 G_8 F_0^6 \mathrm{g}_{\mathrm{weak}} \langle \lambda_{23} U^\dagger Q U \rangle \qquad \mathcal{L}_4^{\Delta S=1;EM} = e^2 G_8 F_0^4 \sum_{i=1}^{14} Z_i \mathcal{O}_i^{\Delta S=1;EM}$$

J. Bijnens, M. B. Wise, Phys. Lett. B 137, 245 (1984)

G. Ecker, G. Isidori, Müller, H. Neufeld, A. Pich, Nucl. Phys. 591, 1419 (2000)

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$\longrightarrow$ need to also include the light leptons in the low-energy EFT

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q,U] + iU\sum_\ell(\bar{\ell}\gamma_\mu\nu_{\ell L}Q_{\mathrm{w}} + \overline{\nu_L}\gamma_\mu\ell Q_{\mathrm{w}}^\dagger) \quad Q_{\mathrm{w}} = -2\sqrt{2}G_F\begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{L}^{\mathrm{lept}} = \mathcal{L}_2^{\mathrm{lept}}(0) + \mathcal{L}_4^{\mathrm{lept}}(5) + \cdots \qquad \mathcal{L}_2^{\mathrm{lept}}(0) = \sum_\ell[(\bar{\ell}(i\not{\partial} + e\not{A} - m_\ell)\ell + \overline{\nu_{\ell L}}i\not{\partial}\nu_{\ell L}]$$

$$\mathcal{L}_4^{\mathrm{lept}} = \sum_{i=1}^{5} X_i\mathcal{O}_i^{\mathrm{lept}}$$

M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)

Crucial issue: determination of low-energy constants

- K_i

- identify the corresponding QCD correlators (two-, three- and four-point functions), in the chiral limit, convoluted with the free photon propagator

- study their short-distance behaviour

- write spectral sum rules

- saturate with lowest-lying narrow-width resonances

B. Moussallam, Nucl. Phys. B 504, 391 (1997) [hep-ph/9701400]

B. Ananthanarayan, B. Moussallam, JHEP06, 047 (2004) [hep-ph/0405206]

Analogous to the DGMLY sum-rule for C

$$C = -\frac{1}{16\pi^2}\frac{3}{2\pi}\int_0^\infty ds\, s\,\ln\frac{s}{\mu^2}\,[\rho_{VV}(s) - \rho_{AA}(s)]$$

T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, Phys. Rev. Lett. 18, 759 (1967)

B. Moussallam, Eur. Phys. J. C 6, 681 (1999) [hep-ph/9804271]

Crucial issue: determination of low-energy constants

• X_i

- two-step matching procedure:

i) compute radiative corrections to $\bar{q}q' \to \ell\nu$ in the SM and in the four-fermion theory

ii) match the radiatively corrected four-fermion theory to the chiral lagrangian, by identifying the QCD correlators (convoluted with the free photon propagator) that describe the X_i's
Saturate the resulting spectral sum rules with lowest-lying resonance states

S. Descotes-Genon, B. Moussallam, Eur. Phys. J. C 42, 403 (2005) [hep-ph/0505077]

• $\mathrm{g}_{\mathrm{weak}}$ and Z_i

Have been estimated in the large-N_c limit

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)

For instance

$$(g_8 e^2 \mathrm{g}_{\mathrm{weak}})^{\infty} = -\left(\frac{\langle\bar{\psi}\psi\rangle}{F_0^3}\right)^2 \left[3C_8(\mu) + \frac{16}{3}e^2 C_6(\mu)(K_9 - 2K_{10})\right]$$

Crucial issue: determination of low-energy constants

The dependence on the short-distance scale vanishes at leading-order in the large-N_c limit. A scale dependence remains at subleading order in $1/N_c$. The (subleading order) contribution of Q_7 can also be computed,

$$(g_8 e^2 \mathrm{g}_{\mathrm{weak}})^{1/N_c;Q_7} = -\frac{9}{8\pi^2} C_7(\mu) \frac{M_\rho^2}{F_0^2} \left[\ln \frac{\mu^2}{M_\rho^2} + \frac{1}{3} - 2 \ln 2 \right]$$

M. K., S. Peris, E. de Rafael, Phys. Lett. B 457, 227 (1999)

but this does not completely remove the residual scale dependence

Applications to many examples (non-exhaustive list)

- $\pi \to \ell\nu_\ell(\gamma)$ and $K \to \ell\nu_\ell(\gamma)$
  M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)
  V. Cirigliano, I. Rosell, JHEP 0710, 005 (2007)
  J. Gasser, G. R. S. Zarnauskas, Phys. Lett. B 693, 122 (2010)
  V. Cirigliano, H. Neufeld, Phys. Lett. B 700, 7 (2011)

- $K \to \pi\ell\nu_\ell(\gamma)$
  V. Cirigliano, M. K., H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. C 23, 121 (2002)
  A. Kastner, H. Neufeld, Eur. Phys. J. C 57, 541 (2008)
  V. Cirigliano, M. Giannotti, H. Neufeld, JHEP 0811, 006 (2008)
  J. Gasser, B. Kubis, N. Paver, M. Verbeni, Eur. Phys. J. C 40, 205 (2005)

- $\pi^+ \to \pi^0 e\nu_e$
  V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003)

- $K^+ \to \pi^+\pi^-\ell\nu_\ell$
  V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274
  A. Nehme, Nucl. Phys. B 682, 289 (2004)
  P. Stoffer, Eur. Phys. J. C 74, 2749 (2014)

- $K \to \pi\pi$
  V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Phys. Rev. Lett. 91, 162001 (2003)
  V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)
  V. Cirigliano, G. Ecker, A. Pich, Phys. Lett. B 679, 445 (2009)

- $K \to \pi\pi\pi$
  J. Bijnens, F. Borg, Nucl Phys. B 697, 319 (2004); Eur. Phys. J. C 39, 347 (2005); C 40, 383 (2005)

- $\ldots$
  V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84, 399 (2012)

Case study I: the pion β decay

Possible source of information on $|V_{ud}|$

Many advantages:

- pure vector transition (like super-allowed Fermi transitions, but in contrast to neutron β decay)

- no problem with nuclear transition matrix elements in evaluation of radiative corrections (like neutron β decay, but in contrast to super-allowed Fermi transitions)

- protected from first-order isospin breaking corrections (Ademollo-Gatto-Sirlin theorem)

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V. Cirigliano, M. Knecht, H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003) [hep-ph/0209226]

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$\longrightarrow$ cleanest way to extract V_{ud}

Serious drawback: $\Gamma_{\pi_\beta}/\Gamma_{\rm tot} \sim 1 \cdot 10^{-8}$

PIBETA exp. at PSI: $\Gamma_{\pi_\beta}/\Gamma_{\rm tot} = [1.036 \pm 0.004_{\rm stat} \pm 0.004_{syst} \pm 0.003_{\pi_{e2}}] \cdot 10^{-8}$

$\sim 10^6 \pi^+/{\rm sec}, 6.4 \cdot 10^4$ events $\longrightarrow V_{ud}^{\rm PIBETA} = 0.9739(30)$

D. Pocanić et al. (PIBETA Coll.), Phys. Rev. Lett. 93, 181803 (2004) [hep-ex/0312030]

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PIONEER proposal at PSI could deliver $\sim 7 \cdot 10^5$ ($\sim 7 \cdot 10^6$) events during phase II (III)

W. Altmannshofer *et al.* [PIONEER], [arXiv:2203.01981 [hep-ex]]

Case study II: K_{e4}^{00}

NA48/2 has measured the two $K_{e4}^{\pm}$ channels:

K_{e4}^{+-} [i.e. $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e$], about 10^6 events

J. R. Batley et al. [NA48/2 Coll.], Phys. Lett. B 715, 105 (2012)

K_{e4}^{00} [i.e. $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu_e$], about $6.5 \cdot 10^4$ events (unitarity cusp in $M_{\pi^0\pi^0}$ seen)

J. R. Batley et al. [NA48/2 Coll.], JHEP 1408, 159 (2014)

In the isospin limit, there is a form factor common to the two matrix elements, whose normalization f_s can thus be measured in both decay distribution

$$|V_{us}|f_s[K_{e4}^{+-}] = 1.285 \pm 0.001_{\rm stat} \pm 0.004_{\rm syst} \pm 0.005_{\rm ext},$$

$$(1+\delta_{EM})|V_{us}|f_s[K_{e4}^{00}] = 1.369 \pm 0.003_{\rm stat} \pm 0.006_{\rm syst} \pm 0.009_{\rm ext}$$

i.e.

$$(1+\delta_{EM})\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010$$

where δ_{EM} is an unspecified coefficient supposed to account for unknown radiative corrections

Can one understand this 6.5% effect in terms of isospin breaking?

$\longrightarrow$ need to understand how radiative corrections were treated in the K_{e4}^{+-} mode...

Treatment of radiative corrections in the data analyses:

K_{e4}^{00}: no radiative corrections whatsoever applied (hence the factor $\delta_{\rm EM}$!)

K_{e4}^{+-}:

– Sommerfeld-Gamow-Sakharov factors applied to each pair of charged legs

– Corrections induced by emission of real photons treated with PHOTOS

Z. Wąs et al., Comp. Phys. Comm. 79, 291 (1994); Eur. Phys. J. C 45, 97 (2006); C 51, 569 (2007);
Q.-J. Xu, Z. Wąs, Chin. Phys. C 34, 889 (2010)

– PHOTOS also implements (1 loop QED) w.f.r. on the external charged legs and virtual photon exchanges between charged external legs [$\longrightarrow$ no IR divergences], based on

Y. M. Bystritskiy, S. R. Gevorkian, E. A. Kuraev, Eur. Phys. J. C 67, 47 (2009)

– All structure-dependent corrections are discarded (gauge invariant truncation)

– UV divergences not treated

$$\left(|V_{us}|^2 G_F^2\right)^{\text{bare}} \left(1 - \frac{9}{4}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right) = |V_{us}|^2 G_F^2 \qquad [K_{e4}^{+-}]$$

– UV divergences not treated

$$\left(|V_{us}|^2 G_F^2\right)^{\text{bare}} \left(1 - \frac{9}{4}\frac{\alpha}{\pi} \ln \frac{\Lambda^2}{M_\pi^2}\right) = |V_{us}|^2 G_F^2 \quad [K_{e4}^{+-}]$$

– but

$$\left(|V_{us}|^2 G_F^2\right)^{\text{bare}} \left(1 - \frac{3}{4}\frac{\alpha}{\pi} \ln \frac{\Lambda^2}{M_\pi^2}\right) = |V_{us}|^2 G_F^2 \quad [K_{e4}^{00}]$$

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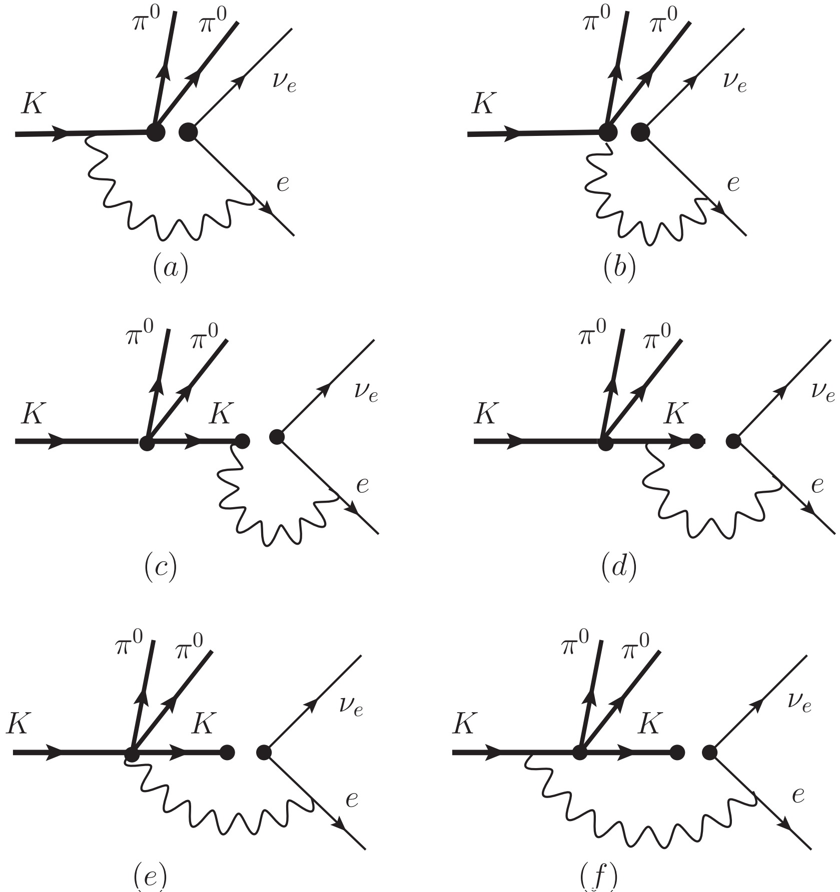
$$\left(|V_{us}|^2 G_F^2\right)^{\rm bare}\left(1-\frac{3}{4}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right) = |V_{us}|^2 G_F^2 \qquad [K_{e4}^{00}]$$

– correct treatment is to include the counterterms K_i and X_i and to renormalize the form factors

$$\left(1-\frac{9}{8}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right) f_s^{\rm bare}[K_{e4}^{+-}] = f_s[K_{e4}^{+-}] \qquad \left(1-\frac{3}{8}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right) f_s^{\rm bare}[K_{e4}^{00}] = f_s[K_{e4}^{00}]$$



Non factorizable radiative corrections

Besides w.f. factors of QED, only diagram (a) is considered in a PHOTOS-like treatment of radiative corrections [diagrams (b), (c), and (d) vanish for $m_e \to 0$]

Adding the diagrams for the emission of a soft photon, one obtains

$$\Gamma^{\mathrm{tot}} = \Gamma(K_{e4}^{00}) + \Gamma^{\mathrm{soft}}(K_{e4\gamma}^{00}) = \Gamma_0(K_{e4}^{00}) \times (1 + 2\delta_{EM})$$

with $\delta_{EM} = 0.018 \longrightarrow \quad \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.047 \pm 0.010$

V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 75, 145 (2015) can one understand the origin of the remaining $\sim 4.5\%$ effect?

can one understand the origin of the remaining $\sim 4.5\%$ effect?

$\longrightarrow$ isospin breaking in the quark masses

$$\left.\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]}\right|_{\mathrm{LO}} = \left(1 + \frac{3}{2R}\right) = 1.039 \pm 0.002$$

V. Cuplov, PhD Thesis (2004); A. Nehme, Nucl. Phys. B 682, 289 (2004)

$$R = \frac{m_s - m_{ud}}{m_d - m_u} = 38.2(1.1)(0.8)(1.4)$$

Z. Fodor et al. [BMW Coll.], Phys. Rev. Lett. 117, 082001 (2016)

Conclusions

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ($m_u \neq m_d$ and $\alpha \neq 0$) unavoidable

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The effects due to $M_\pi \neq M_{\pi^0}$ are important (especially for $K \to \pi\pi\pi$ and for K_{e4}). ChPT at NLO is not always sufficient.
$\longrightarrow$ This issue can be dealt with through more elaborate/adapted approaches, like NREFT, dispersive representations,...

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Treatment of radiative corrections in K_{e4} rather rudimentary, does not match the quality of the data
$\longrightarrow$ Improvements should be possible

There are many interesting situations where low-energy effective theory does not apply (hadronic tau decays, semi-leptonic decays of B and D mesons,...)

The SM itself provides a framework to handle radiative corrections to semi-leptonic decays in these cases; genuine $\mathcal{O}(\alpha G_F)$ effects can be disantagled from $\mathcal{O}(G_F^2)$ contributions in a systematic (and gauge invariant) manner

A. Sirlin, Rev. Mod. Phys. 50 (1978)
S. Weinberg, Phys. Rev. D 8 (1973)
G. Preparata, W. I. Weisberger, Phys Rev. 175 (1968)
Abers et al., Phys. Rev. 167 (1968)
C. Y. Seng, Particles 4, 397 (2021)

where $\mathcal{O}(G_F^2)$ also means

$$\mathcal{O}(\alpha G_F \times \frac{m_\ell^2}{M_{W,Z}^2}) \qquad \mathcal{O}(\alpha G_F \times \frac{m_q m_{q'}}{M_{W,Z}^2}) \qquad \mathcal{O}(\alpha G_F \times \frac{\Lambda_H^2}{M_{W,Z}^2})$$

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Result is finite and involves three-current and two-current correlation functions of QCD, whose evaluation requires nonperturbative approaches (lattice, large-N_C)

In the case of light pseudoscalar mesons, alternative identification of the low-energy constants in terms of these QCD correlators

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In studying radiative corrections, one encounters a large variety of situations, for each situation the appropriate framework must be found

Thanks for your attention!