Structure-dependent QED effects in B decays from effective theory

M. Beneke (TU München)

Workshop "QED in Weak Decays"
Edinburgh, June 22 - 24, 2022

MB, Bobeth, Szafron, 1708.09152, 1908.07011 [$B_s \to \mu^+\mu^-$]

MB, Böer, Toelstede, Vos, 2008.10615 [$B \to \pi K$, charmless]

MB, Böer, Finauri, Vos, 2107.03819 [$B \to D_{(s)}^{(*)+} L^-$, colour-allowed + semi-leptonic]

MB, Böer, Toelstede, Vos, 2108.05589 + 2204.09091 [LCDAs of light and heavy mesons]
$\longrightarrow$ Philipp Böer's talk

TECHNISCHE UNIVERSITÄT MÜNCHEN

Sino-German CRC110 "Emergence of Structure in QCD"

Motivation: Theory

- Precision: Traditionally focus on hadronic uncertainties. Time to look at QED. QED effects violate isospin symmetry and can cause large "lepton-flavour violating" logarithms, $\log m_\ell$.

- Photons couple weakly to strongly interacting quarks $\rightarrow$ probe of hadronic physics, requires factorization theorems, which mostly don't exist yet.

- Photons have long-range interactions with the charged particles in the initial/final state $\rightarrow$ QED factorization is more complicated than QCD factorization.

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Observables

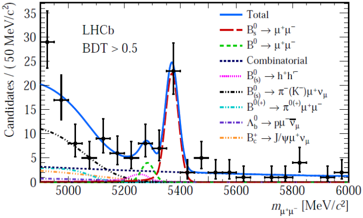
IR finite observable is

$$\Gamma_{\text{phys}} = \sum_{n=0}^{\infty} \Gamma(B \to f + n\gamma, \sum_n E_{\gamma,n} < \Delta E)$$
$$\equiv \omega(\Delta E) \times \Gamma_{\text{non-rad.}}(B \to f)$$

Signal window $|m_B - m_f| < \Delta \quad \Longrightarrow \quad \Delta E = \Delta$

Assume $\Delta \ll \Lambda_{\text{QCD}} \sim$ size of hadrons

Large $\ln \Delta E$.

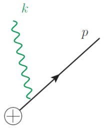


[LHCb, $B_s \to \mu^+\mu^-$, 1703.05748]

(Ultra-) Soft photons and the point-like approximation

Universal soft radiative amplitude

$$A^{i\to f+\gamma}(p_j,k) = A^{i\to f}(p_j)\times\sum_{j=\text{legs}}\frac{-eQ_j p_j^\mu}{\eta_j p_j\cdot k + i\epsilon}$$



The amplitude implies that the charged particles (B-meson, pion, lepton, ...) are treated as point-like. Exponentiates for the decay rate, but the virtual correction is UV divergent in the soft limit. Cut-off Λ.

$$\Gamma = \Gamma^{i\to f}_{\text{tree}}\times\left(\frac{2\Delta E}{\Lambda}\right)^{-\frac{\alpha}{\pi}\sum_{i,j}Q_iQ_jf(\beta_{ij})}$$

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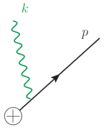

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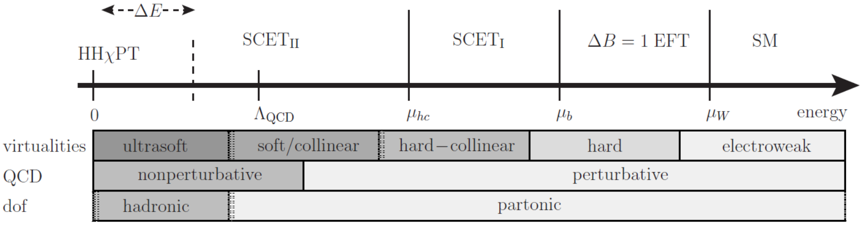
What is Λ?

- Present treatment of QED effects sets $\Lambda = m_B$ (e.g. using a theory of point-like mesons)
- Experimental analyses uses the PHOTOS Monte Carlo [Golonka, Was, 2005], which in addition neglects radiation from charged initial state particles.

However, the derivation implies that $\Lambda \ll \Lambda_{\text{QCD}} \sim$ size of the hadron (B-meson). Otherwise virtual corrections resolve the structure of the hadron and higher-multipole couplings are unsuppressed.

Scales and Effective Field theories (EFTs)

Multiple scales: m_W, m_b, $\sqrt{m_b\Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$, m_μ, ΔE



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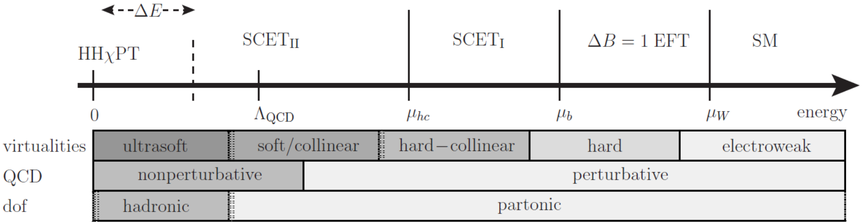
Short-distance QED at $\mu \gtrsim m_b$ can be included in the usual weak effective Lagrangian (extended Fermi theory) + renormalization group.

Far IR (ultrasoft scale) described by theory of point-like hadrons.



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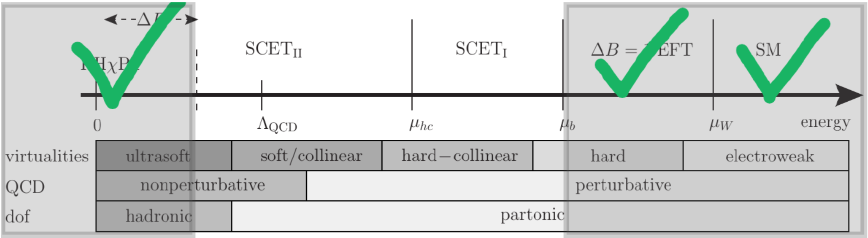
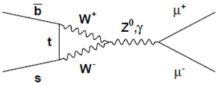


Goal: Theory for QED corrections between the scales m_b and $\Lambda_{\rm QCD}$ (structure-dependent effects).

$B_s \to \mu^+\mu^-$

1708.09152, 1908.07011, with C. Bobeth and R. Szafron

Some facts about $B_s \to \mu^+\mu^-$



"Instantaneous", "non-radiative" branching fraction

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}$$
$$\times \left\{ \left| \frac{2m_\mu}{m_{B_s}}(C_{10}-C_{10}') + (C_P - C_P') \right|^2 + \left(1-\frac{4m_\mu^2}{m_{B_s}^2}\right) |C_S - C_S'|^2 \right\}$$

- Long-distance QCD effects are very simple. Local annihilation. Only $\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle = i f_{B_q} p^\mu$ Task for lattice QCD (1.5% [Aoki et al. 1607.00299], 0.5% [FNAL/MILC 1712.09262]).
- Only the operator Q_{10} from the weak effective Lagrangian enters.
- No scalar lepton current $\bar{\ell}\ell$, only $\bar{\ell}\gamma_5\ell \Longrightarrow$

$$\mathcal{A}_{\Delta\Gamma}^{\lambda} = 1 \qquad C_\lambda = S_\lambda = 0$$

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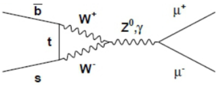

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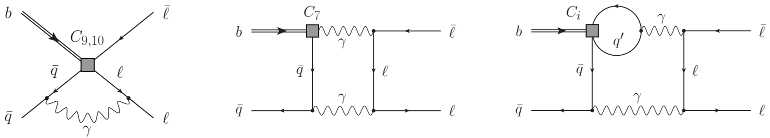
$$\mathcal{A}^\lambda_{\Delta\Gamma} = 1 \qquad C_\lambda = S_\lambda = 0$$

None of these are exactly true in the presence of electromagnetic corrections

Enhanced electromagnetic effect

m_B/Λ power-enhanced and (double) logarithmically enhanced, purely virtual correction



$$i\mathcal{A} = m_\ell f_{B_q}\mathcal{N}\, C_{10}\,\bar{\ell}\gamma_5\ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell f_{B_q}\mathcal{N}\,\bar{\ell}(1+\gamma_5)\ell$$

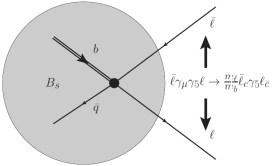
$$\times\Bigg\{\int_0^1 du\,(1-u)\,C_9^{\rm eff}(um_b^2)\,m_B\int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega)\left[\ln\frac{m_b\omega}{m_\ell^2}+\ln\frac{u}{1-u}\right]$$

$$-\,Q_\ell C_7^{\rm eff} m_B\int_0^\infty\frac{d\omega}{\omega}\,\phi_{B+}(\omega)\left[\ln^2\frac{m_b\omega}{m_\ell^2}-2\ln\frac{m_b\omega}{m_\ell^2}+\frac{2\pi^2}{3}\right]\Bigg\}+\ldots$$

The virtual photon probes the B meson structure. B-meson LCDA and $1/\lambda_B$ enters.

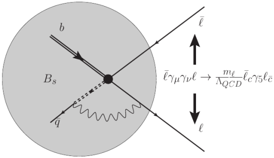
$$\frac{m_B}{\lambda_B}\equiv m_B\int_0^\infty\frac{d\omega}{\omega}\,\phi_{B+}(\omega)\sim 20 \qquad \ln\frac{m_b\omega}{m_\mu^2}\sim 6$$

Interpretation of the m_B/Λ-enhanced QED correction



$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$$

Local annihilation and helicity flip.



$$\langle 0|\int d^4x\, T\{j_{\mathrm{QED}}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$$

Helicity-flip and annihilation delocalized by a hard-collinear distance

The virtual photon probes the B meson structure. Annihilation/helicity-suppression is “smeared out” over light-like distance $1/\sqrt{m_B\Lambda}$ [→ B-LCDA]. Still short-distance.

Logarithms are not the standard soft logarithms, but due to hard-collinear, collinear and soft regions, including final-state soft lepton exchange.

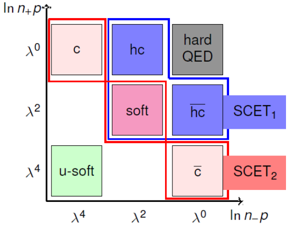
All orders, EFT, summation of logarithms

Back-to-back energetic lepton pair
Collinear (lepton $n_+ p_\ell$ large) and anti-collinear (anti-lepton $n_- p_{\bar\ell}$ large) modes

$$n_+^2 = n_-^2 = 0, \quad n_+ \cdot n_- = 2, \qquad p^\mu = n_+ p \, \frac{n_-^\mu}{2} + n_- p \, \frac{n_+^\mu}{2} + p_\perp^\mu$$

$$p = (n_+ p, p_\perp, n_- p), \qquad \lambda \sim \frac{\Lambda_{\rm QCD}}{m_b} \sim \frac{m_\mu}{m_b}$$

- Modes in the EFT classified by virtuality and rapidity
- Matching QCD+QED $\to$ SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$

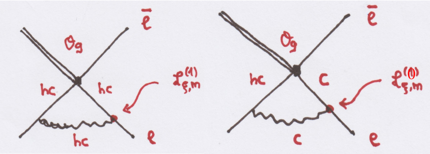


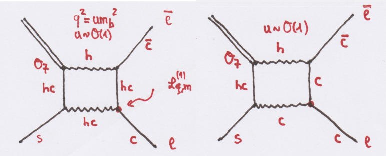
mode	relative scaling	absolute scaling	virtuality k^2
hard	$(1,1,1)$	(m_b, m_b, m_b)	m_b^2
hard-collinear	$(1,\lambda,\lambda^2)$	$(m_b, \sqrt{m_b\Lambda_{\rm QCD}}, \Lambda_{\rm QCD})$	$m_b\Lambda_{\rm QCD}$
anti-hard-collinear	$(\lambda^2,\lambda,1)$	$(\Lambda_{\rm QCD}, \sqrt{m_b\Lambda_{\rm QCD}}, m_b)$	$m_b\Lambda_{\rm QCD}$
collinear	$(1,\lambda^2,\lambda^4)$	$(m_b, m_\mu, m_\mu^2/m_b)$	m_μ^2
anticollinear	$(\lambda^4,\lambda^2,1)$	$(m_\mu^2/m_b, m_\mu, m_b)$	m_μ^2
soft	$(\lambda^2,\lambda^2,\lambda^2)$	$(\Lambda_{\rm QCD}, \Lambda_{\rm QCD}, \Lambda_{\rm QCD})$	$\Lambda_{\rm QCD}^2$

SCET interpretation of the one-loop QED correction to $B_s \to \mu^+\mu^-$

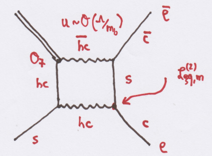
- After tree-level matching to SCET_I need matrix element of

$$\stackrel{\text{SCET}_\text{I}}{\to} \int_0^1 du \left(C_9^{\text{eff}}(u) + \frac{C_7^{\text{eff}}}{u} \right) \bar{\chi}_{\text{hc}}(\bar{u} p_\ell) \Gamma h_v \, \bar{\ell}_{\text{hc}}(u p_\ell) \Gamma' \ell_{\overline{\text{hc}}}(p_{\bar{\ell}})$$
$$+ \, \bar{\chi}_{\text{hc}}(p_\ell) \gamma_\perp^\mu h_v \, \mathcal{A}^\gamma_{\text{hc},\perp\mu}(p_{\bar{\ell}})$$

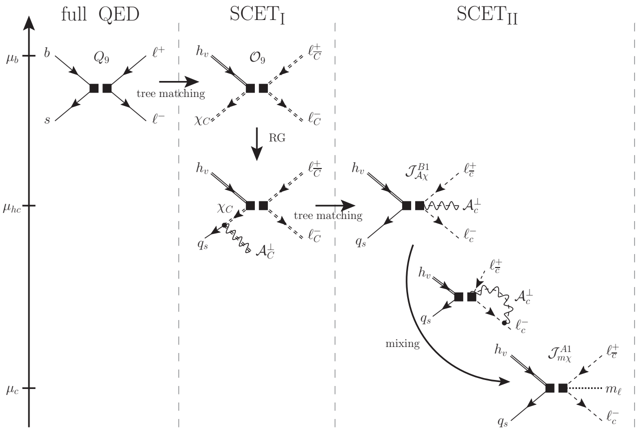




- Sum of hard-collinear and collinear loop in SCET_II gives a structure-dependent collinear logarithm $\ln(m_b\Lambda/m_\mu^2)$
- Endpoint (rapidity) divergence for $u \to 0$ in C_7^{eff} term. Cancelled by soft lepton exchange.



Matching, RGE, leading-(double) log resummation – sketch

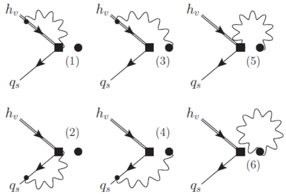


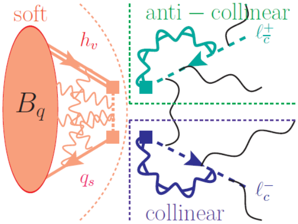
- Factorization and resummation of logs only understood for the Q_9 operator up to now [MB, Bobeth, Szafron, 2019]

SCET$_{\rm II}$ factorization and soft rearrangement

$$\widetilde{\mathcal{J}}^{B1}_{\mathcal{A}\chi}(v,t) = \bar{q}_s(vn_-)Y(vn_-,0)\frac{\not{n}_-}{2}P_L h_v(0)\left[Y_+^\dagger\, Y_-\right](0)\,\left[\bar{\ell}_c(0)(2\mathcal{A}_{c\perp}(tn_+)P_R)\ell_{\bar{c}}(0)\right] = \widehat{\mathcal{J}}_s \otimes \widehat{\mathcal{J}}_c \otimes \widehat{\mathcal{J}}_{\bar{c}}$$

- s, c, $\bar{c}$ do not interact in SCET$_{\rm II}$. Sectors are factorized.
  Anomalous dimension should be separately well defined.
- But the anomalous dimension of the soft graphs is IR divergent.





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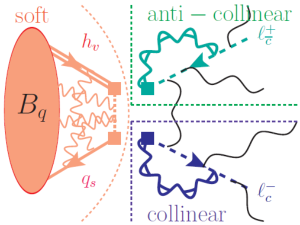

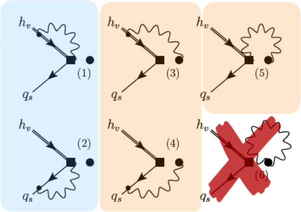

$$\langle 0|\left[Y_+^\dagger\, Y_-\right](0)|0\rangle \equiv R_+R_-$$

- Soft rearrangement $\quad \widehat{\mathcal{J}}_s \otimes \widehat{\mathcal{J}}_c \otimes \widehat{\mathcal{J}}_{\bar{c}} = \dfrac{\widehat{\mathcal{J}}_s}{R_+R_-} \otimes R_+\widehat{\mathcal{J}}_c \otimes R_-\widehat{\mathcal{J}}_{\bar{c}}$

  Soft matrix element defines a generalized B-LCDA

Structure of the final result

Amplitude [evolved to μ_c]

$$i\mathcal{A}_9 = e^{S_\ell(\mu_b,\,\mu_c)}\, T_+(\mu_c) \times \int_0^1 du\, e^{S_q(\mu_b,\,\mu_{hc})}\, 2H_9(u;\mu_b) \int_0^\infty d\omega\, U_s^{\rm QED}(\mu_{hc},\mu_s;\omega)\, m_{B_q} F_{B_q}(\mu_{hc})\, \phi_+(\omega;\mu_{hc})$$

$$\times \left[J_m(u;\omega;\mu_{hc}) + \int_0^1 dw\, J_A(u;\omega,w;\mu_{hc}) \left(M_A(w;\mu_c) - \frac{Q_\ell \overline{w}}{\beta_{0,\rm em}} \ln \eta_{\rm em} \right) \right]$$

$$\equiv e^{S_\ell(\mu_b,\,\mu_c)} \times A_9\, [\bar{u}_c(1+\gamma_5)v_{\bar{c}}]$$

– defines the non-radiative amplitude A_9 by extracting universal final state virtual logs.
QED+QCD Logs between m_b and μ_c summed.
Similarly for the A_{10} standard amplitude.

Including ultrasoft photon radiation (+ virtual ultrasoft)

$$\mathcal{N}_{\Delta B=1} C_i \left\langle \ell\bar{\ell} X_s \middle| Q_i \middle| \overline{B}_s \right\rangle = \mathcal{A}_i(\mu_c) \left\langle X_s \middle| S^\dagger_{v_\ell}(0) S_{v_{\bar{\ell}}}(0) \middle| 0 \right\rangle(\mu_c)\,, \qquad i = 9, 10$$

– $\mathcal{A}_i$ is an exclusive amplitude, therefore IR divergent and scale-dependent.
This includes the QED-generalized LCDAs (here: B-meson), which are also IR divergent.
Should be viewed as a matching coefficient of SCET$_{\rm II}$ onto the theory of point-like hadrons at a scale $\mu_f \lesssim \Lambda_{\rm QCD}$. This matching is non-perturbative.

Structure of the final result (II)

Decay rate [including ultrasoft photon radiation]

$$\Gamma[B_q \to \mu^+\mu^-](\Delta E) = \underbrace{\frac{m_{B_q}}{8\pi}\beta_\mu \left(\left|A_{10}+A_9+A_7\right|^2 + \beta_\mu^2 \left|A_9+A_7\right|^2\right)}_{\text{non-radiative rate}} \times \underbrace{\left|e^{S_\ell(\mu_b,\,\mu_c)}\right|^2 \mathcal{S}(v_\ell, v_{\overline{\ell}}, \Delta E)}_{\text{ultrasoft radiation}}$$

$$= \Gamma^{(0)}[B_q \to \mu^+\mu^-] \left(\frac{2\Delta E}{m_{B_q}}\right)^{-\frac{2\alpha}{\pi}\left(1+\ln\frac{m_\mu^2}{m_{B_q}^2}\right)}$$

$$\mathcal{S}(v_\ell, v_{\overline{\ell}}, \Delta E) = \sum_{X_s} \left|\langle X_s|S^\dagger_{v_\ell}(0) S_{v_{\overline{\ell}}}(0)|0\rangle\right|^2 \theta(\Delta E - E_{X_s})$$

Ultrasoft function

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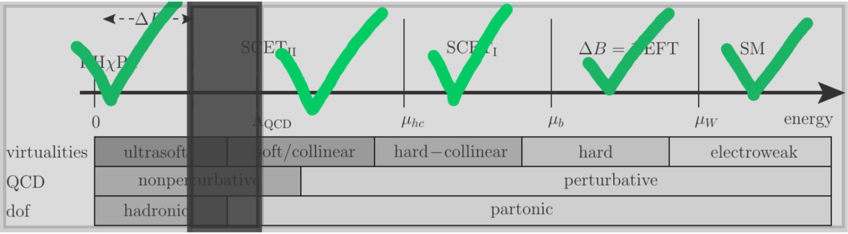
$\Gamma^{(0)}[B_q \to \mu^+\mu^-]$ is not the point-like amplitude, but the represents the composite B meson including structure-dependent effects. This is an explicit counterexample to the statement [Isidori, Nabeebaccus, Zwicky, 2009.00929] that "hard-collinear" logarithms in structure-dependent terms cancel ("Any gauge invariant addition (to the point-like approximation) can at most lead to logs of the form $\mathcal{O}(\alpha)m_\ell^2 \ln m_\ell$" [2205.06194])

Example: World with larger $\alpha_{\rm em}$ and larger m_B. The power-enhanced amplitude then dominated the $B_s \to \mu^+\mu^-$ decay width.



Can sum leading logs, and calculate all QED effects between scale m_b and a few times Λ_{QCD}.

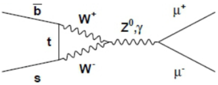
BUT: matching of SCET_{II} to the ultrasoft theory of point-like hadrons at a scale $\mu_c \sim \Lambda_{\text{QCD}}$ must be done non-perturbatively.

Hadronic B two-body decays
($B \to \pi K$ ..., $D^+ L^-$, ...)

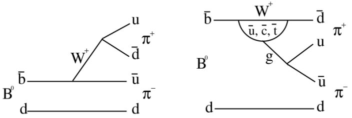
2008.10615 (charmless + semi-leptonic $B \to D$)
and 2107.03819 (heavy-light + semi-leptonic $B \to D$),
with P. Böer, G. Finauri, J. Toelstede and K. Vos

Charmless decays, $B \to \pi^+\pi^-$ vs. $\mu^+\mu^-$

- Same kinematics, charges, composite pions instead of elementary leptons. QED effects similar, identical for ultrasoft photons.
- But QCD dynamics is very different.



$B \to \mu^+\mu^-$
$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}\rangle$



$B \to \pi^+\pi^-$
$\langle \pi^+\pi^-|Q_i|\bar{B}\rangle$

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V_{pD}^* \Big(C_1 \mathcal{O}_1^p + C_2 \mathcal{O}_2^p + \sum_{i=({\rm EW}){\rm pen,\ mag}} C_i \mathcal{O}_i \Big)$$

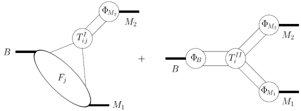
$$\mathcal{O}^p_{1,2} = (\bar{p}\Gamma b)\,(\bar{D}\Gamma' p) \qquad \mathcal{O}_{i,{\rm QCD\ pen}} = (\bar{D}\Gamma b) \sum_{q=u,d,s,c,b} (\bar{q}\Gamma' q)$$

- Different CKM amplitudes, strong rescattering in $\langle \pi^+\pi^-|Q_i|\bar{B}\rangle \Rightarrow$ (direct) CP violation, determination of CKM angles, search for new physics
- Branching fractions 10^{-5}, first measured by CLEO in the late 1990s, now $\mathcal{O}(50-100)$ different two-body final states M_1M_2 measured.

QCD theory

"QCD factorization" [MB, Buchalla, Neubert, Sachrajda, 1999-2001], later understood and formulated as a $\mathrm{SCET_{II}}$ problem:

$$\mathrm{QCD} \overset{\text{remove h}}{\longrightarrow} \mathrm{SCET_I} \overset{\text{remove hc}}{\longrightarrow} \mathrm{SCET_{II}}(c, \bar{c}, s)$$



$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = \underbrace{F^{BM_1}(0)}_{\text{form factor}} \int_0^1 du\, T_i^I(u) \Phi_{M_2}(u)$$

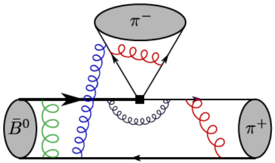
$$+ \int_0^1 dz du\, H_i^{\mathrm{II}}(z, u) \int_0^\infty d\omega \int_0^1 dv\, J(\omega, u, v)\, \underbrace{\Phi_B(\omega) \Phi_{M_1}(v) \Phi_{M_2}(u)}_{\text{LCDAs}}$$

- Rigorous at leading power in $\Lambda_{\mathrm{QCD}}/m_b$
- Strong rescattering phases are $\delta \sim \mathcal{O}(\alpha_s(m_b), \Lambda/m_b)$. $\mathrm{SCET_I}$ matching coefficients only. Direct CP asymmetry is calculable at LP

$$A_{\mathrm{CP}}(M_1 M_2) = \underbrace{a_1 \alpha_s}_{1999} + \underbrace{a_2 \alpha_s^2}_{2020} + \ldots + \mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$$

[Bell, MB, Huber, Li]

Including virtual QED effects into the factorization theorem



SCET$_\mathrm{I}$ operators

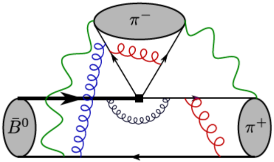
$$\mathcal{O}^\mathrm{I}(t) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C h_v]$$

$$\mathcal{O}^\mathrm{II}(t,s) = \underbrace{[\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]}_{M_2}\;\underbrace{[\bar{\chi}_C \mathcal{A}\!\!\!/_{C,\perp}(sn_+)h_v]}_{B\to M_1}$$

QCD factorization formula

$$\langle M_1 M_2|Q_i|\bar{B}\rangle = F^{BM_1}(0)\int_0^1 du\, T_i^{\mathrm{I,QCD}}(u)\, f_{M_2}\phi_{M_2}(u)$$
$$+\int_0^\infty d\omega \int_0^1 dudv\, T_i^{\mathrm{II,QCD}}(z,u)\, f_B\phi_B(\omega) f_{M_1}\phi_{M_1}(v) f_{M_2}\phi_{M_2}(u)$$

Including virtual QED effects into the factorization theorem



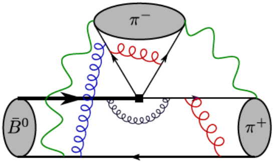
SCET$_\mathrm{I}$ operators

$$\mathcal{O}^{\mathrm{I}}(t) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$\mathcal{O}^{\mathrm{II}}(t,s) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\not{\mathcal{A}}_{C,\perp}(sn_+)\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$S_{n_\pm}^{(q)} = \exp\left\{-iQ_q e\int_0^\infty ds\, n_\pm A_s(sn_\pm)\right\}$$

Including virtual QED effects into the factorization theorem



SCET$_\text{I}$ operators

$$\mathcal{O}^{\text{I}}(t) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$\mathcal{O}^{\text{II}}(t,s) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\not{\mathcal{A}}_{C,\perp}(sn_+)\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$S_{n_\pm}^{(q)} = \exp\left\{-iQ_q e\int_0^\infty ds\, n_\pm A_s(sn_\pm)\right\}$$

QCD + QED factorization formula

$$\langle M_1M_2|Q_i|\bar{B}\rangle_{|\text{non-rad.}} = \mathcal{F}_{Q_2}^{BM_1}(0)\int_0^1 du\, T_{i,Q_2}^{\text{I,QCD+QED}}(u)\, f_{M_2}\Phi_{M_2}(u)$$
$$+\int_{-\infty}^{\infty} d\omega\int_0^1 dudv\, T_{i,\otimes}^{\text{II,QCD+QED}}(z,u)\, f_B\Phi_{B,\otimes}(\omega) f_{M_1}\Phi_{M_1}(v) f_{M_2}\Phi_{M_2}(u)$$

- Formula retains its form, but the hadronic matrix elements are generalized. They become process-dependent through the directions and charges of the *other* particles.
- Computation of $\mathcal{O}(\alpha_{\text{em}})$ corrections to the h and hc short-distance coefficient (all poles cancel).

Example of a hard-scattering kernel

Same generalization for colour-allowed decays to D^+L^-.
Hard scattering kernel

$$
\begin{aligned}
H_-^{(1)} = &- Q_d^2\left\{\frac{L_b^2}{2} + L_b\left(\frac{5}{2} - 2\ln(u(1-z))\right) + h\big(u(1-z)\big) + \frac{\pi^2}{12} + 7\right\} \\
&- Q_u^2\left\{\frac{L_c^2}{2} + L_c\left(\frac{5}{2} + 2\pi i - 2\ln\Big(\bar{u}\,\frac{1-z}{z}\Big)\right) + h\Big(\bar{u}\Big(1-\frac{1}{z}\Big)\Big) + \frac{\pi^2}{12} + 7\right\} \\
&+ Q_d Q_u\left\{\frac{L_b^2}{2} + \frac{L_c^2}{2} - 6L_\nu + 2L_b\Big(2 - \ln(\bar{u}(1-z))\Big)\right. \\
&\left. - 2L_c\left(1 - i\pi + \ln\Big(u\,\frac{1-z}{z}\Big)\right) + g\big(\bar{u}(1-z)\big) + g\Big(u\Big(1-\frac{1}{z}\Big)\Big) + \frac{\pi^2}{6} - 12\right\} \\
&+ Q_d Q_u f(z)\,, \\
H_+^{(1)} = &- Q_d^2\sqrt{z}\,w\big(u(1-z)\big) - Q_u^2\frac{1}{\sqrt{z}}\;w\Big(\bar{u}\Big(1-\frac{1}{z}\Big)\Big) - Q_d Q_u\sqrt{z}\,\frac{\ln z}{1-z}\,,
\end{aligned}
$$

$$
z \equiv \frac{m_c^2}{m_b^2}\,, \qquad L_c \equiv \ln\frac{\mu^2}{m_c^2} = L_b - \ln z\,, \qquad L_\nu \equiv \ln\frac{\nu^2}{m_b^2}\,,
$$

$$
h(s) \equiv \ln^2 s - 2\ln s + \frac{s\ln s}{1-s} - 2\mathrm{Li}_2\Big(\frac{s-1}{s}\Big)\,,
$$

$$
f(z) \equiv \left(1 - \frac{1+z}{1-z}\ln z\right)L_b + \frac{\ln z}{1-z}\left(\frac{1}{2}(1+z)\ln z - 2 - z\right), \ldots
$$

Size of QED effects

2009.10615 (πK)and 2107.03819 (colour-allowed DL and semi-leptonic $B \to D$),
with P. Böer, G. Finauri, J. Toelstede and K.K. Vos

Numerical estimate of QED effects for πK and D^+L^- final states

Up to now virtual corrections to the non-radiative amplitude.
Add (ultra)soft photon radiation.

- Electroweak scale to m_B: QED corrections to Wilson coefficients included
- m_B to μ_c: $\mathcal{O}(\alpha_{\rm em})$ corrections to short-distance kernels included.
  QED effects in form factors and LCDA not included.
- Ultrasoft photon radiation included (same formalism as for $\mu^+\mu^-$ with $m_\mu \to m_\pi, m_K$)

$$U(M_1M_2) = \left(\frac{2\Delta E}{m_B}\right)^{-\frac{\alpha_{\rm em}}{\pi}\left(Q_B^2 + Q_{M_1}^2\left[1+\ln\frac{m_{M_1}^2}{m_B^2}\right] + Q_{M_2}^2\left[1+\ln\frac{m_{M_2}^2}{m_B^2}\right]\right)} \qquad (M_1, M_2 \text{ light mesons})$$

$$U(\pi^+K^-) = 0.914$$
$$U(\pi^0K^-) = U(K^-\pi^0) = 0.976$$
$$U(\pi^-\bar{K}^0) = 0.954 \qquad [\text{for } \Delta E = 60\,\text{MeV}]$$
$$U(\bar{K}^0\pi^0) = 1$$

$$U(D^+K^-) = 0.960$$
$$U(D^+\pi^-) = 0.938$$

Isospin-protected ratios / sum rules for the πK final states

Consider ratios / sums where some QCD uncertainties drop out.

[MB, Neubert, 2003]

$$R_L = \frac{2\mathrm{Br}(\pi^0 K^0) + 2\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- K^0) + \mathrm{Br}(\pi^+ K^-)} = R_L^{\mathrm{QCD}} + \cos\gamma \mathrm{Re}\, \delta_{\mathrm{E}} + \delta_U$$

$$R_L^{\mathrm{QCD}} - 1 \approx (1 \pm 2)\% \qquad \delta_E \approx 0.1\% \qquad \delta_U = 5.8\%$$

QED correction larger than QCD and QCD uncertainty, but short-distance QED negligible.

[Gronau, Rosner, 2006]

$$\Delta(\pi K) \equiv A_{\mathrm{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 K^-)$$
$$- \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{\mathrm{QCD}} + \delta\Delta(\pi K)$$

$$\Delta(\pi K)^{\mathrm{QCD}} = (0.5 \pm 1.1)\% \qquad \delta\Delta(\pi K) \approx -0.4\%$$

QED correction of similar size but small. Ultrasoft factors cancel.

Numerical estimate of QED effects for $D^{(*)+}L^-$ final states

$$R_L^{(0),(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+}L^-)(\Delta E)}{d\Gamma^{(0)}(\bar{B}_d \to D^{(*)+}\mu^-\bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}}$$

$$R_L^{(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+}L^-)(\Delta E)}{d\Gamma(\bar{B}_d \to D^{(*)+}\mu^-\bar{\nu}_\ell)(\Delta E)/dq^2 \,|_{q^2=m_L^2}}$$

- Short-distance QED effects $\approx -1\%$, ultrasoft up to $\approx -7\%$ for pions, depending on the semi-leptonic normalization.
- Not large enough to explain the apparent amplitude deficit of -15% [Bordone et al., 2020], but highlights the importance of proper treatment of ultrasoft radiation effects.

$R_L^{(*)}$	LO	QCD NNLO	$+\delta_{\rm QED}$	$+\delta_{\rm U}$ ($\delta_{\rm U}^{(0)}$)
R_π	0.969 ± 0.021	$1.078^{+0.045}_{-0.042}$	$1.069^{+0.045}_{-0.041}$	$1.074^{+0.046}_{-0.043}$ $(1.003^{+0.042}_{-0.039})$
R_π^*	0.962 ± 0.021	$1.069^{+0.045}_{-0.041}$	$1.059^{+0.045}_{-0.041}$	$1.065^{+0.047}_{-0.042}$ $(0.996^{+0.043}_{-0.039})$
$R_K \cdot 10^2$	7.47 ± 0.07	$8.28^{+0.27}_{-0.26}$	$8.21^{+0.27}_{-0.26}$	$8.44^{+0.29}_{-0.28}$ $(7.88^{+0.26}_{-0.25})$
$R_K^* \cdot 10^2$	6.81 ± 0.16	$7.54^{+0.31}_{-0.29}$	$7.47^{+0.30}_{-0.29}$	$7.68^{+0.32}_{-0.30}$ $(7.19^{+0.29}_{-0.28})$

Table 3: Theoretical predictions for $R_L^{(*)}$ expressed in GeV^2 at LO, NNLO QCD and subsequently adding $\delta_{\rm QED}$ given in (82) and the ultrasoft effects $\delta_{\rm U}$ (or in brackets $\delta_{\rm U}^{(0)}$). The last column presents our final results.

Summary

- QED factorization is more complicated than QCD due to charged external states. SCET applies and we now understand how to systematically include QED effects, but it requires new non-perturbative matrix elements, generalizing the familiar hadronic matrix elements.

Summary

(I) QED factorization is more complicated than QCD due to charged external states. SCET applies and we now understand how to systematically include QED effects, but it requires new non-perturbative matrix elements, generalizing the familiar hadronic matrix elements.

(II) For $B_s \to \mu^+\mu^-$ there is a power-enhanced virtual electromagnetic correction.

- More long-distance QCD than f_B
- Effect of the same order as the non-parametric uncertainty, larger than previously estimated QED uncertainty

(III) For charmless hadronic decays the QCD $\times$ QED factorization formula takes a similar form as in QCD alone, but the generalized pion (etc.) and B-meson LCDA exhibit novel properties (asymmetric evolution, soft rescattering phases in the B-LCDA)

(IV) Structure-dependent logarithms turn out to be small

(V) Comparison to experiment now requires precise statements how QED effects are treated in the analysis. Ideally compare theoretically well-defined and calculable *radiative* branching fractions and use Monte Carlo generators only to estimate efficiencies.

Back-up slides

Numerical size of the QED correction to $B_s \to \mu^+\mu^-$

Include through the substitution

$$\overline{\mathcal{B}}(B_s \to \ell^+\ell^-) = \frac{\tau_{B_q} m_{B_q}^3 f_{B_q}^2}{8\pi} |\mathcal{N}|^2 \frac{m_\ell^2}{m_{B_q}^2} \sqrt{1 - \frac{4m_\ell^2}{m_{B_q}^2}} |C_{10}|^2, \qquad C_{10} \to C_{10} + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \Delta_{\rm QED}$$

where

$$\Delta_{\rm QED} = (33 \ldots 119) + i\,(9 \ldots 23)$$

- Reduction of the branching fraction by 0.3–1.1 %
  Uncertainty entirely due to B-meson LCDA.

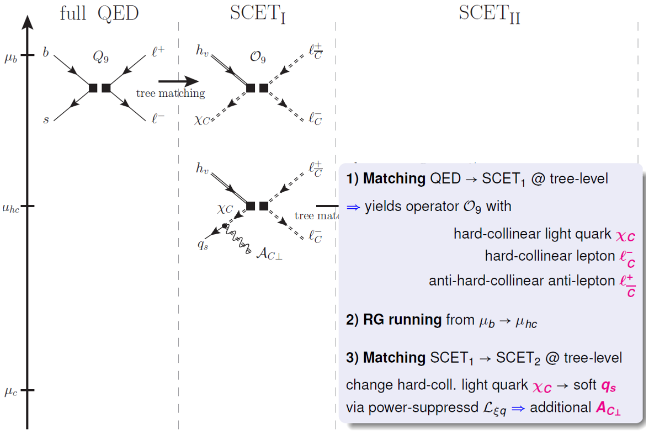
- Cancellation of a factor of three between the $C_9^{\rm eff}(u m_b^2)$ and double-log enhanced $C_7^{\rm eff}$ term:

$$-0.6\% = 1.1\%\,(C_9^{\rm eff}) - 1.7\%\,(C_7^{\rm eff})$$

- Significantly larger than previously estimated QED correction.
  QED uncertainty almost as large as other non-parametric uncertainties (1.2%)

- Small time-dependent rate asymmetries are generated.
  [$C_\lambda = -\eta_\lambda\, 2r\, {\rm Re}(\Delta_{\rm QED}) \approx \eta_\lambda\, 0.6\%$]

Matching, RGE, leading-(double) log resummation – sketch



1) Matching QED → SCET$_1$ @ tree-level

⇒ yields operator $\mathcal{O}_9$ with

hard-collinear light quark χ_C

hard-collinear lepton ℓ^-_C

anti-hard-collinear anti-lepton $\ell^+_{\bar{C}}$

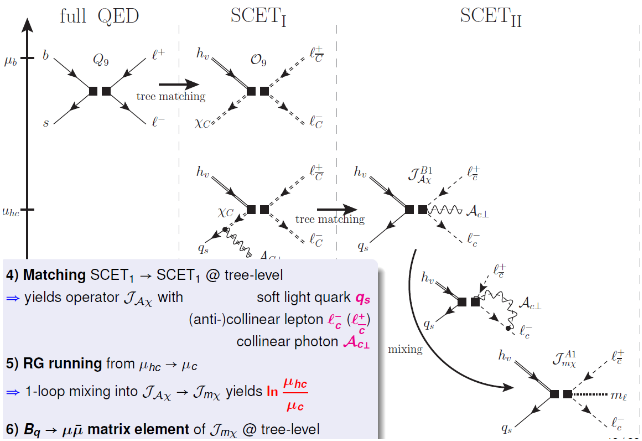
2) RG running from $\mu_b \to \mu_{hc}$

3) Matching SCET$_1$ → SCET$_2$ @ tree-level

change hard-coll. light quark χ_C → soft q_s

via power-suppressd $\mathcal{L}_{\xi q}$ ⇒ additional $A_{C\perp}$

Matching, RGE, leading-(double) log resummation – sketch



- Operator mixing in SCET$_{II}$
  RGE with cusp anomalous dimension → double logarithms $\alpha \times \alpha^n_{(s)} \ln^{2n+1}$