Light-Cone Distribution Amplitudes with QED Effects

Philipp Böer

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with M. Beneke, J. N. Toelstede and K. K. Vos

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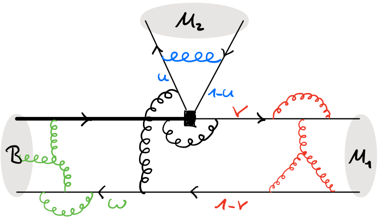
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Outline

1. Introduction (Recap of QCD LCDAs)
2. Light-meson LCDA
3. *B*-meson LCDA/Soft functions

1. Introduction

Hard-Scattering Picture



- **hard** partonic interaction at short distances of $\mathcal{O}(1/m_B)$ and $\mathcal{O}(1/\sqrt{\Lambda_{\mathrm{QCD}}\, m_B})$
- hadronic physics (soft, coll., anti-coll) characterized by wavelengths of $\mathcal{O}(1/\Lambda_{\mathrm{QCD}})$

Decouple sectors in the heavy-quark & large-energy limit $E \sim m_B \gg \Lambda_{\mathrm{QCD}} \quad \Rightarrow$ QCDF

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{B \to M_1} \int_0^1 \mathrm{d}u\, \mathbf{T}_i^{\mathrm{I}}(u)\, f_{M_2} \phi_{M_2}(u) + \int_0^\infty \mathrm{d}\omega \int_0^1 \mathrm{d}u\, \mathrm{d}v\; \mathbf{T}_i^{\mathrm{II}}(u,v,\omega)\, f_{M_1} \phi_{M_1}(v)\, f_{M_2} \phi_{M_2}(u)\, f_B \phi_B(\omega)$$

→ process-dependent but perturbative hard-scattering kernels **T**

→ universal but non-perturbative light-cone distribution amplitudes ϕ_B^+, ϕ_{M_1}, ϕ_{M_2}

Light-Meson LCDA in QCD

$$\langle \pi(p)|\bar{q}(tn_+)[tn_+,0]\not{n}_+\gamma_5 q(0)|0\rangle = -2iEf_\pi \int_0^1 du\, e^{iu(n_+p)t}\phi_\pi(u;\mu)$$

$\rightarrow$ delocalized quark fields connected by finite-length Wilson line

$\rightarrow$ normalized to decay constant f_π $\Rightarrow$ $\int_0^1 du\, \phi_\pi(u;\mu) = 1$

One-loop evolution kernel ("ERBL") known up to 3loop [Braun et al '17]

$$\gamma(u,v) = -\frac{\alpha_s C_F}{\pi}\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\theta(u-v)\right]_+$$

becomes multiplicative and diagonal in Gegenbauer space:

$$\phi(u;\mu) = 6u(1-u)\left(1+\sum_{n=1}^{\infty} a_n(\mu) C_n^{(3/2)}(2u-1)\right)$$

- Asymptotic form: $\phi(u;\mu\to\infty) = 6u(1-u)$
  - $\rightarrow$ symmetric in $u \leftrightarrow 1-u$ and vanishes linearly at both endpoints
- a_n non-perturbative, become smaller for larger n $\Rightarrow$ truncate series
- For pions: $a_n = 0$ for odd n (isospin symmetry)

B-Meson LCDA in QCD

$$\langle 0|\bar{q}(tn_-)[tn_-,0]\not{n}_-\gamma_5 h_v(0)|\bar{B}_v\rangle = iF_{\rm stat}(\mu)\int_0^\infty d\omega\; e^{-i\omega t}\phi_B^+(\omega;\mu)$$

- $\rightarrow$ scale-dependent static decay constant in HQET
- $\rightarrow$ support $\omega \in [0,\infty)$ as hard scale $m_b \rightarrow \infty$ integrated out

One-loop evolution kernel (“Lange/Neubert”) 2loop known [Braun et al ’19]

$$\gamma(\omega,\omega') = \frac{\alpha_s C_F}{\pi}\left\{\left(\ln\frac{\mu}{\omega}-\frac{1}{2}\right)\delta(\omega-\omega') - \omega\left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)}\right]_+ - \left[\frac{\theta(\omega-\omega')}{\omega-\omega'}\right]_+\right\}$$

becomes multiplicative and diagonal in dual space [Bell et al ’13]

- $\rightarrow$ continuous integral transform involving Bessel functions $J_1(2\sqrt{\omega/\omega'})$
- $\rightarrow$ not characterized by discrete set of numbers. In l.p. factorization theorems usually need

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\phi_B^+(\omega;\mu)$$

- $\rightarrow$ $\phi_B^+(\omega;\mu)$ vanishes linearly for $\omega \rightarrow 0$

Including QED

1. The LCDAs discussed here appear in the factorization of the non-radiative amplitude when the energy of real radiation is $\Delta E \ll \Lambda_{\rm QCD}$.
   - $\rightarrow$ exclusive matrix elements that include arbitrary number of virtual photon exchanges above $\Lambda_{\rm QCD}$ from $\mathcal{L}_{\rm QCD+QED}$

2. Non-decoupling: Soft photons sensitive to total charge (monopole) of final-state mesons
   - $\rightarrow$ B-LCDA becomes process dependent (whereas Φ_π remains universal)
   - $\rightarrow$ different support properties, complex-valued due to soft rescattering, ...
   - $\rightarrow$ soft physics qualitatively different: "Soft functions" of the process

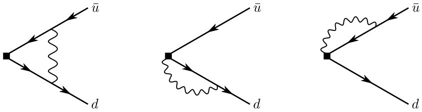
3. The LCDAs for charged mesons are IR divergent and can be viewed as (non-perturbative) matching coefficients for a theory of ultrasoft photons coupling to pointlike mesons.
   - $\rightarrow$ not part of this talk
   - $\rightarrow$ we only study the (IR finite) UV-scale evolution with perturbative RG methods

2. Light-Meson LCDA with QED Effects

Definition and Anomalous Dimension

$$\langle \pi^-(p)| \quad (\bar{d} W^{(d)})(tn_+) \not{n}_+ \gamma_5 (W^{\dagger(u)} u)(0)|0\rangle = -2iE f_\pi \int_0^1 du\, e^{iu(n_+ p)t} \Phi_{\pi^-}(u;\mu)$$

- $\rightarrow$ normalized to QCD decay constant
- $\rightarrow$ QED Wilson extend on infinite light-ray: $W^{(d)}(tn_+) W^{\dagger(u)}(0) = [tn_+, 0]^{(d)} W^{Q_M}(0)$
- $\rightarrow$ UV-scale evolution IR-divergent



$$\gamma(u,v) = -\frac{\alpha_{\rm em}}{\pi}\delta(u-v) Q_M \left(Q_d \ln\frac{\mu^2}{-k_d^2} - Q_u \ln\frac{\mu^2}{-k_u^2} + \frac{3}{4}\right)$$
$$- \left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\rm em}}{\pi} Q_u Q_d\right)\left[\left(1 + \frac{1}{v-u}\right)\frac{u}{v}\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\theta(u-v)\right]_+$$

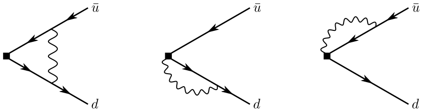
Definition and Anomalous Dimension

$$\langle \pi^-(p)|R_c(\bar{d}W^{(d)})(tn_+)\not{n}_+\gamma_5(W^{\dagger(u)}u)(0)|0\rangle = -2iEf_\pi \int_0^1 du\, e^{iu(n_+p)t}\Phi_{\pi^-}(u;\mu)$$

→ normalized to QCD decay constant

→ QED Wilson extend on infinite light-ray: $W^{(d)}(tn_+)W^{\dagger(u)}(0) = [tn_+,0]^{(d)}\, W^{Q_M}(0)$

→ UV-scale evolution IR-divergent; well-defined after soft rearrangement



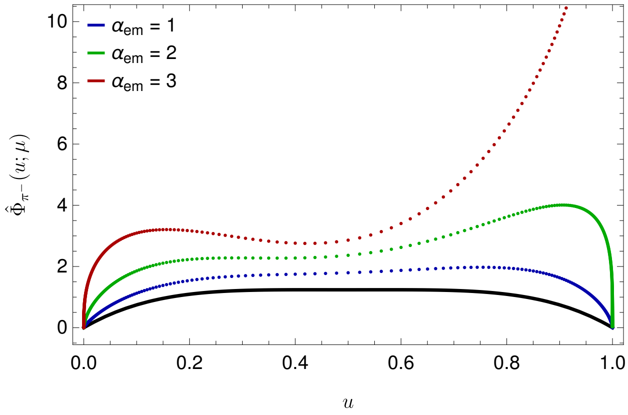
$$\gamma(u,v) = -\frac{\alpha_{\rm em}}{\pi}\delta(u-v)Q_M\left(Q_M \ln\frac{\mu}{2E} - Q_d \ln u + Q_u \ln(1-u) + \frac{3}{4}Q_M\right)$$
$$-\left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\rm em}}{\pi}Q_uQ_d\right)\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\theta(u-v)\right]_+$$

Evolution kernel depends on the large energy $2E = m_B$ and on $\log(u)$ and $\log(1-u)$ terms

→ Gegenbauer polynomials no longer eigenfunctions

→ LCDA becomes asymmetric due to different quark charges

Discretization

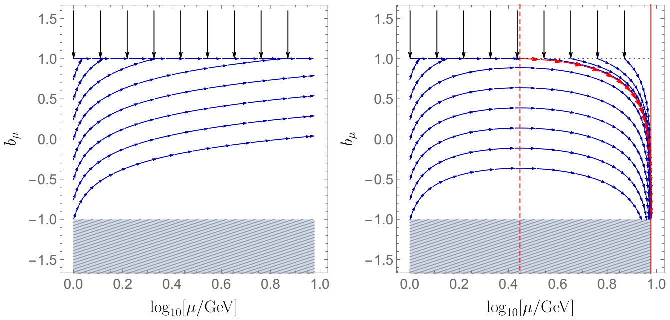


Discretize integro-differential evolution equation in QCD$\times$QED

1. LCDA becomes asymmetric ($|Q_u| > |Q_d|$)
2. LCDA becomes divergent at endpoints for (unphysical) high scales!
3. Normalization no longer invariant under scale evolution

Endpoint Behaviour

- for asymptotically small momenta $u \to 0$: ERBL kernel $\to$ Lange-Neubert kernel
- study RG-flow of exponent b_μ for $\Phi_{\pi^-}(u \to 0; \mu) \sim u^{b_\mu}$ with Mellin-space techniques:



(plot for unrealistic values: $\alpha_s(1\mathrm{GeV}) = 4\pi$, $\alpha_{\mathrm{em}}(1\mathrm{GeV}) = \pi/4$ in theory with one generation of quarks and leptons).

$\rightarrow$ RG evolution in μ drives $\Phi(u; \mu)$ to a singular form that is incompatible with the RGE!
(Convolution $\gamma(u, v) * \Phi(v; \mu)$ becomes divergent!)

$\rightarrow$ for realistic α_{em}: $\sim u \ln^p(u) \rightarrow$ inverse moments in l.p. factorization theorems well-defined!

Gegenbauer Coefficients

Normalize to point-like limit $Z_\ell(\mu)$:

$$\Phi_M(u;\mu) = Z_\ell(\mu) \times 6u\bar{u} \sum_{n=0}^{\infty} a_n(\mu) C_n^{(3/2)}(2u-1)$$

Gegenbauer coefficients mix under QED evolution (for $Q_M \neq 0$) with mixing matrix

$$f_{nm} = \frac{4(2n+3)}{(n+2)(n+1)} \int_0^1 du\,(Q_d \ln u - Q_u \ln \bar{u})\, u\bar{u}\, C_n^{(3/2)}(2u-1) C_m^{(3/2)}(2u-1) = \begin{pmatrix} 0.83 & 0.25 & 0.30 & \dots \\ 0.14 & 1.03 & 0.28 & \dots \\ 0.12 & 0.19 & 1.13 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

→ no triangular structure contrary to QCD

→ $a_0(\mu) \neq 1$ not stable under RG evolution $\Rightarrow$ $\int_0^1 du\, \Phi_M(u;\mu) \neq 1$

→ non-vanishing isospin violating $a_1(\mu)$ for pions

Analytic first-order $\mathcal{O}(\alpha_{\rm em})$ solution sums QCD logs on top of a fixed-order $\alpha_{\rm em}$ expansion

$$a_n(\mu) = a_n^{\rm QCD}(\mu) + \frac{\alpha_{\rm em}(\mu)}{\pi} a_n^{(1)}(\mu) + \mathcal{O}(\alpha_{\rm em}^2)$$

→ used for numerical estimates

Numerical Impact of QED Evolution

Reminder: we study the UV-scale evolution in QCD×QED using $a_2^\pi = 0.116$ @ 2GeV

$\mu = 5.3$ GeV (B decays)

$$a_0 = 1 + 0.0035\big|_{\rm QED}\,, \qquad a_1 = 0 + 0.0006\big|_{\rm QED}$$
$$a_2 = 0.0951\big|_{\rm LL} - 0.0084\big|_{\rm NLL} + 0.0001\big|_{\rm NNLL} + 0.0010\big|_{\rm QED}$$

$\mu = 80.4$ GeV ($W^- \to \pi^-\gamma$)

$$a_0 = 1 + 0.0094\big|_{\rm QED}\,, \qquad a_1 = 0 + 0.0015\big|_{\rm QED}\,,$$
$$a_2 = 0.0657\big|_{\rm LL} - 0.0098\big|_{\rm NLL} + 0.0002\big|_{\rm NNLL} + 0.0021\big|_{\rm QED}$$

Inverse moments relevant in l.p. factorization theorems

$$\left\langle \bar{u}^{-1} \right\rangle (5.3\,\text{GeV}) = 0.9997\big|^{\rm QED}_{\text{point charge}} \left(3.285^{+0.05}_{-0.05}\big|_{\rm LL} - 0.020\big|_{\rm NLL} + 0.017\big|^{\rm QED}_{\rm partonic}\right)$$
$$\left\langle \bar{u}^{-1} \right\rangle (80.4\,\text{GeV}) = 0.985\big|^{\rm QED}_{\text{point charge}} \left(3.197^{+0.03}_{-0.03}\big|_{\rm LL} - 0.022\big|_{\rm NLL} + 0.042\big|^{\rm QED}_{\rm partonic}\right)$$

→ For inverse moments: QED as important as two-loop QCD evolution!

→ both are $\mathcal{O}(1\%)$ effects $\sim$ uncertainty from lattice determinations

3. B-Meson LCDA aka Soft Functions

Definition

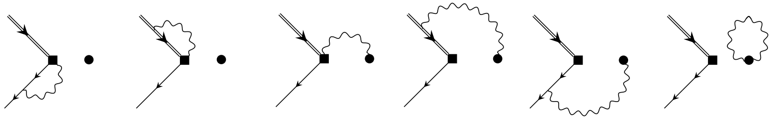
Definition for $\bar{B}^0 \to L^+ L^- \quad (L = \pi, K, \mu, \dots)$

$$iF_{\rm stat}(\mu) \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \Phi_{B,+-}(\omega;\mu) = \frac{1}{R_c R_{\bar{c}}} \langle 0| \bar{q}_s^{(d)}(tn_-)[tn_-,0]^{(d)} \not{n}_- \gamma_5 h_v(0)\, (S^\dagger_{n_+} S^\dagger_{n_-})(0)\, |\bar{B}_v\rangle$$

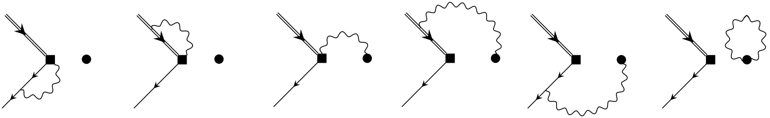
- soft photons sensitive to charge and direction of final-state particles
  - $\rightarrow$ soft functions process dependent!
- again normalized to static decay constant in absence of QED

Most surprising new feature:

- energy of outgoing anti-coll. meson M_2 is of order $m_B \to \infty$
- soft photons still couple to charged M_2 via $S^\dagger_{n_+}$; can cary away arbitrary fraction of light-cone momentum from the specator quark
  - $\rightarrow$ QED B-LCDA has support $\omega \in (-\infty,\infty)$



Anomalous Dimension



Anomalous dimension:

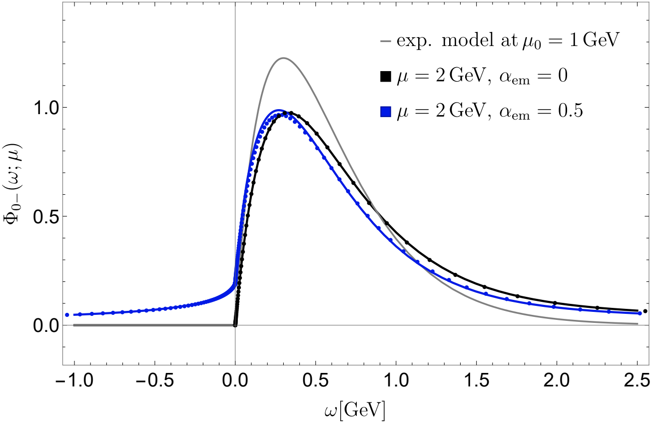
$$\gamma_{\otimes}(\omega,\omega') = \frac{\alpha_s C_F}{\pi}\left[\left(\ln\frac{\mu}{\omega - i0} - \frac{1}{2}\right)\delta(\omega-\omega') - H_+(\omega,\omega')\right]$$
$$+ \frac{\alpha_{\rm em}}{\pi}\left[\left((Q_{\rm sp}^2 + 2Q_{\rm sp}Q_{M_1})\ln\frac{\mu}{\omega - i0} - \frac{3}{4}Q_{\rm sp}^2 - \frac{1}{2}Q_d^2 + i\pi(Q_{\rm sp}+Q_{M_1})Q_{M_2}\right)\delta(\omega-\omega')\right.$$
$$\left. - Q_{\rm sp}Q_d H_+(\omega,\omega') + Q_{\rm sp}Q_{M_2}H_-(\omega,\omega')\right]$$

$\rightarrow$ contains modified plus-distributions that mix positive with negative ω, e.g.

$$\left[\frac{\theta(\omega-\omega')}{\omega-\omega'}\right]_{\oplus} \qquad \text{with} \qquad \int d\omega' \, [\ldots]_{\oplus} f(\omega') = \int d\omega' \, [\ldots]\Big(f(\omega') - \theta(\omega')f(\omega)\Big)$$

$\rightarrow$ contains explicit phases as well as phases from logarithms for $\omega < 0$

Discretization



- discretize integro-differential evolution equation e.g. for $\otimes = (0,-)$
- evolve exp. model $\phi(\omega,\mu_0) = \theta(\omega)\omega/\omega_0^2 e^{-\omega/\omega_0}$ from $\mu_0 = 1\mathrm{GeV}$ to $\mu = 2\mathrm{GeV}$

$\mathcal{O}(\alpha_{\rm em})$ Solution in ω-Space

- Full analytic all-order solution in Mellin space of conceptual importance
  $\rightarrow$ analytic properties, existence of inverse moments, ...
- for numerical estimate sufficient to use fixed-order $\mathcal{O}(\alpha_{\rm em})$ solution in ω-space:

$$
\begin{aligned}
\Phi^{>(1)}_{B,\otimes}(\omega,\mu) &= \frac{1}{\omega_0} e^{V+2\gamma_E a} \left(\frac{\mu_0}{\omega_0}\right)^a \Bigg[-Q_{\rm sp} Q_{M_2} \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \frac{\alpha_s(\mu')C_F}{\pi} G^{33}_{34}\left(\frac{\omega}{\omega_0}\right) \ln\frac{\mu'}{\mu_0} \\
&+ \int_0^1 dx \left[\frac{1}{1-x}\right]_+ \Bigg(-Q_{\rm sp}Q_{M_1} \left\{ h(x) F_a\left(\frac{x\omega}{\omega_0}\right) + h(x^{-1}) F_a\left(\frac{\omega}{x\omega_0}\right) \right\} \\
&+ Q_{\rm sp} Q_{M_2} \left\{ h(x) F_a\left(\frac{x\omega}{\omega_0}\right) - x^{-1} h(x^{-1}) F_a\left(\frac{\omega}{x\omega_0}\right) \right\} \\
&+ (Q_{\rm sp}^2 + 2Q_{\rm sp}Q_{M_1}) x^a F_a\left(\frac{x\omega}{\omega_0}\right) \ln\frac{\mu}{\mu_0} \Bigg) \\
&+ \Bigg\{ -(Q_{\rm sp}^2 + 2Q_{\rm sp}Q_{M_1}) \left(\frac{1}{2}\ln^2\frac{\mu}{\mu_0} + \ln\frac{\mu_0 e^{\gamma_E}}{\omega_0} \ln\frac{\mu}{\mu_0} \right) \\
&+ \left(\frac{3}{4} Q_{\rm sp}^2 + \frac{1}{2} Q_d^2 - i\pi Q_{M_1} Q_{M_2} \right) \ln\frac{\mu}{\mu_0} \Bigg\} F_a\left(\frac{\omega}{\omega_0}\right) \Bigg], \\
\Phi^{<(1)}_{B,\otimes}(\omega,\mu) &= -\frac{Q_{\rm sp}Q_{M_2}}{\omega_0} e^{V+2\gamma_E a} \left(\frac{\mu_0}{\omega_0}\right)^a \Gamma(1+a)\Gamma(2+a)\, U\!\left(1+a, 0, -\frac{\omega}{\omega_0}\right) \ln\frac{\mu}{\mu_0},
\end{aligned}
$$

Numerical Impact of QED Evolution

Inverse moments in leading-power factorization theorems:

$$\lambda_B^{-1}(\mu) = \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i0} \Phi_B(\omega,\mu) \qquad \text{and} \qquad \sigma_n(\mu) = \lambda_B(\mu) \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i0} \ln^n \frac{\tilde{\mu}}{\omega - i0} \Phi_B(\omega,\mu)$$

- → $i0$ prescription appears naturally from hard-coll. jet function
- → λ_B acquires an imag. part if M_2 is charged!
- → of conceptual importance: λ_B^{-1} exists, but higher moments endpoint-divergent (as in QCD)

For exp. model $\phi(\omega,\mu_0) = \omega/\omega_0^2 e^{-\omega/\omega_0}$ at $\mu_0 = 1$GeV with $\omega_0 = \lambda_B(\mu_0) = 0.3$ GeV:

	$\mu_0 = 1$ GeV	$\mu = 2$ GeV				
	initial	QCD	(0,0)	(−,0)	(0,−)	(+,−)
λ_B^{-1}	3.333	2.792	2.792	2.802	$2.790 + 0.010i$	$2.798 + 0.010i$
σ_1	0	−0.213	−0.213	−0.210	−0.214	−0.211

- → QED evolution of soft function at most 1.2% effect; smaller than two-loop QCD evolution

Conclusion

- The factorization of QED effects in non-leptonic B decays between m_B and Λ_{QCD} requires new non-perturbative hadronic matrix elements.
- Generalized heavy- and light-meson LCDAs exhibit novel properties (semi-universality, support properties, re-scattering phases, etc.).
  - $\rightarrow$ non-decoupling of soft photons from charged mesons
  - $\rightarrow$ different quark charges of meson constituents

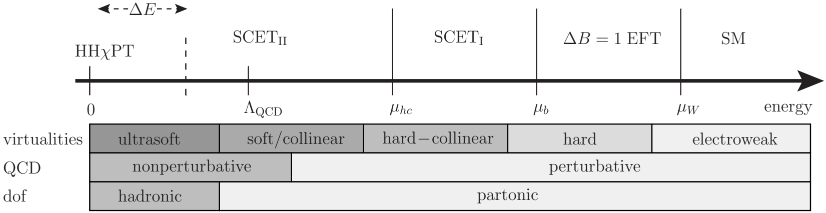
  In particular soft physics is qualitatively different from standard hard-scattering picture.
- Numerically small, but can compete with QCD uncertainty. For light mesons QED $\sim$ two-loop QCD running.
- $\mathcal{O}(\alpha_{\mathrm{em}})$ RG evolution completes the analysis of QED factorization in non-leptonic B decays at scales greater than a few times Λ_{QCD}.
  - $\rightarrow$ non-perturbative matching onto an effective theory of ultrasoft photons coupling to infinitely heavy, but ultra-relativistic point-like mesons would complete the theoretical understanding of QED effects in exclusive B-meson decays from the EFT viewpoint.

Backup-Slides

Scales and EFTs

Hierarchy of energy scales:

$M_W \sim 80\,\text{GeV} \gg m_b \sim 4.2\,\text{GeV} \gg \text{few times}\ \Lambda_{\text{QCD}} \gg \Delta E \sim 60\,\text{MeV}$



Soft Rearrangement

- effective SCET operator composend of soft and (anti-)collinear fields
  - $\rightarrow$ has well-defined UV-scale evolution ✓
- But: Factorization requires renormalization of each individual mode

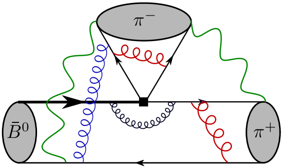
↯ Problem: UV-scale evolution of individual pieces IR-divergent! ("factorization anomaly")

- $\rightarrow$ Anomalous dimension depends on IR-regulator (off-shellness, quark masses, . . .)
- $\rightarrow$ can be cured by a "soft rearrangement" that removes the soft overlap

cf. [Beneke, Bobeth, Szafron]

$$\mathcal{O} = \mathcal{O}_{\bar{c}} \times \mathcal{O}_{s,C} \rightarrow (\mathcal{O}_{\bar{c}} R_{\bar{c}}) \times \left(\frac{\mathcal{O}_{s,C}}{R_{\bar{c}}}\right) \quad \text{with} \quad \left|\langle 0| S_{n_+}^{\dagger(Q_2)} S_{n_-}^{(Q_2)} |0\rangle\right| \equiv R_{\bar{c}} R_c$$

QCD×QED Factorization Formula



SCET$_\mathrm{I}$ operators:

$$\mathcal{O}^\mathrm{I} \sim [\bar\chi_C S_{n_+}^{Q_2} h_v] \, [\bar\chi_{\bar C}(tn_-) \not{n}_- \gamma_5 \chi_{\bar C}]$$

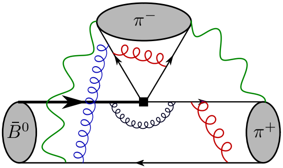
$$\mathcal{O}^\mathrm{II} \sim [\bar\chi_C \mathcal{A}_{C,\perp}(sn_+) S_{n_+}^{Q_2} h_v] \, [\bar\chi_{\bar C}(tn_-) \not{n}_- \gamma_5 \chi_{\bar C}]$$

$$\langle M_1 M_2 | Q_i | \bar B \rangle\big|_{\text{non-rad.}} = \mathcal{F}_{Q_2}^{B\to M_1}(q^2=0) \int_0^1 \mathrm{d}u\, \mathbf{T}^\mathrm{I}_{i,Q_2}(u)\, f_{M_2} \Phi_{M_2}(u) + \int \mathrm{d}\omega \int_0^1 \mathrm{d}u\,\mathrm{d}v\, \mathbf{T}^\mathrm{II}_{i,\otimes}(u,v,\omega)\, f_{M_1}\Phi_{M_1}(v)\, f_{M_2}\Phi_{M_2}(u)\, f_B \Phi_{B,\otimes}(\omega)$$

$\otimes = (Q_{M_1}, Q_{M_2})$

- we consider $\langle M_1 M_2 | Q_{1,2} | \bar B\rangle$ with current-current operators $Q_{1,2}$ only
- retains its form but non-perturbative objects need to be generalized
  - $\to$ decay constants in pure QCD
  - $\to$ similarly for heavy-to-heavy $B \to D$ transitions
- form factor $F^{B\to\pi} \to$ semi-leptonic amplitude $A^{B\to\pi\ell\bar\nu_\ell}$ for charged M_2

QCD×QED Factorization Formula



SCET$_{\rm I}$ operators:

$$\mathcal{O}^{\rm I} \sim \quad [\bar{\chi}_C S_{n_+}^{Q_2} h_v] \quad [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]$$

$$\mathcal{O}^{\rm II} \sim [\bar{\chi}_C \not{A}_{C,\perp}(sn_+) S_{n_+}^{Q_2} h_v] \, [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]$$

$$\langle M_1 M_2 | Q_i | \bar{B}\rangle\big|_{\text{non-rad.}} = \mathcal{F}_{Q_2}^{B\to M_1}(q^2=0) \int_0^1 \mathrm{d}u \, \mathbf{T}^{\rm I}_{i,Q_2}(u) \, f_{M_2}\Phi_{M_2}(u)$$
$$+ \int \mathrm{d}\omega \int_0^1 \mathrm{d}u \, \mathrm{d}v \, \mathbf{T}^{\rm II}_{i,\otimes}(u,v,\omega) \, f_{M_1}\Phi_{M_1}(v) \, f_{M_2}\Phi_{M_2}(u) \, f_B \Phi_{B,\otimes}(\omega)$$

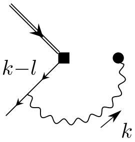
$$\otimes = (Q_1, Q_2)$$

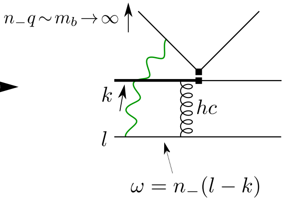
Important feature: effective operators inherit soft Wilson lines from electrically charges mesons!

$$S_{n_+}^{(q)}(x) = \exp\left\{-iQ_q e \int_0^\infty \mathrm{d}s \, n_+ \cdot A_s(x+sn_+)\right\}$$

⇒ Soft functions become process-dependent!

On the Support of QED B LCDAs





- even for on-shell massive partons with $\Phi^{(0)}(\omega) = \delta(\omega - m)$ the one-loop soft photon exchange with the anti-coll. π^- generates a support for $\omega < 0$
- diagram has e.g. the following contribution

$$\int \mathrm{d}^d k \, \frac{\delta(\omega - n_-\ell + n_-k)}{(k^2 + i0)[(k-\ell)^2 - m^2 + i0]\,(n_+k - i0)} \quad \Big| \text{pick up residues in } (n_+k)$$

$$\sim \Gamma(\epsilon) \int_{n_-\ell}^{\infty} \mathrm{d}(n_-k)\,(n_-k)^{-1-\epsilon} \delta(\omega - m + n_-k) = \Gamma(\epsilon)\,(m-\omega)^{-1-\epsilon}\theta(-\omega)$$

- QED B LCDA no longer linear in ω as $\omega \to 0$ but rather const.
  - $\to$ no endpoint singularity in first inverse moment

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{\omega - i0} \Phi_{B,+-}(\omega)$$