QED Corrections in Leptonic B-Meson Decays

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Invitation - a simple computation for a simple process

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In the exclusive case, one puts a **cut on photon energy**: $E_\gamma < E_s/2$. This gives additional logarithms $L_s = \log E_s^2/m_\ell^2$.

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$$\Gamma_{\rm NLO} = \Gamma_{\rm LO}\left\{1 + \frac{\alpha}{2\pi}\left[\frac{3}{2}L_\mu - L_\ell^2 - L_\ell L_s - \frac{7}{2}L_\ell - 2L_s - \frac{\pi^2}{3} + 2\right]\right\}$$

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Yes, but things will get **much** more complicated (and fun!).

All the scales

If we are being precise, the process $B \to \ell\nu$ depends on a multitude of scales (we focus on $\ell = \mu$ for now)

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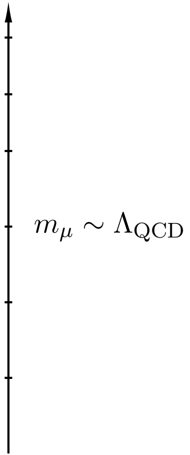

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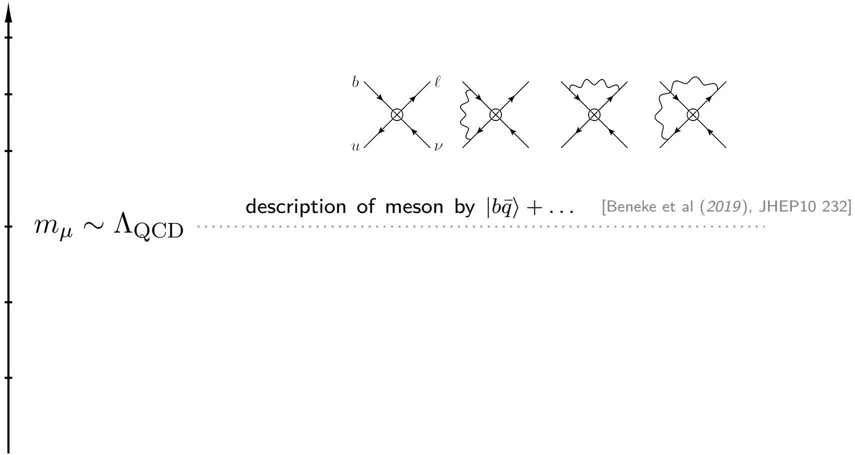

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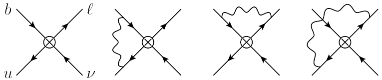

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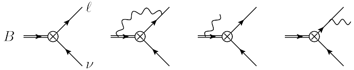
$m_\mu \sim \Lambda_{\mathrm{QCD}}$

description of meson by point-like scalar



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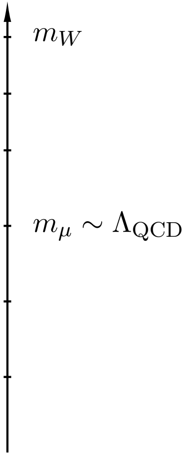

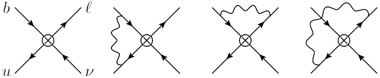

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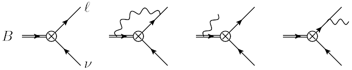

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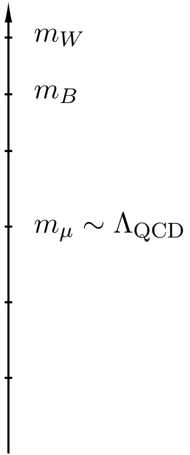

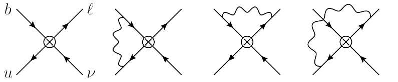

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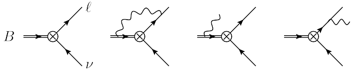

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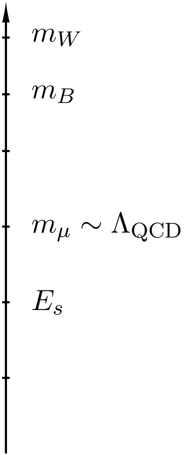

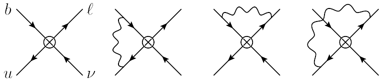

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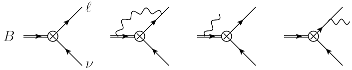

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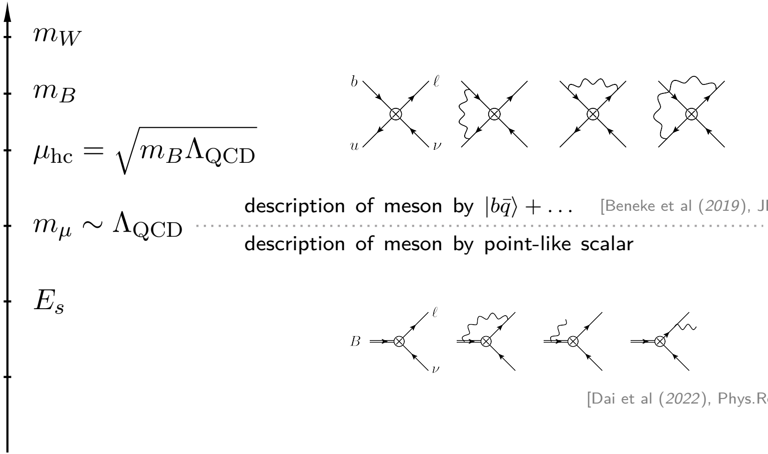

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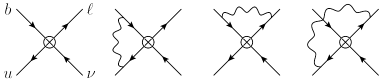

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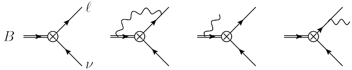

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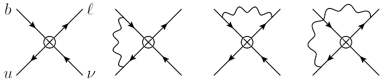

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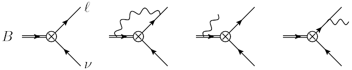

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Want to disentangle all of these scales $\Rightarrow$ EFTs!

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- Derive a **factorization theorem** to break the multiscale process into a **product of single-scale objects**

- Use the **renormalization group** to evaluate each object at its natural scale and evolve them to a common scale to **resum logarithms**

Matching step 1 - the hard scale: $\mu \sim m_b$

Below the hard scale, $\mu \sim m_b$, radiation is too soft to significantly change the b momentum. The appropriate description of the b quark is **HQET**:

$$b(x) \to e^{im_b(v\cdot x)} h_v(x)$$

Consider b at rest, up to fluctuations of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$.

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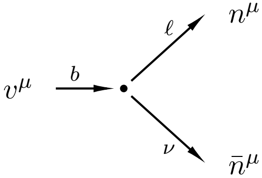
The light fields can have **large momenta**, but **small invariant mass**. Momentum scalings by direction of large energy flow:



$n^\mu = (1,0,0,1)$
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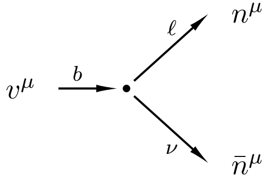

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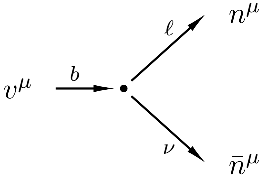

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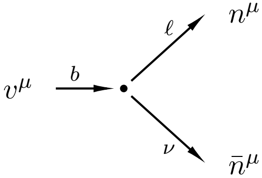

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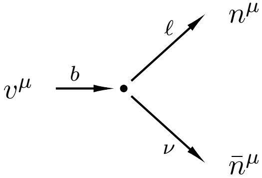

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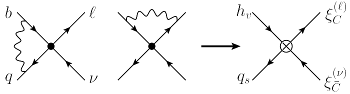
Note: $p_C^2 \sim \mathcal{O}(\lambda) \ll p_s^2 \sim \mathcal{O}(\lambda^2)$

$\Rightarrow$ The appropriate EFT here is **SCET I**.

Method of regions - the hard-collinear scale

Loops with virtuality $p_h^2 \sim \mathcal{O}(m_b^2)$ match onto four-fermion operators in SCET I:



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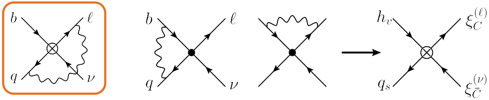

The graph $\propto Q_\ell Q_u$ has two "collinear" regions. The on-shell lepton momentum would scale as:

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p^\mu & = & \frac{\bar{n}\cdot p}{2} n^\mu & + & \frac{n\cdot p}{2}\bar{n}^\mu & + & p_\perp^\mu \\
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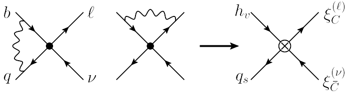

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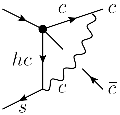

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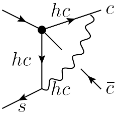

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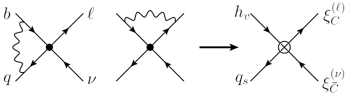

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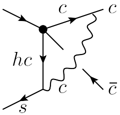

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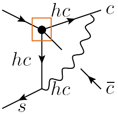

$\Rightarrow$ need operators with hard-collinear fermions!

SCET I operators

With SCET I power-counting:

$$h_v, q_s \sim \mathcal{O}\left(\lambda^{3/2}\right) \qquad \chi_C \sim \mathcal{O}\left(\lambda^{1/2}\right) \qquad \mathcal{A}_C^{\perp} \sim \mathcal{O}\left(\lambda\right)$$

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we can define our operator basis. Two types:

direct (A-type) $\mathcal{O}_A \sim (\bar{q}_s \dots h_v)\left(\bar{\chi}_C^{(l)} \dots \bar{\chi}_{\bar{C}}^{(\nu)}\right)$

indirect (B-type) $\mathcal{O}_B \sim (\bar{\chi}_C \dots h_v)\left(\bar{\chi}_C^{(l)} \dots \bar{\chi}_{\bar{C}}^{(\nu)}\right)$

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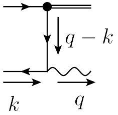
Some **indirect** operators are **power-enhanced**, but the matrix elements need an insertion of the power-suppressed soft-collinear interactions.

To understand the emergence of a hard-collinear scale, consider the virtual quark in the following subgraph:



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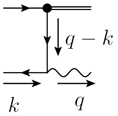

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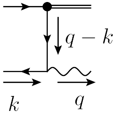

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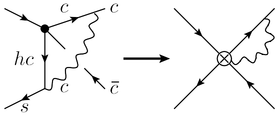

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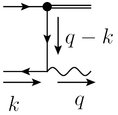

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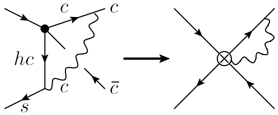

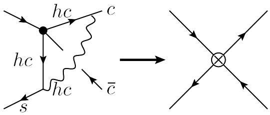

We now **lower the virtuality**, removing all hard-collinear modes. In the new EFT, the collinear and soft modes live at the same virtuality:

$$p_c \sim (1, \lambda^2, \lambda)\,, \qquad p_s \sim (\lambda, \lambda, \lambda)\,, \qquad p_c^2 \sim p_s^2 \sim \mathcal{O}\left(\lambda^2\right)\,.$$

$\Rightarrow$ SCET II type construction.

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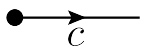

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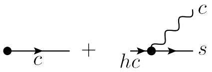

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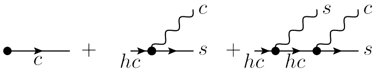

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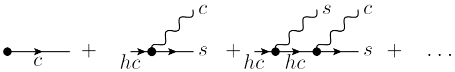

The intermediate propagators introduce **non-localities**, even in soft operator products:

$$\frac{1}{n \cdot \partial} q_s\,, \quad \left(\frac{1}{n \cdot \partial} \mathcal{A}^{\mu}_{\perp s}\right) \left(\frac{1}{n \cdot \partial} q_s\right), \quad \ldots$$

$\Rightarrow$ more fields, same order

SCET II – non-localities

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For us $v = a$ so this is evanescent.

Of course, the $\mathcal{O}(\epsilon)$ structures **can still contribute** to physical processes, once these operators are inserted into loop graphs:

$$\mathcal{O}(\epsilon) \cdot \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^4} \quad = \quad \mathcal{O}(1)$$

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- ... and fully cancel between matrix elements and matching coefficients $\rightarrow$ means they vanish identically.

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$$O_A^{(5)} = \left(\bar{q}_s(sn)\gamma_\mu^\perp P_L h_v(0)\right)\left(\bar{\chi}_c^{(\ell)}(t\bar{n})\gamma_\perp^\mu P_L \chi_{\bar{c}}^{(\nu)}(un)\right)$$

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SCET II operators

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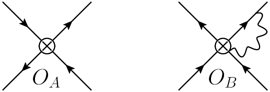
Matrix elements $\langle O_A \rangle$ and $\langle O_B \rangle$ start at tree-level and one-loop, respectively:



Matching step 3 - the hadronic scale

Below $\mu \sim \Lambda_{\mathrm{QCD}}$ we are passing to an **effective description** of a Yukawa theory:

$$\mathcal{L}_{\mathrm{y}} = y\, e^{-im_B(v\cdot x)} \varphi_B \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) + \mathrm{h.c.}$$

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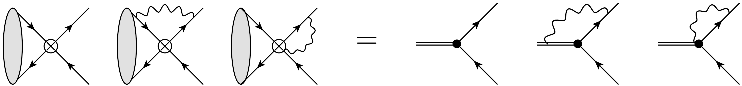
$$\langle \ell\, \nu \,|\, \mathcal{L}_{\rm SCET\,II\otimes HQET} \,|\, B\rangle \quad = \quad \langle \ell\, \nu \,|\, \mathcal{L}_{\rm SCET\,II\otimes HSET} \,|\, B\rangle$$



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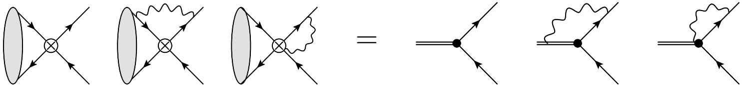
$$\langle \ell\, \nu \,|\, \mathcal{L}_{\text{SCET II}\otimes\text{HQET}} \,|\, B \rangle \quad = \quad \langle \ell\, \nu \,|\, \mathcal{L}_{\text{SCET II}\otimes\text{HSET}} \,|\, B \rangle$$



But there is a second matching step.

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This means, there are **no virtual corrections** in this EFT!

In order to understand the **degrees of freedom** of the low-energy theory, a region analysis is in order.

Remember the cut on extra photons was: $E_\gamma < E_s/2 \sim \mathcal{O}\left(\lambda^2\right)$.

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$$q_{\rm sc} \sim m_B(\lambda^2, \lambda^4, \lambda^3) \quad \to \quad q'_{\rm sc} \sim m_B(\lambda^3, \lambda^3, \lambda^3) \sim m_\ell(\lambda^2, \lambda^2, \lambda^2)$$

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This new region gives rise to logarithms $L_\mu^{\rm sc} = \log \mu^2/\mu_{\rm sc}^2$ of the **soft-collinear scale** $\mu_{\rm sc} = \frac{E_s m_\ell}{m_B}$:

$$\Gamma_{(\rm sc)} = \Gamma_0 \frac{\alpha}{2\pi}\left[-\frac{1}{\epsilon^2} + \frac{1 - L_\mu^{\rm sc}}{\epsilon} - \frac{1}{2}\left(L_\mu^{\rm sc}\right)^2 + L_\mu^{\rm sc} - \frac{\pi^2}{12}\right]$$

The low-energy theory is now given by (HSET $\otimes$ bHLET), with the fields:

$$\ell(x) = e^{-im_\ell\,(v_\ell\cdot x)}\chi_{v_\ell}(x)\,, \qquad \Phi_B(x) = e^{-im_B\,(v\cdot x)}\varphi_B$$

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$$Y_v^{(\mathrm{s})}(x) = \mathcal{P} \exp\left\{ ie \int_{-\infty}^{0} ds\, v \cdot A_s(x + sv) \right\}$$

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leading to the operator:

$$O_\varphi = Y_n^{(\mathrm{s})}(x_-)Y_v^{(\mathrm{s})\dagger}(x)Y_{v_\ell}^{(\mathrm{sc})}(x)Y_{\bar{n}}^{(\mathrm{sc})\dagger}(x_+)\cdot\varphi_B(x)\left(\bar{\chi}_{v_\ell}P_L\xi_{\bar{c}}^{(\nu)}\right)$$

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$$W_s(\omega_s,\mu) = \left[\sum_{n_s=0}^{\infty}\prod_{i=1}^{n_s}\int d\Pi_i(q_i)\right]\left|\langle n_s\gamma_s(q_i)|\, Y_v^{(\mathrm{s})} Y_n^{(\mathrm{s})\dagger}\,|0\rangle\right|^2 \delta\left(\omega_s - q_0^{(\mathrm{s})}\right)\,,$$

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When integrated over a **measurement function**, they combine to the **soft function** of the process:

$$S(E_s,\mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc}\; \theta\left(\frac{E_s}{2} - \omega_s - \omega_{sc}\right) W_s(\omega_s,\mu) W_{sc}(\omega_{sc},\mu)$$

Soft function at one-loop

This can be integrated with the measurement function over $\omega_{s,sc}$ in Laplace space:

$$\tilde{S}_0(s,\mu) = \int_0^\infty dE_s e^{-sE_s} S(E_s,\mu) = \frac{1}{s}\tilde{W}_{\rm s}(2s,\mu)\tilde{W}_{\rm sc}(2s,\mu)\,,$$

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with:

$$\tilde{W}_s^0(2s,\mu) = 1 + \frac{\alpha}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{1+\tilde{L}_s}{\epsilon} + \frac{\tilde{L}_s^2}{2} + \tilde{L}_s + \frac{\pi^2}{12} + 1\right)\,,$$

$$\tilde{W}_{sc}^0(2s,\mu) = 1 + \frac{\alpha}{2\pi}\left(-\frac{1}{\epsilon^2} + \frac{1-\tilde{L}_{sc}}{\epsilon} - \frac{\tilde{L}_{sc}^2}{2} + \tilde{L}_{sc} - \frac{5\pi^2}{12}\right)\,,$$

$$\tilde{L}_s = \log \mu^2 s^2 e^{2\gamma_E}\,, \qquad \tilde{L}_{sc} = \log\frac{\mu^2 s^2 e^{2\gamma_E}}{r_l^2}.$$

Each of these can now be renormalized to perform the resummation of the soft and soft-collinear logs.

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Thank you for your attention!

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Bonus slides

There are no bonus slides.