YFS soft-photon resummation in hadron decays, systematics and ME corrections

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The SHERPA event generator framework

Two multi-purpose Matrix Element (ME) generators
AMEGIC++ JHEP02(2002)044
COMIX JHEP12(2008)039

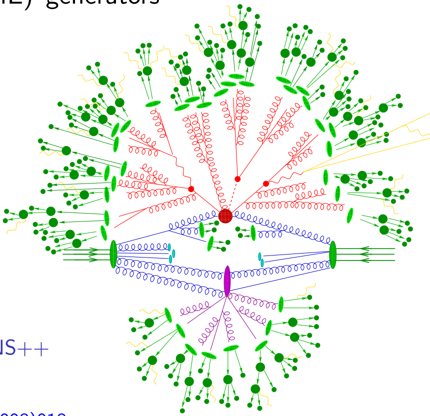
Two Parton Shower (PS) generators
CSSHOWER++ JHEP03(2008)038
DIRE EPJC75(2015)461

A multiple interaction simulation
à la PYTHIA AMISIC++ hep-ph/0601012

A cluster fragmentation module
AHADIC++ EPJC36(2004)381

A hadron and τ decay package HADRONS++

A higher order QED generator using
YFS-resummation PHOTONS++ JHEP12(2008)018



Sherpa's traditional strength is the perturbative part of the event

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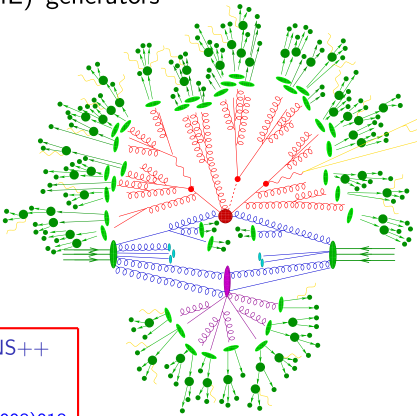

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SHERPA

HADRONS++ package for hadron and τ decays

- decay tables for τ and $\mathcal{O}(200)$ hadrons
- in total > 2.6k decay channels
- many have multiple form factor models available
- accounts for $D-\overline{D}$-, $B-\overline{B}$-, $B_s-\overline{B}_s$-mixing, off-shell decays, ...
- new calculations for specific decay channels can be introduced without modifying the existing release through dynamically loaded libs

QED corrections

- through soft-photon resummation in YFS framework

  Yennie, Frauschi, Suura Annals Phys. 13 (1961) 379-452

- universal collinear logarithms supplied at fixed-order

  $\Rightarrow$ **details in the following**

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Overview

YFS soft photon resummation

1. YFS soft photon resummation
2. Hard emission corrections
3. Some results
4. Conclusions

YFS algorithm

Yennie, Frauschi, Suura Annals Phys. 13 (1961) 379-452

General ideas:

- identify the infrared divergence structure of QED to all orders
- all charged particles (leptons, hadrons, etc) are considered massive
  $\rightarrow$ no collinear singularities
  $\rightarrow$ no γ-splittings in soft-photon limit
  ↳ radiator function the same to all orders
- all divergences associated with emission off external legs
  $\rightarrow$ all large (soft) logs can be resummed to all orders can be constructed universally without knowledge of the hard interaction
- soft-photon limit is spin-independent,
  same soft limit in scalar, fermion, vector QED

YFS algorithm

Divergences in loop diagrams

Construct off-shell Eikonal B to subtract all infrared divergences at amplitude level

$$\mathcal{M}_0^1 = M_0^1 + \alpha B M_0^0,$$

$$\mathcal{M}_0^2 = M_0^2 + \alpha B M_0^1 + \frac{(\alpha B)^2}{2!} M_0^0$$

$$\vdots$$

$$\mathcal{M}_0^{\bar{n}_\gamma} = \sum_{r=0}^{\bar{n}_\gamma} M_0^{\bar{n}_\gamma - r} \frac{(\alpha B)^r}{r!}$$

The $M_0^{\bar{n}_\gamma}$ are now free of divergences due to r of the $\bar{n}_\gamma$ virtual photons becoming soft. This holds for any simultaneous real-emission photons n_γ.

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Divergences in real-emission diagrams

Construct on-shell Eikonal $\tilde{S}$ to subtract all infrared divergences at squared amplitude level

$$\left|M_1^{\frac{1}{2}}\right|^2 = \tilde{\beta}_1^1 + \alpha\tilde{S}\left|M_0^0\right|^2$$

$$\left|M_2^1\right|^2 = \tilde{\beta}_2^2 + \alpha\tilde{S}\left|M_1^{\frac{1}{2}}\right|^2 + (\alpha\tilde{S})^2\left|M_0^0\right|^2$$

$$\vdots$$

The $\tilde{\beta}_{n_\gamma}^{\bar{n}_\gamma + n_\gamma}$ are now free of any (real and virtual) infrared divergence.

Additional $\frac{1}{n_\gamma!}$ from symmetry factor, introduce $\tilde{B}(\omega_{\text{cut}}) = \int_0^{\omega_{\text{cut}}} \mathrm{d}\Phi\, \tilde{S}$.

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YFS algorithm

Differential decay rate

$$\mathrm{d}\Gamma^{\mathrm{YFS}} = e^{\alpha Y(\omega_{\mathrm{cut}})} \cdot \sum_{n_\gamma} \frac{1}{n_\gamma!} \, \mathrm{d}\Phi \left[\prod_{i=1}^{n_\gamma} \mathrm{d}\Phi_{k_i} \cdot \alpha \, \tilde{S}_{\omega_{\mathrm{cut}}}(k_i) \right] \times \left(\tilde{\beta}_0^0 + \tilde{\beta}_0^1 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1^1(k_i)}{\alpha \tilde{S}(k_i)} + \ldots \right)$$

The YFS form factor $Y(\omega_{\mathrm{cut}}) = B + \tilde{B}(\omega_{\mathrm{cut}})$ resums all soft-photon logarithms to all orders, and all $\tilde{\beta}_{n_\gamma}^{\bar{n}_\gamma + n_\gamma}$ are IR finite.

So far, the perturbative series has only been reordered. There is no matching procedure, etc, needed.
Compute the hard emission corrections $\tilde{\beta}$ as far as possible/needed.

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Hard emission corrections

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Full hard emission corrections are process dependent.

Universal collinear hard emission approximation:

- use collinear approximation (DGLAP splitting functions) D and subtract fraction of soft Eikonal associated with each coll. region D^{soft}

$$\tilde{\beta}_1^1 = \alpha \sum_i \left[D_i - D_i^{\text{soft}} \right] \tilde{\beta}_0^0$$

- in SHERPA use Catani-Seymour splitting functions for D as they conserve momentum locally

⇒ essentially restores $\log \frac{M}{m_\ell}$ to $\mathcal{O}(\alpha)$

Hard emission corrections for $B \to X\ell\nu$

Full hard emission corrections are process dependent.

Computed NLO QED corrections to semileptonic B decays.
$B \to D\ell\nu$, $B \to D^*\ell\nu$, $B \to \pi\ell\nu$ (B^0, $B^\pm$) Bernlochner, MS '10

LO **hadronic current** is given by

$$H^\mu(p_B, p_X; q^2) = (p_B + p_X)^\mu\, f_+(q^2) + (p_B - p_X)^\mu\, f_-(q^2)$$

For constant form factors one can assign a **Lagrangian** for the weak decay

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}}\, V_{xb}\left[(f_+ + f_-)\phi_X \partial^\mu \phi_B + (f_+ - f_-)\phi_B \partial^\mu \phi_X\right]\, \overline{\psi}_\nu P_R \gamma_\mu \psi_\ell + \text{h.c.}$$

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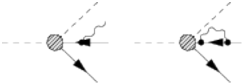
Hard emission corrections for $B \to D\ell\nu$

Requiring QED gauge invariance gives interaction terms

$$\mathcal{L}_{\text{int}}^{\text{QED}} = -e\overline{\psi}_\ell\gamma^\mu\psi_\ell A_\mu - ieQ_\phi\left(\phi^+\partial^\mu\phi^- - \phi^-\partial^\mu\phi^+\right)A_\mu + e^2Q_\phi^2\phi^+\phi^- A_\mu A^\mu + ie\sqrt{2}G_F V_{xb}f_\pm(Q_B \pm Q_X)\phi_B\phi_X\,\overline{\psi}_\nu P_R\gamma^\mu\psi_\ell\,A_\mu + \text{h.c.}$$

In addition to usual QED and scalar QED interactions an additional terms describing emissions off the vertex arises.



What is missing?

- off-shell form-factors in hadronic current
- proper treatment of hadronic resonances, eg. $B \to D^*\ell\nu \to D\gamma\,\ell\nu$
- hadron-photon interaction beyond point-like scalar QED

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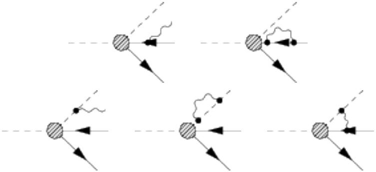

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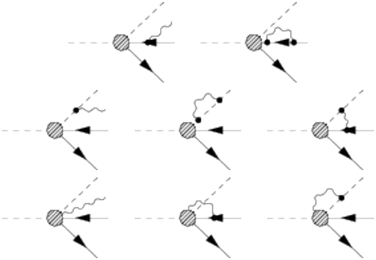

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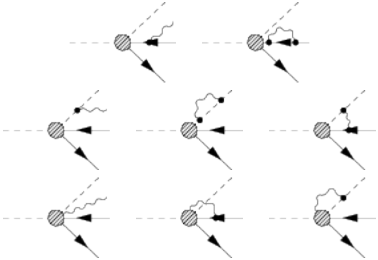

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Some results

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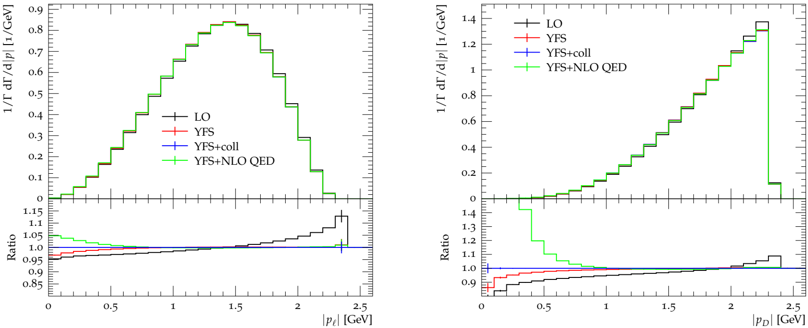
Some results – $B^- \to D^- e^+ \nu_e$

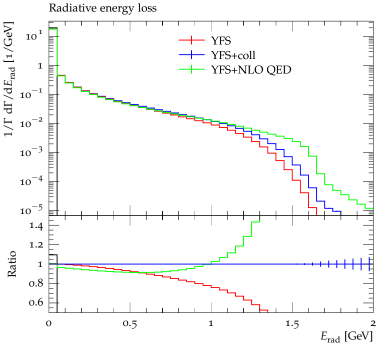


- bulk of the effect capture already by pure soft-photon resummation, universal collinear corrections needed at small $|p|$
- though large enhancement through full NLO QED hard emission corrections at small $|p|$, physical or artifact?

Some results – $B^- \to D^- e^+ \nu_e$

Useful observable:
total radiative energy loss

Kinematic restrictions limit energy taken by single photon, higher losses only through multi-photon radiation



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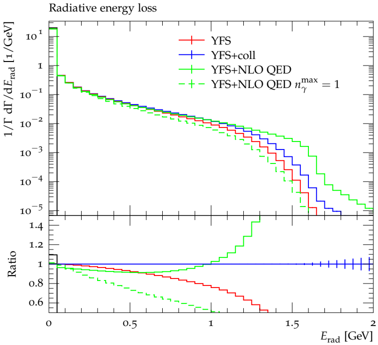

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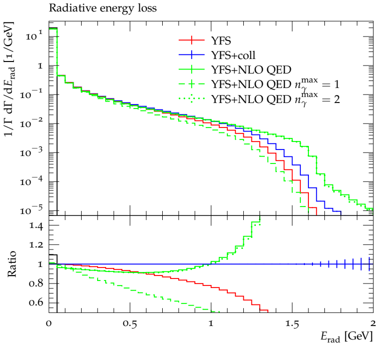

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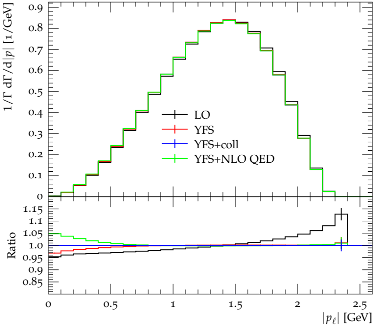

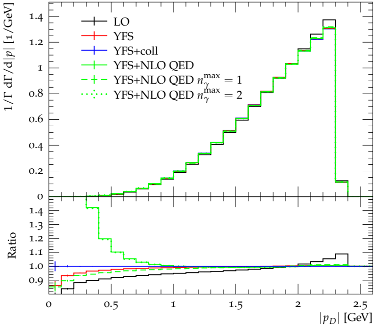

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- cannot say, no NNLO corrections impl., investigate systematics

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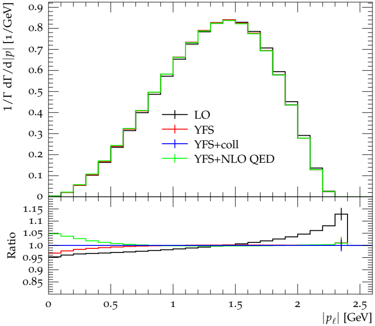

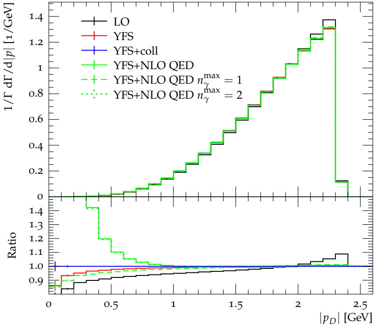

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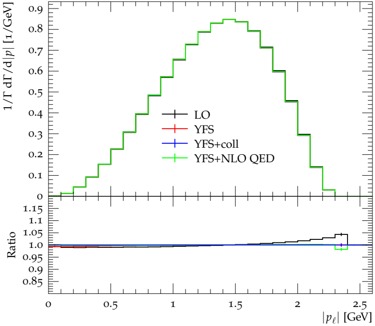
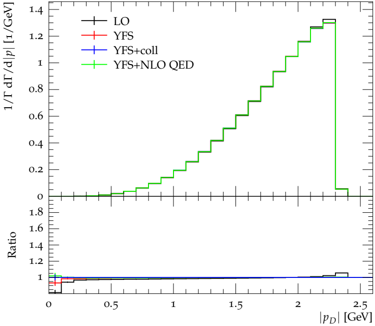
Systematics

Not much in the YFS soft-photon resummation framework, no free scales or choices. Only some freedom how to distribute recoil for photon emissions away from the soft limit.

Improvements: calculate NLO QED hard emission corrections including off-shell currents and full treatment of resonances. Calculate dominant NNLO QED hard emission corrections, ie. in the two-hard-photon phase space.

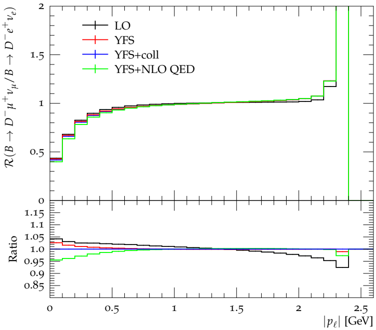
Some results – $B^- \to D^- \mu^+ \nu_\mu$

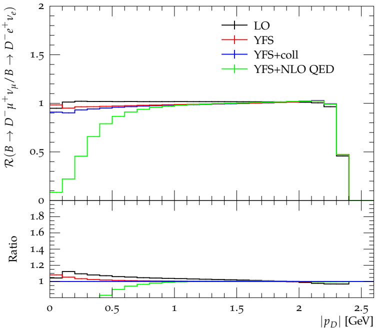




- due to $m_\mu \gg m_e$ radiative corrections much smaller
- no enhancement at small $|p|$

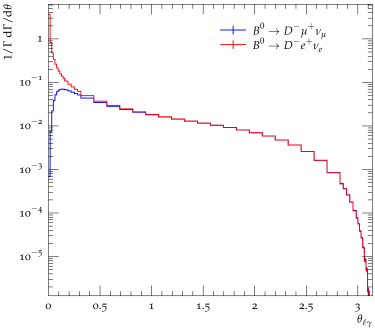
Some results – ratio μ/e

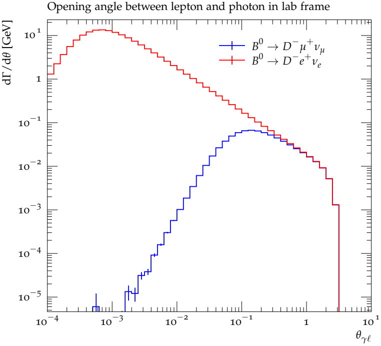




- lepton universality broken at LO due to $m_\mu \gg m_e$ in phase space
- QED induces non-universal effect at small $|p_D|$, $|p_\ell|$

Some results – $B^- \to D^- \mu^+ \nu_\mu$ vs. $B^- \to D^- e^+ \nu_e$





- spectra coincide away from mass suppressed collinear region
- deadcone of both electron numerically stable

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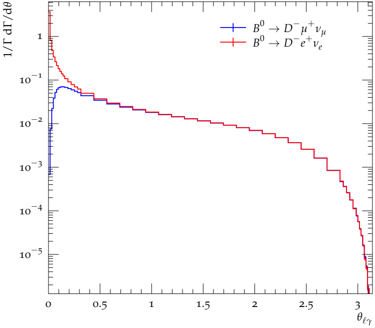

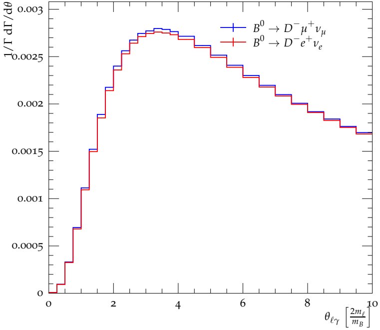

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Conclusions

- soft-photon resummation, however, captures the bulk of the QED corrections
- hard emission corrections can universally be constructed through collinear splitting functions
- some indication that larger effects induced by vertex emissions may be present at small $|p_B|$, and may be significantly affected by structure-dependent terms
- dedicated hard emission corrections available for generic $S \to SS$, $S \to \ell\ell$, $V \to SS$, $V \to \ell\ell$, $S \to S\ell\nu$, $S \to V\ell\nu$, and $\tau \to \ell\nu\nu$ decays using point-like scalar and vector QED implemented in SHERPA
- $\gamma \to e^+e^-, \mu^+\mu^-, \pi^+\pi^-, \ldots$ currently in development (L. Flower) $\to$ finite NNLO correction to soft-photon resummation

Thank you!