

BERNHARD MISTLBERGER



APPROXIMATIONS FOR LHC CROSS SECTIONS USING COLLINEAR EXPANSIONS

With **Gherardo Vita**

[arXiv:2006.03055](https://arxiv.org/abs/2006.03055)

And To Appear ...

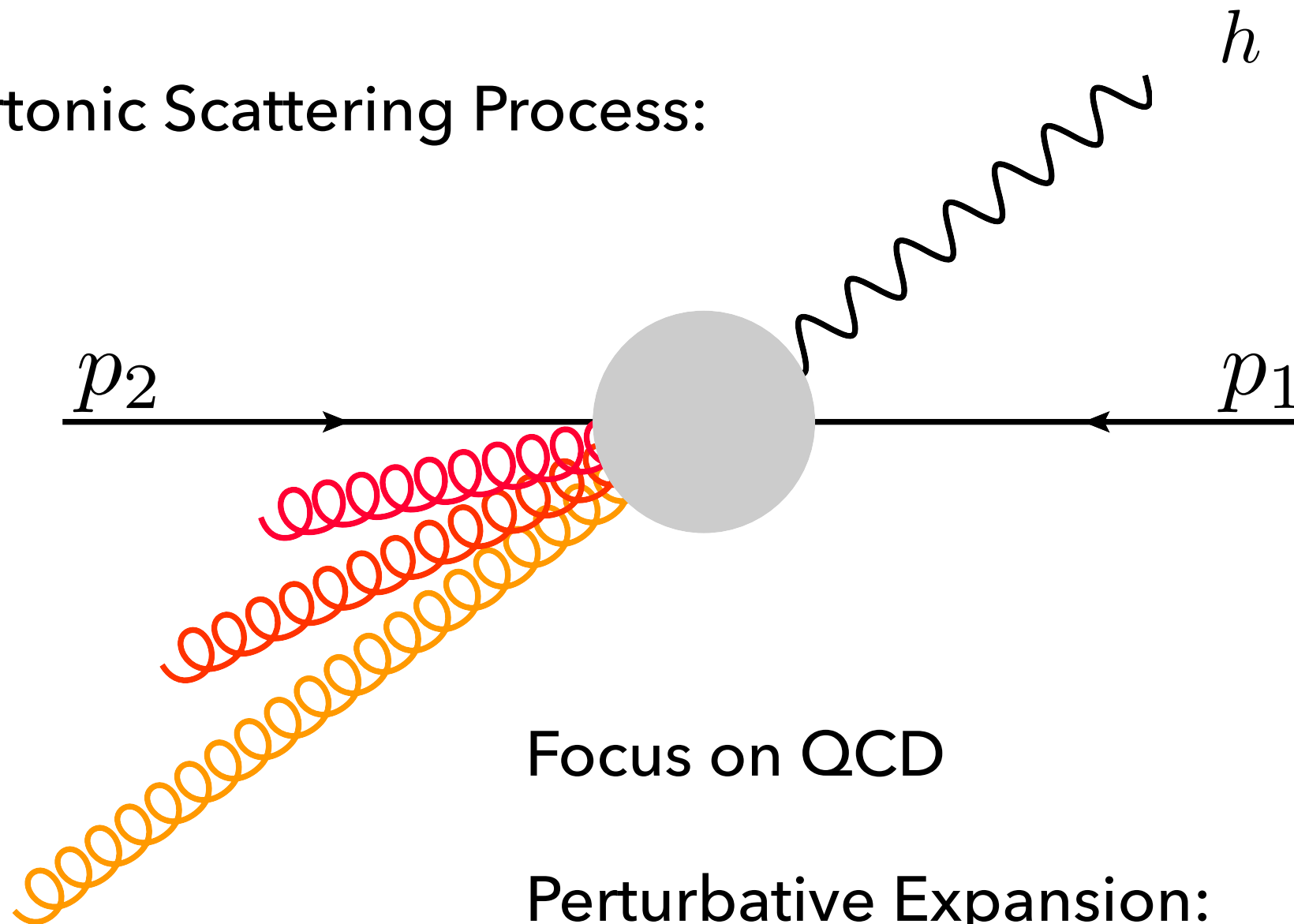


Formulate and test a useful approximation for inclusive color singlet observables.

$$Q^2 \frac{d\sigma_{P \rightarrow h+X}}{dY dQ^2}$$

- ▶ Improve fixed order precision predictions by obtaining ingredients and understanding for universal structures of QFT.
- ▶ Learn about structure of kinematic limits.
- ▶ Obtain universal quantities for resummation.
- ▶ Find a new way to approximate perturbative cross sections.
- ▶ Study the universal structure of QCD beyond the leading term in kinematic expansions (beyond leading power)
- ▶ ...

Partonic Scattering Process:

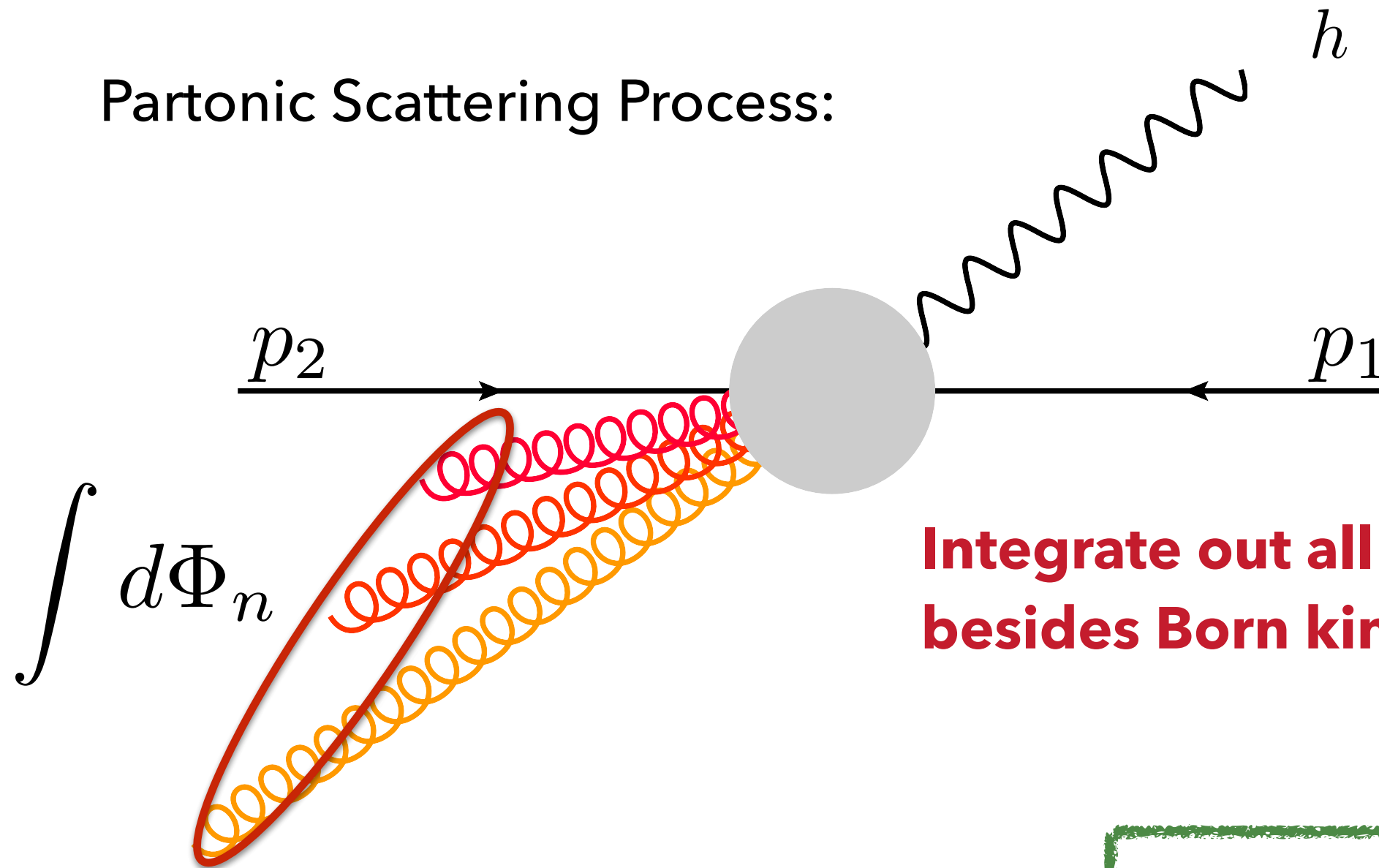


Focus on QCD

Perturbative Expansion:

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \dots$$

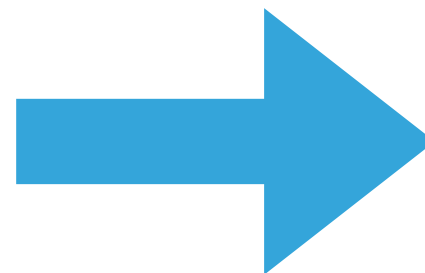
Partonic Scattering Process:



Integrate out all radiation besides Born kinematics

Born degrees of freedom for h :

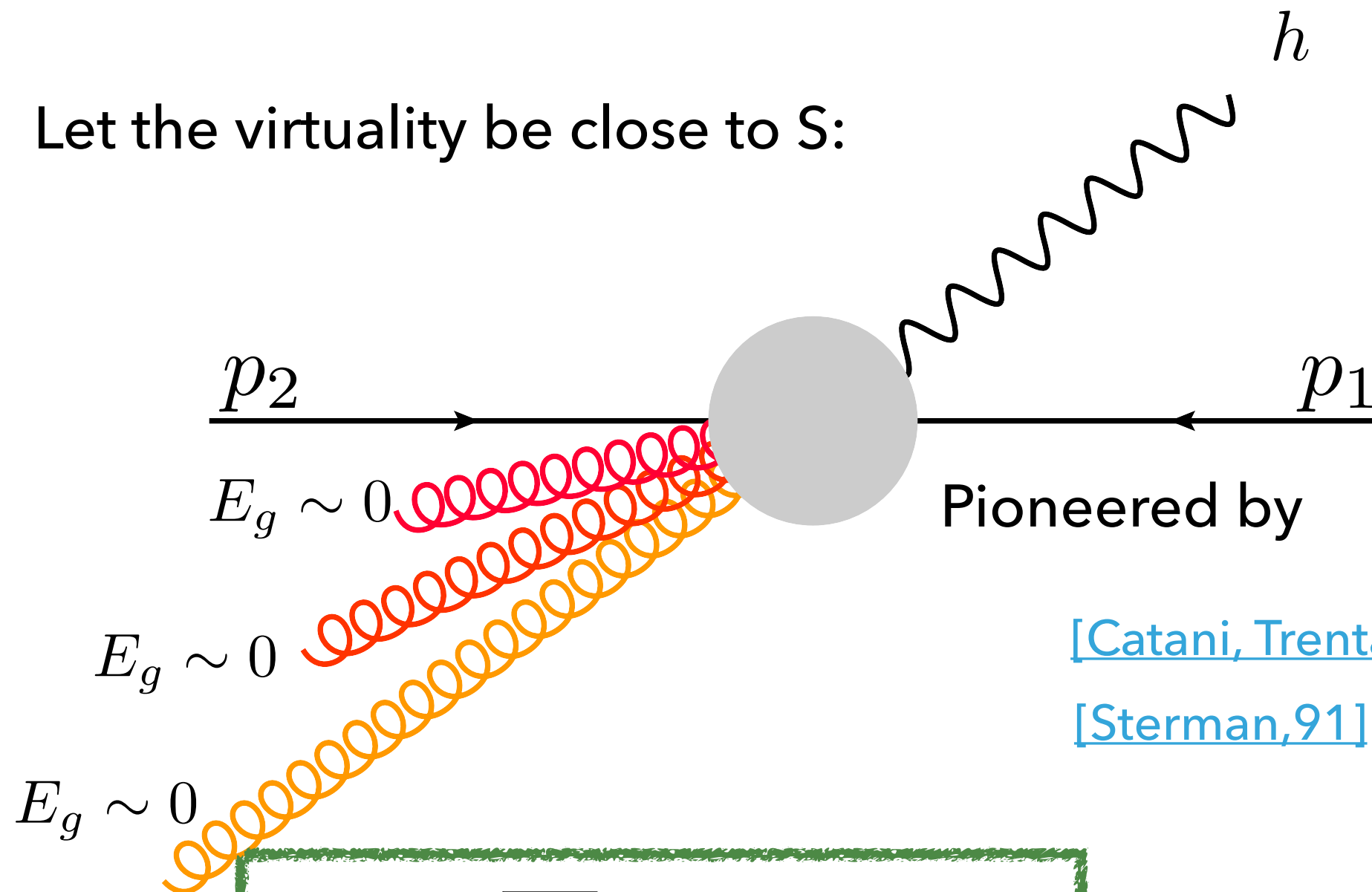
- ▶ Virtuality Q^2
- ▶ Rapidity Y
- ▶ Hadronic c.o.m energy S



$$\xi_1 = \sqrt{\frac{Q^2}{S}} e^{-Y}$$

$$\xi_2 = \sqrt{\frac{Q^2}{S}} e^Y$$

Let the virtuality be close to S :



Pioneered by

[\[Catani, Trentadue,89\]](#)

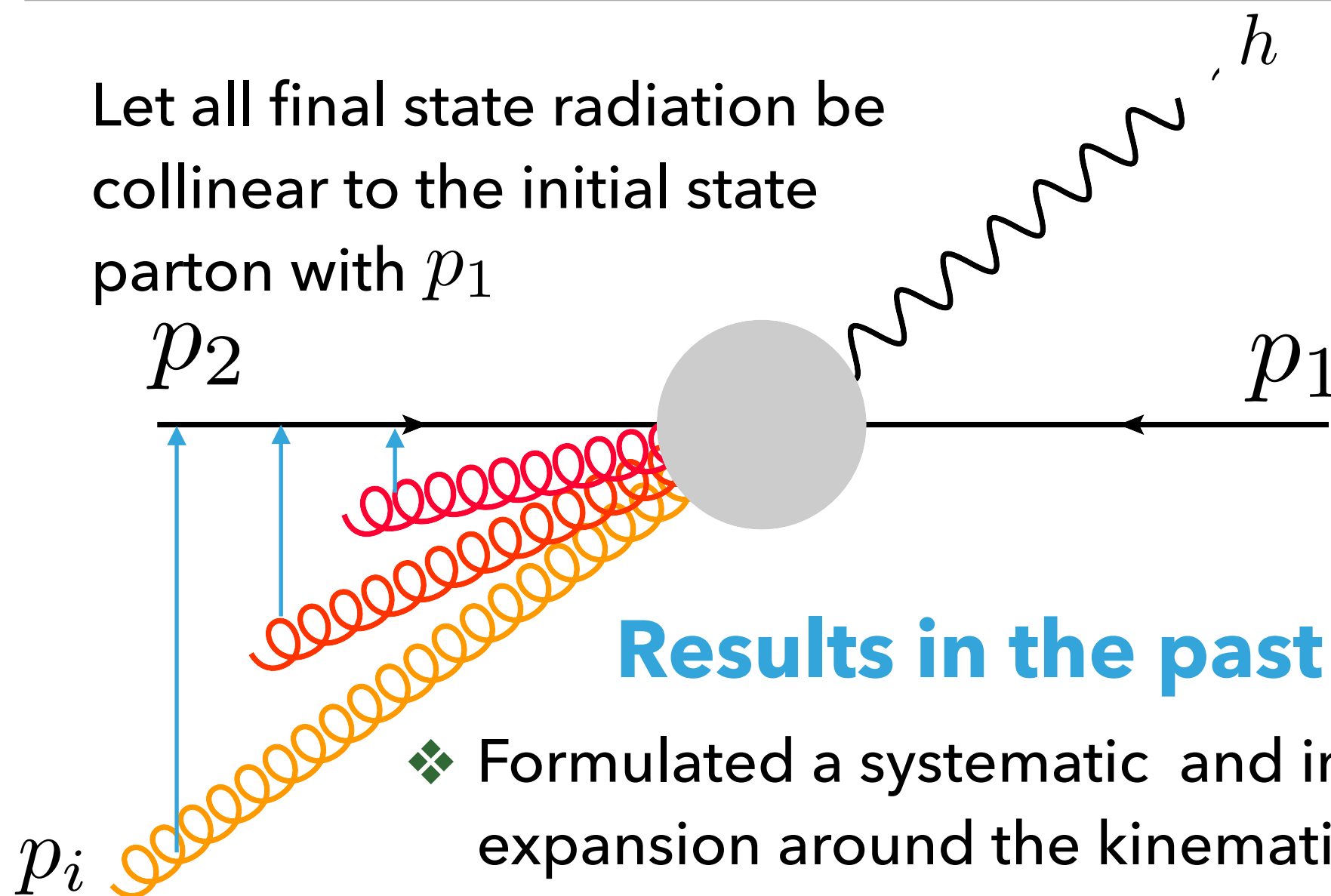
[\[Sternan,91\]](#)

$$\xi_1 = \sqrt{\frac{Q^2}{S}} e^{-Y} \rightarrow 1$$

$$\xi_2 = \sqrt{\frac{Q^2}{S}} e^Y \rightarrow 1$$

**Threshold
Factorization
and Resummation**

Let all final state radiation be collinear to the initial state parton with p_1

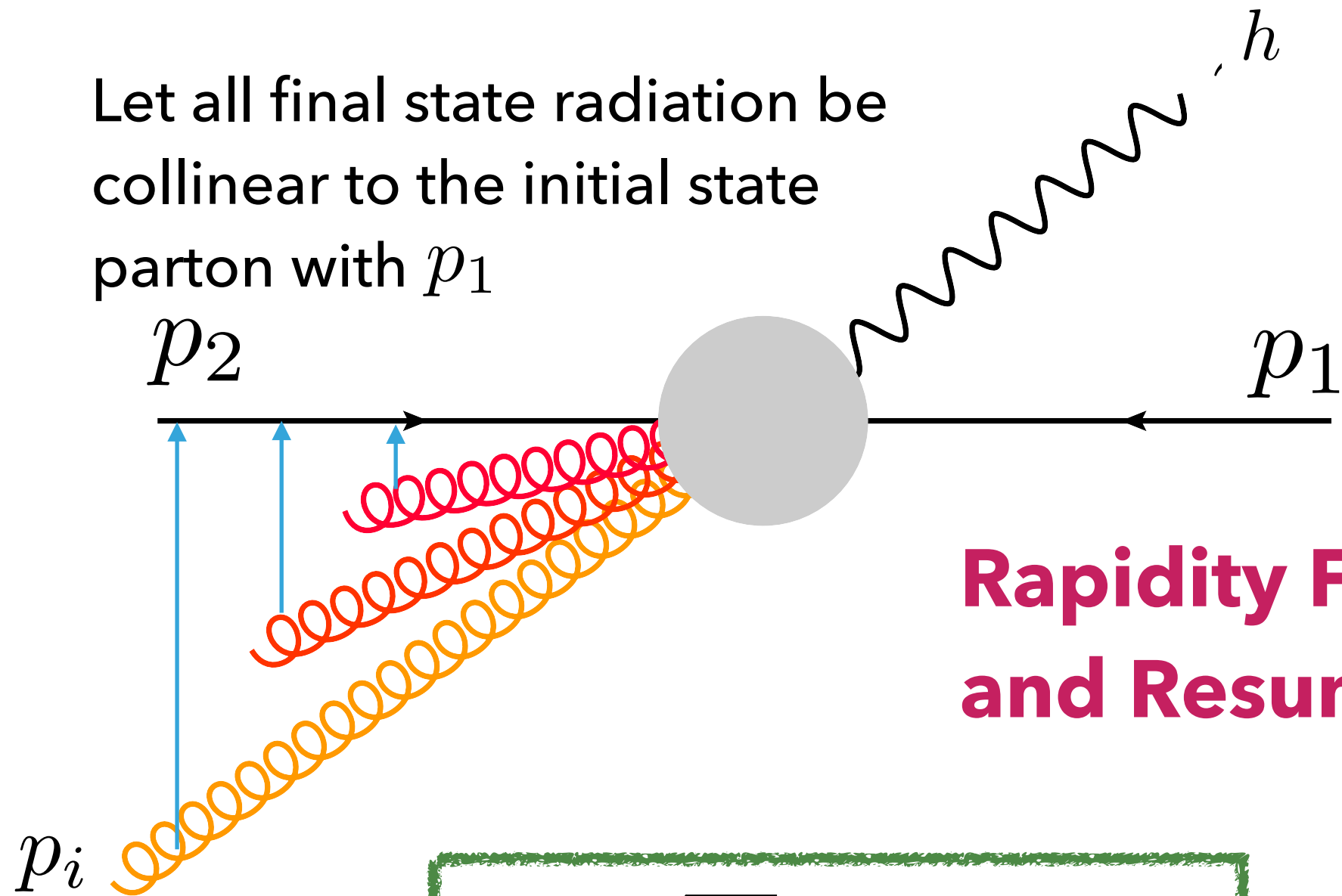


Results in the past years:

- ❖ Formulated a systematic and improvable expansion around the kinematic limit. [arXiv:2006.03055](https://arxiv.org/abs/2006.03055)
- ❖ Computed N3LO beam function for N-Jettiness and qT factorization. [arXiv:2006.03056](https://arxiv.org/abs/2006.03056) [arXiv:2006.05329](https://arxiv.org/abs/2006.05329)
- ❖ Related DY - DIS - e^+e^- and derived new predictions for EEC.

[arXiv:2012.07853](https://arxiv.org/abs/2012.07853) [arXiv:2012.07859](https://arxiv.org/abs/2012.07859)

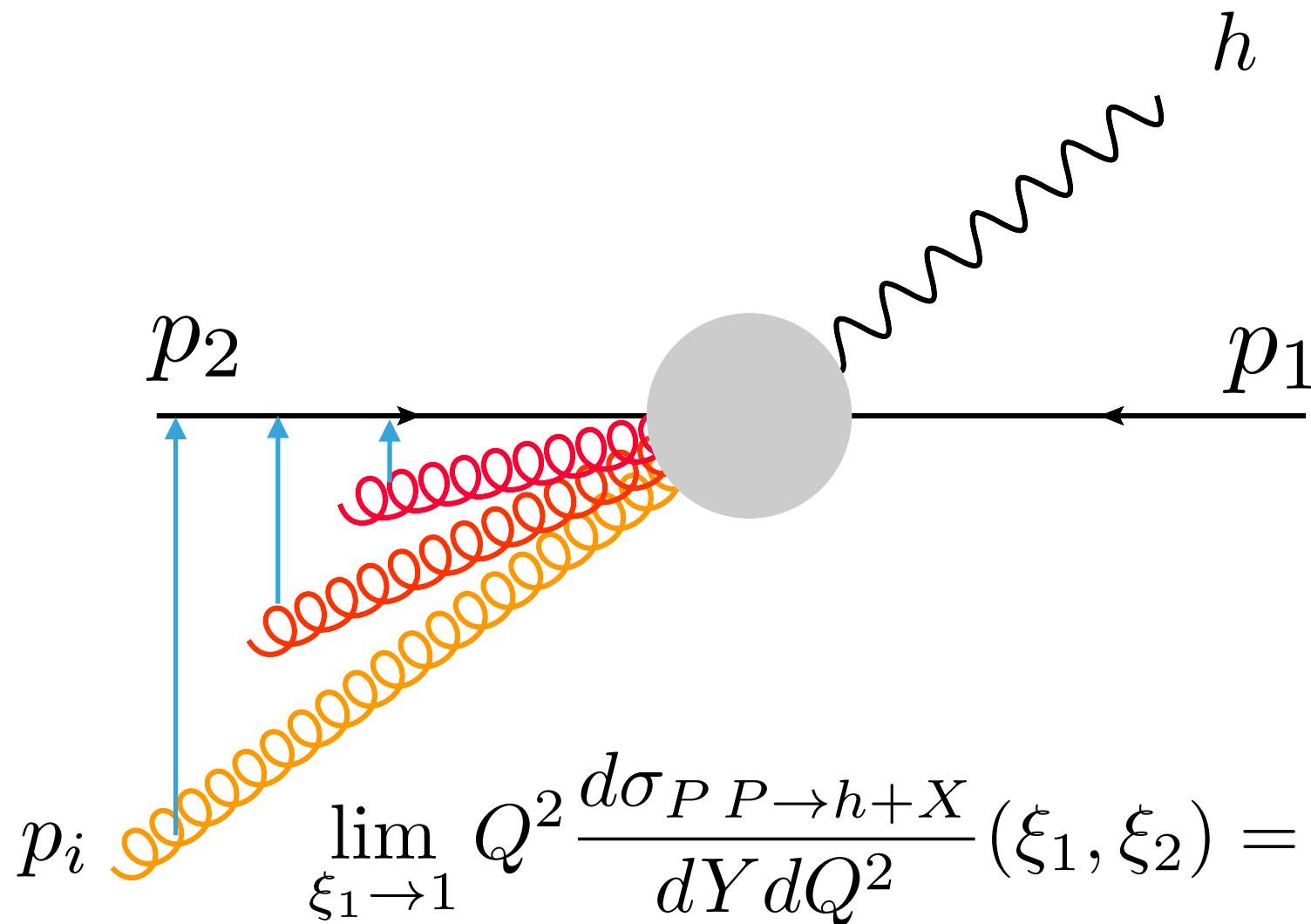
Let all final state radiation be collinear to the initial state parton with p_1



Rapidity Factorization and Resummation

$$\xi_1 = \sqrt{\frac{Q^2}{S}} e^{-Y} \rightarrow 1$$

$$\xi_2 = \sqrt{\frac{Q^2}{S}} e^Y$$



$$\xi_1 = \sqrt{\frac{Q^2}{S}} e^{-Y} \rightarrow 1$$

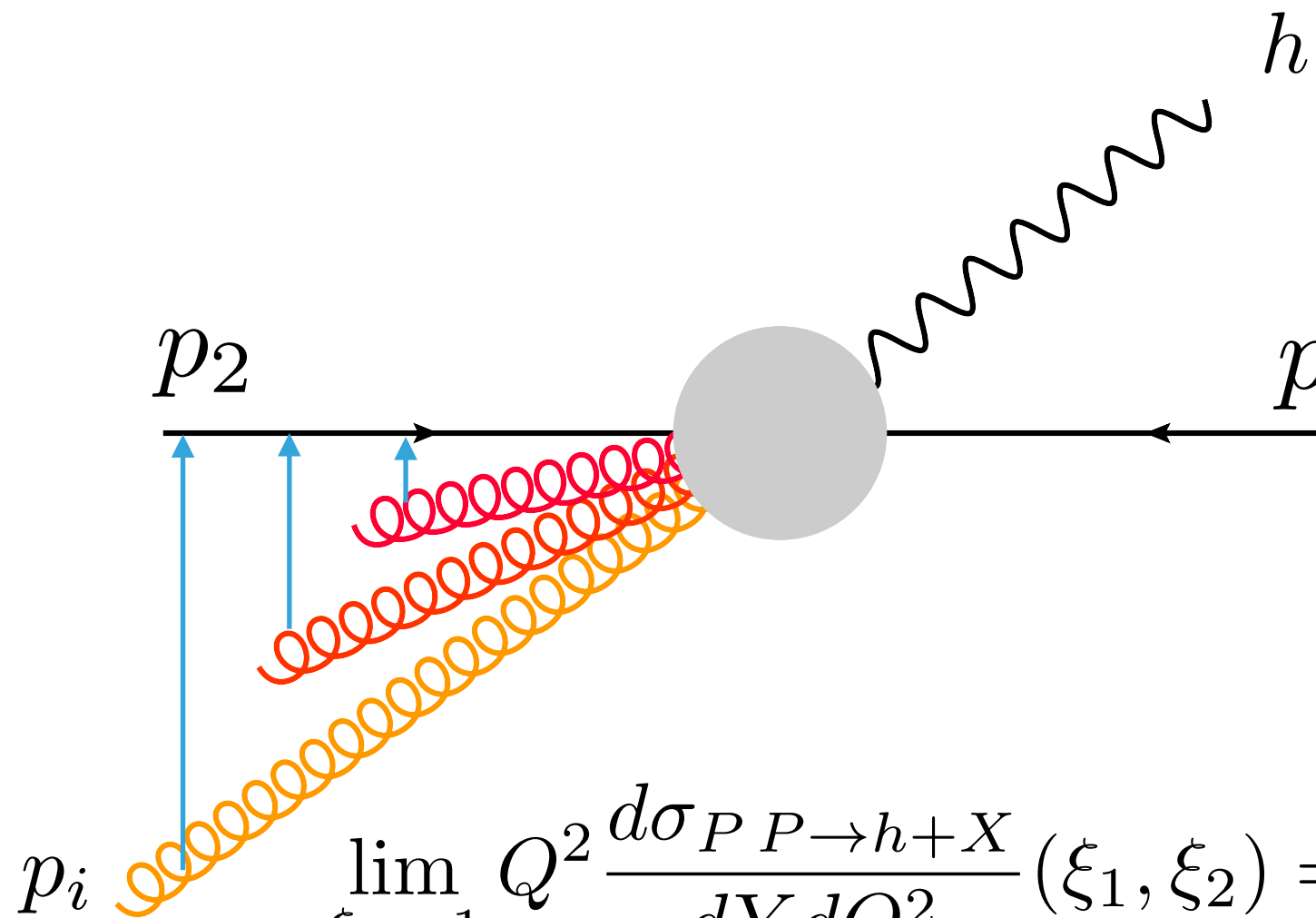
$$\xi_2 = \sqrt{\frac{Q^2}{S}} e^Y$$

$$\bar{\xi} = 1 - \xi$$

Hard Function: Describes the hard born process (Higgs production vs 12W production)

PDF: Parton Distributions

Rapidity Beam Function: Describes collinear dynamics of the process.



$$\xi_1 = \sqrt{\frac{Q^2}{S}} e^{-Y} \rightarrow 1$$

$$\xi_2 = \sqrt{\frac{Q^2}{S}} e^Y$$

$$\bar{\xi} = 1 - \xi$$

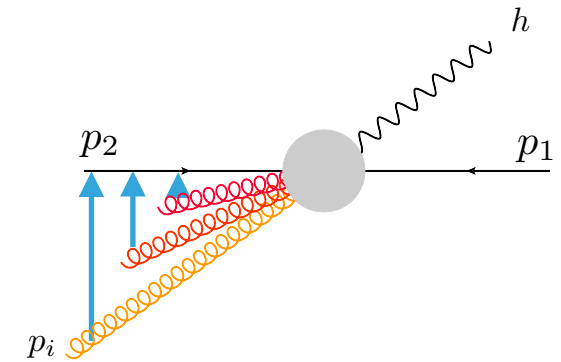
$$p_i \lim_{\xi_1 \rightarrow 1} Q^2 \frac{d\sigma_{P \rightarrow h+X}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} f_i(\xi_1) \otimes_{\xi_1} B_j^Y(\xi_2; \bar{\xi}_1)$$

- ❖ Collinear limit pioneered by CSS, SCET, Catani-Grazzini ...
- ❖ Factorization theorem ("generalized threshold") realized and studied by [\[Lustermans, Michel, Tackmann, 19\]](#)!
- ❖ Explored factorization in [arXiv:2006.03055](#) [arXiv:1810.09462](#)

Let's build an approximation:

Collinear 1

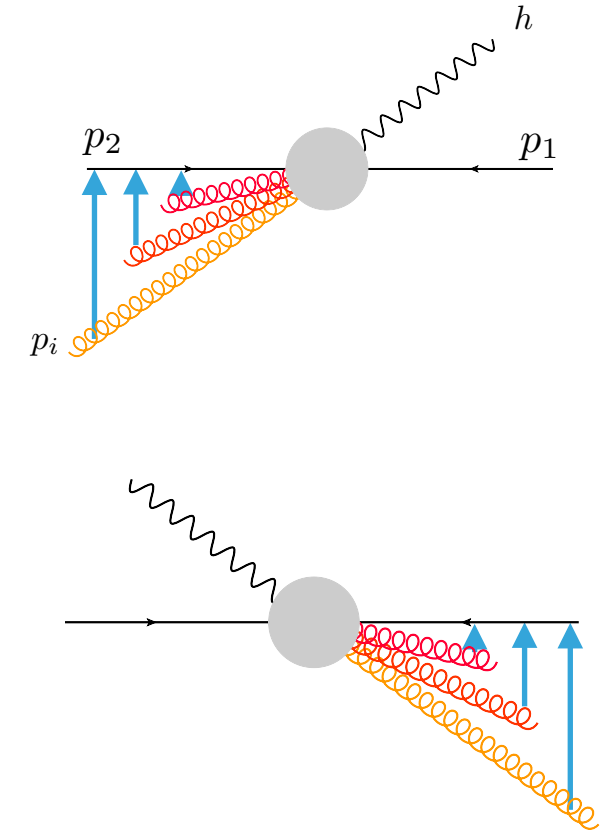
$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} = \lim_{\xi_1 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}$$



Let's build an approximation:

$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} = \lim_{\xi_1 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} \quad \text{Collinear 1}$$

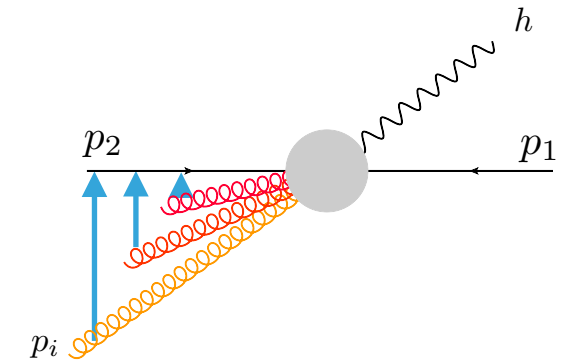
$$+ \lim_{\xi_2 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} \quad \text{Collinear 2}$$



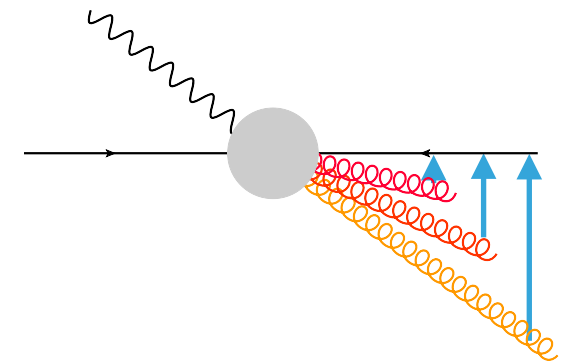
Let's build an approximation:

$$\begin{aligned}
 Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} &= \lim_{\xi_1 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} \\
 &+ \lim_{\xi_2 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} \\
 &- \lim_{\xi_1, \xi_2 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}
 \end{aligned}$$

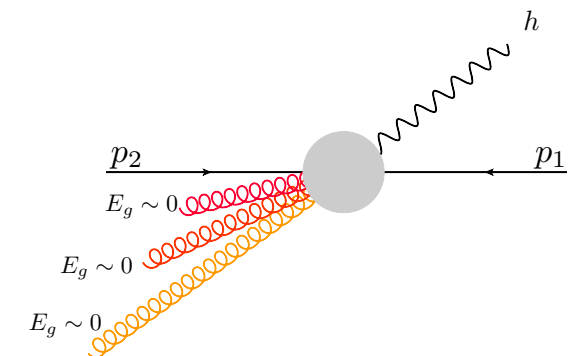
Collinear 1



Collinear 2



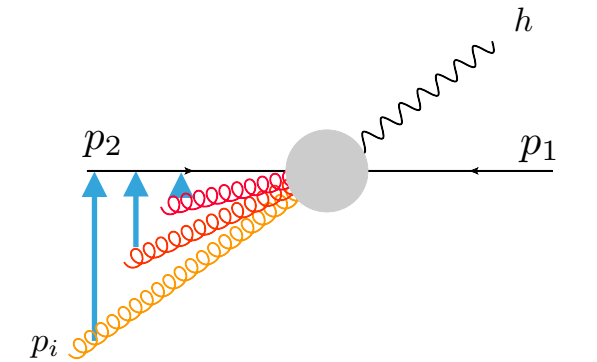
Soft



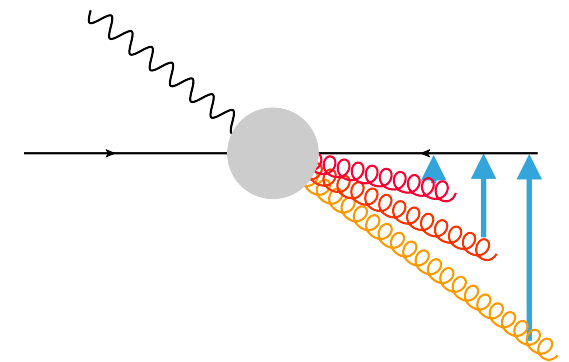
Let's build an approximation:

$$\begin{aligned}
 Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} &= \lim_{\xi_1 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} \\
 &+ \lim_{\xi_2 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2} \\
 &- \lim_{\xi_1, \xi_2 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}
 \end{aligned}$$

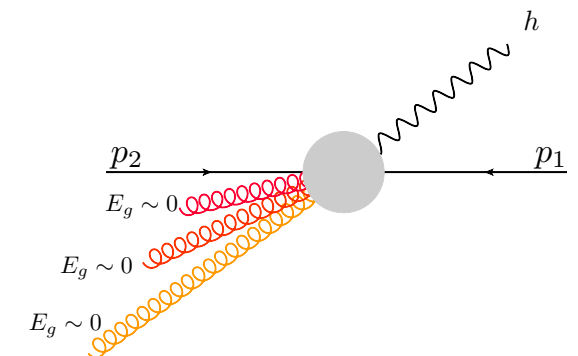
Collinear 1



Collinear 2



Soft



A little bit of magic ...

$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} \boxed{H_{ij}} \left[\boxed{B_i^Y(\xi_1; \bar{\xi}_2)} \otimes_{\xi_1; \xi_2} \boxed{S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2)} \otimes_{\xi_2; \xi_1} \boxed{B_j^Y(\xi_2; \bar{\xi}_1)} \right]$$

Hard Function: Describes the hard Born process (Higgs production vs 12 W production).

Rapidity Beam Function: Collinear dynamics for radiation collinear to Proton 1.

Rapidity Beam Function: Collinear dynamics for radiation collinear to Proton 2.

Soft Function: Describes soft radiation.

$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} [B_i^Y(\xi_1; \bar{\xi}_2) \otimes_{\xi_1; \xi_2} S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2) \otimes_{\xi_2; \xi_1} B_j^Y(\xi_2; \bar{\xi}_1)]$$

Fact 1:

We can make these objects very concrete:

$$B_i^Y(\xi_2; \bar{\xi}_1) = \sum_j I_{ij}^Y(\xi_2; \bar{\xi}_1) \otimes_{\xi_2} f_j(\xi_2).$$

BF related to usual PDF via perturbative matching kernel, which is calculable from the strict collinear limit of the partonic coefficient function.

$$I_{ij}^Y(\xi_2; \bar{\xi}_1) = \int_0^1 dx \int_0^\infty dw_1 dw_2 \delta[\xi_2 - (1 - w_2)] \\ \times \lim_{\text{strict } n\text{-coll.}} \left\{ \delta \left[\bar{\xi}_1 - \frac{2 - w_2 - w_2 x}{2(1 - w_2)} w_1 \right] \frac{d\eta_{ji}}{dQ^2 dw_1 dw_2 dx} \right\}$$

$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} [B_i^Y(\xi_1; \bar{\xi}_2) \otimes_{\xi_1; \xi_2} S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2) \otimes_{\xi_2; \xi_1} B_j^Y(\xi_2; \bar{\xi}_1)]$$

Fact 2:

There are some non-trivial things

Mixed, two variable convolutions. All Laplace convolutions are performed first.

$$f(x_1, x_2) \otimes_{x_1; x_2} g(x_1, x_2) = \int_{x_1}^1 \frac{dy_1}{y_1} \int_{x_2}^1 dy_2 f(y_1, y_2) g\left(\frac{x_1}{y_1}, 1 + x_2 - y_2\right)$$

$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} \boxed{H_{ij}} \left[\boxed{B_i^Y(\xi_1; \bar{\xi}_2)} \otimes_{\xi_1; \xi_2} \boxed{S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2)} \otimes_{\xi_2; \xi_1} \boxed{B_j^Y(\xi_2; \bar{\xi}_1)} \right]$$

Fact 3:

In the double limit we reproduce exactly threshold factorization.

$$\lim_{\xi_1, \xi_2 \rightarrow 1} Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \text{Threshold XS}$$



Rapidity Factorization

- ❖ Why should this give anything good?
- ❖ How well does it work?
- ❖ Can you compute resummed cross sections?

Rapidity Factorization

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For LHC cross sections the kinematic limit sounds like nonsense.

For central rapidities ($Y=0$) and typical hard interactions ($Q=100$ GeV) we find

$$\xi \sim 0.01 \quad \text{Very far away ...}$$

We can go to large invariant mass:

$$\xi = 0.9 \quad \longrightarrow$$

$$Q = 12.24 \text{ TeV}$$

Very large ...

We can go forward:

$$\xi = 0.9 \quad \longrightarrow$$

$$Y = -4.8$$

Very forward ...

$$\xi_1 = \sqrt{\frac{Q^2}{S}} e^{-Y}$$
$$\xi_2 = \sqrt{\frac{Q^2}{S}} e^Y$$

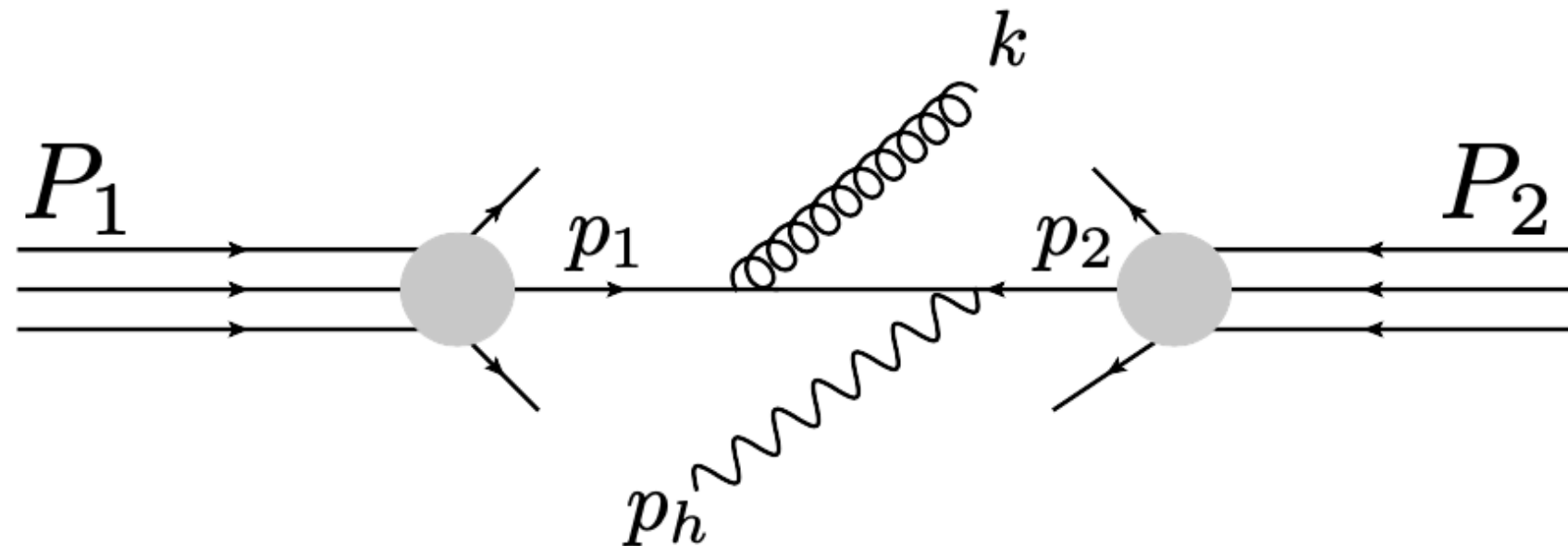
Our approximation performs at the partonic level!

$$\xi_1 = \sqrt{\frac{Q^2}{S}} e^{-Y}$$

$$\xi_2 = \sqrt{\frac{Q^2}{S}} e^Y$$

$$Q^2 \frac{d\sigma_{PP \rightarrow h+X}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} \int_{\xi_1}^1 \frac{dx_1}{x_1} f_i \left(\frac{\xi_1}{x_1} \right) B_j^Y(\xi_2, \mu^2; \bar{x}_1) + \mathcal{O}(\bar{\xi}_1^0).$$

First term in the expansion around $x_1 \rightarrow 1$



$$\xi \sim 0.01$$

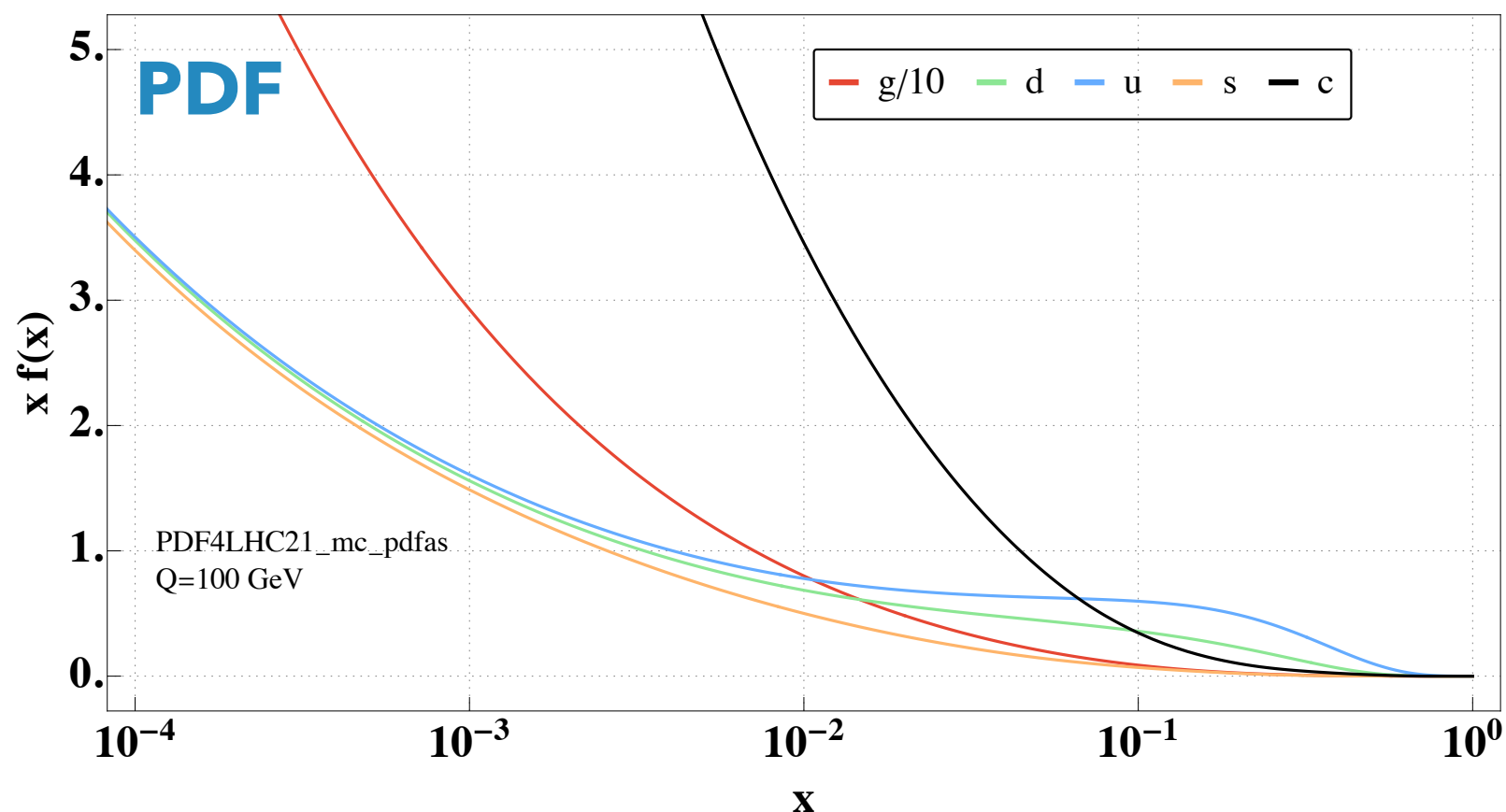
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First term in the expansion around $x_1 \rightarrow 1$



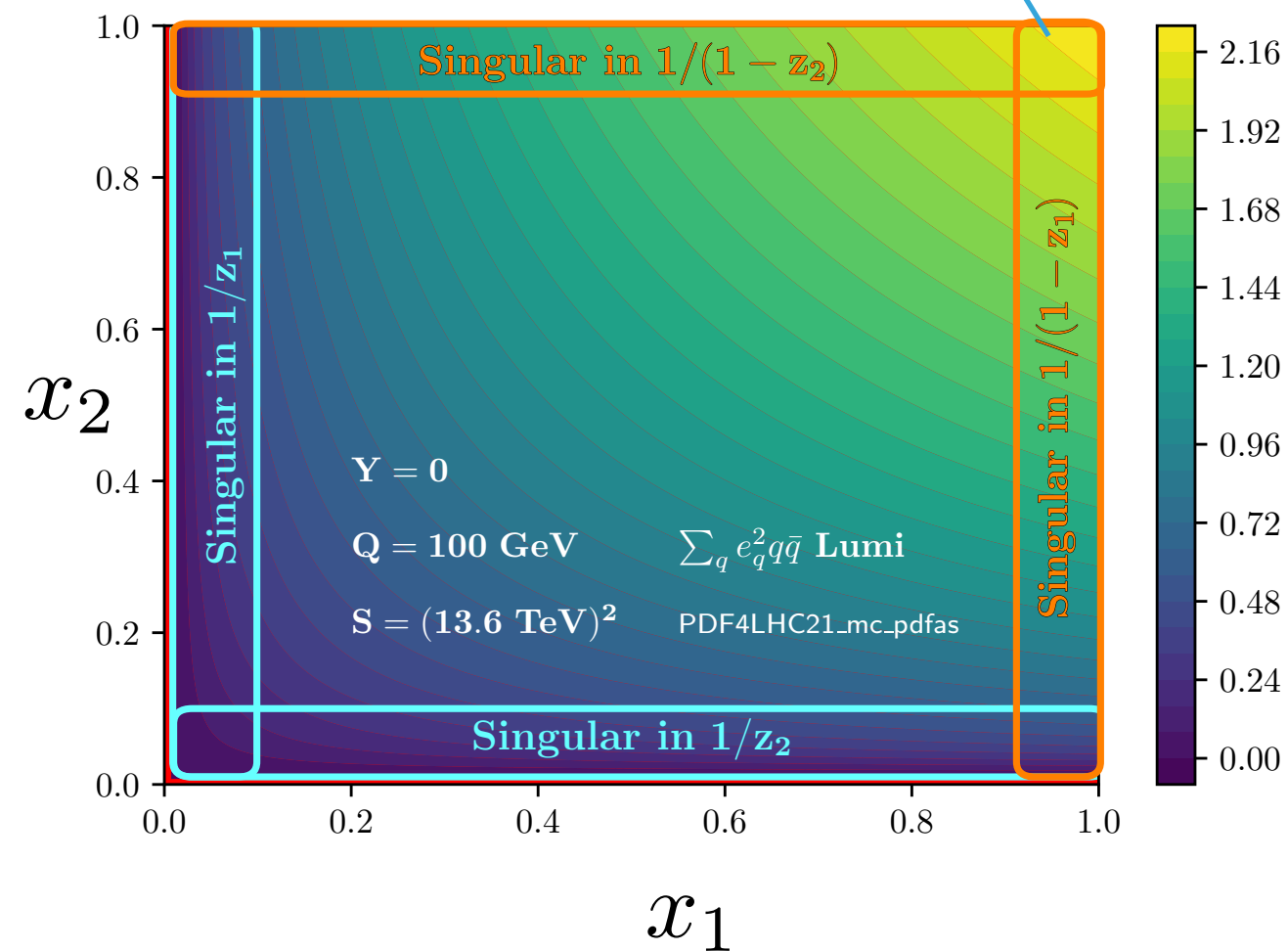
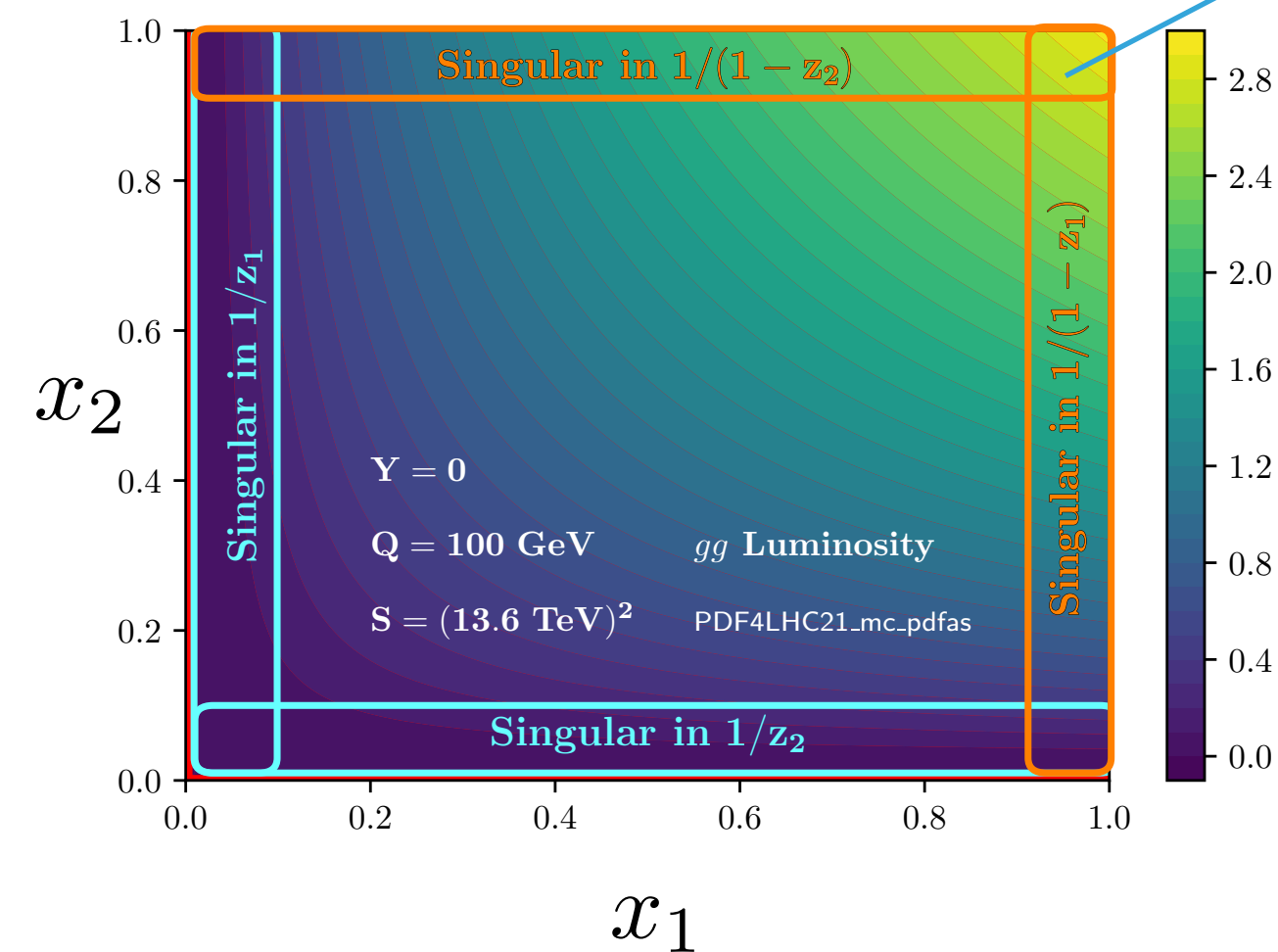
Our approximation performs at the partonic level!

Luminosities:

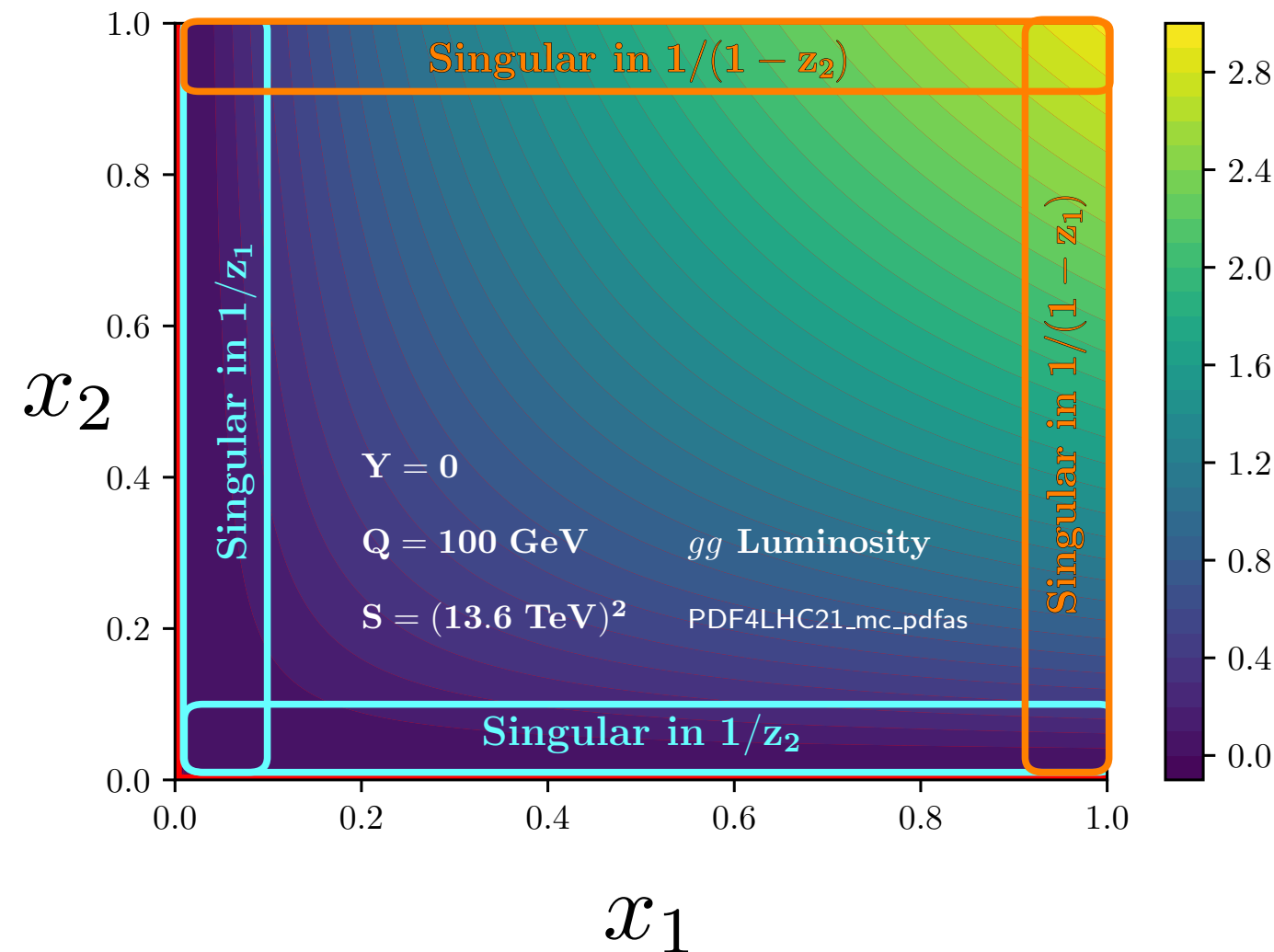
Production Threshold

$g g$ (Higgs)

$q \bar{q}$ (DY)



Our approximation performs at the partonic level!



The partonic coefficient function is enhanced by:

$$\frac{\log^m(1-x)}{1-x}$$

Our approx.
contains all this!

$$\frac{\log^n(x)}{x}$$

Kinematically
suppressed

Rapidity Factorization has a chance to describe LHC pheno due to enhancement + PDF suppression.

Rapidity Factorization

- ❖ Why should this give anything good?
- ❖ How well does it work?
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$$Q^2 \frac{d\sigma_{P P \rightarrow H+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} [B_i^Y(\xi_1; \bar{\xi}_2) \otimes_{\xi_1; \xi_2} S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2) \otimes_{\xi_2; \xi_1} B_j^Y(\xi_2; \bar{\xi}_1)]$$

$$= \tau \sum_{i,j} f_i(\xi_1) \otimes_{\xi_1} \eta_{ij}^{Y_{\text{approx.}}}(\xi_1, \xi_2) \otimes_{\xi_2} f_j(\xi_2)$$

✓ Correct rapidity limit!

A regular partonic cross section!

✓ Correct threshold limit!

✓ Correct Next-to-Leading Power (NLP)
in threshold counting at fixed order!

$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} [B_i^Y(\xi_1; \bar{\xi}_2) \otimes_{\xi_1; \xi_2} S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2) \otimes_{\xi_2; \xi_1} B_j^Y(\xi_2; \bar{\xi}_1)]$$

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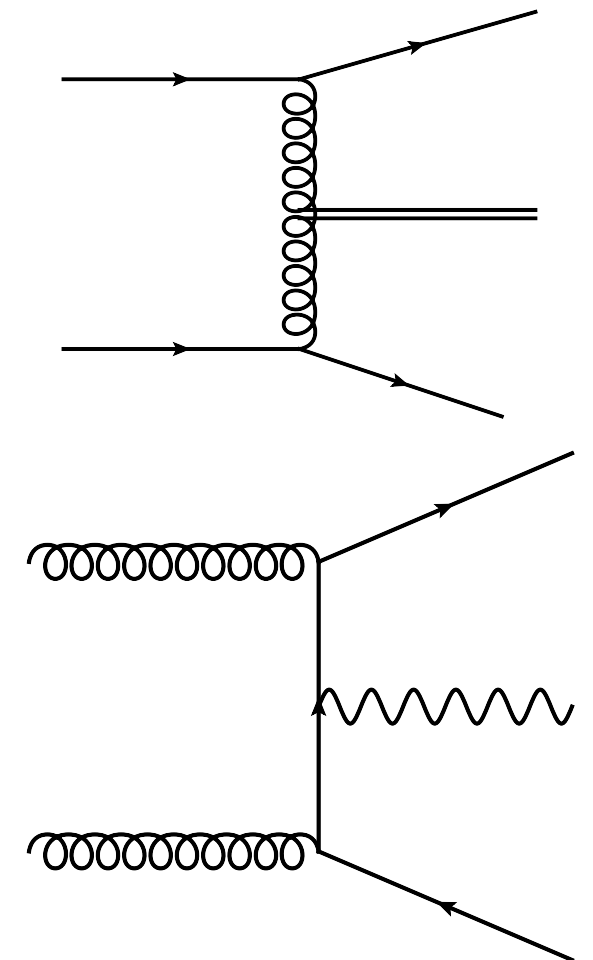
✓ Correct rapidity limit!

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✓ Correct threshold limit!

✓ Correct Next-to-Leading Power (NLP) in threshold counting at fixed order!

✓ Correct $q\bar{q} \rightarrow H + X$ and $gg \rightarrow \gamma^* + X$ at NNLP at NNLO in threshold counting.



$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} [B_i^Y(\xi_1; \bar{\xi}_2) \otimes_{\xi_1; \xi_2} S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2) \otimes_{\xi_2; \xi_1} B_j^Y(\xi_2; \bar{\xi}_1)]$$

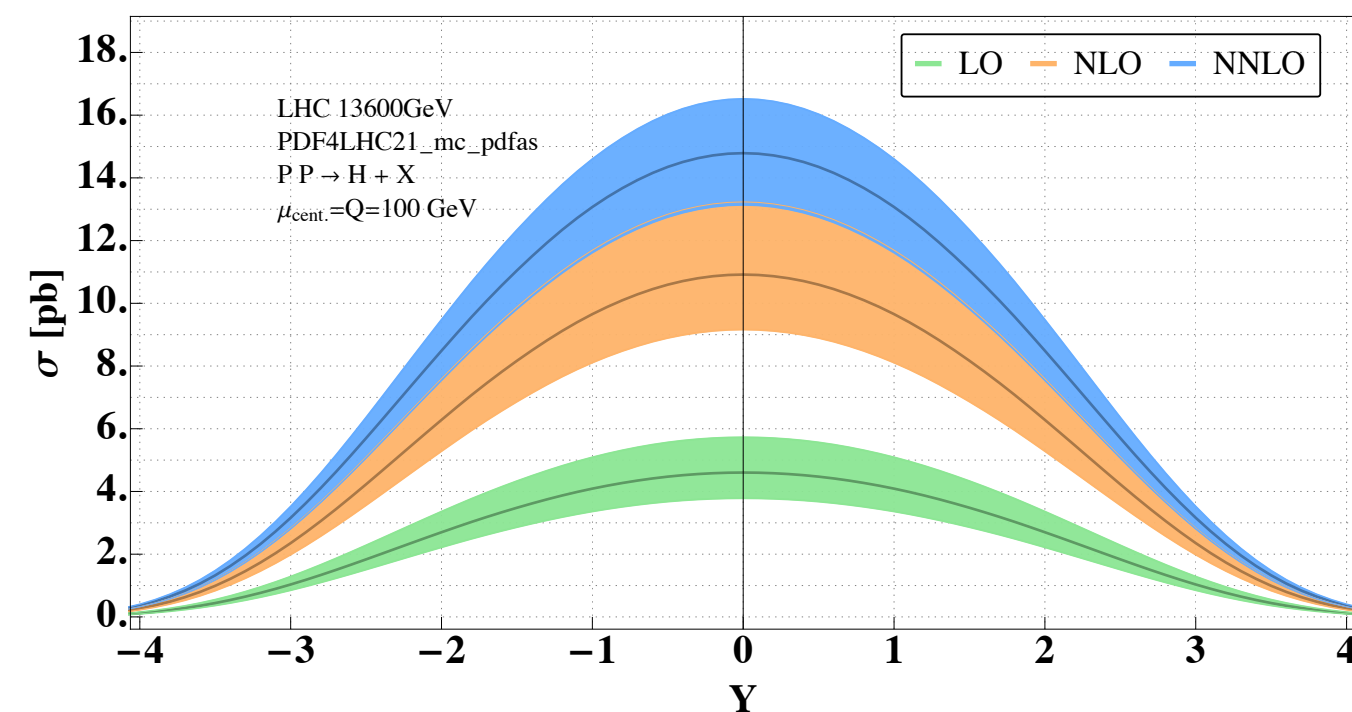
$$= \tau \sum_{i,j} f_i(\xi_1) \otimes_{\xi_1} \eta_{ij}^{Y_{\text{approx.}}}(\xi_1, \xi_2) \otimes_{\xi_2} f_j(\xi_2)$$

- ✓ Correct rapidity limit! A regular partonic cross section!
- ✓ Correct threshold limit!
- ✓ Correct Next-to-Leading Power (NLP) in threshold counting at fixed order!
- ✓ Correct $q\bar{q} \rightarrow H + X$ and $gg \rightarrow \gamma^* + X$ at NNLP at NNLO in threshold counting.
- ✓ At fixed order, we know how to compute systematically higher power terms for our approximation!

Test based on example production cross sections for the LHC:

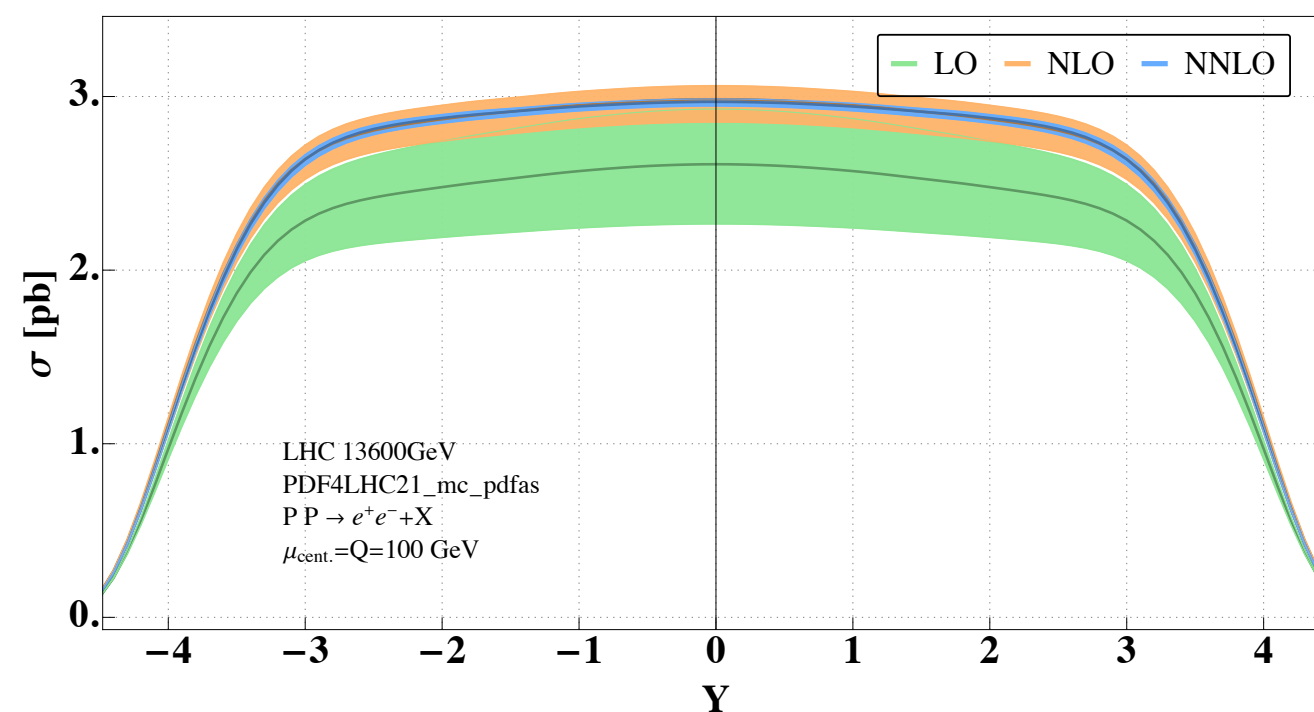
Gluon Fusion Higgs Production

$$P P \rightarrow H + X$$



Drell-Yan Production

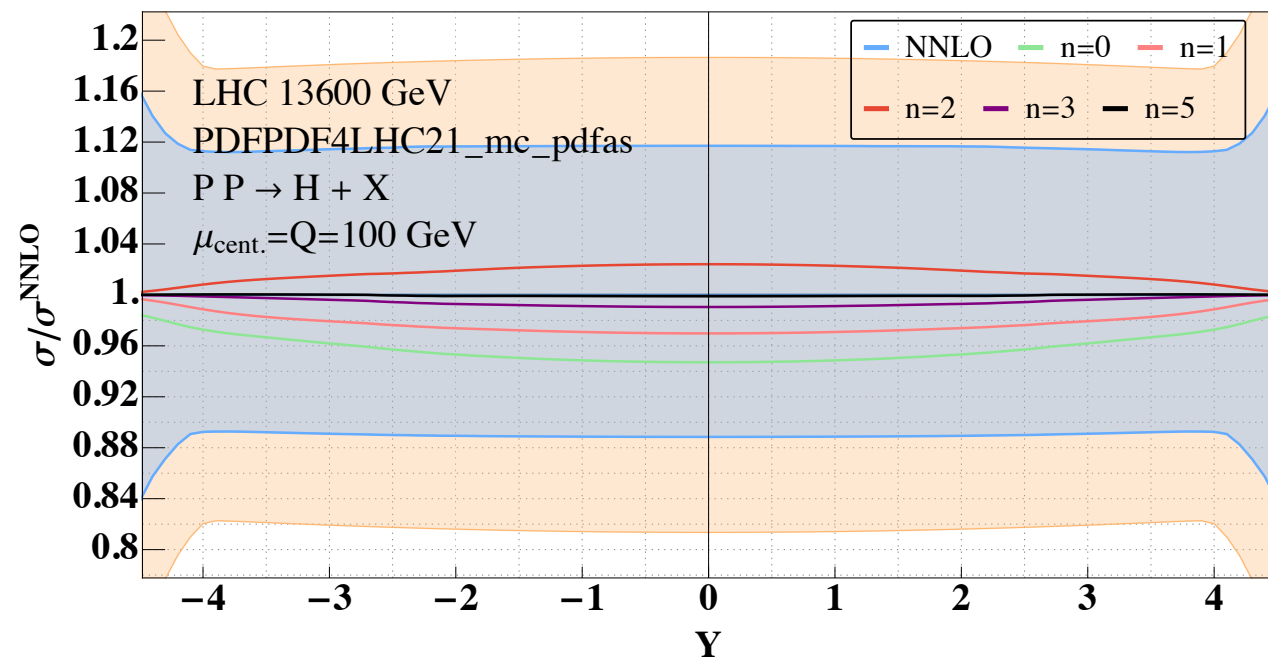
$$P P \rightarrow \gamma^* + X \rightarrow e^+ e^- + X$$



Use collinear expansion to compute NnLP terms to the rapidity approximation:

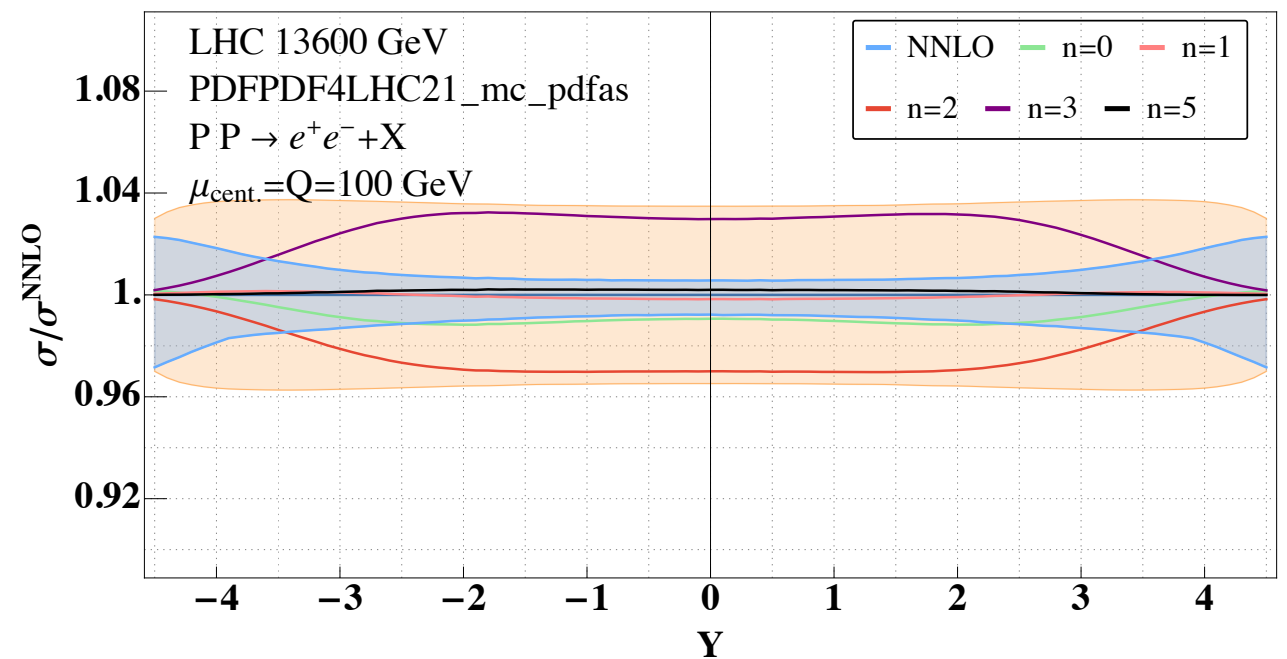
Gluon Fusion Higgs Production

$$P P \rightarrow H + X$$



Drell-Yan Production

$$P P \rightarrow \gamma^* + X \rightarrow e^+ e^- + X$$

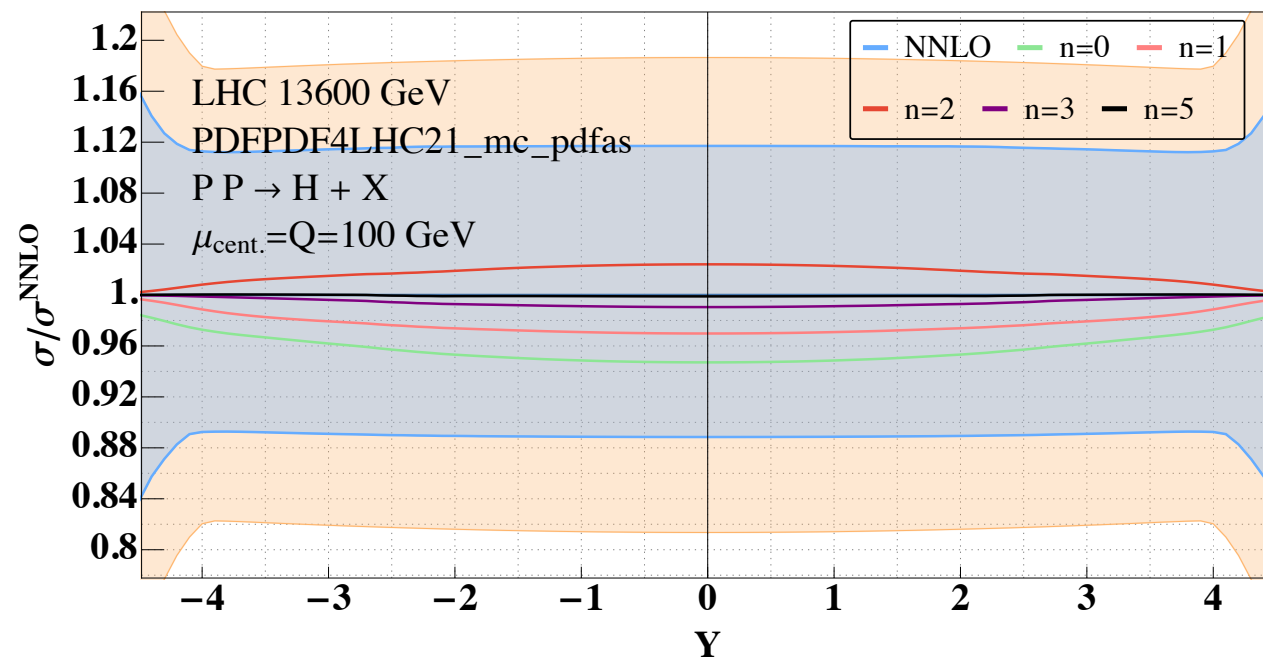


✓ Great for large Y !

✓ Within ~5% of NNLO - good for gluons!

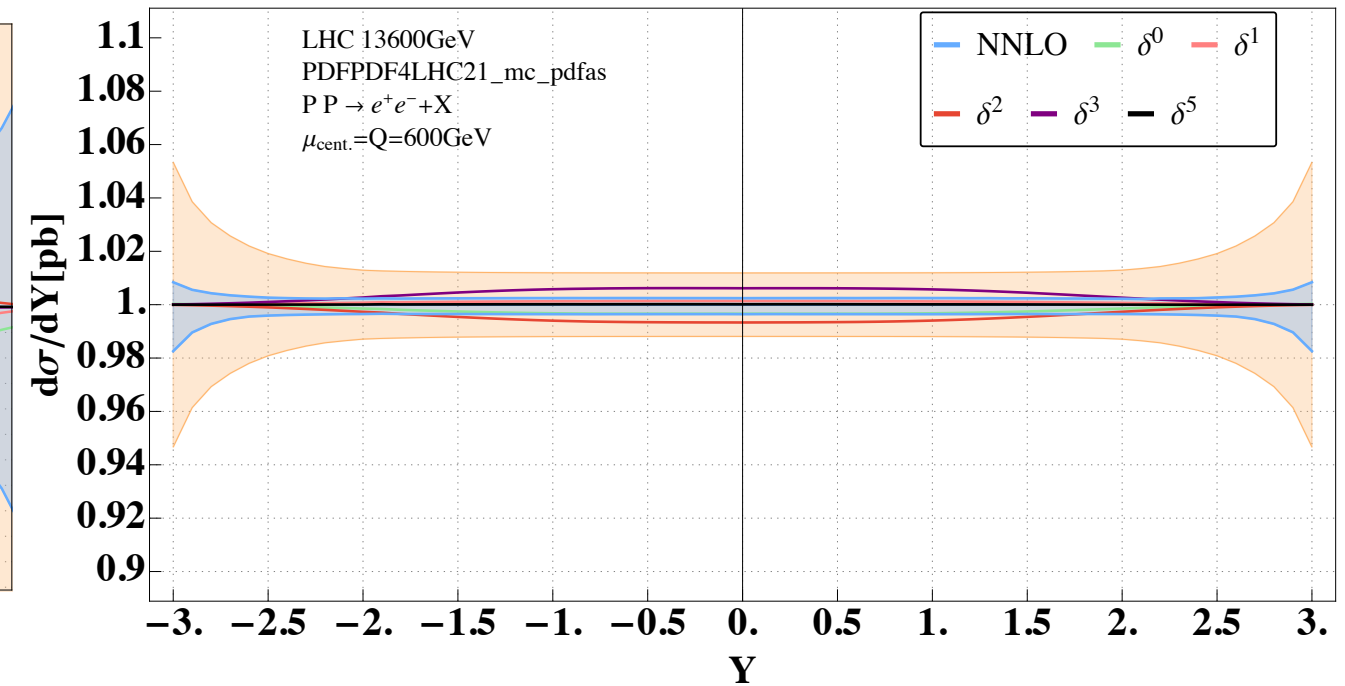
$$P P \rightarrow H + X$$

Q=600 GeV

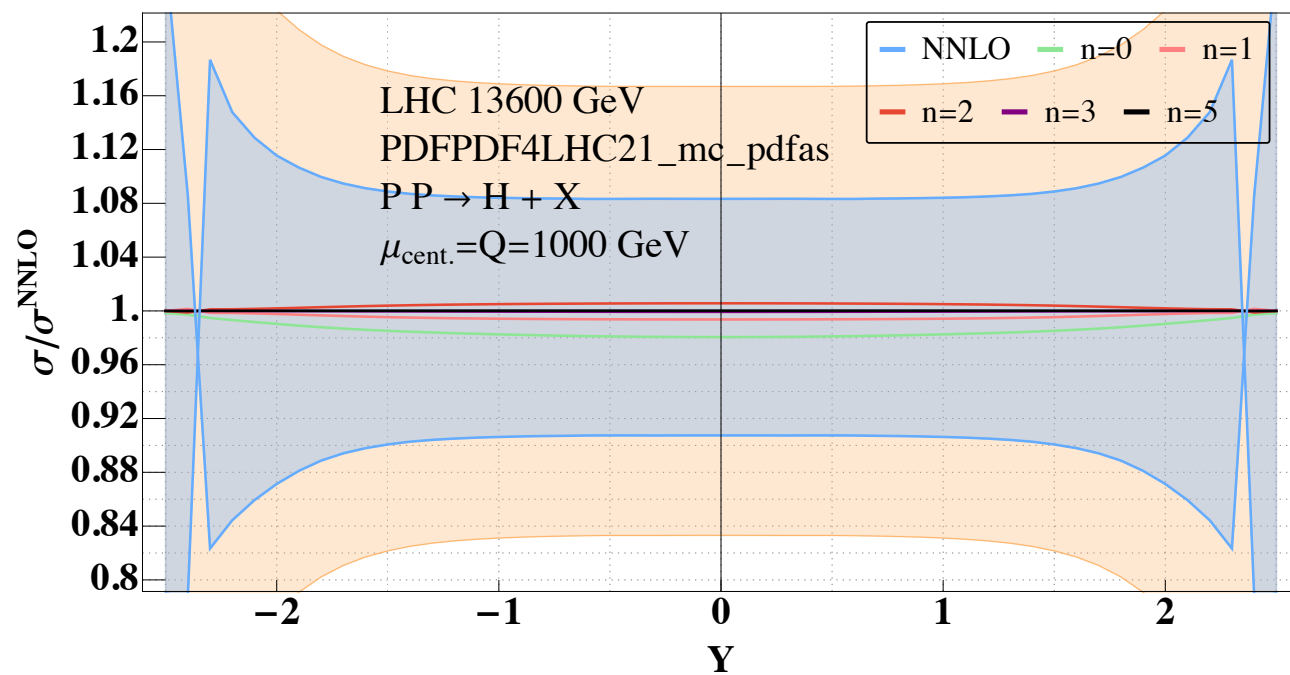


$$P P \rightarrow \gamma^* + X \rightarrow e^+ e^- + X$$

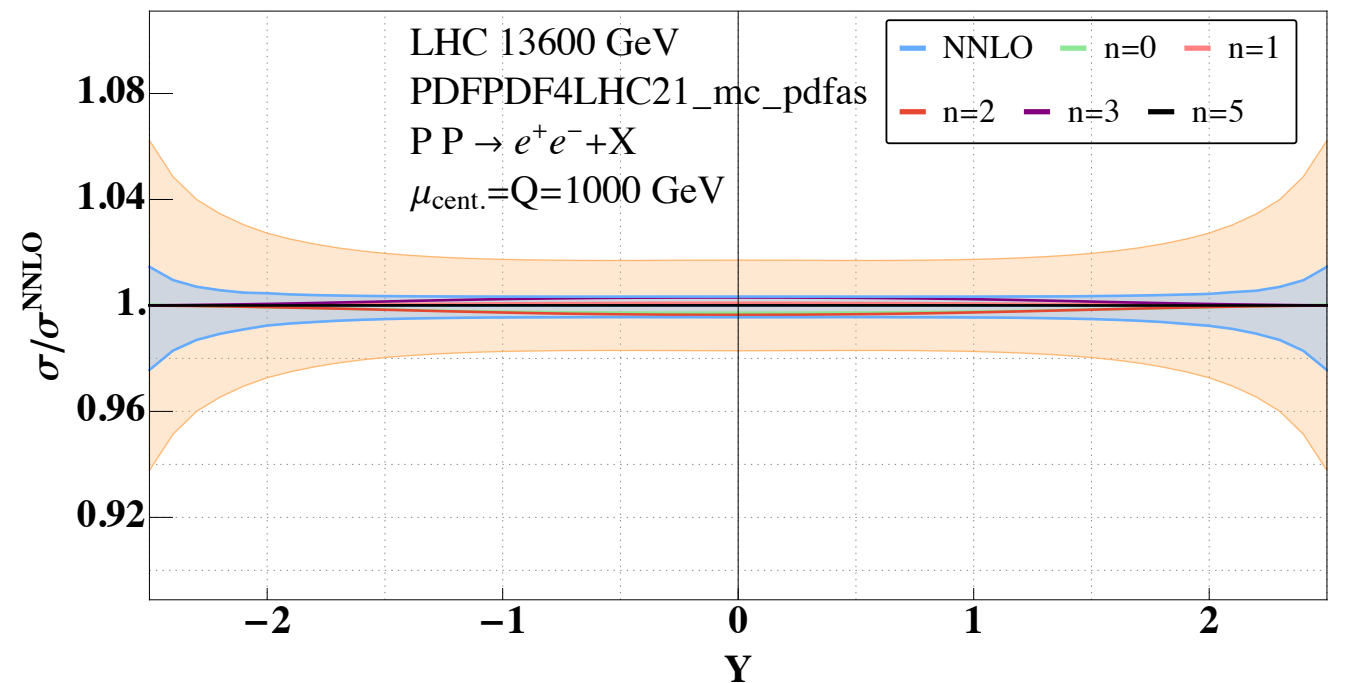
Q=600 GeV



Q=1000 GeV

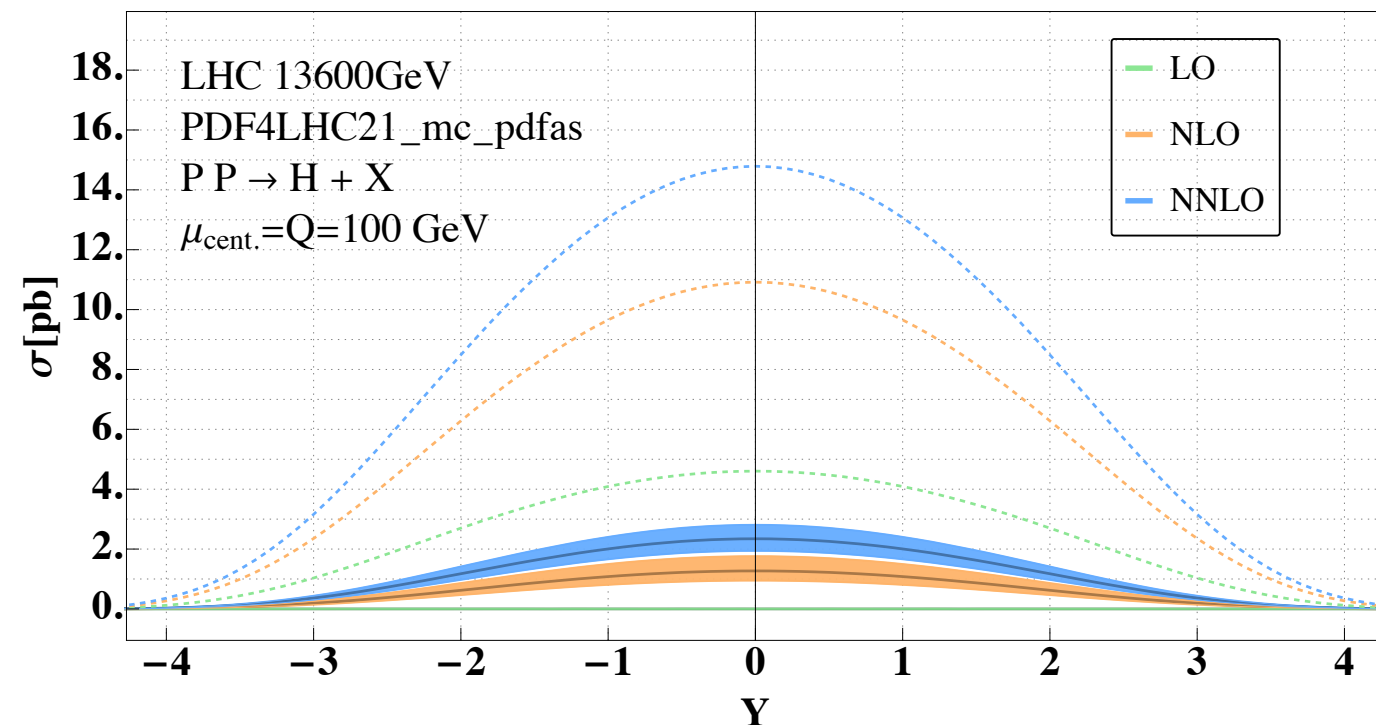


Q=1000 GeV

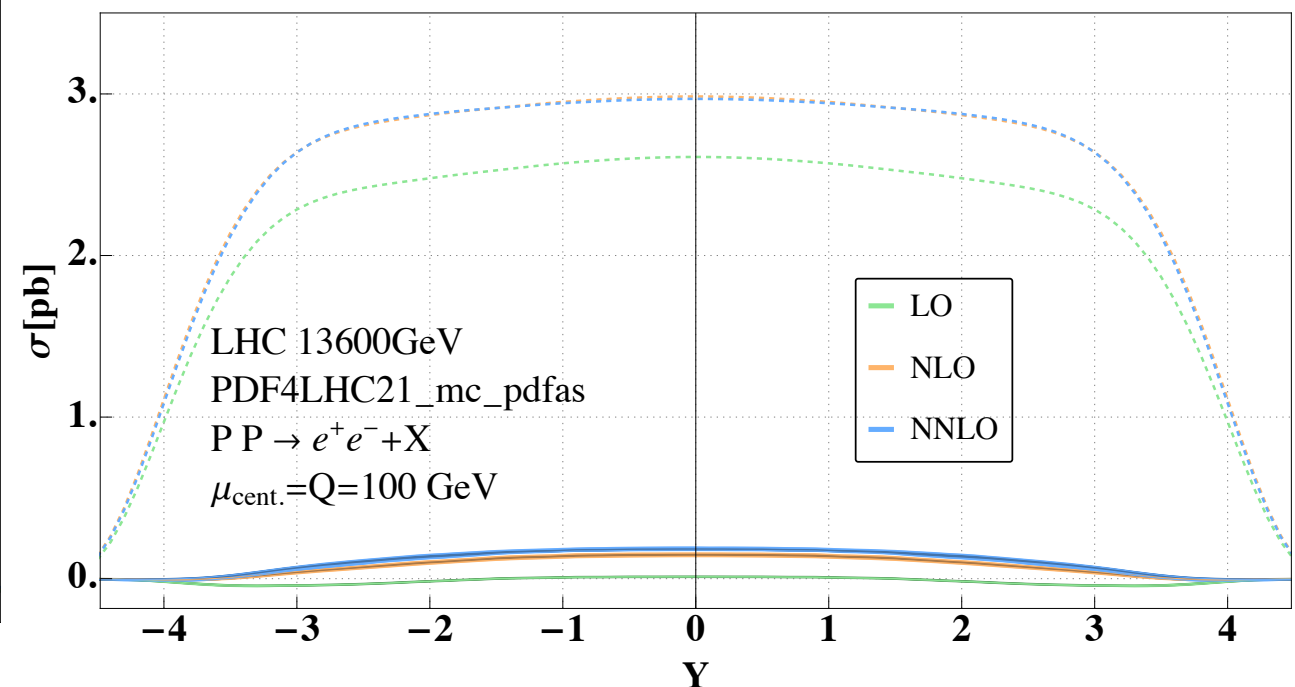


Subtracting the rapidity approximation
from the exact cross section:

$$P P \rightarrow H + X$$



$$P P \rightarrow \gamma^* + X \rightarrow e^+ e^- + X$$



Relatively small residuals beyond the leading power term of the rapidity approximation.

Rapidity Factorization

- ❖ Why should this give anything good?
- ❖ How well does it work?
- ❖ Can you compute resummed cross sections?

$$Q^2 \frac{d\sigma_{P P \rightarrow H+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2) = \tau \sum_{i,j} H_{ij} [B_i^Y(\xi_1; \bar{\xi}_2) \otimes_{\xi_1; \xi_2} S^{r_{ij}}(\bar{\xi}_1 \bar{\xi}_2) \otimes_{\xi_2; \xi_1} B_j^Y(\xi_2; \bar{\xi}_1)]$$

All our functions satisfy renormalization group equations of the form:

$$\tilde{F} \in \{H_{ij}, \tilde{B}_i^Y, \tilde{S}_r\}.$$

$$\frac{d}{d \log \mu^2} \tilde{F} \left(\log \frac{\kappa \mu^2}{Q^2}, \mu^2 \right) = \left[\Gamma(\alpha_S(\mu^2)) \log \left(\frac{\kappa \mu^2}{Q^2} \right) + \frac{1}{2} \gamma(\alpha_S(\mu^2)) \right] \tilde{F} \left(\log \frac{\kappa \mu^2}{Q^2}, \mu^2 \right)$$

With the anomalous dimensions:

	H_{ij}	\tilde{B}_i^Y	\tilde{S}_r
Γ_F^r	$-\Gamma_{\text{cusp}}^r$	Γ_{cusp}^r	$-\Gamma_{\text{cusp}}^r$
$\gamma^{\text{F,r}}$	γ_{H}^r	$\gamma_J^r = \frac{1}{2} (\gamma_{\text{thr.}}^r - \gamma_{\text{coll.}}^r)$	$-\gamma_{\text{thr.}}^r$

Following relative standard SCETty technology we find:

$$Q^2 \frac{d\sigma_{P P \rightarrow h+X}^{\text{approx.}}}{dY dQ^2}(\xi_1, \xi_2, \mu^2, \mu_S^2, \mu_{C_1}^2, \mu_{C_2}^2) \\ = \tau \sum_{i,j} H_{ij}(\mu^2, \mu_H^2) [B_i^Y(\xi_1, \mu^2, \mu_{C_1}^2; \bar{\xi}_2) \otimes_{\xi_1; \xi_2} S^{r_{ij}}(\mu^2, \mu_S^2; \bar{\xi}_1 \bar{\xi}_2) \otimes_{\xi_2; \xi_1} B_i^Y(\xi_2, \mu^2, \mu_{C_2}^2; \bar{\xi}_1)]$$

Canonical scale choices:

$$\hat{\mu}_S^2 = Q^2(1 - \xi_1)(1 - \xi_2), \quad \hat{\mu}_{C_1}^2 = Q^2(1 - \xi_2), \quad \hat{\mu}_{C_2}^2 = Q^2(1 - \xi_1).$$

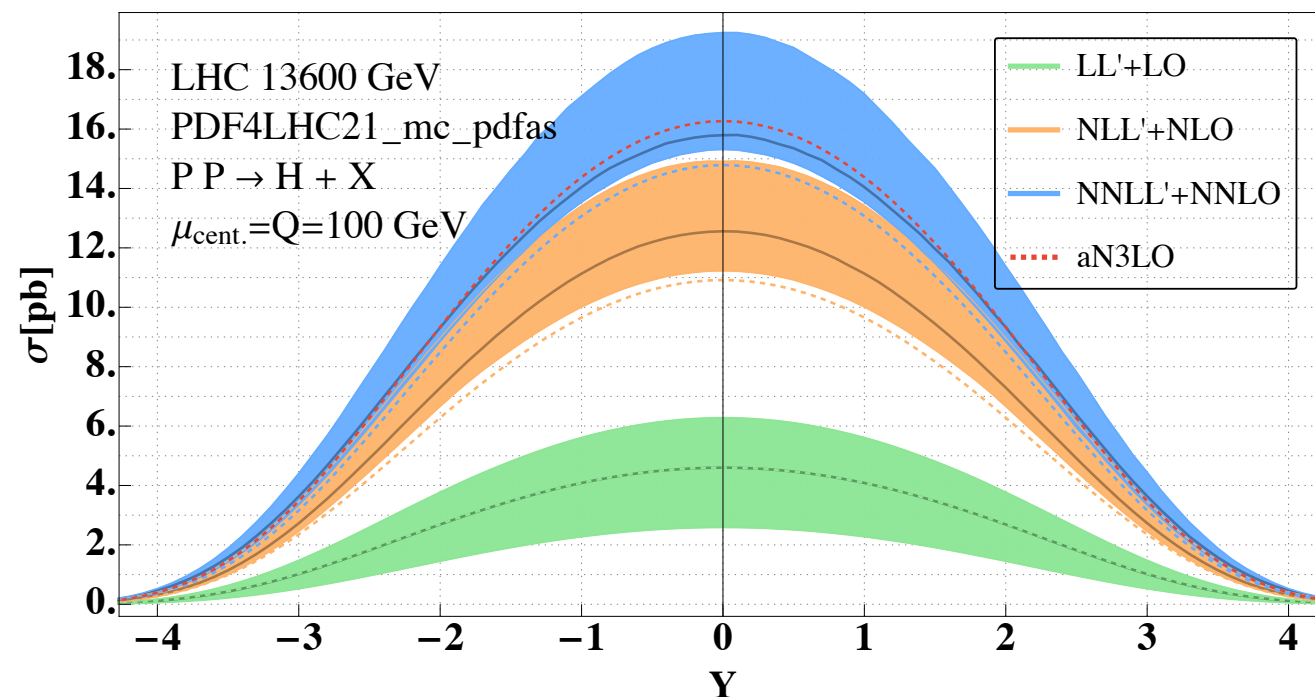
Results logs of

$$(1 - \xi_1)(1 - \xi_2), \quad (1 - \xi_2), \quad (1 - \xi_1).$$

We implemented and matched to fixed order.

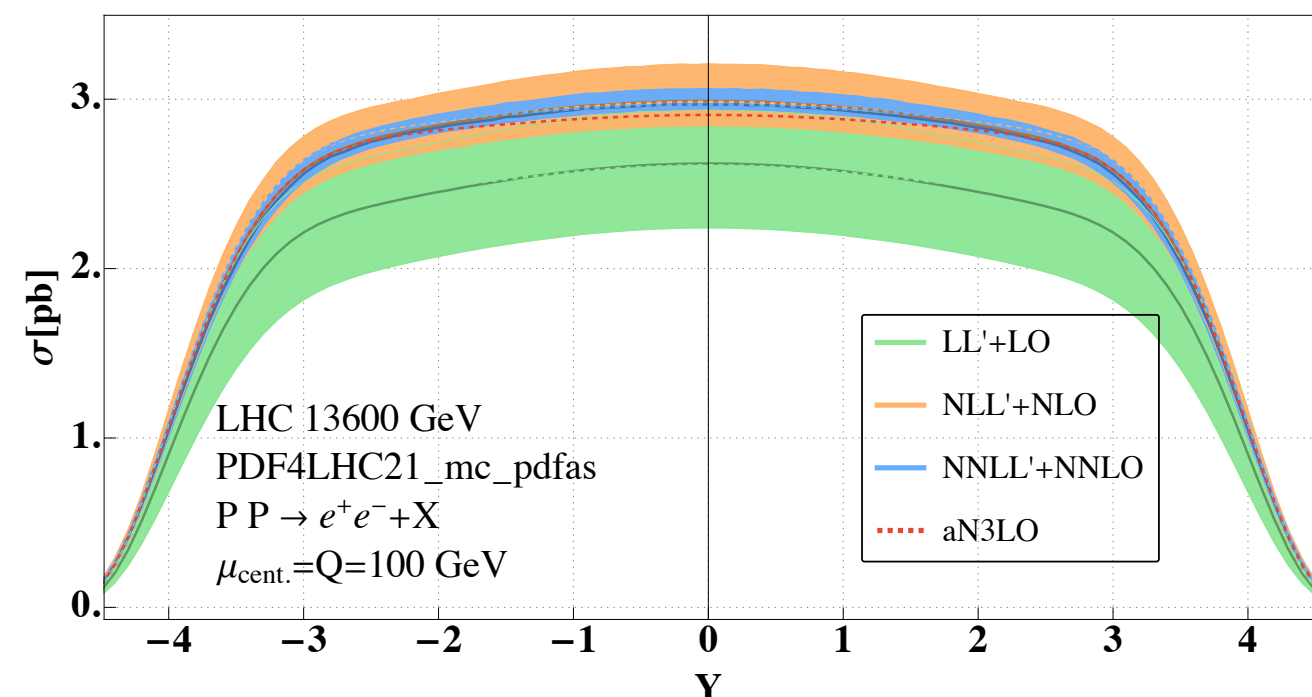
Gluon Fusion Higgs Production

$$P P \rightarrow H + X$$



Drell-Yan Production

$$P P \rightarrow \gamma^* + X \rightarrow e^+ e^- + X$$



- ❖ Resummation scale uncertainty comparable in size to FO.
- ❖ Resummation adds almost nothing for DY and is positive for Higgs.
- ❖ Resummation certainly possible but probably more just an alternative representation of the fixed order accurate cross section. (MHOU).

- ▶ $N^3\text{LO}+N^3\text{LL}'$ in progress -> Gherardo's talk!
- ▶ Apply to anything we can think about and do some phenomenology.
- ▶ Extend to top quarks?
- ▶ Extend to Next-to-Leading Power?
- ▶ Extend to color-charged final states?
- ▶ ...

- ▶ Rapidity Factorization:
A framework to approximate color-singlet LHC cross sections.
- ▶ Rapidity Factorization contains all universal soft and collinear radiation factorizing to the Born cross section.
- ▶ Approximate cross sections contain a good bit of analytic information and intriguing patterns (NNLP terms).
- ▶ This framework should supersede threshold approximations / resummation.
- ▶ We demonstrated our results for Higgs Boson and Drell-Yan production and computed the cross sections at NNLO+NNLL' accuracy.

Thank you!

