Power corrections to collider observables

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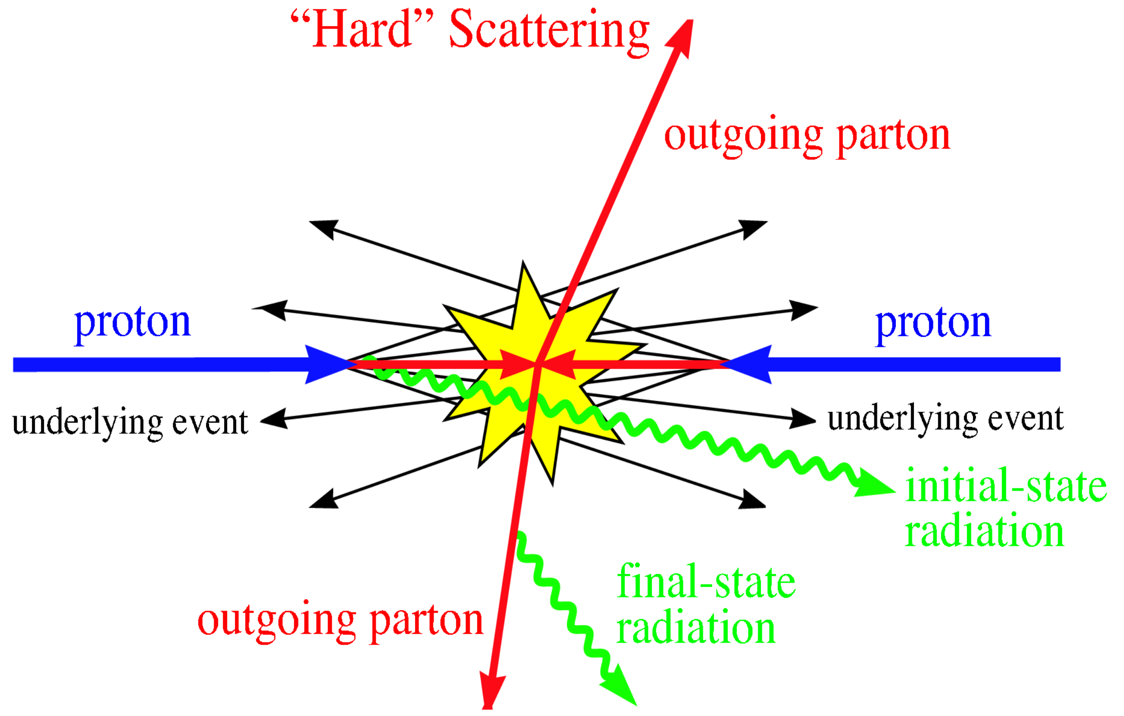
In particle physics, we deduce information about the SM Lagrangian by comparing properties of hadrons, produced in collider processes, with theoretical predictions obtained using quark and gluon degrees of freedom. This mismatch leads to differences between partonic and hadronic cross sections.



$$d\sigma^H = \sum_{ij} \int dx_1\, dx_2\, f_i(x_1) f_j(x_2) d\sigma_{ij}^{\rm part}(x_1 P_1, x_2 P_2) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^p}{Q^p} \right) \right].$$

There is no theory of power corrections and even the exponent $p > 0$ cannot be predicted from first principles for a given process or observable.

$$\Lambda_{\rm QCD} \sim 300\ {\rm MeV}, \quad Q \sim 30\ {\rm GeV} \quad \Rightarrow \quad \frac{\Lambda_{\rm QCD}}{Q} \sim 10^{-2}.$$

Numerically, such corrections cannot be large but linear $p = 1$ power corrections may still be relevant for ``standard-candle" processes and for studies of high-precision observables (strong coupling constant, mass of the top quarks and of EW bosons etc.).

We will study linear power corrections within the renormalon model where perturbatively-induced Landau singularity in the running QCD coupling constant is the only source of non-perturbative effects.

$$\int \mathrm{d}k\; k^{p-1} \alpha_s(\mu)\; F(k) \Rightarrow \int \mathrm{d}k\; k^{p-1} \alpha_s(k) F(k)\,. \qquad \alpha_s(k^2) = \approx \frac{1}{2\beta_0} \frac{\Lambda_{\rm QCD}^2}{k^2 - \Lambda_{\rm QCD}^2}, \quad k^2 \approx \Lambda_{\rm QCD}^2\,.$$

Within this model, linear power correction to a process X can be easily computed provided that the linear term in the expansion of the NLO QCD corrections to the cross section of this process in the small gluon mass is known.

Linear gluon-mass corrections are special because they are none-analytic. They only arise from the emission of real and virtual gluons with energies comparable to their masses. But if the masses are small, the gluons are soft. Hence, to find a systematic way to compute linear corrections, we need to understand soft-gluon emissions.

This set up only works for processes which do not contain gluons at leading order, i.e. non-abelian vertices are not allowed through NLO QCD.

Our goal is to understand how linear dependences of partonic NLO QCD cross sections and observables on the tiny gluon mass λ arise.

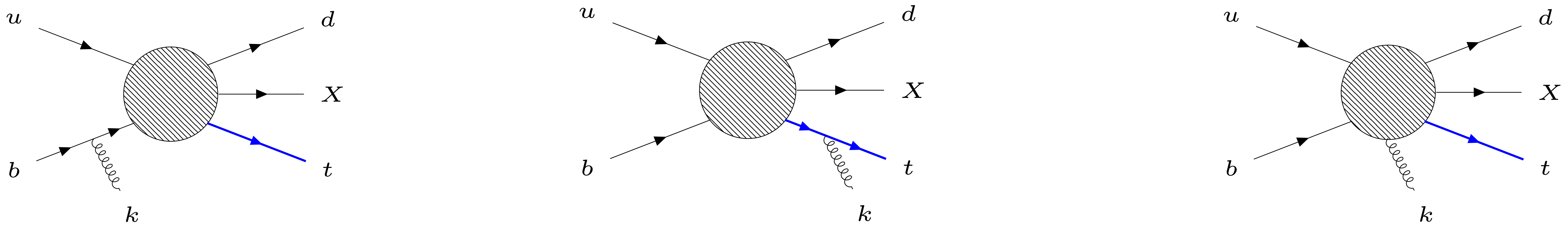
The leading term is independent of the gluon mass; the linear term is the first correction. To compute it, we need to understand next-to-leading terms in the soft expansion.

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_s \, \Delta\sigma(\lambda) \, , \qquad \Delta\sigma(\lambda) \approx \Delta\sigma(0) + \lambda \Delta\sigma' + \mathcal{O}(\lambda^2).$$

Calculation of NLO QCD cross sections requires well-defined ingredients such as real and virtual matrix elements, phase space parametrization and an infra-red safe observable; all these quantities depend on the gluon mass and we need to understand their (soft) expansions through next-to-leading power.

$$\mathrm{d}\sigma = \mathrm{dLips}_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}|^2_{\mathcal{O}(k)} \times \mathcal{O}_{\mathcal{O}(k)} \, .$$

It follows from Burnett-Kroll-Low theorem that next-to-leading soft corrections to the real-emission contribution can be computed in a process-independent manner. The BKL theorem fully exposes the dependence of the amplitude on the soft-gluon momentum.



$$|\mathcal{M}|^2 = -J^\mu J_\mu F_{\rm LO}(q_t, p_b, q_d, ...) - J_\mu L^\mu F_{\rm LO}(q_t, p_b, q_d, ...) + \mathcal{O}(k)\,.$$

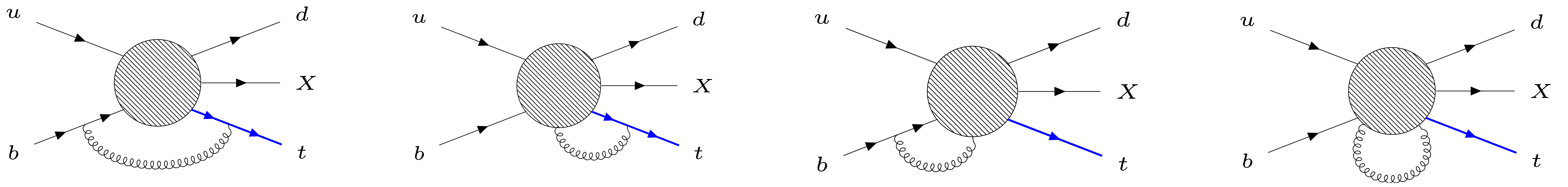
$$d_b = (p_b - k)^2$$
$$d_t = (q_t + k)^2 - m_t^2$$

$$J^\mu = J_t^\mu + J_b^\mu, \quad L^\mu = L_t^\mu - L_b^\mu, \qquad J_t^\mu = \frac{2q_t + k^\mu}{d_t} \qquad J_b^\mu = \frac{2p_b^\mu - k^\mu}{d_b} \qquad L_t^\mu = J_t^\mu k^\nu \frac{\partial}{\partial q_t^\nu} - \frac{\partial}{\partial q_t^\mu} \qquad L_b^\mu = J_b^\mu k^\nu \frac{\partial}{\partial p_b^\nu} + \frac{\partial}{\partial p_b^\mu}.$$

$$M^\mu = \bar{u}(q_t)\gamma^\mu \frac{\not{q}_t + \not{k} + m_t}{d_t}\mathbf{N}(q_t + k, p_b, q_d, ...)u(p_b) + \bar{u}(q_t)\mathbf{N}(q_t, p_b - k, q_d, ...)\frac{\not{p}_b - \not{k}}{d_b}\gamma^\mu u(p_b) + \mathcal{M}^\mu_{\rm reg}(q_t, p_b, q_d, ..|k).$$

$$k_\mu M^\mu = 0 \quad \longrightarrow \quad \mathcal{M}^\mu_{\rm ext}(q_t, p_b, q_d, ...|k=0) = -\bar{u}_t \left[\frac{\partial \mathbf{N}(q_t, p_b, q_d, ...)}{\partial q_{t\mu}} + \frac{\partial \mathbf{N}(q_t, p_b, q_d, ...)}{\partial p_{b\mu}}\right] u_b.$$

A similar analysis can be performed for the virtual corrections. We split diagrams into groups according to how many times a virtual gluon couples to external lines. The expansion is then constructed similar



$$\mathcal{M}_{\rm virt} = \int \frac{{\rm d}^4k}{(2\pi)^4}\frac{-i}{k^2-\lambda^2}\Big[J_t^\alpha J_{b,\alpha}\,\bar{u}_t\left(\mathbf{N}(p_t,p_b,..)+k^\mu D_{p,\mu}\mathbf{N}(p_t,p_b,..)\right)u_b$$
$$-J_t^\alpha \bar{u}_t\mathbf{N}(p_t,p_b,..)\mathbf{S}_{b,\alpha}u_b + J_b^\alpha\bar{u}_t\mathbf{S}_{t,\alpha}\mathbf{N}(p_t,p_b,..)u_b - (J_t^\alpha+J_b^\alpha)\bar{u}_t D_{p,\alpha}\mathbf{N}u_b\Big].$$

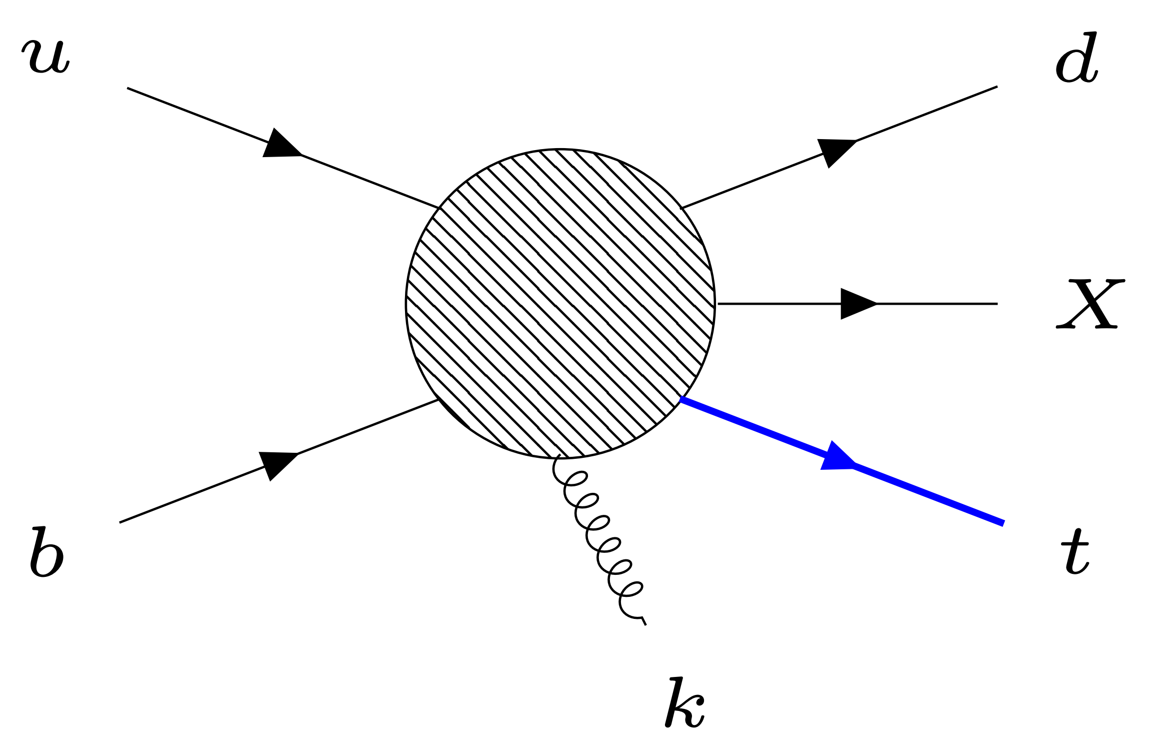
$$J_t^\alpha = \frac{2p_t^\alpha+k^\alpha}{d_t},\quad \mathbf{S}_t^\alpha = \frac{\sigma^{\alpha\beta}k_\beta}{d_t},$$
$$J_b^\alpha = \frac{2p_b^\alpha+k^\alpha}{d_b},\quad \mathbf{S}_b^\alpha = \frac{\sigma^{\alpha\beta}k_\beta}{d_b}.$$

$$d_t = (p_t+k)^2 - m_t^2$$
$$d_b = (k+p_b)^2$$
$$D_p^\mu = \frac{\partial}{\partial p_{t,\mu}} + \frac{\partial}{\partial p_{b,\mu}}$$

$$\delta[\mathcal{M}\mathcal{M}^+]_{\rm virt} = \int \frac{{\rm d}^4k}{(2\pi)^4}\frac{-i}{k^2-\lambda^2}\Big[2J_t^\alpha J_{b,\alpha}F_{\rm LO} + J_t^\alpha J_{b,\alpha}k^\mu\, D_{p,\mu}F_{\rm LO} - (J_t^\alpha+J_b^\alpha)D_{p,\alpha}F_{\rm LO}$$
$$+ J_t^\alpha {\rm Tr}\left[(D_{p,\alpha}\not{p}_t)\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right] + J_b^\alpha {\rm Tr}\left[(\not{p}_t+m_t)\mathbf{N}(D_{p,\alpha}\not{p}_b)\bar{\mathbf{N}}\right]\Big] + \mathcal{O}(\lambda^2)\ .$$

The dependence on the loop momentum is (again) clearly exposed and the integration over loop momentum can be performed in a process-independent manner.

However, integration of the real-emission contribution involves the phase space that depends on the soft-gluon momentum. One can factorize this dependence in the phase space with (linear) power accuracy by redefining momenta of hard particles. Once this is done, it becomes possible to integrate over the gluon momentum in a process-independent fashion.



Original momenta

Redefined momenta (no dependence on k!)

$$q_t + q_d + k + p_X = p_t + p_d + p_X. \quad k$$

$$q_t = p_t - k + \frac{p_t k}{p_t p_d} p_d, \qquad q_d = p_d - \frac{p_t k}{p_t p_d} p_d$$

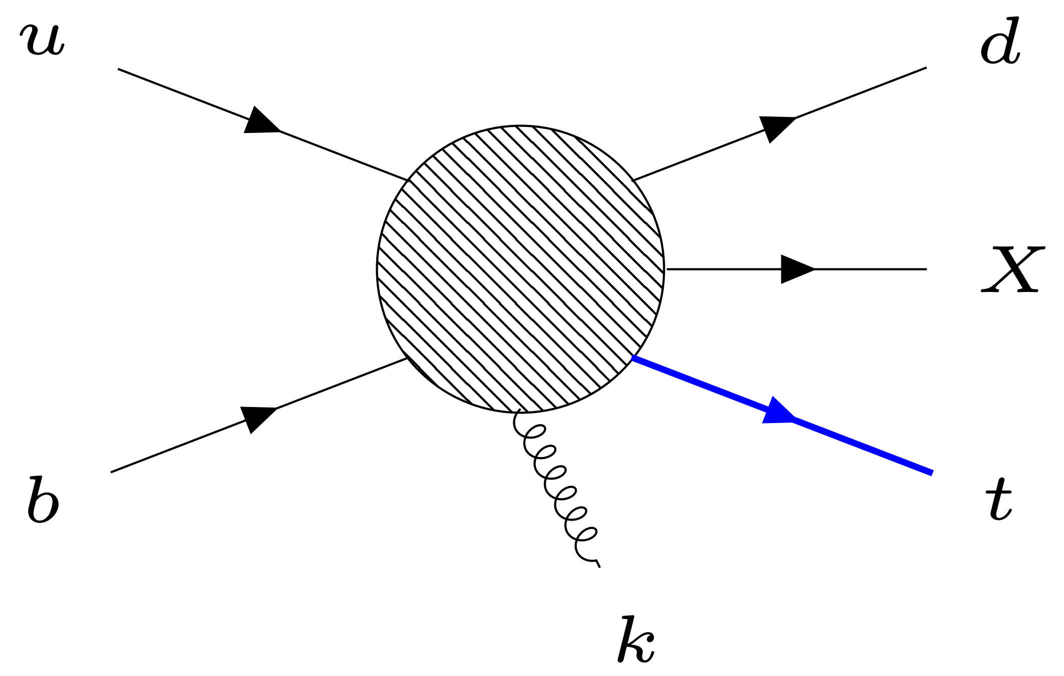
$$\mathrm{dLips}(p_u, p_b; q_d, q_t, p_X, k) = \mathrm{dLips}(p_u, p_b; p_d, p_t, p_X) \frac{\mathrm{d}^4 k}{(2\pi)^3} \delta_+(k^2 - \lambda^2) \times \left[1 + \frac{k p_d}{p_t p_d} - \frac{p_t k}{p_t p_d} \right] + \mathcal{O}(k^2).$$

Original phase space with the original momenta

Born phase space with the redefined momenta

Factorized dependence on the gluon momentum

To recap: using BKL theorem, the momenta mapping and the phase-space factorization, we compute $\mathcal{O}(\lambda)$ contributions due to real and virtual gluon emissions for an arbitrary process of a single-top-production type.



$$|\mathcal{M}|^2 = -J^\mu J_\mu F_{\rm LO}(q_t, p_b, q_d, ...) - J_\mu L^\mu F_{\rm LO}(q_t, p_b, q_d, ...).$$

$$q_t = p_t - k + \frac{p_t k}{p_t p_d} p_d\,, \qquad q_d = p_d - \frac{p_t k}{p_t p_d} p_d$$

$$\mathrm{dLips}(p_u, p_b; q_d, q_t, p_X, k) = \mathrm{dLips}(p_u, p_b; p_d, p_t, p_X) \frac{\mathrm{d}^4 k}{(2\pi)^3} \delta_+(k^2 - \lambda^2) \times \left[1 + \frac{k p_d}{p_t p_d} - \frac{p_t k}{p_t p_d}\right] + \mathcal{O}(k^2).$$

$$\mathcal{T}_\lambda\left[\sigma_{\rm R}\right] = \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int \mathrm{dLips}(p_u, p_b; p_d, p_t, p_X) \left[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b} \right) - \frac{m_t^2}{p_d p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\rm LO}.$$

$$\mathcal{T}_\lambda\left[\sigma_V\right] = -\frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int \mathrm{dLips}_{\rm LO} \left[\mathrm{Tr}\left[\not{p}_t \mathbf{N} \not{p}_b \bar{\mathbf{N}} \right] + \left(\frac{2 p_t p_b - m_t^2}{p_t p_b} - \frac{m_t^2}{p_t p_b} p_b^\mu D_{p,\mu} \right) F_{\rm LO} \right].$$

$$D_p^\mu = \frac{\partial}{\partial p_{t,\mu}} + \frac{\partial}{\partial p_{b,\mu}}$$

Real and virtual corrections is not the whole story. In principle, when massive particles are involved, linear power corrections arise because of the renormalization (the mass and the wave-function, in the on-shell scheme).

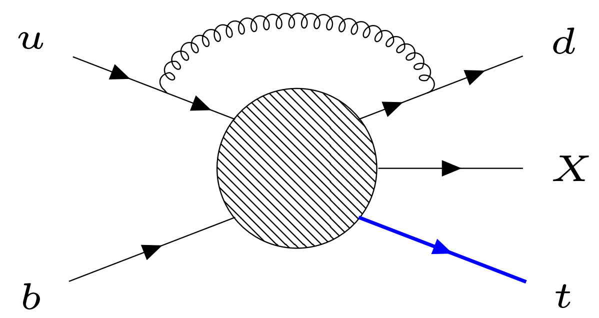
$$Z_m = 1 + \frac{C_F g_s^2 m_t^{-2\epsilon}\Gamma(1+\epsilon)}{(4\pi)^{d/2}}\left[-\frac{3}{\epsilon} - 4 + \frac{2\pi\lambda}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right)\right],$$
$$Z_2 = 1 + \frac{C_F g_s^2 m_t^{-2\epsilon}\Gamma(1+\epsilon)}{(4\pi)^{d/2}}\left[-\frac{1}{\epsilon} - 4 + 4\ln\frac{m_t}{\lambda} + \frac{3\lambda\pi}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right)\right].$$

It is also well-motivated to expect that we need to “undo” the on-shell mass renormalization and express physical quantities in terms of short-distance masses which are free of linear power corrections.

$$m_t = \tilde{m}_t\left(1 - \frac{C_F\alpha_s}{2\pi}\frac{\pi\lambda}{m_t}\right).$$

It is an interesting technical question how to do that for a general process, i.e. without using explicit form of matrix elements. This can be done by performing momenta redefinitions in the leading order cross section.

$$\sigma_{\rm LO}(m_t) - \sigma_{\rm LO}(\tilde{m}_t) = \frac{C_F\alpha_s}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips}_{\rm LO}\left[\frac{m_t^2}{p_d p_t}\left[1 + p_d^\mu\left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu}\right)\right]F_{\rm LO} - m_t{\rm Tr}\left[\mathbf{1}\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right]\right.$$
$$\left. - m_t{\rm Tr}\left[(\not{p}_t + m_t)\left(\frac{\partial\mathbf{N}}{\partial m_t}\not{p}_b\bar{\mathbf{N}} + \mathbf{N}\not{p}_b\frac{\partial\bar{\mathbf{N}}}{\partial m_t}\right)\right]\right].$$



QCD emissions of real and virtual gluons off the upper (massless) line do not induce linear power corrections; hence focusing on the massive line is sufficient.

Combining four different contributions: real, virtual, renormalization and the mass redefinition, we observe that cross sections for arbitrary single-top-like production processes are free of linear power corrections.

$$\sigma = \sigma_{\rm LO}(m_t) + \sigma_R + \sigma_V + \sigma_{\rm ren} = \sigma_{\rm LO}(\tilde{m}_t) + \delta\sigma_{\rm NLO}, \qquad \delta\sigma_{\rm NLO} = \sigma_R + \sigma_V + \sigma_{\rm ren} + \delta\sigma_{\rm mass}^{\rm expl} + \delta\sigma_{\rm mass}^{\rm impl}.$$

$$\mathcal{T}_\lambda\left[\delta\sigma_{\rm mass}^{\rm expl} + \delta\sigma_{\rm mass}^{\rm impl}\right] = \frac{C_F\alpha_s}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips}_{\rm LO}\left[\frac{m_t^2}{p_d p_t}\left[1 + p_d^\mu\left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu}\right)\right]F_{\rm LO} - m_t{\rm Tr}\left[\mathbf{1}\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right] - m_t{\rm Tr}\left[(\not{p}_t + m_t)\left(\frac{\partial\mathbf{N}}{\partial m_t}\not{p}_b\bar{\mathbf{N}} + \mathbf{N}\not{p}_b\frac{\partial\bar{\mathbf{N}}}{\partial m_t}\right)\right]\right],$$

$$\mathcal{T}_\lambda\left[\sigma_{\rm R}\right] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips}(p_u, p_b; p_d, p_t, p_X)\left[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b}\right) - \frac{m_t^2}{p_d p_t}p_d^\mu\left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu}\right) - \frac{m_t^2}{p_t p_b}p_b^\mu\left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu}\right)\right]F_{\rm LO},$$

$$\mathcal{T}_\lambda\left[\sigma_V\right] = -\frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips}_{\rm LO}\left[{\rm Tr}\left[\not{p}_t\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right] + \left(\frac{2p_t p_b - m_t^2}{p_t p_b} - \frac{m_t^2}{p_t p_b}p_b^\mu D_{p,\mu}\right)F_{\rm LO}\right],$$

$$\mathcal{T}_\lambda\left[\sigma_{\rm ren}\right] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips}_{\rm LO}\left[\frac{3}{2}F_{\rm LO} + m_t{\rm Tr}\left[(\not{p}_t + m_t)\frac{\partial\mathbf{N}}{\partial m_t}\not{p}_b\bar{\mathbf{N}}\right] + m_t{\rm Tr}\left[(\not{p}_t + m_t)\mathbf{N}\not{p}_b\frac{\partial\bar{\mathbf{N}}}{\partial m_t}\right]\right],$$

$$\mathcal{T}_\lambda\left[\delta\sigma_{\rm NLO}\right] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips}_{\rm LO}\left(F_{\rm LO} - {\rm Tr}\left[\not{p}_t\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right] - m_t{\rm Tr}\left[\mathbf{1}\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right]\right) = 0.$$

Similar results should hold for many other (abelian) processes. In particular, we find that for processes with massless coloured particles unrestricted loop and phase space integrals over gluon momenta do not induce linear power corrections.

We have not talked about observables, but it is clear that they can lead to an appearance of linear power corrections. In what follows, I would like to discuss two examples of such cases, one for single top production and another one for abelianized three jet production at a lepton collider.

We focus on the single top production and consider kinematic distributions that only depend on the top quark four-momentum. We repeat the same steps as discussed earlier. The new element is that momentum mapping induces changes in the observable.

momentum mapping induces changes in an observable

$$O_X = \int \mathrm{d}\sigma \; X(q_t) \longrightarrow X(q_t) = X(p_t) + \frac{\partial X(p_t)}{\partial p_t^\mu}\left(\frac{p_t k}{p_t p_d} p_d^\mu - k^\mu\right). \qquad \mathcal{T}_\lambda[O_X] = \mathcal{T}_\lambda[O_X^{(1)}] + \mathcal{T}_\lambda[O_X^{(2)}].$$

$$\mathcal{T}_\lambda[O_X^{(1)}] = \mathcal{T}_\lambda\left[\int \mathrm{d}\sigma \; \left(X(p_t) + \frac{\partial X(p_t)}{\partial p_t^\mu} \frac{p_t k}{p_t p_d} p_d^\mu\right)\right]$$

“inclusive” cancellation leads to

$$\mathcal{T}_\lambda[O_X^{(1)}] = 0 \;.$$

$$\mathcal{T}_\lambda[O_X^{(2)}] = -\mathcal{T}_\lambda\left[\int \mathrm{d}\sigma \; \frac{\partial X(p_t)}{\partial p_t^\mu} \; k^\mu\right]$$

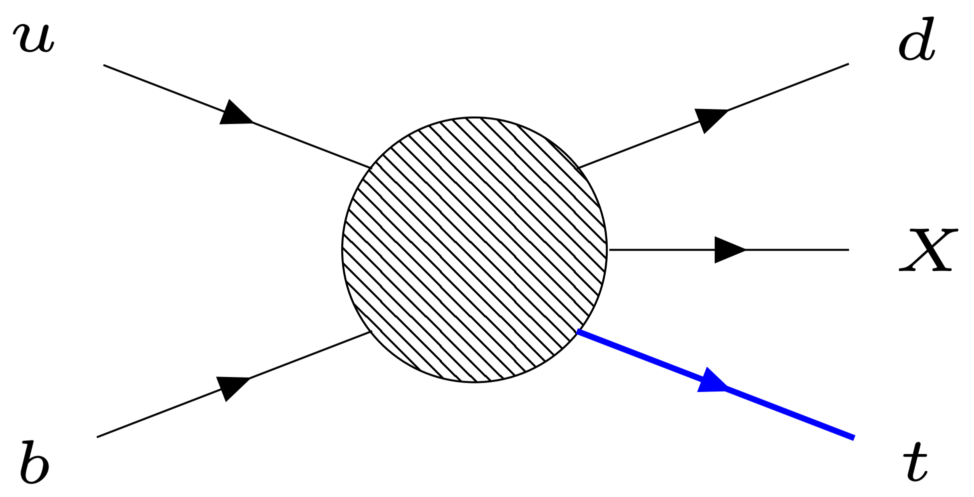
only need the cross section in the leading soft approximation

$$\mathcal{T}_\lambda[O_X^{(2)}] = \mathcal{T}_\lambda\left[C_F g_s^2 \int \mathrm{d}\sigma_{\mathrm{LO}} \frac{\partial X(p_t)}{\partial p_t^\mu} \int \frac{\mathrm{d}^4 k}{(2\pi)^3} \delta_+(k^2 - \lambda^2) \; J^\nu J_\nu \; k^\mu\right].$$

Making use of the “inclusive cancellations”, we derive a universal formula for the non-perturbative mometum shift.

$$\mathcal{O}_X = \int \mathrm{d}\sigma_{\mathrm{LO}} \; X\left(p_t + \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t}\left(p_t - \frac{2m_t^2}{p_b p_t} p_b\right)\right).$$

It is straightforward to apply the general formula to compute the non-perturbative shifts in transverse momentum and rapidity of the final-state top quark. The shifts are not large although if the top quark mass is to be extracted from the transverse momentum distribution, it may be marginally relevant.



$$\mathcal{O}_X = \int \mathrm{d}\sigma_{\mathrm{LO}}\; X\left(p_t + \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\left(p_t - \frac{2m_t^2}{p_b p_t}p_b\right)\right).$$

Transverse momentum

$$\frac{\delta_{\mathrm{NP}}\left[\,p_{t\perp}\right]}{p_{t\perp}} = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\,,$$

$$\frac{\delta_{\mathrm{NP}}\left[\,p_{t\perp}\right]}{p_{t\perp}} = \frac{\delta_{\mathrm{NP}}[m_t]}{m_t}\,,$$

$$\delta_{\mathrm{NP}}\left[\,p_{t\perp}\right] \approx (0.1 - 0.2)\;\frac{p_{t\perp}}{m_t}\;\mathrm{GeV}.$$

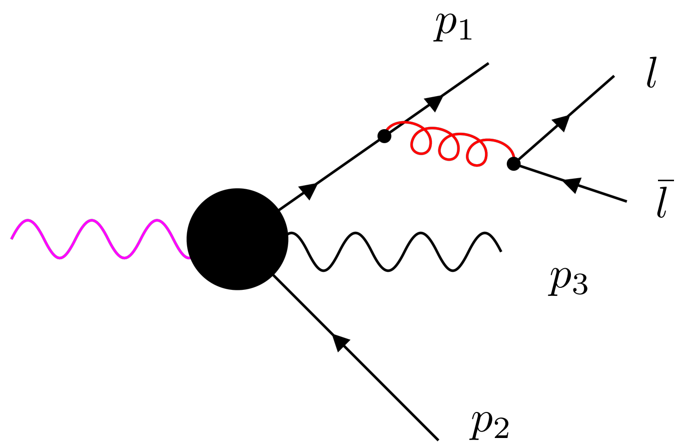
Rapidity

$$\delta_{\mathrm{NP}}\left[y_t\right] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\frac{(p_u p_b)m_t^2}{(p_u p_t)(p_b p_t)}$$

$$= \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\frac{8m_t^2 s\,\mathrm{ch}^2(y_t)}{(s+m_t^2)^2} \approx 10^{-3}$$

$$.$$

The second example that I want to discuss, is the calculation of power corrections to shape variables, such as the C-parameter and thrust, in the 3-jet region. Such power corrections are important for the extraction of the strong coupling constant from shape variables.



$$\Sigma(v) = \sum_F \int \mathrm{d}\sigma_F \; \theta(V(\Phi_F) - v).$$

$$C = 3 - 3\sum_{ij}^{N} \frac{(p_i p_j)^2}{(p_i q)(p_j q)}.$$

Since we cannot deal with gluons in Born diagrams, we consider production of two hard quarks and a hard photon as a gluon proxy.

Shape variables include sums over all final state partons. In the context of large-N_f calculation, we need to include fermions from the soft gluon splitting.

The non-vanishing result only appears because of the dependence of the observable on the soft quark momenta; this allows us to discard virtual contributions, phase-space modifications and next-to-soft corrections to matrix elements, and generalize the result to true QCD jets.

$$\mathcal{T}_\lambda[\Sigma(v;\lambda)] = \int \mathrm{d}\sigma^{\mathrm{b}}(\tilde{\Phi}_{\mathrm{b}})\delta(V(\{\tilde{p}\}) - v) \times \Big[\mathcal{N}\mathcal{T}_\lambda\left[I_V(\{\tilde{p}\},\lambda)\right]\Big].$$

$$C(\{\tilde{p}\},l,\bar{l}) - C(\{\tilde{p}\}) = \sum_{i=1}^{3} \frac{(\tilde{p}_i l)^2}{(\tilde{p}_i q)(lq)} + (l \to \bar{l}) .$$

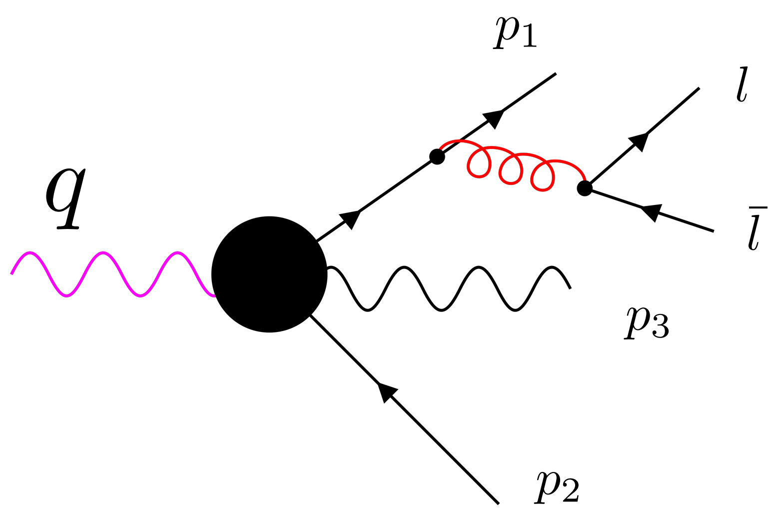
$$I_V(\{\tilde{p}\},\lambda) = \int [\mathrm{d}k]\frac{J^\mu J^\nu}{\lambda^2} \int [\mathrm{d}l][\mathrm{d}\bar{l}](2\pi)^4\delta^{(4)}(k - l - \bar{l}) \; \mathrm{Tr}\left[\hat{l}\gamma^\mu\hat{\bar{l}}\gamma^\nu\right]\left[V(\{\tilde{p}\},l,\bar{l}) - V(\{\tilde{p}\})\right].$$

$$J^\mu = \frac{p_1^\mu}{p_1 k} - \frac{p_2^\mu}{p_2 k}.$$

It turns out that this integral can be written in a factorized form by choosing a particular order of integration. The so-called Milan factor emerges in a natural way and the relations between power corrections to different shape variables become exposed.

$$I_C = \lambda\, F \times W_C$$

$$I_V(\{\tilde p\},\lambda) = \int [\mathrm{d}k] \frac{J^\mu J^\nu}{\lambda^2} \int [\mathrm{d}l][\mathrm{d}\bar l](2\pi)^4 \delta^{(4)}(k - l - \bar l)\ \mathrm{Tr}\left[\hat l \gamma^\mu \hat{\bar l} \gamma^\nu\right] \left[V(\{\tilde p\}, l, \bar l) - V(\{\tilde p\})\right]$$



$$\tilde C_{\alpha\beta} = \sum_{i=1}^{3} \frac{p_i^\alpha p_i^\beta}{(p_i q)}$$

$$\tilde l^\mu = \frac{p_1^\mu}{\sqrt{s}} e^{\eta} + \frac{p_2^\mu}{\sqrt{s}} e^{-\eta} + n^\mu.$$

A universal, observable-independent constant that turns out to be the (famous) Milan factor

$$F = 16\pi \int [\mathrm{d}k] \frac{J_\mu J_\nu}{\lambda^3} \left\{ -2 \tilde l^\mu \tilde l^\nu \frac{\lambda^8}{(2k\tilde l)^5} - \frac{g^{\mu\nu}\lambda^6}{2(2k\tilde l)^3} \right\} = -\frac{5}{64\pi}.$$

Observable-dependent function

$$W_C = -3 \int \frac{\mathrm{d}\eta\, \mathrm{d}\varphi}{2(2\pi)^3}\ \tilde C_{\alpha\beta} \frac{\tilde l^\alpha \tilde l^\beta}{(\tilde l q)}\ .$$

$$I_C = \frac{15}{128\pi^3 q} \frac{s_{12}^3}{1 - z_3} \left[\frac{1 + z_3}{2} K(c_{12}^2) - (1 - z_1 z_2) E(c_{12}^2) \right].$$

$$c_{12} = \cos\frac{\theta_{12}}{2},$$

$$q p_i = \frac{q^2}{2}(1 - z_i).$$

To summarise:

1) linear non-perturbative corrections $\mathcal{O}(\Lambda_{\rm QCD}/Q)$ may become relevant for collider physics thanks to a very high precision of perturbative predictions and experimental measurements;

2) there is no theory to calculate such power corrections from first principles;

3) nevertheless, for processes without gluons in Born diagrams, they can be studied within the renormalon model for power correction;

4) linear power corrections can be easily computed once linear dependence of NLO QCD corrections on infinitesimal gluon mass is known.

5) such a dependence can be derived for arbitrary processes using Burnett-Kroll-Low theorem for next-to-soft emissions, its generalization to virtual corrections, and the momenta mappings that factorise the dependence of the phase space on soft gluon momentum with next-to-soft accuracy.

6) this approach can be used to prove cancellation of linear power corrections to arbitrary (abelian) processes at collliders without the need to compute the one-loop corrections with the gluon mass exactly.

7) calculation of linear corrections to kinematic distributions in various processes can also be performed very efficiently using this method; such corrections are rather small in general but they can be relevant for, e.g., the top quark mass measurement and for the extraction of the strong coupling constant from shape variables.