Locally finite two-loop amplitudes for multiple Higgs boson production in gluon fusion

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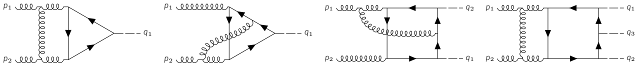
Introduction

INTRODUCTION

- Last few years: massive theoretical progress towards the NNLO cross-sections and amplitudes for $2 \to 3$ processes: e.g. (...; Badger et al. 2018; D. Chicherin et al. 2019; Chawdhry et al. 2020; Kallweit, Sotnikov, and Wiesemann 2021; Abreu et al. 2021; Kardos et al. 2022; Hartanto et al. 2022; Dmitry Chicherin, Sotnikov, and Zoia 2022)
- Up to now: Computable $2 \to 3$ process at NNLO have no internal mass and only one external massive particle.
- Computational complexity grows fast when adding more kinematic or mass scales.
- Idea: numerical integration directly in momentum space: fixed number of integrations for each loop.
- Remove infrared (IR) and ultraviolet (UV) singularities locally before integrating.
- Program started by investigating amplitudes with incoming quarks and colorless final states. Locally finite amplitude for generic electroweak productions in quark scattering at two loops: (Anastasiou, Haindl, et al. 2021; Anastasiou and G. Sterman 2022)

$$q\bar{q} \to V_1 \ldots V_n \text{ with } V_i \in \{\gamma^*, W, Z\}$$

- Next step: incoming gluons with colorless final states e.g. Higgs (NLO)



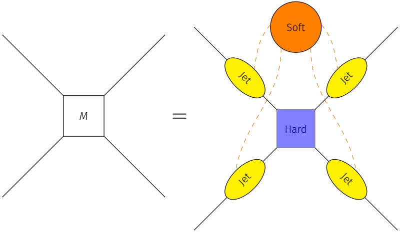
Introduction

Factorization and subtraction

Infrared factorization

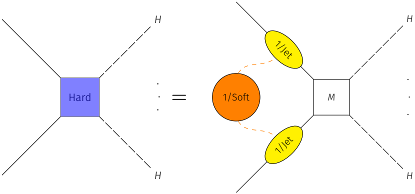
UV renormalised scattering amplitudes factorize: (Ma 2020; Erdoğan and G. Sterman 2015; Dixon, Magnea, and G. F. Sterman 2008; Catani 1998; Sen 1983)

$$\text{Amplitude} = \text{Hard} \cdot \text{Soft} \cdot \prod_i \text{Jet}_i,$$



- Hard function is process-dependent and receives contributions from non-singular regions; in this case the heavy top-quark loop.
- Soft and Jet_i functions contain all singularities and are universal functions
- For new process should only need to compute Hard function.

Using factorization for Higgs production



Derive Hard function for amplitudes as an integrand free of IR and UV singularities.

$$M = \int [dk]\ \mathcal{M} = \underbrace{\int [dk]\ \mathcal{S} \prod_i \mathcal{J}_i}_{\substack{\text{IR divergent} \\ \text{universal}}} \int [dk]\ \underbrace{\mathcal{M}\ \mathcal{S}^{-1} \prod_i \mathcal{J}_i^{-1}}_{\substack{=\mathcal{H} \\ \text{finite, process dependent}}}$$

Using factorization

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Identify and subtract the singular parts point-by-point in the integration domain at each order in the perturbative expansion. For $gg \to nH$:

$$\mathcal{H}^{(1)} = \mathcal{M}^{(1)}$$
$$\mathcal{H}^{(2)} = \mathcal{M}^{(2)} - \mathcal{S}\mathcal{M}^{(1)} - \mathcal{J}\mathcal{M}^{(1)}$$

- Naive construction has non-local cancellations $\longrightarrow$ cannot be integrated numerically.
- All IR and UV singularities need to cancel locally on the integrand level!
- This is a challenge!

Challenges

- Arrange loop momentum flows for each diagram such that cancellations due to Ward identities arise locally.
- Take care of shift mismatches arising in the collinear limits.
- Modify the usual Feynman rules for some diagrams.

FACTORIZATION AND SUBTRACTION

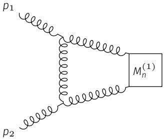
Challenges

- Arrange loop momentum flows for each diagram such that cancellations due to Ward identities arise locally.
- Take care of shift mismatches arising in the collinear limits.
- Modify the usual Feynman rules for some diagrams.

Main result: Local Factorization

Factorization based construction of a local IR counterterm for two-loop gluon-fusion amplitude integrand for an arbitrary number of Higgs.

$$\mathcal{M}^{(2)}_{n,\text{IR-finite}} = \mathcal{M}^{(2)}_n - \mathcal{F}\left(\mathcal{M}^{(1)}_n\right) \quad \text{with} \quad \mathcal{F}\left(\mathcal{M}^{(1)}_n\right) =$$

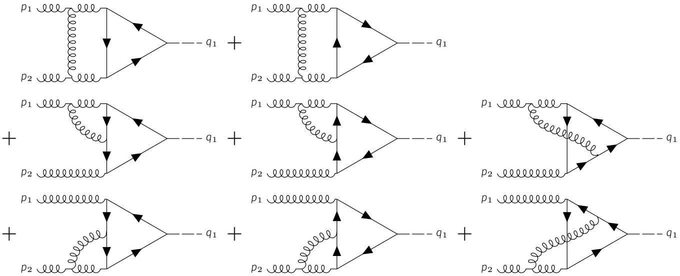


Jet and Soft function are exactly the same for the generic gluon fusion form factor amplitude $2 \to 1 \longrightarrow$ subtract to remove all IR singularities.

Single Higgs production amplitude in gluon fusion

SINGLE HIGGS PRODUCTION

Infrared singular diagrams of the two-loop amplitude for the single Higgs production in gluon fusion:

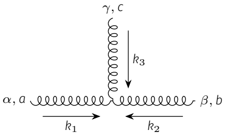


- Diagrams with triple gluon vertices are the origin of collinear singularities.
- Different terms from the triple gluon vertex have a different behaviour and play a different role in the local factorization.
- Analyze each contribution separately $\longrightarrow$ decompose triple gluon vertex.

Single Higgs production amplitude in gluon fusion

"Scalar decomposition"

Diagrammatic decomposition of a triple-gluon vertex:

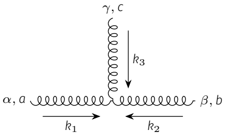


$$= (-g_s) f_{abc} \big((k_1 - k_2)^\gamma \eta^{\alpha\beta} + (k_2 - k_3)^\alpha \eta^{\beta\gamma} + (k_3 - k_1)^\beta \eta^{\alpha\gamma} \big)$$

Each term resembles a tree-level Feynman rule for the interaction of a colour-octet scalar and a gluon. $\longrightarrow$ Decompose all IR singular diagrams and separately introduce a momentum labels.

“SCALAR DECOMPOSITION”

Diagrammatic decomposition of a triple-gluon vertex:



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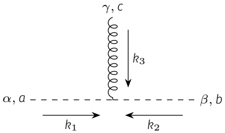
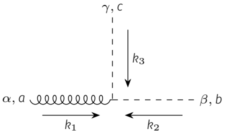
$=$



$+$



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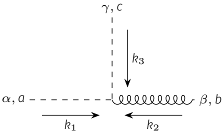
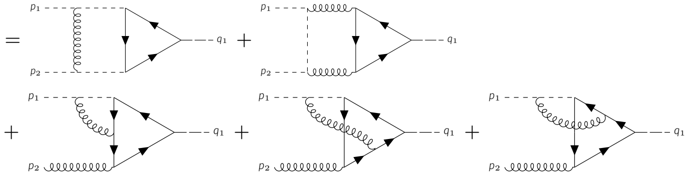


Each term resembles a tree-level Feynman rule for the interaction of a colour-octet scalar and a gluon. $\longrightarrow$ Decompose all IR singular diagrams and separately introduce a momentum labels.

“SCALAR DECOMPOSITION”

Collinear divergent $k||p_1$ diagrams after “Scalar decomposition”

$$\mathcal{M}^{(2)}_{1,\text{coll }p_1} =$$



$+$ other charge flow

- Some “scalar”-decomposed diagrams are IR-finite, due to the choice of polarization vectors.
- Now: Ready to introduce loop momentum labels.

“SCALAR DECOMPOSITION”

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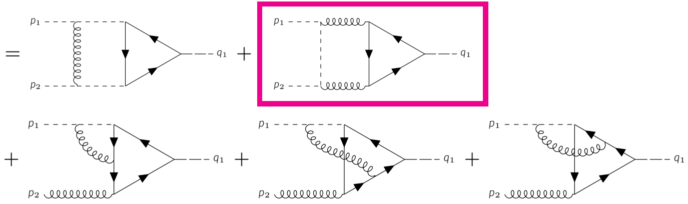


$+$ other charge flow

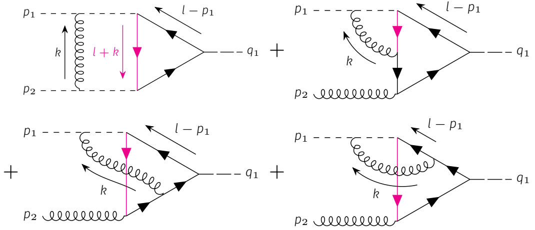
Diagrams with “scalar” in t-channel do not contribute to the local factorization in the collinear limits. Only IR singular on the local level and finite when integrated!

- Solution: add counterterms, which vanish due to Ward identities on the integrated level, but make diagrams IR finite on the local level.
- $\rightarrow$ modified the integrand of the diagrams to regularize it in all its soft and collinear limits, without changing its integrated value.

Impose a local loop momentum routing for all the different scalar decomposed diagrams.

- k momentum label is introduced such that we always have $\frac{1}{k^2}$, $\frac{1}{(k+p_1)^2}$, $\frac{1}{(k-p_2)^2}$ propagators or a subset thereof.
- l momentum label flows out of gqq-vertex next to external p_1.
- k momentum label flows in the direction of the charge flow when entering the top-quark loop.

All diagrams for one charge flow, which are singular in the collinear limit $k \to p_1$ with specified loop momentum labels l, k:



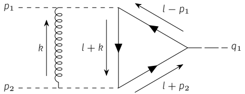
Single Higgs production amplitude in gluon fusion

Infrared singularities

SOFT SINGULARITY $k \to 0$

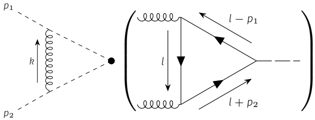
Soft singular diagrams for one charge flow after "scalar" decomposition:

$$\mathcal{M}_{SS}^{(2)} =$$



The form factor counterterm

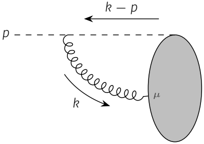
$$\mathcal{F}_{SS}^{(2)} \left(\widetilde{\mathcal{M}}^{(1)} \right) =$$



cancels the soft singularity $k \longrightarrow 0$.
Does this counterterm also cancel the collinear limits?

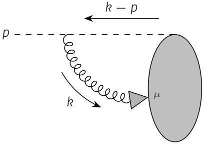
COLLINEAR SINGULARITIES

In the collinear limits the virtual gluons become longitudinally polarized.

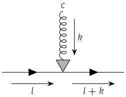


$$= \dots \frac{(k-2p)^{\mu}}{k^2(k-p)^2} \dots \xrightarrow[x=\frac{k\cdot\eta}{p\cdot\eta}]{k=xp} \dots \frac{x-2}{x}\,\frac{k^{\mu}}{k^2(k-p)^2}$$

We represent this by an arrow:



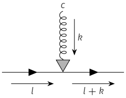
Tree-level Ward identity for longitudinally polarised gluon in a quark-quark-gluon vertex:



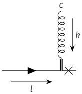
$$= \frac{i}{\not{l} + \not{k} - m} (-ig_s) T_c \not{k} \, \frac{i}{\not{l} - m} = g_s T_c \left(\frac{i}{\not{l} - m} - \frac{i}{\not{l} + \not{k} - m} \right)$$

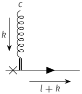
WARD IDENTITIES

In a diagrammatic representation:

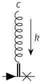


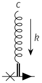
$$= \frac{i}{\not{l} + \not{k} - m} \, (-ig_s) T_c \not{k} \, \frac{i}{\not{l} - m} = g_s T_c \left(\frac{i}{\not{l} - m} - \frac{i}{\not{l} + \not{k} - m} \right)$$

$=$ 

$+$ 

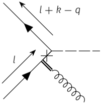
with the vertices defined as

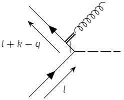


$= g_s T_c,$


$= -g_s T_c$

We can see that the following identity holds:

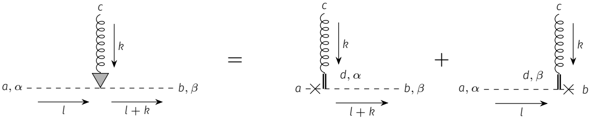


$+$ 

$= 0$

WARD IDENTITIES

Diagrammatic representation of the tree-level Ward identity of the “scalar”-“scalar”-gluon vertex is:



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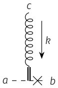
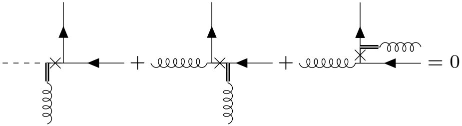


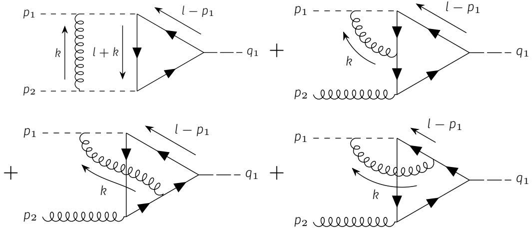
$$= i g_s f_{abc}$$

In the collinear limits of the amplitude the following identity is useful:

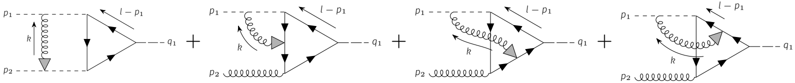


Example

All diagrams for one charge flow, which are singular in the collinear limit $k \to p_1$ with specified loop momentum labels l, k:



Examine the limit $k \to p_1$ for one charge flow:



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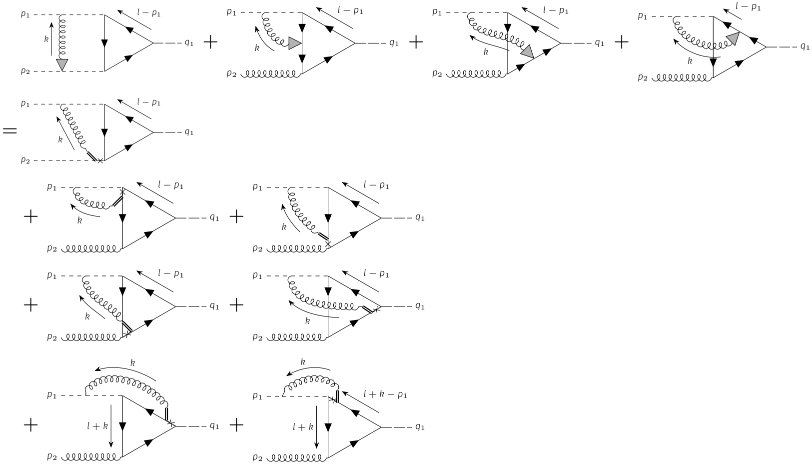

COLLINEAR LIMIT

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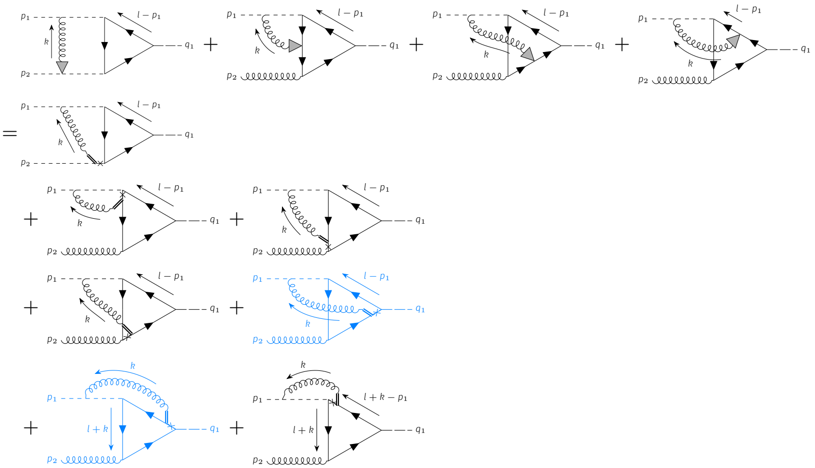

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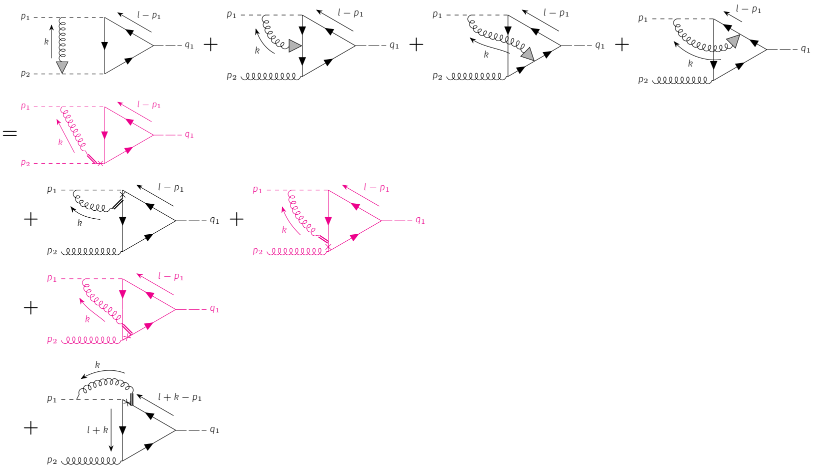
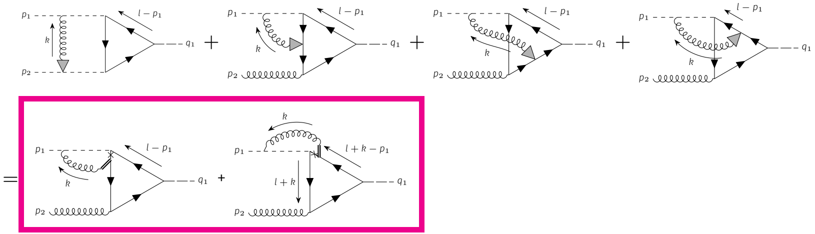


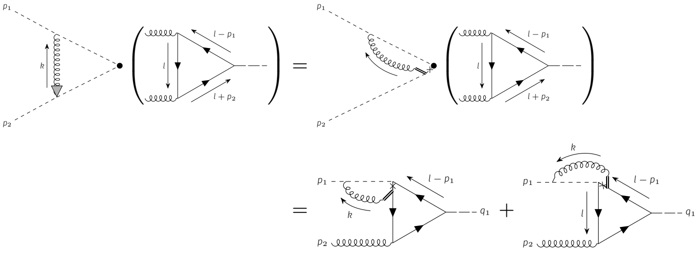
The remaining diagrams in the collinear limit are two external leg corrections $\longrightarrow$ **factorization!**



- NOTE: The top-quark loop momentum routing differs by a shift.
- Does the form factor counterterm introduced to remove the soft singularity also remove the collinear singularity?

Collinear limit of the form factor counterterm

The form factor counterterm for one charge flow contribution of the one-loop diagram in the collinear limit $k \to p_1$:



which can be seen by factorizing out the color factors.

Collinear limit of the form factor counterterm

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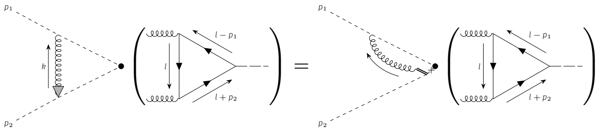
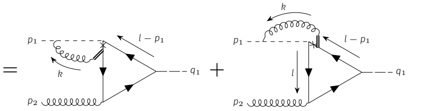
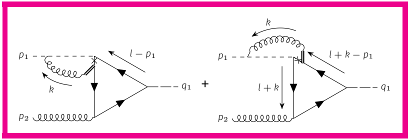




not locally equal to $=$

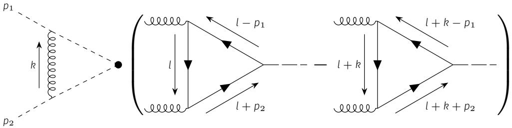


$\rightarrow$ The form factor counterterm needs a loop momentum shift in the quark-loop to remove the the two external leg corrections in the collinear limit.

Shift counterterm

We shift the quark-loop contribution of the form factor counterterm with a local counterterm to remove the collinear limits locally.

$$\mathcal{CT}_{shift} = \frac{1}{2}$$



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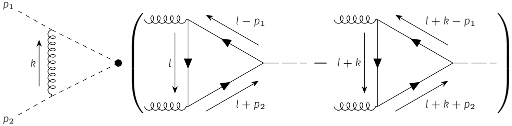
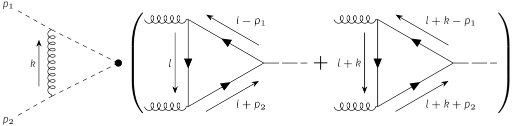


The shift counterterm integrates to zero but ensures local factorization!

Shifted form factor counterterm for one charge flow

$$\mathcal{F}_{ss}^{(1)}\left(\widetilde{\mathcal{M}}^{(1)}(l)\right) - \mathcal{CT}_{shift} = \frac{1}{2}\mathcal{F}_{ss}^{(1)}\left(\widetilde{\mathcal{M}}^{(1)}(l) + \widetilde{\mathcal{M}}^{(1)}(l+k)\right)$$

$$= \frac{1}{2}$$



Removes the soft limit as well as the collinear $k \to p_1$ and $k \to p_2$ limit.

Summary and Conclusion

SUMMARY AND CONCLUSION

- With the introduced loop momentum routing for scalar decomposed diagrams, all IR singularities of the multiple Higgs production two-loop amplitude are removed by the shifted form factor counterterm:

$$\mathcal{M}^{(2)}_{n,\text{IR-finite}} = \mathcal{M}^{(2)}_{n} - \frac{1}{2}\mathcal{F}^{(1)}_{ss}\left(\widetilde{\mathcal{M}}^{(1)}_{n}(l) + \widetilde{\mathcal{M}}^{(1)}_{n}(l+k)\right).$$

- IR finiteness of $\mathcal{M}^{(2)}_{n,\text{IR-finite}}$ checked numerically up to 3 Higgs.

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- Local UV counterterms are constructed using the R-operation forest formula (BPHZ) (Bogoliubov and Parasiuk 1957; Hepp 1966; Zimmermann 1969) $\longrightarrow$ fully finite amplitude on the integrand level ready for numerical integration in $D = 4$:

$$\mathcal{M}^{(2)}_{n,\text{finite}} = \mathcal{M}^{(2)}_{n,\text{ UV-finite}} - \frac{1}{2}\mathcal{F}^{(1)}_{ss,\text{ UV-finite}}\left(\widetilde{\mathcal{M}}^{(1)}_{n,\text{ finite}}(l) + \widetilde{\mathcal{M}}^{(1)}_{n,\text{ finite}}(l+k)\right),$$

with e.g. $\mathcal{M}^{(2)}_{n,\text{ UV-finite}} = R(\mathcal{M}^{(2)}_{n})$.

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- IR finiteness of $\mathcal{M}^{(2)}_{n,\text{IR-finite}}$ checked numerically up to 3 Higgs.
- Local UV counterterms are constructed using the R-operation forest formula (BPHZ) (Bogoliubov and Parasiuk 1957; Hepp 1966; Zimmermann 1969) $\longrightarrow$ fully finite amplitude on the integrand level ready for numerical integration in $D = 4$:

$$\mathcal{M}^{(2)}_{n,\text{finite}} = \mathcal{M}^{(2)}_{n,\text{ UV-finite}} - \frac{1}{2}\mathcal{F}^{(1)}_{ss,\text{ UV-finite}}\left(\widetilde{\mathcal{M}}^{(1)}_{n,\text{ finite}}(l) + \widetilde{\mathcal{M}}^{(1)}_{n,\text{ finite}}(l+k)\right),$$

  with e.g. $\mathcal{M}^{(2)}_{n,\text{ UV-finite}} = R(\mathcal{M}^{(2)}_{n})$.
- Next challenges:
  - numerical integration in $D = 4$
  - extend method to colorful final states