RADCOR-2023, Crieff, Scotland

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DI-HIGGS AT N3LO+N3LL

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LPTHE, Paris

Based on *(JHEP 02 (2023) 067)*

In collaboration with Hua-Sheng Shao

OUTLINE

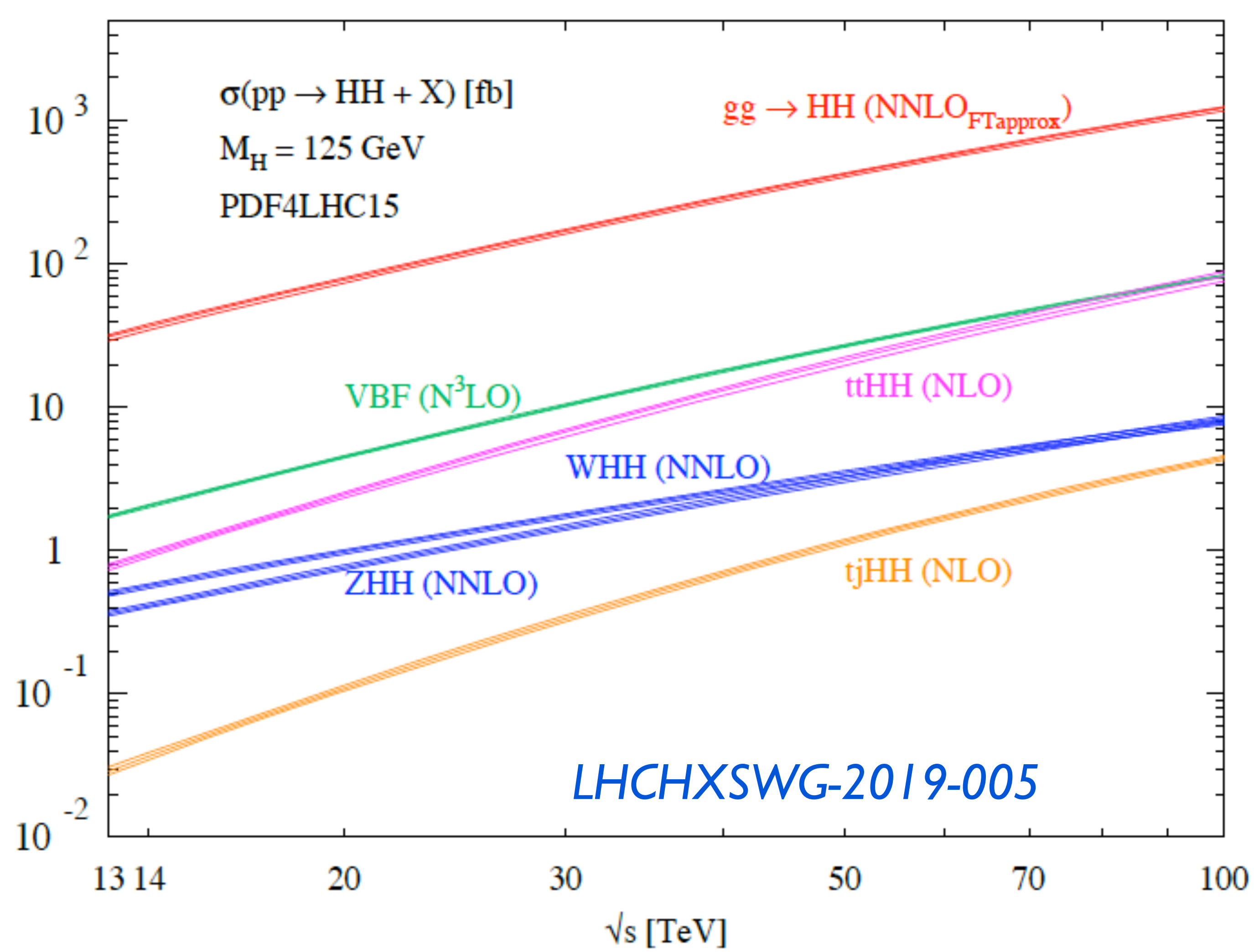
- Brief Introduction
- Infinite top quark approximation:
  - Overview on N3LO computation
  - Threshold resummation
- Resummation - scheme dependence
- Results for the inclusive and differential cross section in the infinite top quark limit
- Resummation with finite top quark mass dependence
- Summary & Outlook

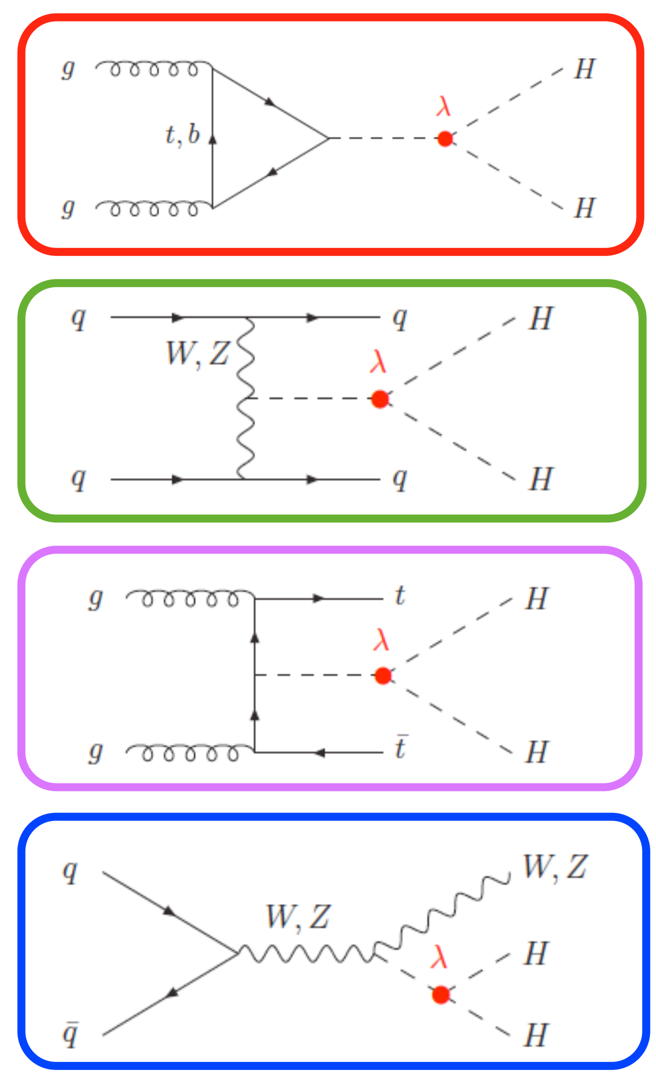
BRIEF INTRODUCTION

- Why study Higgs pair productions?
  - directly probing trilinear coupling
  - Indirect constrain to quartic couping

} Constrain the Higgs field potential

Main production modes





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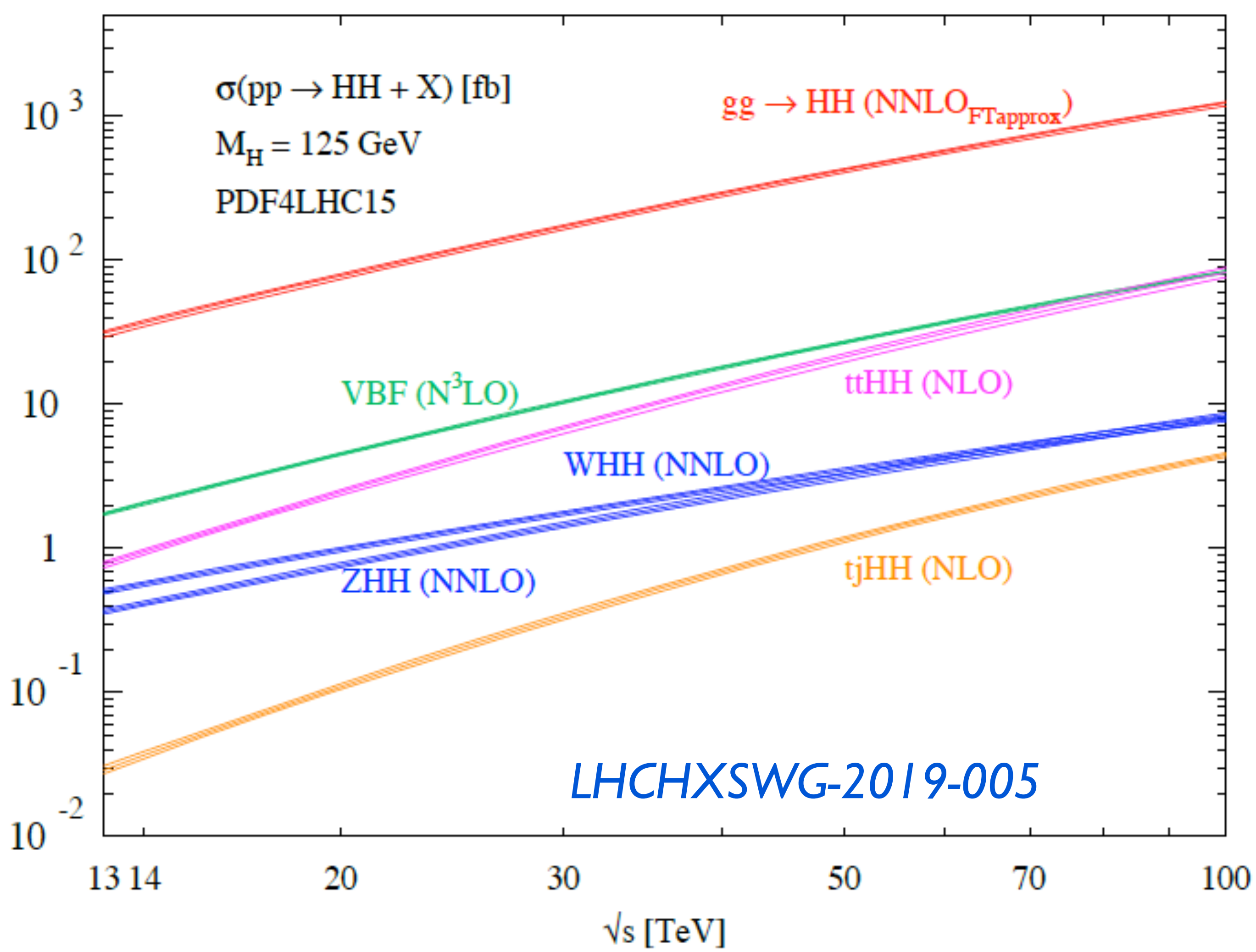
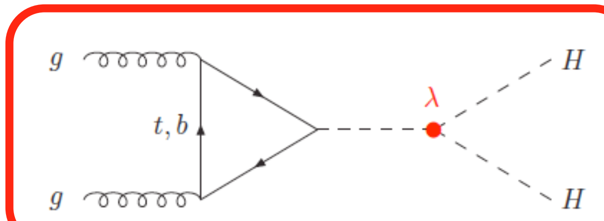
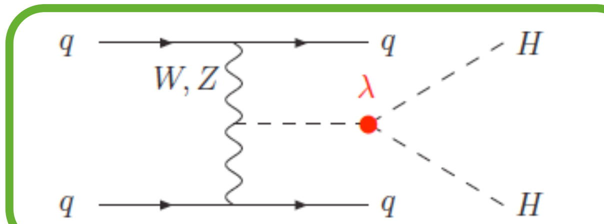
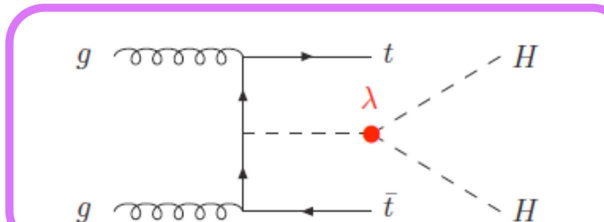
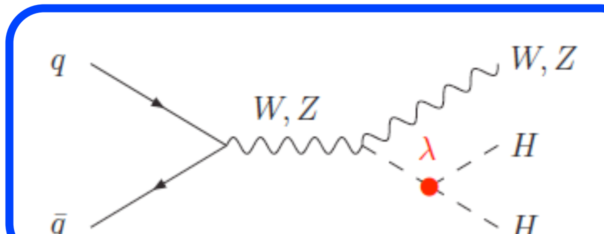










at 14 TeV

$\sigma_h^{\mathrm{N^3LO}} = 54.72 \text{ pb}$

arXiv:1902.00134

$\sigma_{hh}^{\mathrm{NNLO_{FTapprox}}} = 36.69 \text{ fb}$

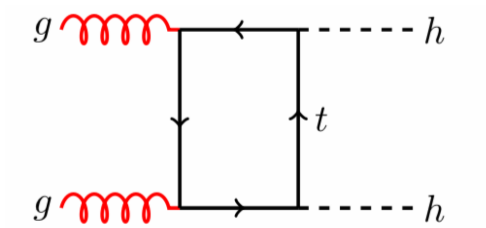
arXiv:1910.00012

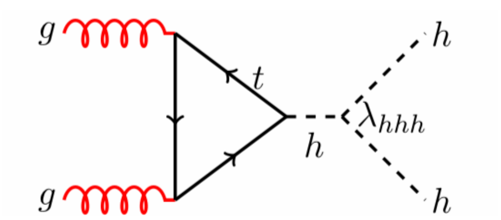
$\sigma_{hhh}^{\mathrm{NLO_{FTapprox}}} = 89.4 \text{ ab}$

arXiv:1408.6542

BRIEF INTRODUCTION

- Loop induced LO: destructively interfering box and triangle diagrams





Higher order perturbative computations are quite challenging

State-of-the-art : NLO

further improved by Soft gluon resummation/parton shower matching

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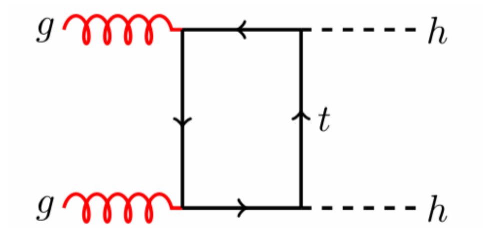
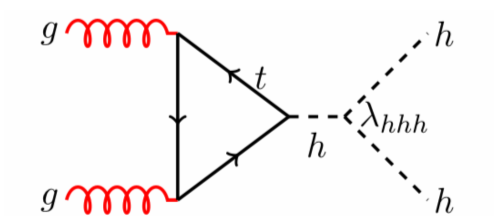
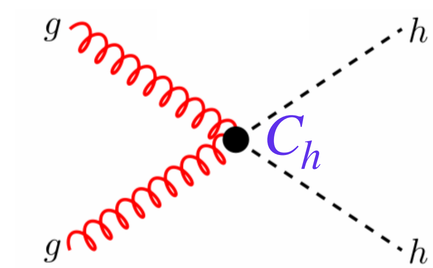




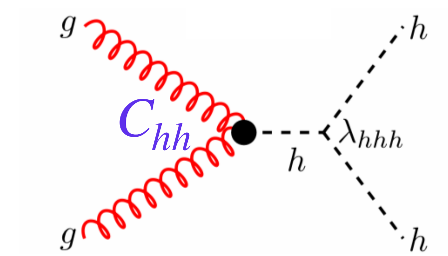
Infinite top quark approximation

$m_t \to \infty$



$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}\left(C_h \frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right) G^a_{\mu\nu} G^{a\ \mu\nu}$$

$m_h \ll m_t$



Integrate out the top quarks ($m_T \to \infty$) : introduces the effective vertices C_h & C_{hh} between Higgs and gluons

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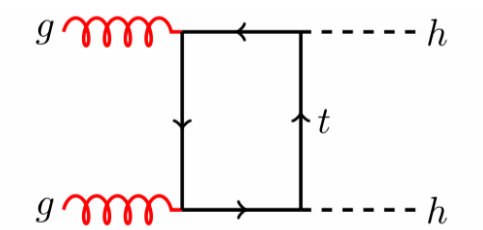

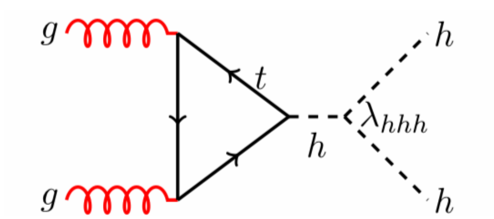

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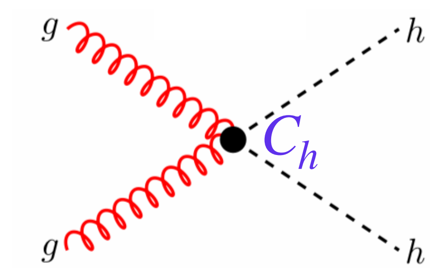

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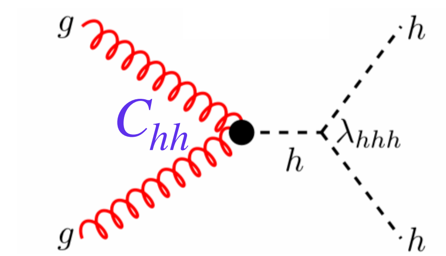

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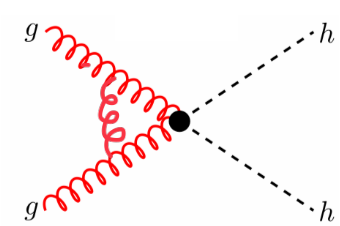
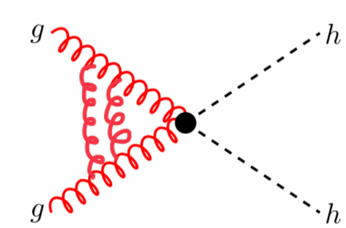
No internal mass : higher order perturbative computation are more feasible, in this limit



$K \approx 2$

Dawson,Dittmair, Spira ‘98



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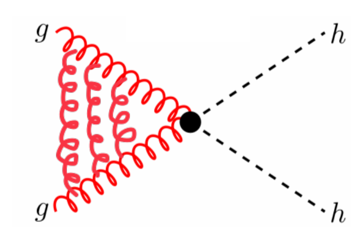

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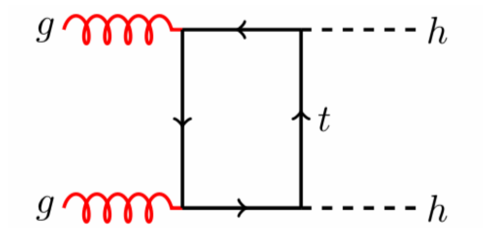

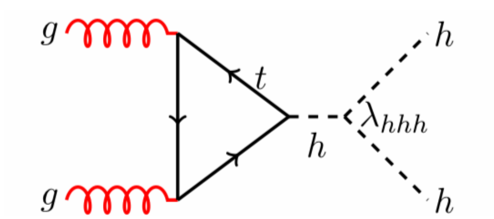

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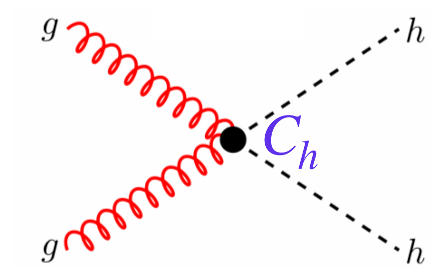

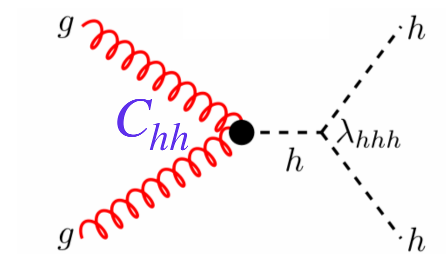

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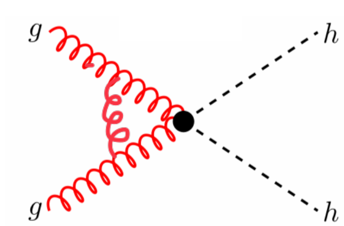

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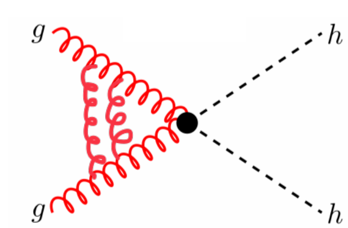

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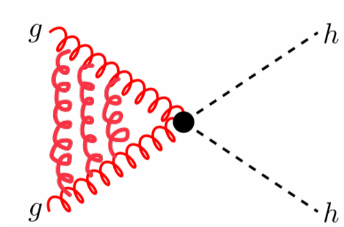

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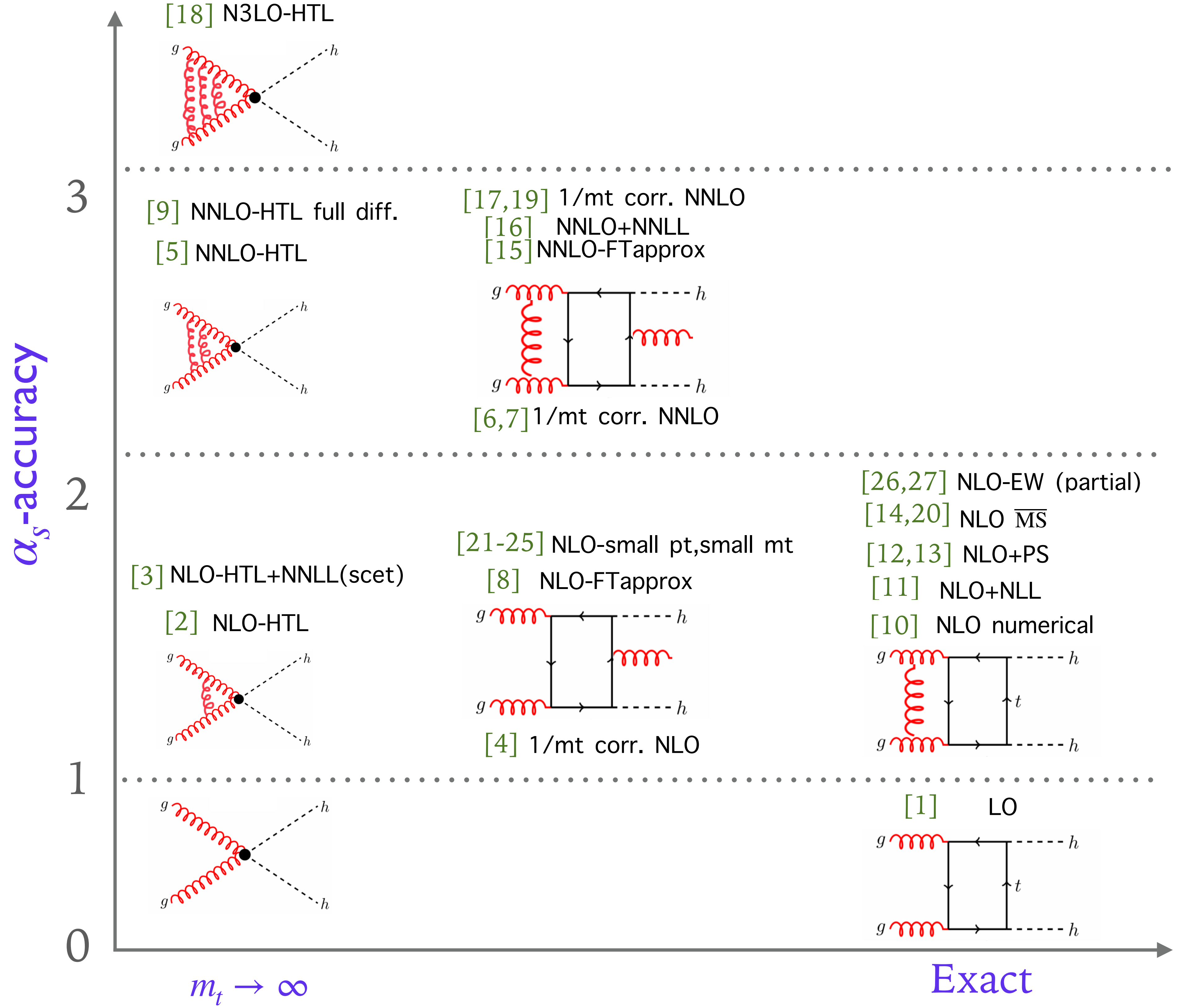
Chen, Li, Shao, Wang ‘19

Though useful, insufficient for phenomenological applications.

Numerous efforts to include the finite top mass corrections to this approximation

$gg \to HH$: SUMMARY



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[9] Florian, et.al '16
[10] Borowka, et.al '16, '16
[11] Ferreira, Pires '16
[12] Heinrich et.al '17
[13] Jones, Kuttimalai '17
[14] Baglio,et.al '18
[15] Grazzini,et.al '18
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[17] Davies, Steinhauser '19
[18] Chen, Li, Shao, Wang '19, '19
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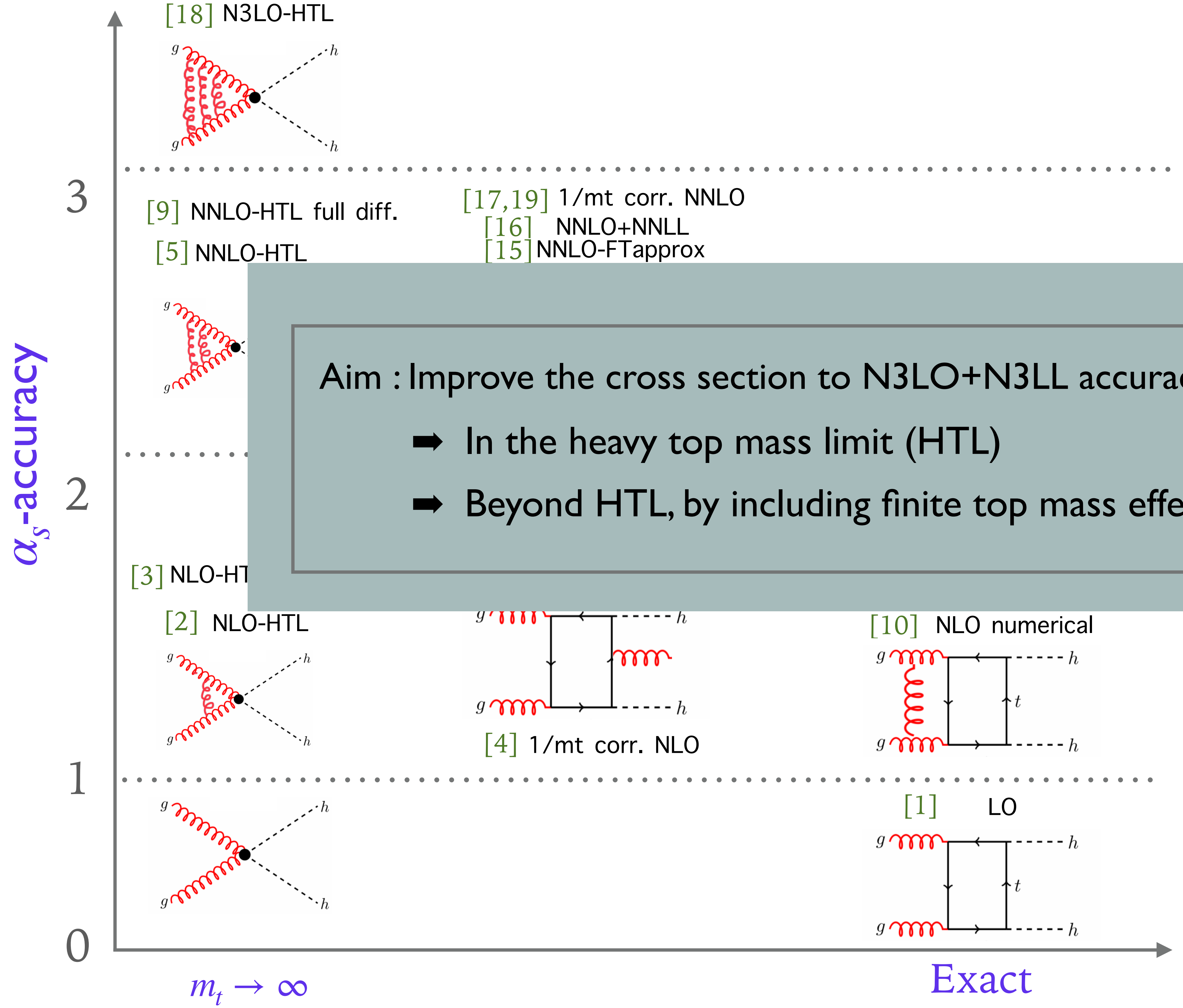

Aim : Improve the cross section to N3LO+N3LL accuracy

- ➡ In the heavy top mass limit (HTL)
- ➡ Beyond HTL, by including finite top mass effects

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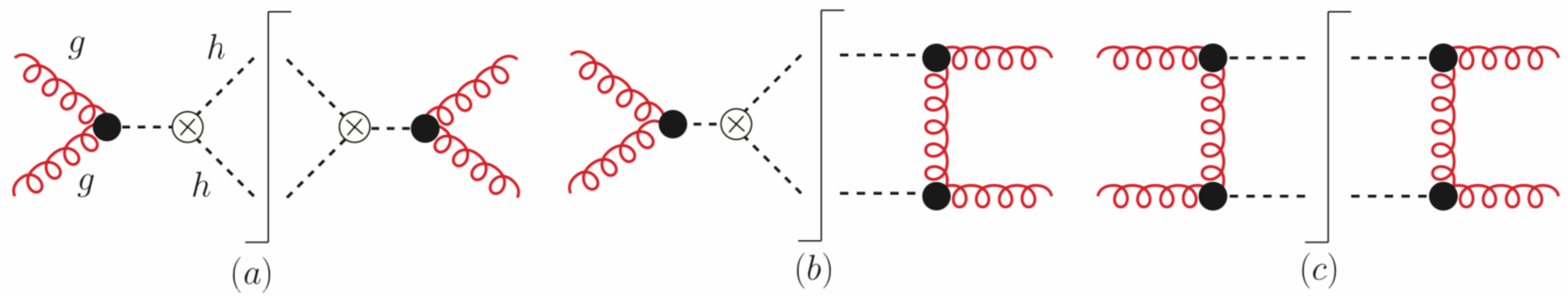
INFINITE TOP QUARK MASS LIMIT : OVERVIEW

- Breakdown to 3 channels : depending on number of effective vertices

$$d\sigma_{hh} = d\sigma_{hh}^{a} + d\sigma_{hh}^{b} + d\sigma_{hh}^{c}.$$



	LO	NLO	NNLO	N^{3}LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
class-a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
class-b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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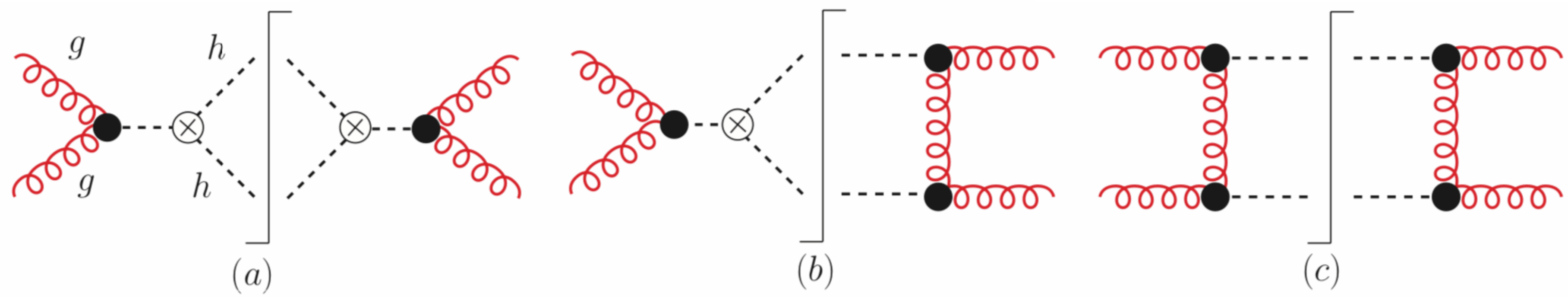

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at N^{3}LO

Class-a $\rightarrow$ N^{3}LO

Class-b $\rightarrow$ NNLO

Class-c $\rightarrow$ NLO

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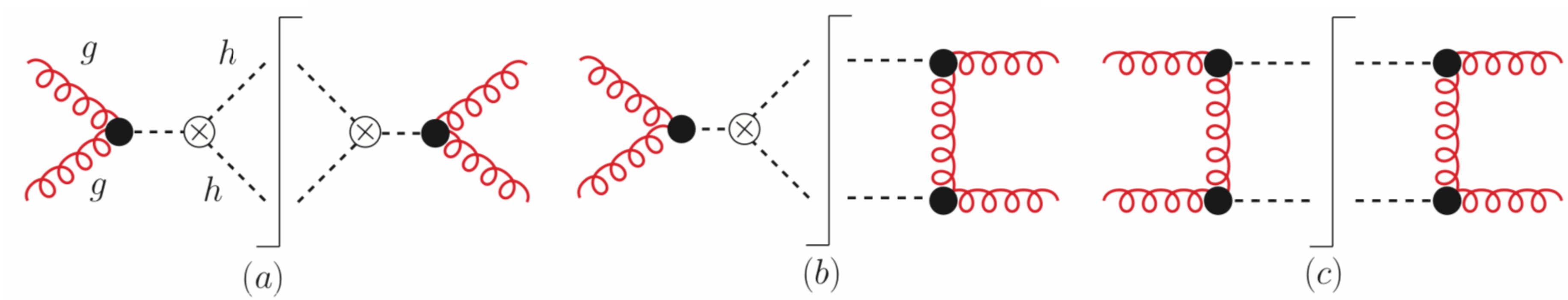

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at N^{3}LO

Class-a $\rightarrow$ N^{3}LO

Class-b $\rightarrow$ NNLO

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Chen, Li, Shao, Wang(PLB'20, JHEP'20)

- Class-a : share same topology as ggH at the large mass limit

$$\frac{d\sigma^{a}_{hh}}{dm_{hh}} = f_{h\to hh}\left(\frac{C_{hh}}{C_h} - \frac{6\lambda_{hhh}v^2}{m_{hh}^2 - m_h^2}\right)^2 \times \left(\sigma_h\big|_{m_h\to m_{hh}}\right)$$

Phase space factor mapping from ggH $\longrightarrow$ ggHH

$$f_{h\to hh} = \frac{\sqrt{m_{hh}^2 - 4m_h^2}}{16\pi^2 v^2}$$

$m_{hh} \longrightarrow$ Higgs pair invariant mass

at N^{3}LO

From iHixs2

Dulat, Lazopoulos, Mistlberger, CPC'18

Anastasiou, Duhr, Dulat, Herzog, Mistlberger PRL'15

INFINITE TOP QUARK MASS LIMIT : OVERVIEW

- **Class-b : to NNLO accuracy - use q_t-subtraction method** Catani & Grazzini PRL'07

$$d\sigma^b_{hh} = d\sigma^b_{hh}\Big|_{p_T^{hh}<p_T^{\rm veto}} + d\sigma^b_{hh}\Big|_{p_T^{hh}>p_T^{\rm veto}}$$

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q_t-resummation formalism

$$\frac{d\sigma^b_{hh}}{dp_T^{hh}} = H^b \otimes B_g \otimes B_g \otimes S \times \left(1 + \mathcal{O}\left(\frac{(p_T^{hh})^2}{Q^2}\right)\right)$$

Hard function

Banerjee, Borowka, Dhani, Gehrmann, Ravindran, JHEP'18

Process independent

TMD beam function

Soft functions

Gehrmann et al.PRL'12, JHEP'14; Luebbert et al., JHEP'16; Echevarria et al. JHEP'16; Luo et al., '19

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HH + jet at NLO for class-b

MG5_aMC@NLO

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- Class-c : to NLO accuracy - MG5_aMC@NLO

THRESHOLD RESUMMATION : OVERVIEW

- Resummation is relevant at production threshold, defined as $z = \frac{m_{hh}^2}{\hat{s}} \to 1$,
- Required due to threshold enhanced logarithms $\left(\frac{\ln(1-z)}{1-z}\right)_+$, arising from soft gluon emissions

$m_{hh} \longrightarrow$ Higgs pair invariant mass

$\hat{s} \longrightarrow$ partonic center of mass energy square

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$m_{hh} \longrightarrow \text{Higgs pair invariant mass}$

$\hat{s} \longrightarrow \text{partonic center of mass energy square}$

- At production threshold, the partonic cross section shows an exponential behaviour for the color singlet productions:

Naively : the partonic cross section,

Convoluted exponential

$$\hat{\sigma}_{hh}(m_{hh}^2, z) = C_0\left(m_{hh}^2\right)\delta(1-z) \quad \mathscr{C} \exp\left(\mathscr{Q}\left(m_{hh}^2, z\right)\right)$$

3-loop corrections

Virtual +
non-logarithmic soft contributions

Universal

Soft logarithms
Universal, depends only on initial partons

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- Resummation is convenient to perform in Mellin-N space, where the convolutions become normal products

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$z \to 1 \longrightarrow \mathrm{N} \to \infty$

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \to \frac{\ln^2(\overline{N}) + \zeta_2}{2} + \mathcal{O}\left(\frac{1}{\overline{N}}\right)$$

$$\overline{N} = N \exp(\gamma_E)$$

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- In the N-space, the master formula :

$$\bar{\omega} = \frac{\alpha_s}{2\pi}\beta_0 \log \overline{N}$$

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N-independent

Process dependent

N-independent

$f(\log N)$

THRESHOLD RESUMMATION : OVERVIEW

- Resummation is convenient to perform in Mellin-N space, where the convolutions become normal products

$$\Delta_{hh}^{res}(N) = \int_0^1 dz\ z^{N-1} \hat{\sigma}_{hh}(z, m_{hh}^2)$$

$z \to 1 \longrightarrow \mathrm{N} \to \infty$

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \to \frac{\ln^2(\overline{N}) + \zeta_2}{2} + \mathcal{O}\left(\frac{1}{\overline{N}}\right) \qquad \overline{N} = N \exp(\gamma_E)$$

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[Sterman]

[Catani, Trentedue]

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[Sterman]

[Catani, Trentedue]

Leading logarithm

LL : resum terms $\alpha_s^n \log^{n+1} N$ highest logarithms at each α_s order

THRESHOLD RESUMMATION : OVERVIEW

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[Sterman]

[Catani, Trentedue]

Next-to-leading logarithm

NLL : resum terms $\alpha_s^n \log^n N$

Next-to-highest logarithms at each α_s order

THRESHOLD RESUMMATION : OVERVIEW

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[Sterman]

[Catani, Trentedue]

Next-to-next-to-leading logarithm

NNLL : resum terms $\alpha_s^n \log^{n-1} N$

Next-to-next-to-highest logarithms at each α_s order

and so on…

THRESHOLD RESUMMATION : OVERVIEW

- Resummation schemes : freedom to choose some N-independent part inside/outside the exponent, up to a given logarithmic accuracy
- Depending on that we propose 4 schemes :

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  - $\overline{\mathrm{N}}_1$ scheme :

N-independent term outside exponent

$$\Delta^{\mathrm{res}}_{\overline{N}_1}(N, m_{hh}^2, \mu_F^2)\Big|_{\mathrm{N^kLL}} = \left(\overline{\tilde{g}}_{0,0} + \alpha_s \overline{\tilde{g}}_{0,1} + \alpha_s^2 \overline{\tilde{g}}_{0,2} + \cdots\right)\Big|_{\mathrm{N^kLO}} \exp\left(g_1(\bar{\omega})\ \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s\ g_3(\bar{\omega}) + \cdots\right)$$

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- $\overline{\mathrm{N}}_2$ scheme :

Part of N-independent term, coming from Mellin transformation, is kept within the exponent

$$\Delta^{\mathrm{res}}_{\overline{N}_2}(N, m_{hh}^2, \mu_F^2)\Big|_{\mathrm{N^kLL}} = \left(\bar{g}_{0,0} + \alpha_s \bar{g}_{0,1} + \alpha_s^2 \bar{g}_{0,2} + \cdots\right)\Big|_{\mathrm{N^kLO}} \exp\left(\tilde{C}_{0,\zeta_2}(\alpha_s) + g_1(\bar{\omega})\ \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s\ g_3(\bar{\omega}) + \cdots\right)$$

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Resum $\log N$ terms instead of $\log \overline{N}$, with $\overline{N} = N \exp(\gamma_E)$

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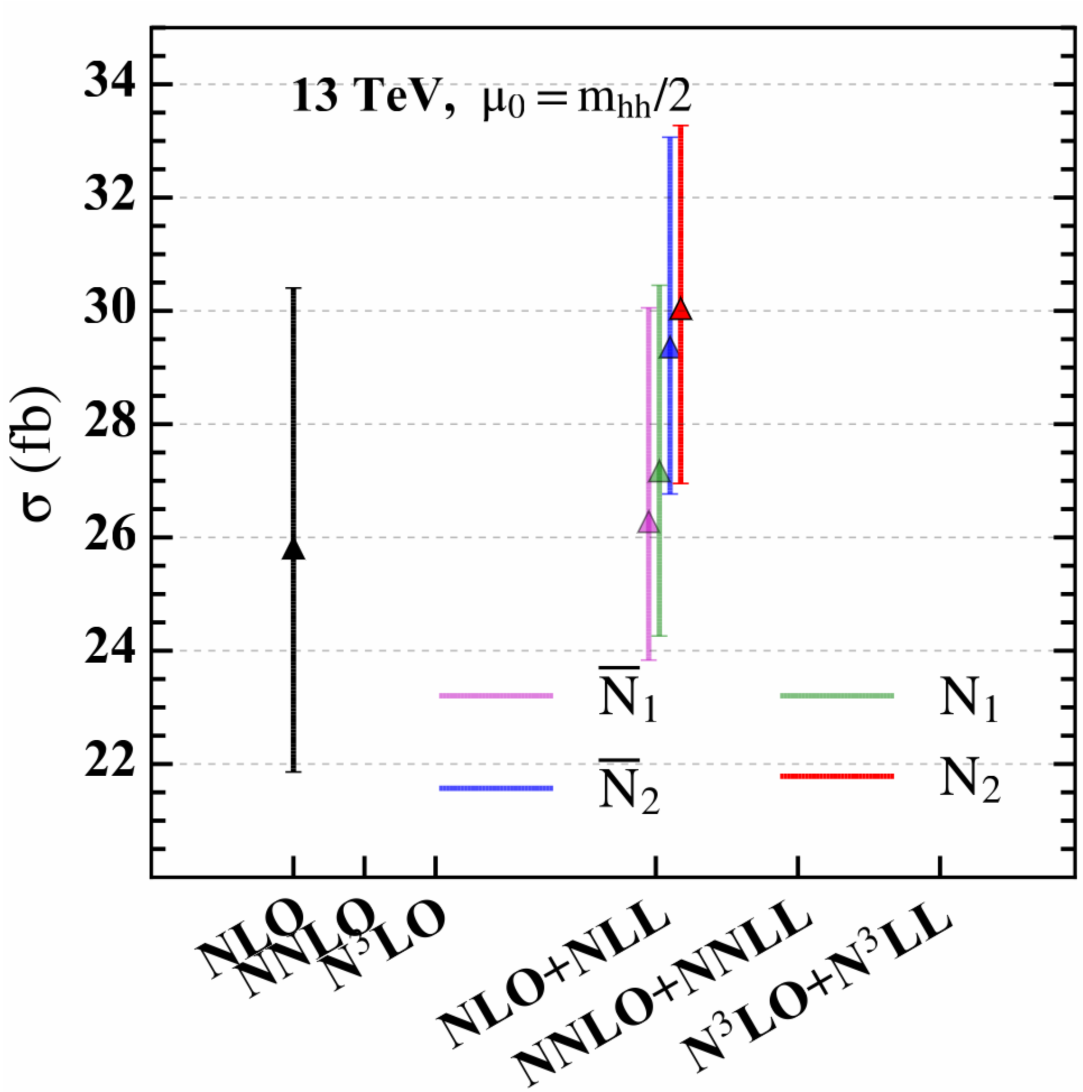
  - N_2 scheme :

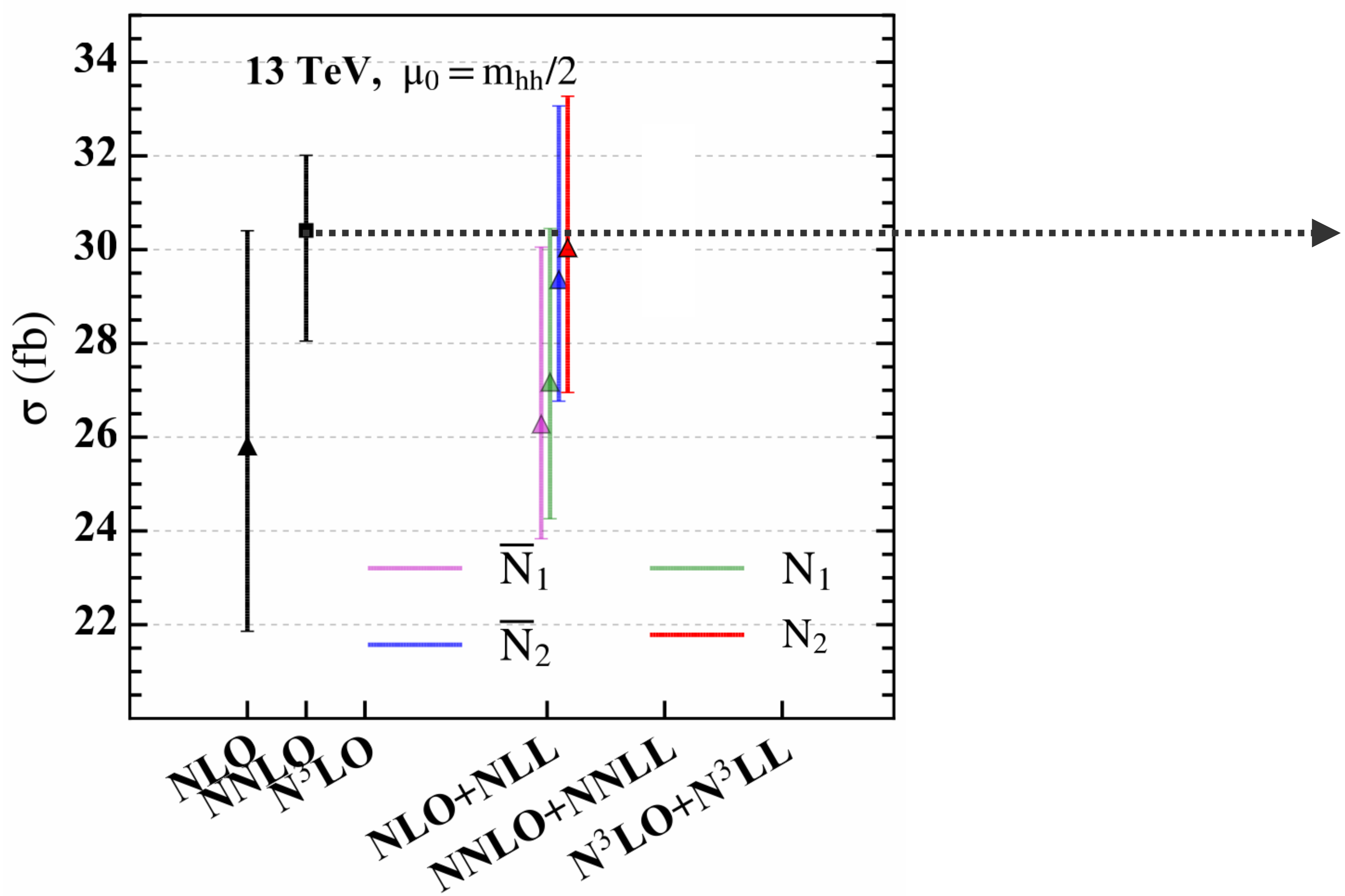
N_2- scheme with resuming $\log N$ terms

$$\Delta^{\mathrm{res}}_{N_2}(N, m_{hh}^2, \mu_F^2)\Big|_{\mathrm{N^kLL}} = \left(g_{0,0} + a_s g_{0,1} + a_s^2 g_{0,2} + \cdots\right)\Big|_{\mathrm{N^kLO}} \exp\Big(\tilde{C}_{0,\zeta_2}(a_s) + g_1(\omega) \ \ln N + g_2(\omega) + a_s \ g_3(\omega) + \cdots\Big)$$

RESUMMATION - SCHEME AMBIGUITIES

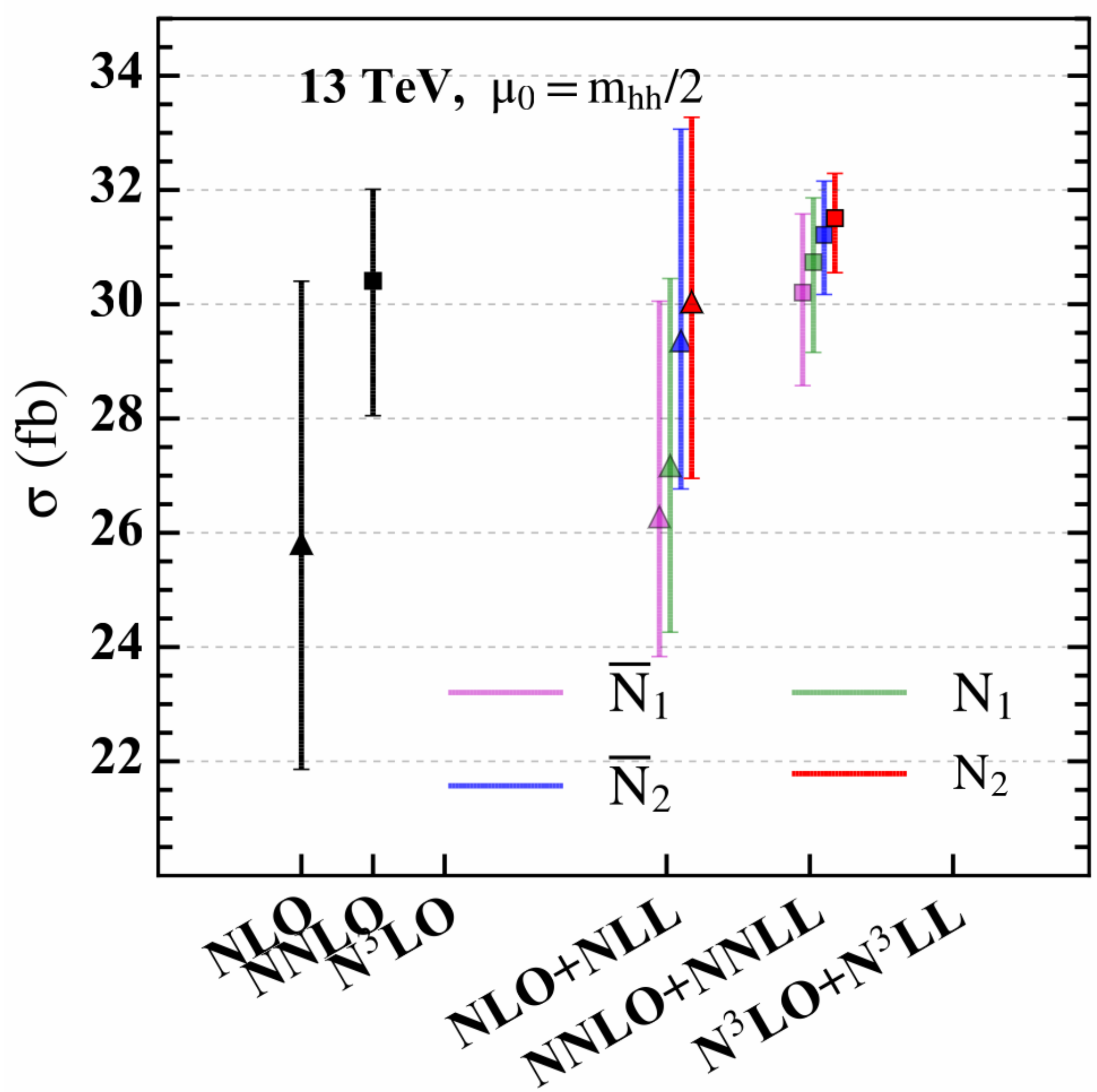


RESUMMATION – SCHEME AMBIGUITIES



NLO+NLL is close to NNLO for N_2 & $\overline{N}_2$ schemes

RESUMMATION – SCHEME AMBIGUITIES



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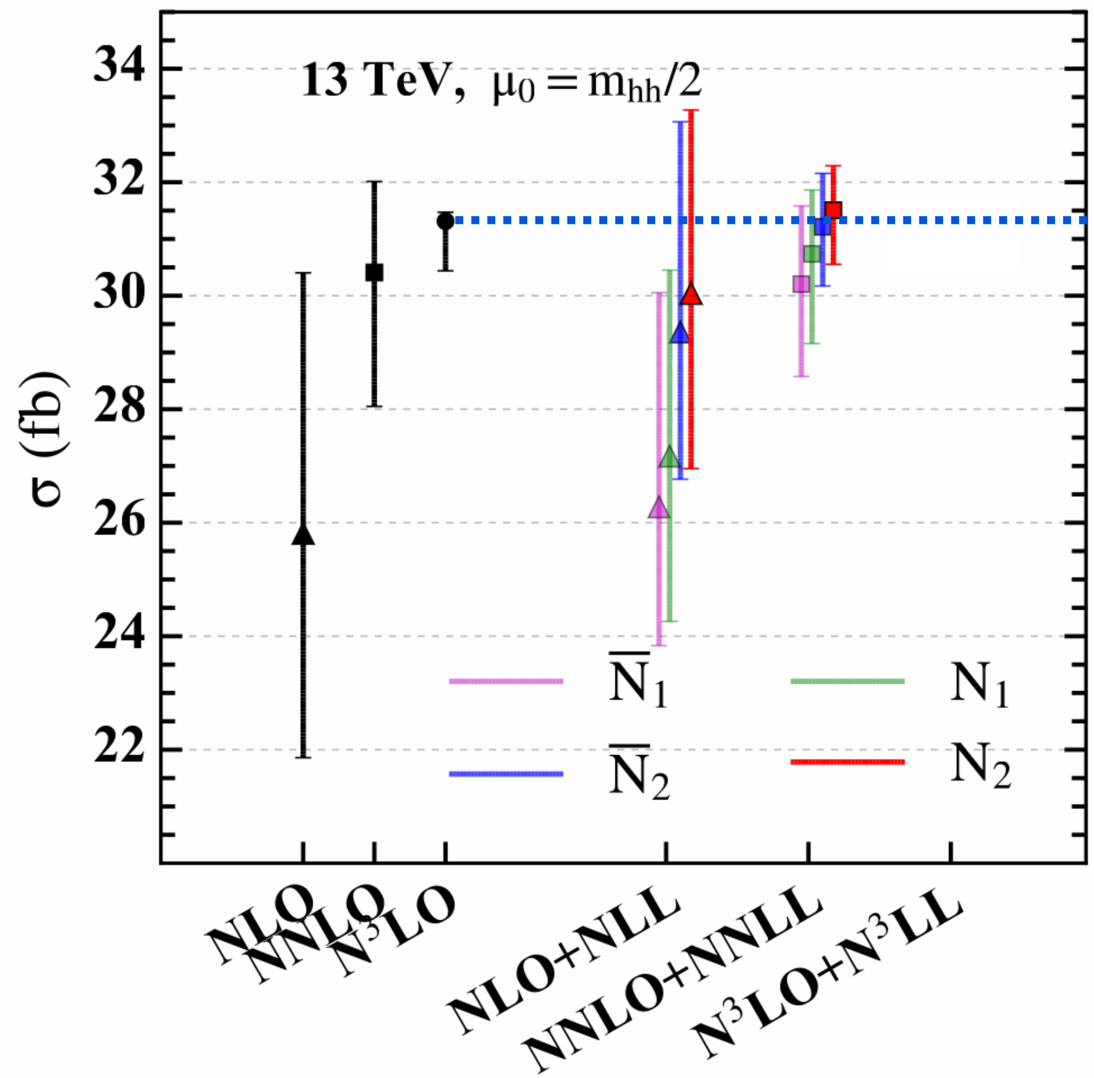
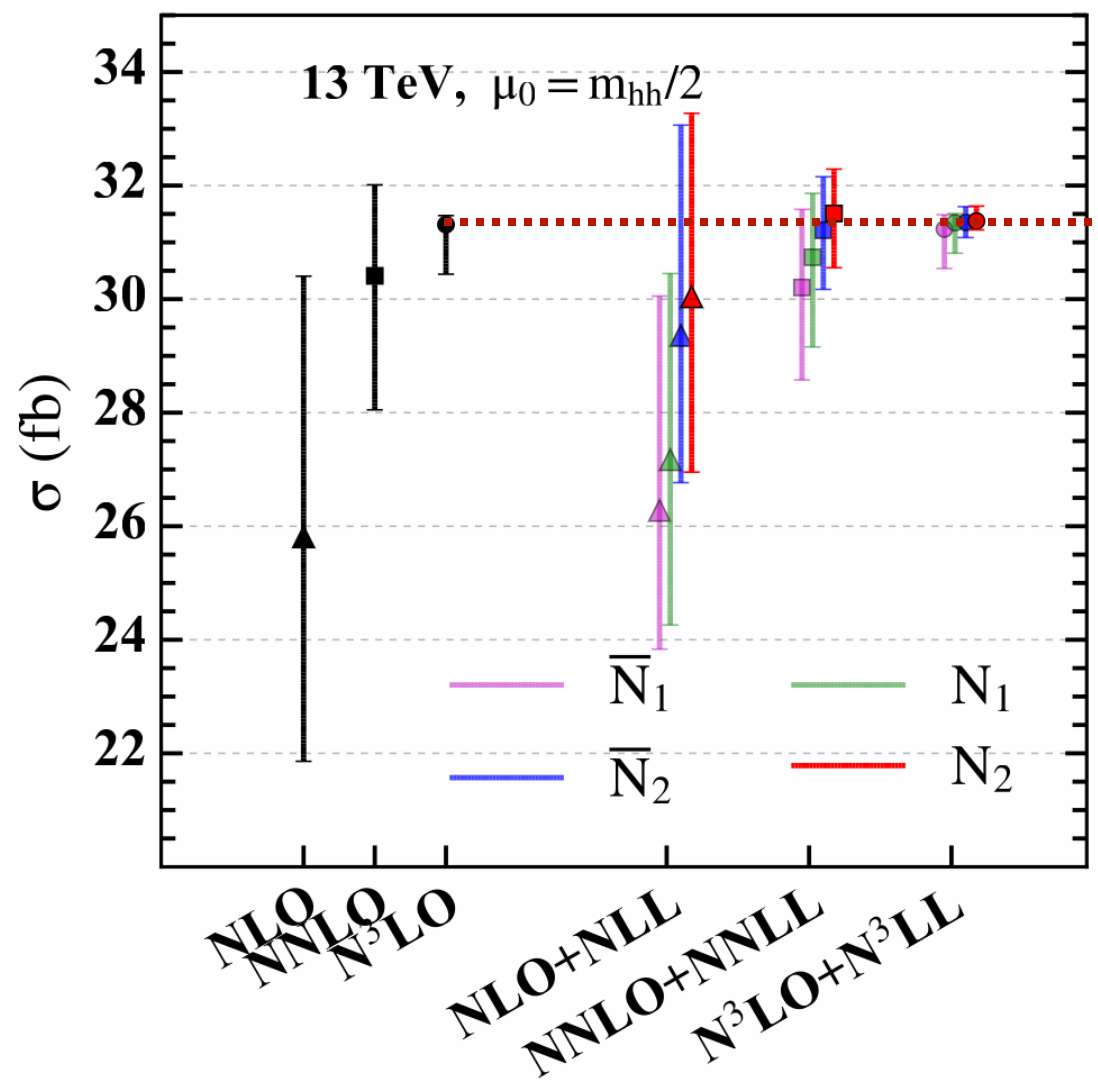


N2LO+N2LL very close to N3LO in N_2 & $\bar{N}_2$ schemes

RESUMMATION – SCHEME AMBIGUITIES

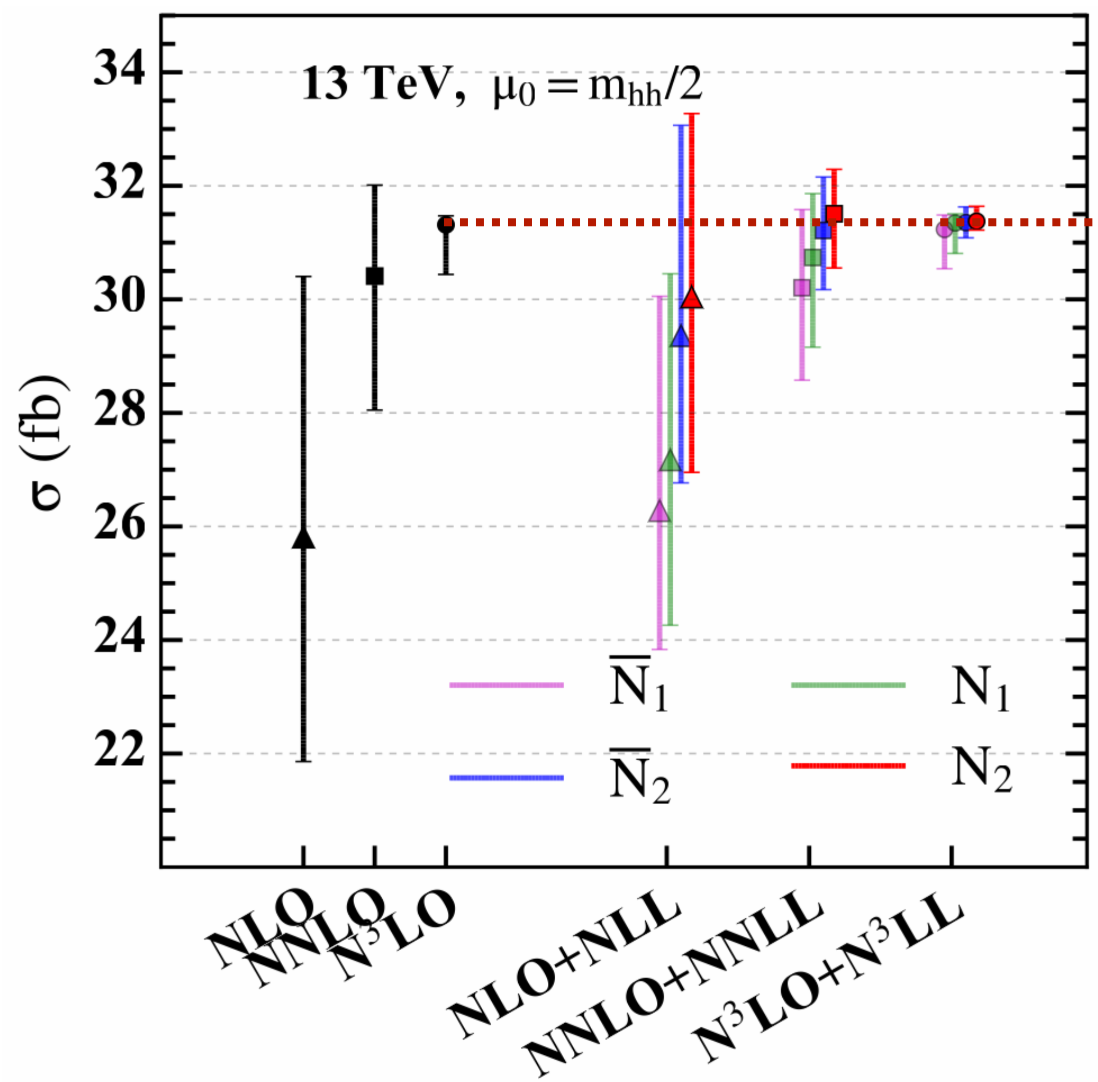


Good perturbative convergence at N3LO

N_2 & $\overline{N}_2$ *are on equal footing.*
We choose to work with $\overline{N}_2$ *scheme.*
Symmetric error bar.

$N^3LO + N^3LL$ *in* $\overline{N}_2$ *scheme could be extrapolated to predict* N^4LO

RESUMMATION - SCHEME AMBIGUITIES



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- **Our results at NNLL in N_1 scheme are verified with 1505.07122** Florian, Mazzitelli JHEP'15
- **With same numerical setup, the resum results for $gg \to H$ are verified with 1603.08000** Bonvini, Marzani, Muselli, Rottoli JHEP'16

INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section

in unit of fb central scale $\mu_0 = \frac{m_{hh}}{2}$

$\sqrt{s}$ [TeV]	Order k	N^{k}LO	N^{k}LO+N^{k}LL	
			N$_2$ scheme	$\overline{\text{N}}_2$ scheme
13	0	$13.80^{+31\%}_{-22\%}$	$16.01^{+32\%}_{-23\%}$	$21.02^{+36\%}_{-24\%}$
	1	$25.81^{+18\%}_{-15\%}$	$30.04^{+10.8\%}_{-10.3\%}$	$29.36^{+12.6\%}_{-8.8\%}$
	2	$30.41^{+5.3\%}_{-7.8\%}$	$31.51^{+2.5\%}_{-3.0\%}$	$31.21^{+3.0\%}_{-3.3\%}$
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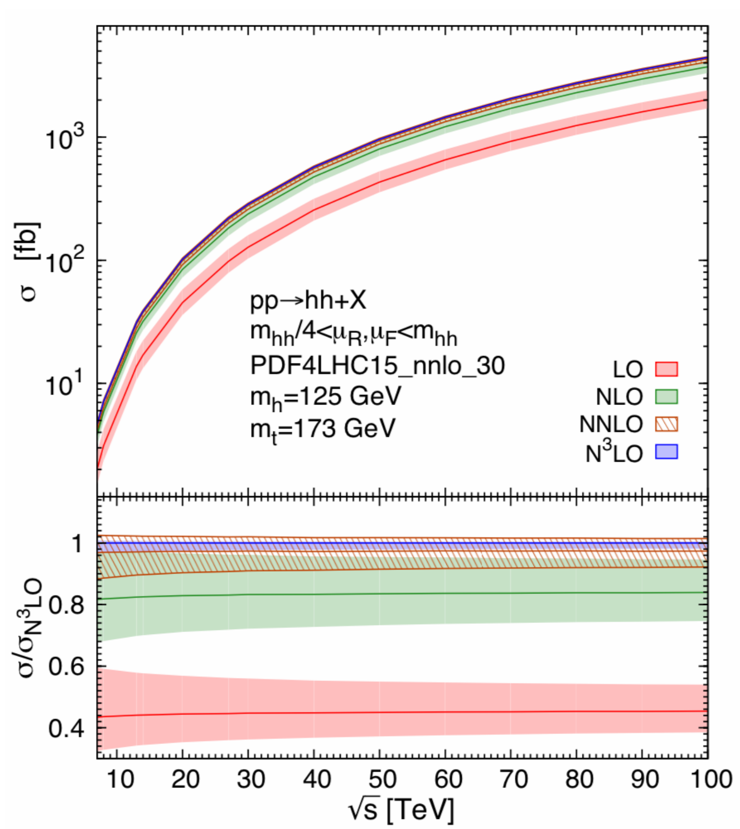
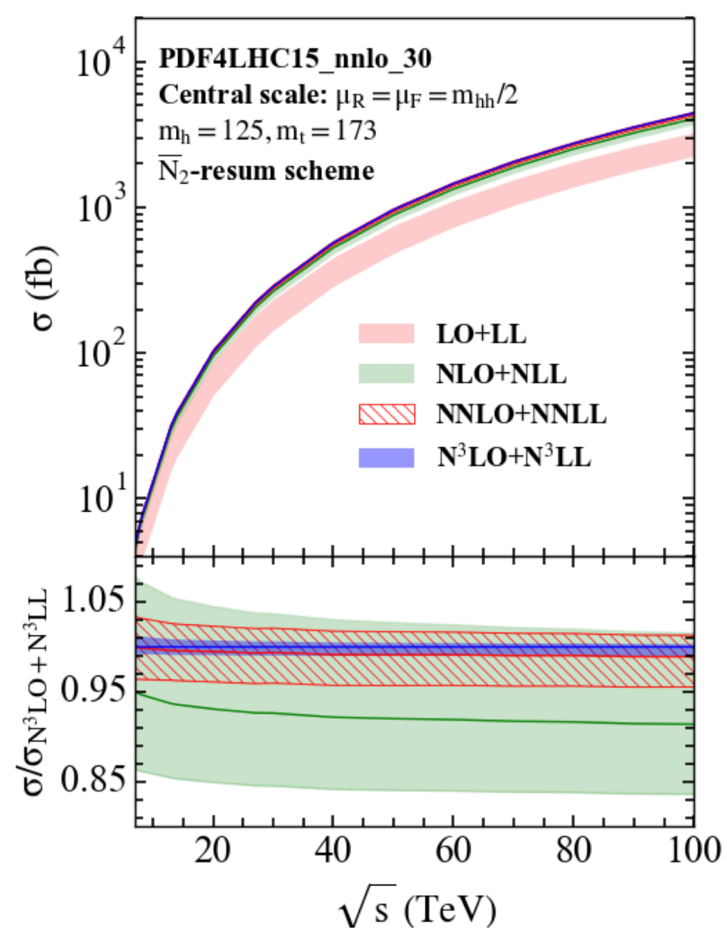
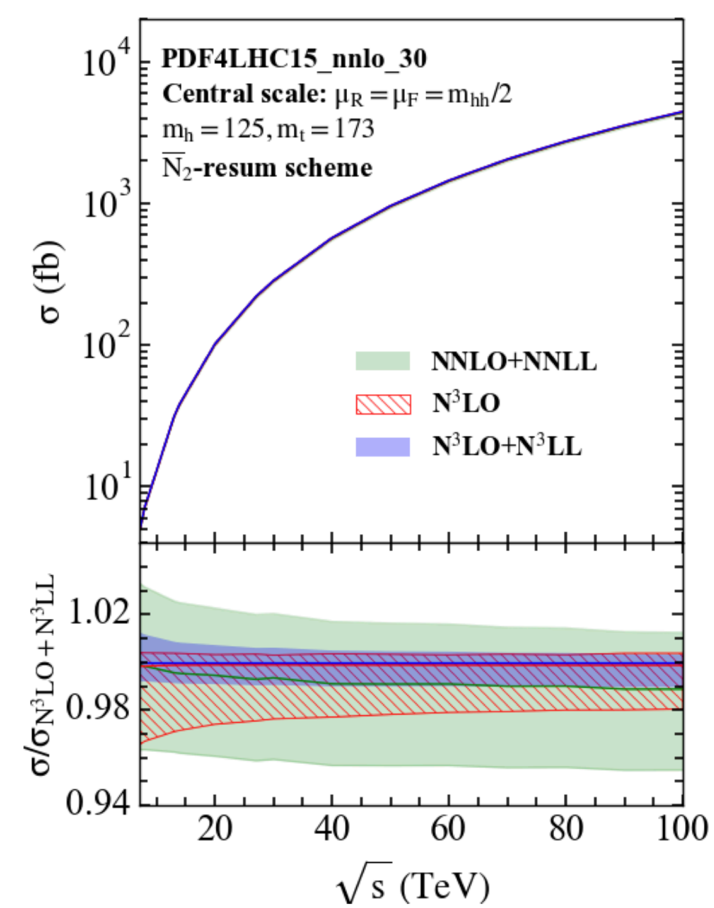


Chen, Li, Shao,
Wang(PLB'20, JHEP'20)





INFINITE TOP QUARK MASS LIMIT : RESULTS

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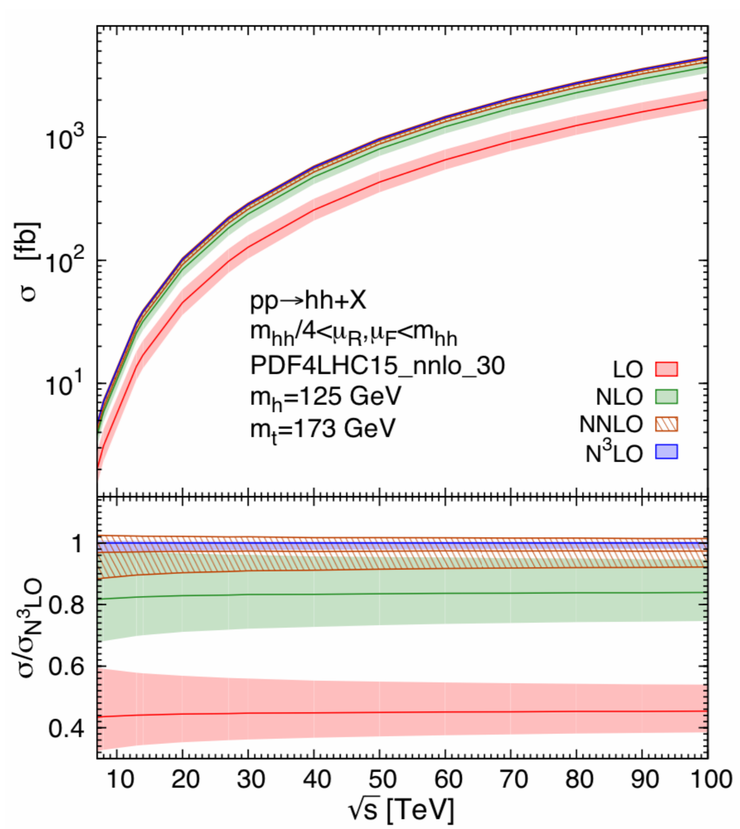
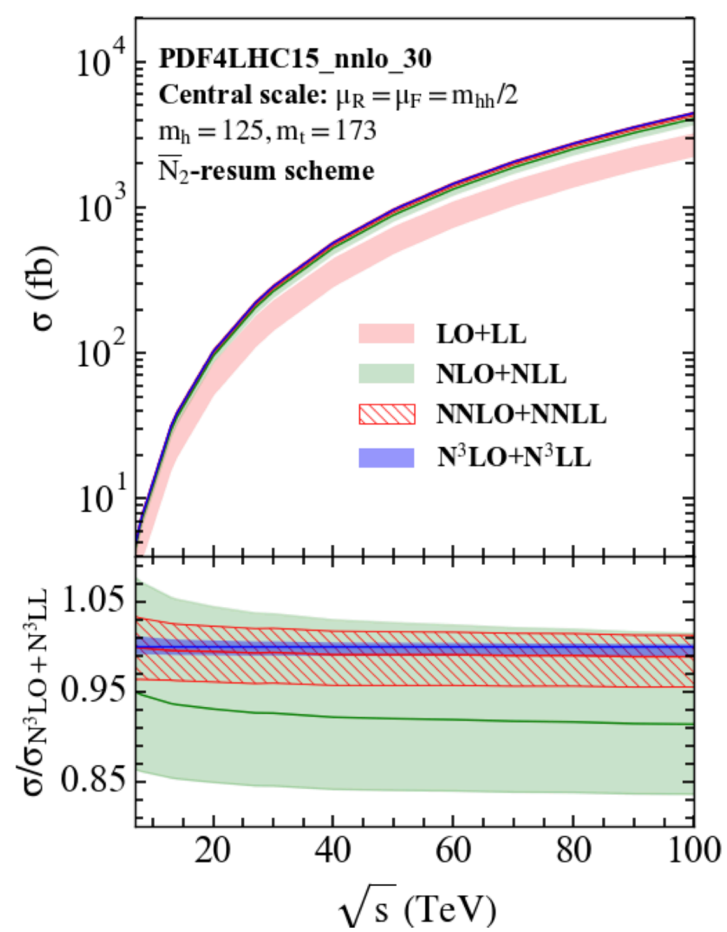
➤ QCD corrections

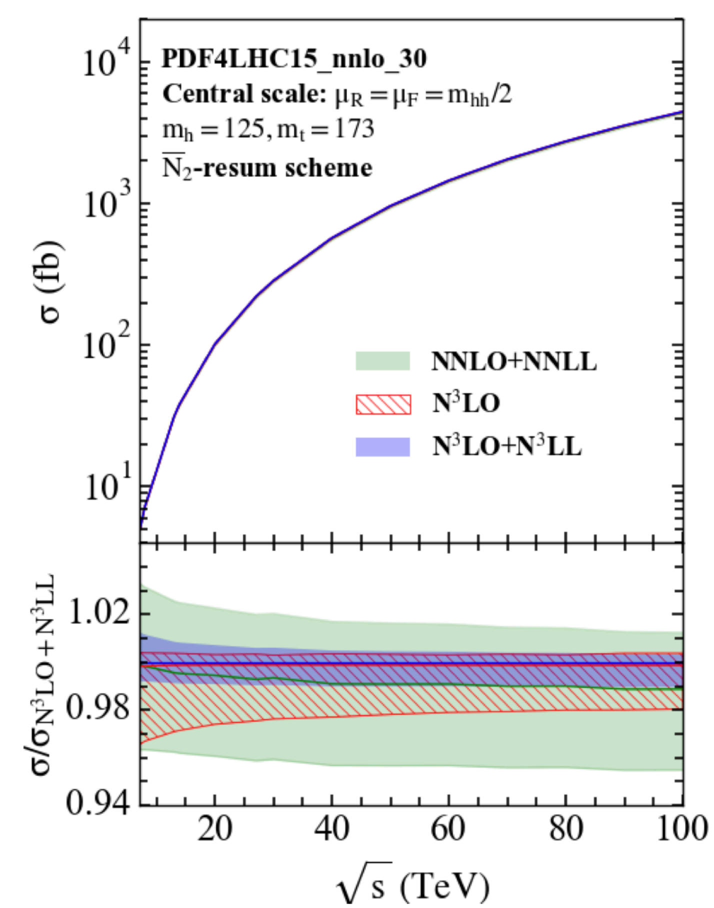
at (13, 100) TeV

- NNLO $\rightarrow$ $\mathrm{N}^3\mathrm{LO}$: (3% , 2.7%)
- NNLO $\rightarrow$ NNLO + NNLL : (3% , 1.7%)
- NNLO + NNLL $\rightarrow$ $\mathrm{N}^3\mathrm{LO}$:(0.3% , 1.02%)
- $\mathrm{N}^3\mathrm{LO}$ $\rightarrow$ $\mathrm{N}^3\mathrm{LO}$ + $\mathrm{N}^3\mathrm{LL}$: (0.13% , 0.11%)



Chen, Li, Shao, Wang(PLB'20, JHEP'20)





INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section

in unit of fb central scale $\mu_0 = \frac{m_{hh}}{2}$

$\sqrt{s}$ [TeV]	Order k	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$	
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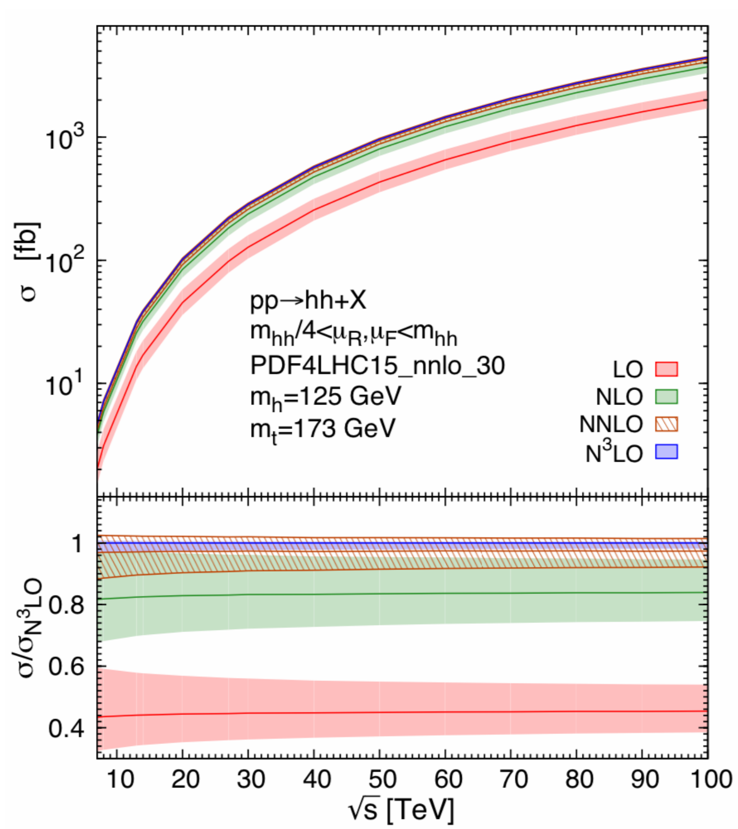
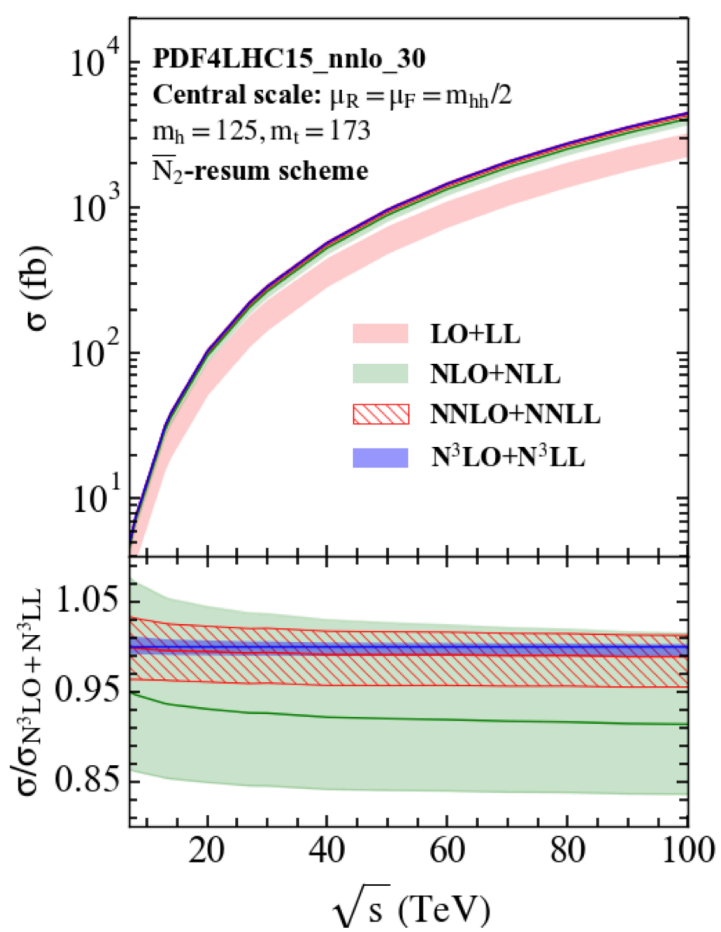
➤ Scale reduction to percent-level

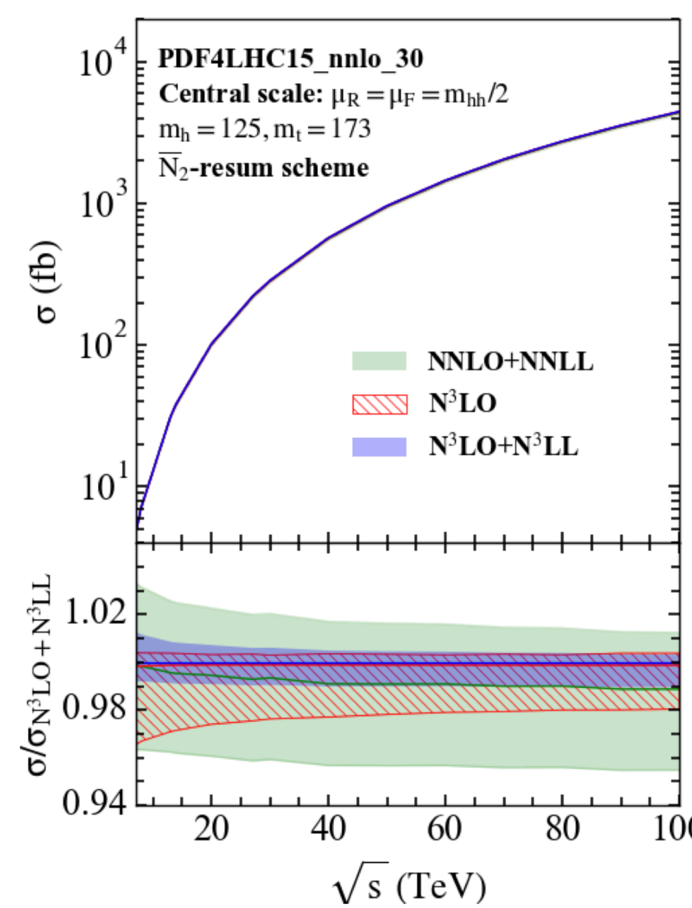
- $\mathrm{N^3LO} \rightarrow \mathrm{N^3LO} + \mathrm{N^3LL}$: factor 2 reduction
- $\mathrm{NNLO} + \mathrm{NNLL} \rightarrow \mathrm{N^3LO} + \mathrm{N^3LL}$: factor 4 reduction

Scale uncertainty: sub-percent level



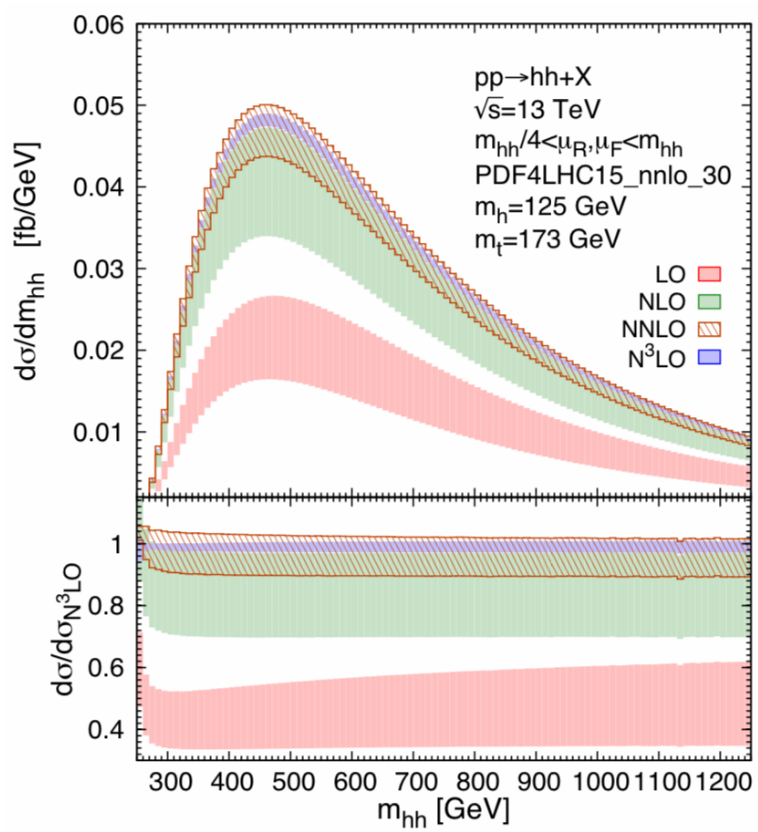
Chen, Li, Shao, Wang(PLB'20, JHEP'20)





INFINITE TOP QUARK MASS LIMIT : RESULTS

Invariant Mass distributions



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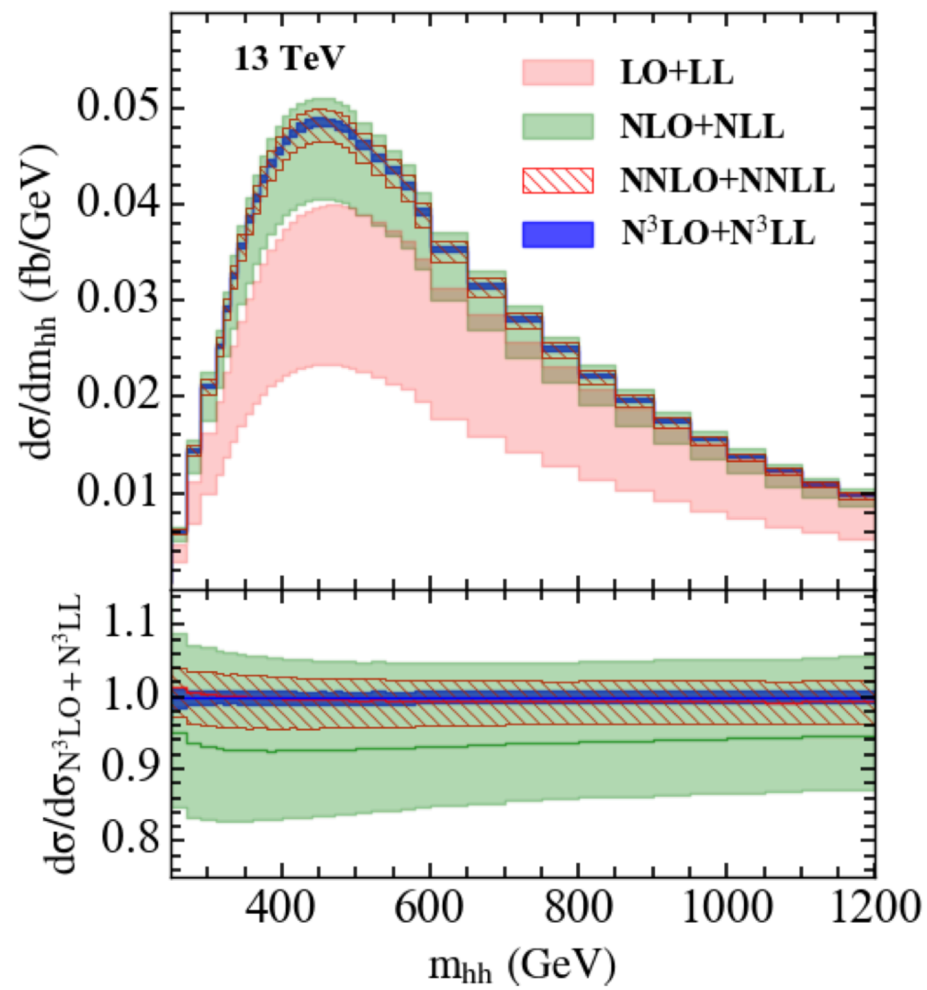
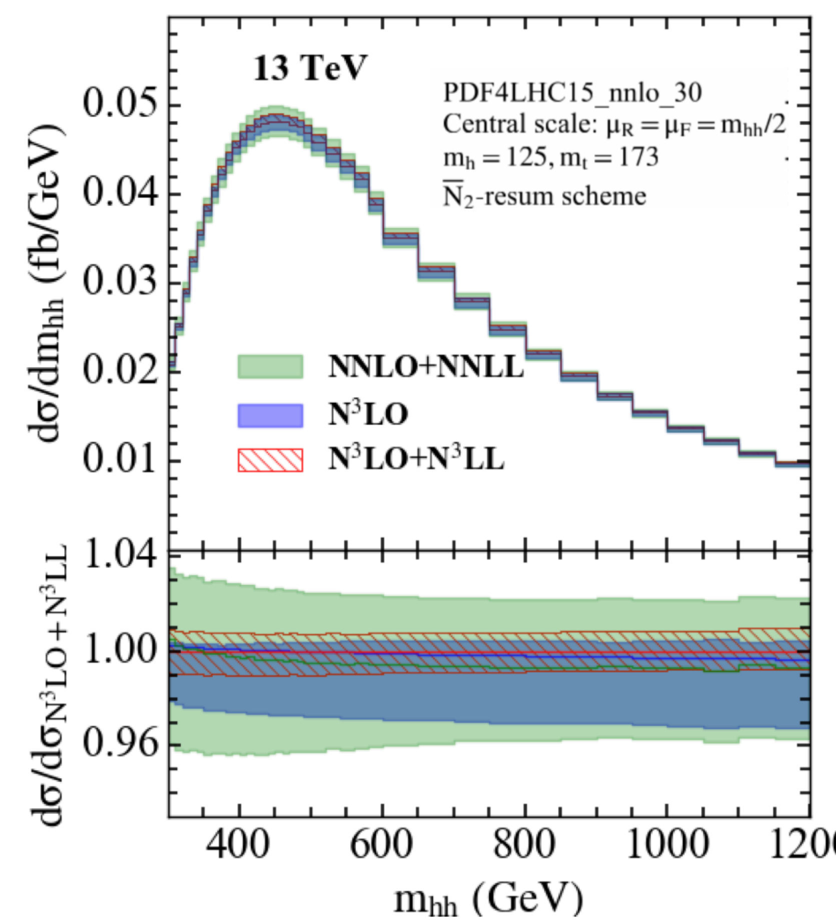
- Shape of the distribution is almost unchanged
- Significant scale reductions with good perturbative convergence

Inclusion of higher order stabilises the invariant mass distribution





BEYOND INFINITE TOP QUARK LIMIT

- To improve the results at large m_t-limit, they are combined with the finite top quark mass effects.

Not unique! *Different approximations*

- For N^{3}LO following approximations are considered

With $\left\{ \begin{array}{ll} \text{N}^k\text{LO} & \text{infinite top-quark mass limit} \\ \text{N}^l\text{LO} & \text{full top-quark mass dependence} \end{array} \right.$ $k > l$

Chen, Li, Shao, Wang (JHEP'20)

BEYOND INFINITE TOP QUARK LIMIT

- To improve the results at large m_t-limit, they are combined with the finite top quark mass effects.

Not unique! *Different approximations*

- For N^3LO following approximations are considered

With $\left\{ \begin{array}{ll} N^kLO & \text{infinite top-quark mass limit} \\ N^lLO & \text{full top-quark mass dependence} \end{array} \right.$ $k > l$

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Only improve leading mt expansion term

- $\mathbf{N^kLO \oplus N^lLO_{m_t}}$: $d\sigma^{\mathbf{N^kLO \oplus N^lLO_{m_t}}} = d\sigma^{\mathrm{N}^l\mathrm{LO}}_{m_t} + d\sigma^{\mathrm{N}^k\mathrm{LO}}_{m_t \to \infty} - d\sigma^{\mathrm{N}^l\mathrm{LO}}_{m_t \to \infty}$

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NLO-improved. Same K factor for mass correction

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$N^3LO \otimes NLO_{m_t}$ *: most accurate out of three above approximations*

BEYOND INFINITE TOP QUARK LIMIT : RESULTS

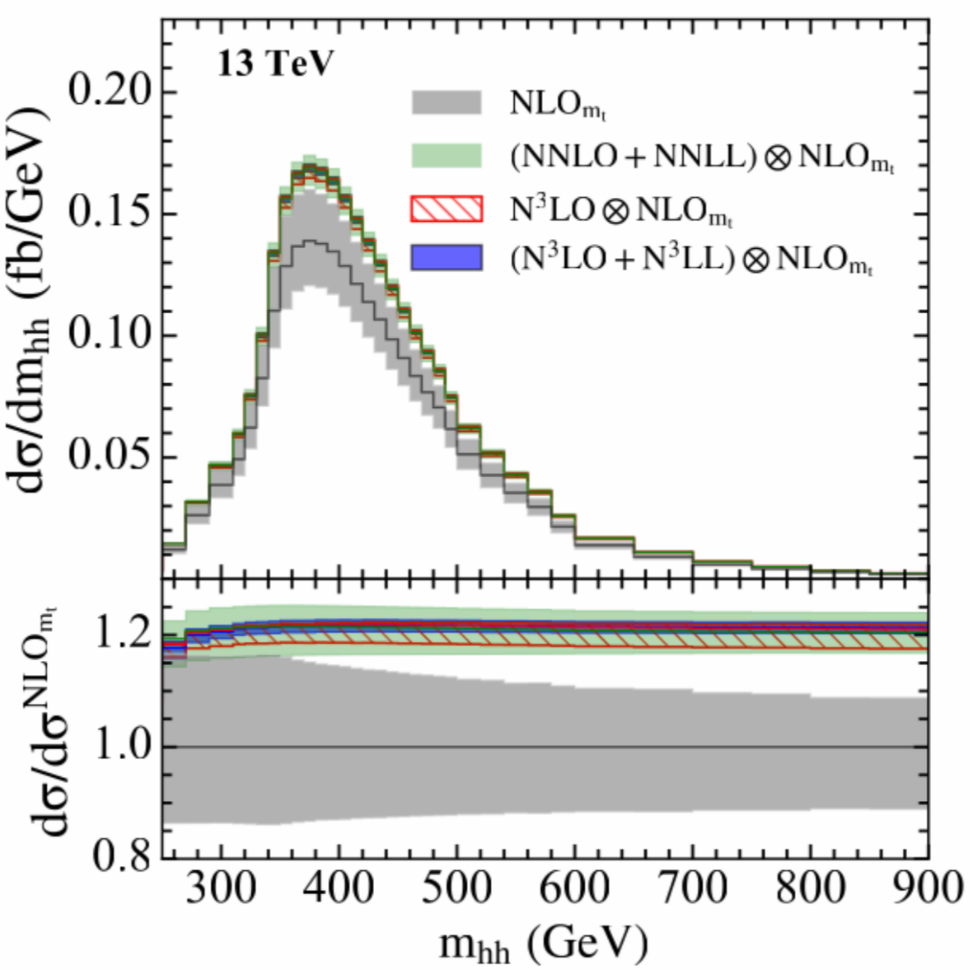
N3LO+N3LL

For N3LO+N3LL, we consider only the NLO-improved approximation:

m_t-dependent full NLO (NLO_{m_t}) is obtained using POWHEG.

Heinrich et al. JHEP'19

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO_{m_t}	$27.56^{+13.9\%}_{-12.7\%}$	$32.64^{+13.5\%}_{-12.47\%}$	$126.1^{+11.5\%}_{-10.4\%}$	$1119^{+10.7\%}_{-9.9\%}$
$(NNLO + NNLL) \otimes NLO_{m_t}$	$33.33^{+3.0\%}_{-3.3\%}$	$39.42^{+3.0\%}_{-3.4\%}$	$150.8^{+2.7\%}_{-3.4\%}$	$1320^{+2.4\%}_{-3.4\%}$
$N^3LO \otimes NLO_{m_t}$	$33.43^{+0.50\%}_{-2.8\%}$	$39.56^{+0.50\%}_{-2.7\%}$	$151.7^{+0.46\%}_{-2.3\%}$	$1333^{+0.51\%}_{-1.8\%}$
$(N^3LO + N^3LL) \otimes NLO_{m_t}$	$33.47^{+0.88\%}_{-0.85\%}$	$39.60^{+0.85\%}_{-0.87\%}$	$151.9^{+0.63\%}_{-0.94\%}$	$1335^{+0.35\%}_{-1.0\%}$



- **Enhancement :** At 13 TeV
  - 21% for $NLO_{m_t} \rightarrow (NNLO + NNLL) \otimes NLO_{m_t}$
  - 0.4% for $\rightarrow (NNLO + NNLL) \otimes NLO_{m_t} \rightarrow (N^3LO + N^3LL) \otimes NLO_{m_t}$
- **Scale uncertainty :**
  - $(NNLO + NNLL) \otimes NLO_{m_t} \sim 3\%$
  - $(N^3LO + N^3LL) \otimes NLO_{m_t}$: sub-percent level

SUMMARY & OUTLOOK

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- Observations :
    - QCD corrections: 3% improvement in central value at N3LO from NNLO, scale reduce by factor > 4!
    - Further reduction by factor 2 when we include N3LL to N3LO corrections.
    - $N^3LO + N^3LL$ achieve sub-percent level scale uncertainty
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- To improve the finite top quark mass corrections, we reweight the NLO_{m_t} with higher order K-factors:
  - $N^3LO \otimes NLO_{m_t}$ captures the best scale uncertainty, with 3% improvement in QCD corrections
  - $\left(N^3LO + N^3LL\right) \otimes NLO_{m_t}$ captures sub-percent level scale uncertainty.

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- Further improvement in the results could be achieved by combining our results with the most advanced finite top mass approximation - $\mathrm{NNLO_{FT}}$

$$\frac{\left(\mathrm{N^3LO + N^3LL}\right)}{\mathrm{NNLO}} \times \mathrm{NNLO_{FT}} = 31.79^{+0.88\%}_{-0.85\%}$$

At 13 TeV

Scale reduction by factor > 4

$$\mathrm{NNLO_{FT}} : 31.05^{+2.2\%}_{-5.0\%}$$

5% missing top mass corrections

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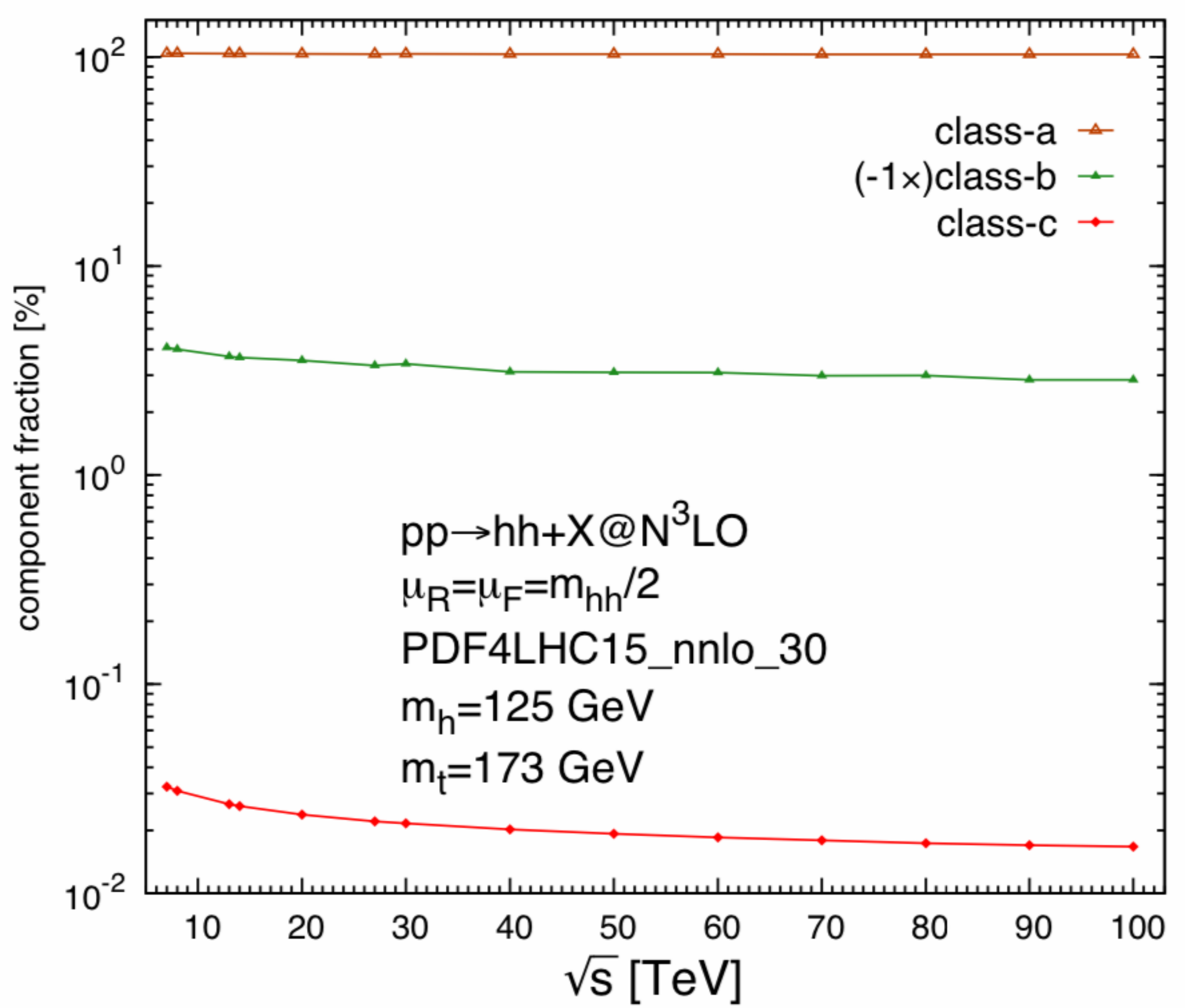
THANKS FOR THE ATTENTION !

Back up slides

CLASS A,B,C



class-a
(-1×)class-b
class-c
pp→hh+X@N3LO
μR=μF=mhh/2
PDF4LHC15_nnlo_30
mh=125 GeV
mt=173 GeV
component fraction [%]
√s [TeV]
10
20
30
40
50
60
70
80
90
100

INTEGRAL REPRESENTATION – ALL ORDER STRUCTURE

- With the knowledge on the structure, we can formulate an Integral representation for Coefficient function, which gives an understanding on the all order structure.

$$\Delta_c(q^2,\mu_R^2,\mu_F^2,z) = C_0^c(q^2,\mu_R^2,\mu_F^2)\ \mathscr{C}\exp\left(\int_{\mu_F^2}^{q^2(1-z)^2}\frac{d\lambda^2}{\lambda^2}P_{cc}(a_s(\lambda^2),z) + \mathscr{Q}^c(a_s(q^2(1-z)^2),z)\right)$$

$\mathscr{C}$ **denotes convoluted exponential.**

- C_0 is proportional to $\delta(1-z)$. Finite part after cancelling of poles between F & S.

- The integrant is the finite part after cancellation poles between splitting kernels and soft collinear function

$$P'_{cc}\left(z,a_s(\mu_F^2)\right) = 2\left[A^c(a_s(\mu_F^2))\mathcal{D}_0(z) + C^c(a_s(\mu_F^2))\ln(1-z) + D^c(a_s(\mu_F^2))\right]$$

- The finite contribution comes completely from soft-collinear function

$$\mathscr{Q}^c(a_s(q^2(1-z)^2),z) = \left(\frac{1}{1-z}2\mathscr{G}^{SV,c}(a_s(q^2(1-z)^2))\right)_+ + 2\mathscr{G}^{NSV,c}(a_s(q^2(1-z)^2),z)$$

THRESHOLD RESUMMATION : OVERVIEW

- Perform Inverse mellin transfrom numerically to get the real space cross section
- To avoid double counting, Matching procedure :

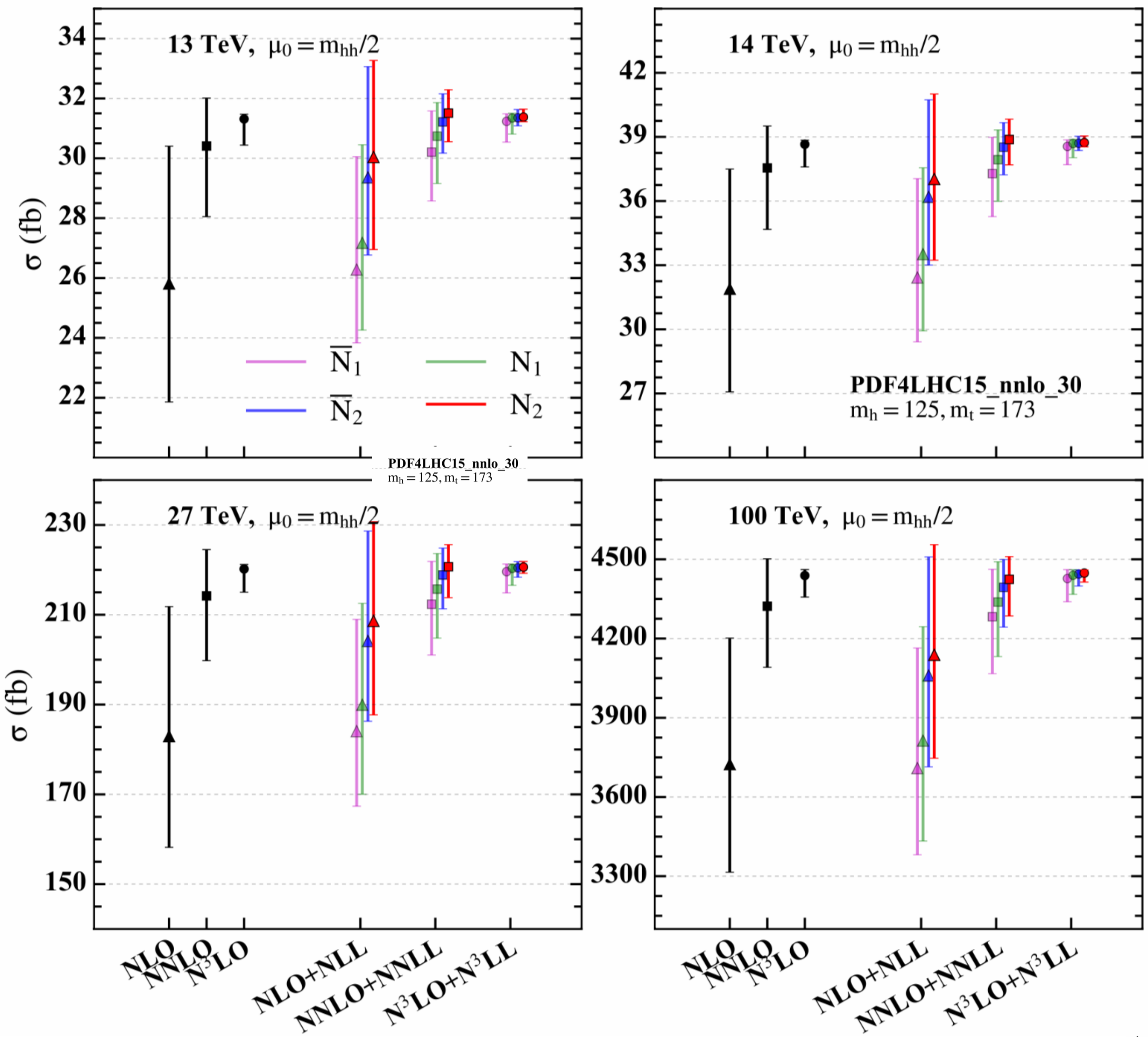
$$\sigma^{\mathrm{N^3LO+N^3LL}} = \left\{ \sigma^{\mathrm{N^3LL}} - \sigma^{\mathrm{N^3LL}} \Big|_{\mathcal{O}(\alpha_s^5)} \right\} + \sigma^{\mathrm{N^3LO}}$$

↓

Improves the predictions with missing higher order logarithmic terms.

INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section



at 14 TeV

$$\sigma_{\mathrm{N^3LO+N^3LL}} = 38.70 \begin{pmatrix} +0.85\% \\ -0.87\% \end{pmatrix}_{\mathrm{scale}} \begin{pmatrix} +0.08\% \\ -0.39\% \end{pmatrix}_{\mathrm{scheme}} \mathrm{fb}$$

$\sqrt{s}$ [TeV]	Order k	N^kLO	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$			
			N_1 scheme	N_2 scheme	$\overline{N}_1$ scheme	$\overline{N}_2$ scheme
13	0	$13.80^{+31\%}_{-22\%}$	$16.01^{+32\%}_{-23\%}$	$16.01^{+32\%}_{-23\%}$	$21.02^{+36\%}_{-24\%}$	$21.02^{+36\%}_{-24\%}$
	1	$25.81^{+18\%}_{-15\%}$	$27.17^{+12.1\%}_{-10.7\%}$	$30.04^{+10.8\%}_{-10.3\%}$	$26.30^{+14.4\%}_{-9.3\%}$	$29.36^{+12.6\%}_{-8.8\%}$
	2	$30.41^{+5.3\%}_{-7.8\%}$	$30.74^{+3.7\%}_{-5.1\%}$	$31.51^{+2.5\%}_{-3.0\%}$	$30.20^{+4.6\%}_{-5.4\%}$	$31.21^{+3.0\%}_{-3.3\%}$
	3	$31.31^{+0.50\%}_{-2.8\%}$	$31.34^{+0.51\%}_{-1.7\%}$	$31.37^{+0.84\%}_{-0.49\%}$	$31.23^{+0.81\%}_{-2.2\%}$	$31.35^{+0.88\%}_{-0.85\%}$
14	0	$17.06^{+31\%}_{-22\%}$	$19.72^{+32\%}_{-23\%}$	$19.72^{+32\%}_{-23\%}$	$25.80^{+35\%}_{-24\%}$	$25.80^{+35\%}_{-24\%}$
	1	$31.89^{+18\%}_{-15\%}$	$33.52^{+12\%}_{-10.7\%}$	$37.03^{+10.7\%}_{-10.3\%}$	$32.42^{+14.3\%}_{-9.3\%}$	$36.19^{+12.5\%}_{-8.8\%}$
	2	$37.55^{+5.2\%}_{-7.6\%}$	$37.93^{+3.7\%}_{-5.1\%}$	$38.88^{+2.4\%}_{-3.0\%}$	$37.28^{+4.5\%}_{-5.4\%}$	$38.52^{+3.0\%}_{-3.4\%}$
	3	$38.65^{+0.50\%}_{-2.7\%}$	$38.69^{+0.50\%}_{-1.7\%}$	$38.73^{+0.81\%}_{-0.51\%}$	$38.55^{+0.80\%}_{-2.2\%}$	$38.70^{+0.85\%}_{-0.87\%}$
27	0	$98.22^{+26\%}_{-19\%}$	$110.6^{+27\%}_{-20\%}$	$110.6^{+27\%}_{-20\%}$	$141.1^{+29\%}_{-21\%}$	$141.1^{+29\%}_{-21\%}$
	1	$183.0^{+16\%}_{-14\%}$	$190.0^{+11.9\%}_{-10.5\%}$	$208.6^{+10.6\%}_{-10\%}$	$18.41^{+13.5\%}_{-9.1\%}$	$204.2^{+12.0\%}_{-8.8\%}$
	2	$214.2^{+4.8\%}_{-6.7\%}$	$215.7^{+3.7\%}_{-5.1\%}$	$220.7^{+2.2\%}_{-3.1\%}$	$212.3^{+4.5\%}_{-5.3\%}$	$218.9^{+2.7\%}_{-3.4\%}$
	3	$220.2^{+0.46\%}_{-2.3\%}$	$220.3^{+0.44\%}_{-1.7\%}$	$220.6^{+0.57\%}_{-0.62\%}$	$219.6^{+0.77\%}_{-2.2\%}$	$220.4^{+0.63\%}_{-0.94\%}$
100	0	$2015^{+19\%}_{-15\%}$	$2195^{+19\%}_{-15\%}$	$2195^{+19\%}_{-15\%}$	$2697^{+21\%}_{-17\%}$	$2697^{+21\%}_{-17\%}$
	1	$3724^{+13\%}_{-11\%}$	$3813^{+11.3\%}_{-10\%}$	$4138^{+10.1\%}_{-9.5\%}$	$3709^{+12.3\%}_{-8.8\%}$	$4060^{+11.1\%}_{-8.5\%}$
	2	$4322^{+4.2\%}_{-5.3\%}$	$4338^{+3.5\%}_{-4.8\%}$	$4424^{+1.9\%}_{-3.1\%}$	$4283^{+4.2\%}_{-5.0\%}$	$4394^{+2.4\%}_{-3.4\%}$
	3	$4439^{+0.51\%}_{-1.8\%}$	$4440^{+0.47\%}_{-1.6\%}$	$4448^{+0.26\%}_{-0.77\%}$	$4427^{+0.76\%}_{-2.0\%}$	$4444^{+0.35\%}_{-1.02\%}$

BEYOND INFINITE TOP QUARK LIMIT : RESULTS

N3LO

m_t-dependent full NLO (NLO_{m_t}) is obtained using POWHEG. Heinrich et JHEP'19

NLO_{m_t} is 6.8% larger than $\mathrm{NLO}|_{m_t\to\infty}$ at 13TeV - POWHEG

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO_{m_t}	$27.56^{+14\%}_{-13\%}$	$32.64^{+14\%}_{-12\%}$	$126.2^{+12\%}_{-10\%}$	$1119^{+13\%}_{-13\%}$
$\mathrm{NNLO}\oplus\mathrm{NLO}_{m_t}$	$32.16^{+5.9\%}_{-5.9\%}$	$38.29^{+5.6\%}_{-5.5\%}$	$157.3^{+3.0\%}_{-4.7\%}$	$1717^{+5.8\%}_{-12\%}$
$\mathrm{NNLO}_{\mathrm{B-i}}\oplus\mathrm{NLO}_{m_t}$	$33.08^{+5.0\%}_{-4.9\%}$	$39.16^{+4.9\%}_{-5.0\%}$	$150.8^{+4.6\%}_{-5.7\%}$	$1330^{+4.0\%}_{-7.2\%}$
$\mathrm{NNLO}\otimes\mathrm{NLO}_{m_t}$	$32.47^{+5.3\%}_{-7.8\%}$	$38.42^{+5.2\%}_{-7.6\%}$	$147.6^{+4.8\%}_{-6.7\%}$	$1298^{+4.2\%}_{-5.3\%}$
$\mathrm{N^3LO}\oplus\mathrm{NLO}_{m_t}$	$33.06^{+2.1\%}_{-2.9\%}$	$39.40^{+1.7\%}_{-2.8\%}$	$163.3^{+4.0\%}_{-8.3\%}$	$1833^{+14\%}_{-20\%}$
$\mathrm{N^3LO}_{\mathrm{B-i}}\oplus\mathrm{NLO}_{m_t}$	$34.17^{+1.9\%}_{-4.6\%}$	$40.44^{+1.9\%}_{-4.7\%}$	$155.5^{+2.3\%}_{-5.0\%}$	$1372^{+2.8\%}_{-5.0\%}$
$\mathrm{N^3LO}\otimes\mathrm{NLO}_{m_t}$	$33.43^{+0.66\%}_{-2.8\%}$	$39.56^{+0.64\%}_{-2.7\%}$	$151.7^{+0.53\%}_{-2.4\%}$	$1333^{+0.51\%}_{-1.8\%}$

$\mathrm{N^3LO}\otimes\mathrm{NLO}_{m_t}$ *: most accurate out of three above approximations*

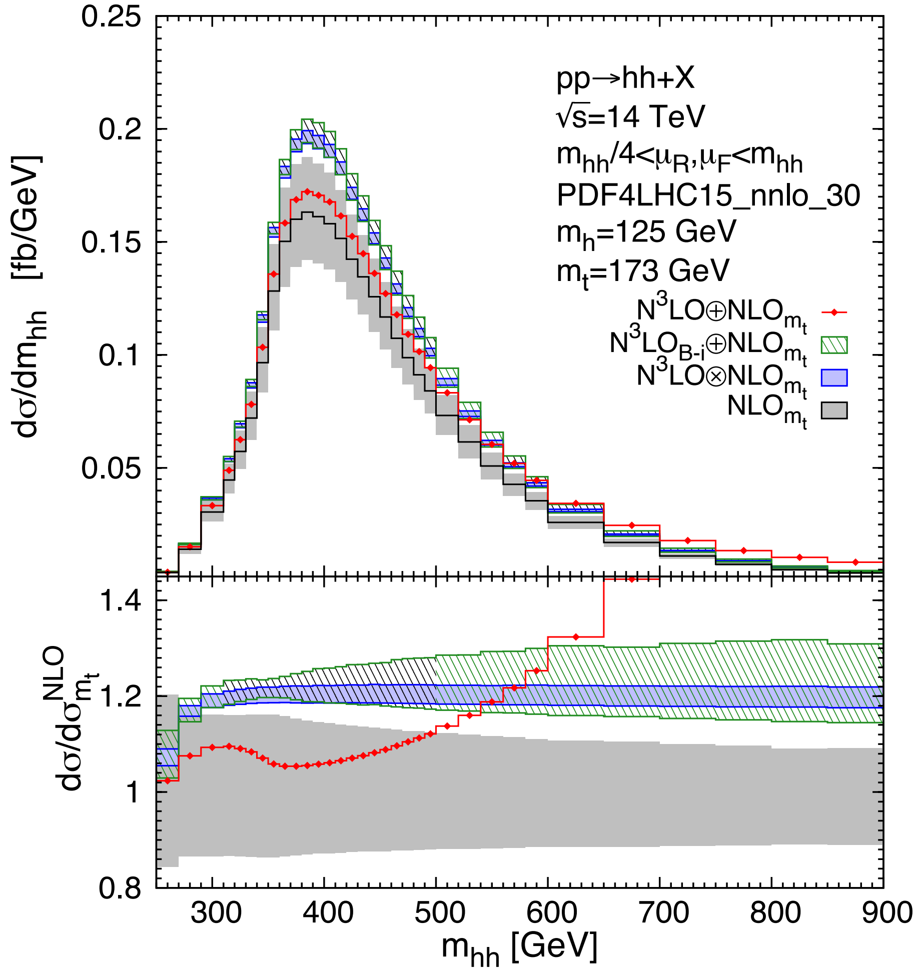
- **Enhancement :**
  - 20% for $\mathrm{NLO}_{m_t} \to \mathrm{NNLO}\otimes\mathrm{NLO}_{m_t}$ — At 13 TeV
  - 3% for $\to \mathrm{NNLO}\otimes\mathrm{NLO}_{m_t} \to \mathrm{N^3LO}\otimes\mathrm{NLO}_{m_t}$
- **Scale uncertainty within 3%**

Chen, Li, Shao, Wang(PLB'20, JHEP'20)

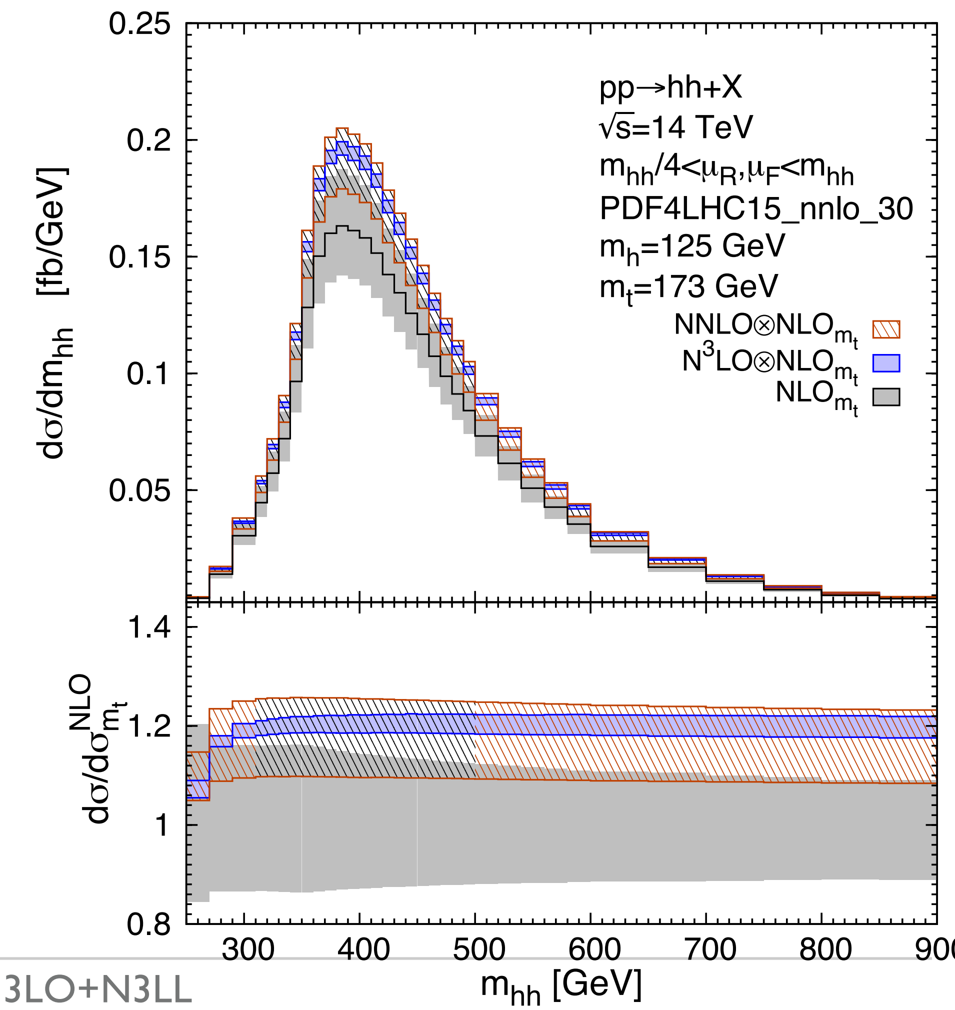
WITH TOP QUARK MASS EFFECTS : RESULTS

Invariant Mass distributions

Chen, Li, Shao, Wang
(PLB'20, JHEP'20)



- $\text{N}^3\text{LO} \oplus \text{NLO}_{\text{m}_\text{t}}$ overshoots for $m_{hh} > 600$ GeV
- $\text{N}^3\text{LO}_{\text{B}-\text{i}} \oplus \text{NLO}_{\text{m}_\text{t}}$ degraded to NLO accuracy for $m_{hh} > 2m_t$
- Relative scale uncertainties of $\text{N}^3\text{LO} \otimes \text{NLO}_{\text{m}_\text{t}} \sim \text{N}^3\text{LO}$



- Higher order corrections for $\text{NNLO} \otimes \text{NLO}_{\text{m}_\text{t}}$ and $\text{N}^3\text{LO} \otimes \text{NLO}_{\text{m}_\text{t}}$ are quite small near $m_{hh} \simeq 2m_h$
- K-factor : $\dfrac{\text{N}^3\text{LO} \otimes \text{NLO}_{\text{m}_\text{t}}}{\text{NLO}_{\text{m}_\text{t}}} \rightarrow 1.2$ for large m_h