Karlsruher Institut für Technologie

CARLSBERG FOUNDATION

Non-factorisable contributions to *t*-channel single-top and Higgs VBF production

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RADCOR
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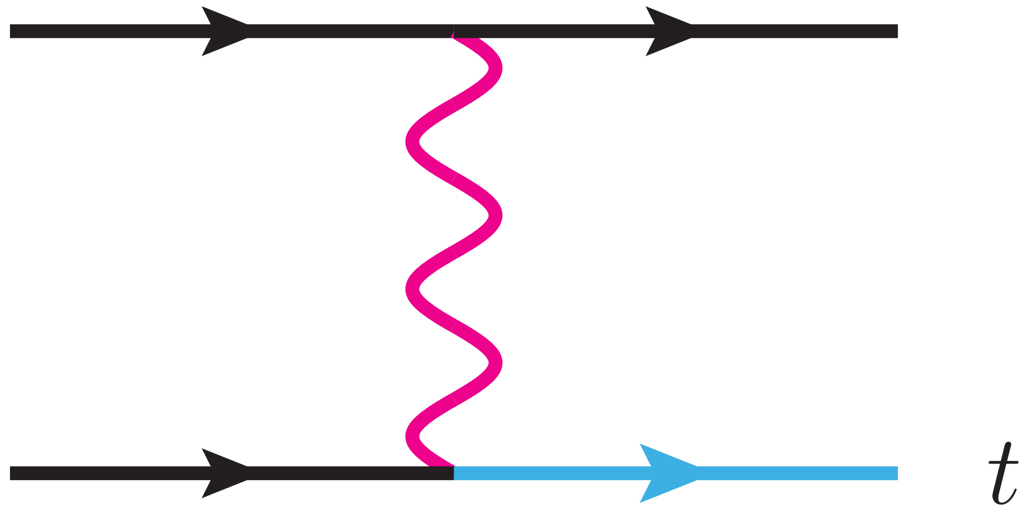
based on work with
Konstantin Asteriadis, Kirill Melnikov, Jérémie Quarroz, Chiara Signorile-Signorile, and Chen-Yu Wang
presented in 2108.09222, 2204.05770, and 2305.08016

Motivation

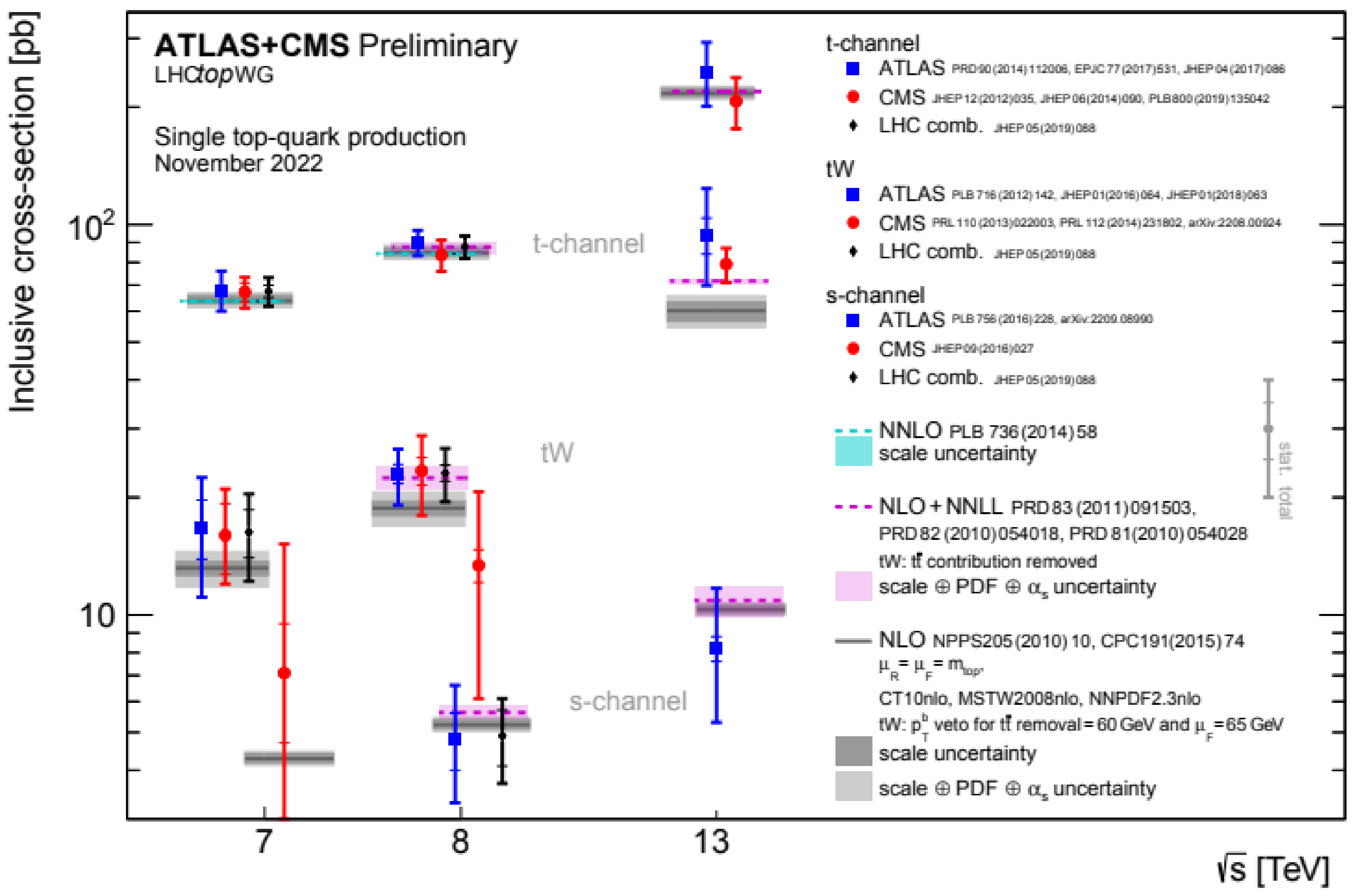
- large (but subdominant) cross sections
- colourless exchange in the t-channel involving weak gauge bosons
  - study of electroweak parameters and vertex structures

$$q\, b \to q'\, t$$

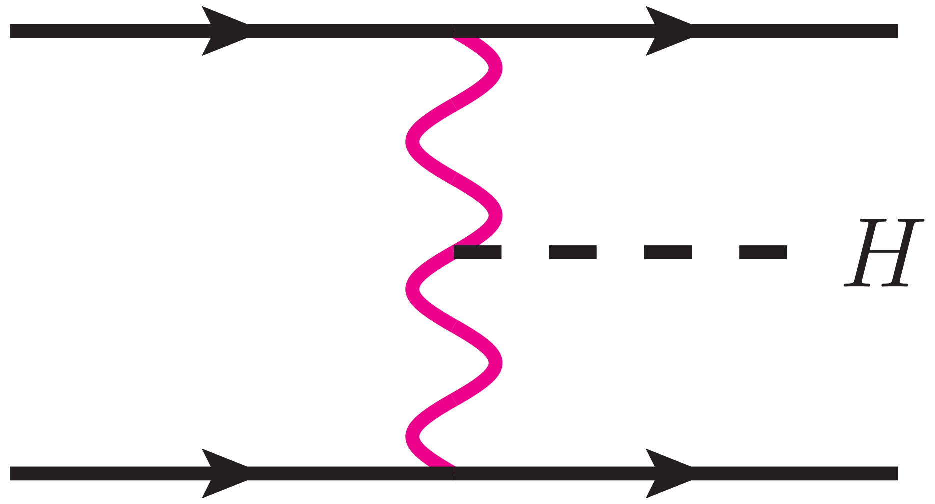


$\sigma_t \sim 25\%\, \sigma_{t\bar{t}}$

constrain CKM + PDF
top quark properties

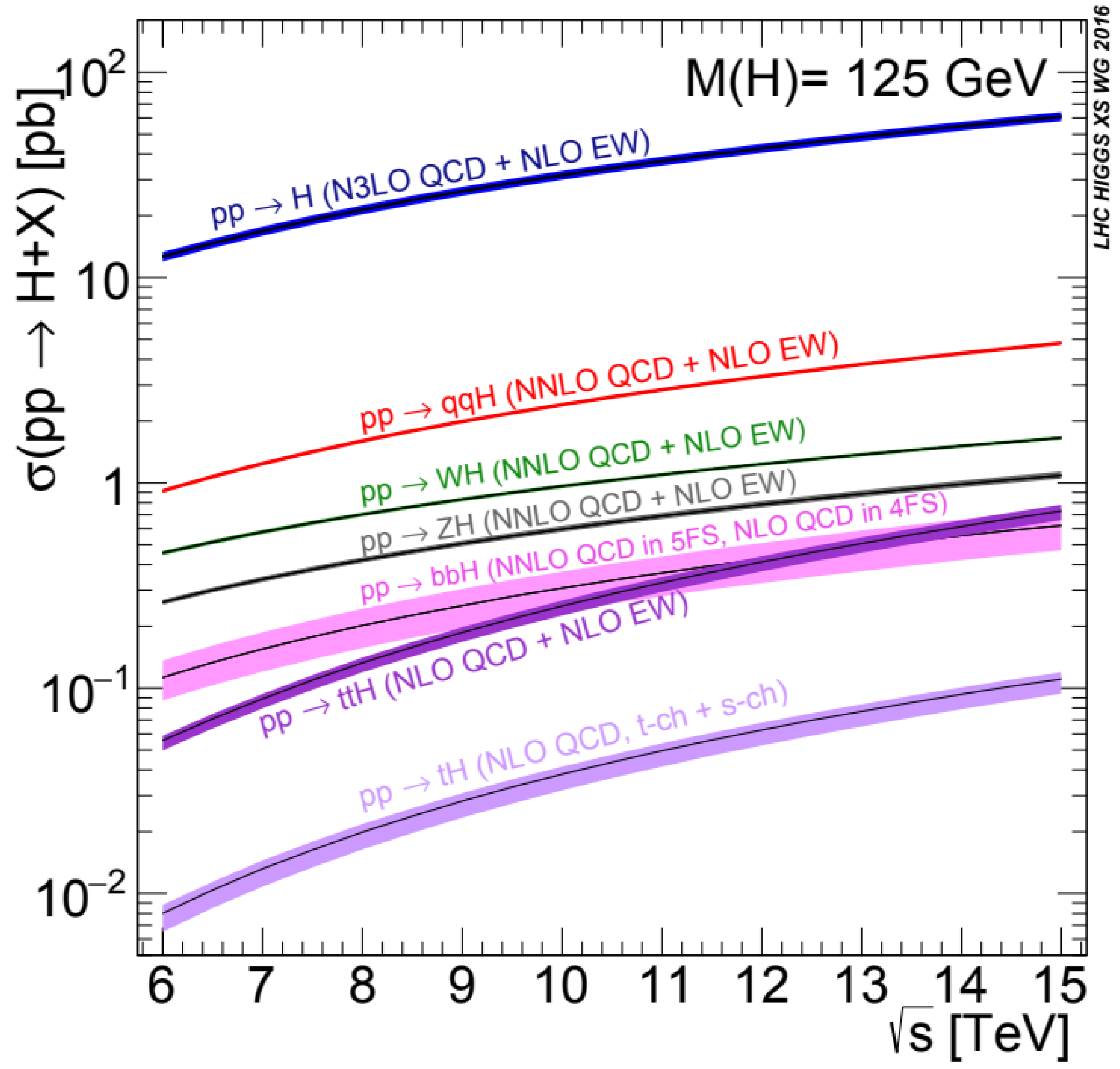


$$q\, Q \to q'\, Q' + H$$



$\sigma_{\mathrm{VBF}} \sim 10\%\, \sigma_{ggH}$

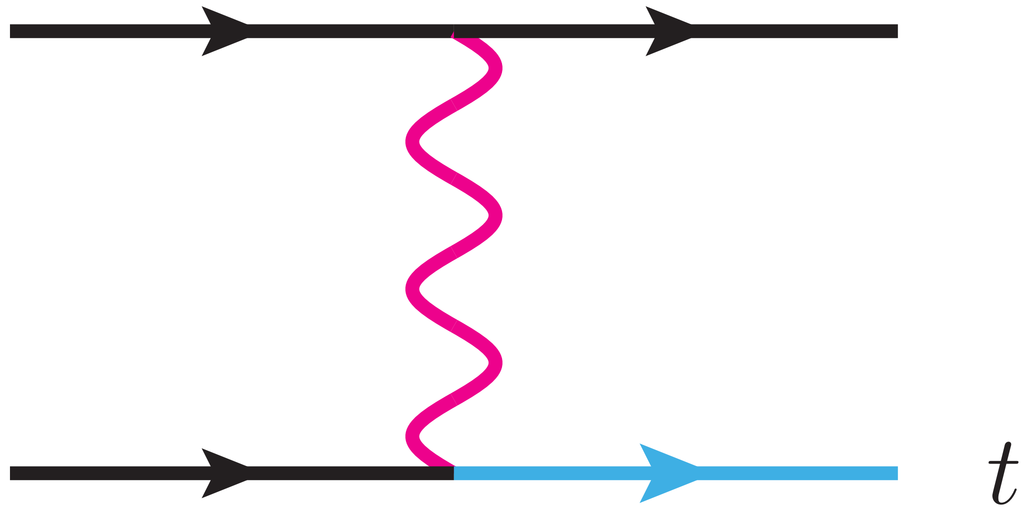
clear signature
Higgs searches



Motivation – theoretical developments (I)

- large (but subdominant) cross sections
- colourless exchange in the t-channel involving weak gauge bosons
  - study of electroweak parameters and vertex structures

$$q\, b \to q'\, t$$



$\sigma_t \sim 25\%\, \sigma_{t\bar{t}}$

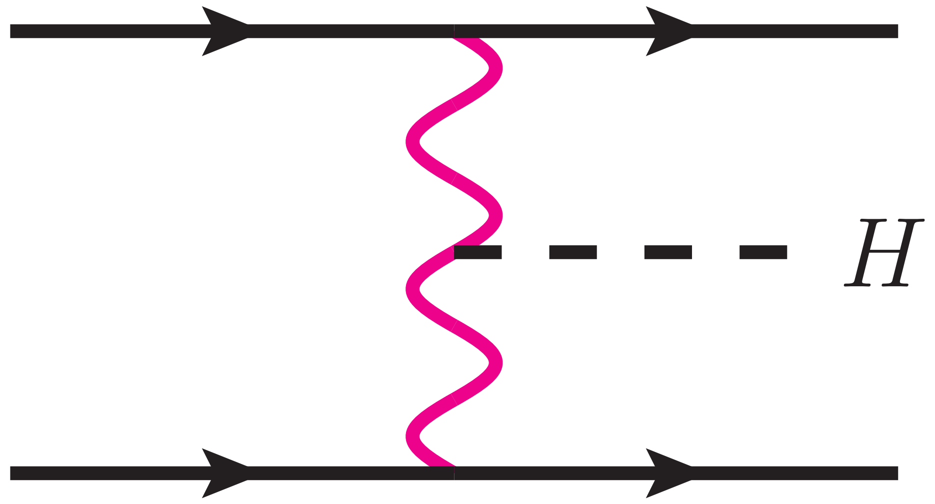
constrain CKM + PDF
top quark properties

NLO QCD [Bordes, van Eijk 1995] [Campbell, Ellis, Tramontano 2004] [Cao, Yuan 2005] [Cao, Schwienhorst, Benitez, Brock, Yuan 2005] [Harris, Laenen, Phaf, Sullivan, Weinzierl 2002] [Schwienhorst, Yuan, Mueller, Cao 2011]

NLO QCD+EW [Frederix, Pagani, Tsinikos 2019]

NNLO QCD [Brucherseifer, Caola, Melnikov 2014] [Berger, Gao, Yuan, Zhu 2016] [Berger, Gao, Zhu 2017] [Campbell, Neumann, Sullivan 2021]

$$q\, Q \to q'\, Q' + H$$



$\sigma_{\mathrm{VBF}} \sim 10\%\, \sigma_{ggH}$

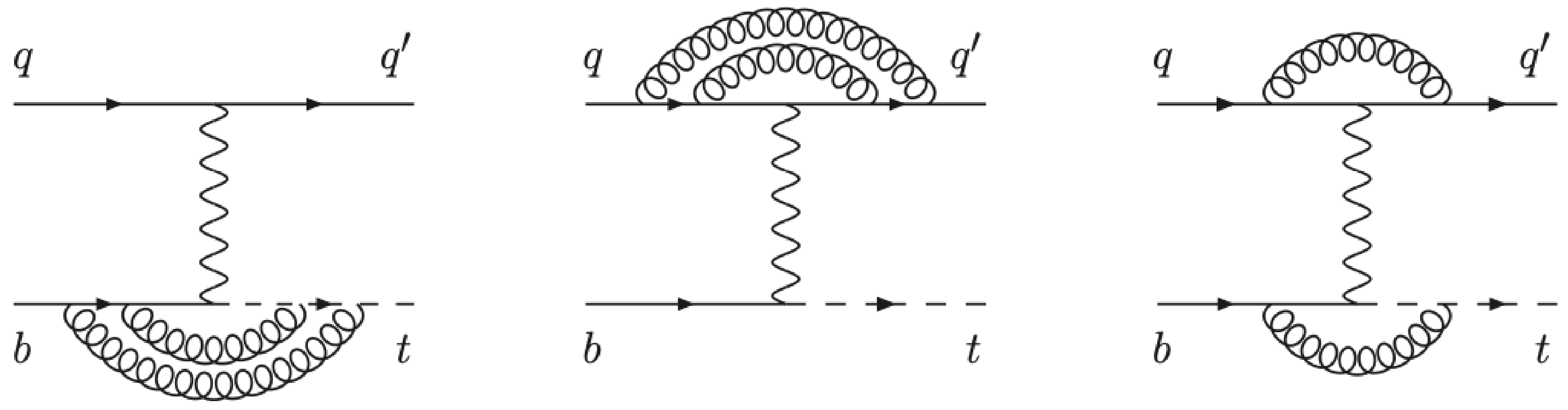
clear signature
Higgs searches

NLO QCD [Figy, Oleari, Zeppenfeld 2003] [Berger, Campbell 2004] [Figy, Zeppenfeld 2004]

NLO EW [Ciccolini, Denner, Dittmaier 2007 & 2008] [Figy, Palmer, Weiglein 2012]

NNLO QCD [Bolzoni, Maltoni, Moch, Zaro 2010 & 2012] [Cacciari, Dreyer, Karlberg, Salam, Zanderighi 2015] [Cruz-Martinez, Gehrmann, Glover, Huss 2018] [Asteriadis, Caola, Melnikov, Röntsch 2022 & 2023]

NNNLO QCD [Dreyer, Karlberg 2016]



$\leftarrow$ **factorisable diagrams**

Motivation – theoretical developments (II)

- large (but subdominant) cross sections
- NNLO (factorisable) corrections are at the level of a few %

$$q\, b \to q'\, t$$

MSTW2008, lo, nlo, nnlo PDF, $\mu_R = \mu_F = m_t = 173$ GeV @ 8 TeV

$p_\perp$	$\sigma_{\rm LO}$, pb	$\sigma_{\rm NLO}$, pb	$\delta_{\rm NLO}$	$\sigma_{\rm NNLO}$, pb	$\delta_{\rm NNLO}$
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	+2.4%	$54.2^{+0.5}_{-0.2}$	−1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	+4.9%	$48.3^{+0.3}_{-0.02}$	−1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	+9.3%	$36.5^{+0.1}_{+0.1}$	−0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	+13.6%	$25.4^{-0.1}_{+0.2}$	+1.6%

[Brucherseifer, Caola, Melnikov 2014]

Table 7. Fully inclusive results in pb for pp at 7 TeV and 14 TeV (LHC), as well as $p\bar{p}$ at 1.96 TeV (Tevatron) with scales $\mu_R = \mu_F = m_t$ and DDIS scales and using CT14 PDFs. Uncertainties next to the cross section in super- and subscript are from a six-point scale variation, while PDF uncertainties are below.

	7 TeV pp		14 TeV pp		1.96 TeV $\bar{p}p$
	top	anti-top	top	anti-top	$t+\bar{t}$
$\sigma^{\mu=m_t}_{\rm LO}$	$37.1^{+7.1\%}_{-9.5\%}$	$19.1^{+7.3\%}_{-9.7\%}$	$134.6^{+10.0\%}_{-12.1\%}$	$78.9^{+10.4\%}_{-12.6\%}$	$2.09^{+0.8\%}_{-3.1\%}$
$\sigma^{\rm DDIS}_{\rm LO}$	$39.5^{+6.4\%}_{-8.6\%}$	$19.9^{+7.0\%}_{-9.3\%}$	$140.9^{+9.4\%}_{-11.4\%}$	$80.7^{+10.2\%}_{-12.3\%}$	$2.31^{-0.3\%}_{-1.8\%}$
$\sigma^{\mu=m_t}_{\rm NLO}$	$41.4^{+3.0\%}_{-2.0\%}$	$21.5^{+3.1\%}_{-2.0\%}$	$154.3^{+3.1\%}_{-2.3\%}$	$91.4^{+3.1\%}_{-2.2\%}$	$1.96^{+3.1\%}_{-2.3\%}$
$\sigma^{\rm DDIS}_{\rm NLO}$	$41.8^{+3.3\%}_{-2.0\%}$	$21.5^{+3.4\%}_{-1.6\%}$	$154.4^{+3.7\%}_{-1.4\%}$	$91.2^{+3.1\%}_{-1.8\%}$	$2.00^{+3.6\%}_{-3.4\%}$
	PDF $^{+1.7\%}_{-1.4\%}$	PDF $^{+2.2\%}_{-1.5\%}$	PDF $^{+1.7\%}_{-1.1\%}$	PDF $^{+1.9\%}_{-0.9\%}$	PDF $^{+4.3\%}_{-5.3\%}$
$\sigma^{\mu=m_t}_{\rm NNLO}$	$41.9^{+1.2\%}_{-0.7\%}$	$21.9^{+1.2\%}_{-0.7\%}$	$153.3(2)^{+1.0\%}_{-0.6\%}$	$91.5(2)^{+1.1\%}_{-0.9\%}$	$2.08^{+2.0\%}_{-1.3\%}$
$\sigma^{\rm DDIS}_{\rm NNLO}$	$41.9^{+1.3\%}_{-0.8\%}$	$21.8^{+1.3\%}_{-0.7\%}$	$153.4(2)^{+1.1\%}_{-0.7\%}$	$91.2(2)^{+1.1\%}_{-0.9\%}$	$2.07^{+1.7\%}_{-1.1\%}$
	PDF $^{+1.3\%}_{-1.1\%}$	PDF $^{+1.4\%}_{-1.3\%}$	PDF $^{+1.2\%}_{-1.0\%}$	PDF $^{+1.0\%}_{-1.0\%}$	PDF $^{+3.7\%}_{-5.0\%}$

[Campbell, Neumann, Sullivan 2021]

$$q\, Q \to q'\, Q' + H$$

	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032\,^{+0.057}_{-0.069}$	$0.957\,^{+0.066}_{-0.059}$
NLO	$3.929\,^{+0.024}_{-0.023}$	$0.876\,^{+0.008}_{-0.018}$
NNLO	$3.888\,^{+0.016}_{-0.012}$	$0.844\,^{+0.008}_{-0.008}$

TABLE I: Cross sections at LO, NLO and NNLO for VBF Higgs production, fully inclusively and with VBF cuts. The quoted uncertainties correspond to scale dependence, while statistical errors at NNLO are about 0.1% with VBF cuts and much smaller without.

[Cacciari, Dreyer, Karlberg, Salam, Zanderighi 2015]

TABLE I. Inclusive cross sections at LO, NLO, NNLO, and N^{3}LO for VBF Higgs production. The quoted uncertainties correspond to scale variations $Q/2 < \mu_R, \mu_F < 2Q$, while statistical uncertainties are at the level of 0.2‰.

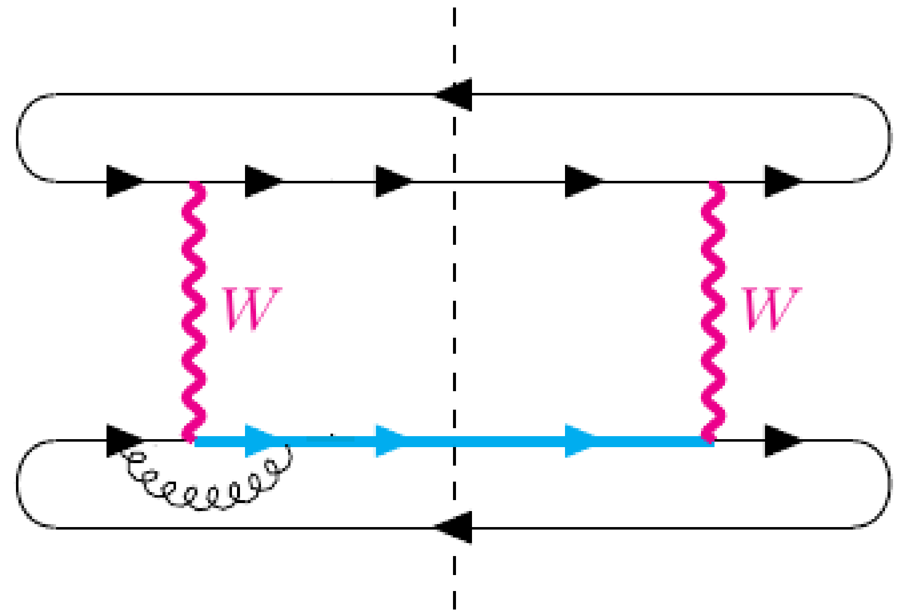
	$\sigma^{(13\,\text{TeV})}$ (pb)	$\sigma^{(14\,\text{TeV})}$ (pb)	$\sigma^{(100\,\text{TeV})}$ (pb)
LO	$4.099^{+0.051}_{-0.067}$	$4.647^{+0.037}_{-0.058}$	$77.17^{+6.45}_{-7.29}$
NLO	$3.970^{+0.025}_{-0.023}$	$4.497^{+0.032}_{-0.027}$	$73.90^{+1.73}_{-1.94}$
NNLO	$3.932^{+0.015}_{-0.010}$	$4.452^{+0.018}_{-0.012}$	$72.44^{+0.53}_{-0.40}$
N^{3}LO	$3.928^{+0.005}_{-0.001}$	$4.448^{+0.006}_{-0.001}$	$72.34^{+0.11}_{-0.02}$

[Dreyer, Karlberg 2016]

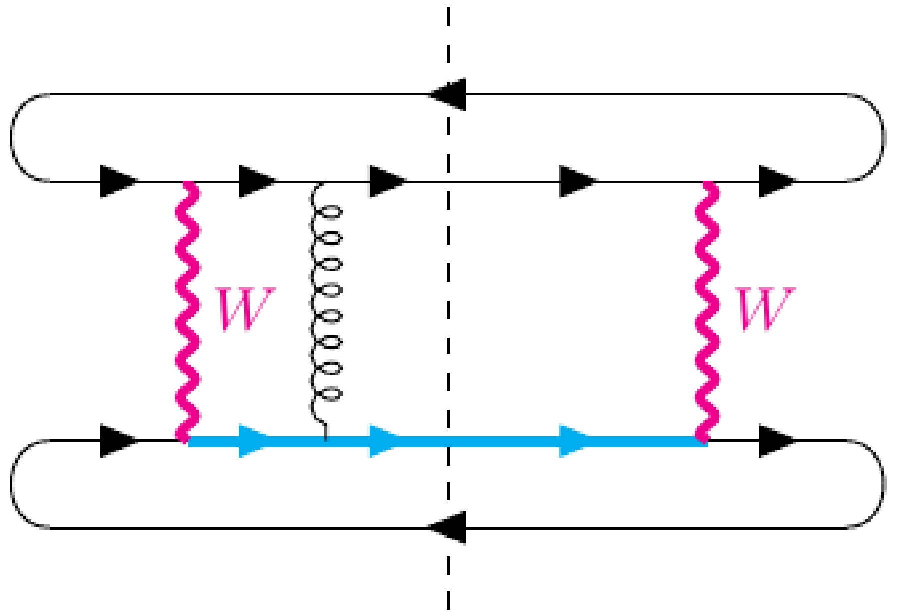
Motivation – Non-factorisable contribution (I)

Factorisable contributions | **Non-factorisable contributions**

NLO

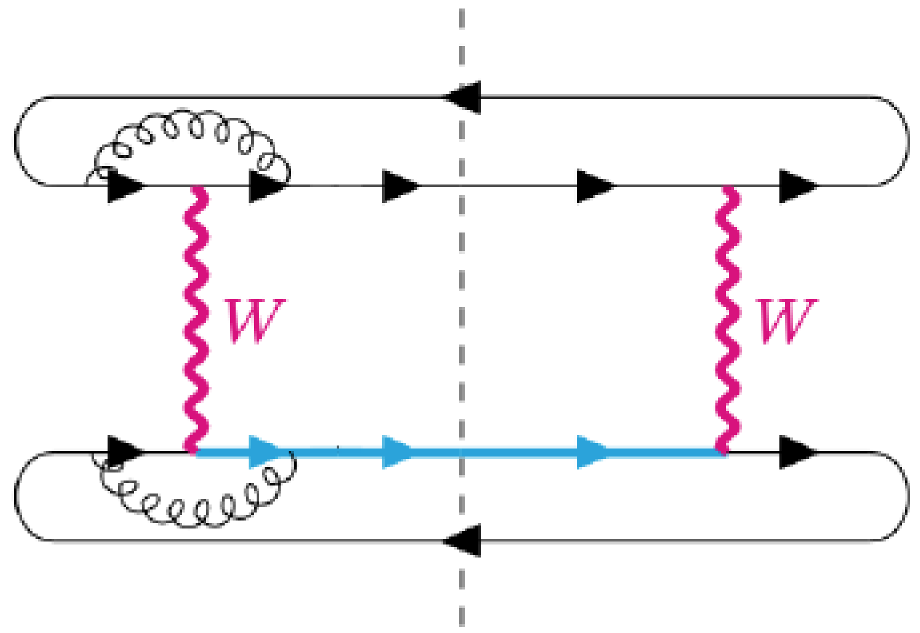


$$\mathrm{tr}\left(t^a t^a\right) = \frac{N_c^2 - 1}{2}$$

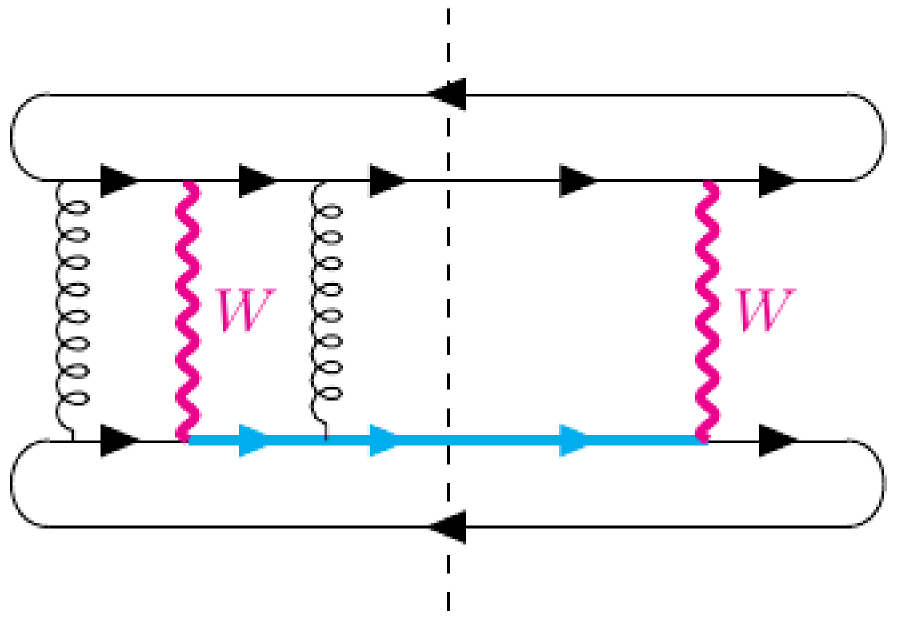


$$\mathrm{tr}\left(t^a\right) = 0$$

NNLO



$$\mathrm{tr}\left(t^a t^a\right)\,\mathrm{tr}\left(t^b t^b\right) = \frac{(N_c^2 - 1)^2}{4}$$



$$\mathrm{tr}\left(t^a t^b\right)\,\mathrm{tr}\left(t^a t^b\right) = \frac{N_c^2 - 1}{4}$$

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

Motivation – Non-factorisable contribution (II)

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

However:

Factorisable predictions are already small, a few %

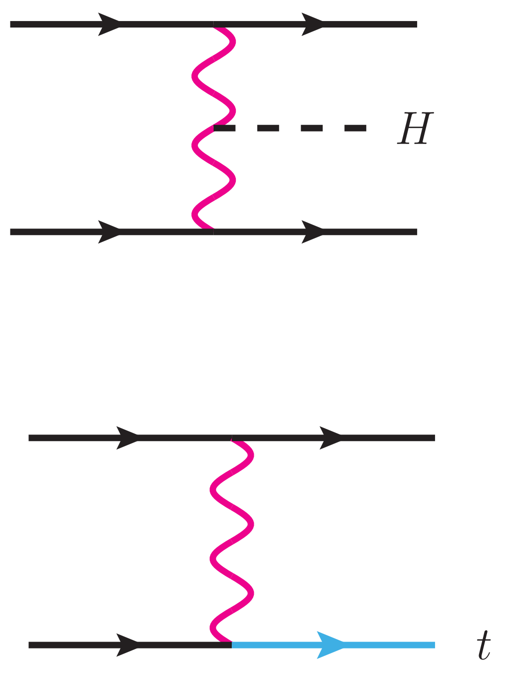
The actual size of NNLO non-factorisable corrections **cannot be inferred from NLO contributions**

Non-factorisable corrections could be enhanced by a factor $\pi^2 \simeq 10$ related to a Glauber phase

- shown for Higgs production in weak boson fusion in the eikonal approximation [Liu, Melnikov, Penin 2019]
- a loop effect that, in principle, does not require a scattering to occur

$\boxed{\pi^2}$

$$\sigma = \underbrace{\sigma_0 + \overbrace{\left(\frac{p_\perp}{\sqrt{s}}\right)^2 \sigma_1}^{\text{real contribution}} + \dots}_{\text{virtual contribution}}$$



$p_\perp^j \sim 100\,\mathrm{GeV}$ $\quad \sqrt{s} \sim 600\,\mathrm{GeV}$

$p_\perp^t \sim 40\,\mathrm{GeV}$ $\quad \sqrt{s} \sim 300\,\mathrm{GeV}$

Focus on single top for now, arguments translate (almost) verbatim to VBF Higgs

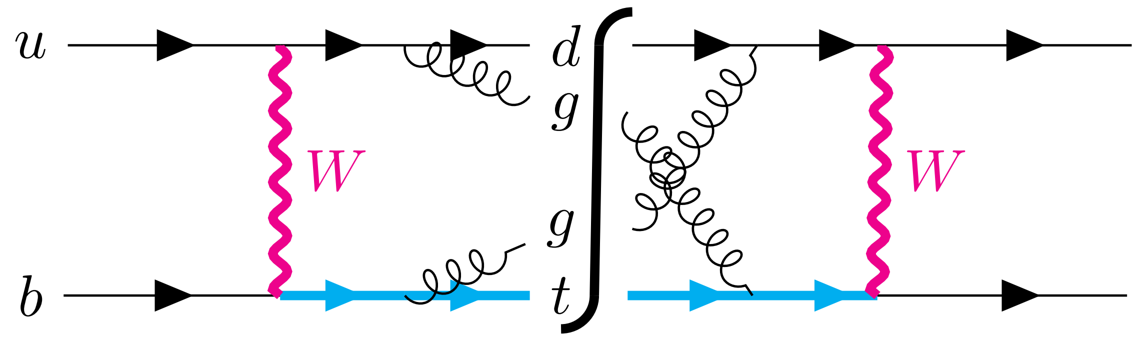
Non-factorisable contribution – ingredients of the calculation (I)

Three terms contribute to the non-factorisable single top cross section at NNLO

$$\mathrm{d}\sigma^{\mathrm{nf}}_{pp\to X+t} = \sum_{i,j} \int \mathrm{d}x_1\, \mathrm{d}x_2\; f_i(x_1,\mu_F)\, f_j(x_2,\mu_F)\; \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{ij\to X+t}(x_1,x_2)$$

$$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RR}} + \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RV}} + \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{VV}}$$

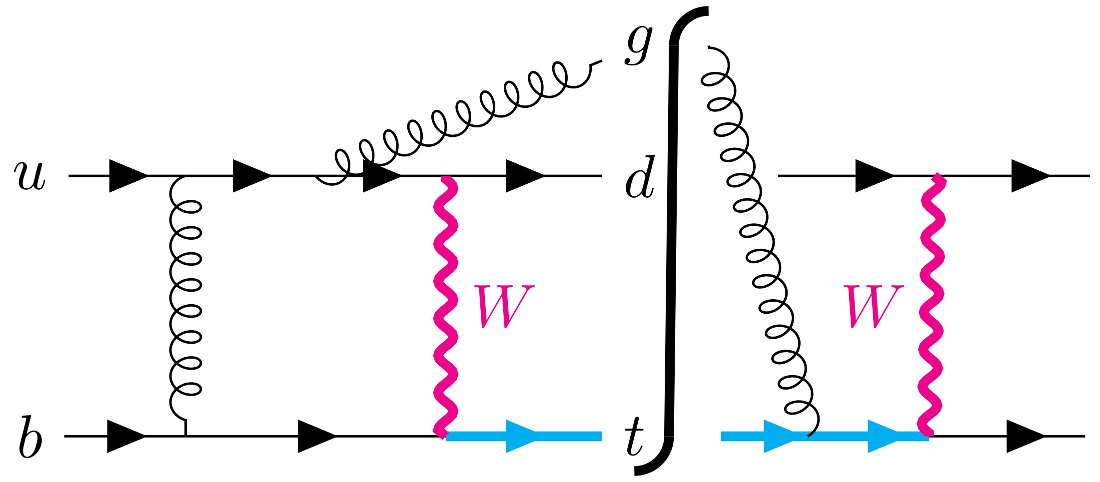
$$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RR}} \;:\; \mathscr{M}^{(0)}_6 \otimes \mathscr{M}^{(0)}_6$$



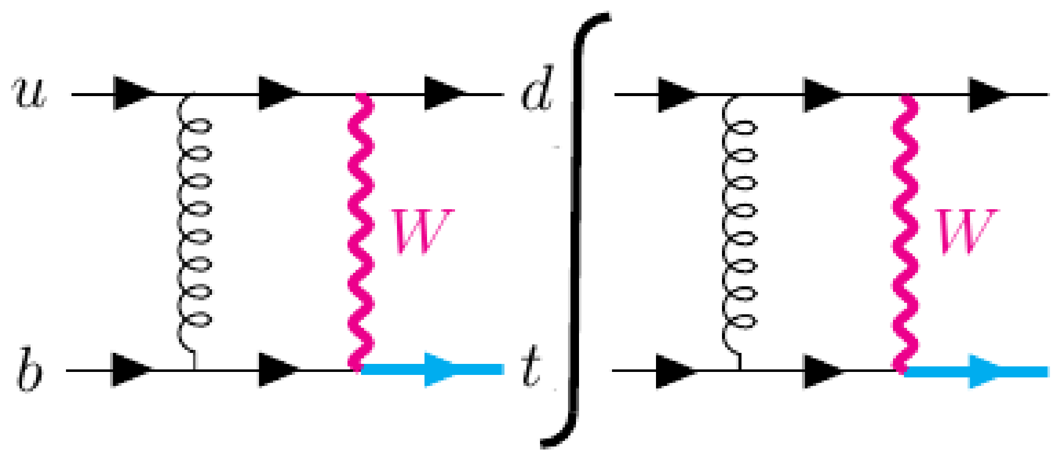
Each ingredient requires **individual treatment** with different challenges:

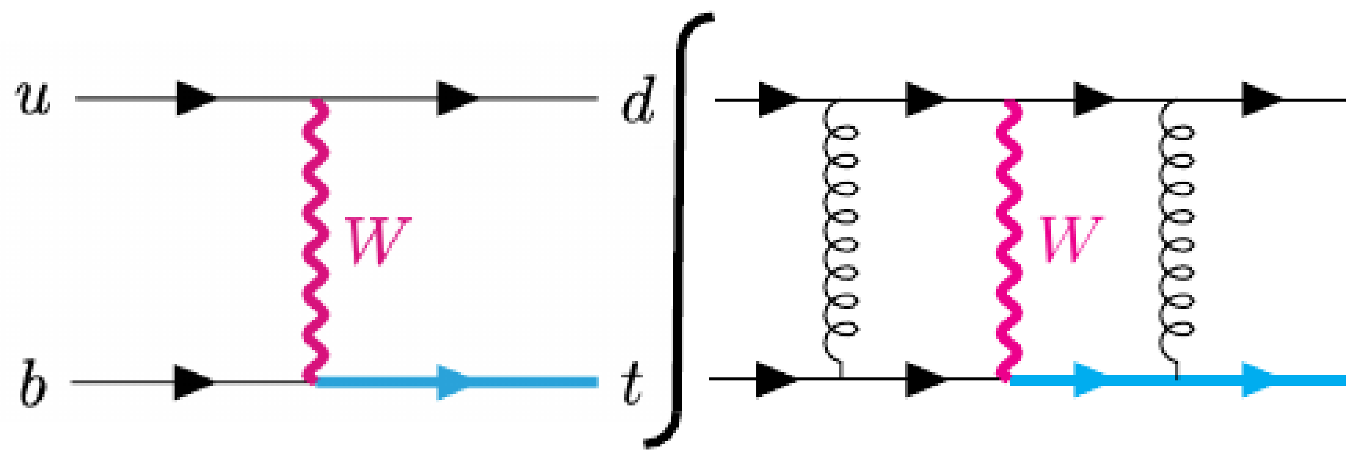
- Pole cancellation
- Loop amplitudes

$$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RV}} \;:\; \mathscr{M}^{(1)}_5 \otimes \mathscr{M}^{(0)}_5$$



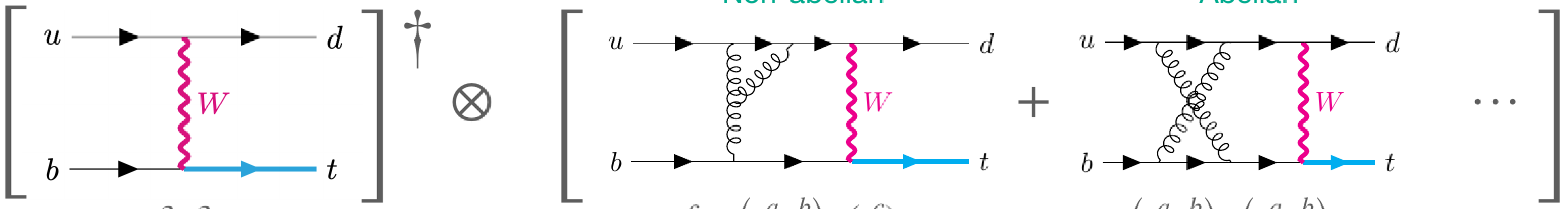
$$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{VV}} \;:\; \mathscr{M}^{(1)}_4 \otimes \mathscr{M}^{(1)}_4 \,,\; \mathscr{M}^{(0)}_4 \otimes \mathscr{M}^{(2)}_4$$





Non-factorisable contribution – ingredients of the calculation (II)

Non-factorisable contributions have to **connect upper and lower quark lines** and are effectively **Abelian**

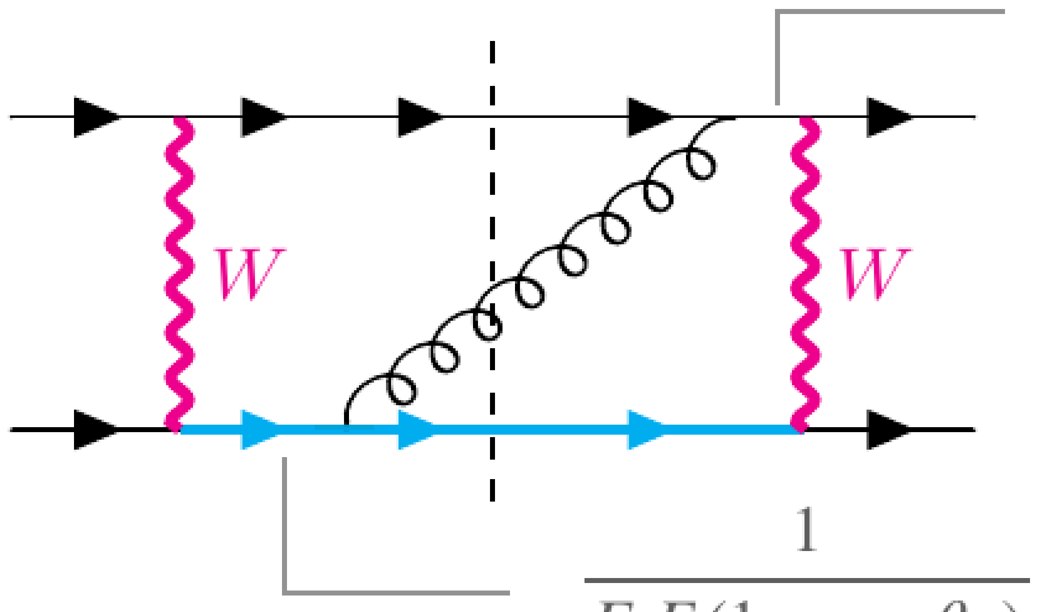


$\delta_{ij}\,\delta_{kl}$

$f_{abc}\left(t^a t^b\right)_{ij}\left(t^c\right)_{kl} \rightarrow 0$

$\left(t^a t^b\right)_{ij}\left(t^a t^b\right)_{kl} \rightarrow \frac{N_c^2-1}{4}$

The **infrared structure is simplified**: **no collinear singularities**

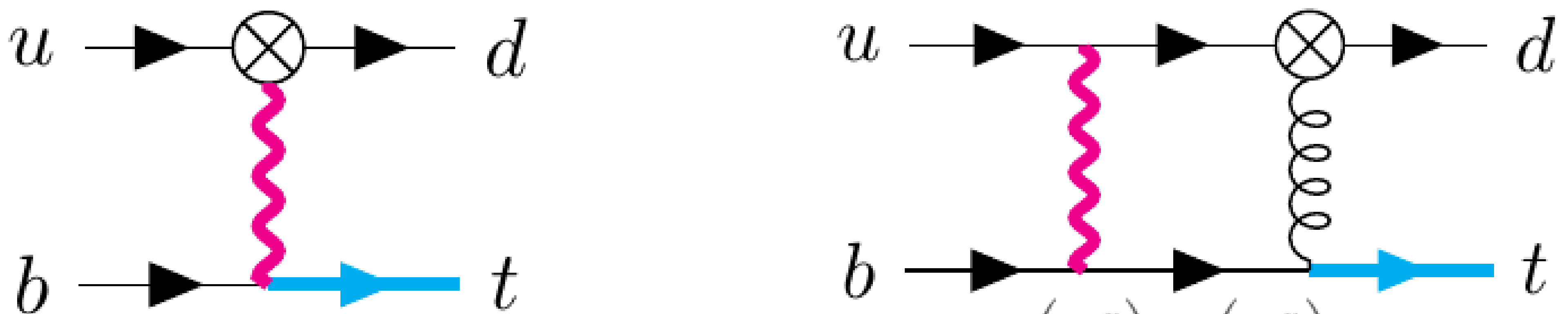


$\frac{1}{E_g E_q (1-\cos\theta_{gq})}$

$\frac{1}{E_g E_t (1-\cos\theta_{gt})}$

All IR singularities are of soft origin.

Non-factorisable contributions are **UV finite**



$\delta_{ij}\,\delta_{kl}$

$\left(t^a\right)_{ij}\left(t^a\right)_{kl}$

Renormalisation simply consists of $\alpha_s^{\text{bare}} = \alpha_s \mu^{2\epsilon} S_\epsilon$

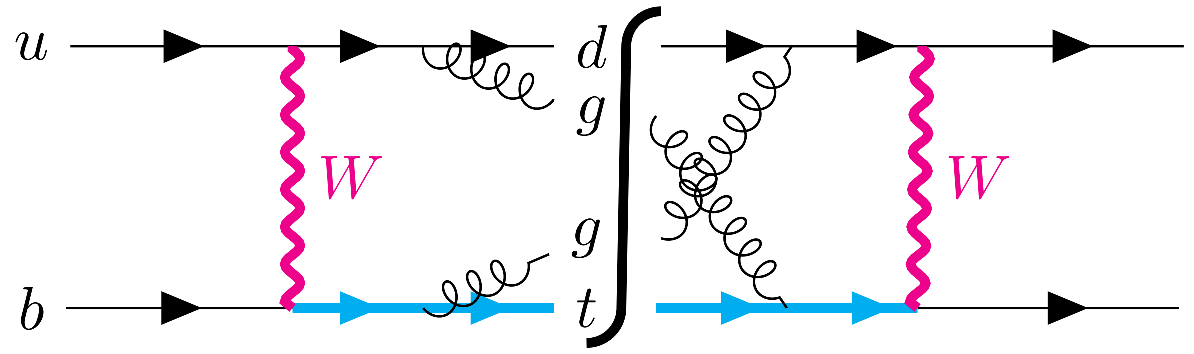
Double-real emission

Main issue of the **double-real contribution: extract IR singularities** while preserving the fully-differential nature of the calculation

→ **simplified nested soft-collinear subtraction scheme** [Caola, Melnikov, Röntsch 2017]

- fully factorised emissions, due to Abelian nature
- abscence of collinear singularities

$$F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right) = \mathcal{N} \int \mathrm{dLips}_{34}\, (2\pi)^d\, \delta^{(d)}\Big(p_1 + p_2 - \sum_{i=3}^{6} p_i\Big) \times \left|\mathcal{M}_0\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right)\right|^2_{\mathrm{nf}}$$



integration over potentially unresolved phase space

$$2s \cdot \sigma_{\mathrm{RR}}^{\mathrm{nf}} = \frac{1}{2!} \int [\mathrm{d}p_5]\,[\mathrm{d}p_6]\, F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right) \equiv \left\langle F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right) \right\rangle$$

$$[\mathrm{d}p] = \frac{\mathrm{d}^{d-1}p}{(2\pi)^{d-1} 2E_p}\, \theta\big(E_{\max} - E_p\big)$$

Separate the **soft-divergent part** from the **soft-finite contribution**

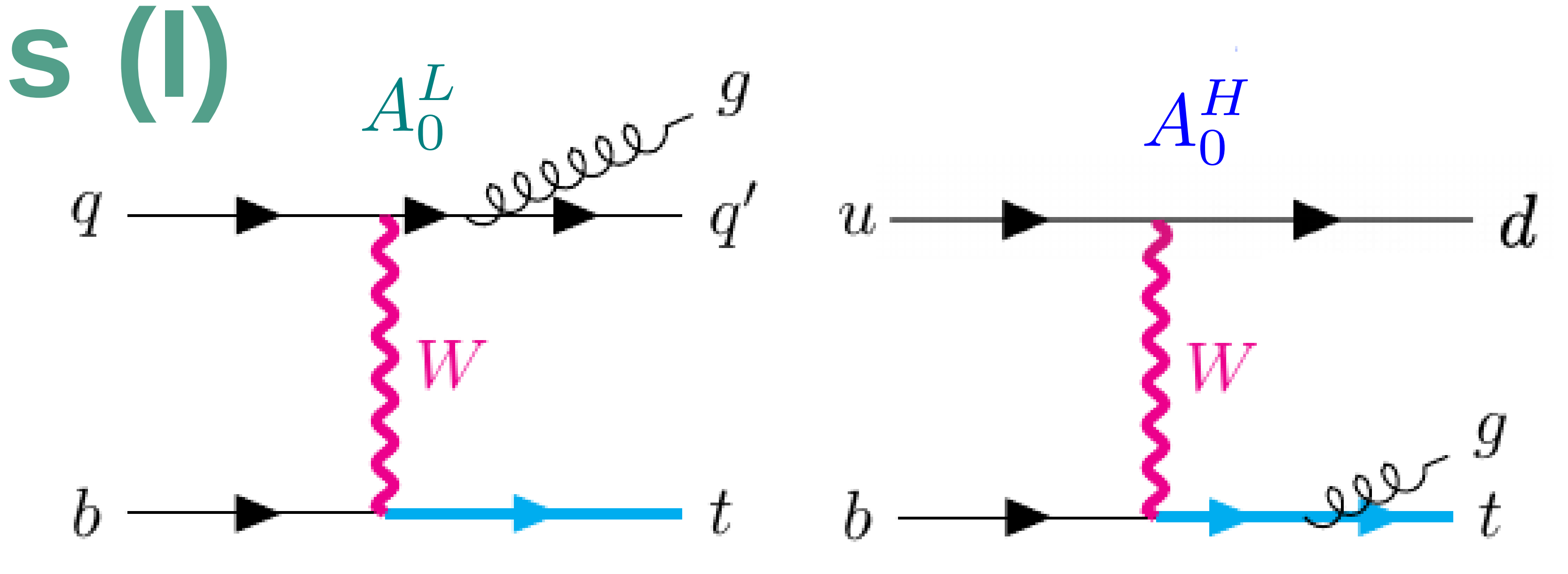
$$\begin{aligned}\left\langle F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right) \right\rangle = &\left\langle S_5 S_6\, F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right) \right\rangle \\ &+ 2\left\langle S_6\, (I - S_5)\, F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right) \right\rangle \\ &+ \left\langle (I - S_5)\,(I - S_6)\, F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g\right) \right\rangle\end{aligned}$$

Double-soft counterterm

Single-soft counterterm

Resolved contribution

Extracting soft singularities from real corrections (I)



Consider **single emission**: simpler bookkeeping, clear procedure

Decompose the amplitude into **colour-stripped, sub-amplitudes**

$$\mathcal{M}_5^{(0)}(1_q, 2_b, 3_{q'}, 4_t; 5_g) = g_{s,b}\left[t^{c_5}_{c_3 c_1}\delta_{c_4 c_2} A_0^L(5_g) + t^{c_5}_{c_4 c_2}\delta_{c_3 c_1} A_0^H(5_g)\right]$$

In the **soft limit**, sub-amplitudes factorise into **universal eikonal factors** and **lower-multiplicity amplitudes**

$$S_5\, A_0^L(5_g) = \varepsilon_\mu^{(\lambda)}(5) J^\mu(3,1;5)\, A_0(1_q, 2_b, 3_{q'}, 4_t)$$

$$J^\mu(i,j;k) = \frac{p_i^\mu}{p_i \cdot p_k} - \frac{p_j^\mu}{p_j \cdot p_k}$$

Contract sub-amplitudes to connect different quark lines

$$\begin{aligned} S_5\, 2\mathrm{Re}\left[A_0^L(5_g) A_0^{H\star}(5_g)\right] &= \sum_\lambda \varepsilon_\mu^{(\lambda)}(5)\varepsilon_\nu^{\star(\lambda)}(5) J^\mu(3,1;5) J^\nu(4,2;5)\, |A_0(1_q, 2_b, 3_{q'}, 4_t)|^2 \\ &= -\mathrm{Eik}_{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; k_g) |A_0(1_q, 2_b, 3_{q'}, 4_t)|^2 \end{aligned}$$

$$\mathrm{Eik}_{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; k_g) = J^\mu(3,1;k) J_\mu(4,2;k) = \sum_{\substack{i\in[1,3] \\ j\in[2,4]}} \frac{\lambda_{ij}\, p_i \cdot p_j}{(p_i \cdot p_k)(p_j \cdot p_k)}$$

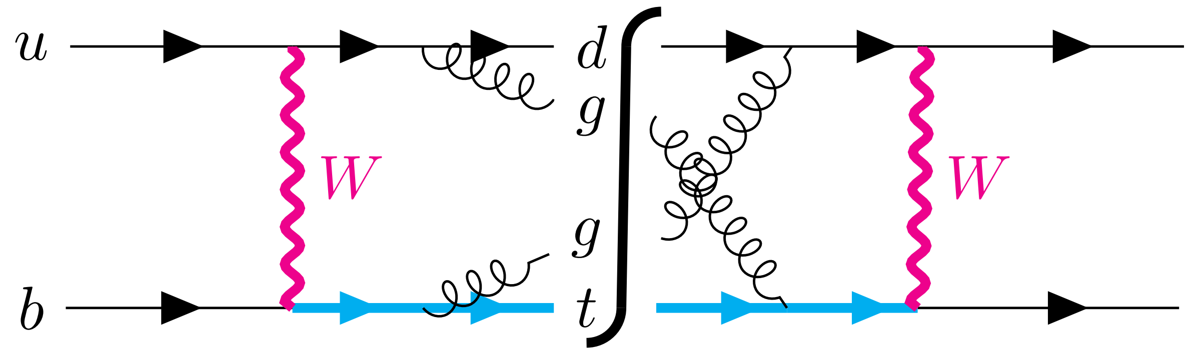
Extracting soft singularities from real corrections (II)

Integrate the eikonal factor over the radiation phase space

$$g_{s,b}^2 \int [\mathrm{d}p_k]\, \mathrm{Eik}_{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; k_g) \equiv \frac{\alpha_s}{2\pi}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} K_{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; \epsilon) = \frac{\alpha_s}{2\pi}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} \frac{1}{\epsilon}\left[\log\left(\frac{p_1\cdot p_4\; p_2\cdot p_3}{p_1\cdot p_2\; p_3\cdot p_4}\right) + \mathcal{O}(\epsilon^0)\right]$$

Double-real correction treated in the same fashion:

- Independent emissions
- Factorised double-soft limit



$$S_6 \left|\mathcal{M}_0(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g)\right|^2_{\mathrm{nf}} = -g_{s,b}^4 \frac{N^2-1}{2} \mathrm{Eik}_{\mathrm{nf}}(6_g)\left[A_0^L(5_g)\, A_0^{H*}(5_g) + \mathrm{c.c.}\right]$$

$$S_5 S_6 \left|\mathcal{M}_0(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g)\right|^2_{\mathrm{nf}} = g_{s,b}^4 (N^2-1)\, \mathrm{Eik}_{\mathrm{nf}}(5_g)\, \mathrm{Eik}_{\mathrm{nf}}(6_g)\left|A_0(1_q, 2_b, 3_{q'}, 4_t)\right|^2$$

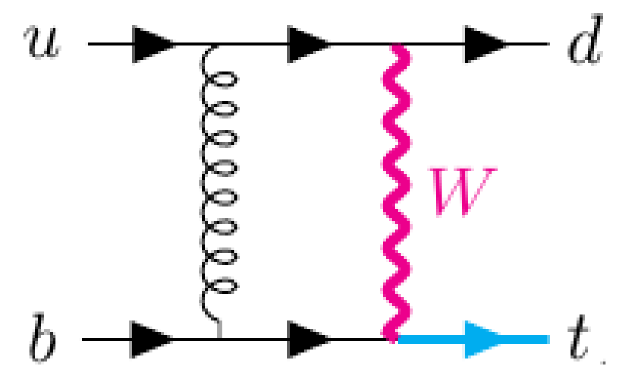
Double-real at **cross-section level** results in a remarkably simple object

$$2s\cdot\sigma_{RR} = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{2N^2}\left(\frac{2E_{\max}}{\mu}\right)^{-4}\left\langle K_{\mathrm{nf}}^2(\epsilon)\, F_{\mathrm{LM}}(1_q, 2_b, 3_{q'}, 4_t)\right\rangle$$
$$-\left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{2}\left(\frac{2E_{\max}}{\mu}\right)^{-2}\left\langle K_{\mathrm{nf}}(\epsilon)\,(I-S_5)\, \widetilde{F}_{\mathrm{LM}}^{\mathrm{nf}}(1_q, 2_b, 3_q, 4_t; 5_g)\right\rangle + \left\langle (I-S_5)(I-S_6)\, F_{\mathrm{LM}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g, 6_g)\right\rangle$$

Extracting soft singularities from virtual corrections

Extract IR singularities from virtual radiation and compute finite contributions.

- One-loop correction to the 4-point amplitude



$$\mathcal{M}_1(1_q, 2_b, 3_{q'}, 4_t) = \frac{\alpha_s}{2\pi}\left(\cdots + t^a_{c_3c_1} t^a_{c_4c_2}\, B_1(1_q, 2_b, 3_{q'}, 4_t)\right)$$

The amplitudes are UV-finite, but IR-divergent:

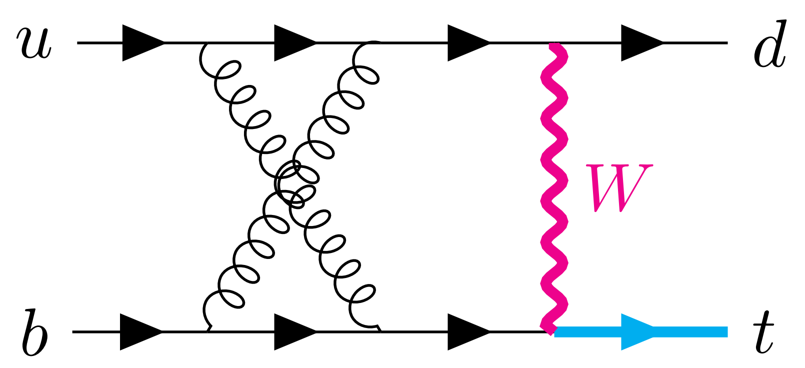
$$B_1(1_q, 2_b, 3_{q'}, 4_t) = I_1(\epsilon)\; A_0(1_q, 2_b, 3_{q'}, 4_t) + B_{1,\text{fin}}(1_q, 2_b, 3_{q'}, 4_t)$$

$$I_1(\epsilon) \equiv \frac{1}{\epsilon}\left[\log\left(\frac{p_1\cdot p_4\; p_2\cdot p_3}{p_1\cdot p_2\; p_3\cdot p_4}\right) + 2\pi i\right]$$

- Two-loop correction, the Abelian nature of the correction leads to the simple pole structure

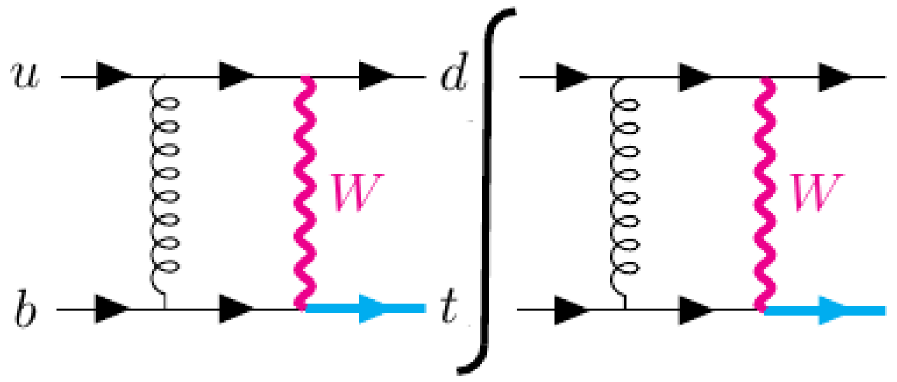
$$\mathcal{M}_2(1_q, 2_b, 3_{q'}, 4_t) = \left(\frac{\alpha_s}{2\pi}\right)^2\left(... + \frac{1}{2}\{t^a, t^b\}_{c_3c_1}\frac{1}{2}\{t^a, t^b\}_{c_4c_2}\; B_2(1_q, 2_b, 3_{q'}, 4_t)\right)$$

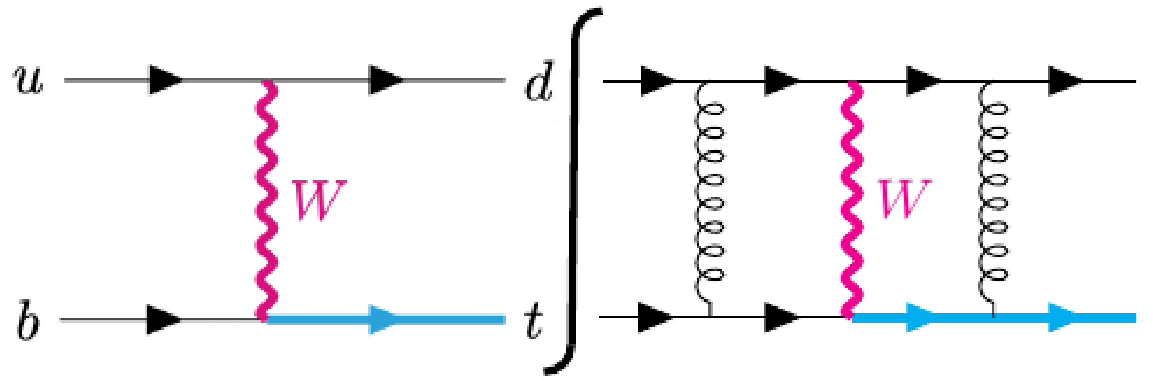
$$B_2(1_q, 2_b, 3_{q'}, 4_t) = -\frac{I_1^2(\epsilon)}{2}\; A_0(1_q, 2_b, 3_{q'}, 4_t) + I_1(\epsilon)\; B_1(1_q, 2_b, 3_{q'}, 4_t) + B_{2,\text{fin}}(1_q, 2_b, 3_{q'}, 4_t)$$



Finite contributions to B_2 comes from both second and last term, we need to know B_1 to $O(\epsilon)$

Double-virtual contributions (I)





Complete double-virtual cross section cast in a very compact expression

$$2s\cdot\sigma_{\rm VV} = \left\langle F^{\rm nf}_{\rm LVV}(1_q,2_b,3_{q'},4_t)\right\rangle = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{4}\left[\frac{2}{N^2}\left\langle \left({\rm Re}\left[I_1(\epsilon)\right]\right)^2 F_{\rm LM}(1_q,2_b,3_{q'},4_t)\right\rangle \right.$$
$$\left. + 2\left\langle {\rm Re}\left[I_1(\epsilon)\right]\widetilde{F}^{\rm nf}_{\rm LV,fin}(1_q,2_b,3_{q'},4_t)\right\rangle + \left\langle \widetilde{F}^{\rm nf}_{\rm VV,fin}(1_q,2_b,3_{q'},4_t)\right\rangle\right]$$

Finite contributions built of one- and two-loop colour-stripped amplitudes

$$\widetilde{F}^{\rm nf}_{\rm LV,fin}(1_q,2_b,3_{q'},4_t) = \mathcal{N}\int {\rm dLips}_{34}\,(2\pi)^d\,\delta^{(d)}(p_1+p_2-p_3-p_4)\,2{\rm Re}\Big[A_0^*(1_q,2_b,3_{q'},4_t)B_{1,\rm fin}(1_q,2_b,3_{q'},4_t)\Big]$$

$$\widetilde{F}^{\rm nf}_{\rm VV,fin}(1_q,2_b,3_{q'},4_t) = \mathcal{N}\int {\rm dLips}_{34}\,(2\pi)^d\,\delta^{(d)}(p_1+p_2-p_3-p_4)\Big\{\left|B_{1,\rm fin}(1_q,2_b,3_{q'},4_t)\right|^2 + 2{\rm Re}\Big[A_0^*(1_q,2_b,3_{q'},4_t)B_{2,\rm fin}(1_q,2_b,3_{q'},4_t)\Big]\Big\}$$

Pole cancellation

We combine RR, RV, and VV but split into contributions of **different multiplicities**

$$\sigma_{\rm nf} = \sigma_{\rm nf}^{(0g)} + \sigma_{\rm nf}^{(1g)} + \sigma_{\rm nf}^{(2g)}$$

and we introduce a finite combination of two the divergent functions

$$\mathcal{W}(1,2,3,4) = \left(\frac{2E_{\rm max}^{-2\epsilon}}{\mu}\right) K_{\rm nf}(\epsilon) - {\rm Re}\left[(I_1(\epsilon)\right] = \mathcal{O}(\epsilon^0)$$

no gluon emissions

$$2s \cdot \sigma_{\rm nf}^{(0g)} = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N_c^2-1}{2N_c^2}\langle \mathcal{W}^2 F_{\rm LM}(1_q,2_b,3_{q'},4_t)\rangle - \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N_c^2-1}{2}\langle \mathcal{W}\widetilde{F}^{\rm nf}_{\rm LV,fin}(1_q,2_b,3_{q'},4_t)\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N_c^2-1}{4}\langle \widetilde{F}^{\rm nf}_{\rm VV,fin}(1_q,2_b,3_{q'},4_t)\rangle$$

one emitted gluon

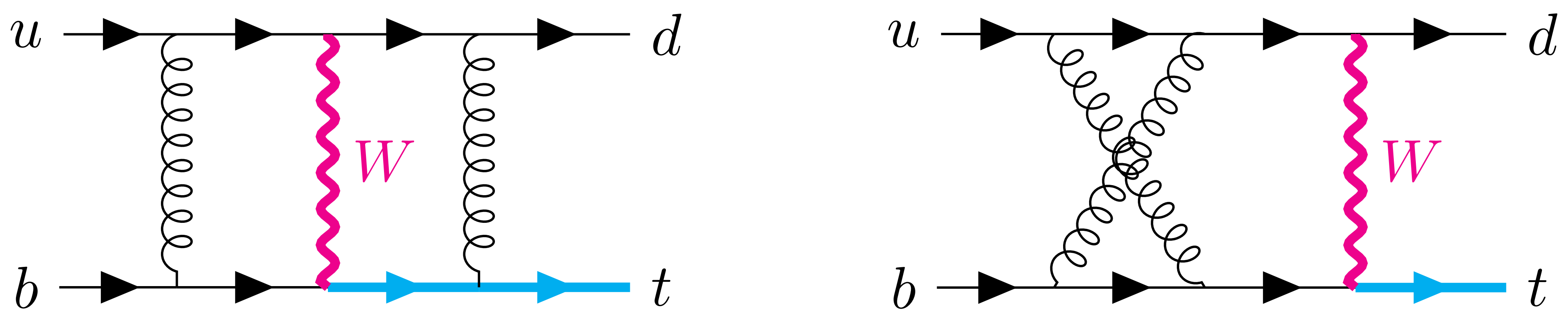
$$2s \cdot \sigma_{\rm nf}^{(1g)} = -\left(\frac{\alpha_s}{2\pi}\right) \frac{N_c^2-1}{2}\langle \mathcal{W}(I-S_5)\widetilde{F}^{\rm nf}_{\rm LM}(1_q,2_b,3_{q'},4_t;5_g)\rangle + \left(\frac{\alpha_s}{2\pi}\right) \frac{N_c^2-1}{4}\langle (I-S_5)\widetilde{F}^{\rm nf}_{\rm LV,fin}(1_q,2_b,3_{q'},4_t;5_g)\rangle$$

two resolved gluons

$$2s \cdot \sigma_{\rm nf}^{(2g)} = \langle (I-S_5)(I-S_6)F^{\rm nf}_{\rm LM}(1_q,2_b,3_{q'},4_t;5_g,6_g)\rangle$$

Achievement: **local pole cancellation** and simple form due to **abelian nature of non-factorisable contribution**

Double-virtual contributions (II)



$$q\, b \to q'\, t$$

4 kinematic scales: s, t, m_W, m_t

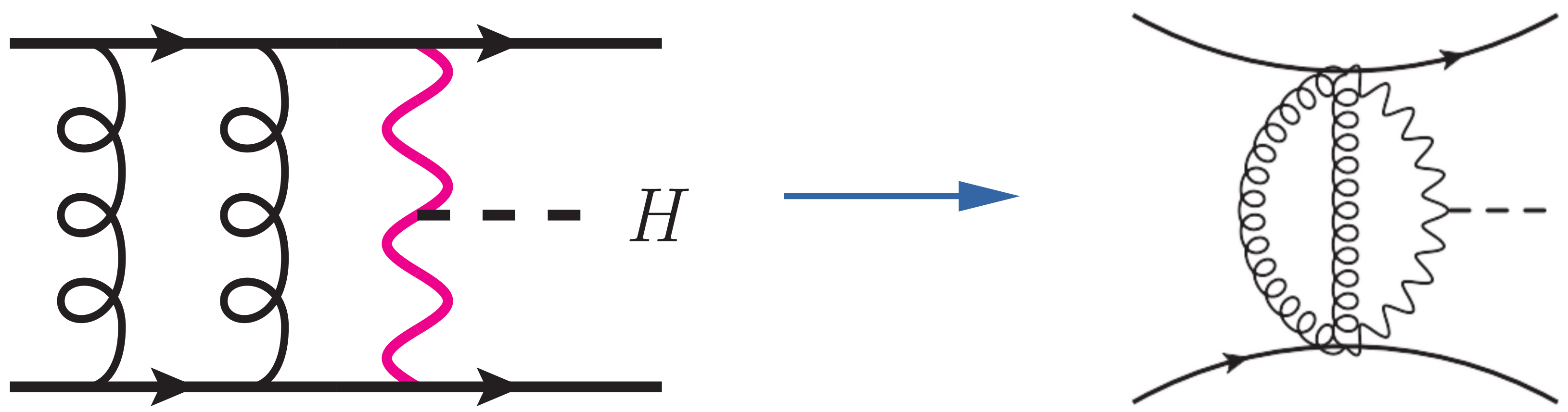
18 diagrams: all topologies maximal

analytic reduction with KIRA [Klappert, Lange, Maierhöfer, Usovitsch 2021]

428 masters evaluated numerically using the auxiliary mass flow [Liu, Ma, Wang 2018] *

10 sets of 10k points extracted from a VEGAS grid prepared for the LO process

details in [Brønnum-Hansen, Melnikov, Quarroz, Wang 2021]



$$q\, Q \to q'\, Q' + H$$

7 kinematic scales: $s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_W, m_t$

eikonal approximation** [Liu, Melnikov, Penin 2019] [Gates 2023]

validity ensured by kinematic signature and VBF cuts

anti-k_t	2 jets, $R = 0.4$
jet transverse momentum	$p_{\perp,j} > 25$ GeV
jet rapidity	$\lvert y_j \rvert < 4.5$
jet separation	$\lvert y_{j_1} - y_{j_2} \rvert > 4.5$
invariant mass of jets	$M_{jj} > 600$ GeV
separate hemispheres	$y_{j_1} y_{j_2} < 0$

* recent progress with analytic results for master integrals [Syrrakos 2023] & [Wu, Long, 2303.08814]

** double-virtual in *eikonal approximation* here, recent result for sub-eikonal correction [Long, Melnikov, Quarroz 2305.12937]

Single top production: Results at 13 TeV (I)

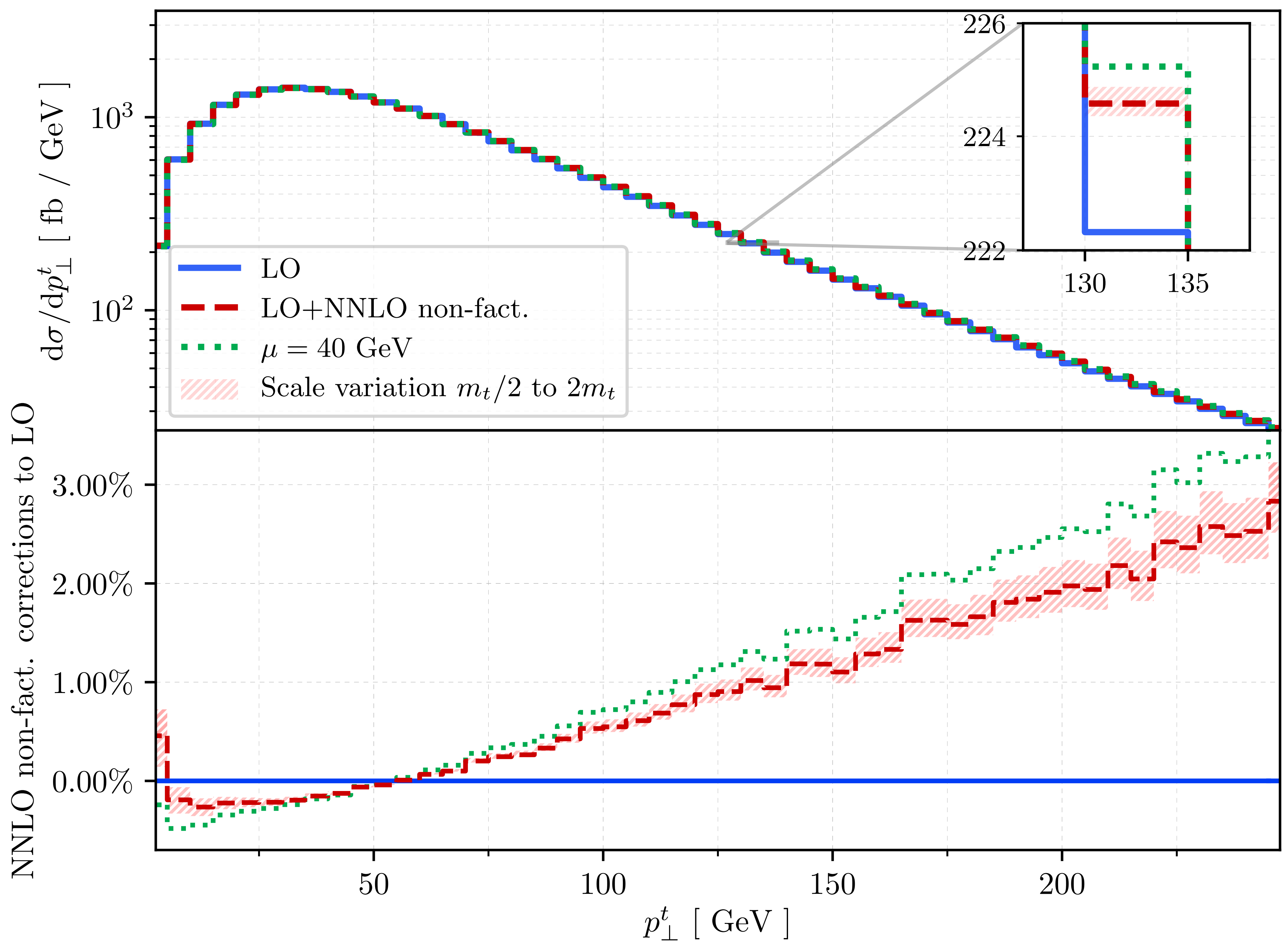
pp collision: $\sqrt{s} = 13\,\mathrm{TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO $m_W = 80.379\,\mathrm{GeV},\ m_t = 173.0\,\mathrm{GeV}, \alpha_S(m_t) = 0.108,\ \mu_F = m_t$

$$\frac{\sigma_{pp\to X+t}}{1\ \mathrm{pb}} = 117.96 + 0.26\left(\frac{\alpha_s(\mu_R)}{0.108}\right)^2$$

- Non-factorisable corrections are $0.22^{-0.04}_{+0.05}\,\%$ LO for $\mu_R = m_t$.
- **Theoretical uncertainties** are estimated through **scale variation**: $\mu_R \in [m_t/2,\ 2m_t]$.
- **Unclear optimal scale choice**: non-factorisable corrections appear for the first time at NNLO → no indication from lower orders.
- For $\mu_R = 40$ GeV (typical **momentum transfer scale** of top quark) non-factorisable corrections are $0.35\,\%$ LO.
- In comparison, **NNLO factorisable** corrections to **NLO cross section** are around $0.7\,\%$.

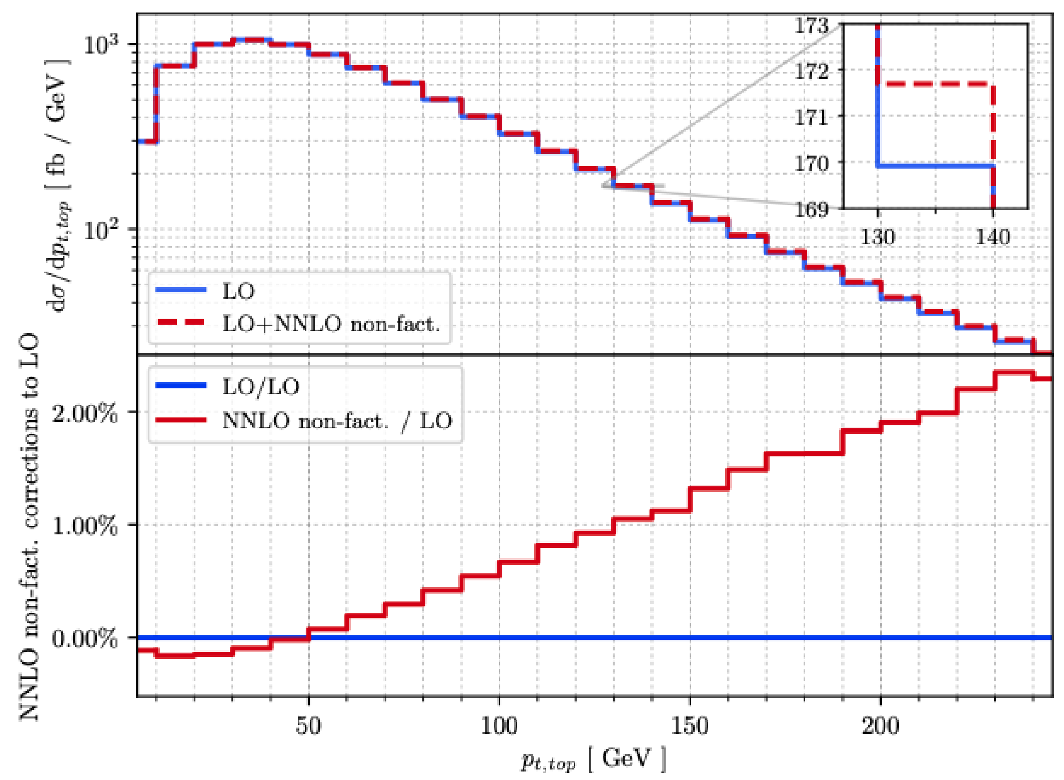
Single top production: Results at 13 TeV (II)

pp collision: $\sqrt{s} = 13\,\mathrm{TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO $m_W = 80.379\,\mathrm{GeV},\ m_t = 173.0\,\mathrm{GeV}, \alpha_S(m_t) = 0.108,\ \mu_F = m_t$



[Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile,Wang 2022]

- Non-factorisable corrections are $p_\perp^t$ dependent.
- Non-factorisable corrections are small and negative at low values of $p_\perp^t$
- They vanish at $p_\perp^t \sim 50\,\mathrm{GeV}$ (in agreement with results for virtual corrections)
- **Factorisable corrections** vanish around $p_\perp^t \sim 30\,\mathrm{GeV}$.
- Factorisable and non-factorisable corrections are comparable in the region around the maximum of the $p_\perp^t$ distribution.



[Brønnum-Hansen, Melnikov, Quarroz, Wang 2021]

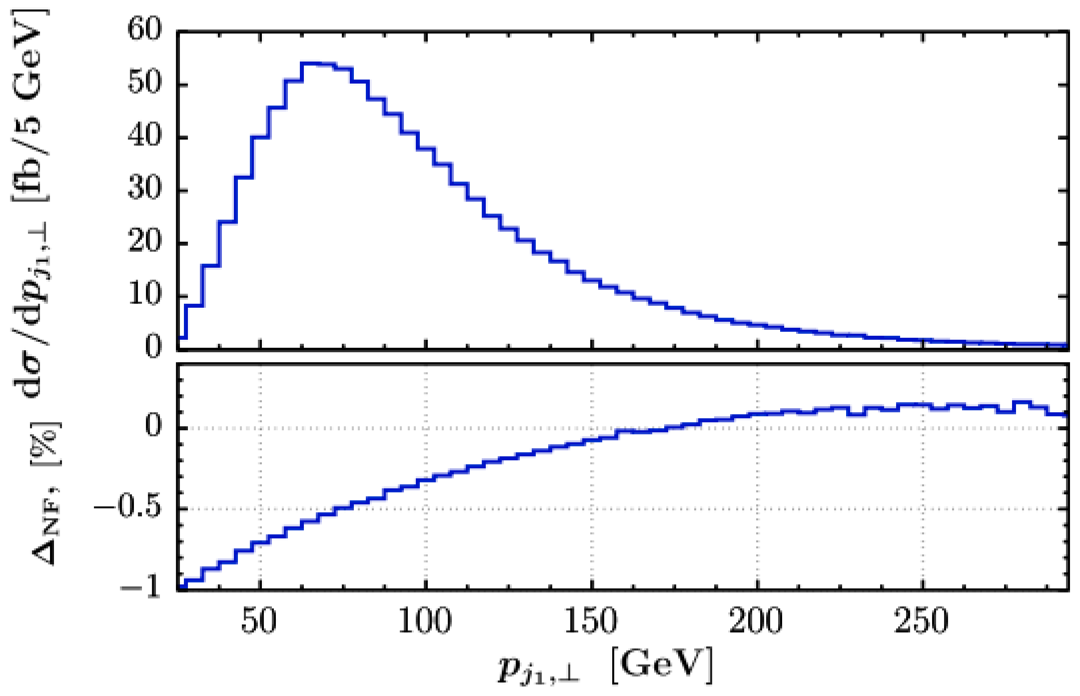
VBF Higgs production: Results at 13 TeV (I)

pp collision: $\sqrt{s} = 13\,\text{TeV}$, PDFs: NNPDF31-nnlo-as-118

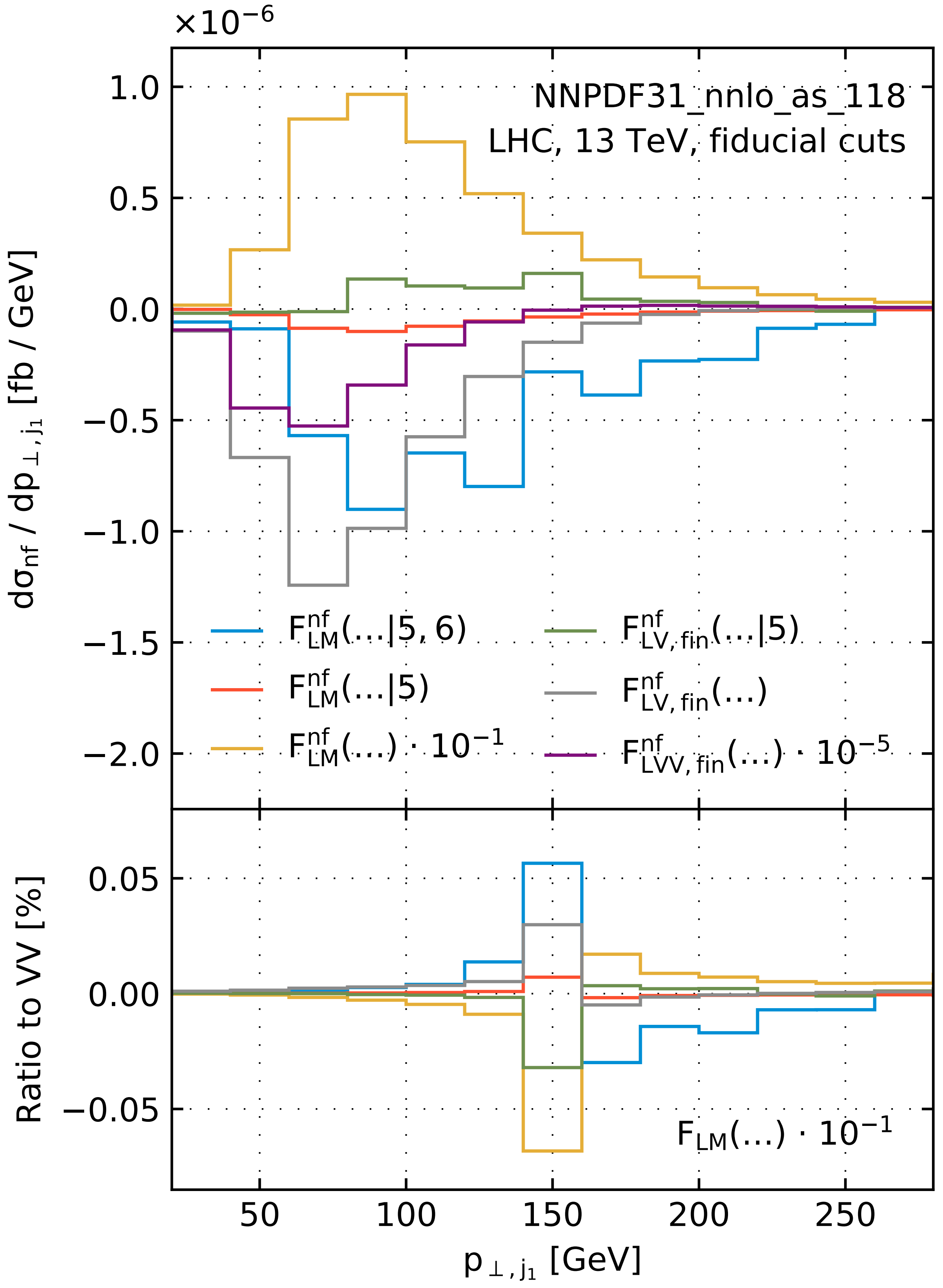
$$\sigma_{\text{nf}} = -3.1\,\text{fb} \qquad \mu_R = \mu_F = \sqrt{\frac{m_H}{2}\sqrt{\frac{m_H^2}{4} + p_{\perp,H}^2}}$$

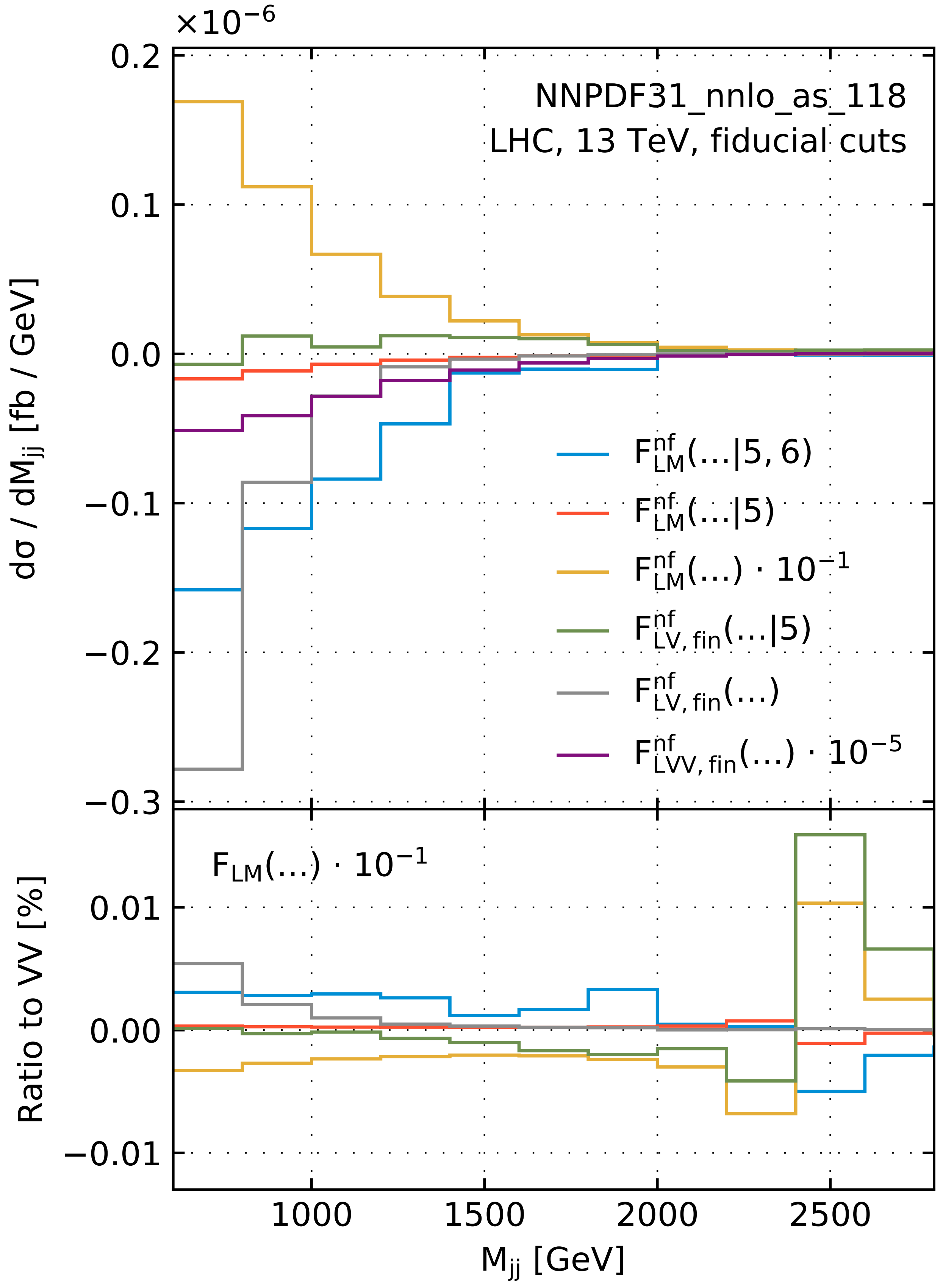
$$m_H = 125.0\,\text{GeV},\ m_W = 80.398\,\text{GeV},\ m_Z = 91.1876\,\text{GeV}, \alpha_S(m_Z) = 0.118$$

- Non-factorisable corrections are $0.5\,\%$ of factorisable through NNLO
- Double-virtual accounts for $99.99\,\%$
- For $\mu_R = \mu_F$ scale variation is $\mathcal{O}(40)\,\%$



[Liu, Melnikov, Penin 2019]





[Asteriadis, Brønnum-Hansen, Melnikov 2305.08016]

VBF Higgs production: Results at 13 TeV (II)

To **estimate the real emission contributions** to the cross section, we consider the quantity

$$L(1,2,3,4) = \ln\left(\frac{p_1\cdot p_4\; p_3\cdot p_2}{p_1\cdot p_2\; p_3\cdot p_4}\right)$$

VBF kinematics is **characterised by two hard jets** that are nearly collinear to the beam axis

$$p_3 = \alpha_3\, p_1 + \beta_3\, p_2 + p_{3,\perp} \qquad \alpha_3, \beta_4 \sim 1 \qquad \beta_3 \sim \frac{p_{3,\perp}^2}{s} \ll 1\,, \quad \alpha_4 \sim \frac{p_{4,\perp}^2}{s} \ll 1$$
$$p_4 = \alpha_4\, p_1 + \beta_4\, p_2 + p_{4,\perp}$$

With this approximation, we get

$$L(1,2,3,4) = -\ln\left(1 + \frac{\beta_3\,\alpha_4}{\alpha_3\,\beta_4} - \frac{2\,\vec{p}_{3,\perp}\cdot\vec{p}_{4,\perp}}{s\,\alpha_3\,\beta_4}\right)$$
$$\approx \frac{2\vec{p}_{3,\perp}\cdot\vec{p}_{4,\perp}}{s} \approx 10^{-2}$$

typical values

$$|\vec{p}_{i,\perp}| \sim 60\,\mathrm{GeV}$$
$$\sqrt{s} \sim 600\,\mathrm{GeV}$$

and we **estimate the contribution from two soft gluons** and compare to the Glauber phase enhanced double-virtual

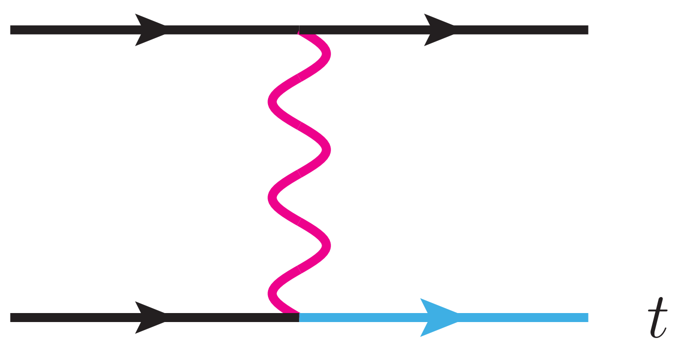
$$\sigma_{RR} \sim N_c^2\langle L^2(1,2,3,4) F_{\rm LM}^{\rm nf}(1_q,2_q,3_q,4_q)\rangle \sim 10^{-4}\,\sigma_{\rm LO}\,,$$

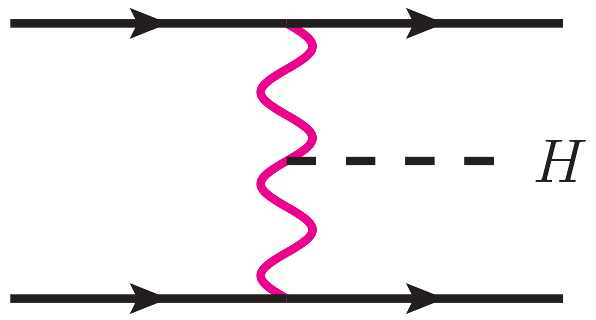
$$\sigma_{VV} \sim N_c^2\langle\, \chi_{\rm nf}(1,2,3,4)\; F_{\rm LM}^{\rm nf}(1,2,3,4)\rangle \approx 10\,\sigma_{\rm LO}$$

$$\boxed{\pi^2}$$

Summary

	$q\,b \to q'\,t$	$q\,Q \to q'\,Q' + H$
NNLO complete	YES	YES*
dominant double-virtual	YES	YES (by 10^5)
size of inclusive correction	+0.22 % of LO	-0.39 % of LO
size of differential corrections	$\mathcal{O}(1\%)$	$\mathcal{O}(1\%)$





* double-virtual in *eikonal approximation* here, recent result for sub-eikonal correction in [Long, Melnikov, Quarroz 2305.12937] reductions to leading eikonal of $\mathcal{O}(20\%)$

- In certain regions of phase space, the **non-factorisable contributions are comparable to the factorisable ones**.
- If percent precision for these processes can be reached, the **non-factorisable effects will have to be taken into account.**

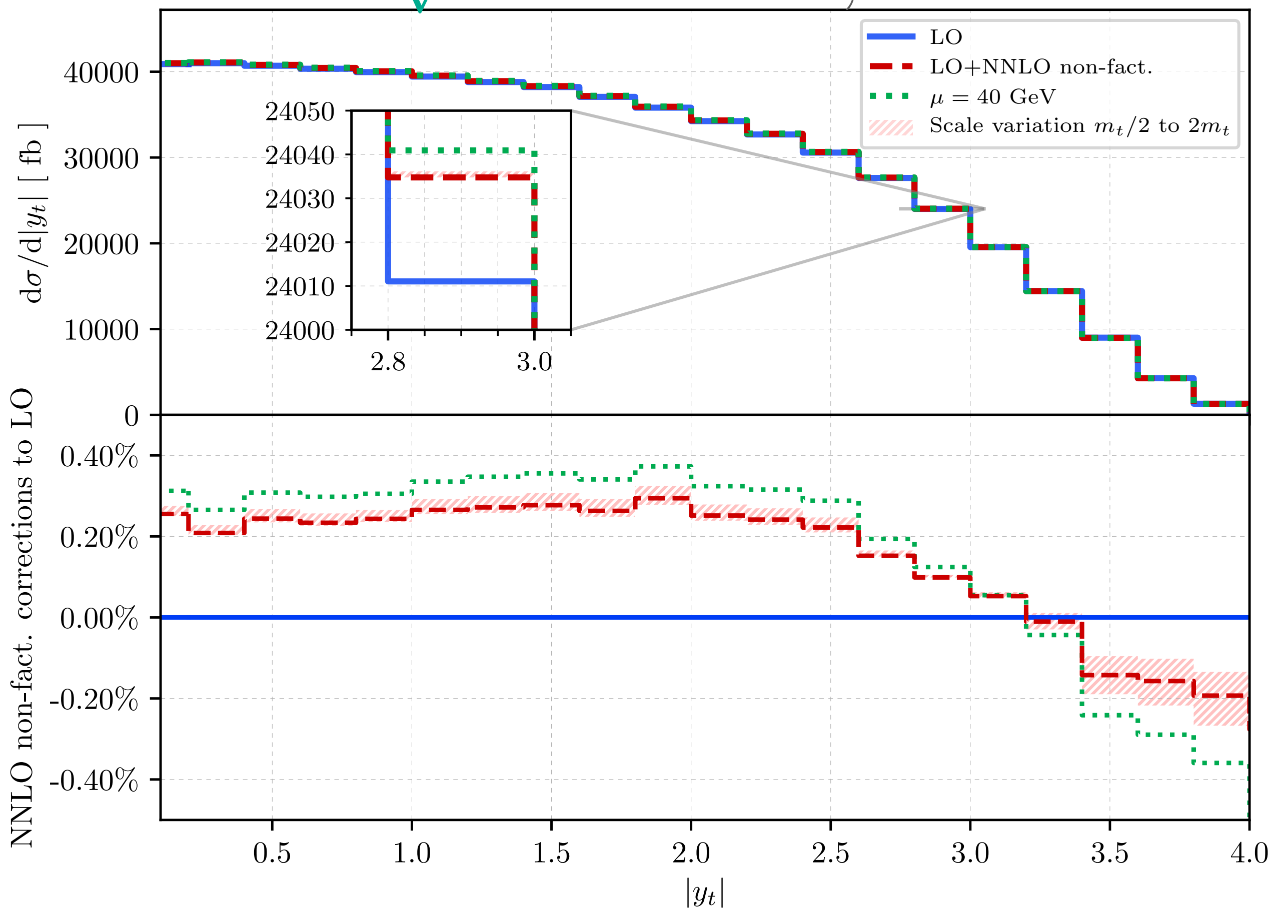
Thank you for your attention

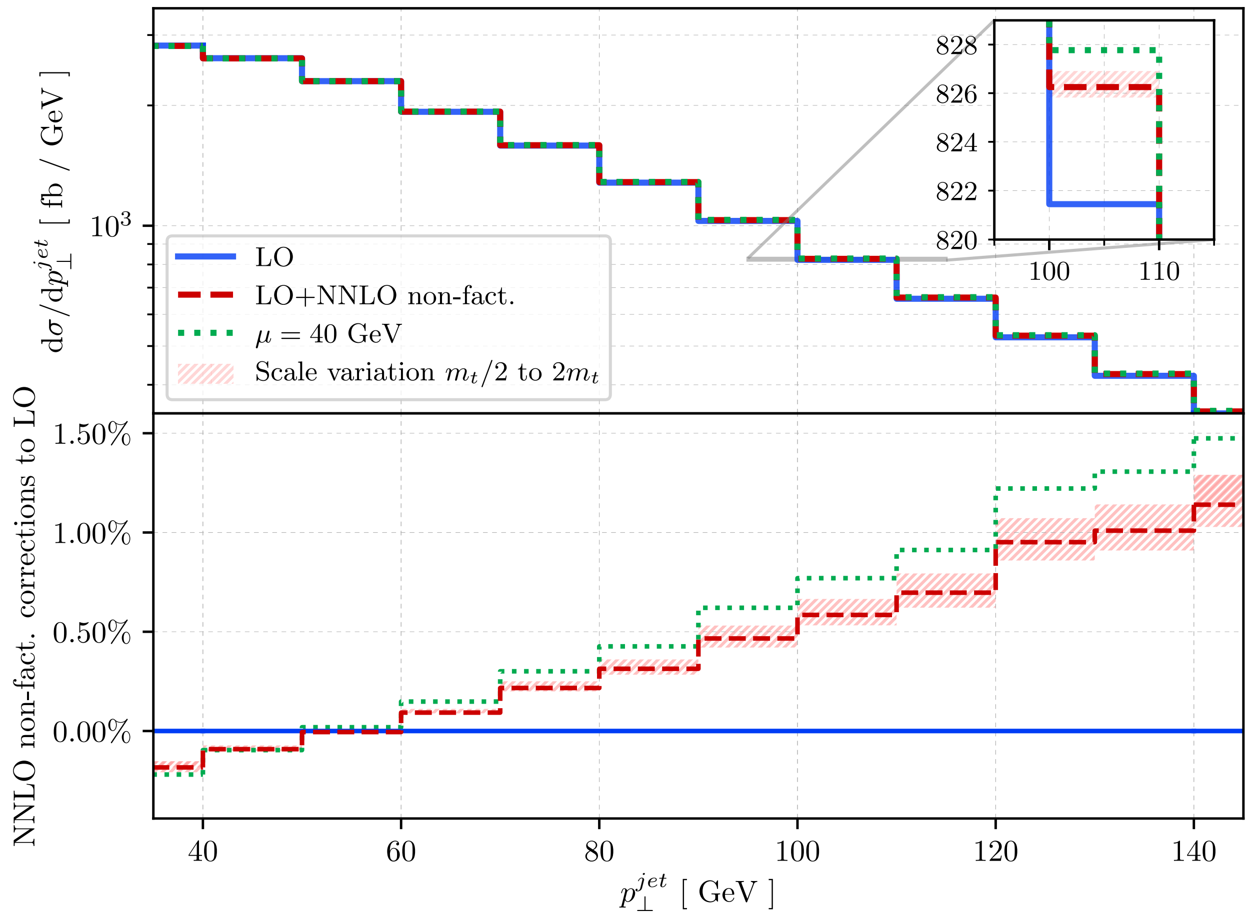
Results at 13 TeV (III)

Differential cross section:

pp collision: $\sqrt{s} = 13$ TeV, PDFs: CT14_lo@LO, CT14_nnlo@NNLO

$$m_W = 80.379\,\mathrm{GeV},\ m_t = 173.0\,\mathrm{GeV}, \alpha_S(m_t) = 0.108,\ \mu_F = m_t$$





- Relative non-factorisable **correction** to **top-quark rapidity fairly** $|y_t| \leq 2.5$, $\mathcal{O}(0.25\%)$
- **Sign change** around $|y_t| \sim 3$
- **Factorisable** corrections change sign around $|y_t| \sim 1.2$
- For some top-quark rapidity values, **factorisable and non-factorisable** correction become **quite comparable**.
- k_t**-algorithm** to define jets $p_\perp^{\mathrm{jet}} > 30\,\mathrm{GeV}$, $R = 0.4$.
- **Non-factorisable** corrections reach 1.2 % at $p_\perp^{\mathrm{jet}} \sim 140\,\mathrm{GeV}$

Results at 100 TeV

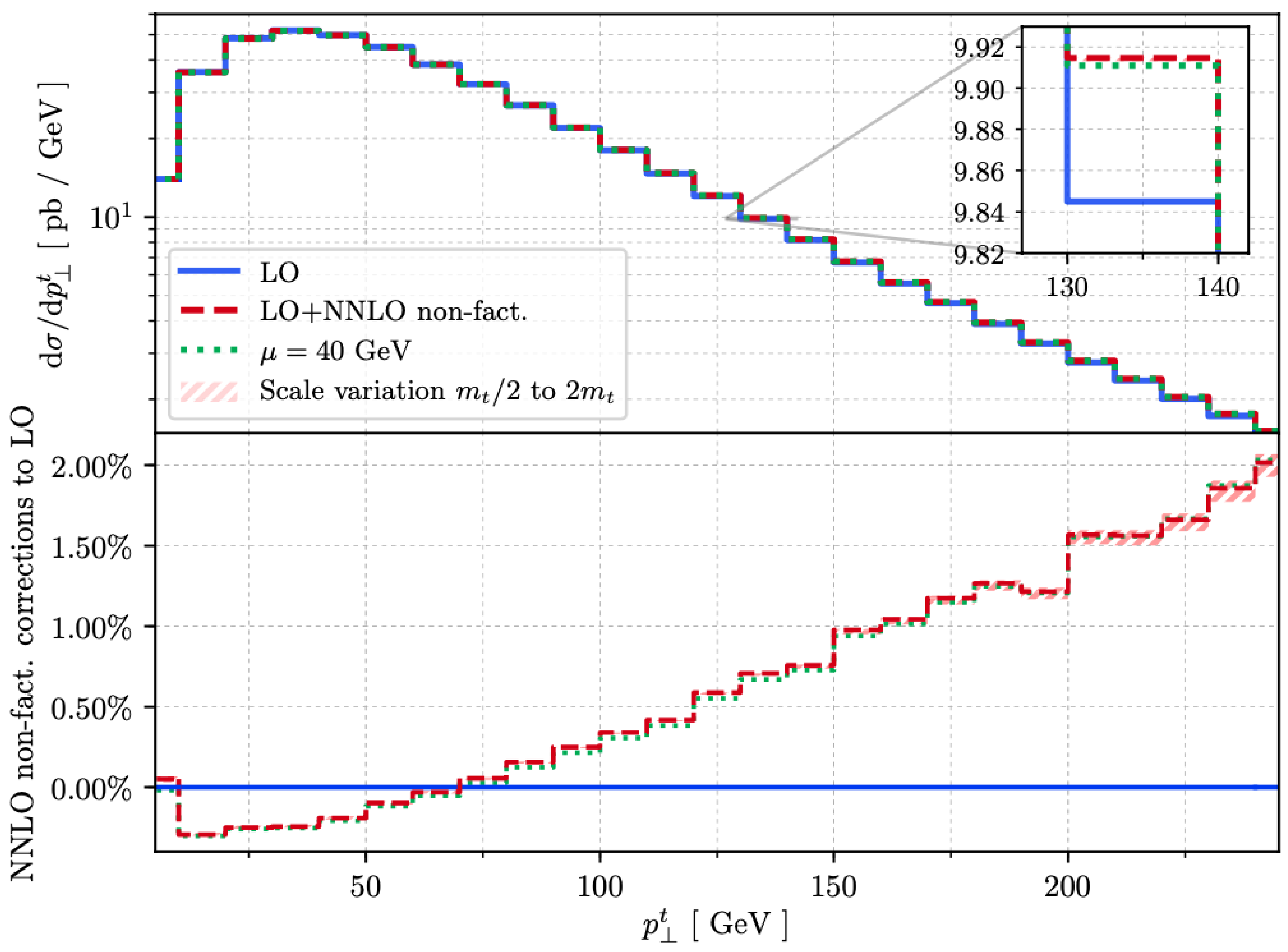
Differential cross section:

pp collision: $\sqrt{s} = 100\ \mathrm{TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO $m_W = 80.379\,\mathrm{GeV},\ m_t = 173.0\,\mathrm{GeV}, \alpha_S(m_t) = 0.108,\ \mu_F = m_t$

$$\frac{\sigma_{pp\to X+t}}{1\ \mathrm{pb}} = 2367.0 + 3.8\left(\frac{\alpha_s(\mu_R)}{0.108}\right)^2$$

- Non-factorisable corrections are $0.16\ \%$ LO for $\mu_R = m_t$.
- $p_\perp^t$ peaks around 40 GeV, changes sign around 70 GeV.
- For $\mu_R = 40$ GeV non-factorisable corrections are $0.25\ \%\ LO$.

		$\mu_R = m_t$		$\mu_R = 40$ GeV	
$p_\perp^{t,\mathrm{cut}}$	σ_{LO} (pb)	$\sigma^{\mathrm{nf}}_{\mathrm{NNLO}}$ (pb)	δ_{NNLO} [%]	$\sigma^{\mathrm{nf}}_{\mathrm{NNLO}}$ (pb)	δ_{NNLO} [%]
0 GeV	2367.02	$3.79^{-0.63}_{\ 0.84}$	$0.16^{-0.03}_{\ 0.04}$	5.95	0.25
20 GeV	2317.03	$3.89^{-0.64}_{\ 0.86}$	$0.17^{-0.03}_{\ 0.04}$	6.11	0.26
40 GeV	2216.61	$4.14^{-0.69}_{\ 0.92}$	$0.19^{-0.03}_{\ 0.04}$	6.50	0.29
60 GeV	2121.88	$4.28^{-0.71}_{\ 0.95}$	$0.20^{-0.03}_{\ 0.04}$	6.71	0.32



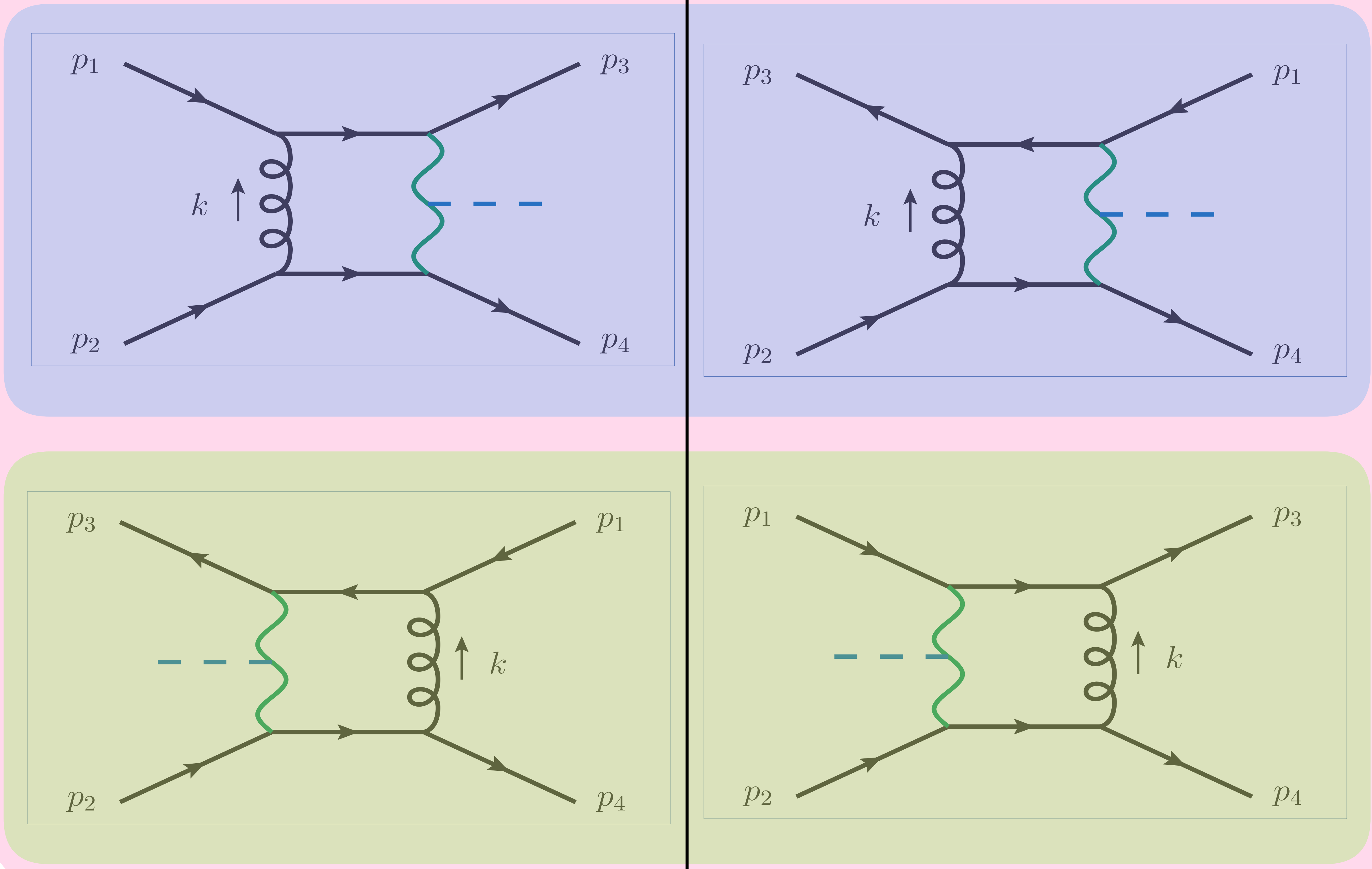
Non-factorisable corrections: why? (III)

Eikonal approximation of virtual contribution to VBF Higgs *[Liu, Melnikov, Penin '19]*

$$\sqrt{s} \gtrsim 600 \text{ GeV}$$
$$p_{j,\perp} \sim 100 \text{ GeV}$$
$$k_\perp \sim m_h \ll \sqrt{s}/2$$

$$\frac{\gamma_\mu\left(k^\mu + p^\mu_{1,2}\right)}{\left(k+p_{1,2}\right)^2 + i\varepsilon} \approx \frac{\gamma_\mu p^\mu_{1,2}}{2k\cdot p_{1,2} + i\varepsilon} = \frac{\gamma^\mp}{2k^\mp + i\varepsilon}$$

$$\frac{\gamma_\mu\left(k^\mu - p^\mu_{1,2}\right)}{\left(k-p_{1,2}\right)^2 + i\varepsilon} \approx \frac{-\gamma_\mu p^\mu_{1,2}}{-2k\cdot p_{1,2} + i\varepsilon} = \frac{\gamma^\mp}{2k^\mp - i\varepsilon}$$



$$\frac{1}{2k^- + i\varepsilon} - \frac{1}{2k^- - i\varepsilon} \to -i\pi\delta\left(k^-\right)$$

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{-i\pi\delta(k^-)}{k^2(2k^+ - i\varepsilon)[(k+p_1-p_3)^2 - m_W^2][(k-p_2+p_4-m_W^2)]}$$

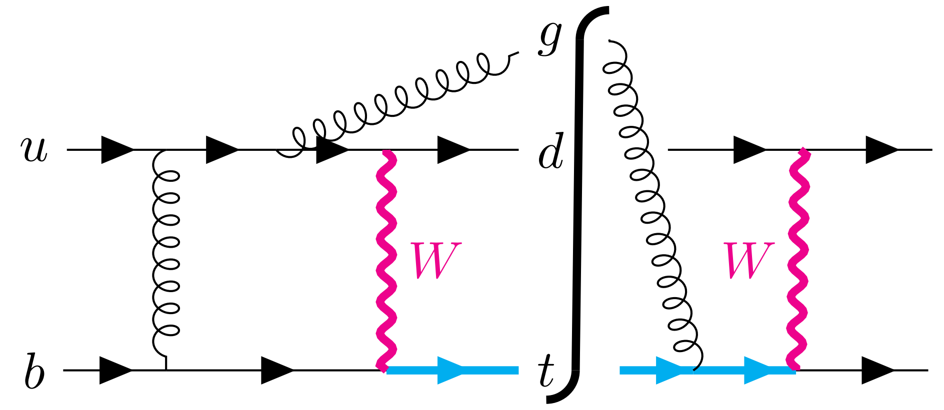
$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{+i\pi\delta(k^-)}{k^2(2k^+ + i\varepsilon)[(k+p_1-p_3)^2 - m_W^2][(k-p_2+p_4-m_W^2)]}$$

$$\frac{1}{2k^+ + i\varepsilon} - \frac{1}{2k^+ - i\varepsilon} \to -i\pi\delta\left(k^+\right)$$

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\boxed{\pi^2}\delta(k^-)\delta(k^+)}{k^2[(k+p_1-p_3)^2 - m_W^2][(k-p_2+p_4-m_W^2)]}$$

Could single top show similar enhancement?

Real-virtual contribution



Complete real-virtual cross section cast in a very compact expression

$$2s \cdot \sigma_{\mathrm{RV}} = \int [\mathrm{d}p_5]\, F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) = \left\langle S_5 F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g)\right\rangle + \left\langle (I - S_5) F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g)\right\rangle$$

$$\left\langle S_5\, F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g)\right\rangle = -\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{N^2} \left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} \left\langle K_{\mathrm{nf}}(\epsilon)\, \mathrm{Re}[I_1(\epsilon)]\, F_{\mathrm{LM}}(1_q, 2_b, 3_{q'}, 4_t)\right\rangle - \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{2} \left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} \left\langle K_{\mathrm{nf}}(\epsilon)\, \widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t)\right\rangle$$

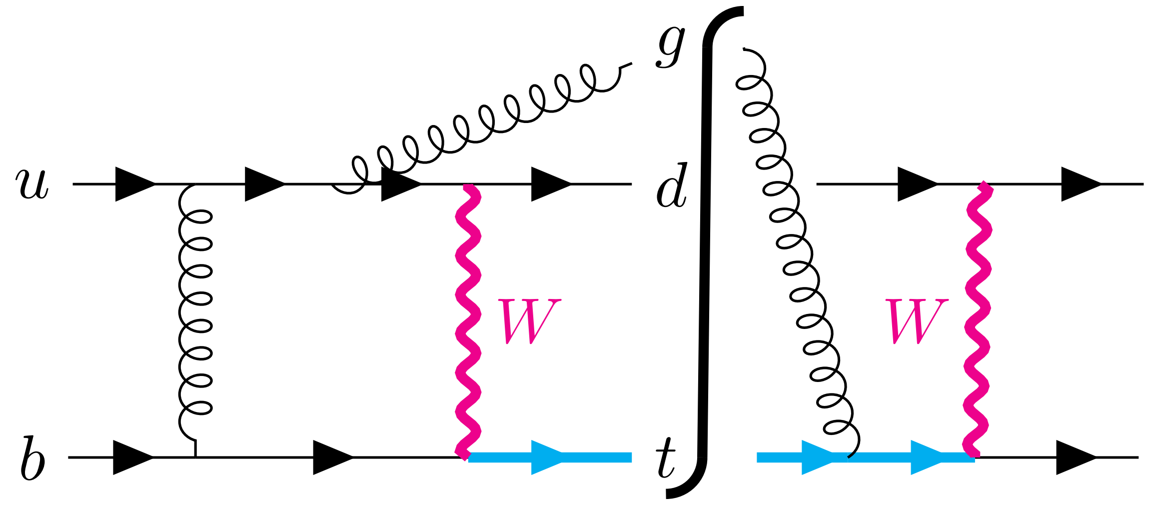
$$\left\langle (I - S_5) F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g)\right\rangle = \left(\frac{\alpha_s}{2\pi}\right) \frac{N^2-1}{2} \left\langle \mathrm{Re}[I_1(\epsilon)] \left\langle (I - S_5)\, \widetilde{F}_{\mathrm{LM}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g)\right\rangle + \left(\frac{\alpha_s}{2\pi}\right) \frac{N^2-1}{4} \left\langle (I - S_5)\, \widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g)\right\rangle\right.$$

Finite contributions built on one-loop, 4-point and 5-point colour-stripped amplitudes

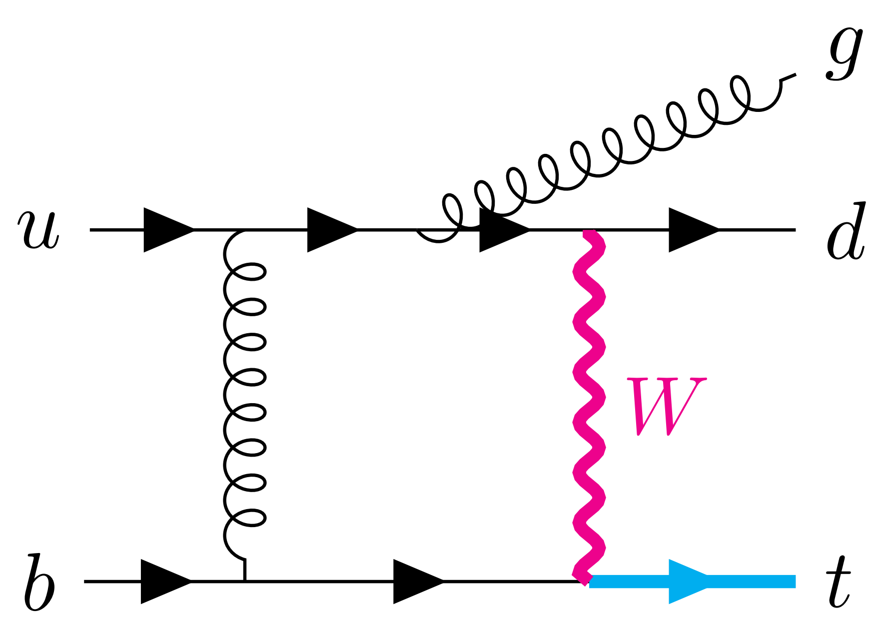
$$\widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t) = \mathcal{N} \int \mathrm{dLips}_{34}\, (2\pi)^d\, \delta^{(d)}(p_1 + p_2 - p_3 - p_4)\, 2\mathrm{Re}\Big[A_0^*(1_q, 2_b, 3_{q'}, 4_t) B_{1,\mathrm{fin}}(1_q, 2_b, 3_{q'}, 4_t)\Big]$$

$$\widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) = \mathcal{N} \int \mathrm{dLips}_{34}\, (2\pi)^d\, \delta^{(d)}\Big(p_1 + p_2 - \sum_{i=3}^{5} p_i\Big) g_{s,b}^2 \Big(A_0^{L*}(5_g) B_{1,\mathrm{fin}}^{sH}(5_g) + A_0^{H*}(5_g) B_{1,\mathrm{fin}}^{sL}(5_g) + \mathrm{c.c.}\Big)$$

Real-virtual contribution: increasing numerical stability



$d\hat{\sigma}^{\text{nf}}_{\text{RV}} : \mathscr{M}_5^{(1)} \otimes \mathscr{M}_5^{(0)}$



$\mathscr{M}_5^{(1)}(1_u, 2_b, 3_d, 4_t, 5_g)$

The real-virtual amplitude is written in terms of **109** box, triangle, and bubble integrals

We switch to a basis of **finite box integrals**, e.g.

$$I_{4,1} = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k-p_1)^2 (k-p_1-p_2)^2 (k-p_1-p_2+p_5)^2}$$

This integral is IR divergent when one of the propagators go on-shell. We can regulate these divergences by insertion of a numerator *[Badger, Mogull, Peraro '16]*

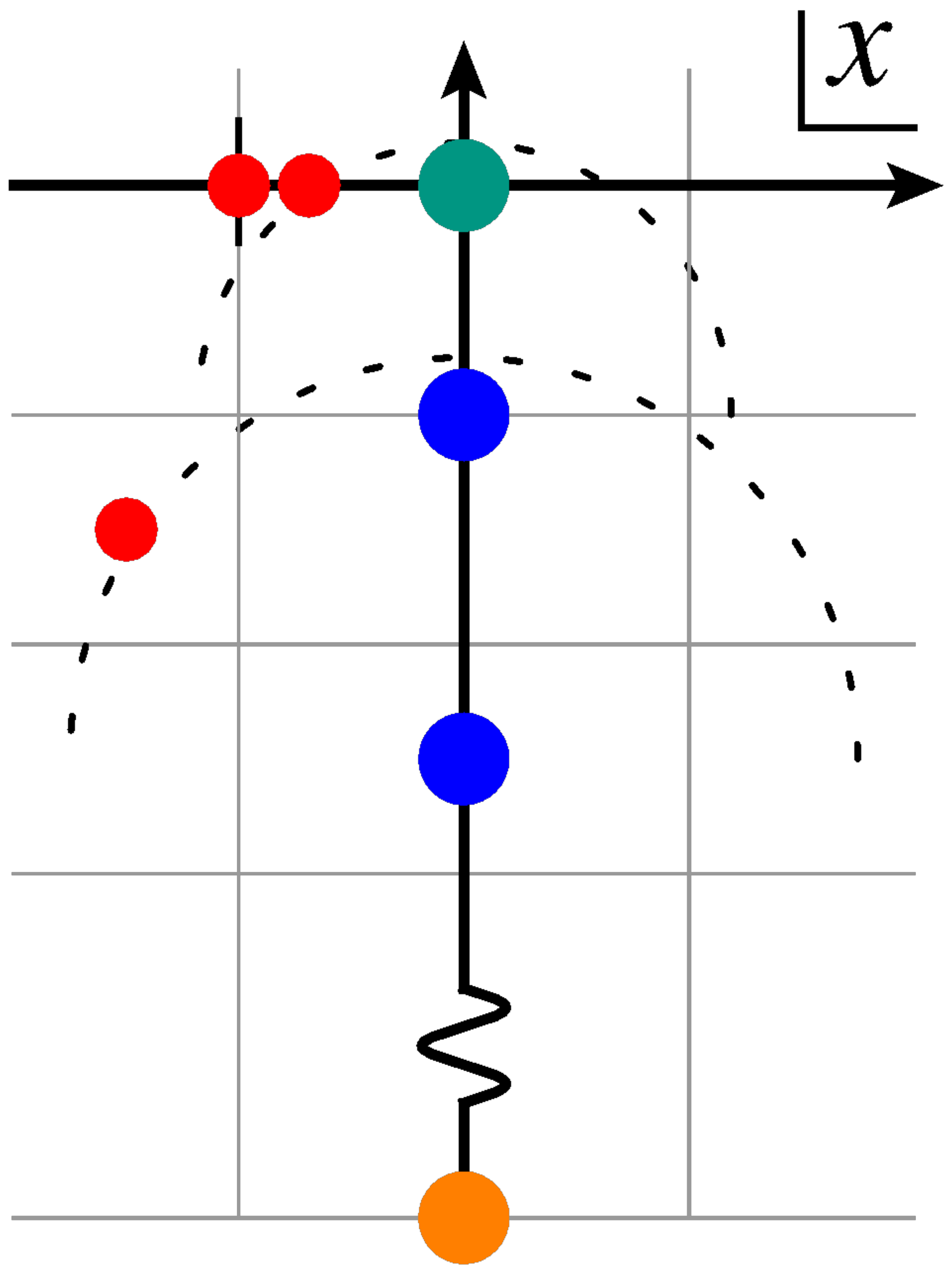
$$\begin{aligned} \text{tr}\left(p_1(k-p_1)(k-p_1-p_2)p_5\right) = & \, s_{12}(s_{12}+s_{15}-s_{34}) - (s_{12}+s_{15}-s_{34})\, k^2 \\ & + (s_{12}-s_{34})\,(k-p_1)^2 - (s_{12}+s_{15})\,(k-p_1-p_2)^2 \\ & + s_{12}\,(k-p_1-p_2+p_5)^2 \end{aligned}$$

In finite box coefficients we can set $\epsilon \to 0$ and triangle coefficients become ϵ - independent

Further stability achieved by switching to variables

$$s_{12}\,, \quad s_{23}\,, \quad \delta_1 = s_{34} - s_{12}\,, \quad \delta_2 = s_{45} - m_t^2\,, \quad \delta_3 = s_{15}$$

Double-virtual contribution: amplitude calculation 2108.09222



Expand integrals $\boldsymbol{I}$ around the boundary in variable $y = x^{-1} = 0$

$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_k^N \sum_l \boldsymbol{c}_{jkl} y^k \ln^l y + \dots$$

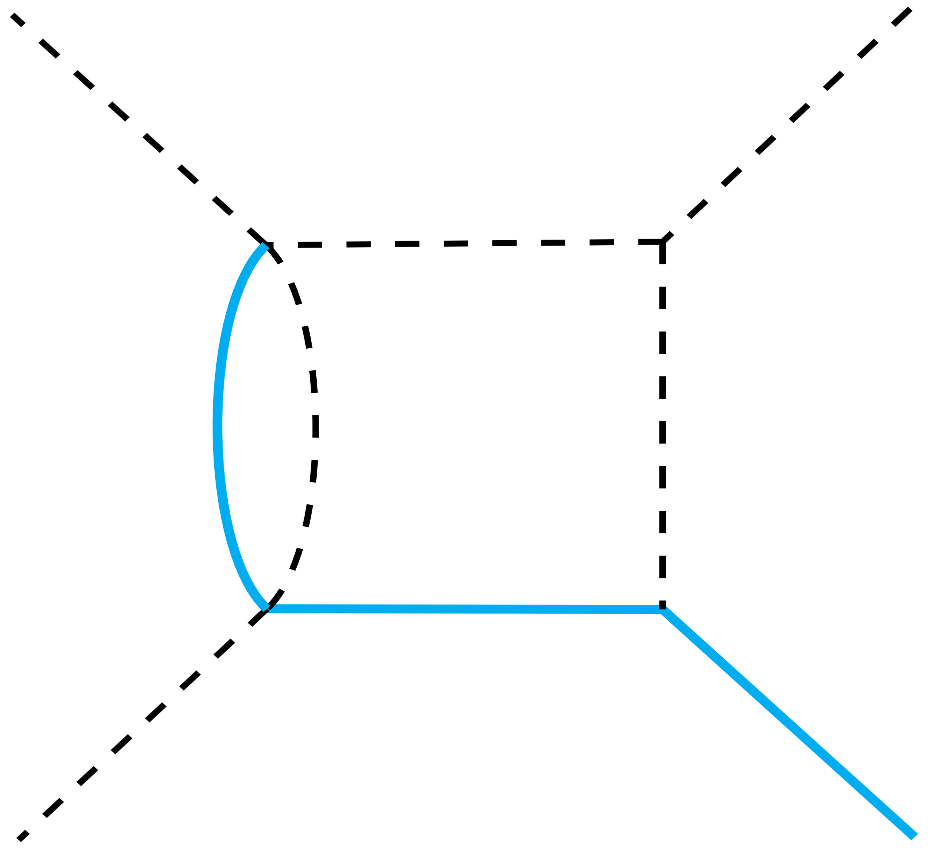
Evaluate and expand around regular points x:

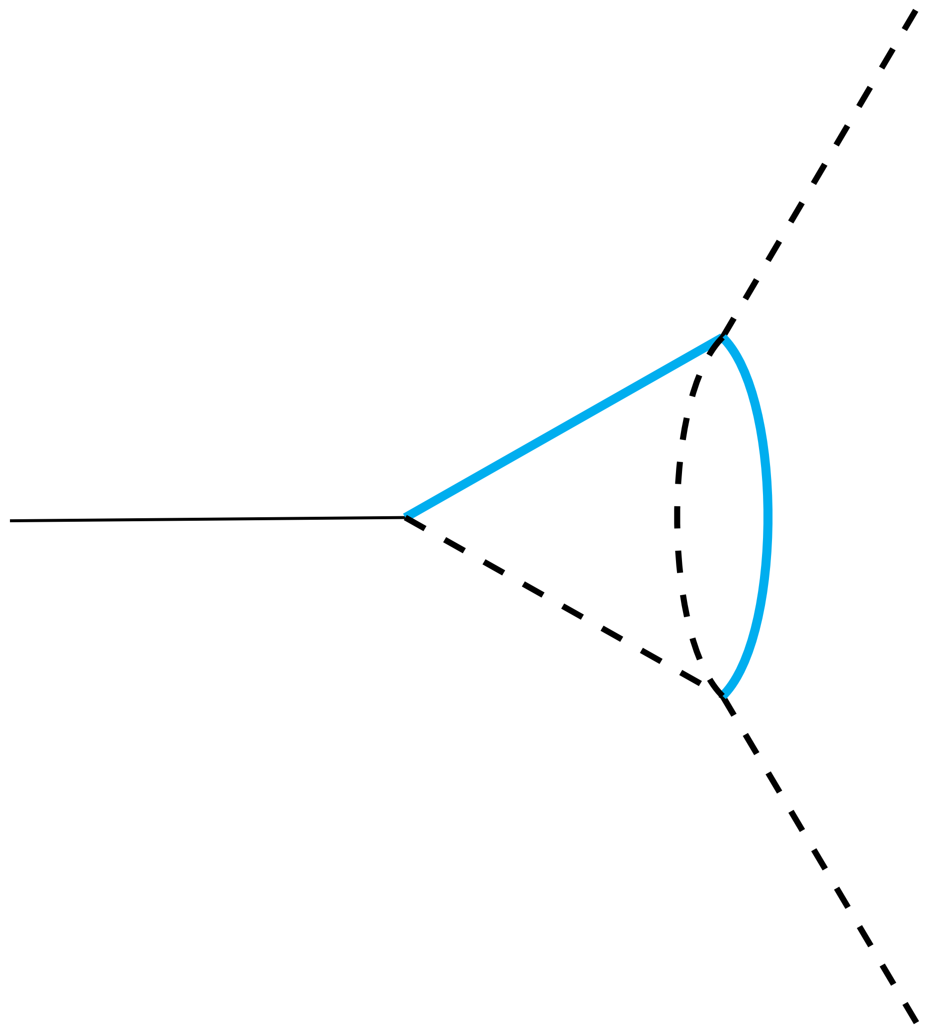
$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_k^N \boldsymbol{c}_{jk} x^k + \dots$$

Repeat previous step until reaching the physical mass

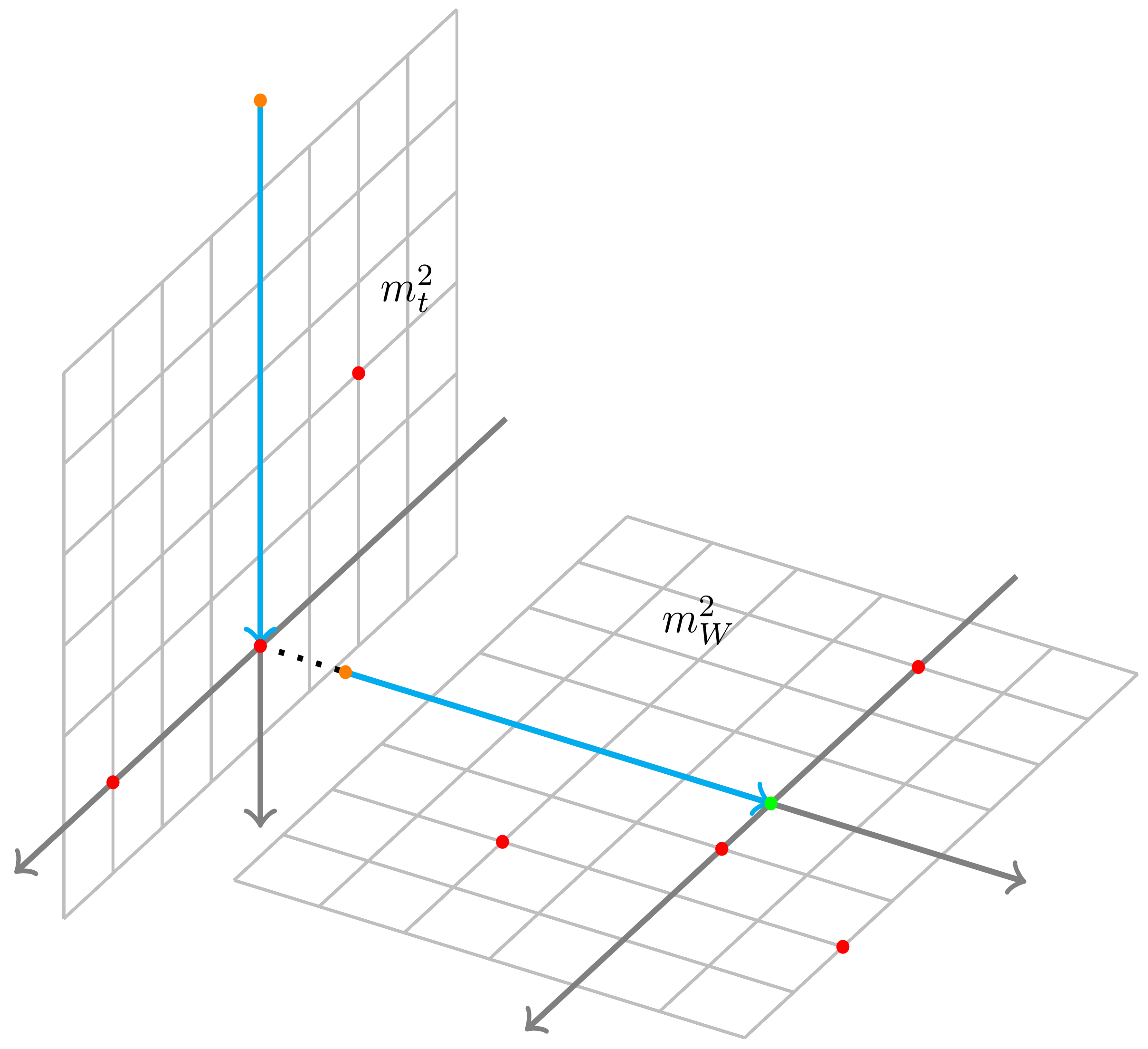
Path fixed by singularities and desired precision

Expected relative error is $(\Delta \,/\, R)^N$





Double-virtual contribution: amplitude calculation *2108.09222*



Add imaginary part to **<u>internal</u> top quark mass**

$$m_t^2 \to m_t^2 - i\eta$$

Boundary condition at $\eta \to \infty$ and physical mass at $\eta = 0$

Due to separation of internal and external mass, the physical point is singular

$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_k^N \sum_l \boldsymbol{c}_{jkl} \eta^k \ln^l \eta + \ldots$$

Separate into branches and pick the relevant one

$$\begin{aligned} I = &\eta^0 \left(b_{10}(\epsilon) + b_{11}(\epsilon) + \ldots\right) \\ &+ \eta^{1-\epsilon} \left(b_{20}(\epsilon) + b_{21}(\epsilon)\eta + \ldots\right) \\ &+ \eta^{3-4\epsilon} \left(\ldots\right) \end{aligned}$$