Next-to-Leading-Order QCD corrections to ZZ production through gluon fusion

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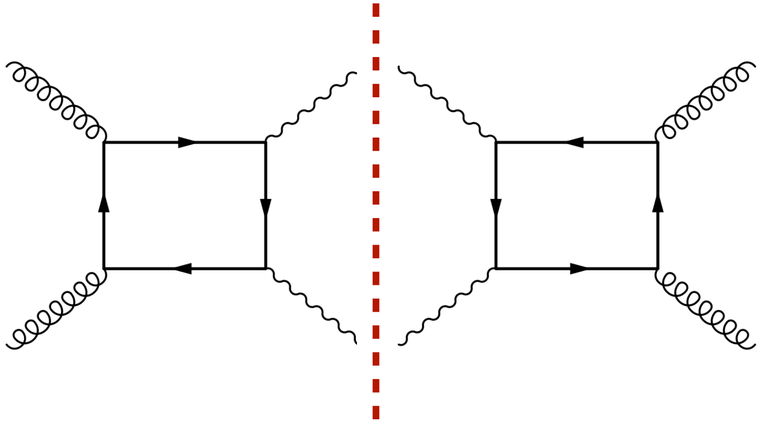
Motivation

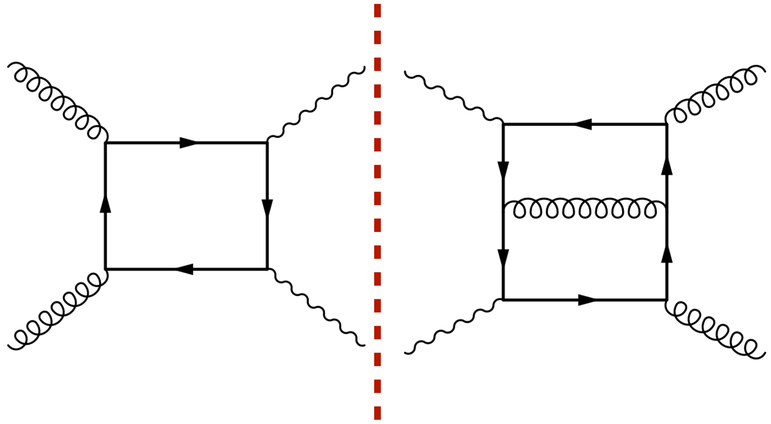
- ZZ production important channel for BSM searches
- Indirectly constrain Higgs width [Caola, Melnikov (2013)]
- Significant contribution to off-shell Higgs production through interference [Kauer, Passarino (2012)]

- $gg \to ZZ$ at the LHC:
  - Loop induced; formally NNLO for $pp \to ZZ$ (starting at $O(\alpha_S^2)$)
  - Large contribution due to high gluon luminosity; $\sim 60\,\%$ of the total NNLO correction [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)]
  - $gg \to ZZ$ at NLO (massless quarks in the loop) increases total $pp \to ZZ$ by $\sim 5\%$ [Grazzini, Kallweit, Wiesemann, Yook (2018)]
- Top quark corrections to $gg \to ZZ$:
  - Expected to be significant, especially for longitudinal modes due to Goldstone boson equivalence theorem
  - Measuring anomalous $t\bar{t}Z$ coupling [Azatov, Grojean, Paul, Salvioni (2016)], [Cao, Yan, Yuan, Zhang (2020)]

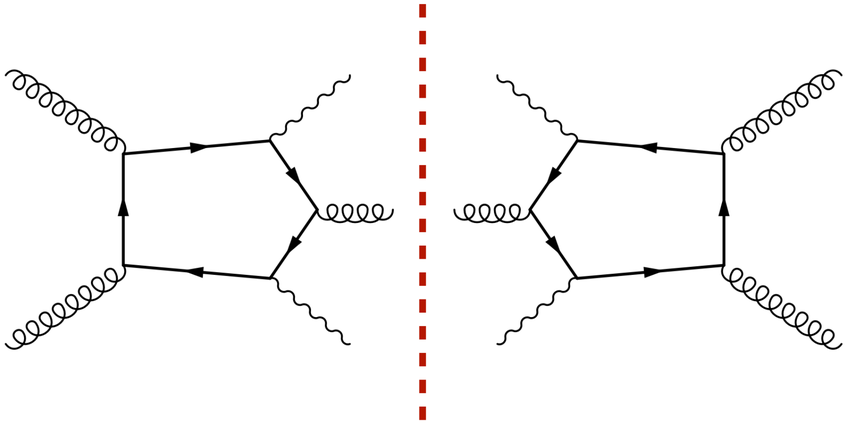
NLO Calculation



Born

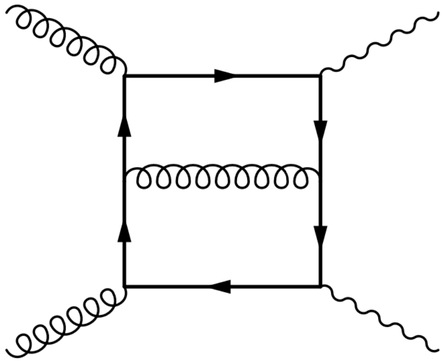


Virtuals



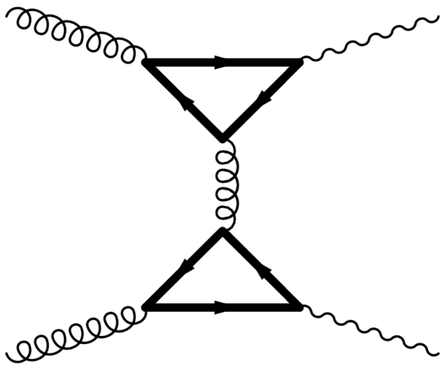
Reals

Two-loop diagrams



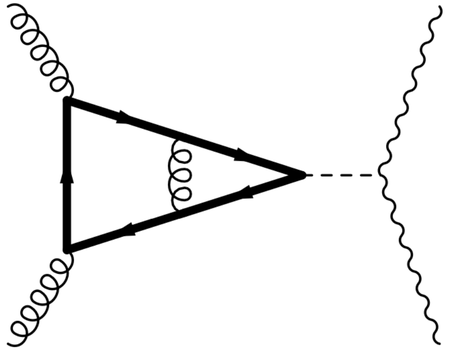
Massless:

[von Manteuffel, Tancredi (2015)] [Caola, Henn, Melnikov, Smirnov, Smirnov (2015)]



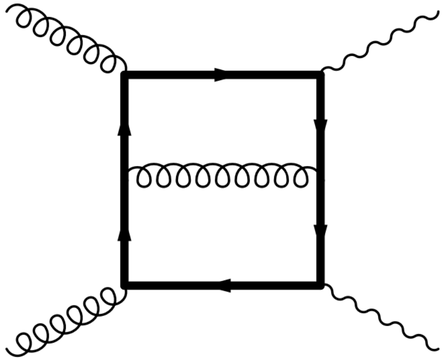
Anomaly type:

[Kniehl, Kühn (1990)] [Cambell, Ellis, Zanderighi (2007)] [Cambell, Ellis, Czakon, Kirchner (2016)]



Higgs mediated:

[Spira et al (1995)] [Harlander & Kant (2005)] [Anastasiou et al (2006)] [Bonciani et al (2006)]



Massive:

[BA, Jones, von Manteuffel (2020)] [Brønnum-Hansen, Wang (2021)] and for various expansions: [Melnikov, Dowling (2015)] [Caola et al (2016)] [Cambell, Ellis, Czakon, Kirchner (2016)] [Gröber, Maier, Rauh (2019)] [Davies, Mishima, Steinhauser, Wellmann (2020)]

Calculating multi-loop amplitudes

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Recipe for a multi-loop amplitude:

1. Generation of unreduced amplitude
2. IBP reduction and back substitution of IBP identities
   - Major bottleneck for processes with many scales and/or legs
   - Significant progress with syzygy based approaches and finite-field methods
   - Use of multivariate partial fractioning to tame the computational complexity and improve numerical performance
3. Evaluation of master integrals
   - Express in terms of multiple polylogarithms; internal masses => Functions beyond multiple polylogarithms
   - Use of numerical methods instead, improved with the use of finite integrals

Form factors and helicity amplitudes

- Decompose the amplitude into a set of basis tensors using Lorentz invariance:

$$\mathscr{M} = \mathscr{M}_{\mu\nu\rho\sigma}\, \epsilon^{\mu}_{\lambda_1}(p_1)\, \epsilon^{\nu}_{\lambda_2}(p_2)\, \epsilon^{*\rho}_{\lambda_3}(p_3)\, \epsilon^{*\sigma}_{\lambda_4}(p_4) = \left(\Sigma\, A_i T_{i,\mu\nu\rho\sigma}\right)\, \epsilon^{\mu}_{\lambda_1}(p_1)\, \epsilon^{\nu}_{\lambda_2}(p_2)\, \epsilon^{*\rho}_{\lambda_3}(p_3)\, \epsilon^{*\sigma}_{\lambda_4}(p_4)$$

T_i are 20 basis tensors obtained using an explicit gauge choice

A_i are form factors, functions of kinematic invariants, masses, and dimension $\left(s, t, m_Z^2, m_t^2, d\right)$, where

$$s = (p_1 + p_2)^2, \;\; t = (p_1 - p_3)^2$$

Use projection operators to obtain A_i from $\mathscr{M}_{\mu\nu\rho\sigma}$

- Can also consider explicit parameterisation of external momenta/polarisation tensors for specific helicities to obtain helicity amplitudes $\mathscr{M}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ in terms of A_i

Only 18 helicity amplitudes; redundancy in A_i due to the use of CDR

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Integration-By-Parts reduction using Syzygies

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General scalar Feynman integral with L-loops and N-edges (propagators in integral family) :

$$I(\nu_1, \ldots, \nu_N) = \int d^D k_1 \ldots d^D k_L \, \Pi_{i=1}^{N} \frac{1}{\left(q_i^2 - m_i^2 + i\epsilon\right)^{\nu_i}}$$

$k_1, \ldots k_L$: Loop momenta

q_i : Momentum of edge i

m_i : Mass of edge i

ν_i : Exponent of edge i

Integration-by-parts identity [Chetyrkin & Tkachov (1981)] :

$$0 = \int d^D k_1 \ldots d^D k_L \, \frac{\partial}{\partial k_\mu} v_\mu \left(\Pi_{i=1}^{N} \frac{1}{\left(q_i^2 - m_i^2 + i\epsilon\right)^{\nu_i}} \right)$$

v_μ is in general a linear combination of loop and external momenta.

Generate a linear system of equations and systematically reduce using Laporta's algorithm [Laporta (2000)] to arrive at a set of basis integrals. Many public codes available *AIR, FIRE6, Kira, LiteRed, Reduze 2, FiniteFlow(+..)* etc.

But, leads to equations with integrals not needed for the amplitude $\Longrightarrow$ very large systems to reduce

Integration-By-Parts reduction using Syzygies (Fix)

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It is desirable to avoid such doubled propagators i.e. propagators with exponents higher than 1:

- Such integrals do not frequently appear in scattering amplitudes, hence reductions are not required
- IBP systems become much larger in size e.g. ~10^8 equations for $gg \to ZZ$

How to avoid these doubled propagators?

- Multiple approaches: [Gluza, Kajda, Kosower (2010)] [Schabinger (2011)]
- Compute module intersection of two syzygy modules [Zhang (2014)] [Larsen, Zhang (2015)] [Boehm, Georgoudis, Larsen, Schoenemann, Zhang (2018)]
- Use of computer algebra packages such as **Singular** for this task.

We use a linear algebra approach based on finite fields [BA, Jones, von Manteuffel (2020)]

- Map the problem of module intersection to row reduction of a matrix; use *Finred* - IBP solver based on finite field methods [von Manteuffel, Schabinger (2014)], [Peraro (2016)] for the linear algebra
- Linear systems generated from syzygies are much smaller than those from conventional approaches. E.g. for $gg \to ZZ$, only ~ $3 \cdot 10^5$ equations compared to ~10^8 before

Calculating multi-loop amplitudes

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Recipe for a multi-loop amplitude:

1. Generation of unreduced amplitude
2. IBP reduction and back substitution of IBP identities
   - Major bottleneck for processes with many scales and/or legs
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3. Evaluation of master integrals
   - Express in terms of multiple polylogarithms; internal masses => Functions beyond multiple polylogarithms
   - Use of numerical methods instead, improved with the use of finite integrals

Numerical evaluation using finite integrals

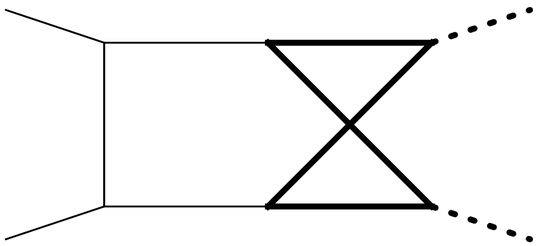
- Sector decomposition standard method to resolve IR poles [Binoth, Heinrich (2000)] [Bogner, Weinzierl (2007)]
- Public codes: *Fiesta5*, *pySecDec*, etc.

Why use finite integrals?

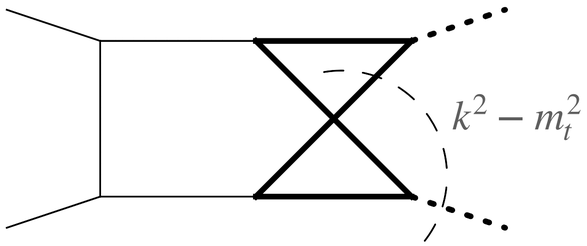
- Much more stable numerics [Borowka et al (2016)] [von Manteuffel, Schabinger (2017)]
- Require fewer orders in epsilon expansion in general
- Poles drop out into the coefficients => Easier to take $d \to 4$ limit, after partial fractioning

Constructing finite integrals:

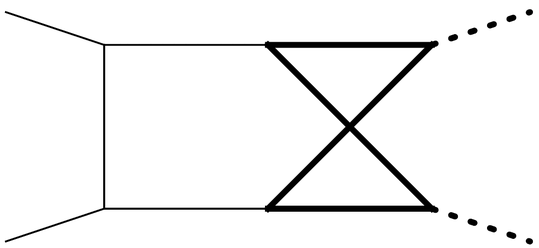
- Dimension shifted integrals [Bern, Dixon, Kosower (1992)]
- Existence of a finite basis [Panzer (2014)] [von Manteuffel, Panzer, Schabinger (2014)]
- *Reduze 2* to find such integrals, usually involving higher propagator powers (dots) and dimension shifts



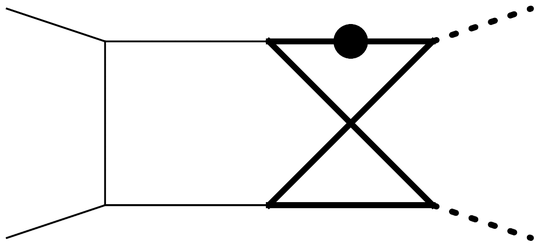
Divergent integral in $d = 4 - 2\epsilon$



Divergent integral in $d = 4 - 2\epsilon$ with a numerator



Finite integral in $d = 6 - 2\epsilon$

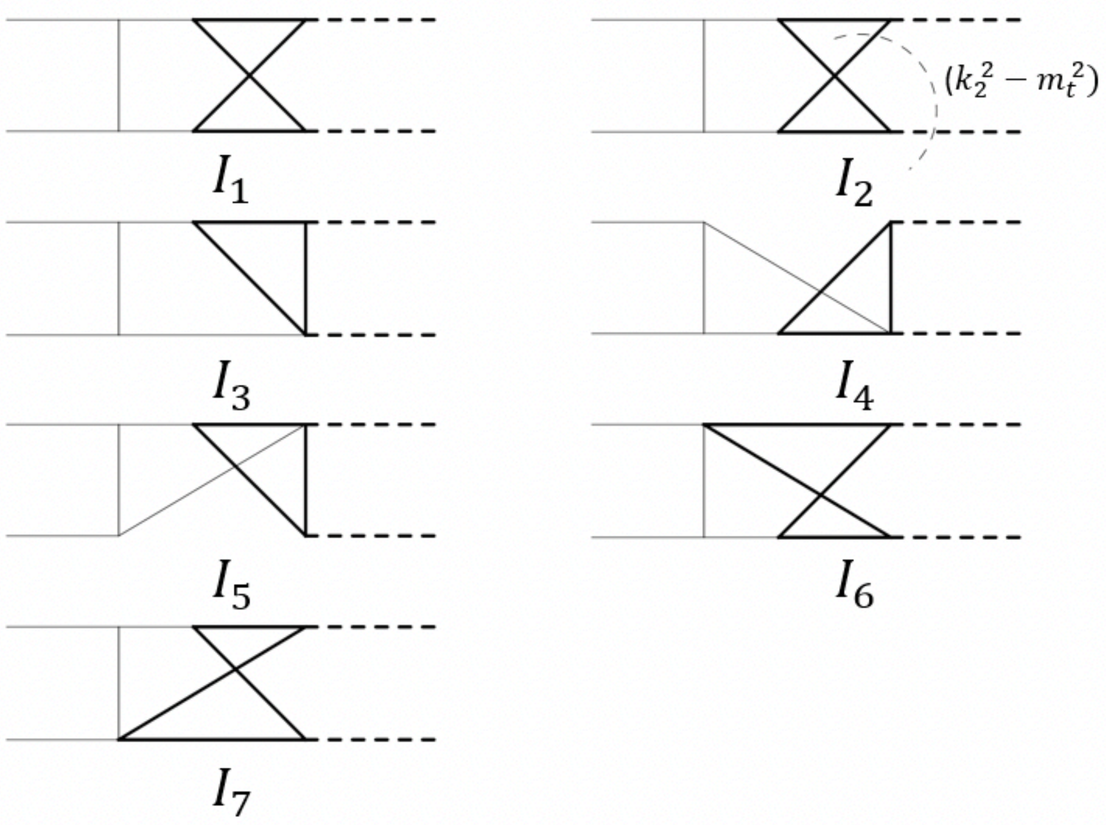


Finite integral in $d = 6 - 2\epsilon$ with a dot

Numerical evaluation using finite integrals

Alternate approach - combining divergent integrals into **finite linear combinations**.

- Integrals often already appearing in the amplitude => avoid computing extra reductions
- More "natural" $d = 4$ representation
- Algorithmically construct finite linear combinations in $d = 4$ from a list of seed integrals (arbitrary integrals with numerators, dots, dimension shifts, subsector integrals) [BA, Jones, von Manteuffel (2020)]
- Numerical performance is very competitive with the conventional finite integrals (naively expected to be much worse)
- We use a combination of two approaches for performance reasons: basis of master integrals contains both integrals with dots/dimension-shifts and finite linear combinations of divergent integrals



$$I = (m_z^2 - s - t)(sI_1 - I_6) + s(I_2 + I_3 - I_4 - I_5) - (m_z^2 - t)I_7$$

Results

- Write the UV and IR finite amplitudes (after renormalisation and IR subtraction respectively) as:

$$\mathscr{M}^{fin}_{\lambda_1\lambda_2\lambda_3\lambda_4} = \left(\frac{\alpha_S}{2\pi}\right)\mathscr{M}^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4} + \left(\frac{\alpha_S}{2\pi}\right)^2\mathscr{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4} + O\left(\alpha_S\right)^3$$

- Define 1-loop squared and interference between 1-loop and 2-loop amplitudes:

$$\mathscr{V}^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4} = |\mathscr{M}^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4}|^2$$

$$\mathscr{V}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4} = 2\,\mathrm{Re}\left(\mathscr{M}^{*(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4}\mathscr{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4}\right)$$

- Note that in the following results, only the pure top-quark contributions are included (i.e. no Higgs mediated diagrams or massless internal quarks)

Results

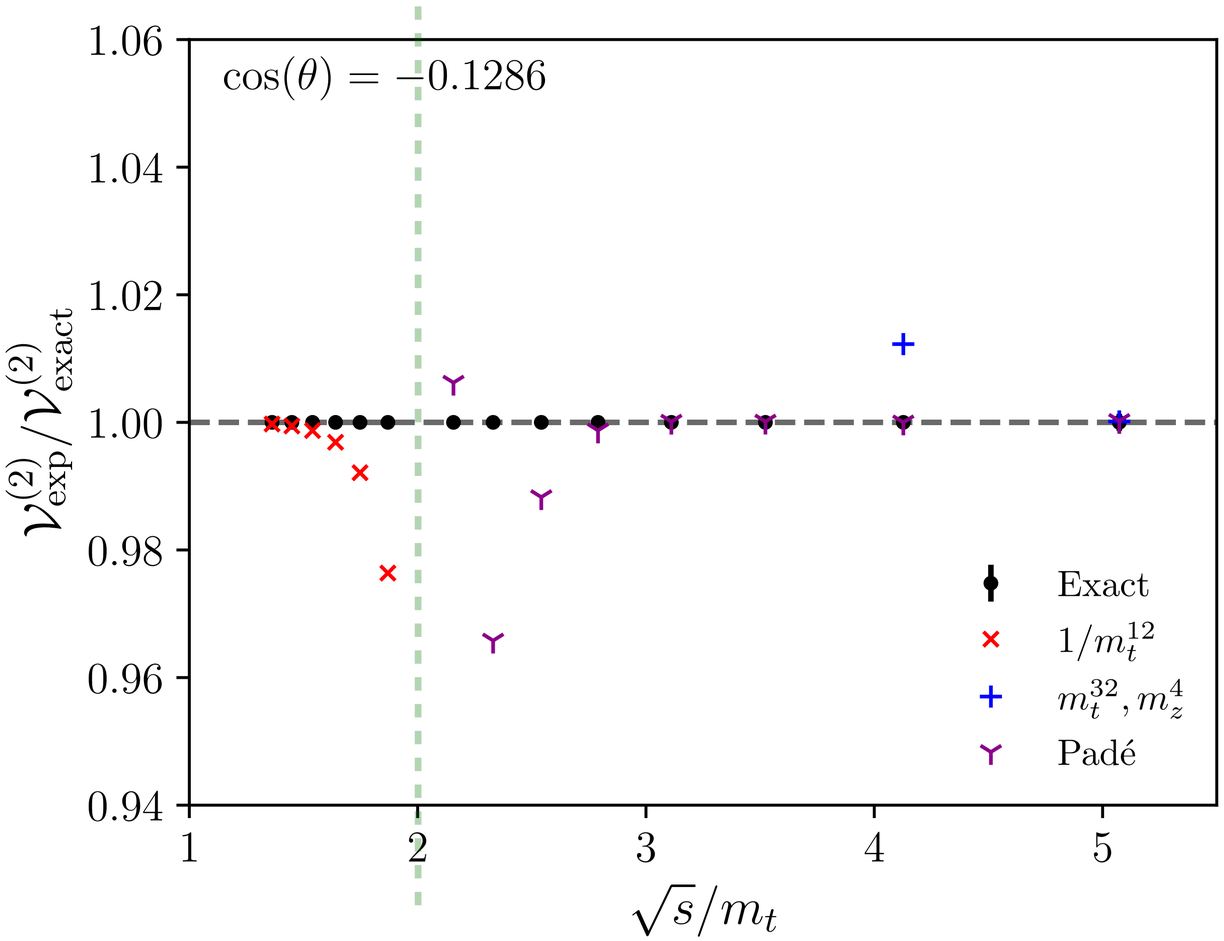
- Integration strategy:
  - Helicity amplitudes $\mathscr{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ written as a linear combination of $\sim O(10^4)$ integrals after sector decomposition i.e. each sector of a master integral is considered and evaluated separately
  - Number of evaluations for each integral set dynamically to minimise the evaluation time for $\mathscr{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ instead of each integral [Borowka et al (2016)]

$$T = \Sigma\, t_i \;+\; \lambda\,(\sigma^2 - \Sigma_i\, \sigma_i^2)$$

T : Total integration time
t_j : Integration time for integral j
σ : Required precision
σ_i : Estimated precision for integral i
λ : Lagrange Multiplier

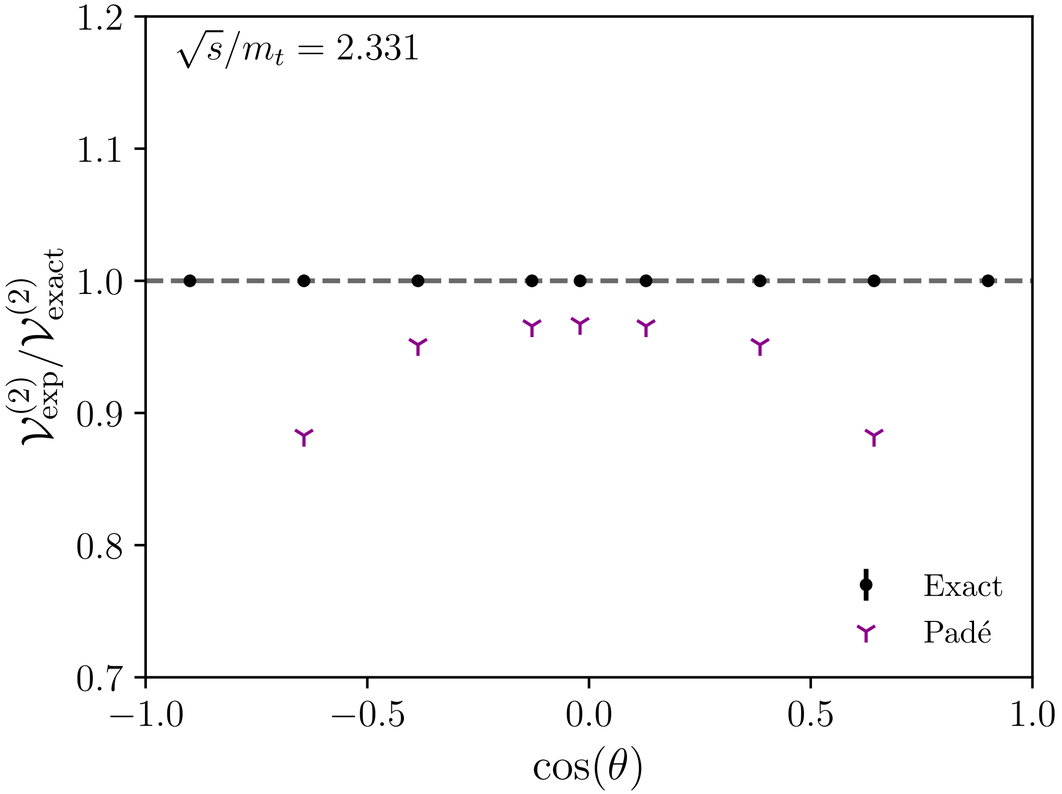
  - Quasi-Monte Carlo algorithm for quadrature [Li, Wang, Zhao (2015)] [Borowka et al (2017)]
  - Request per-cent precision on each helicity amplitude (and ~10% on form factors A_i); much better precision obtained usually

Results: Comparison to expansions

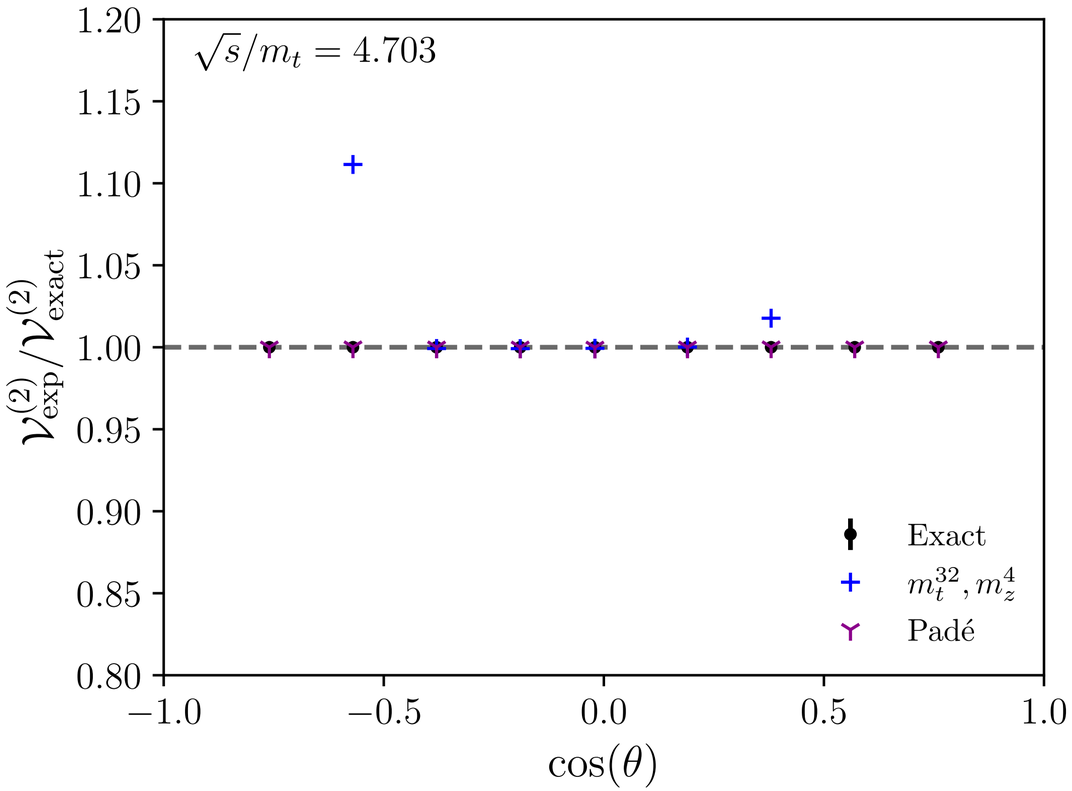


Comparison of $\sqrt{s}$ dependence of the unpolarised interference with expansion results at fixed $\cos\theta = -0.1286$. Exact results from [Agarwal, Jones, von Manteuffel (2020)]. Expansion and Padé results from [Davies, Mishima, Steinhauser, Wellmann (2020)] (see also [Davies, Mishima, Schönwald, Steinhauser (2023)]). Error bars for the exact result are plotted but they are too small to be visible.

Results: Comparison to expansions



Comparison of $\cos\theta$ dependence of the unpolarised interference with expansion results at fixed energy $\sqrt{s} = 403$ GeV. Exact results from [Agarwal, Jones, von Manteuffel (2020)]. Expansion and Padé results from [Davies, Mishima, Steinhauser, Wellmann (2020)] (see also [Davies, Mishima, Schönwald, Steinhauser (2023)]).



Comparison of $\cos\theta$ dependence of the unpolarised interference with expansion results at fixed energy $\sqrt{s} = 814$ GeV. Exact results from [Agarwal, Jones, von Manteuffel (2020)]. Expansion and Padé results from [Davies, Mishima, Steinhauser, Wellmann (2020)] (see also [Davies, Mishima, Schönwald, Steinhauser (2023)]).

Results: Comparison to expansions

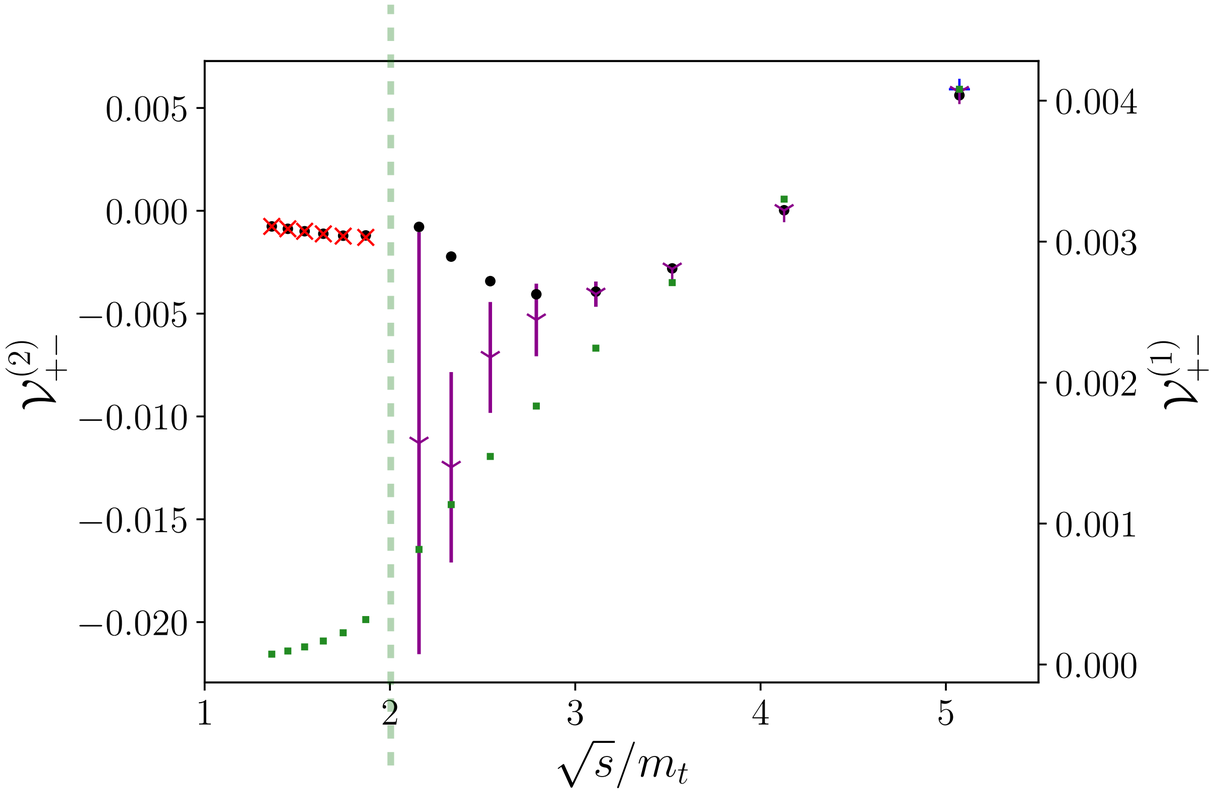
- For previous results, "q_T" subtraction scheme
- Transformation between Catani's original scheme and q_T scheme

$$A_i^{(2),fin,Catani} = A_i^{(2),fin,q_T} + \Delta I_1 A_i^{(1),fin}$$

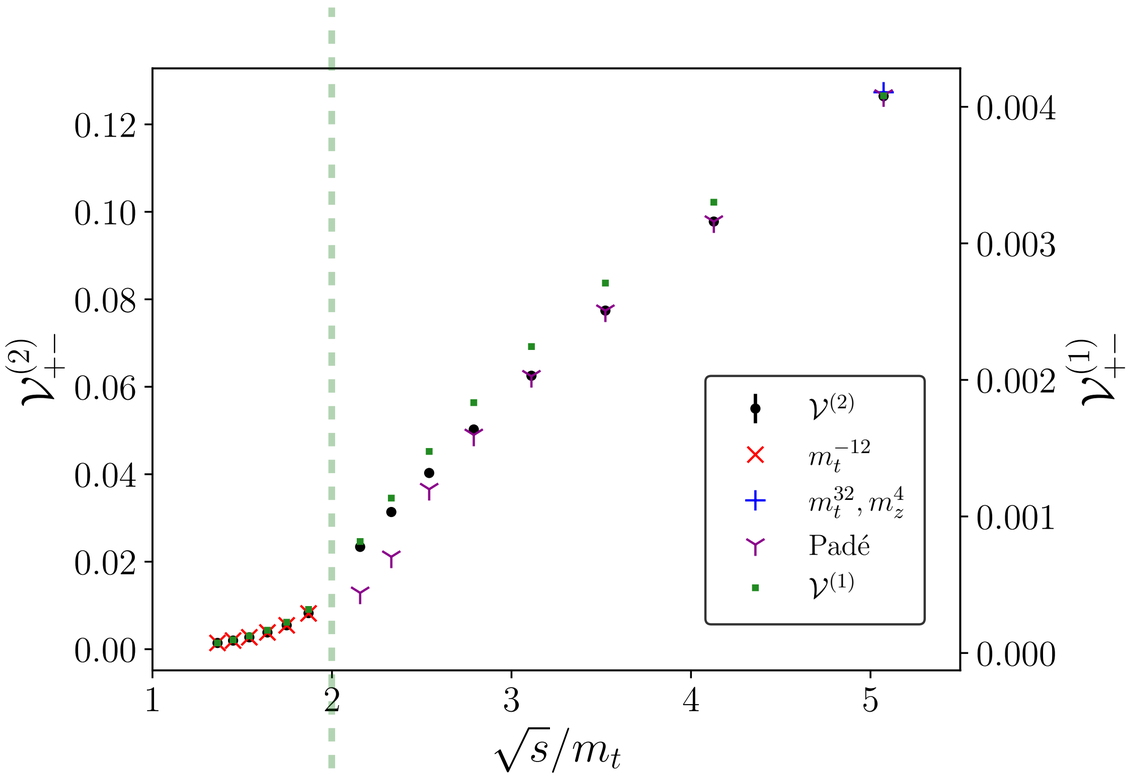
$$\Delta I_1 = -\frac{1}{2}\pi^2 C_A + i\pi\beta_0 \quad \sim 15$$

- For interference terms, 1-loop result multiplied by ~ 30 => Leads to a very different qualitative behaviour
- Relative comparisons highly dependent on IR scheme

Results: Comparison to expansions



Traditional Catani Scheme

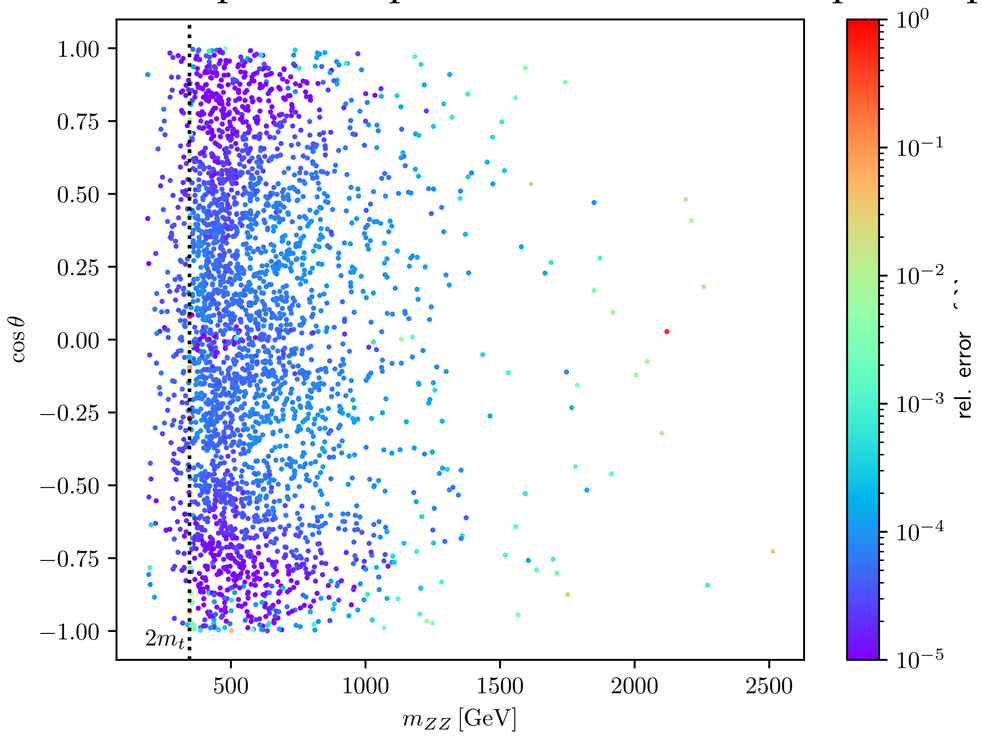


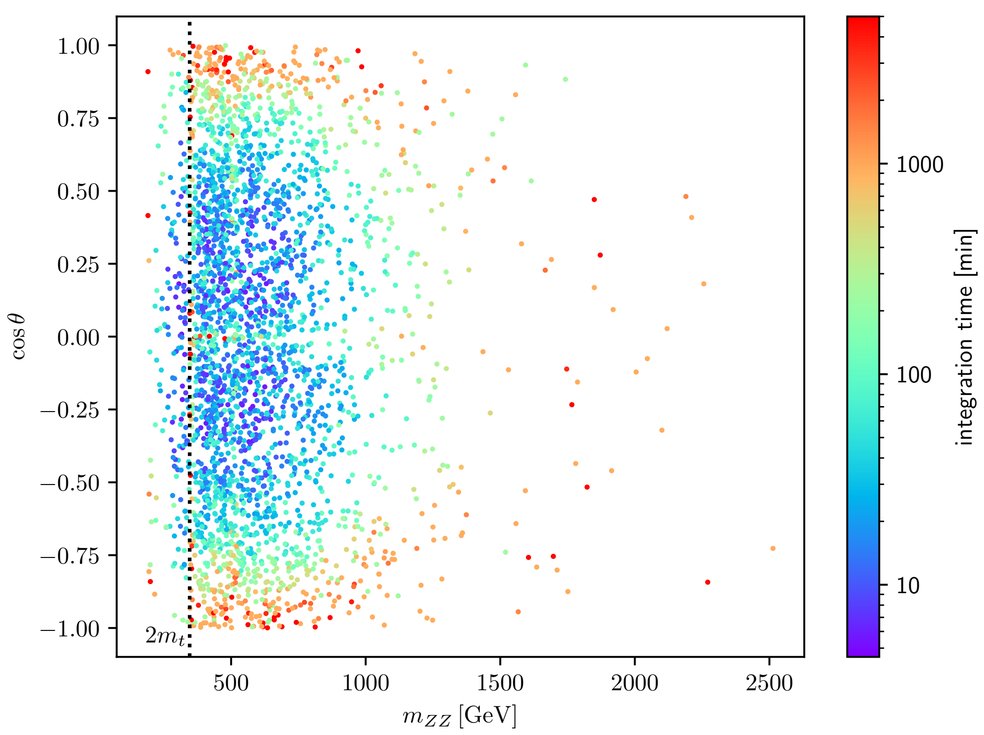
"q_T" scheme

Comparison of $\sqrt{s}$ dependence of the polarised interference with expansion results at fixed $\cos\theta = -0.1286$. Exact results from [Agarwal, Jones, von Manteuffel (2020)]. Expansion and Padé results from [Davies, Mishima, Steinhauser, Wellmann (2020)] (see also [Davies, Mishima, Schönwald, Steinhauser (2023)]).

Results: Virtual corrections

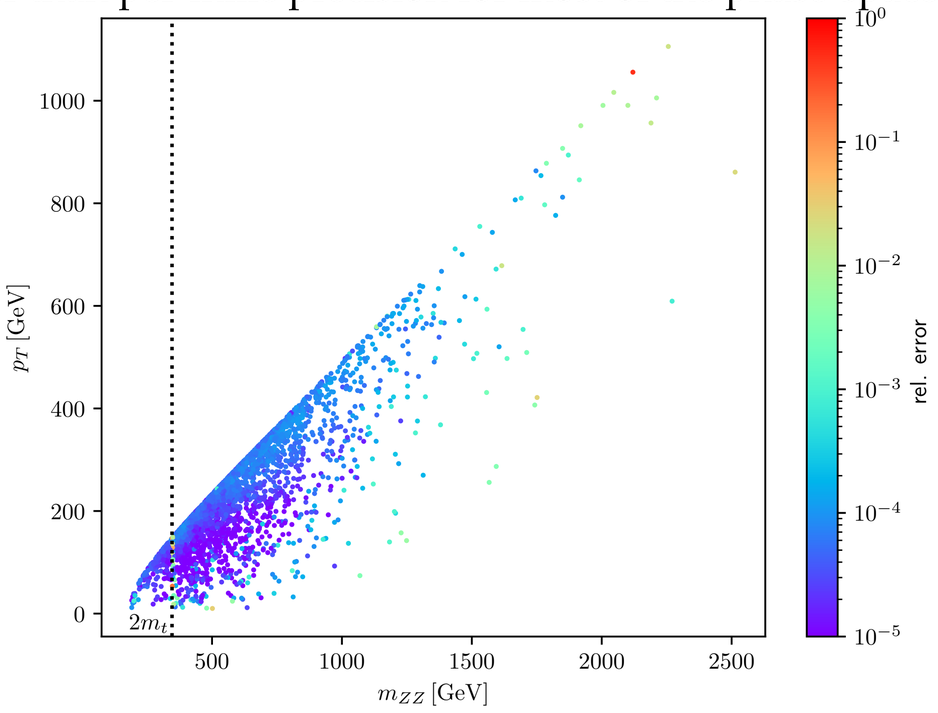
- Use the born calculation (with only top quarks) to generate unweighted events to sample the virtual corrections (~3000 points)
- Good numerical stability in most regions of phase space, in particular around the top-quark threshold
- Runtimes in $O(10)$ min for large part of the phase space with expected difficulties for $|cos\theta| \sim 1$ (very small p_T)
- Better than per-mille precision for most of the phase-space

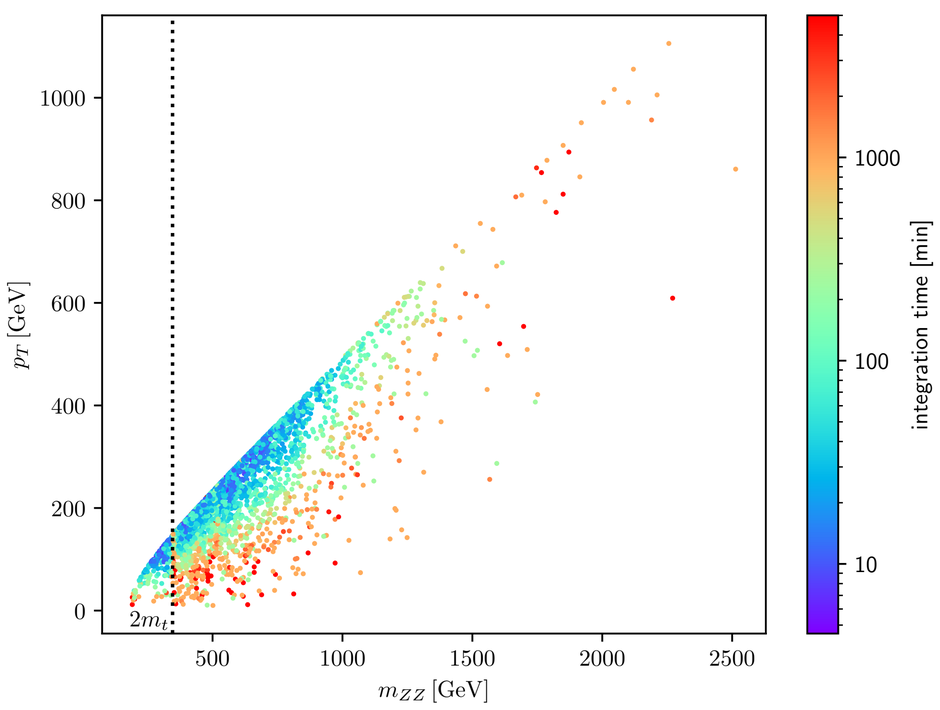




Results: Virtual corrections

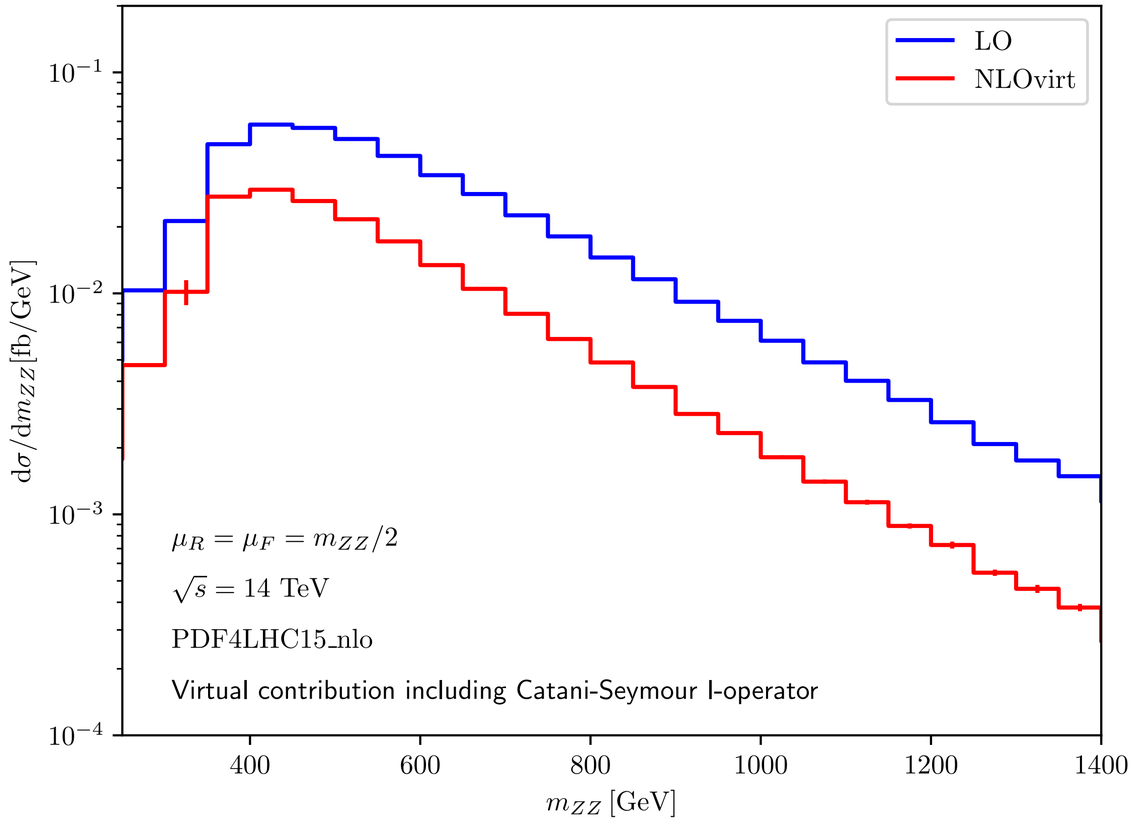
- Good numerical stability in most regions of phase space, in particular around the top-quark threshold (except for small p_T)
- Runtimes in $O(10)$ min for large part of the phase space with expected difficulties for very small p_T
- Can access high energy and high p_T region without much difficulty, but very high energy $\left(\sqrt{s} > 2\,TeV\right)$ challenging
- Better than per-mille precision for most of the phase-space





Results: Virtual corrections

- NLO Virtual corrections compared to Born (only top-quark contribution; does not include reals)
- Significant contribution, especially close to the top-quark production threshold



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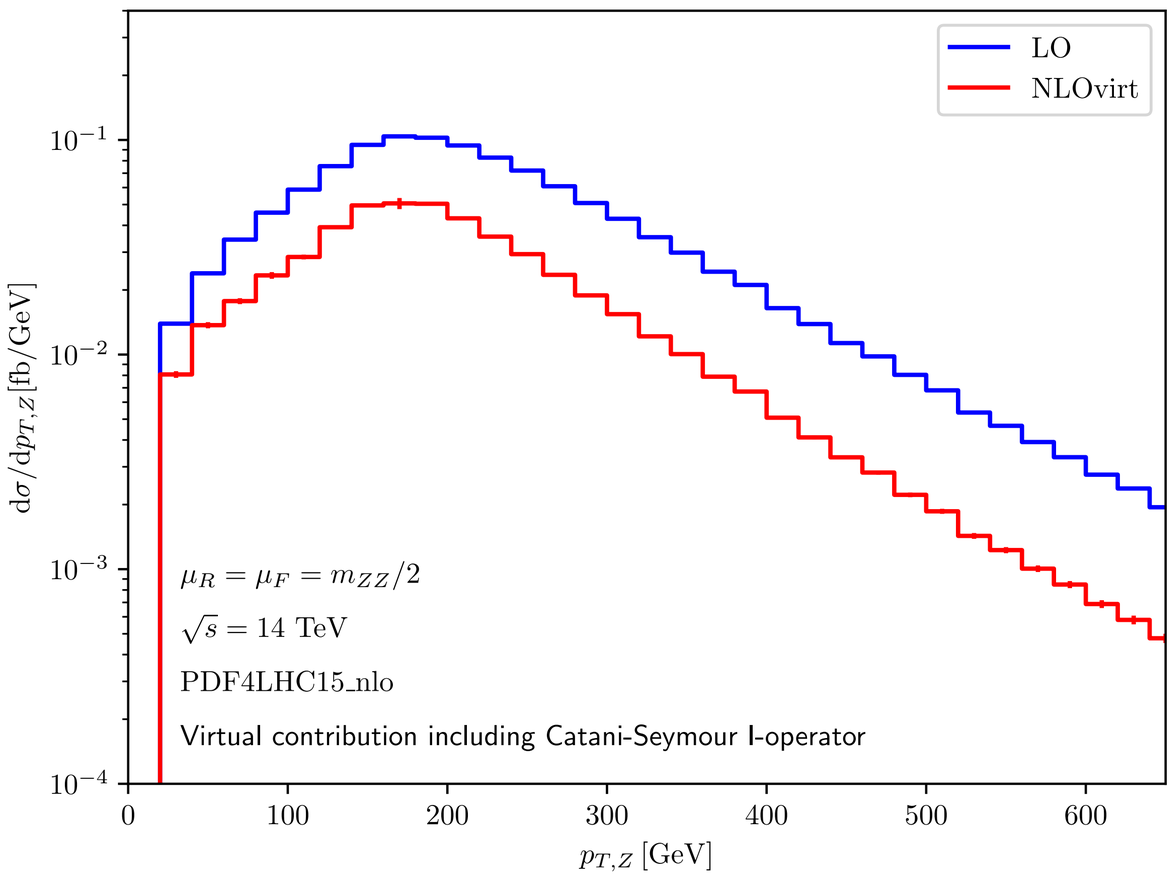

Conclusions and Outlook

- Use of syzygies and finite integrals to facilitate the calculation of the amplitudes
- Efficient integration strategy using sector decomposition to minimise the total integration time (see Vitaly's talk for latest pySecDec developments)
- Numerically very stable in most regions of phase-space, even close to top-quark pair production threshold, at high invariant mass and forward scattering
- Able to get good statistics for distributions; full impact of NLO corrections clearer after the addition of real emission contributions
- All the other ingredients available for full NLO cross-section