Associated production of a W boson and massive bottom quarks in NNLO QCD

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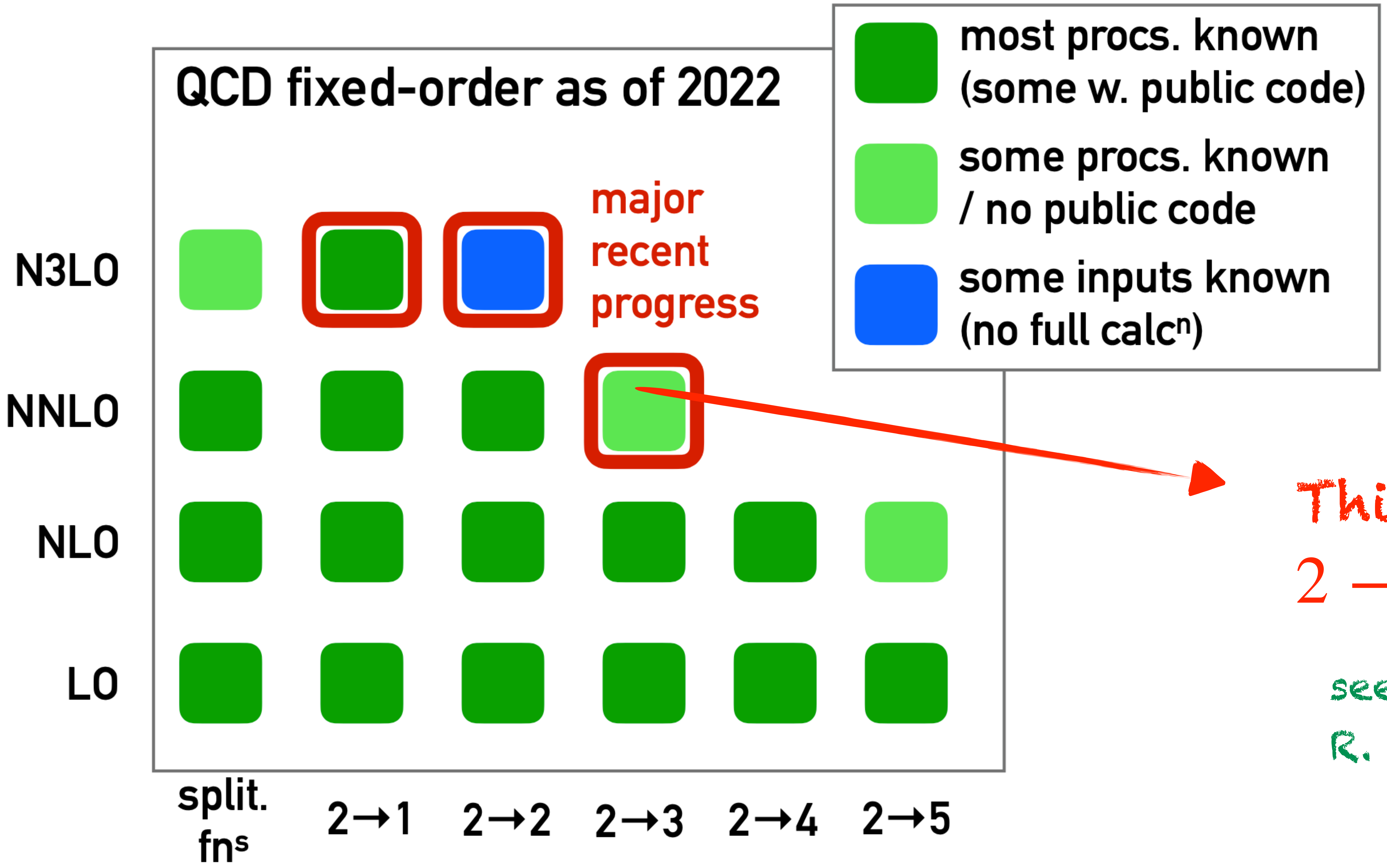
in collaboration with S. Devoto, S. Kallweit, J. Mazzitelli, L. Rottoli and C. Savoini

[*Phys.Rev.D* 107 (2023) 7, 074032, arXiv:2212.04954]

RADCOR 2023

Crieff - 29th May 2023

Introduction



[G. Salam @ ICHEP 2022]

This TALK:
$2 \to 3$ processes

see talks by M. Czakon,
R. Poncelet, C. Savoini

Motivations

W+1bj and W+2bj interesting signatures

- tests of QCD at LHC
- background to $WH(H \to b\bar{b})$ and single top $\bar{b}t(t \to Wb)$
- **bottom quarks modelling:** massive effects, bottom in the PDF, flavour tagging

from VH(->bb) analysis [CMS:arXiv:1808.08242]

Large normalisation corrections with respect to SM simulation

Postfit normalisation corrections

Process	Z($\nu\nu$)H	W($\ell\nu$)H	Z($\ell\ell$)H low-p_T	Z($\ell\ell$)H high-p_T
W + udscg	1.04 ± 0.07	1.04 ± 0.07	–	–
W + b	2.09 ± 0.16	2.09 ± 0.16	–	–
W + $b\bar{b}$	1.74 ± 0.21	1.74 ± 0.21	–	–
Z + udscg	0.95 ± 0.09	–	0.89 ± 0.06	0.81 ± 0.05
Z + b	1.02 ± 0.17	–	0.94 ± 0.12	1.17 ± 0.10
Z + $b\bar{b}$	1.20 ± 0.11	–	0.81 ± 0.07	0.88 ± 0.08
$t\bar{t}$	0.99 ± 0.07	0.93 ± 0.07	0.89 ± 0.07	0.91 ± 0.07

State of the art

NLO corrections (massless bottom quarks)

[Ellis, Veseli, 1999]

NLO corrections (massive bottom quarks)

[Febres Cordero, Reina, Wackeroth, 2006, 2009]

NLO corrections (4FS+5FS)

[Campbell, Ellis, Febres Cordero, Maltoni, Reina, Wackeroth, Willenbrock, 2009] [Campbell, Caola, Febres Cordero, Reina, Wackeroth,2011]

NLO+PS

[Oleari, Reina,2011] [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, 2011]

POWHEG+MiNLO

[Luisoni, Oleari, Tramontano, 2015]

Wbb + up to 3 jets

[Anger, Febres Cordero, Ita, Sotnikov, 2018]

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see talks by S. Zoia, G. Gambuti

Analytical Two-loop W+4 partons amplitude in Leading Colour Approximation (LCA)

[Badger, Hartanto, Zoia, 2021] [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov, 2021]

NNLO corrections (massless bottom quarks)

[Hartanto, Poncelet, Popescu, Zoia, 2022]

First NNLO QCD calculation for massless bottom quarks!

$Wb\bar{b}$ @ NNLO with massless b quarks

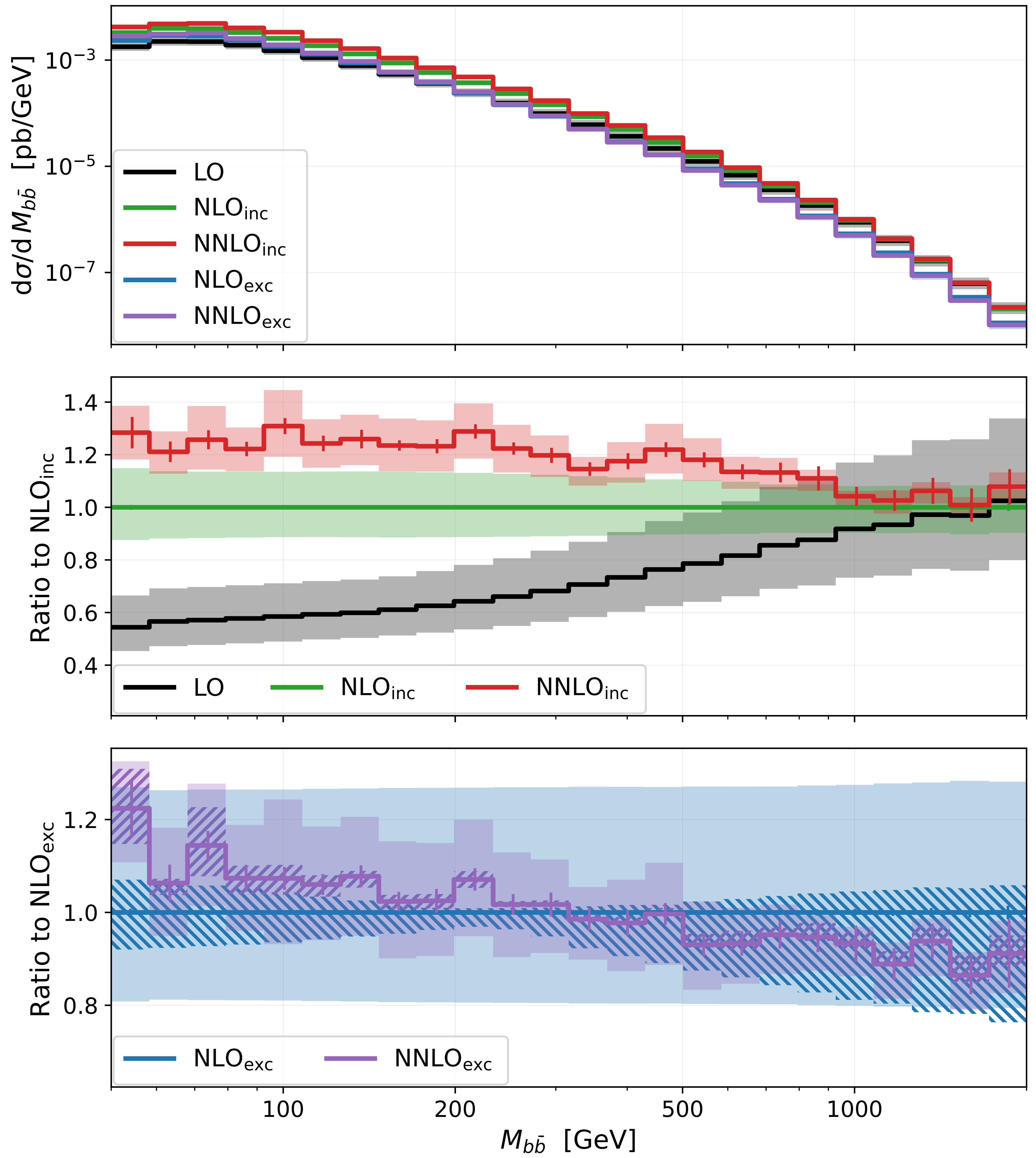
[Hartanto, Poncelet, Popescu, Zoia, 2022]

First computation for Wbb @ NNLO with massless b quarks recently performed

But, massless calculation is subject to ambiguities related to jet-sensitive jet algorithm

Jet algorithm	$\sigma_{\rm NNLO}$ [fb]	$K_{\rm NNLO}$
flavour-$k_{\rm T}$	$445(5)^{+6.7\%}_{-7.0\%}$	1.23
flavour anti-$k_{\rm T}$ $(a=0.05)$	$690(7)^{+10.9\%}_{-9.7\%}$	1.38
flavour anti-$k_{\rm T}$ $(a=0.1)$	$677(7)^{+10.4\%}_{-9.4\%}$	1.36
flavour anti-$k_{\rm T}$ $(a=0.2)$	$647(7)^{+9.5\%}_{-8.9\%}$	1.33

$\mathcal{O}(50\%)$ difference when using flavour k_T algorithm



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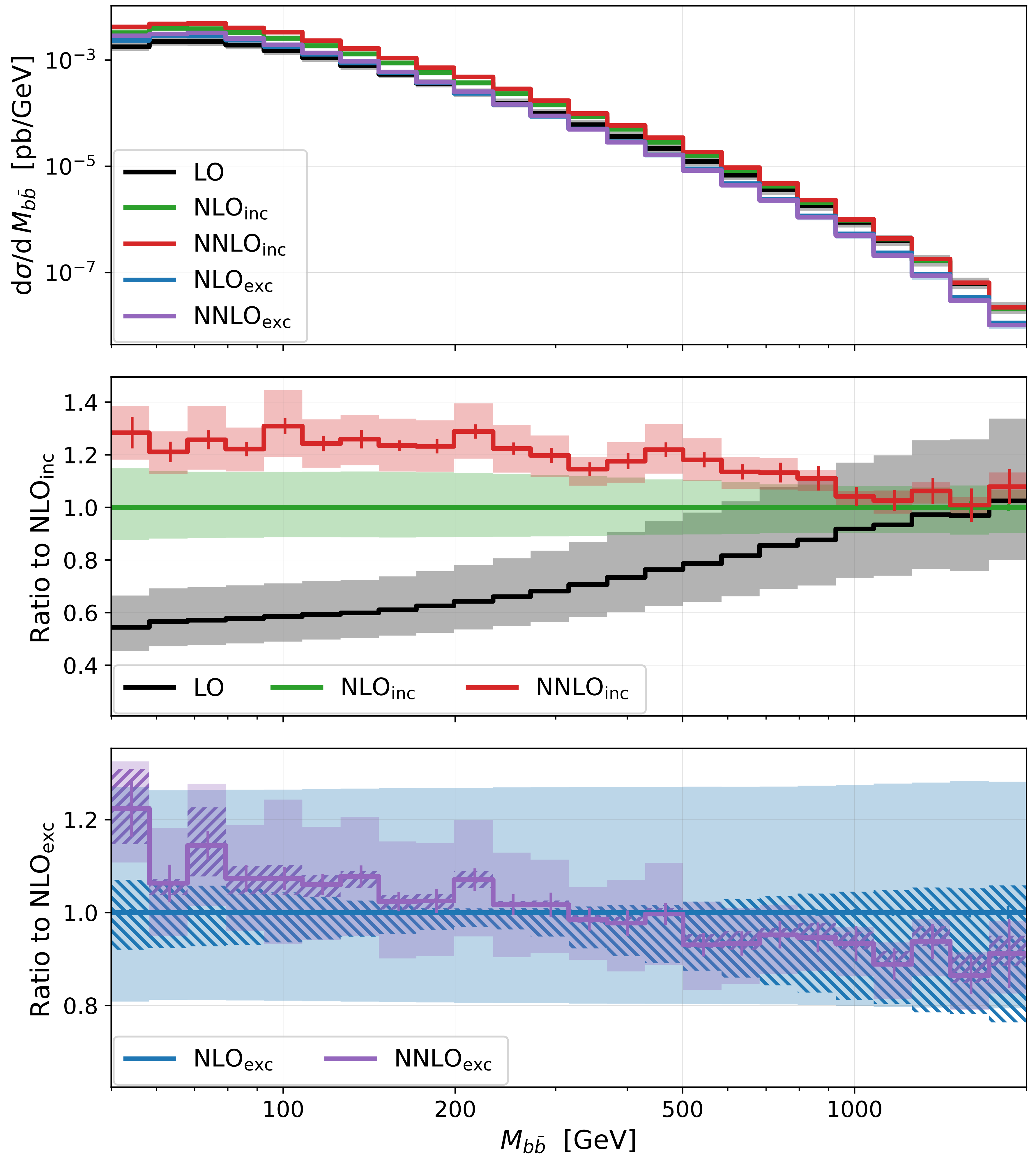
Uncertainties related to the ambiguities reduced when using flavour-aware anti-k_T

[Czakon, Mitov, Poncelet, 2022]



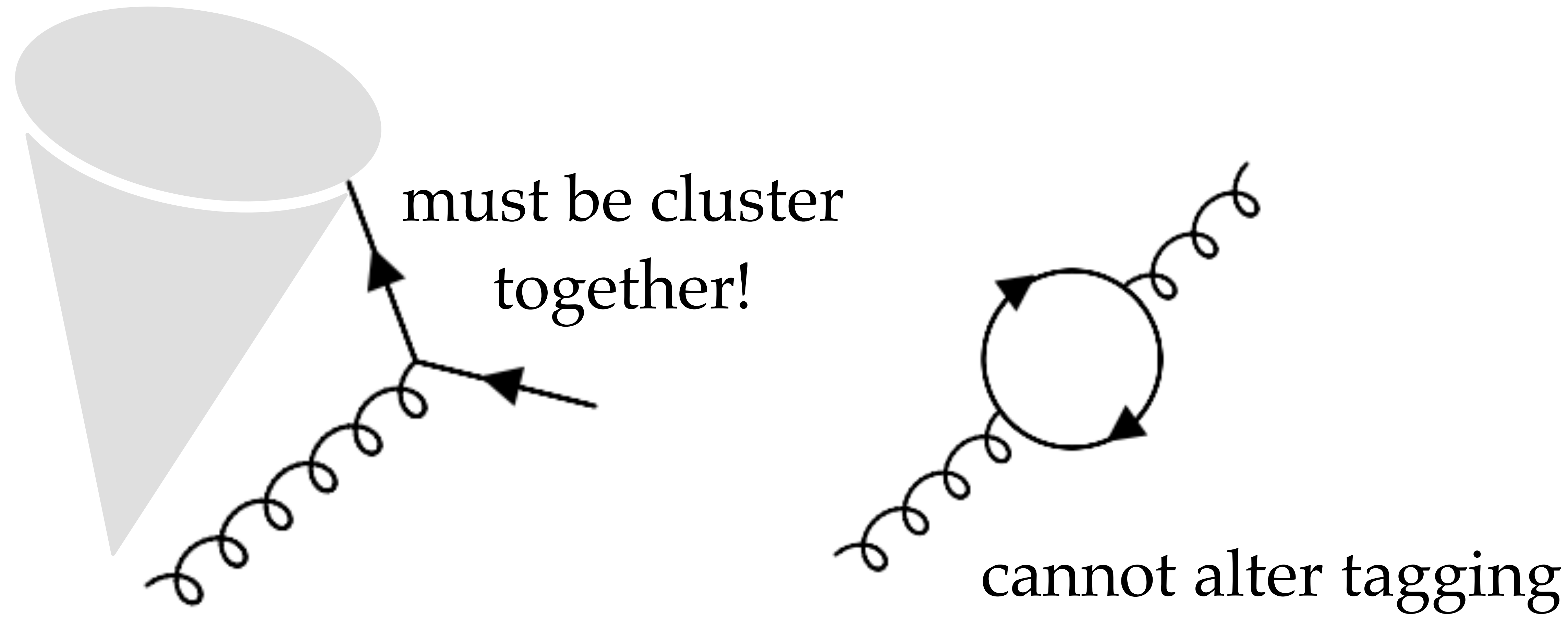
Infrared safety and flavour tagging

Jet clustering algorithms consist in a sequence of two-to-one recombination steps. They are then completely defined once the binary distance d_{ij} and the beam distance d_{iB} are given. For the family of k_T algorithms

$$d_{ij} = \min\left(k_{T,i}^{2\alpha}, k_{T,j}^{2\alpha}\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^{2\alpha} \qquad\qquad R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

For parton level calculation (fixed order), **infrared safety** is a crucial requirement

For observable sensitive to the flavour assignment, **infrared safety can be an issue**, usually associated **to gluon splitting to quarks in the double soft limit** (the problem starts at NNLO)



this may lead to a flavour configuration different from the corresponding virtual one, spoiling KLN cancellation

Flavour aware jet algorithms: flavour k_T

[Banfi, Salam, Zanderighi, 2006]

Theoretically sounded but problematic for data/theory comparison

- experimentally, jet reconstruction and flavour assignment are performed **at the particle level** (not at the parton level)
- **anti-k_T is de-facto the jet algorithm** used in all analysed for its properties

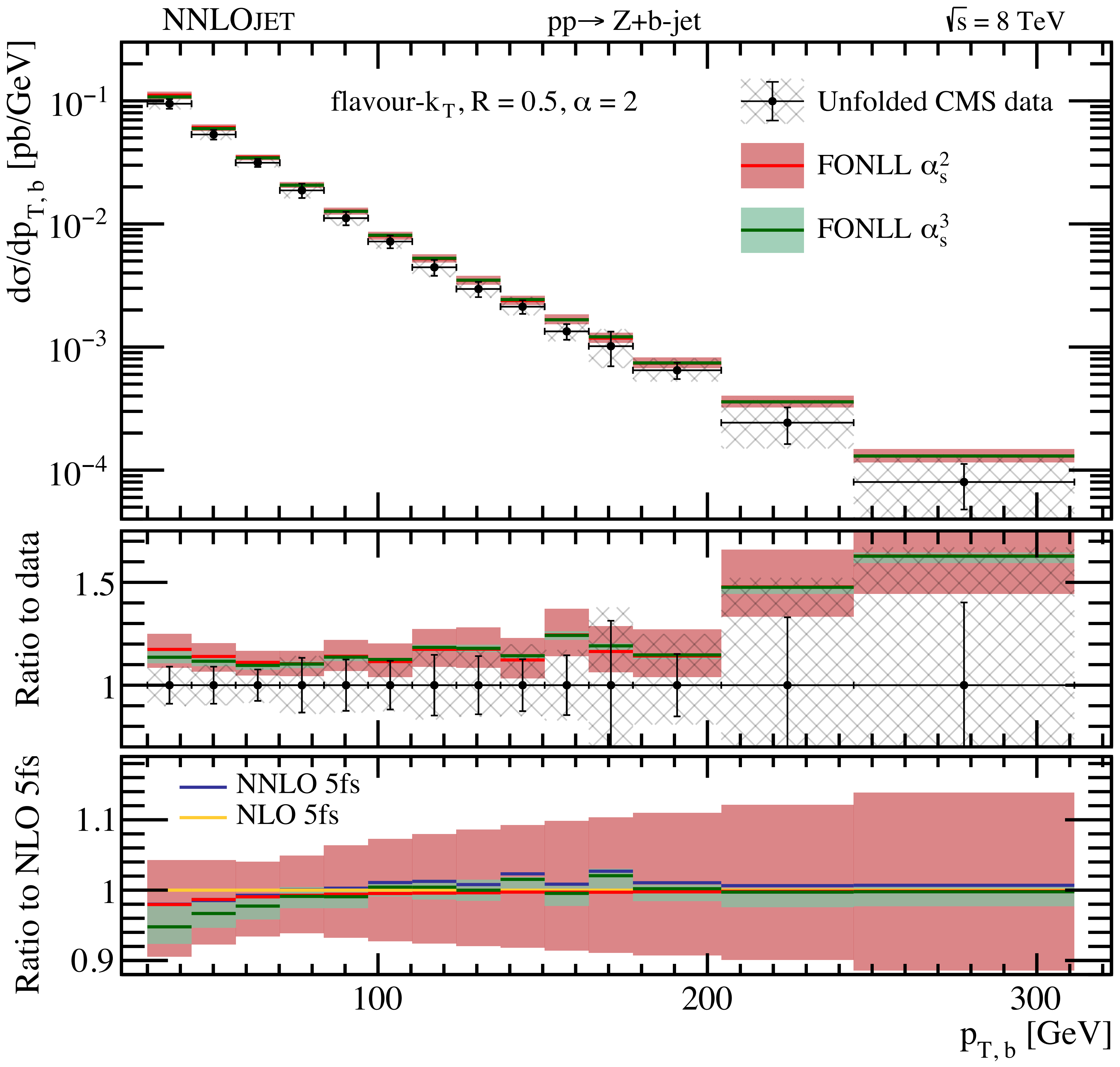
Flavour aware jet algorithms: flavour k_T

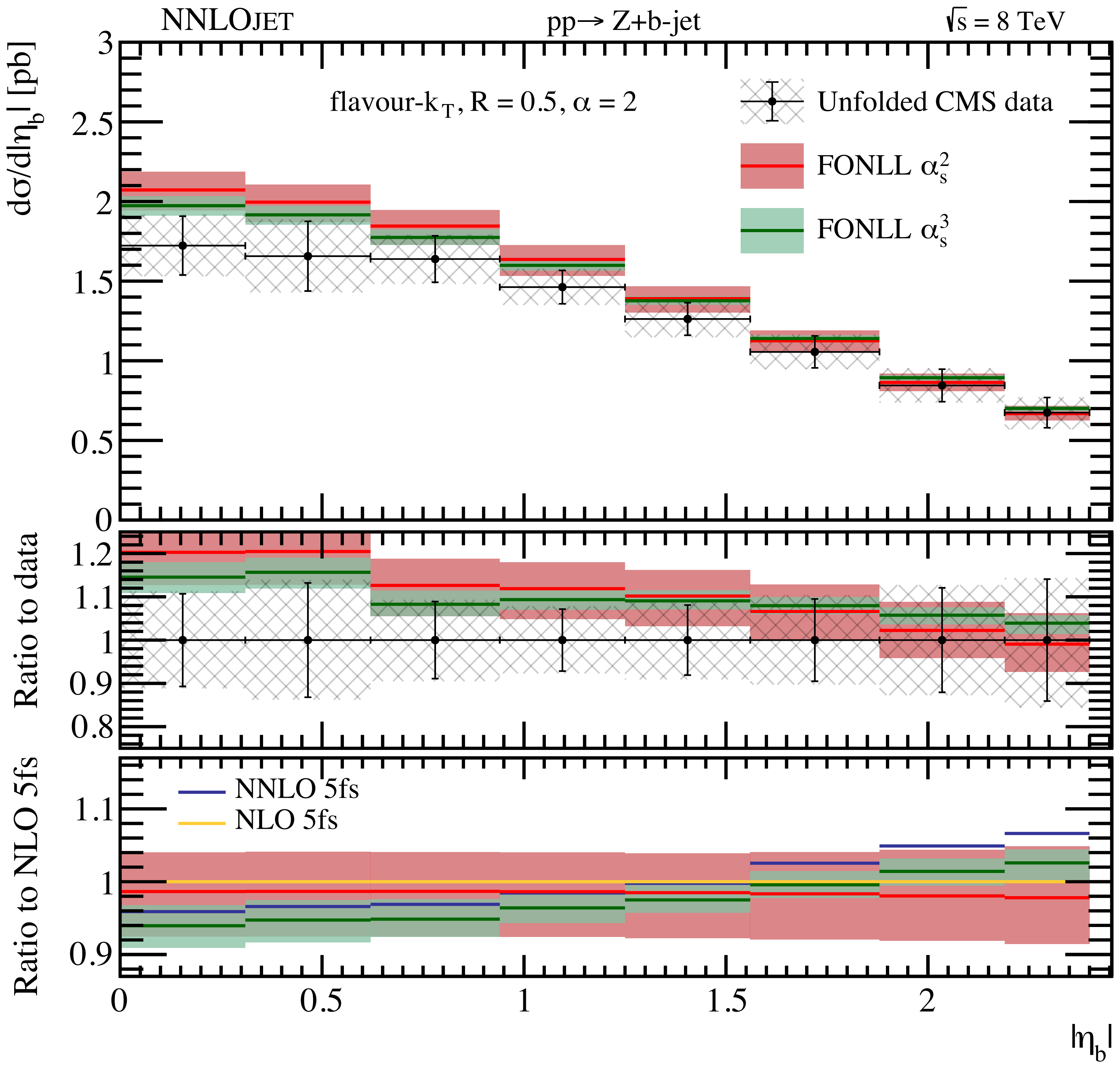
[Banfi, Salam, Zanderighi, 2006]

Theoretically sounded but problematic for data/theory comparison

- requires to unfold the experimental data to the theory calculation performed with the flavour k_T algorithm
- **unfolding corrections** can be **sizeable:** $\sim 12\,\%$ $Z + b$ jet as estimated at NLO+PS accuracy

[Gauld, Gehrmann–De Ridder, Glover, Huss, Majer, 2020]





Flavour aware jet algorithms: flavour anti-k_T

[Czakon, Mitov, Poncelet, 2022]

Standard anti-k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^{-2}, k_{T,j}^{-2}\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^{-2}$$

does not vanish in the double soft limit

Flavour anti-k_T algorithm

$$d_{ij}^{(F)} = d_{ij} \times \begin{cases} \mathcal{S}_{ij}, & \text{if both } i \text{ and } j \text{ have non-zero flavour of opposite sign} \\ 1, & \text{otherwise} \end{cases}$$

$$\mathcal{S}_{ij} = 1 - \theta(1-\kappa)\cos\left(\frac{\pi}{2}\kappa\right), \qquad \kappa = \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,max}^2}$$

$$\mathcal{S}_{ij} \sim E^4 \implies d_{ij}^{(F)} \sim E^2$$

the suppression factor overcompensates the divergent behaviour of d_{ij} in the double soft limit:
in this way, both conditions 1 and 2 hold

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Parameter a control the turning on of the suppression factor: in the limit $a \to 0$, the standard anti-k_T algorithm is recovered. Best choice of the parameter a from comparison at NLO+PS (aiming at minimising unfolding)

Flavour-dependent metric, still needs some (possibly small) unfolding

Flavour aware jet algorithms: new ideas

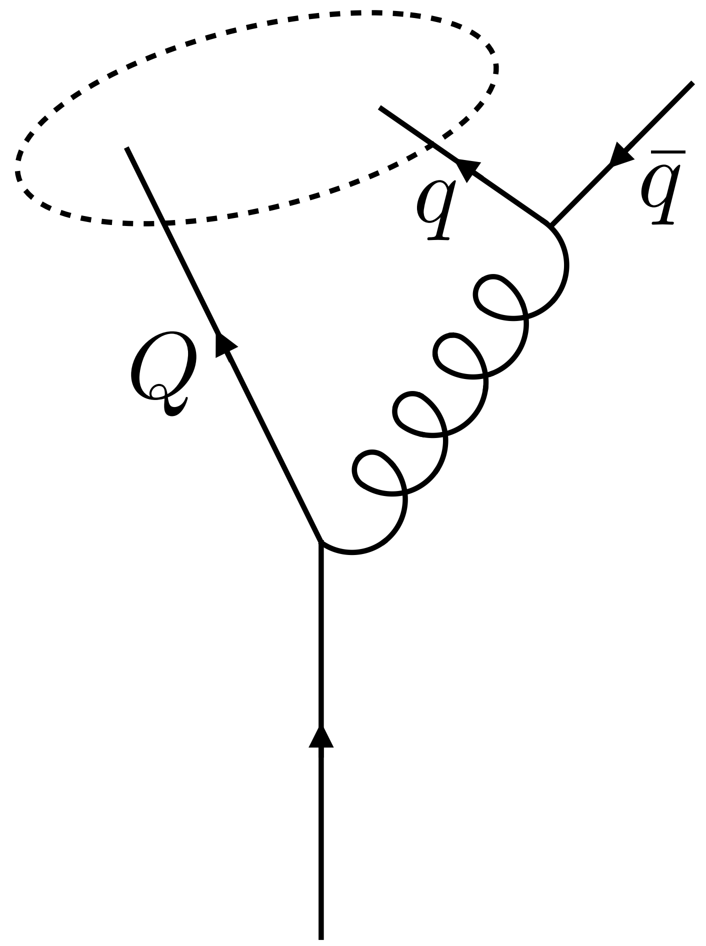
see talk by R. Gauld

Recently, the problem has triggered a lot of activity in the theoretical community!

Use **Soft Drop** to remove soft quarks

No unfolding needed

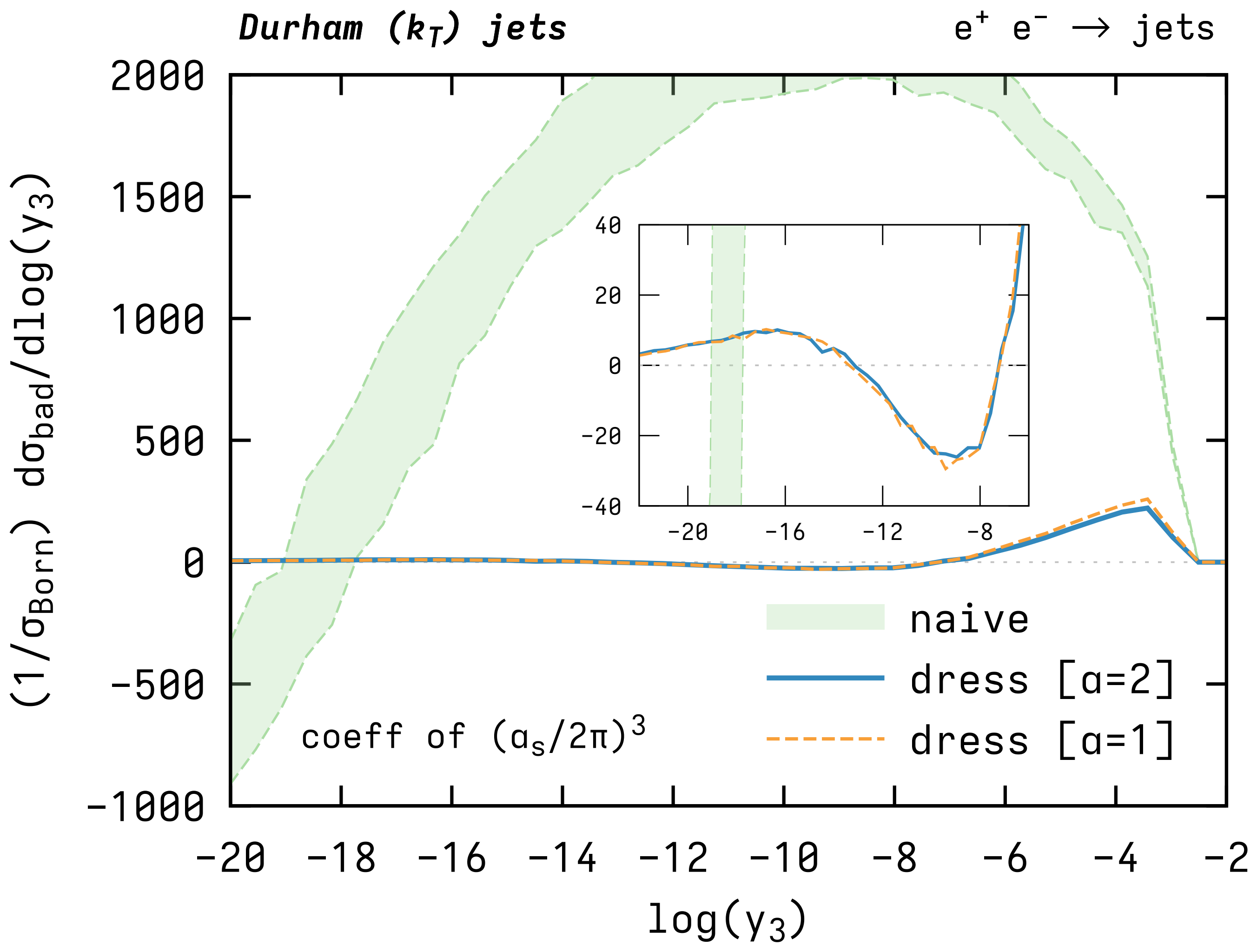
Requires reclustering with JADE (issue with IRC safety beyond NNLO)



[Caletti, Larkoski, Marzani, Reichelt, 2022]

Assign a **flavour dressing** to jets reconstructed with any IRC flavour-blind jet algorithms

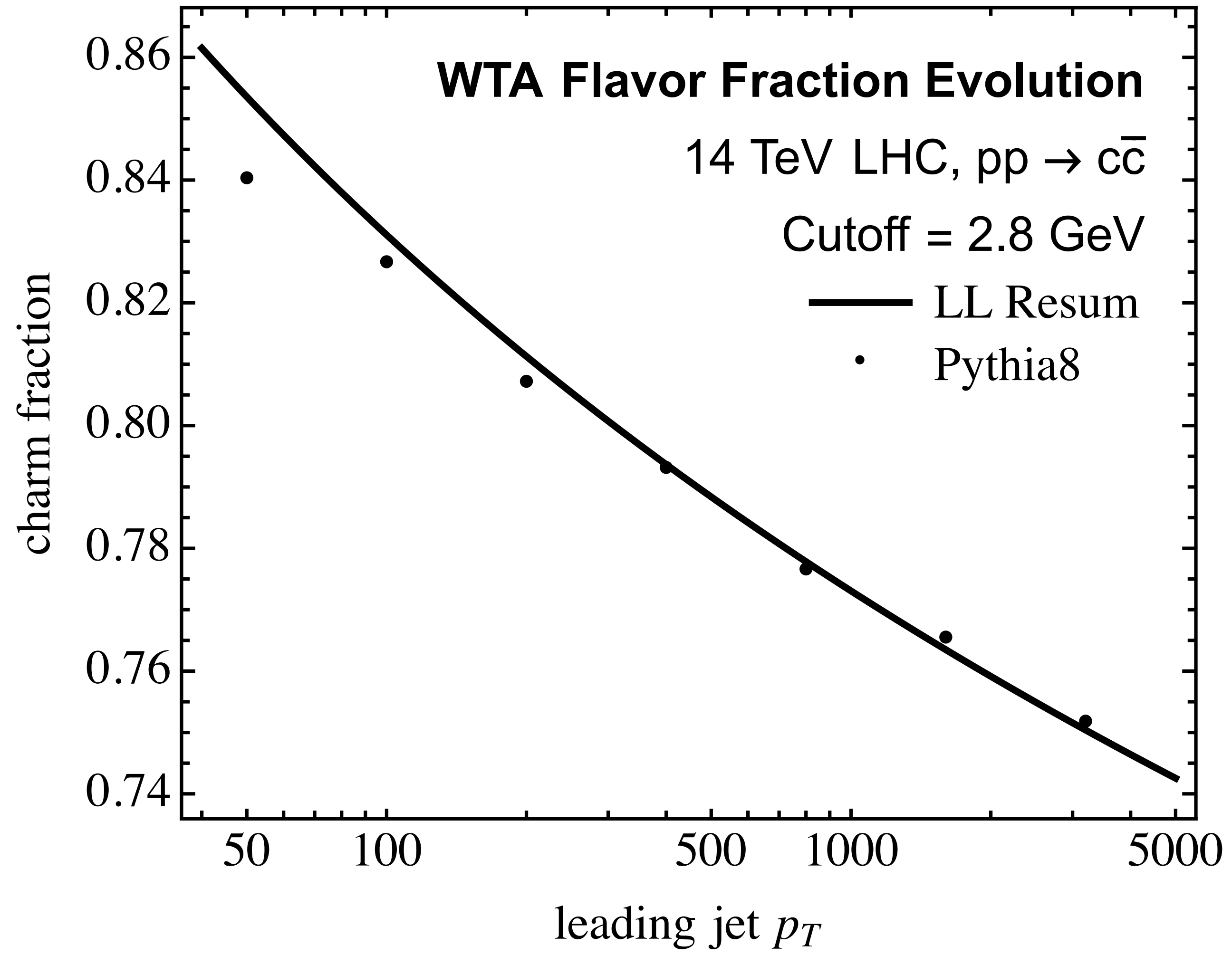
Requires flavour information of many particles in the event



[Gauld, Huss, Stagnitto, 2022]

Recluster using the flavour aware **Winner-Take-All** (WTA) recombination scheme (**soft-safe**)

Requires fully perturbative WTA flavour fragmentation function (for **collinear safety**)



[Caletti, Larkoski, Marzani, Reichelt, 2022]

Flavour aware jet algorithms: massive calculation

Massive bottom quarks

- quark mass is the physical IR regulator: physical suppression in the double-soft limit
- No requirement for flavour-aware jet algorithms: any **flavour-blind algorithm** can be used, in particular **anti** k_T

Direct comparison with experimental data possible
(unfolding corrections limited to non-perturbative modelling and hadronisation)

Caveat

- left over IR sensitivity in the form of logarithms of the heavy quark mass at each order in perturbative theory
- Calculation with massive quarks is challenging

Outline

- Methodology: slicing formalism
- Methodology: two-loop virtual amplitude
- Phenomenological results
- Conclusions

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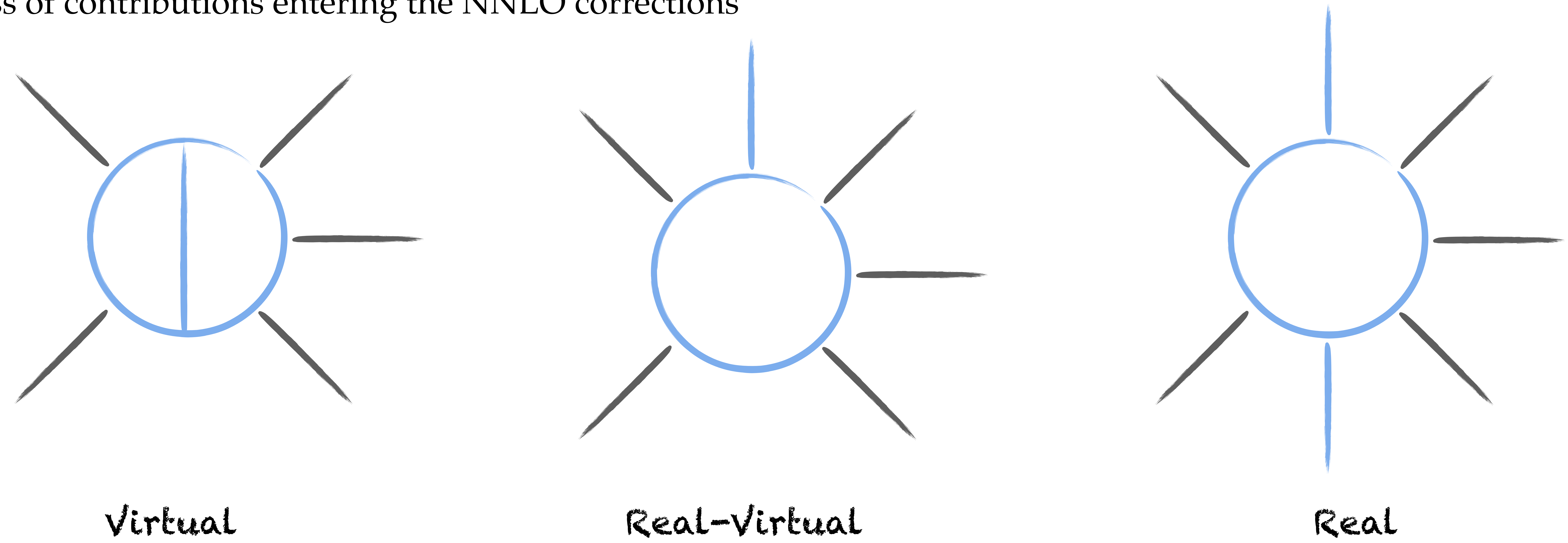
Infrared singularities

Class of contributions entering the NNLO corrections



KLN theorem and collinear factorisation ensure the cancellation of singularities for any infrared safe observables, but virtuals, real-virtual and reals live on different phase spaces and are separately divergent …

Subtraction/Slicing scheme required!

q_T-subtraction formalism

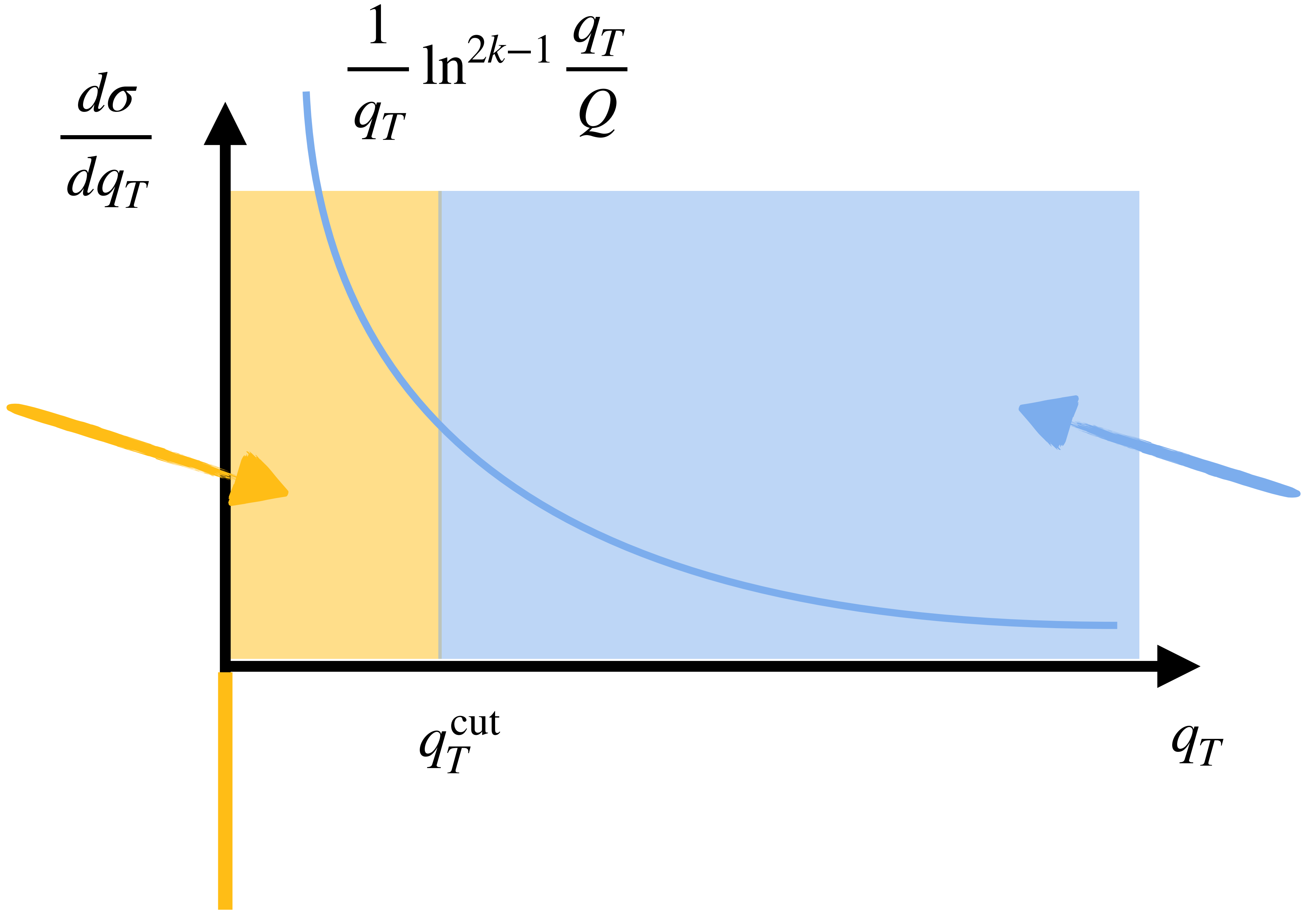
[Catani, Grazzini, 2007]

Cross section for the production of a triggered final state F at N^{k}LO

All emission unresolved; approximate the cross section with its singular part in the soft and/or collinear limits

q_T resummation

- **expand to fixed order**
- **$\mathcal{O}(\alpha_s^k)$ ingredient required**



1 emission always resolved

$F + j$ @ N^{k-1}LO

complexity of the calculation reduced by one order!

$$\int d\sigma_{N^kLO} = \mathcal{H} \otimes d\sigma_{LO} + \int \left[d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T > q_T^{\rm cut}} + \mathcal{O}\left((q_T^{\rm cut})^\ell \right)$$

q_T-subtraction formalism: extension to massive final states

$$\int d\sigma_{N^kLO} = \mathscr{H} \otimes d\sigma_{LO} + \int \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T > q_T^{\mathrm{cut}}} + \mathcal{O}\left((q_T^{\mathrm{cut}})^{\ell} \right)$$

All ingredients for $Wb\bar{b} + j$ @ NLO available:

Required matrix elements implemented in public libraries such as OpenLoops2

[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller '19]

Local subtraction scheme available, for example dipole subtraction

[Catani, Seymour, '98] [Catani, Dittmaier, Seymour, Trocsanyi '02]

Automatised implementation in the MATRIX framework, which relies on the efficient multi-channel Monte Carlo integrator MUNICH

[Grazzini, Kallweit, Wiesemann '17] [Kallweit in preparation]

q_T-subtraction formalism: extension to massive final states

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$\mathcal{H}$ contains virtual correction after subtraction of IR singularities and contribution of soft/collinear origin

- Beam functions ✓
- Soft function

[Catani, Cieri, de Florian, Ferrera, Grazzini '12]
[Gehrmann, Luebbert, Yang '14]
[Echevarria, Scimemi, Vladimirov '16]
[Luo, Wang, Xu, Yang, Yang, Zhu '19]
[Ebert, Mistlberger, Vita]

see talks by B. Mistlberger, G. Vita

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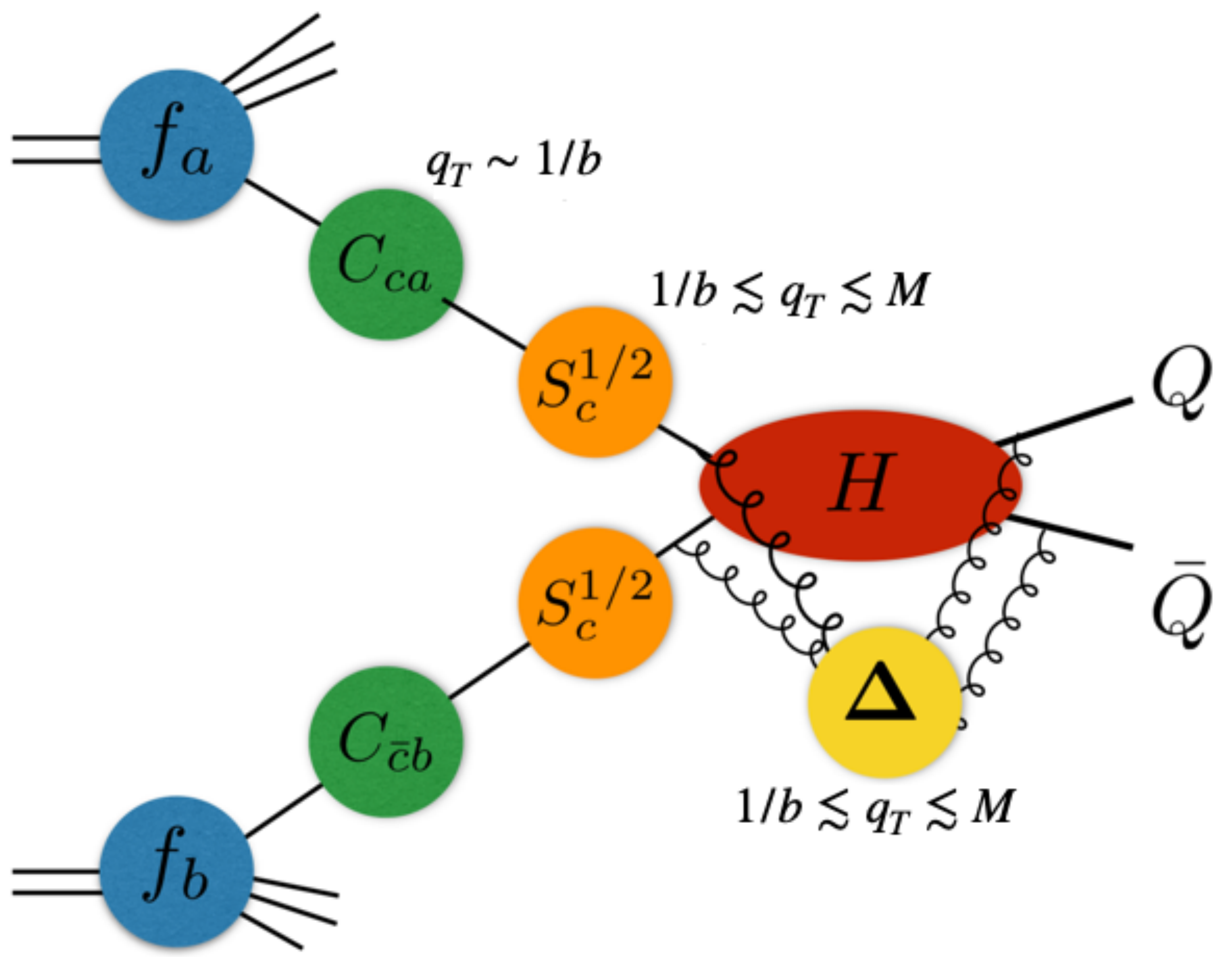

The resummation formula shows a **richer structure** because of additional soft singularities

- Soft logarithms controlled by the **transverse momentum anomalous dimension $\boldsymbol{\Gamma}_t$** known up to NNLO [Mitov, Sterman, Sung, 2009], [Neubert, et al 2009]
- Hard coefficient gets a **non-trivial** colour structure (matrix in colour-space)
- Non trivial azimuthal correlations

q_T-subtraction formalism: extension to massive final states

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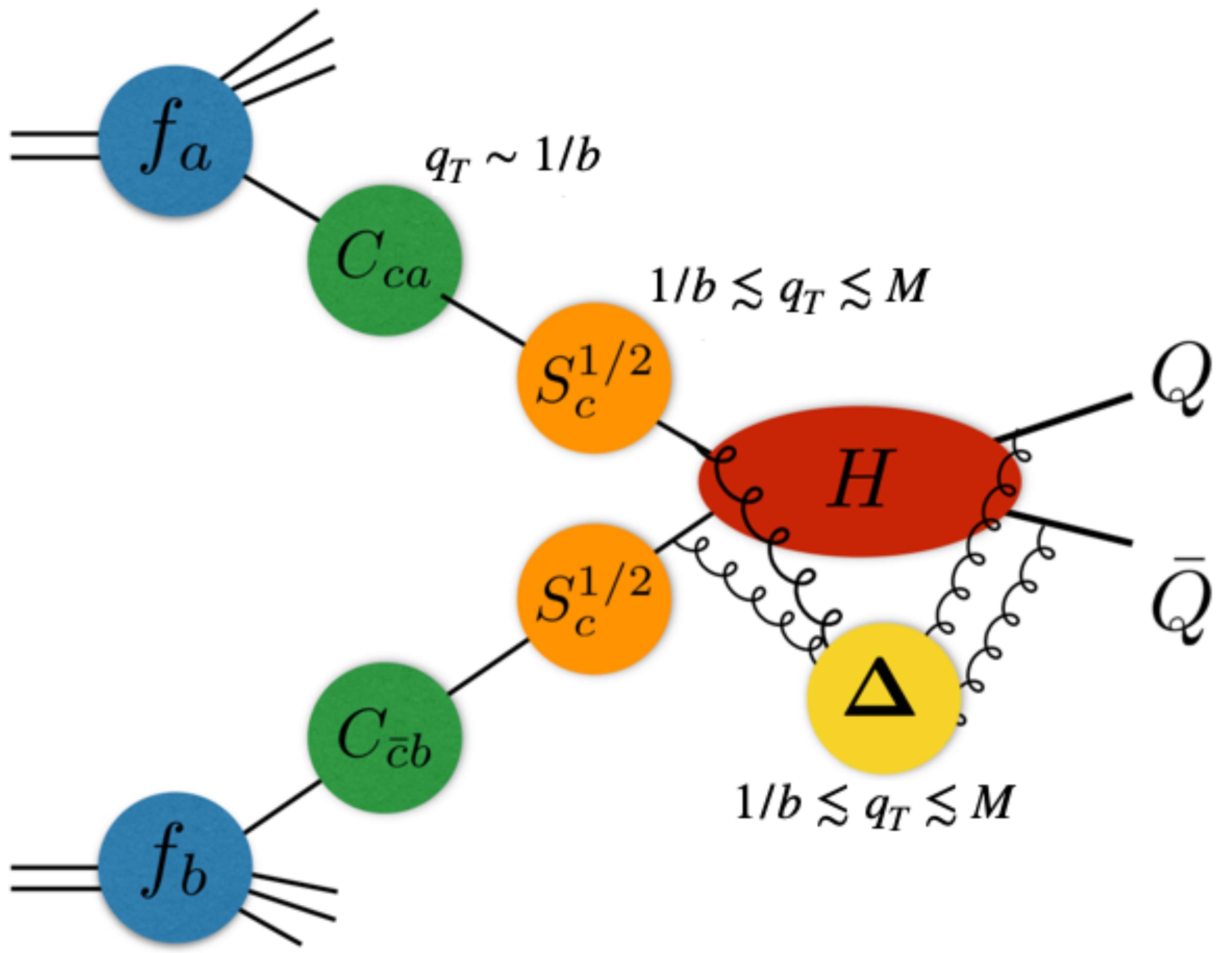

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q_T subtraction formalism extended to the case of **heavy quarks** production [Catani, Grazzini, Torre, 2014]

Successful employed for the computation of NNLO QCD corrections to the production of

- a **top pair** [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan 2019]
- a **bottom pair** production [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, 2021]

q_T-subtraction formalism: extension to massive final states

$$\int d\sigma_{N^kLO} = \mathcal{H} \otimes d\sigma_{LO} + \int \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T > q_T^{\mathrm{cut}}} + \mathcal{O}\left((q_T^{\mathrm{cut}})^{\ell} \right)$$

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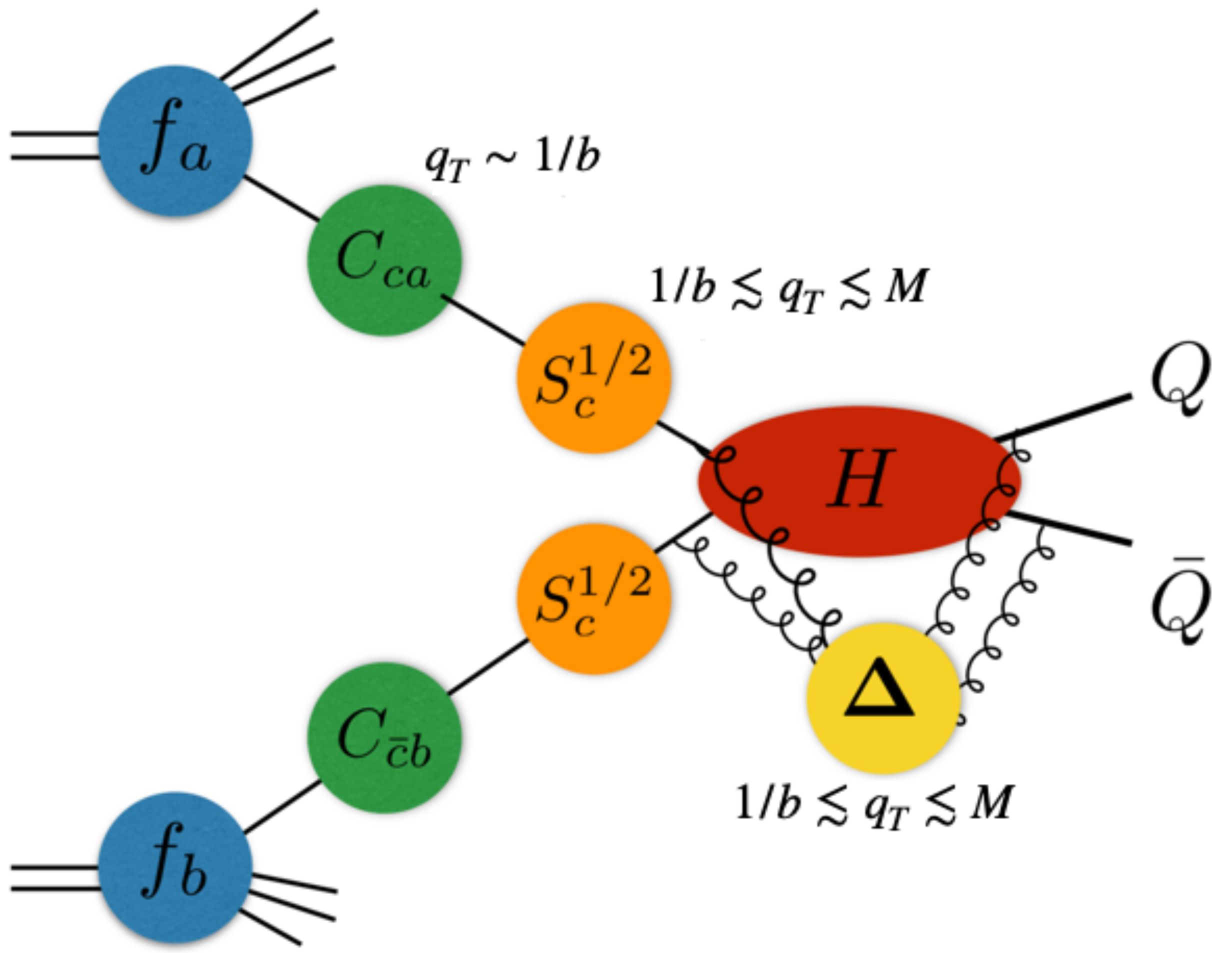

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Non trivial ingredient

- **Two-loop soft function** for heavy-quark (back-to-back Born kinematic) [Catani, Devoto, Grazzini, Mazzitelli,2023]
- Recently generalised to **arbitrary kinematics** [Devoto, Mazzitelli in preparation]

q_T-subtraction formalism: extension to massive final states

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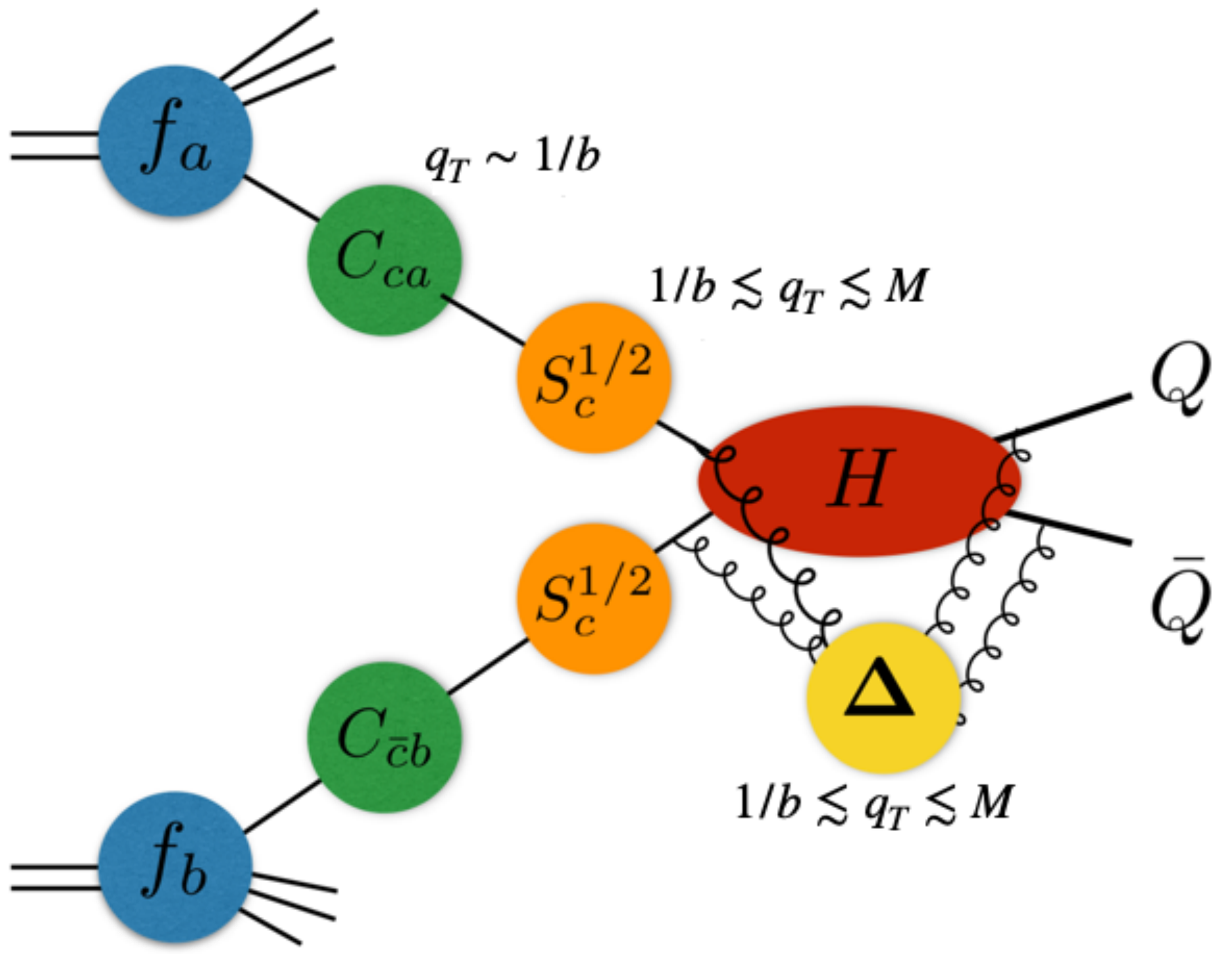

The resummation formula shows a **richer structure** because of additional soft singularities

Once the corresponding two-loop amplitude is available, the framework allows the calculation of the NNLO correction to the production of **a massive heavy-quark pair and a generic color singlet process**

- **First application:** $t\bar{t}H$
  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini, 2023]

see talk by C. Savoini

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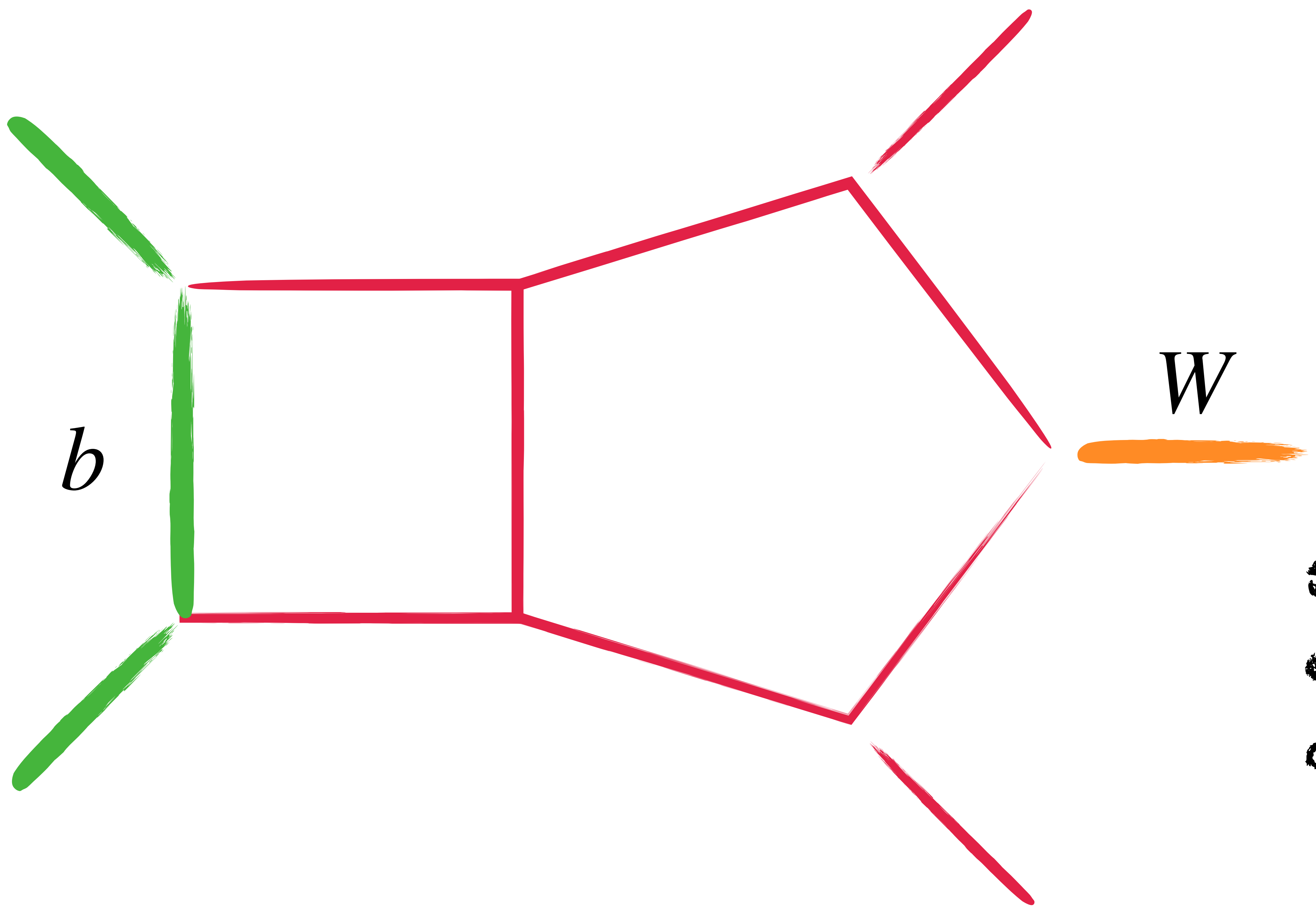
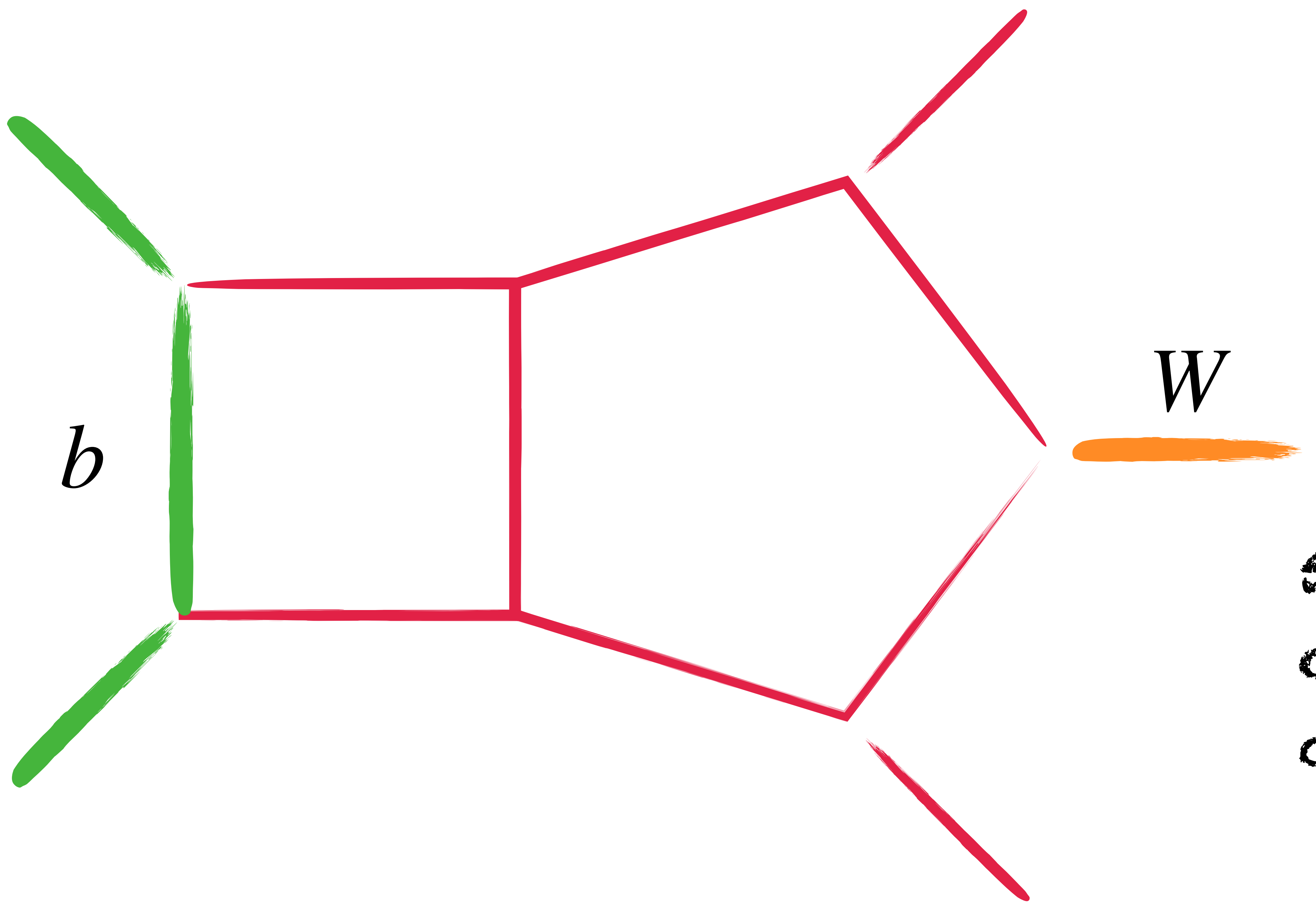
Two-loop virtual amplitude



5-point amplitude with 1 massive particle
current state of the art, more massive legs
out of reach!

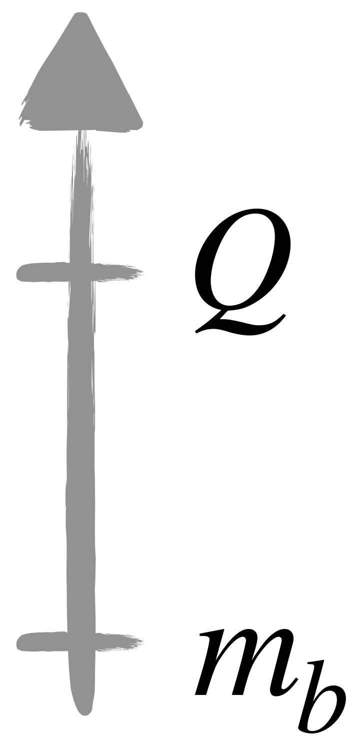
Two-loop virtual amplitude



5-point amplitude with 1 massive particle
current state of the art, more massive legs out of reach!

But m_b is the smallest scale in the game, exploit the hierarchy!

Use MASSIFICATION to relate the massive amplitude to the massless one up to power corrections of the mass



Massification procedure in a nutshell

[Mitov, Moch, 2007]

Amplitude factorisation in massless QCD [Catani, 1998][Sterman, Tejeda-Yeomans, 2003]

$$|\mathscr{M}^{[p]}> = \mathscr{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) \times \mathscr{S}^{[p]}\left(\{k_i\}\frac{Q^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) \times |\mathscr{H}^{[p]}>$$

Jet function: collinear contributions

Soft function: coherent soft radiation

Hard function: short-distance dynamics

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Amplitude factorisation in QCD with a **massive** parton of mass $m^2 \ll Q^2$

$$|\mathscr{M}^{[p],(m)}> = \mathscr{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) \times \mathscr{S}^{[p]}\left(\{k_i\}\frac{Q^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) \times |\mathscr{H}^{[p]}> + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

$$\mathscr{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) = \prod_i \mathscr{J}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) = \prod_i \left(\mathscr{F}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right)\right)^{1/2}$$

space-like massive form factor

Massification procedure in a nutshell

[Mitov, Moch, 2007]

Master formula of "massification"

$$\boxed{\begin{aligned} &|\mathcal{M}^{[p],(m)}> = \prod_i \left[Z_{[i]}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right]^{1/2} \times |\mathcal{M}^{[p]}> + \mathcal{O}\left(\frac{m^2}{Q^2}\right) \\ &Z_{[i]}\left(\frac{m^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) = \mathcal{F}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right)\left[\mathcal{F}^i\left(\frac{Q^2}{\mu^2}, 0, \alpha_S(\mu^2), \epsilon\right)\right]^{-1} \end{aligned}}$$

History & Remarks

- The formula retrieves mass logarithms and constant terms !

[Glover, TauskandJ, VanderBij, 2001]
[Penin 2005-2006]

- Consistent with previous results for NNLO QED correction to Bhabha scattering

[Czakon, Mitov, Moch, 2007]

- Successfully employed to derive and cross check results for $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$ amplitudes
- Recently extended to the case of two different external masses ($M \gg m$)

see talks by M. Rocco and Y. Ulrich

[Engel, Gnendiger, Signer, Ulrich 2019]

Massification procedure for Wbb

[Mitov, Moch, 2007]

The massification procedure is based on the **factorisation properties** of QCD amplitudes

Basic idea: in the small mass limit, the massive amplitude $\mathcal{M}^{[p],(m)}$ and the massless one $\mathcal{M}^{[p],(m=0)}$ are connected as the mass screens some collinear divergences "trading" poles in the dimensional regulator ϵ for logarithms of the mass

This can be viewed as a **change in the renormalisation scheme** which leads to a universal **"multiplicative renormalization"** relation between (*ultraviolet renormalised*) massive and massless amplitudes

$$\mathcal{M}^{[p],(m)} = \prod_{i\in\{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\right)^{\frac{1}{2}} \mathcal{M}^{[p],(m=0)} + \mathcal{O}(m^k)$$

- The function $Z_{[i]}^{(m|0)}$ are universal, depend only on the external parton (quark or gluon) and admit a perturbative expansion in α_s:

$$Z_{[i]} = 1 + \sum_k \left(\frac{\alpha_s}{2\pi}\right)^k Z_{[i]}^k$$

$$\mathcal{M}^{[p],(m)} = \sum_{k=0} \left(\frac{\alpha_s}{2\pi}\right)^k \mathcal{M}_{(k)}^{[p],(m)}$$

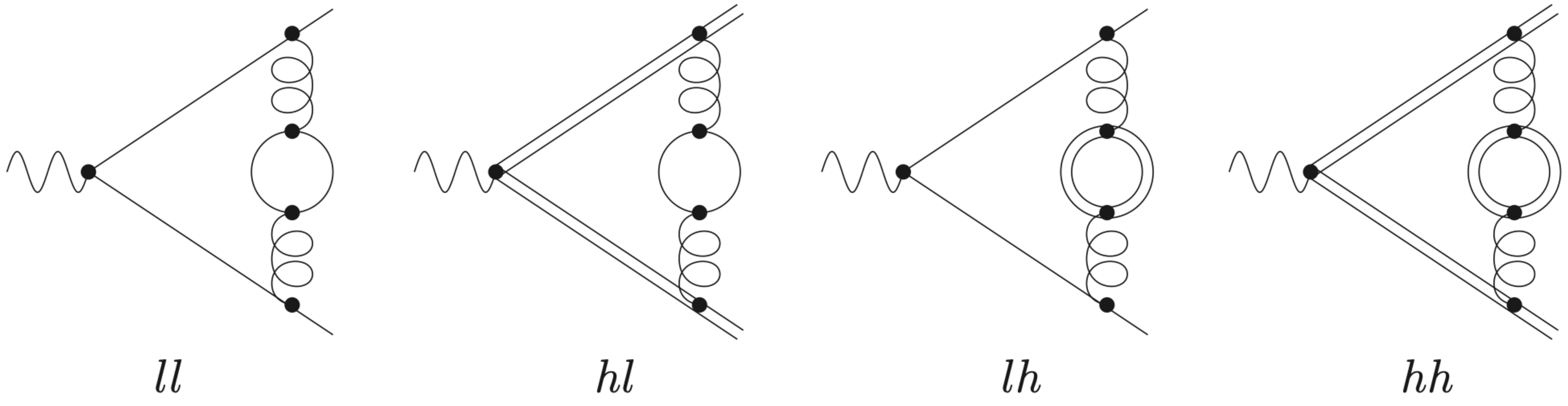
$$\boxed{\begin{aligned}
\mathcal{M}_0^{Wbb,(m)} &= \mathcal{M}_0^{Wbb,(m=0)} \\
\mathcal{M}_{(1)}^{Wbb,(m)} &= \mathcal{M}_{(1)}^{Wbb,(m=0)} + Z_{[q]}^{(1)} \mathcal{M}_{(0)}^{Wbb,(m=0)} \\
\mathcal{M}_{(2)}^{Wbb,(m)} &= \mathcal{M}_{(2)}^{Wbb,(m=0)} + Z_{[q]}^{(1)} \mathcal{M}_{(1)}^{Wbb,(m=0)} + Z_{[q]}^{(2)} \mathcal{M}_{(0)}^{Wbb,(m=0)}
\end{aligned}}$$

Massification procedure for Wbb

[Mitov, Moch, 2007]

$$\mathscr{M}^{[p],(m)} = \prod_{i\in\{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\right)^{\frac{1}{2}} \mathscr{M}^{[p],(m=0)} + \mathcal{O}(m^k)$$

- The $Z_{[i]}^{(m|0)}$ are given by the **ratio of massive and massless form factors** ($\gamma^* qq$ for the quark case)



- Starting from two loops, contributions from **heavy quarks loops** (lh and hh) arise. Their description requires additional process dependent terms and **have been excluded** from the definition of the $Z_{[i]}^{(m|0)}$

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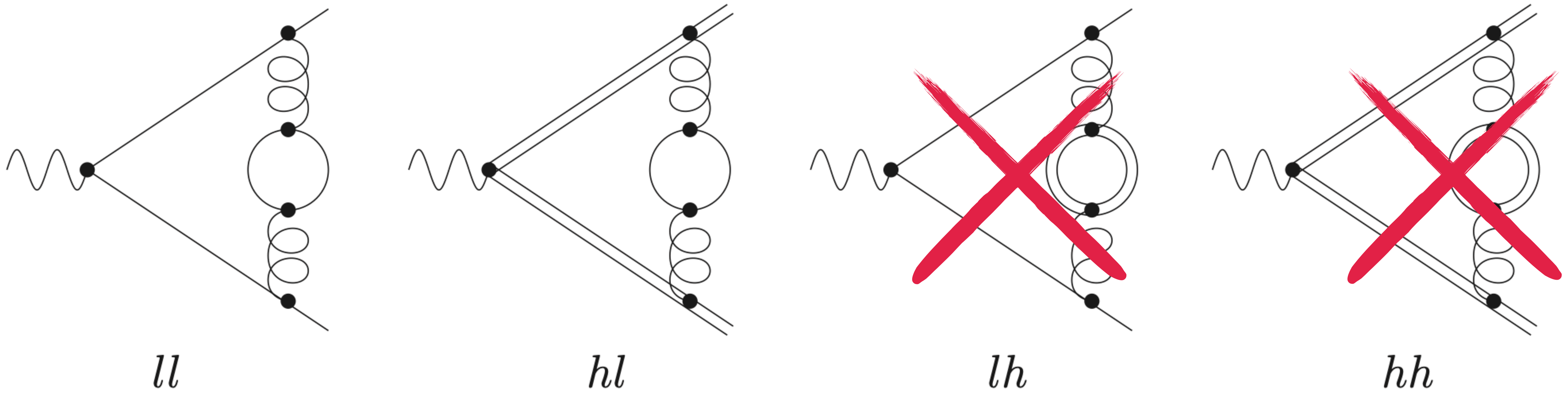

- Starting from two loops, contributions from **heavy quarks loops** (lh and hh) arise. Their description requires additional process dependent terms and **have been excluded** from the definition of the $Z_{[i]}^{(m|0)}$

The massification procedure predicts **poles, logarithms of mass and mass independent terms (constants)** of $\mathscr{M}^{[p],(m)}$ while **power corrections** in the mass and the contribution of **heavy loops** cannot be retrieved using this approach

WQQAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini, https://gitlab.com/lrottoli/WQQAmp]

We have implemented the one-loop and two-loop amplitudes of [Abreu et al, 2022] in a **C++ library** for the efficient numerical evaluation of the **massive amplitudes**

see talk by D. Chicherin

[Chicherin, Sotnikov, Zoia 2021]

PentagonFunctions-cpp

evaluation of pentagons functions

[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller, 2019]

OpenLoops 2

evaluation of exact one-loop amplitudes

$PS = \{p_1, p_2, \ldots, p_6\}$

massive phase space point **mapped** into a massless one (the mapping reduces to the identity in the massless limit)

WbbAmp

Massification

$$\frac{2\mathfrak{R} < M_0 | M_2^{\mathrm{fin}} >}{|M_0|^2}$$

Finite remainder defined subtracting the IR poles as defined in [Ferroglia, Neubert, Pecjac, Yang, 2009]

$\mathcal{O}(4s)$ **per phase space point**

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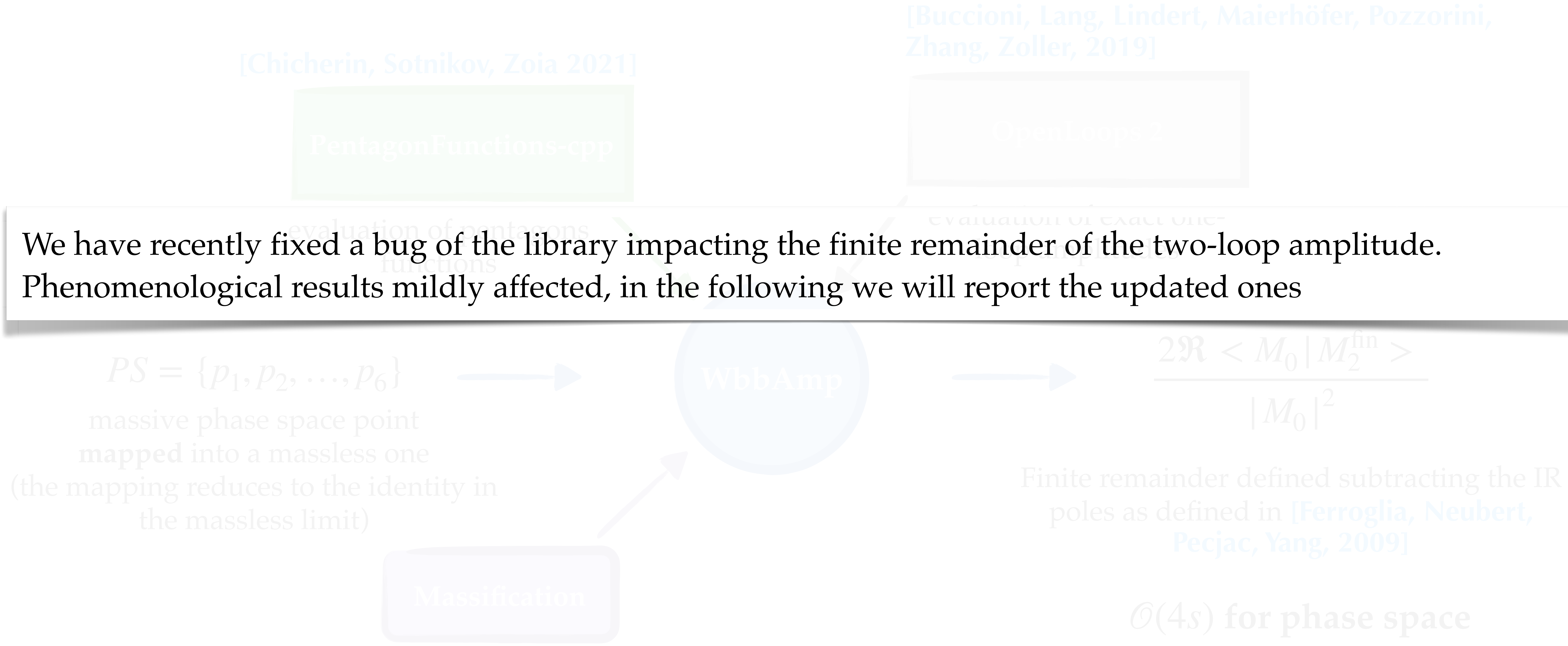


We have recently fixed a bug of the library impacting the finite remainder of the two-loop amplitude.
Phenomenological results mildly affected, in the following we will report the updated ones

Outline

- Methodology: slicing formalism
- Methodology: two-loop virtual amplitude
- Phenomenological results
- Conclusions

Setup

$$W + 2\ b_{\text{(jet)}} + X\ @ \sqrt{s} = 13.6\,\text{TeV}$$

α_s and PDF scheme	4-flavour scheme (4FS), m_b=4.92 GeV
EW	G_μ-scheme, CKM diagonal
Jet clustering algorithm	anti-k_T (and k_T) algorithm with R = 0.4
pdf sets	`NNPDF30_as_0118_nf_4` (LO) `NNPDF31_as_0118_nf_4` (NLO, NNLO)

SETUP

- **fiducial**: inspired by ATLAS $VH(\to b\bar{b})$ **boosted** analysis [ATLAS:arXiv:2007.02873]

$p_{T,\ell} > 25$ GeV $\quad |\eta_\ell| < 2.5 \quad p_T^W > 150\,\text{GeV}$

Jet selection

$p_{T,j} > 20$ GeV and $|\eta_\ell| < 2.5$ or
$p_{T,j} > 30$ GeV and $2.5 < |\eta_\ell| < 4.5$

Requirements on b-tagged jets

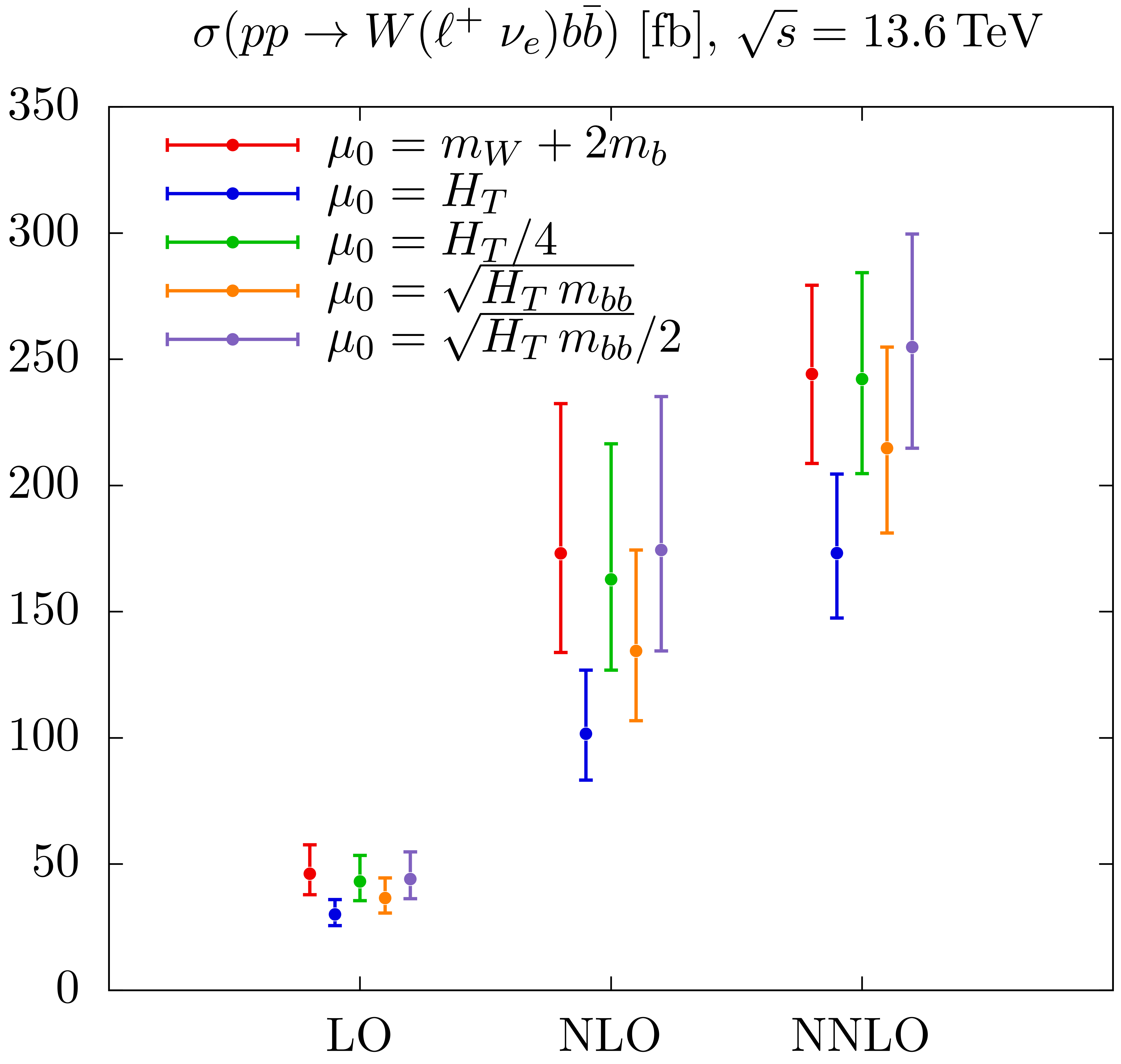
$n_b = 2, \quad p_{T,b_1} > 45$ GeV, $\quad 0.5 < \Delta R_{bb} < 2$

bin I : $150 < p_T^W < 250\,\text{GeV}$
bin II : $p_T^W > 250\,\text{GeV}$

Wbb phenomenology (**bin I+bin II**): scale choice

Behaviour of the perturbative series and scale choice

- A priori, the use of a fixed scale is physically **not very well motivated**



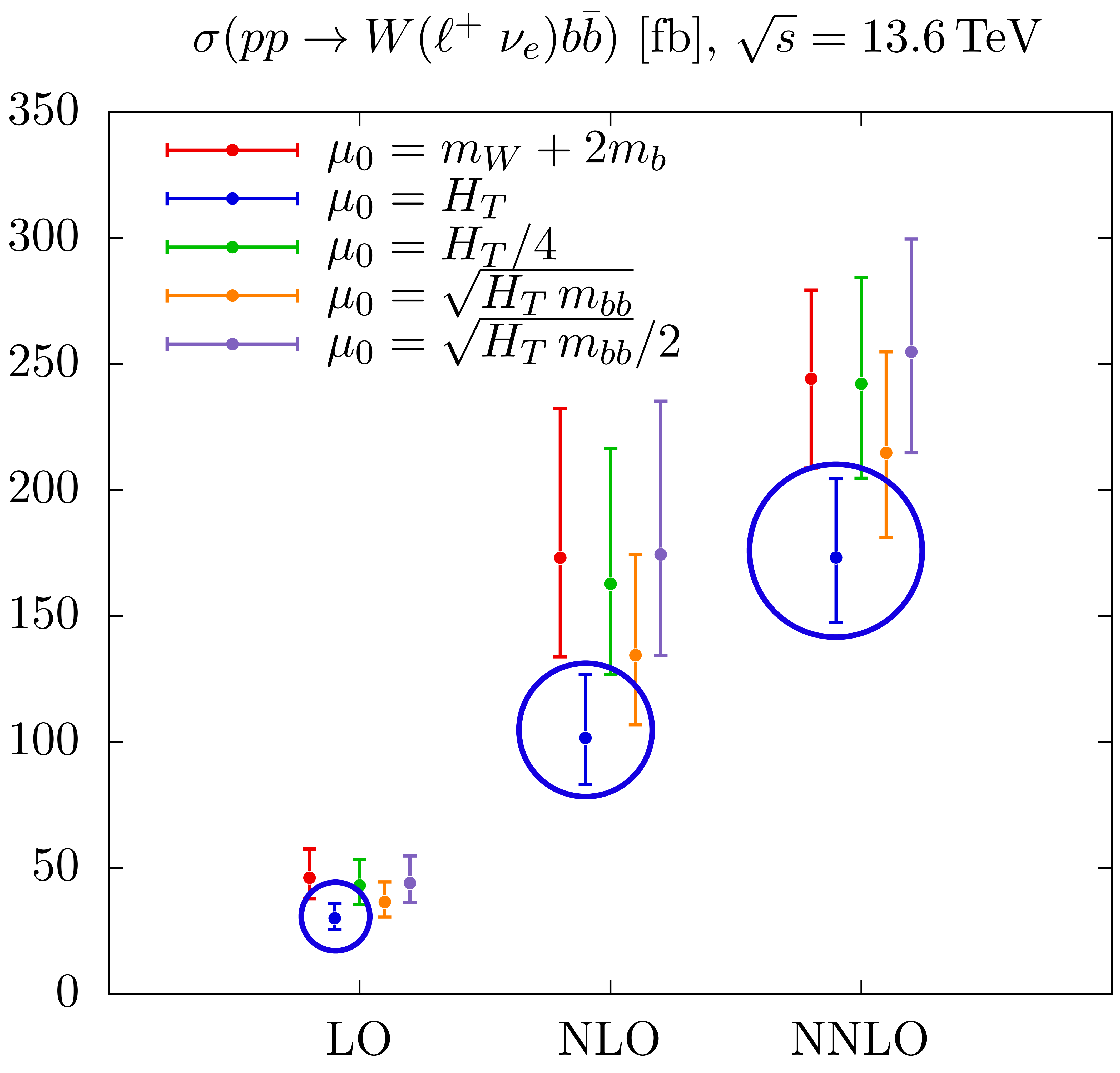
Wbb phenomenology (**bin I+bin II**): scale choice

Behaviour of the perturbative series and scale choice

- A priori, the use of a fixed scale is physically not very well motivated
- Naively, a dynamic scale as H_T would be a better choice. However, it leads to **a poor perturbative convergence with no overlap between NLO and NNLO** within their uncertainties bands

$$H_T = E_T(\ell\nu) + p_T(b_1) + p_T(b_2)$$

$$E_T(\ell\nu) = \sqrt{M^2(\ell\nu) + p_T^2(\ell\nu)}$$



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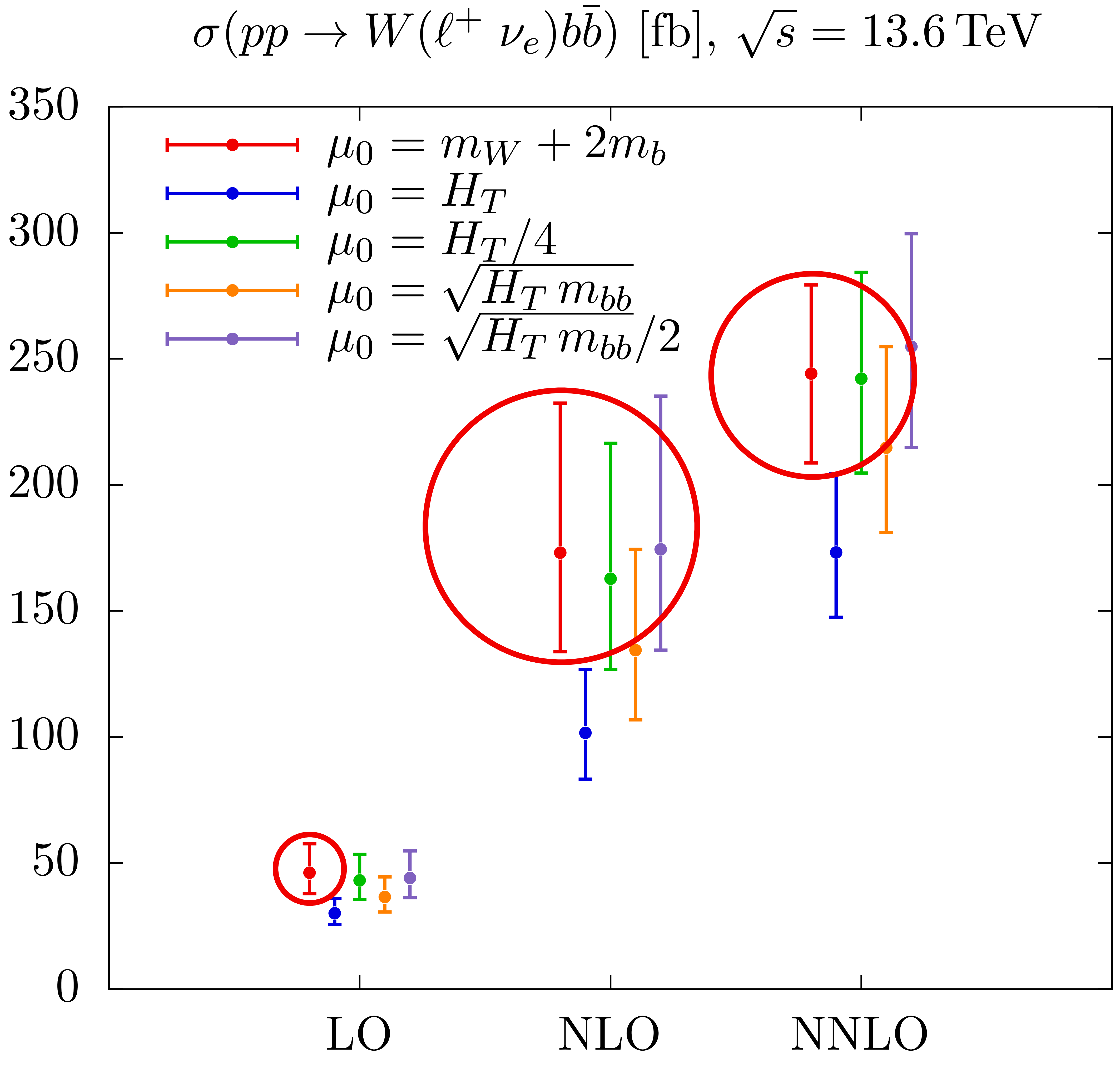

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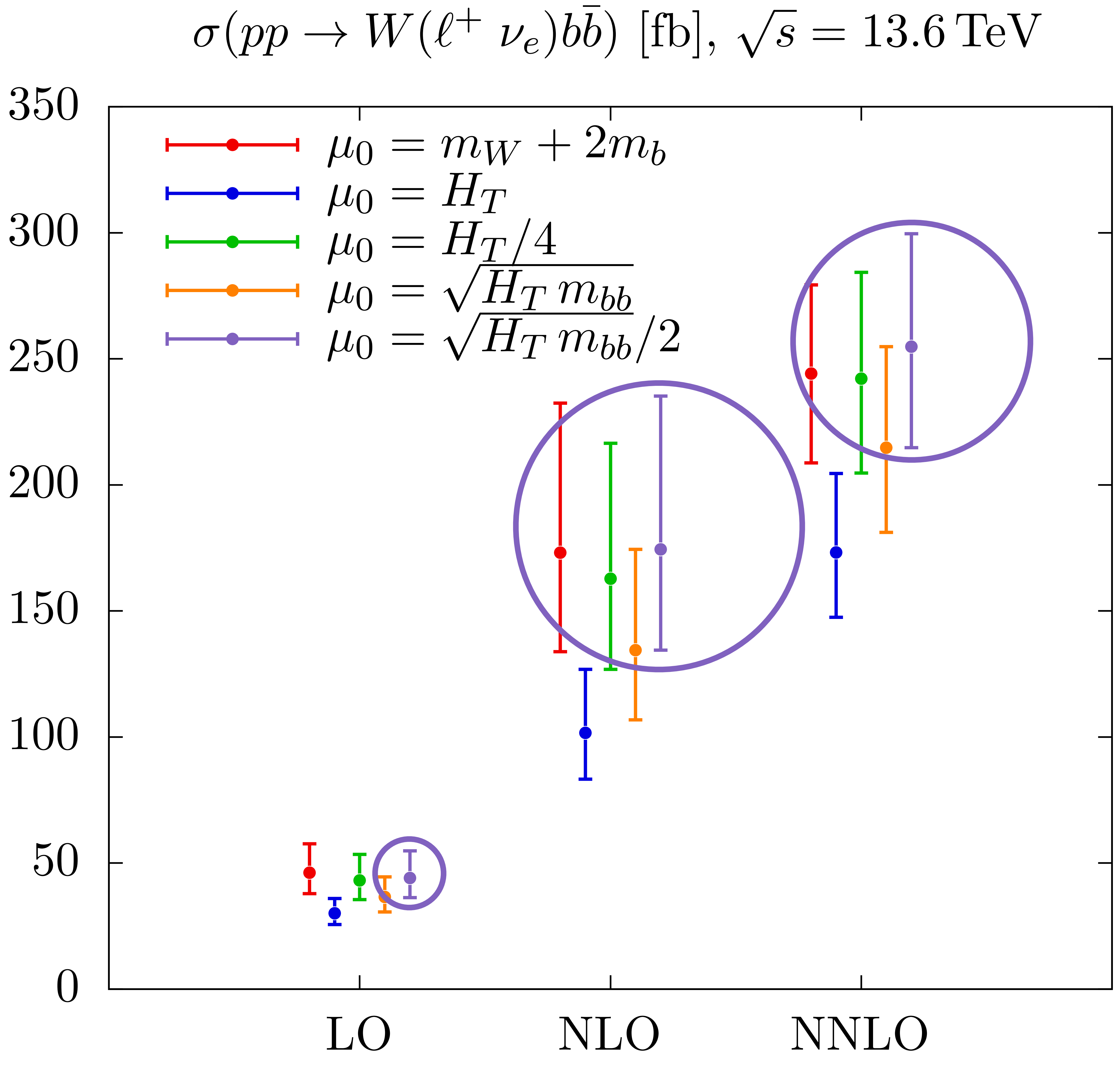
- On the contrary, the choice of a fixed scale leads to a better perturbative convergence, suggesting a preference for smaller scales
- A more detailed analysis should take into account the **"multi-scale" nature** of the process

H_T high-p_T kinematics

m_{bb} gluon splitting kinematics

$$\sqrt{H_T \cdot m_{bb}}$$

possibly divided by a factor of 2



Wbb phenomenology: fiducial cross sections

Results

- Reference scale: $\sqrt{H_T \cdot m_{bb}}/2$
- Large NLO K-factors $K_{\mathrm{NLO}} \gtrsim 3$
- **Relative large positive NNLO corrections**, $K_{\mathrm{NNLO}} \sim 1.5$
- **More reliable** theory uncertainties estimated by scale variations with a reduction to the $15-20\,\%$ level

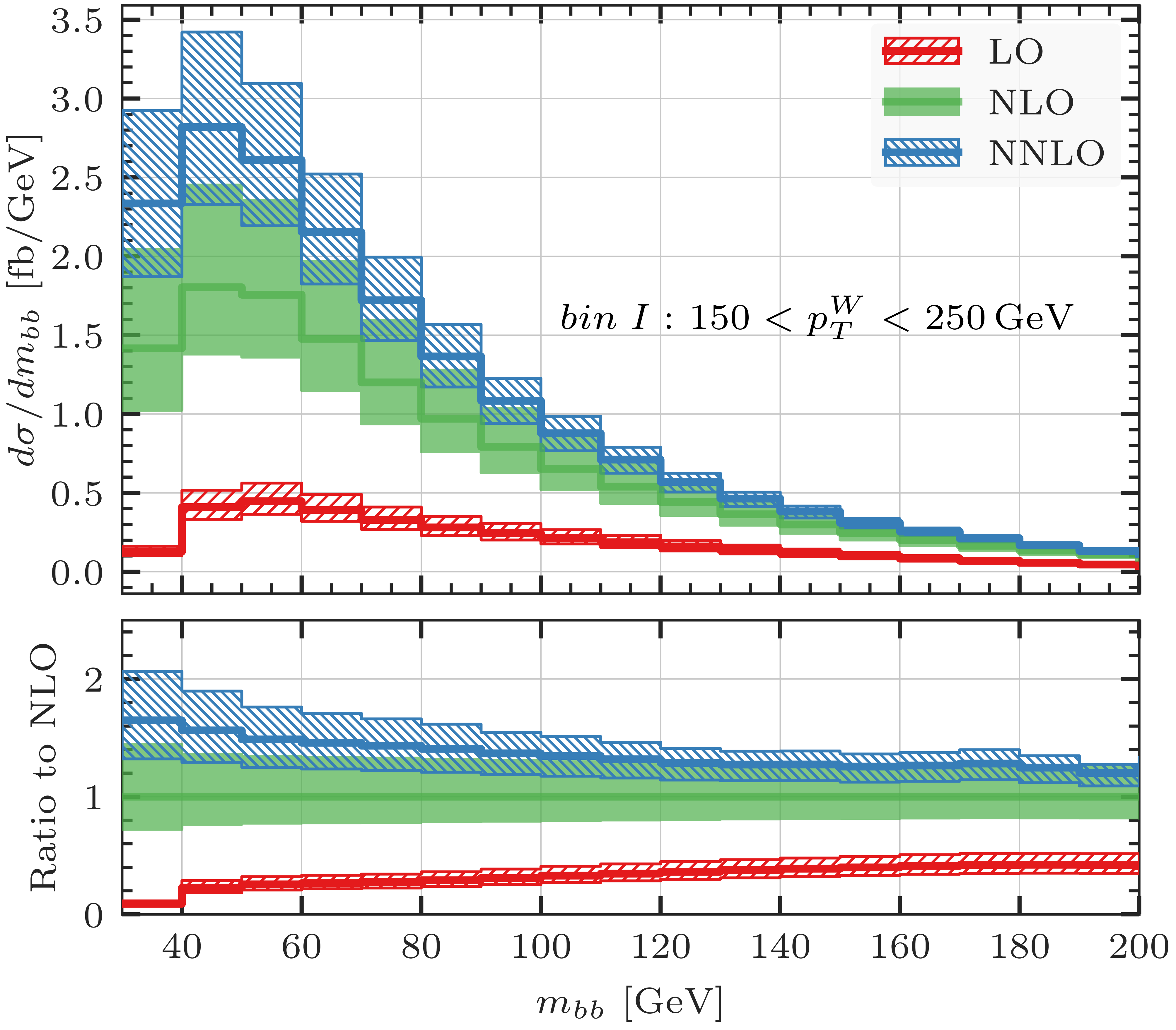
order	$\sigma_{\mathrm{fid}}^{bin\ I}$ [fb]	$\sigma_{\mathrm{fid}}^{bin\ II}$ [fb]
LO	$35.49(1)^{+25\%}_{-18\%}$	$8.627(1)^{+25\%}_{-18\%}$
NLO	$137.20(5)^{+34\%}_{-23\%}$	$37.24(1)^{+38\%}_{-24\%}$
NNLO	$198.9(8)^{+17\%}_{-15\%}$	$55.90(7)^{+19\%}_{-17\%}$

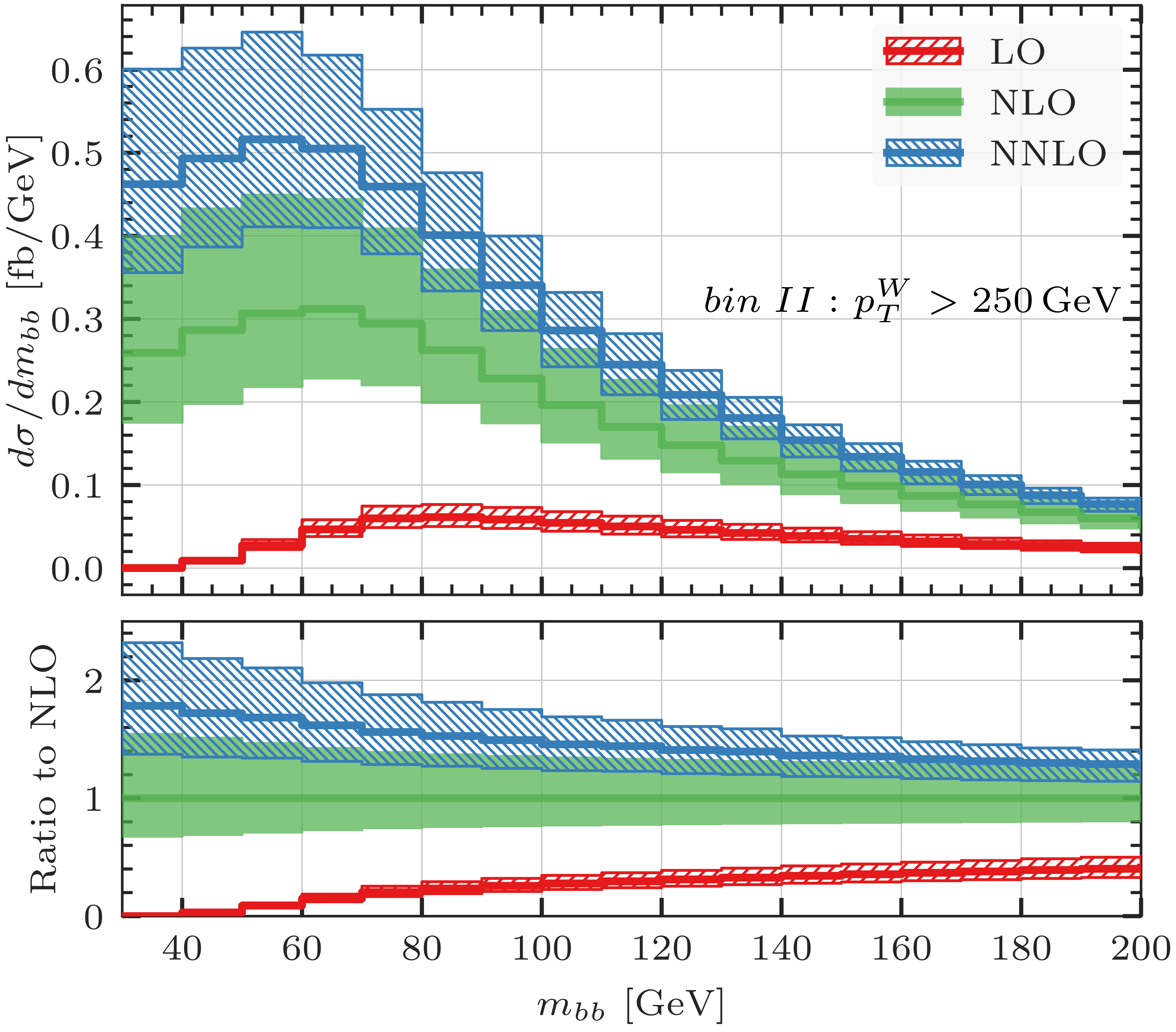
Other theoretical uncertainties are subdominant:

- Variation of bottom mass: $m_b = 4.2\,\mathrm{GeV} \implies \delta\sigma_{\mathrm{NNLO}}/\sigma_{\mathrm{NNLO}} = +\,2\,\%$
- Impact of massification estimated at NLO: $|\,\delta(\Delta\sigma_{\mathrm{NLO}})/\Delta\sigma_{\mathrm{NLO}}^{exact}\,| = 3\,\%$
- The part of the two-loop virtual amplitude computed in LCA contributes at the 2% level of the full NNLO correction

Wbb phenomenology: m_{bb} differential distribution

- Similar pattern of NNLO corrections for the two considered p_T^W bins
- NNLO corrections **not uniform**, larger for smaller invariant-mass values
- **Reduction** of scale uncertainties, **partial overlap** with the NLO bands





Comparison with HPPZ

$$W + 2\ b_{jet} + X\ \textbf{(inclusive)}\ @\ \sqrt{s} = 8\,\mathrm{TeV}$$

[CMS:arXiv:1608.07561]

Selection cuts

$$p_{T,\ell} > 30\ \mathrm{GeV} \qquad |\eta_\ell| < 2.1$$

$$n_b = 2 : p_{T,b} > 25\ \mathrm{GeV} \qquad |\eta_\ell| < 2.4$$

$$p_{T,j} > 25\ \mathrm{GeV} \qquad |\eta_\ell| < 2.4$$

Reference scale

$$H_T = E_T(\ell\nu) + p_T(b_1) + p_T(b_2)$$

$$E_T(\ell\nu) = \sqrt{M^2(\ell\nu) + p_T^2(\ell\nu)}$$

	HPPZ	**This work**
α_s and PDF scheme	5FS	4FS
Jet clustering algorithm	flavour k_T and flavour anti-k_T algorithm (R=0.5)	k_T and anti-k_T algorithm (R=0.5)
pdf sets	`NNPDF31_as_0118` (LO, NLO, NNLO)	`NNPDF30_as_0118_nf_4` (LO) `NNPDF31_as_0118_nf_4` (NLO, NNLO)

Comparison with HPPZ: fiducial cross sections

order	σ^{4FS} [fb]	$\sigma^{5FS}_{a=0.05}$ [fb]	$\sigma^{5FS}_{a=0.1}$ [fb]	$\sigma^{5FS}_{a=0.2}$ [fb]
LO	$210.42(2)^{+21.4\%}_{-16.2\%}$	$262.52(10)^{+21.4\%}_{-16.1\%}$	$262.47(10)^{+21.4\%}_{-16.1\%}$	$261.71(10)^{+21.4\%}_{-16.1\%}$
NLO	$468.01(5)^{+17.8\%}_{-13.8\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$
NNLO	$649.9(1.6)^{+12.6\%}_{-11.0\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	$647(7)^{+9.5\%}_{-9.4\%}$

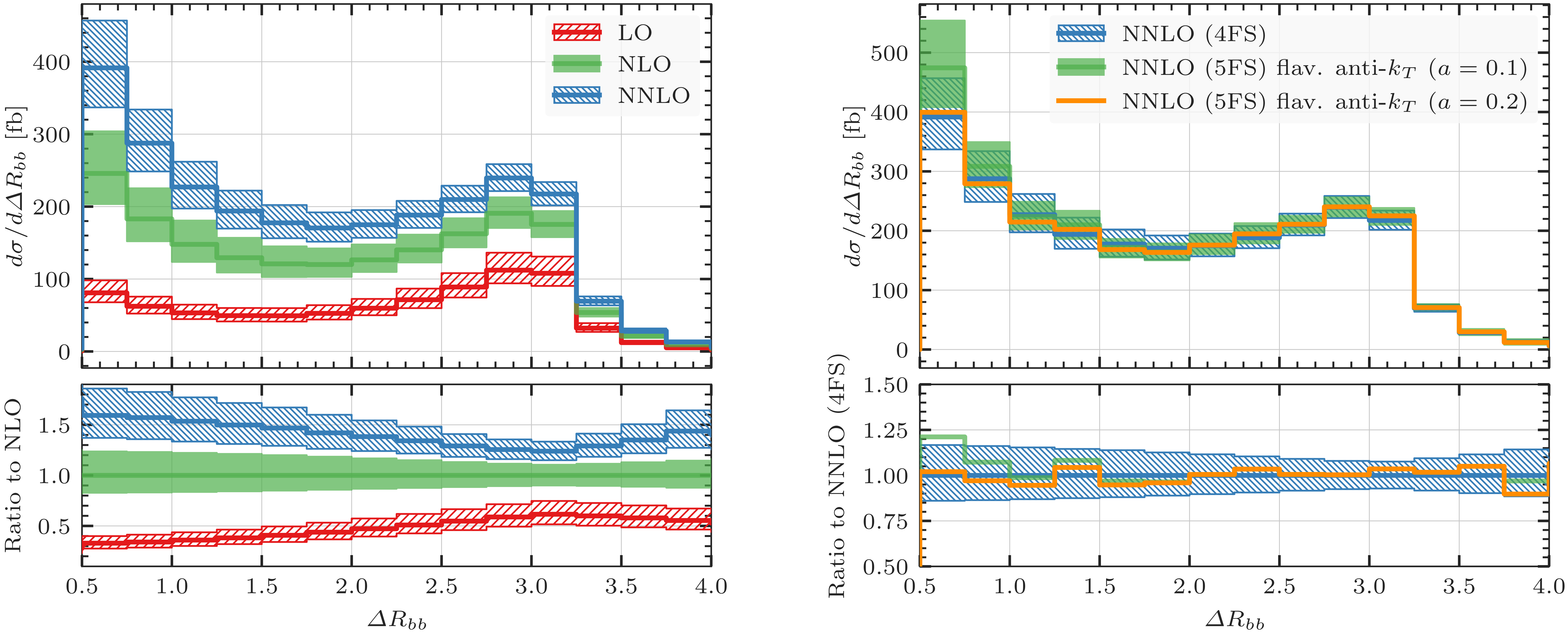
Remarks

- The parameter a of the flavour anti k_T algorithm plays a role similar to m_b in our massive calculation
- Uncertainty estimated by varying $a \in [0.05,0.2]$ amounts to 7 % ; **smaller** uncertainty estimated by varying $m_b \in [4.2,4.92]$, at the 2% level
- General **agreement within scale variations**, but the massive calculation performed in the 4FS **systematically below due to the different flavour scheme**

Comparison with HPPZ: jet clustering algorithms

Sizeable NNLO corrections which lead to a steeper slope at small ΔR_{bb} (where scale uncertainties are larger)

Good agreement between flavour and standard anti-k_T for the largest value $a = 0.2$



Conclusions

Thanks to the progress in

- **subtraction scheme**: the calculation of **soft function for arbitrary kinematics** allows to use the q_T subtraction formalism for the production of **a coloured massive final state + a colour singlet system**, as ttH and Wbb
- **QCD five-point scattering amplitude:** calculation of analytical amplitudes one massive + 4 partons in the leading colour (non planar topology within the reach in the near future)

we have completed the **first calculation of Wbb** production in NNLO QCD in the **4FS (massive b-quarks):**

- **for the missing massive amplitude,** we rely on an approximation based on the **massification procedure** of the corresponding massless amplitude
- NNLO QCD radiative corrections **crucial for precision phenomenology**
- our calculation **minimises ambiguities related to flavour tagging allowing for a more direct comparison to data**

Outlooks

- matching to parton shower to reach NNLO+PS accuracy
- extension to other processes

BACKUP

Flavour aware jet algorithms: flavour k_T

[Banfi, Salam, Zanderighi, 2006]

Standard k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^2$$

condition 1 automatically satisfied

Flavour aware k_T algorithm (usually $\alpha = 2$):

flavour information available at each step of the clustering procedure

$$d_{ij}^{(F)} = R_{ij}^2 \times \begin{cases} \left[\max\left(k_{T,i}^2, k_{T,j}^2\right)\right]^{\alpha} \left[\min\left(k_{T,i}^2, k_{T,j}^2\right)\right]^{2-\alpha}, & \text{if softer of } i,j \text{ is flavoured} \\ \min\left(k_{T,i}^2, k_{T,j}^2\right), & \text{if softer of } i,j \text{ is flavourless} \end{cases}$$

this ensures condition 2 among final state protojets, as soft flavoured quark-anti-quark pair clusters first

Flavour aware jet algorithms: flavour k_T

[Banfi, Salam, Zanderighi, 2006]

Standard k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^2$$

Flavour aware k_T algorithm (usually $\alpha = 2$):
flavour information available at each step of the clustering procedure

Also beam distance problematic:
a soft flavoured parton can be identified as a protojet and removed from the list)

$$d_{iB(\bar{B})}^{(F)} = R_{ij}^2 \times \begin{cases} \left[\max\left(k_{T,i}^2, k_{T,B(\bar{B})}^2\right)\right]^{\alpha} \left[\min\left(k_{T,i}^2, k_{T,B(\bar{B})}^2\right)\right]^{2-\alpha}, & \text{if } i \text{ is flavoured} \\ \min\left(k_{T,i}^2, k_{T,B(\bar{B})}^2\right), & \text{if } i \text{ is flavourless} \end{cases}$$

$$k_{T,B}(y) = \sum_i k_{T,i}\left(\Theta(y_i - y) + \Theta(y - y_i)e^{y_i - y}\right) \qquad k_{T,\bar{B}}(y) = \sum_i k_{T,i}\left(\Theta(y - y_i) + \Theta(y_i - y)e^{y - y_i}\right)$$

Intro: slicing methods: q_T subtraction formalism for massive final states

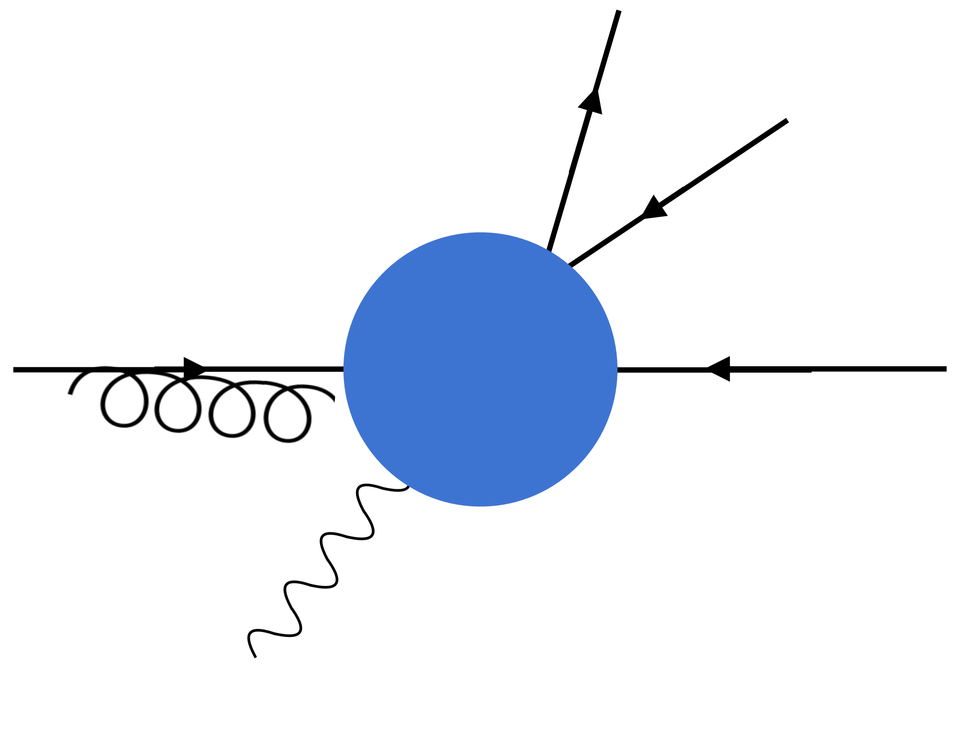
Resolution variable (for example in Drell-Yan)

$q_T :=$ **transverse momentum** of the dilepton final state
$Q :=$ invariant mass of the dilepton final state

Final state must be massive!

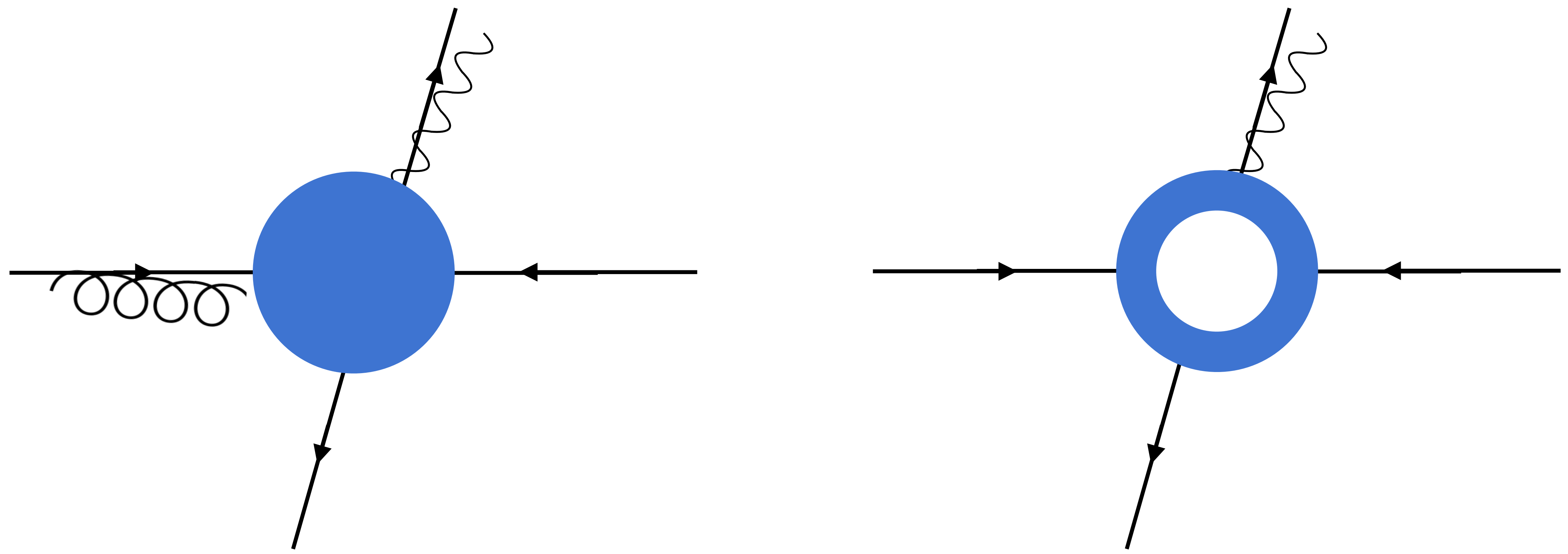
Initial-state radiation

For $q_T/Q > 0$ **one emission is always resolved**



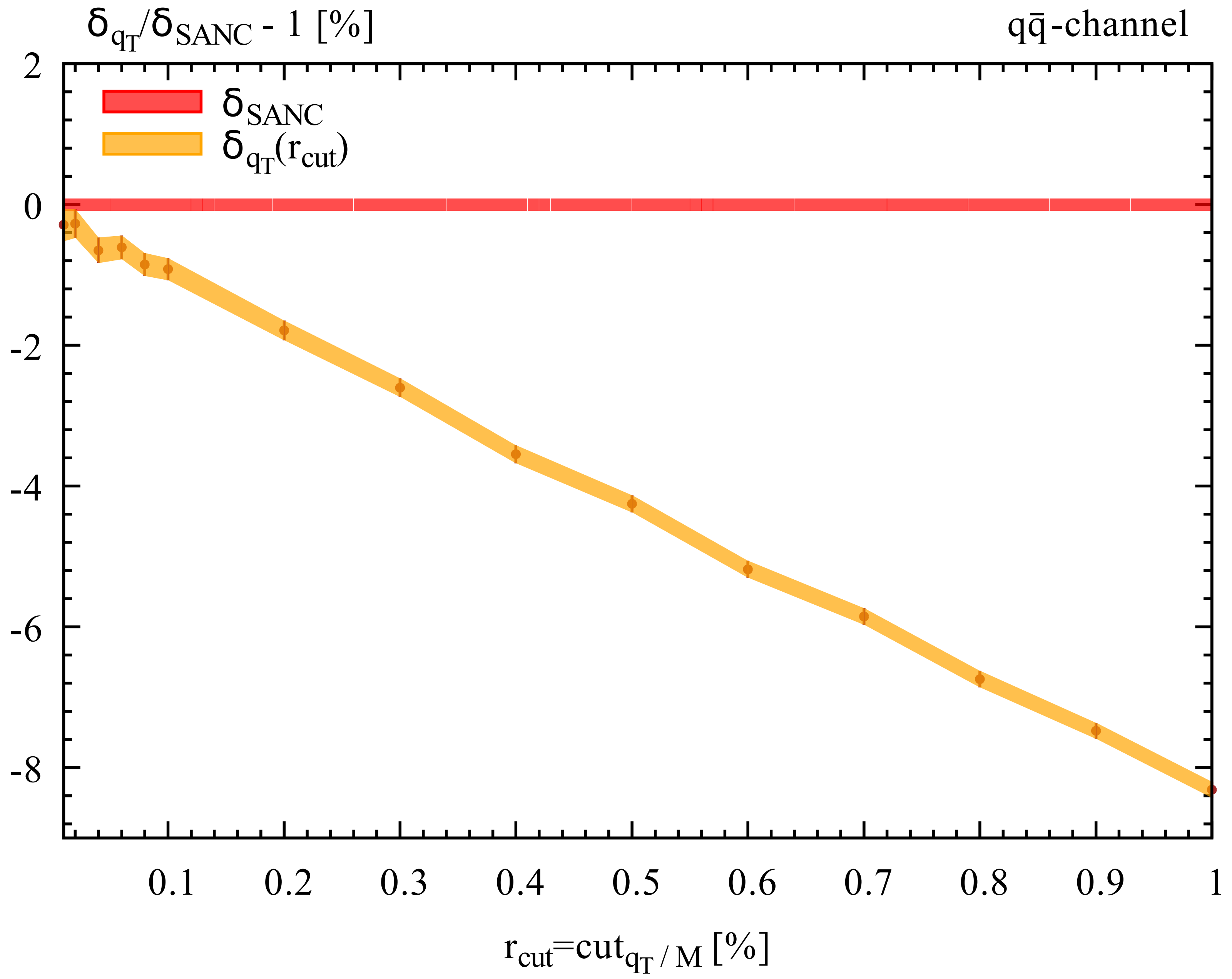
Final-state (collinear) radiation

There are configurations with $q_T/Q > 0$ and **two unresolved emission if leptons are massless**



Intro: slicing methods: q_T subtraction formalism for massive final states

- <u>Massive final state</u> **linear** ($m = 1$) power corrections due to final-state emission



analytical insight for **inclusive cross section** in pure QED

$$\sigma^{\rm NLP}(s; r_{\rm cut}) = -\frac{3\pi}{8}\frac{\alpha}{2\pi} r_{\rm cut}\left[\frac{6(5-\beta^2)}{3-\beta^2} + \frac{-47+8\beta^2+3\beta^4}{\beta(3-\beta^2)}\log\frac{1+\beta}{1-\beta}\right]\sigma_B(s)$$

$$\beta = \sqrt{1-\frac{4m^2}{s}}$$

[LB, Grazzini, Tramontano, 2019]

- <u>At NNLO:</u> linear (m=1) + log enhancement

in general we have to rely on an extrapolation procedure!

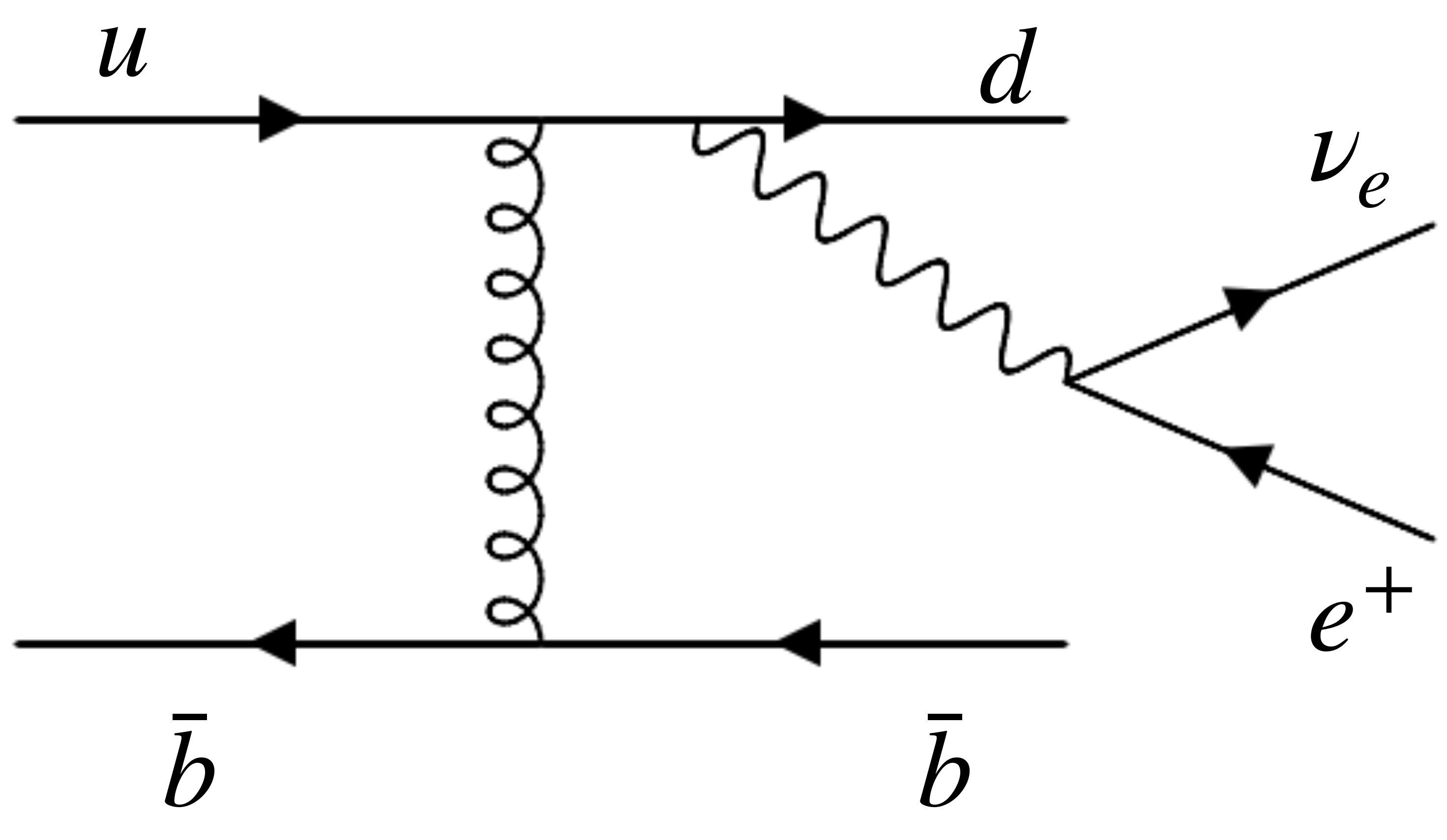
Ingredients: two-loop massless amplitudes [Abreu, Febres-Cordero, Ita, Klinkert, Page, Sotnikov, 2022]

Two-loop helicity virtual amplitudes for **W boson and four partons available in the Leading-colour approximation (LCA)**

- analytical expressions obtained within the framework of numerical unitary (using numerical samples)
- the results are expressed in terms of a basis of **one-mass pentagon functions** [Chicherin, Sotnikov, Zoia 2021]
- **off-shell W boson** including its leptonic decay
- publicly available http://www.hep.fsu.edu/~ffebres/W4partons
- **analytical expressions of the one-loop amplitudes up to** $\mathcal{O}(\epsilon^2)$ available in LCA

Some complications

- Amplitudes provided as analytical expressions that can be processed in Mathematica; **this is not suitable for on-the-fly numerical evaluation** for Monte Carlo integration
- Rather long algebraic expressions akin to numerical round-off errors
- Reference process is $u\bar{b} \to \bar{b}de^+\nu_e$. Initial-final state crossing involves in general **analytic continuation**



WQQAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini, https://gitlab.com/lrottoli/WQQAmp]

LCA and Massification

- we have carried out the massification procedure in LCA to explicitly check the cancellation of the poles
- however, in this way we are artificially introducing **spurious miscancellation** between real and virtual contributions
- moreover, the terms introduced with the massification, being enhanced by large logarithms of μ^2/m^2, are generally the dominant contributions and the difference between Full Colour and Leading Colour can be sizeable $C_F/(N_C/2) \sim 0.89$ and $(C_F/(N_C/2))^2 \sim 0.8$

Retain massification contributions at full colour whenever possible!

$$\mathcal{M}_{(2)}^{Wbb,(m)} = \mathcal{M}_{(2)}^{Wbb,(m=0)} + Z_{[q]}^{(1)} \mathcal{M}_{(1)}^{Wbb,(m=0)} + \boxed{Z_{[q]}^{(2)} \mathcal{M}_{(0)}^{Wbb,(m=0)}}$$

$$\boxed{Z_{[q]}^{(1),2} M_{(1)}^{Wbb,(m=0),-2} + Z_{[q]}^{(1),1} M_{(1)}^{Wbb,(m=0),-1} + Z_{[q]}^{(1),0} M_{(1)}^{Wbb,(m=0),0}} + \boxed{Z_{[q]}^{(1),-1} M_{(1)}^{Wbb,(m=0),1} + Z_{[q]}^{(1),-2} M_{(1)}^{Wbb,(m=0),2}}$$

with **OpenLoops2**

cannot be fully retrieved but it is less problematic, at most a single log

WQQAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini, https://gitlab.com/lrottoli/WQQAmp]

Dealing with the complications

One-Loop amplitudes: $\mathcal{O}(1000)$ source files of small-moderate size (< 100 Kb)

- algebraic expressions (rational function of the invariants) simplified using MultiVariate Apart [Heller, von Manteuffel, 2021] at the level of Mathematica before exporting them
- automatised generation of C++ source files from the Mathematica expressions; very simple optimisation introducing abbreviations (https://github.com/lecopivo/OptimizeExpressionToC)

Two-Loop amplitudes: $\mathcal{O}(3000)$ source files of moderate size (< 250 Kb)

- algebraic expressions **too long and complex**; no pre-simplification step
- breakdown of each expression in small blocks (we found this step to be crucial)
- automatised generation of C++ source files for each block
- handling of **numerical instabilities a posteriori with a simple rescue system** (at integration stage)

Crossing

- simple permutation of the momenta in the algebraic coefficients
- the action of the permutation transforms the **pentagon functions** into each others, no need for analytic continuation. All permutations available in a Mathematica script [Chicherin, Sotnikov, Zoia 2021]

WQQAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini, https://gitlab.com/lrottoli/WQQAmp]

Validation and checks

- two-loop massless amplitudes (stability)
  the C++ (double precision) code reproduces the massless results obtained with (quad precision) Mathematica for different phase space points and crossing of the amplitudes within the single floating-precision (7-9 digits), apart for some points where it badly fails (simple **rescue system**)

- one-loop amplitudes in LCA
  we have tested both the **massless and massive** amplitudes against the **independent implementation available in MCFM**, which allows to extract the LCA

- Poles cancelled!
  the IR singularities of the massive amplitude agree with the ones predicted in [Ferroglia, Neubert, Pecjac, Yang, 2009] (in LCA)

WQQAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini, https://gitlab.com/lrottoli/WQQAmp]

WORKFLOW in a NUTSHELL

Evaluation of One-Loop bare amplitudes and Two-Loop Remainders → Restore all UV and IR poles → Add dependence on dimensional scale μ

↓

Perform (one- and two-loop) UV renormalisation → Massification procedure

↓

Two-loop massive remainder

$\mathcal{O}(4s)$ **for phase space**

Wbb phenomenology (**inclusive**): total cross section

Behaviour of the perturbative series

- fixed scale

$$\mu_0 = m_W + 2m_b$$

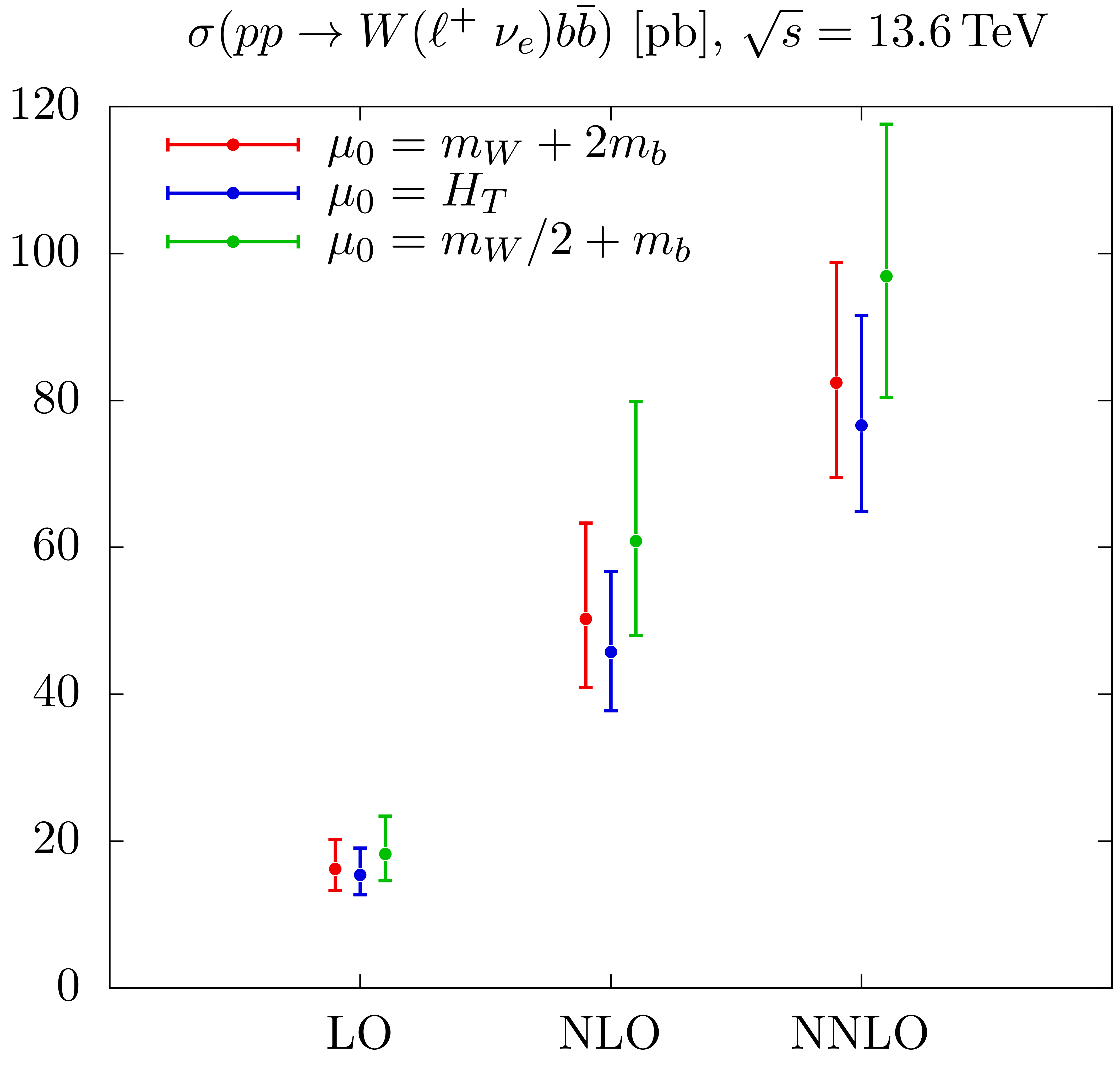
$$\mu_0 = m_W/2 + m_b$$

- dynamic scale

$$H_T = E_T(\ell\nu) + p_T(b_1) + p_T(b_2)$$

$$E_T(\ell\nu) = \sqrt{M^2(\ell\nu) + p_T^2(\ell\nu)}$$

- qualitative similar results



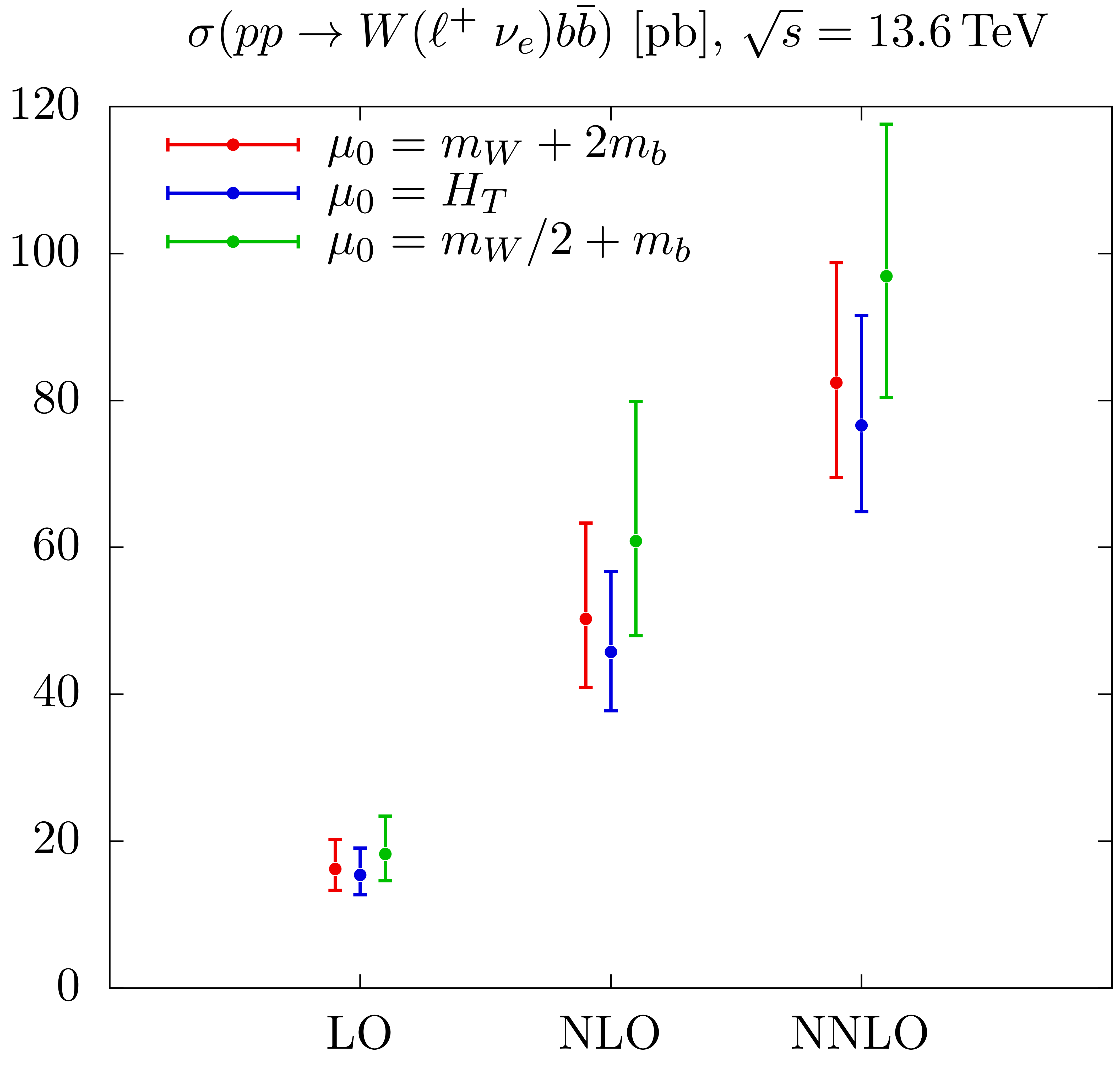
Wbb phenomenology (**inclusive**): total cross section

Behaviour of the perturbative series

- Very large NLO corrections ($K_{\text{NLO}} \sim 5$), due to **opening of the quark-gluon channel**. LO uncertainties bands completely unreliable
- **Starting from NNLO**, one start to see the convergence of the perturbative series with a reduction of both K-factors ($K_{\text{NNLO}} \sim 1.6$) and theoretical uncertainties (scale variations), which **are more reliable**

$$\mu_0 = m_W/2 + m_b$$

order	σ_{incl} [pb]
LO	$18.270(2)^{+28\%}_{-20\%}$
NLO	$60.851(7)^{+31\%}_{-21\%}$
NNLO	$96.91(8)^{+21\%}_{-17\%}$



Comparison with HPPZ: fiducial cross sections

order	σ^{4FS} [fb]	$\sigma^{5FS}_{a=0.05}$ [fb]	$\sigma^{5FS}_{a=0.1}$ [fb]	$\sigma^{5FS}_{a=0.2}$ [fb]
LO	$210.42(2)^{+21.4\%}_{-16.2\%}$	$262.52(10)^{+21.4\%}_{-16.1\%}$	$262.47(10)^{+21.4\%}_{-16.1\%}$	$261.71(10)^{+21.4\%}_{-16.1\%}$
NLO	$468.01(5)^{+17.8\%}_{-13.8\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$
NNLO	$636.4(1.6)^{+11.9\%}_{-10.5\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	$647(7)^{+9.5\%}_{-9.4\%}$

Remarks

Change of scheme @NLO [Cacciari, Nason, Greco, 1998]

1. Use same running coupling and PDF set of the 5FS calculation
2. Add the extra factor (due to the conversion between $\overline{MS}$ and decoupling schemes) : $-\alpha_s \frac{2T_R}{3\pi} \ln \frac{\mu_R^2}{m^2} \sigma_{q\bar{q}}^{LO}$

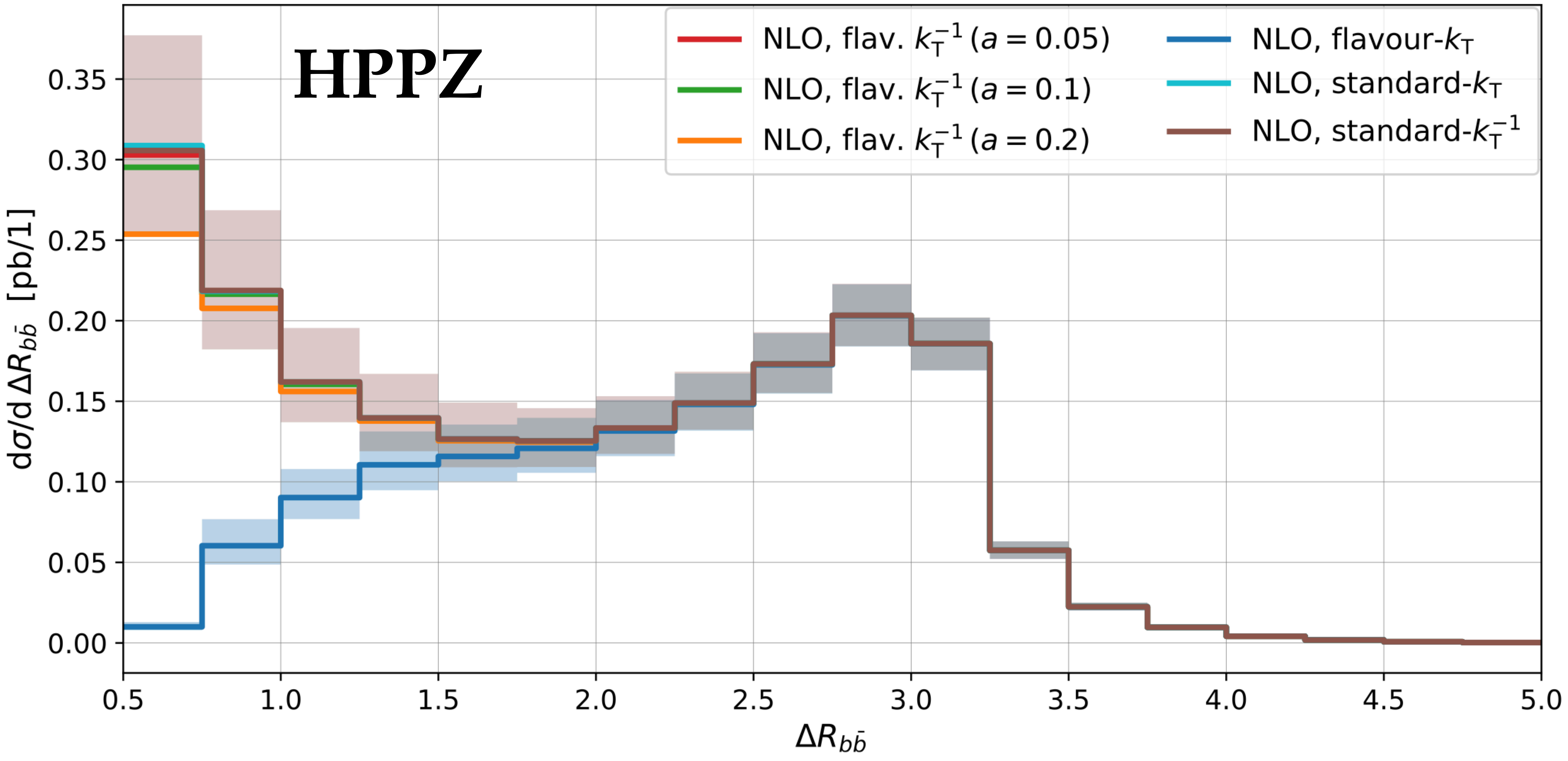
   No corrective term for pdfs at this order
3. Take the massless limit $m_b \to 0$

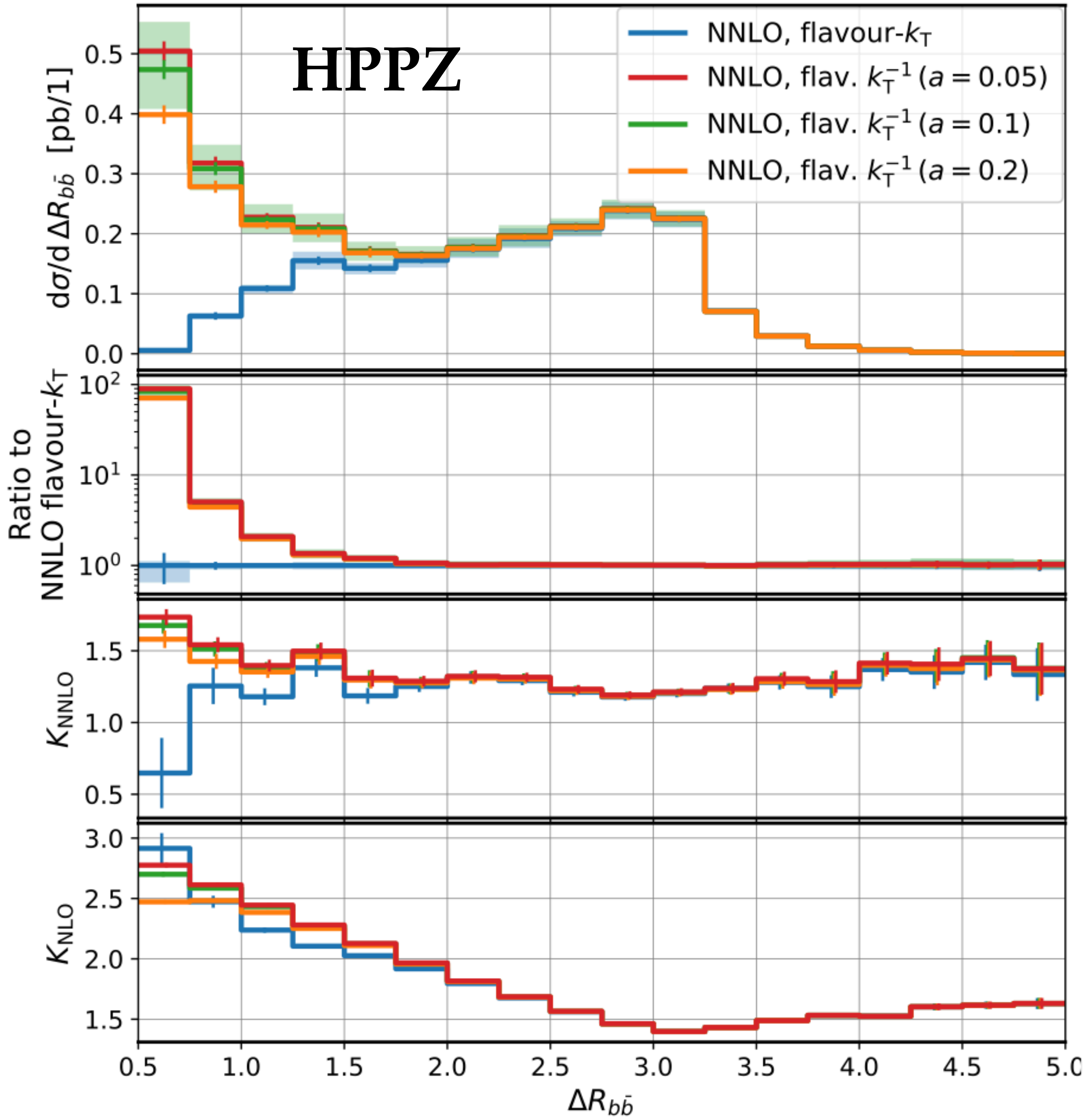
$$\text{NLO 4FS: 468 fb} \xrightarrow{1,2} \text{481 fb} \xrightarrow{3} \text{493 fb}$$

Comparison with HPPZ: jet clustering algorithms

Flavour k_T favours the clustering of the two bottom quarks in the same jet, leading to a suppression at small ΔR_{bb} (largely due to the modified definition of beam distances)

- At NLO, flavour anti k_T reproduces standard anti k_T in the limit $a \to 0$. At NNLO cannot be arbitrarily small because of the infrared problem
- HPPZ choice: $a \in [0.05, 0.2]$

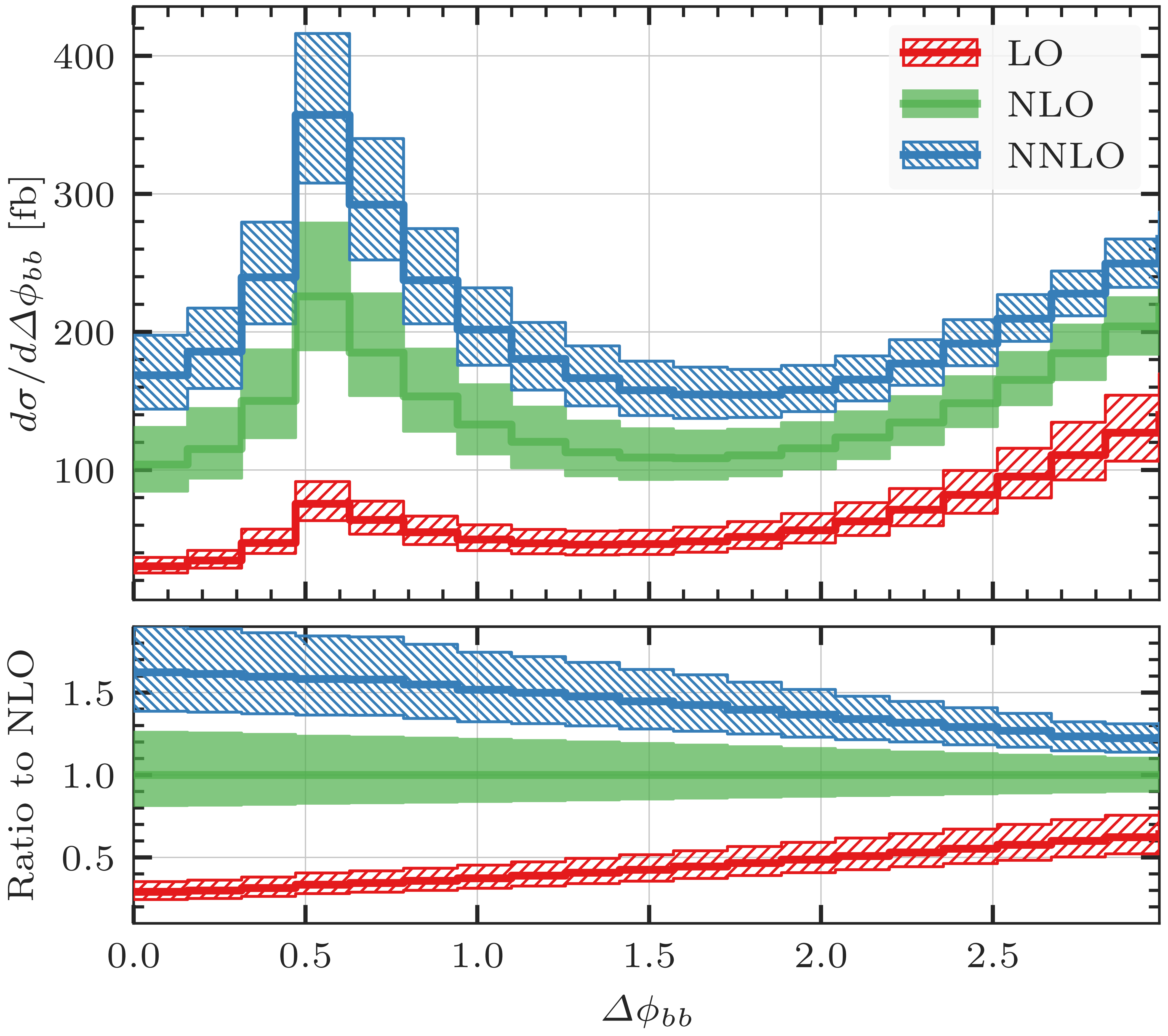


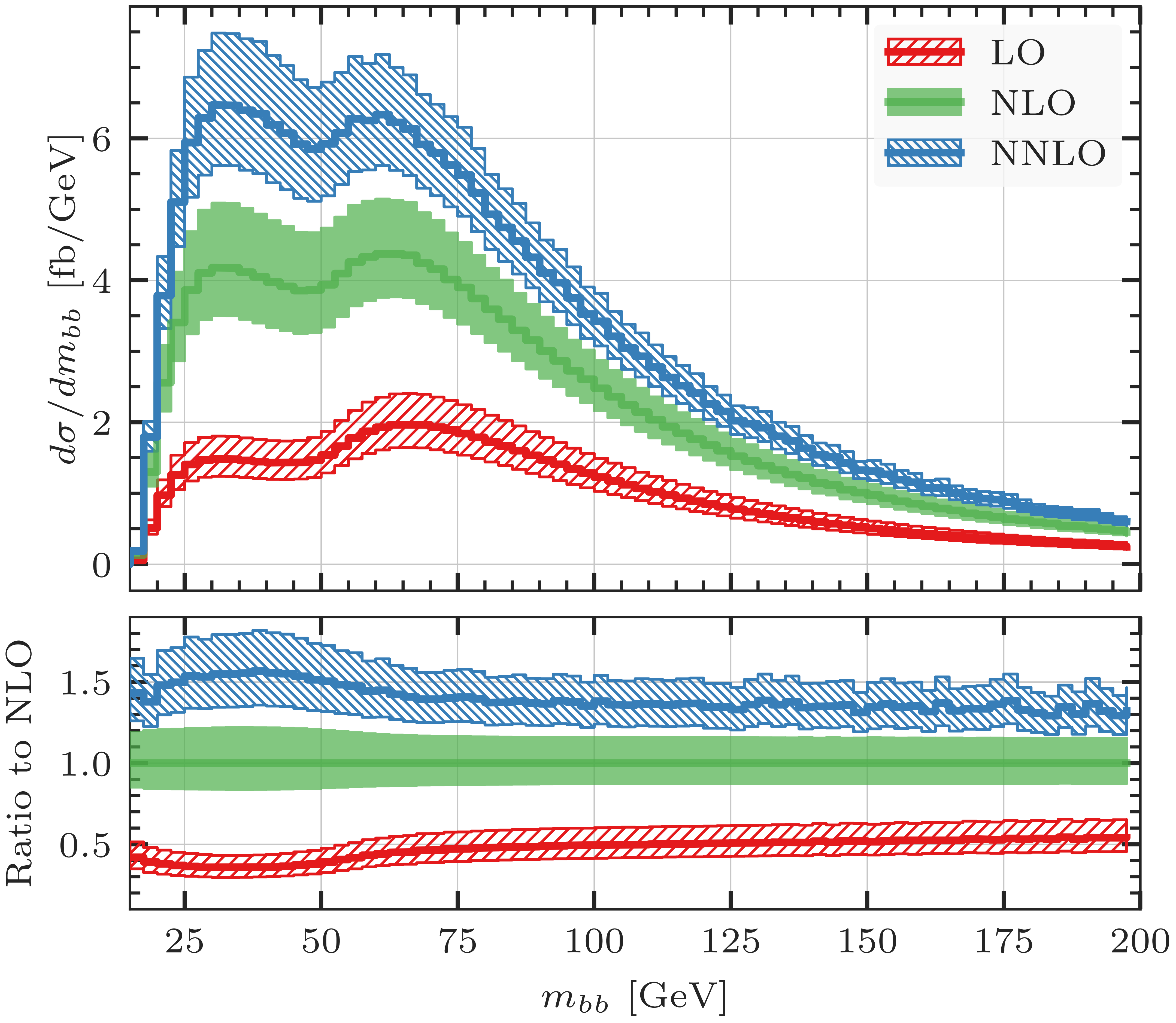


Comparison with HPPZ: additional distributions

Other distributions display similar pattern of the higher-order corrections

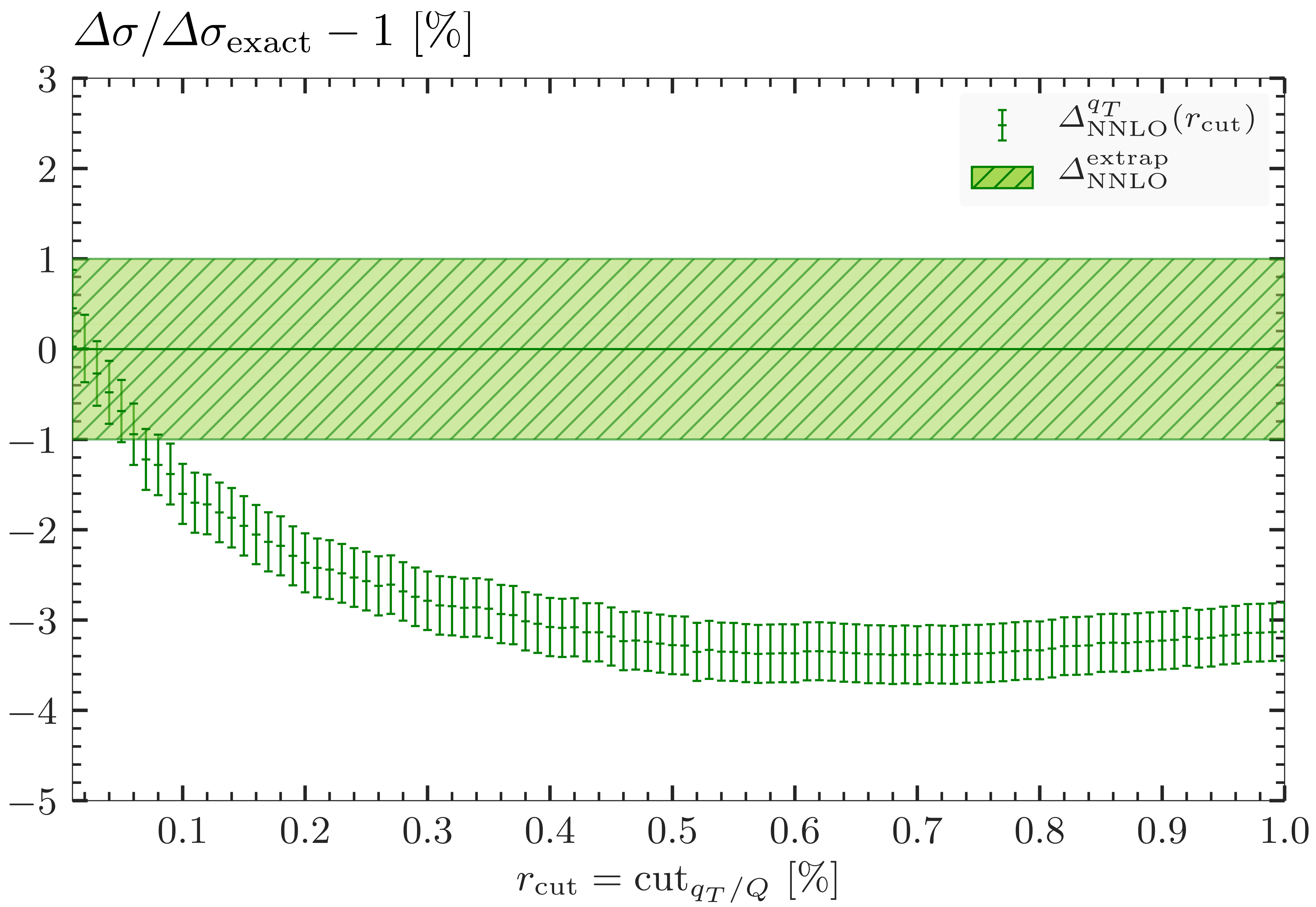
The process features two dominant configurations: **gluon splitting** and **t-channel** enhancement (back-to-back bottom quarks and back-to-back leptons)





Comparison with HPPZ: r_{cut} dependence

$$d\sigma_{N^kLO} = \mathscr{H} \otimes d\sigma_{LO} + \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}\right]_{q_T/Q > r_{\mathrm{cut}}} + \mathcal{O}(r^{\ell}_{\mathrm{cut}})$$



Behaviour of the power corrections compatible with a **linear scaling** as expected from processes with massive final state

Overall mild power corrections

Control of the NNLO correction at $\mathcal{O}(1\%)$
$\rightarrow \mathcal{O}(0.2\%)$ at the level of the total cross section