Feynman amplitudes and the moduli space of graphs

Towards a statistical approach to Feynman integrals and amplitudes
RADCOR 2023

Michael Borinsky - ETH Zürich, Institute for Theoretical Studies

Related to / based on

Feynman integrals and tropical sampling

MB 2020

MB - Sattelberger-Sturmfels-Telen 2022

MB - Munch* - Tellander 2023

Moduli space of graphs

MB - Vogtmann 2019

" 2022

" 2023

MB - Vermaseren (w.p.) 2023*

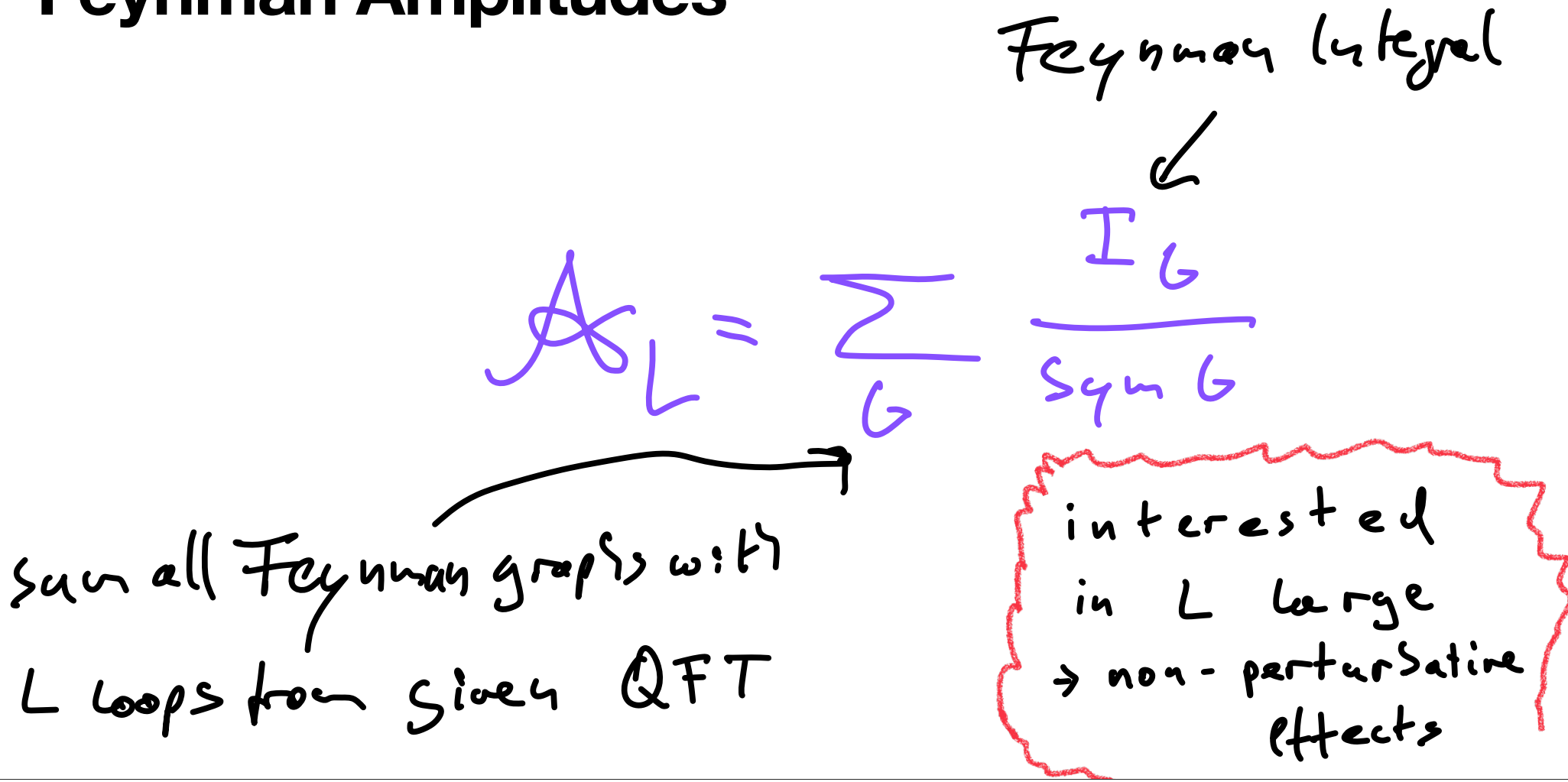
Feynman Amplitudes

Feynman integral

$$\mathcal{A}_L = \sum_G \frac{I_G}{\text{Sym } G}$$

sum all Feynman graphs with L loops from given QFT

Feynman Amplitudes

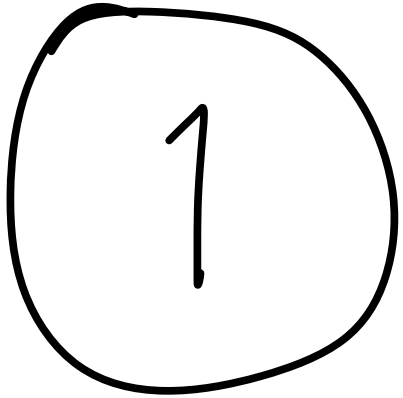


Two problems

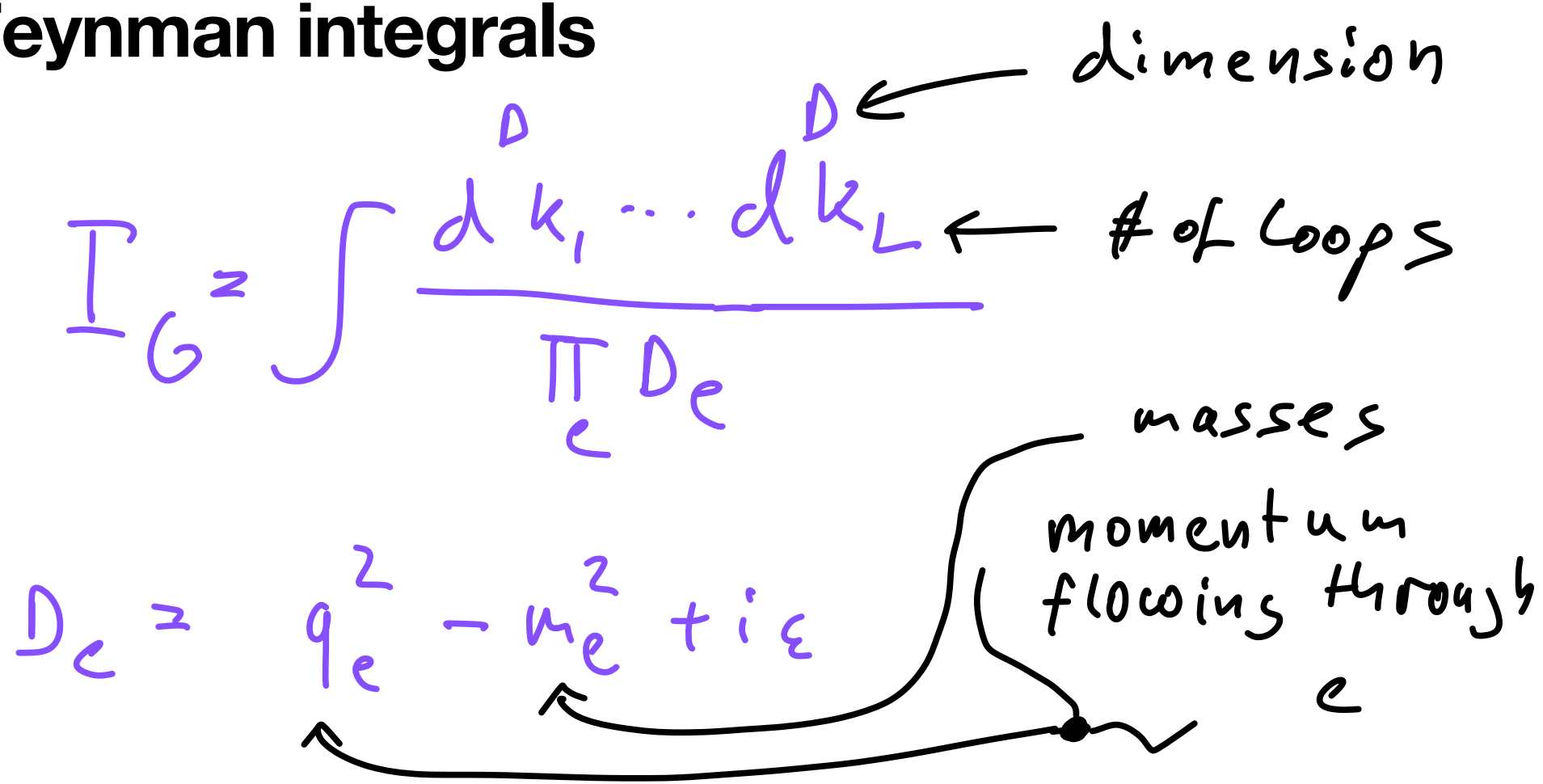
1. Computing I_G is hard
2. Many graphs in $\sum_G$

$\Rightarrow$ Computing A_L is harder

Computing Feynman Integrals



Feynman integrals



Advertisement

IR singularities of Feynman integrals and flow polytopes

MB+Capatti+Laenen+Salas-Bernárdez*

2022

graphical functions and conformal 4-pt functions

MB+Schnetz*

2021+2021+2022

Tropical Feynman integration

Features :

- Works for all* graphs

- Fast with many loops and scales

Caveats

- numerical (Monte Carlo)
- divergencies are complicated

feyntrop

=> MB-Hunch-Tellander 2023

See Henrik's talk for details

Code + Notebooks:

https://github.com/michibo/feyntrop

Parametric representation

$$I_G = \int_{\Delta} \frac{1}{\mathcal{U}^{D/2}} \left(\frac{\mathcal{U}}{\mathcal{F}}\right)^{\omega} \Omega$$

- $\mathcal{U}, \mathcal{F}$: Symanzik polynomials
- Δ: $(E-1)$-dimensional simplex

$$I_G = \int_\Delta \frac{1}{u^{D/2}} \left(\frac{u}{\epsilon}\right)^\omega \Omega$$

tropical approximation

$$= \int_\Delta \frac{\Omega}{(u^{tr})^{D/2}(v^{tr})^\omega} \frac{(u^{tr})^{D/2}(v^{tr})^\omega}{u^{D/2} v^\omega} \qquad \left(v = \frac{\epsilon}{u}\right)$$

Key idea: interpret as probability measure

Integration of Feynman integrals with up to ~17 loops possible

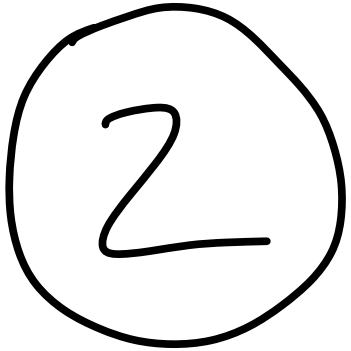
$\Rightarrow$ Study Feynman graphs and integrals as statistical objects.

Next step

1. Computing I_G is ~~hard~~ not quite so hard

2. There are too many graphs in Σ_G

Moduli space of graphs and amplitudes

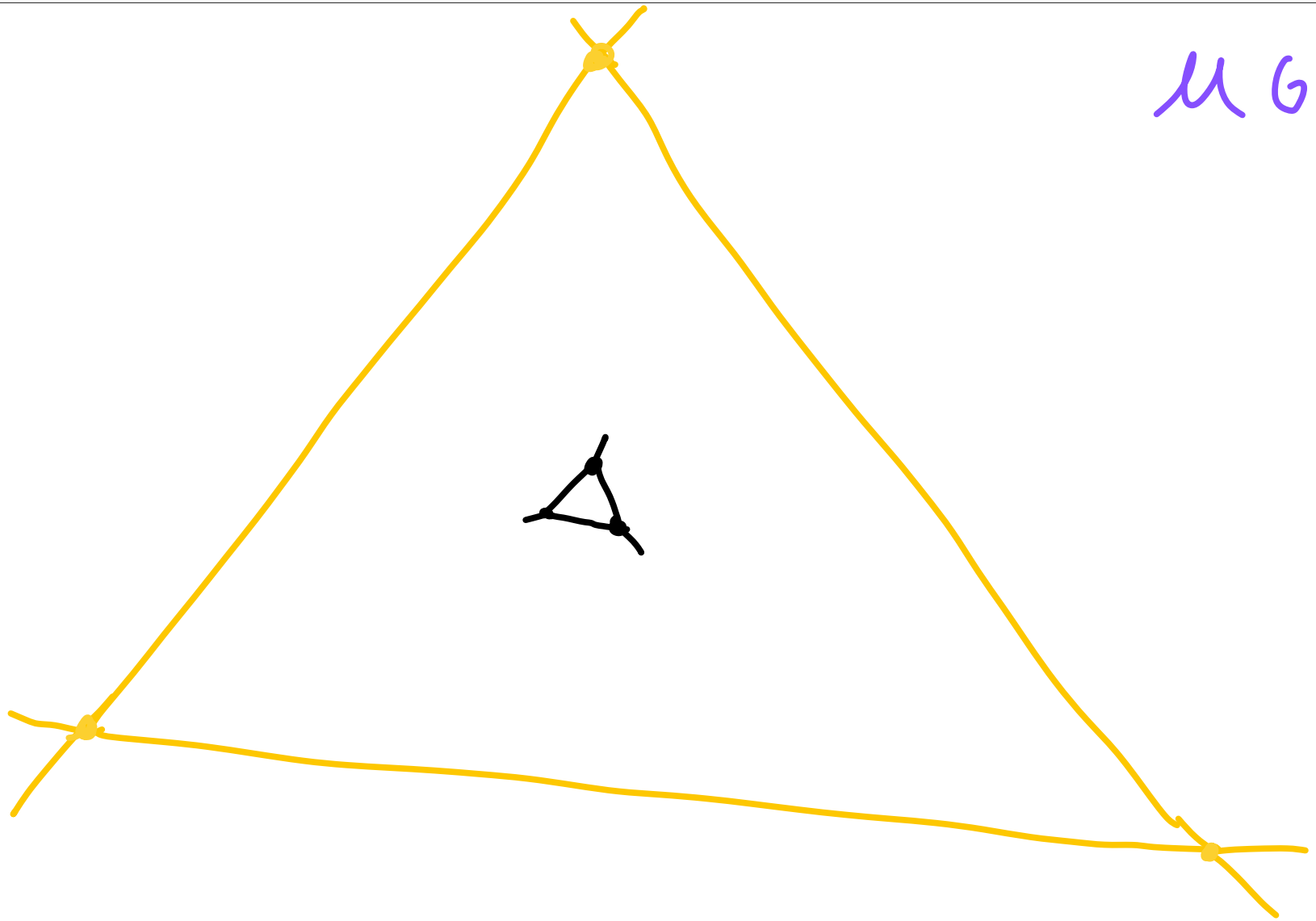


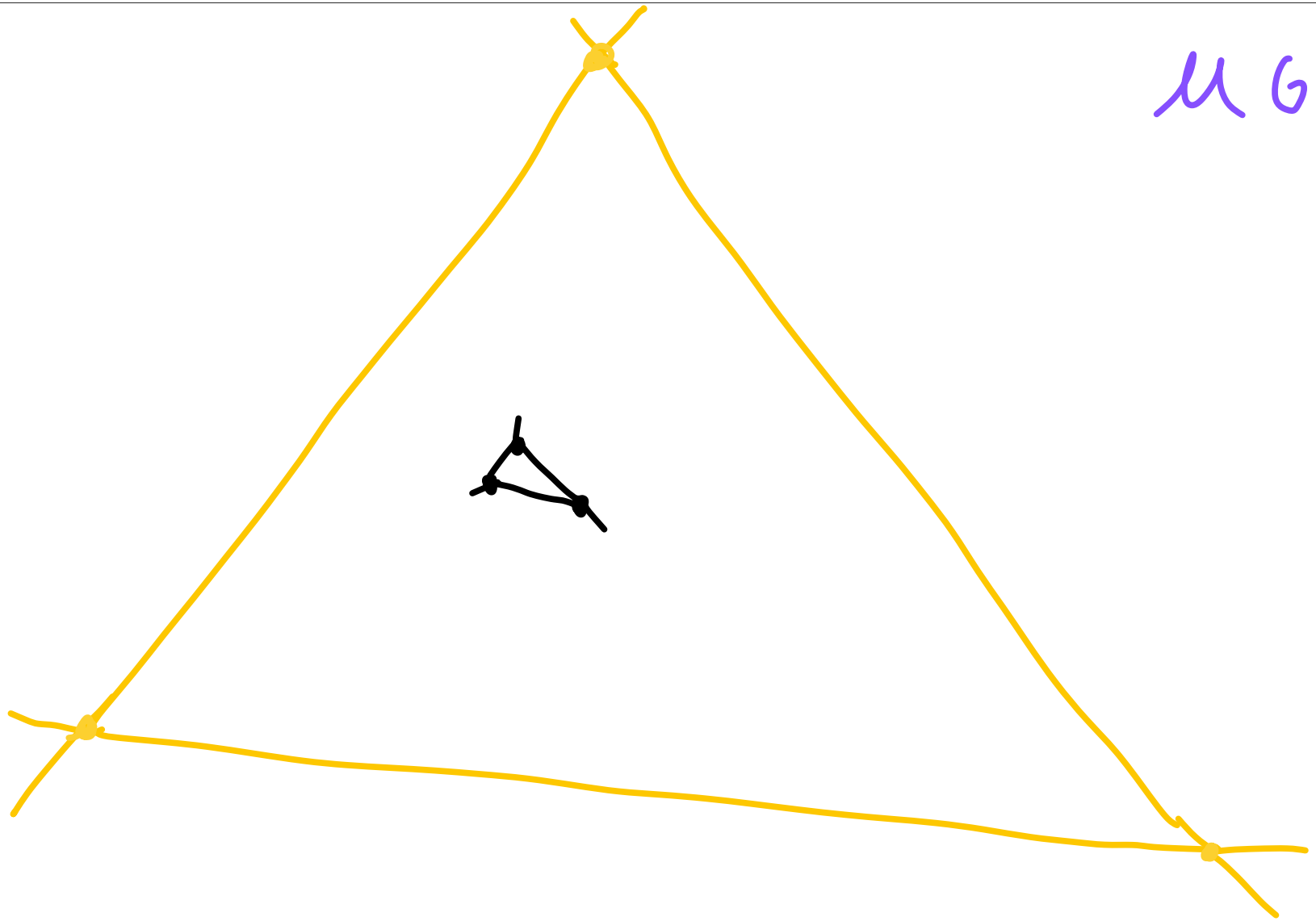
Moduli space of graphs

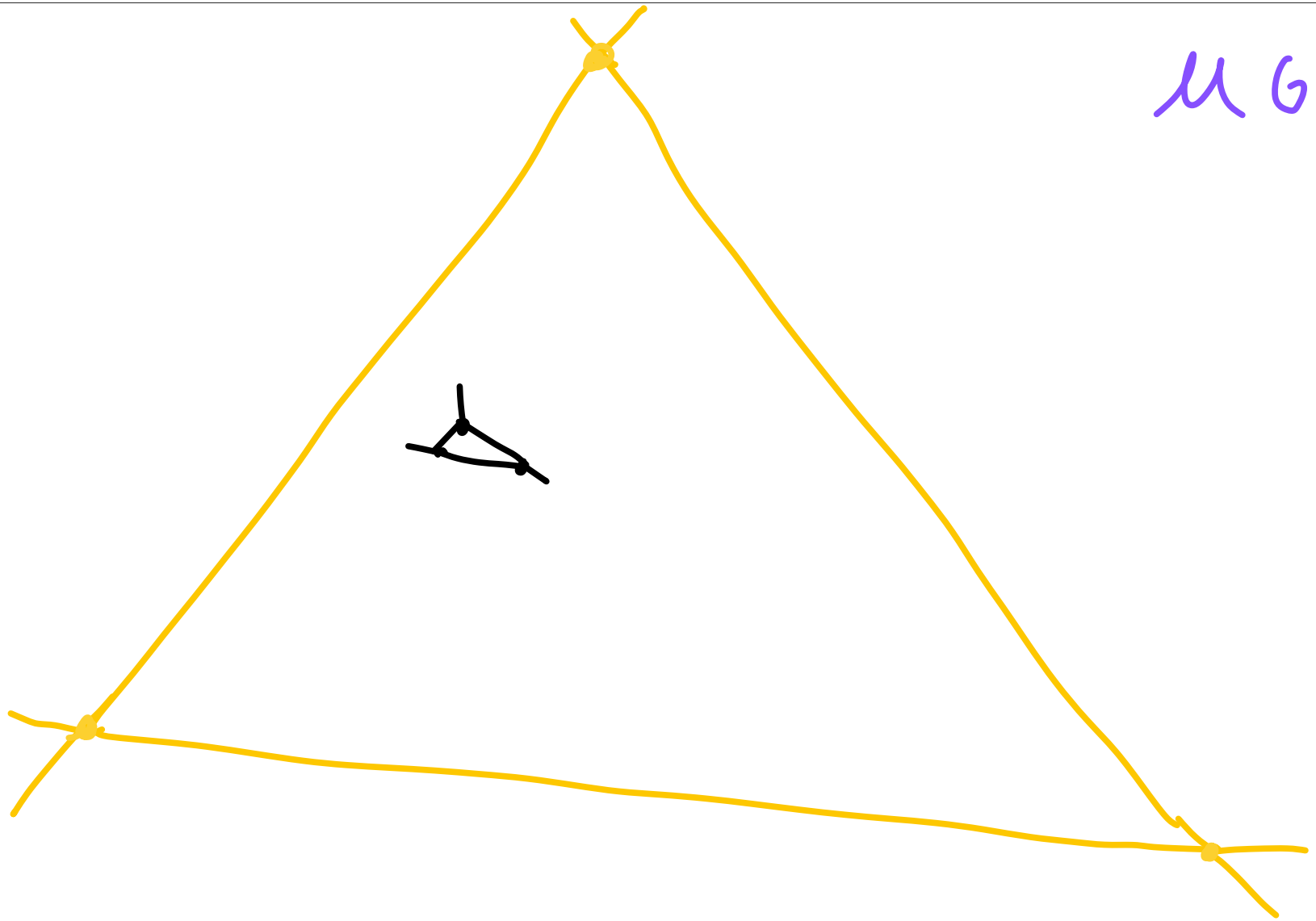
$\mathcal{M}G_{g,n}$ is the space of

metric graphs with g loops and n legs

i.e. graphs with lengths at each edge


$\mathcal{M}_{1,3}$


$\mathcal{M}_{1,3}$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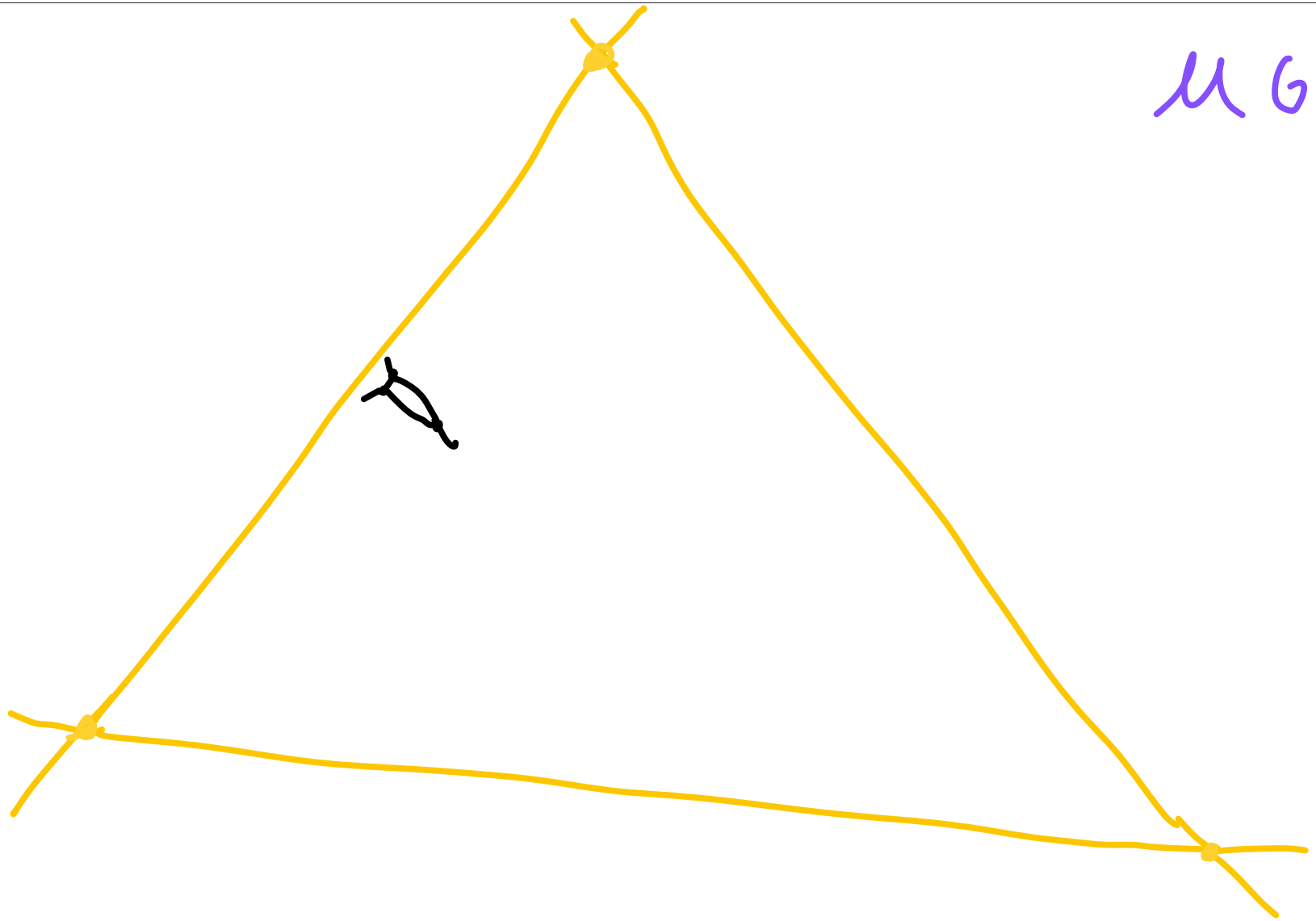
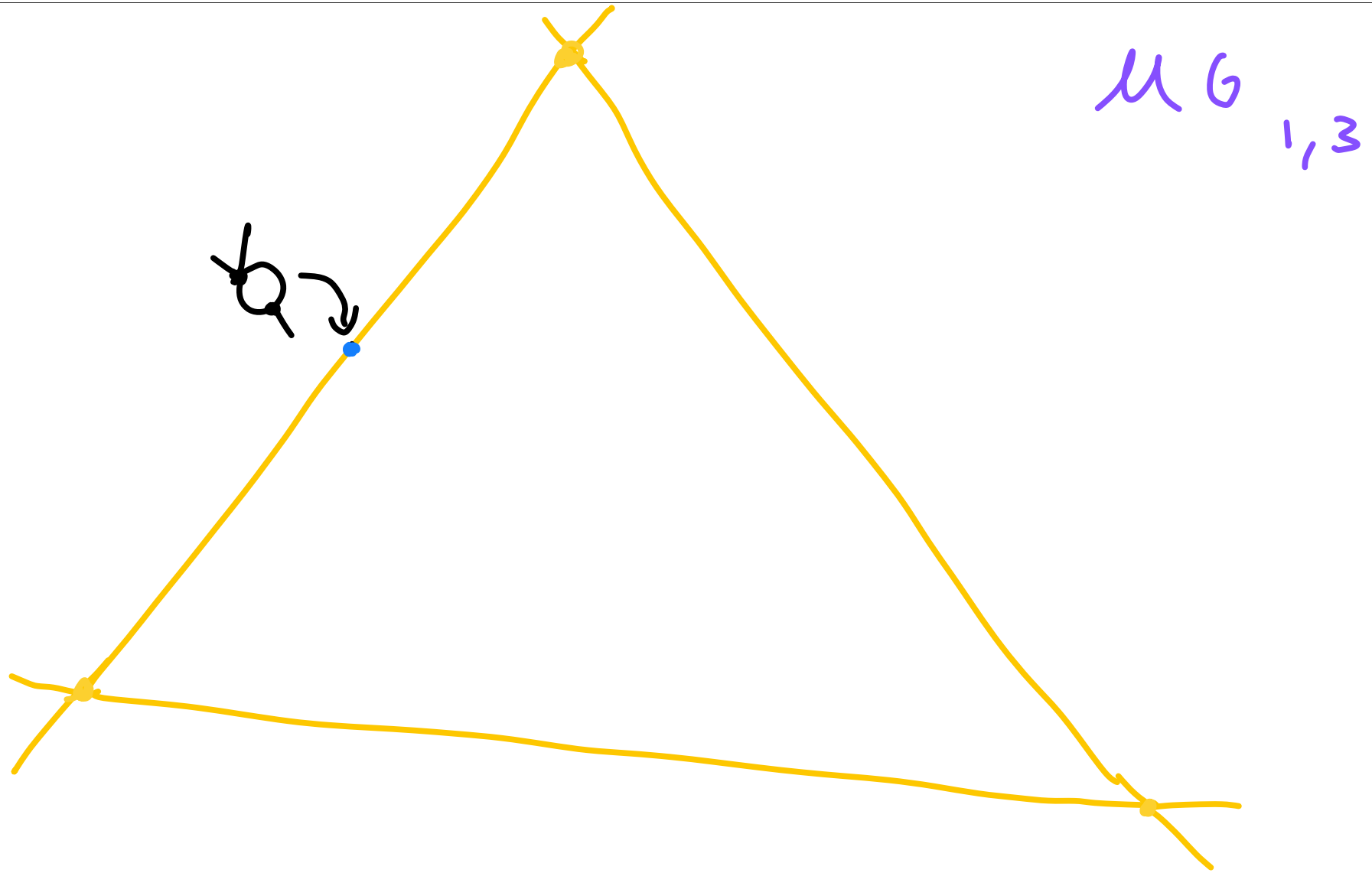

$\mathcal{M}_{1,3}$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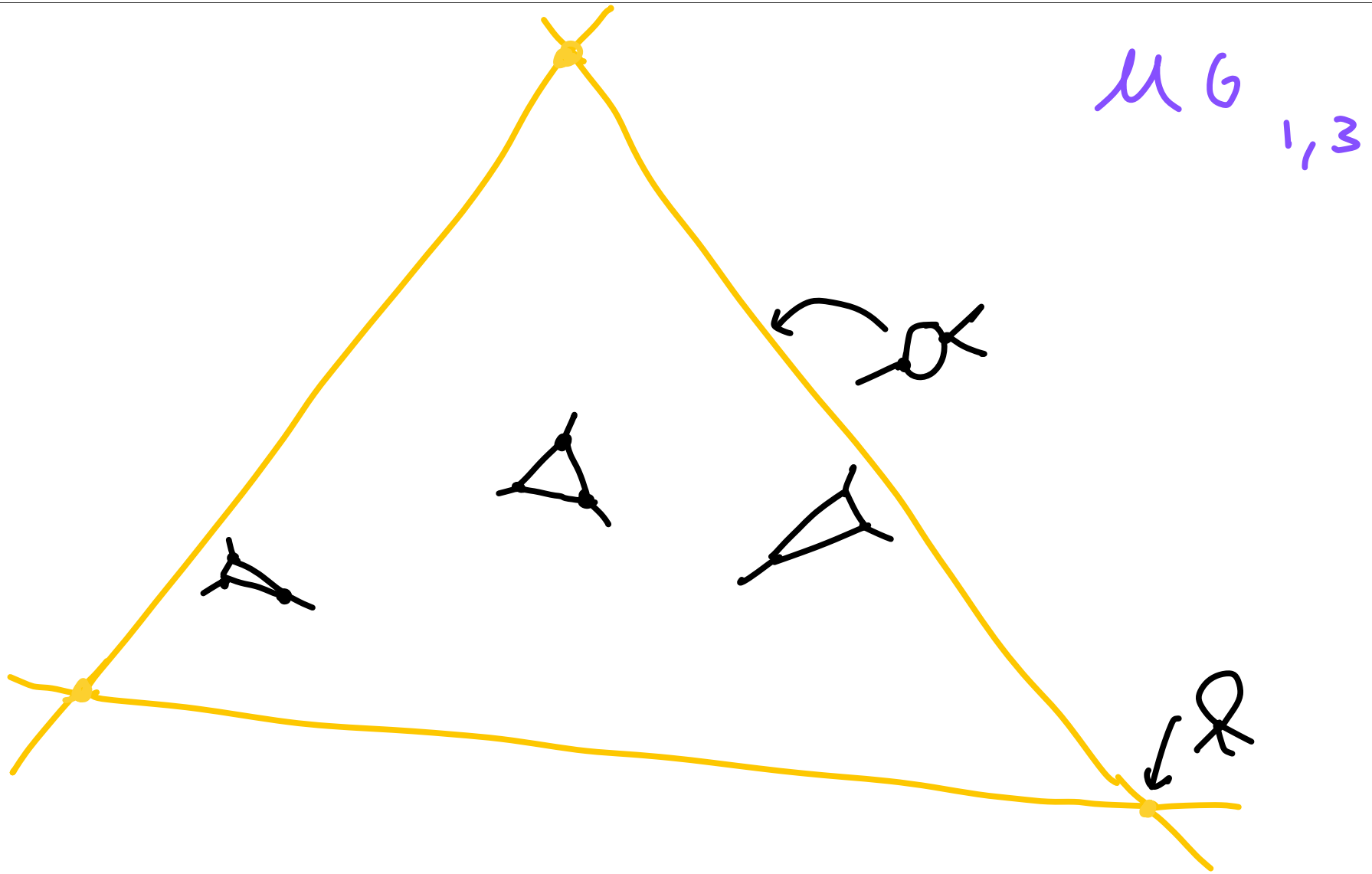
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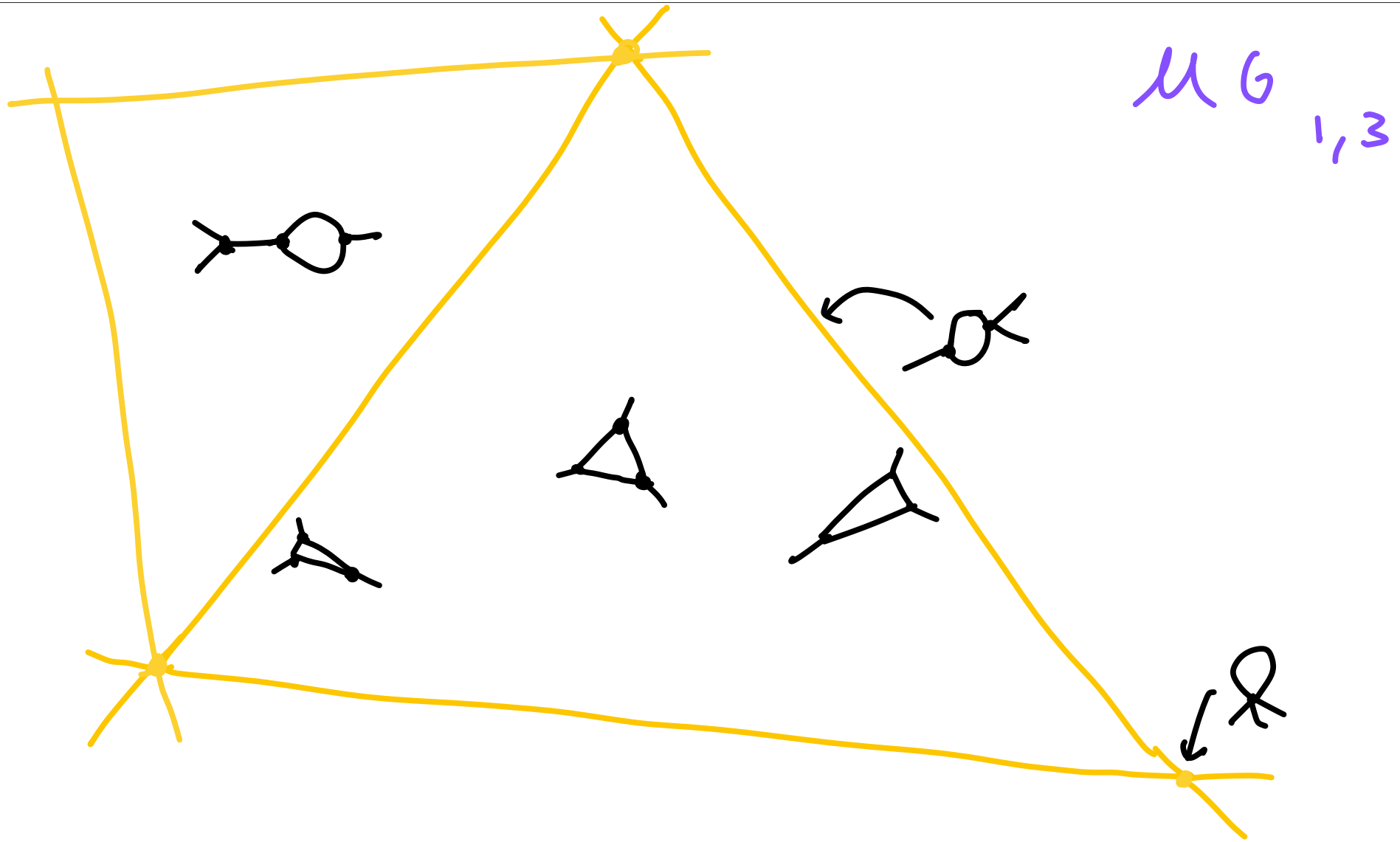

$\mathcal{M}_{1,3}$

Things known about $\mathcal{MG}_{g,n}$

Has a symmetry group associated to it

$$\underline{\underline{Out(F_g)}}$$

(closely related to Outer space)

Culler-Vogtmann '86

Connections to very interesting mathematics

e.g. $\mathcal{M}^{trop}_{g,n}$ is the tropicalization of $\mathcal{M}_{g,n}$ the moduli space of curves

Many applications of QFT ideas in mathematics

QFT tells us about the topology of $\mathcal{MG}_{g,n}$!

MB-Vogtmann 2019
" 2022
" 2023

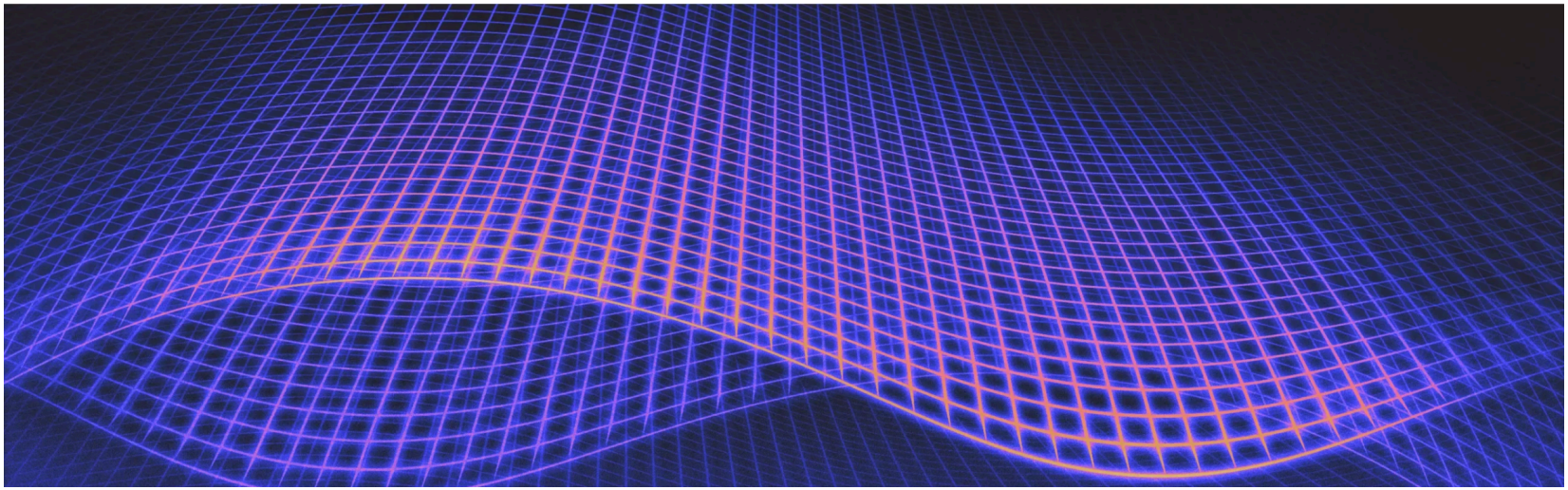
MB-Vermaseren (w.i.p) 2023

Related: Brown 2021

GRAPH THEORY

Quantum Field Theory Pries Open Mathematical Puzzle

Mathematicians have struggled to understand the moduli space of graphs. A new paper uses tools from physics to peek inside.



Applications to physics

Parametric representation of Feynman integrals

$$I_G = \int_{\Delta} \frac{1}{\mathcal{U}^{D/2}} \left(\frac{\mathcal{U}}{\mathcal{F}}\right)^{\omega} \Omega$$

Δ ← $(E-1)$- dimensional simplex

Amplitude

$$\mathcal{A}_L = \sum_G \frac{I_G}{\mathrm{sym}\, G}$$

Amplitude

$$\mathcal{A}_L = \sum_G \frac{I_G}{\mathrm{sym}\, G} =$$

$$= \sum_G \frac{1}{\mathrm{sym}\, G} \int_\Delta \Omega \frac{1}{u_G^{d_2} v_G^{\omega}}$$

Amplitude

$$\mathcal{A}_L = \sum_G \frac{I_G}{\mathrm{sym}\, G} =$$

$$= \underbrace{\sum_G \frac{1}{\mathrm{sym}\, G} \int_\Delta \Omega \, \frac{1}{u_G^{d_2} v_G^{\omega}}}$$

Idea: interpret as a single measure on $\mathcal{M}_{g,n}$!

Amplitude

$$A_g = \int_{\mu_{g_{g,n}}} \Omega \frac{1}{a_G^{D/2} \nu_G^{\omega}}$$

Use cases for physics

- Restrictions of function spaces by homology

Lee-Pomeransky '13

- Feynman amplitudes expressed as single integrals over unified domain.

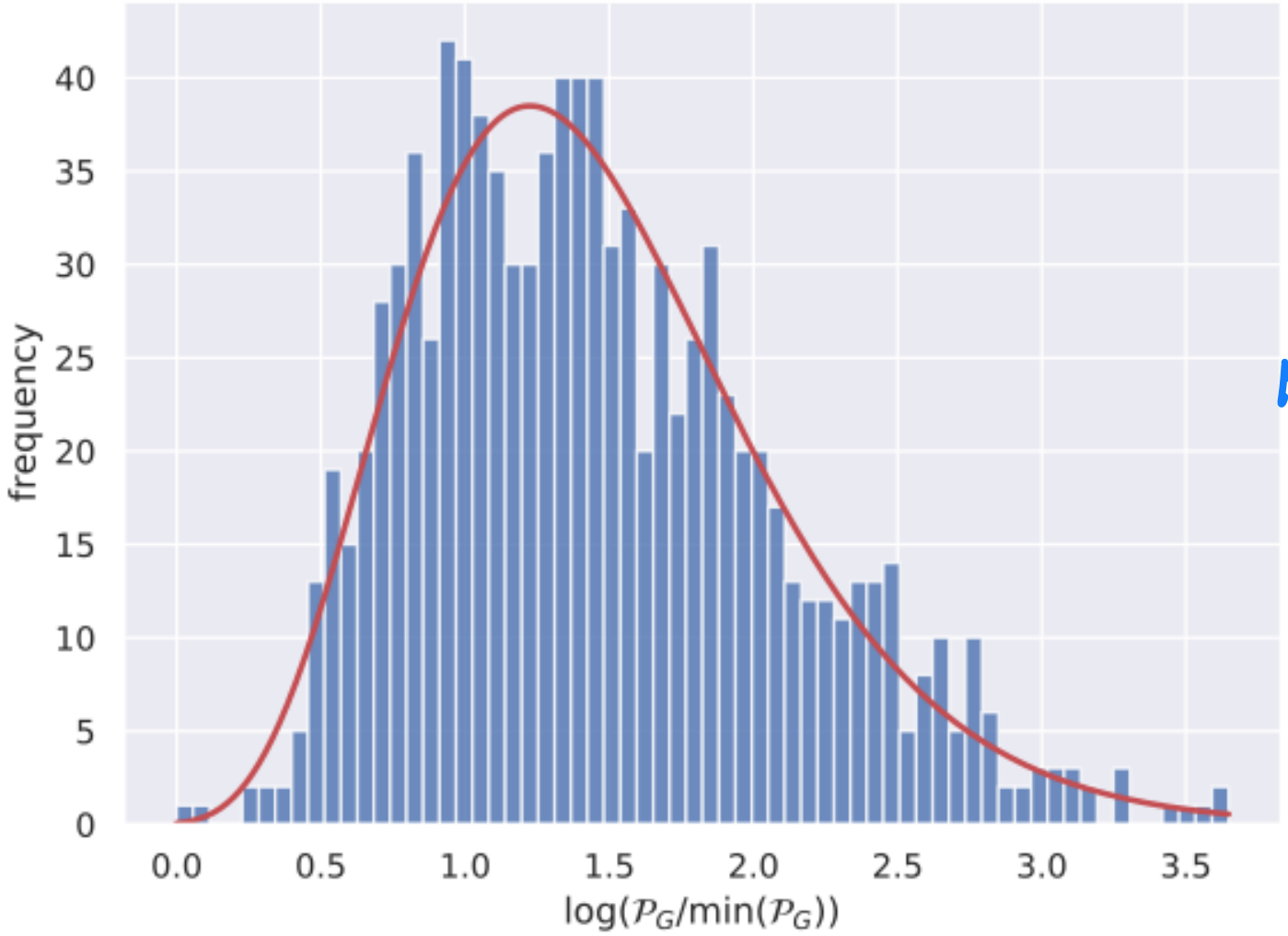
→ opens way to statistical tools

In other words:

What's the value of I_G

for a random graph G?

P_G: "Primitive" part of Feynman integral's contribution to the amplitude



MB-Favorito (wip)

MSc-thesis Favorito 2023

Figure 4.9: Fit of the distribution of $\mathcal{P}_G$ in logarithmic coordinates for $L = 15$.

Towards a statistical treatment of Feynman amplitudes

- The moduli space is a natural habitat for Feynman amplitudes
- Many rich structures
- Tropical Feynman integration allows to probe large order amplitudes numerically

  ⟶ non-perturbative effects