

Five-Point scattering amplitudes in full-colour QCD

In collaboration with: Bakul Agarwal Federico Buccioni Federica Devoto Andreas von Manteuffel Lorenzo Tancredi

Giulio Gambuti - 30/05/2023 - RADCOR, Crieff, Scotland



S. Badger, D. Chicherin, T. Gehrmann, G. Heinrich, J.M. Henn, T. Peraro, P. Wasser, Y. Zhang, S. Zoia: 1905.03733
Herschel A. Chawdhry, Michal Czakon, Alexander Mitov, Rene Poncelet: 2012.13553
Bakul Agarwal, Federico Buccioni, Andreas von Manteuffel, Lorenzo Tancredi: 2102.01820
S. Abreu, F. Febres Cordero, H. Ita, B. Page, V. Sotnikov: 2102.13609
Bakul Agarwal, Federico Buccioni, Andreas von Manteuffel, Lorenzo Tancredi: 2105.04585
Michal Czakon, Alexander Mitov, Rene Poncelet: 2106.05331
Simon Badger, Michał Czakon, Heribertus Bayu Hartanto, Ryan Moodie, Tiziano Peraro: 2304.06682
Samuel Abreu, Giuseppe De Laurentis, Harald Ita, Maximillian Klinkert, Ben Page, Vasily Sotnikov: 2305.17056



Recerci



S. Badger, D. Chicherin, T. Gehrmann, G. Heinrich, J.M. Henn, T. Peraro, P. Wasser, Y. Zhang, S. Zoia: 1905.03733

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The Leading Colour

S. Abreu, F. Febres Cordero, H. Ita, B. Page, V. Sotnikov: 2102.13609

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This talk: Full Colour



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This talk: Full Colour

All non-planar diagrams



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This talk: Full Colour

All non-planar diagrams

Full tower in N_c and n_f



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The All-Plus

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Non-trivial IR

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The All-Plus

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The Leading Colour

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This talk: Full Colour

All non-planar diagrams

Full tower in N_c and n_f

Full amplitude

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Non-trivial IR

Corrections?









 $A^{a_1 a_2 \dots a_5} = \sum_{c=1}^{22} A_c \, \mathscr{C}_c^{a_1 a_2 \dots a_5}$









 a_5

4000000

















$A_1 = a_1 N_c^2 + a_2 1 + a_3 N_c n_f + a_4 N_c^{-1} n_f + a_5 n_f^2$














































 $A^{h_1h_2...h_5} =$ $A^{\mu_1\mu_2...\mu_5} \epsilon^{h_1}_{\mu_1}...\epsilon^{h_5}_{\mu_5}$



 $A^{h_1h_2\dots h_5} =$ $A^{\mu_1\mu_2\ldots\mu_5} \epsilon^{h_1}_{\mu_1}\ldots\epsilon^{h_5}_{\mu_5}$

Projectors & Form Factors

$$A^{h_1h_2\dots h_5} =$$



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Projectors & Form Factors

$$A^{h_1h_2...h_5} = \sum_{j=1}^{32} F^j T_j^{\mu_1...\mu_5} \epsilon_{\mu_1}^{h_1}...\epsilon_{\mu_5}^{h_5}$$







 $A^{h_1h_2...h_5} =$ $A^{\mu_1\mu_2...\mu_5} \epsilon^{h_1}_{\mu_1}...\epsilon^{h_5}_{\mu_5}$



 $A^{h_1h_2...h_5} =$ $A^{\mu_1\mu_2\ldots\mu_5} \epsilon^{h_1}_{\mu_1}\ldots\epsilon^{h_5}_{\mu_5}$

Direct Computation

$$\epsilon_{\mu_i}^+ = \frac{[q_i | \mu_i | p_i \rangle}{\sqrt{2}[p_i q_i]} \qquad \epsilon_{\mu_i}^- = \frac{[p_i | \mu_i | q_i \rangle}{\sqrt{2} \langle p_i q_i \rangle}$$















 $\epsilon_{\mu_{1}}^{+} \dots \epsilon_{\mu_{5}}^{+} = \frac{1}{2^{5/2}} \frac{[q_{1}|\mu_{1}|1\rangle}{[q_{1}p_{1}]} \frac{[q_{2}|\mu_{2}|2\rangle}{[q_{2}p_{2}]} \frac{[q_{3}|\mu_{3}|3\rangle}{[q_{3}p_{3}]} \frac{[q_{4}|\mu_{4}|4\rangle}{[q_{4}p_{4}]} \frac{[q_{5}|\mu_{5}|5\rangle}{[q_{5}p_{5}]}$



$$\epsilon_{\mu_{1}}^{+}...\epsilon_{\mu_{5}}^{+} = \frac{1}{2^{5/2}} \frac{\operatorname{Tr}_{-}\left\{\gamma_{\mu_{1}}p_{1}\gamma_{\mu_{2}}p_{2}\gamma_{\mu_{3}}p_{3}\gamma_{\mu_{4}}p_{4}\gamma_{\mu_{5}}p_{5}\right\}}{[12] [23] [34] [45] [51]}$$



$$\epsilon_{\mu_{1}}^{+}...\epsilon_{\mu_{5}}^{+} = \frac{1}{2^{5/2}} \frac{\operatorname{Tr}_{-}\left\{\gamma_{\mu_{1}}p_{1}\gamma_{\mu_{2}}p_{2}\gamma_{\mu_{3}}p_{3}\gamma_{\mu_{4}}p_{4}\gamma_{\mu_{5}}p_{5}\right\}}{[12] [23] [34] [45] [51]} \operatorname{Tr}_{\pm}\{...\} = \frac{1}{2}\operatorname{Tr}\left\{(1 \pm \gamma_{5})...\right\}$$

 $A^{\mu_1\dots\mu_5} \epsilon^{h_1}_{\bar{\mu}_1}\dots\epsilon^{h_5}_{\bar{\mu}_5}$



't Hooft-Veltman scheme



't Hooft-Veltman scheme



't Hooft-Veltman scheme







Why?



better handle on complexity



better handle on complexity

avoid computation of projectors



better handle on complexity

avoid computation of projectors

reference choice ↔ symmetry of amplitude





T. Gehrmann, J.M. Henn, N.A. Lo Presti: 1511.05409

Costas G. Papadopoulos, Damiano Tommasini, Christopher Wever: **1511.09404**

T. Gehrmann, J.M. Henn, N.A. Lo Presti: 1807.09812

D. Chicherin, T. Gehrmann, J.M. Henn, P. Wasser, Y. Zhang et al: **1812.11160**



Janko Böhm, Alessandro Georgoudis, Kasper J. Larsen, Hans Schönemann, Yang Zhang: **1805.01873**

Samuel Abreu, Ben Page, Mao Zeng: 1807.11522





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D. Chicherin, T. Gehrmann, J.M. Henn, N.A. Lo Presti, V. Mitev et al.: **1809.06240**

Janko Böhm, Alessandro Georgoudis, Kasper J. Larsen, Hans Schönemann, Yang Zhang: **1805.01873**

Samuel Abreu, Ben Page, Mao Zeng: 1807.11522

 $I = \sum c_n(s_{ij}, d) M_n$ $n \in basis$

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FinRed

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FinRed

Finite fields reconstruction

von Manteuffel, Schabinger: **1406.4513** Peraro: **1905.08019**

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Denominator guessing

Abreu, Dormans, Febres Cordero, Ita, Page: **1812.04586** Heller, von Manteuffel: **2101.08283**

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von Manteuffel, Schabinger: **1406.4513** Peraro: **1905.08019**

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FinRed

Syzygy techniques

Gluza, Kadja, Kosower: **1009.0472** Ita: **1510.05626** Larsen, Zhang: **1511.01071** Agarwal, Jones, von Manteuffel: **2011.15113**
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GCD 3GB

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Infrared Structure

$\mathcal{H}_{\rm ren}(\epsilon, \{p\}) = \mathcal{Z}(\epsilon, \{p\}, \mu) \mathcal{H}_{\rm fin}(\mu, \{p\})$

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IR-finite

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IR-finite



$$\boldsymbol{\mathcal{Z}}^{-1} = \mathbf{I} - \left(\frac{\alpha_s}{2\pi}\right)\boldsymbol{\mathcal{I}}_1 - \left(\frac{\alpha_s}{2\pi}\right)^2\boldsymbol{\mathcal{I}}_2 + \mathcal{O}(\alpha_s^3)$$

$$\mathcal{I}_{1} = \frac{e^{\epsilon \gamma_{E}}}{\Gamma(1-\epsilon)} \sum_{i} \left(\frac{1}{\epsilon^{2}} - \frac{1}{2\epsilon} \frac{\gamma_{i}^{(0)}}{C_{i}} \right) \sum_{j \neq i} \frac{\mathbf{T}_{i}^{a} \mathbf{T}_{j}^{a}}{2} \left(-\frac{\mu^{2}}{s_{ij}} \right)^{\epsilon}$$

$$\begin{split} \boldsymbol{\mathcal{I}}_{2} &= \frac{-e^{\epsilon \gamma_{E}} \Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\gamma_{K}^{(1)}}{8} + \frac{\beta_{0}}{2\epsilon} \right) \boldsymbol{\mathcal{I}}_{1}(2\epsilon) \\ &\quad -\frac{1}{2} \boldsymbol{\mathcal{I}}_{1}(\epsilon) \left(\boldsymbol{\mathcal{I}}_{1}(\epsilon) + \frac{\beta_{0}}{\epsilon} \right) + \frac{e^{\epsilon \gamma_{E}}}{4\epsilon \Gamma(1-\epsilon)} \sum_{i} H_{2}^{i} \end{split}$$

$$\mathcal{I}_{1} = \frac{e^{\epsilon \gamma_{E}}}{\Gamma(1-\epsilon)} \sum_{i} \left(\frac{1}{\epsilon^{2}} - \frac{1}{2\epsilon} \frac{\gamma_{i}^{(0)}}{C_{i}} \right) \sum_{j \neq i} \frac{\mathbf{T}_{i}^{a} \mathbf{T}_{j}^{a}}{2} \left(-\frac{\mu^{2}}{s_{ij}} \right)^{\epsilon}$$

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Leading Colour → Diagonal in Colour Space

Catani: 9802439

$\mathbf{T}_1^a \, \mathbf{T}_2^a \, \mathbf{A} =$

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$\sqrt{-\frac{N_c}{2}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	($A_1 $
0	$-\frac{N_c}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0		A_2
0	0.	$-\frac{N_c}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	0	$\overline{0}$	$\frac{1}{2}$		A_3
0	0	0 -	$-\frac{N_c}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$\tilde{0}$		A_4
0	0	0	0 .	$-\frac{N_c}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	$\overline{0}$	$\frac{1}{2}$	0	0		A_5
0	0	0	0	0 .	$-\frac{N_c}{2}$	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0		A_6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{\overline{1}}{2}$	0	0	0	$\frac{1}{2}$	0		A_7
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	0		A_8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$		A_9
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0		A_{10}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{\overline{1}}{2}$	$\frac{\overline{1}}{2}$	0	0		A_{11}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	Ō	0	0	$-\frac{1}{2}$		A_{12}
$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	$-N_c$	0	0	0	0	0	0	0	0	0		A_{13}
0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	$-\frac{N_c}{2}$	0	0	0	0	0	0	0	0		A_{14}
0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0 ·	$-\frac{N_c}{2}$	0	0	0	0	0	0	0		A_{15}
0	0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	0	$-\frac{N_c}{2}$	0	0	0	0	0	0		A_{16}
0	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0		A_{17}
0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0		A_{18}
0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0		A_{19}
0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0		A_{20}
0	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0		A_{21}
0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0 /		(A_{22})

(Preliminary) Results & Workflow

 $A_{\vec{h}}^{a_1...a_5} = \sum_{k} R_{mnc}^{ij} T_j N_c^m n_f^n \mathcal{C}_c^{a_1 a_2 a_3 a_4 a_5}$ *i,m,n,c*



a₁₀₀₀₀₀ a_{1 oood} *a*5 a_5 LEADER $a_2^{a_2a_2}$ a_2 10000 a4 $\partial \partial \partial a_4$ saas a3 a_3 $A_{\vec{h}}^{a_1...a_5} = \sum R_{mnc}^{ij} T_j N_c^m n_f^n \mathscr{C}_c^{a_1 a_2 a_3 a_4 a_5}$ *i,m,n,c* $1, \log, Li_2, g$































UV renormalisation & IR subtraction

a100 a5 a2 a5 a2 a5 a3 a4 a4 a100 a a2 a100 a a2 a100 a a2 a100 a a4







Checks
1 loop



1 loop

Cancellation of poles

2 loops

1 loop

Cancellation of poles

Colour trace identities

2 loops

1 loop

Cancellation of poles

Matched

Zvi Bern, Lance J. Dixon, David A. Kosower: hep-ph/9302280

Colour trace identities

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Leading colour:

S. Abreu, F. Febres Cordero, H. Ita, B. Page, V. Sotnikov: 2102.13609

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All-Plus Yang-Mills:

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Working on it !



Working on it !





 $gg \rightarrow ggg \checkmark$ input for $pp \rightarrow jjj$ N²LO cross section another step towards $pp \rightarrow jj$ at N³LO (real quadrupole radiation!)

gg → ggg √ input for $pp \rightarrow jjj$ N²LO cross section another step towards $pp \rightarrow jj$ at N³LO (real quadrupole radiation!) possibility to study/confirm special kinematical limits

000000000 $gg \rightarrow ggg \checkmark$ input for $pp \rightarrow jjj$ N²LO cross section another step towards $pp \rightarrow jj$ at N³LO (real quadrupole radiation!) possibility to study/confirm **Thanks!** special kinematical limits



The $N_{\mathcal{C}}$ expansion















't Hooft coupling $\lambda = g^2 N_c$











Sphere	$\chi = 2$
Other Manifolds	$\chi = 2 - Holes$



Sphere	$\chi = 2$
Other Manifolds	$\chi = 2 - Holes$

Large Nc limit ↔ Planar diagrams

Example 2



$$\epsilon_{\mu_{1}}^{-}\epsilon_{\mu_{2}}^{-}...\epsilon_{\mu_{5}}^{+} = \frac{1}{2^{5/2}} \frac{\langle q_{1} | \mu_{1} | 1] \langle q_{2} | \mu_{1} | 2] [q_{3} | \mu_{1} | 3 \rangle [q_{4} | \mu_{1} | 4 \rangle [q_{5} | \mu_{1} | 5 \rangle}{\langle q_{1} p_{1} \rangle \langle q_{2} p_{2} \rangle [q_{3} p_{3}] [q_{4} p_{4}] [q_{5} p_{5}]}$$

Example 2



$$\epsilon_{\mu_{1}}^{-}\epsilon_{\mu_{2}}^{-}...\epsilon_{\mu_{5}}^{+} = \frac{1}{2^{5/2}} \frac{\langle q_{1} | \mu_{1} | 1 \rangle \langle q_{2} | \mu_{1} | 2 \rangle [q_{3} | \mu_{1} | 3 \rangle [q_{4} | \mu_{1} | 4 \rangle [q_{5} | \mu_{1} | 5 \rangle}{\langle q_{1} p_{1} \rangle \langle q_{2} p_{2} \rangle [q_{3} p_{3}] [q_{4} p_{4}] [q_{5} p_{5}]}$$

$$\epsilon_{\mu_{1}}^{-}\epsilon_{\mu_{2}}^{-}...\epsilon_{\mu_{5}}^{+} = \frac{1}{2^{5/2}} \frac{\operatorname{Tr}_{+} \left\{ \gamma_{\mu_{1}} p_{1} \gamma_{\mu_{2}} p_{2} \right\} \operatorname{Tr}_{-} \left\{ \gamma_{\mu_{3}} p_{3} \gamma_{\mu_{4}} p_{4} \gamma_{\mu_{5}} p_{5} \right\}}{\langle 12 \rangle \langle 21 \rangle [34] [45] [53]}$$





 $\mathbf{T}^a X^c = -if^a_{cc'} X^{c'}$




$$\mathcal{C}_1 = \delta_{i_1 i_4} \delta_{i_2 i_3} , \qquad \mathcal{C}_2 = \delta_{i_1 i_2} \delta_{i_3 i_4}$$



$$\left(\mathbf{T}_{1}^{a} \mathbf{T}_{2}^{a} \mathcal{C}_{1}\right)_{i_{1} i_{2} i_{3} i_{4}} = -T_{j_{1} i_{1}}^{a} T_{i_{2} j_{2}}^{a} \left(\delta_{j_{1} i_{4}} \delta_{j_{2} i_{3}}\right) = -\frac{1}{2} \left(\delta_{i_{1} i_{2}} \delta_{i_{3} i_{4}} - \frac{1}{N_{c}} \delta_{i_{1} i_{4}} \delta_{i_{2} i_{3}}\right)$$



$$\left(\mathbf{T}_{1}^{a} \mathbf{T}_{2}^{a} \mathcal{C}_{1}\right)_{i_{1} i_{2} i_{3} i_{4}} = -T_{j_{1} i_{1}}^{a} T_{i_{2} j_{2}}^{a} \left(\delta_{j_{1} i_{4}} \delta_{j_{2} i_{3}}\right) = -\frac{1}{2} \left(\delta_{i_{1} i_{2}} \delta_{i_{3} i_{4}} - \frac{1}{N_{c}} \delta_{i_{1} i_{4}} \delta_{i_{2} i_{3}}\right)$$

$$(\mathbf{T}_1^a \ \mathbf{T}_2^a \ \mathcal{C}_2)_{i_1 i_2 i_3 i_4} = -T_{j_1 i_1}^a T_{i_2 j_2}^a \ (\delta_{j_1 j_2} \delta_{i_3 i_4}) = -C_F \ \delta_{i_1 i_2} \delta_{i_3 i_4}$$



$$\begin{aligned} (\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{1})_{i_{1}i_{2}i_{3}i_{4}} &= -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \ (\delta_{j_{1}i_{4}}\delta_{j_{2}i_{3}}) = -\frac{1}{2} \left(\delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}} - \frac{1}{N_{c}} \delta_{i_{1}i_{4}}\delta_{i_{2}i_{3}} \right) \\ (\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{2})_{i_{1}i_{2}i_{3}i_{4}} &= -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \ (\delta_{j_{1}j_{2}}\delta_{i_{3}i_{4}}) = -C_{F} \ \delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}} \end{aligned}$$



$$(\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{1})_{i_{1}i_{2}i_{3}i_{4}} = -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \left(\delta_{j_{1}i_{4}}\delta_{j_{2}i_{3}}\right) = -\frac{1}{2} \left(\delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}} - \frac{1}{N_{c}}\delta_{i_{1}i_{4}}\delta_{i_{2}i_{3}}\right)$$
$$(\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{2})_{i_{1}i_{2}i_{3}i_{4}} = -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \left(\delta_{j_{1}j_{2}}\delta_{i_{3}i_{4}}\right) = -C_{F} \ \delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}}$$

$$v_1 \ \delta_{i_1 i_4} \delta_{i_2 i_3} + v_2 \ \delta_{i_1 i_2} \delta_{i_3 i_4} \rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$



$$(\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{1})_{i_{1}i_{2}i_{3}i_{4}} = -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \left(\delta_{j_{1}i_{4}}\delta_{j_{2}i_{3}}\right) = -\frac{1}{2} \left(\delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}} - \frac{1}{N_{c}}\delta_{i_{1}i_{4}}\delta_{i_{2}i_{3}}\right)$$

$$(\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{2})_{i_{1}i_{2}i_{3}i_{4}} = -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \left(\delta_{j_{1}j_{2}}\delta_{i_{3}i_{4}}\right) = -C_{F} \ \delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}}$$

$$v_1 \ \delta_{i_1 i_4} \delta_{i_2 i_3} + v_2 \ \delta_{i_1 i_2} \delta_{i_3 i_4} \rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\mathbf{T}_1^a \ \mathbf{T}_2^a = \begin{pmatrix} \frac{1}{2N_c} & 0\\ -\frac{1}{2} & -C_F \end{pmatrix}$$



$$(\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{1})_{i_{1}i_{2}i_{3}i_{4}} = -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \ (\delta_{j_{1}i_{4}}\delta_{j_{2}i_{3}}) = -\frac{1}{2} \left(\delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}} - \frac{1}{N_{c}} \delta_{i_{1}i_{4}} \delta_{i_{2}i_{3}} \right)$$

$$(\mathbf{T}_{1}^{a} \ \mathbf{T}_{2}^{a} \ \mathcal{C}_{2})_{i_{1}i_{2}i_{3}i_{4}} = -T_{j_{1}i_{1}}^{a} T_{i_{2}j_{2}}^{a} \ (\delta_{j_{1}j_{2}}\delta_{i_{3}i_{4}}) = -C_{F} \ \delta_{i_{1}i_{2}}\delta_{i_{3}i_{4}}$$

$$v_1 \ \delta_{i_1 i_4} \delta_{i_2 i_3} + v_2 \ \delta_{i_1 i_2} \delta_{i_3 i_4} \rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\mathbf{T}_1^a \ \mathbf{T}_2^a = \begin{pmatrix} \frac{1}{2N_c} & \mathbf{0} \\ -\frac{1}{2} & -C_F \end{pmatrix}$$