Hybrid k_T-factorization at NLO

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in collaboration with

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in memory of
Stanisław (Staszek) Jadach

Collinear factorization in QCD at NLO

$$d\sigma^{LO} = \int \frac{dx_{in}}{x_{in}} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} f_{in}(x_{in})\, f_{\overline{in}}(\bar{x}_{\overline{in}})\, dB(x_{in}, \bar{x}_{\overline{in}})$$

general: $K^\mu = x_K P^\mu + \bar{x}_K \bar{P}^\mu + K^\mu_\perp$

one in-state: $k^\mu_{in} = x_{in} P^\mu$

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$$+ \Bigg[f_{in}(x_{in}) \frac{-\alpha_s}{2\pi\epsilon} \int_{\bar{x}_{\overline{in}}}^1 d\bar{z}\, \mathcal{P}_{\overline{in}}(\bar{z}) f_{\overline{in}}\left(\frac{\bar{x}_{\overline{in}}}{\bar{z}}\right)$$
$$+ f_{\overline{in}}(\bar{x}_{\overline{in}}) \frac{-\alpha_s}{2\pi\epsilon} \int_{x_{in}}^1 dz\, \mathcal{P}_{in}(z) f_{in}\left(\frac{x_{in}}{z}\right) \Bigg] dB(x_{in}, \bar{x}_{\overline{in}}) \Bigg\}$$

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Can I establish the same within hybrid k_T-factorization, for which the LO cross section formula is

$$d\sigma^{LO} = \int \frac{dx_{in}}{x_{in}} \frac{d^2k_\perp}{\pi} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} F_{in}(x_{in}, k_\perp)\, f_{\overline{in}}(\bar{x}_{\overline{in}})\, dB^\star(x_{in}, k_\perp, \bar{x}_{\overline{in}})$$

?

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Auxiliary parton method (tree-level)

$k_{in} = x_{in} P + k_\perp$

We desire to obtain the matrix element with one space-like gluon for the process

$g^\star(k_{in})\ \omega_{\overline{in}}(k_{\overline{in}}) \ \to\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n)$ e.g. $g^\star(k_{in})\ g(k_{\overline{in}}) \ \to\ g(p_1)\ g(p_2)\ g(p_3)$

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and do so by replacing the space-like gluon with an *on-shell auxiliary* quark pair

$q\big(k_1(\Lambda)\big)\ \omega_{\overline{in}}(k_{\overline{in}}) \ \to\ q\big(k_2(\Lambda)\big)\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n)$

with special momenta

$$k_1^\mu = \Lambda P^\mu \quad , \quad k_2^\mu = p_\Lambda{}^\mu = (\Lambda - x_{in}) P^\mu - k_\perp^\mu + \frac{|k_\perp|^2}{(\Lambda - x_{in})\nu^2}\,\bar{P}^\mu$$

such that, while individually on-shell, their difference is

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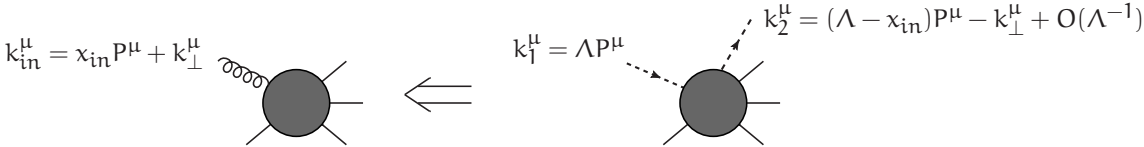

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The matrix element with the space-like gluon is obtained by taking $\Lambda \to \infty$

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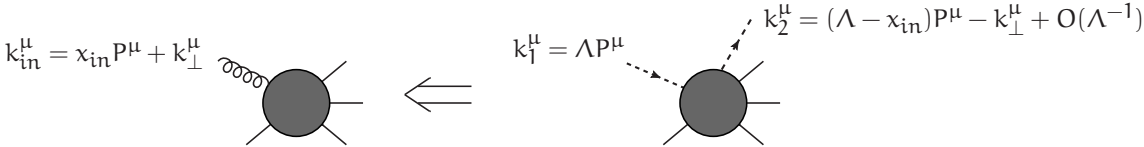

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The factor $x_{in}^2 |k_\perp|^2$ ensures the correct on-shell limit, $1/\Lambda^2$ selects the leading power, $1/g_s^2$ corrects the power of the coupling.

One can use auxiliary quarks, as well as gluons, by including the color-correction factor

$$C_{aux\text{-}q} = \frac{N_c^2 - 1}{N_c} \quad , \quad C_{aux\text{-}g} = 2N_c$$

Auxiliary partons at one loop

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Λ effectively works as a regulator for linear denominators

$$\frac{1}{P\cdot K} \xleftarrow{\Lambda\to\infty} \frac{2\Lambda}{(\Lambda P+K)^2} \implies \ln\Lambda \text{ in loop integrals}$$

One-loop amplitudes turn out to depend non-trivially on the type of auxiliary parton.

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Performing explicit calculations for some simple processes we find for the virtual contribution (Blanco, Giachino, AvH, Kotko 2023)

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For example, apply Λ limit on $A^{\text{loop}}(1_{\bar{Q}}, 6_Q, 2_{\bar{q}}, 3_q, 4_{e^+}, 5_{e^-})$ (Bern, Dixon, Kosower 1998) to get $A^{\text{loop}}(1^\star, 2_{\bar{q}}, 3_q, 4_{e^+}, 5_{e^-})$. The pole-part is proportional to the tree-level amplitude with factor

$$\left\{-\frac{1}{\epsilon^2}\left[\left(\frac{\mu^2}{-s_{p3}}\right)^\epsilon + \left(\frac{\mu^2}{-s_{p2}}\right)^\epsilon\right] - \frac{3}{2\epsilon}\right\} A^{\text{tree}}(1^\star, 2_{\bar{q}}, 3_q, 4_{e^+}, 5_{e^-}) \ ,$$

with s_{p2} and s_{p3} involving only the longitudinal part of $k_1 = p + k_\perp$.

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$$\mathcal{V}_{\text{aux}} = \left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \left[\frac{2}{\epsilon}\ln\frac{\Lambda}{x_{\text{in}}} - i\pi + \bar{\mathcal{V}}_{\text{aux}}\right] + \mathcal{O}(\epsilon) + \mathcal{O}\big(\Lambda^{-1}\big)$$

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$$\bar{\mathcal{V}}_{\text{aux-q}} = \frac{1}{\epsilon}\frac{13}{6} + \frac{\pi^2}{3} + \frac{80}{18} + \frac{1}{N_c^2}\left[\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + 4\right] - \frac{n_f}{N_c}\left[\frac{2}{3}\frac{1}{\epsilon} + \frac{10}{9}\right]$$

$$\bar{\mathcal{V}}_{\text{aux-g}} = -\frac{1}{\epsilon^2} + \frac{\pi^2}{3}$$

Auxiliary partons at one loop

More-or-less proven using known universal collinear limits of one-loop amplitudes (Bern, Chalmers 1995, Bern, Del Duca, Kilgore, Schmidt 1999).

Before the large-Λ, the small-$|k_\perp|$ corresponds to a collinear limit of auxiliary partons. While the large-Λ and small-$|k_\perp|$ limit commute at tree-level, they do not at one loop.

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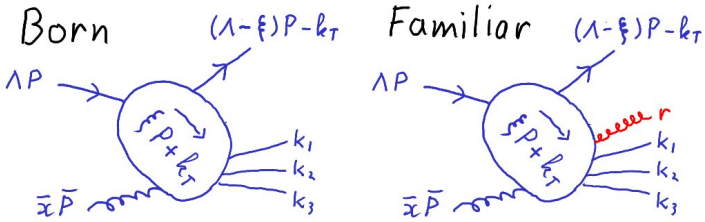

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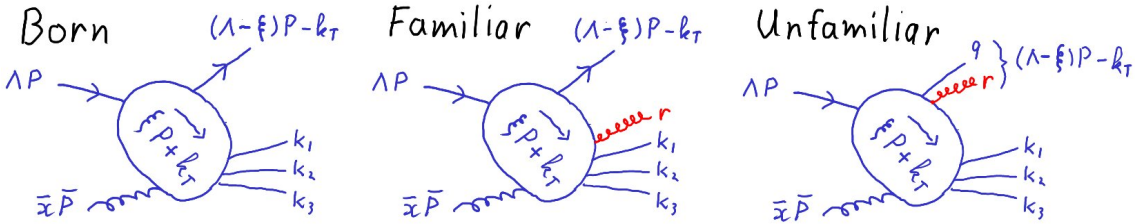
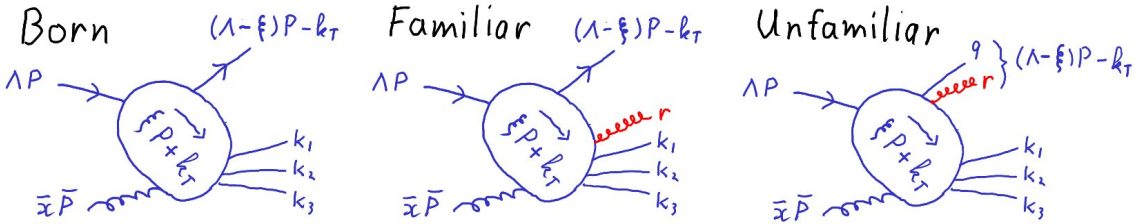
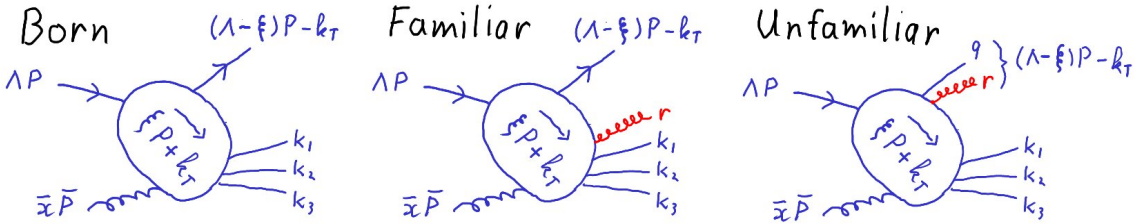


Real radiation with auxiliary partons



The differential phase space and the matrix element factorize for the *unfamiliar* case, where the radiative gluon participates in the consumption of Λ.

Real radiation with auxiliary partons



The differential phase space and the matrix element factorize for the *unfamiliar* case, where the radiative gluon participates in the consumption of Λ.

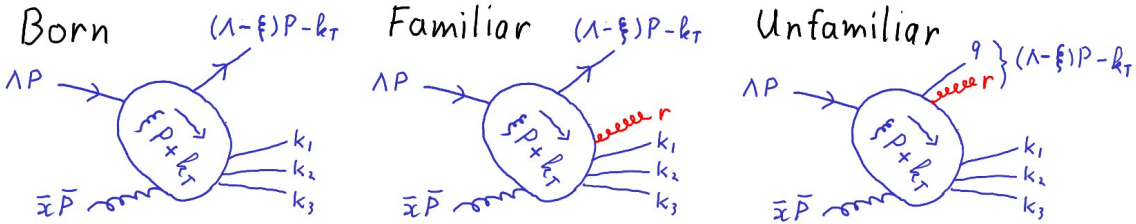
$$\frac{1}{C_{\text{aux}}}\left|\overline{\mathcal{M}}^{\text{aux}}\right|^2\left((\Lambda+x_{\text{in}})P, k_{\overline{\text{in}}}; x_r\Lambda P + r_\perp + \bar{x}_r\bar{P}\,,\, x_q\Lambda P + q_\perp + \bar{x}_q\bar{P}\,,\, \{p_i\}_{i=1}^n\right)$$

$$\xrightarrow{\Lambda\to\infty}\; \mathcal{Q}_{\text{aux}}(x_q, q_\perp, x_r, r_\perp)\,\frac{\Lambda^2\left|\overline{\mathcal{M}}^\star\right|^2\left(x_{\text{in}}P - q_\perp - r_\perp, k_{\overline{\text{in}}}; \{p_i\}_{i=1}^n\right)}{x_{\text{in}}^2|q_\perp + r_\perp|^2}$$

$$\mathcal{Q}_{\text{aux}}(x_q, q_\perp, x_r, r_\perp) = x_q x_r\,\mathcal{P}_{\text{aux}}(x_q, x_r)\,|q_\perp + r_\perp|^2$$
$$\times\left[\frac{c_{\bar{q}}}{|q_\perp|^2|r_\perp|^2} + \frac{1}{x_r|q_\perp|^2 + x_q|r_\perp|^2 - x_q x_r|q_\perp + r_\perp|^2}\left(\frac{c_r\,x_r^2}{|r_\perp|^2} + \frac{c_q\,x_q^2}{|q_\perp|^2}\right)\right]$$

Can be integrated analytically and is proportional to the Born result.
Like the unfamiliar virtual, it is proportional to $\left(\mu^2/|k_\perp|^2\right)^\epsilon$, produces $\ln\Lambda$, and depends on the auxiliary parton types.

Real radiation with auxiliary partons



The differential phase space and the matrix element factorize for the *unfamiliar* case, where the radiative gluon participates in the consumption of Λ.

Precise separation of *familiar* and *unfamiliar* phase space via the demand that in the latter case, the radiation must not become collinear to P in the terms with $1/x_r$

$$\frac{|r_\perp|}{\nu\sqrt{\Lambda}} < x_r < \frac{|r_\perp|}{|r_\perp + k_\perp|} \quad \text{for terms with } 1/x_r$$

Ciafaloni, Colferai 1999

Can be integrated analytically and is proportional to the Born result.
Like the unfamiliar virtual, it is proportional to $\left(\mu^2/|k_\perp|^2\right)^\epsilon$, produces $\ln\Lambda$, and depends on the auxiliary parton types.

Complete unfamiliar contribution

Combining the unfamiliar contributions and organizing them suggestively, we can write

$$dR^{\star\,\mathrm{unf}} + dV^{\star\,\mathrm{unf}} = \Delta_{\mathrm{unf}}\, dB^{\star}\ ,$$

where

$$\Delta_{\mathrm{unf}} = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\left[\mathcal{J}_{\mathrm{aux}} + \mathcal{J}_{\mathrm{univ}} + \mathcal{J}_{\mathrm{univ}} - 2\ln\frac{2P\cdot\bar{P}x_{\mathrm{in}}}{|k_\perp|^2}\right]\ ,$$

with

$$\mathcal{J}_{\mathrm{univ}} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{writing} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}\ ,$$

and

$$\mathcal{J}_{\text{aux-q}} = \frac{3}{2} - \frac{1}{2}(-\epsilon) \quad , \quad \mathcal{J}_{\text{aux-g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon)\ .$$

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- No $\ln\Lambda$ present. $\mathcal{O}(\alpha_s)$ contribution to the space-like gluon Regge trajectory.

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- No $\ln\Lambda$ present. $\mathcal{O}(\alpha_s)$ contribution to the space-like gluon Regge trajectory.
- Target impact factor corrections as in Ciafaloni, Colferai 1999.

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- No $\ln\Lambda$ present. $\mathcal{O}(\alpha_s)$ contribution to the space-like gluon Regge trajectory.
- Target impact factor corrections as in Ciafaloni, Colferai 1999.
- Related to renormalization of the coupling constant (virtual was not UV-subtracted).

Familiar divergencies

Familiar (UV-subtracted) virtual divergencies involving the space-like gluon look as if it were on-shell, with only the longitudinal momentum component $x_{in}P$ in the soft log:

$$-\frac{C_A}{\epsilon^2}\left|\overline{\mathcal{M}}^\star\right|^2+\frac{2}{\epsilon}\sum_{i\neq\star}\ln\left(\frac{\mu^2}{2x_{in}P\cdot p_i}\right)\left(\overline{\mathcal{M}}^\star\right)^2_{i\star}-\frac{11N_c-2n_f}{6\epsilon}\left|\overline{\mathcal{M}}^\star\right|^2$$

Familiar real soft behavior with the space-like gluon acting as "spectator" looks as if it were on-shell, with only the longitudinal momentum component $x_{in}P$ in the eikonal terms:

$$\frac{(x_{in}P\cdot p_i)}{(x_{in}P\cdot r)(r\cdot p_i)}\left(\overline{\mathcal{M}}^\star\right)^2_{i\star}$$

Familiar real collinear singularity

Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

Familiar real collinear singularity

Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

$$\left|\overline{\mathcal{M}}^{\star}\right|^2\left(x_{in}P + k_{\perp}, k_{\overline{in}}; r, \{p_i\}_{i=1}^{n}\right)$$
$$\xrightarrow{r\to x_r P} \frac{2N_c}{P\cdot r}\,\frac{x_{in}^2}{x_r(x_{in}-x_r)^2}\left|\overline{\mathcal{M}}^{\star}\right|^2\left((x_{in}-x_r)P + k_{\perp}, k_{\overline{in}}; \{p_i\}_{i=1}^{n}\right)$$

Collinear splitting function with only the $1/z/(1-z)$ part.

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Collinear splitting function with only the $1/z/(1-z)$ part.

Integrate over relevant phase space with restriction

$$\frac{\bar{x}_r}{\bar{x}_{\overline{in}}} < a\,\frac{x_r}{x_{in}} \quad \text{with} \quad a = \frac{|k_\perp + r_\perp|^2}{S x_{in}\bar{x}_{\overline{in}}} \quad \text{and} \quad |r_\perp|^2 = S x_r \bar{x}_r \quad \Rightarrow \quad |r_\perp| < |k_\perp + r_\perp|\,\frac{x_r}{x_{in}}$$

which is the complement of the restriction on the unfamiliar phase space.

$$\int_0^1 \frac{dx_{in}}{x_{in}} \int d^2k_\perp\, F(x_{in}, k_\perp)\, dR_{coll}^{\star\,fam}\left(x_{in}P_A + k_\perp, k_{\overline{in}}; \{p_i\}_{i=1}^n\right)$$

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$$\int_0^1 \frac{dx_{in}}{x_{in}} \int d^2k_\perp\, F(x_{in}, k_\perp)\, dR^{\star\,\text{fam}}_{\text{coll}}\left(x_{in}P_A + k_\perp, k_{\overline{in}}; \{p_i\}_{i=1}^n\right)$$
$$= \int_0^1 \frac{dx_{in}}{x_{in}} \int d^2k_\perp\, \tilde{F}(x_{in}, k_\perp)\, dB^\star\left(x_{in}, k_\perp, \bar{x}_{\overline{in}}; \{p_i\}_{i=1}^n\right)$$

$$\tilde{F}(x_{in}, k_\perp) = \frac{2a_\epsilon N_c}{\pi_\epsilon \mu^{-2\epsilon}} \int_{x_{in}}^1 \frac{dz}{z(1-z)} \int \frac{d^{2-2\epsilon}r_\perp}{|r_\perp|^2}\,\frac{|k_\perp|^2}{|k_\perp + r_\perp|^2}\, F\left(\frac{x_{in}}{z}, k_\perp + r_\perp\right)\theta_{|r_\perp|<|k_\perp|(1-z)}$$

Essentially identical to formula from Nefedov 2020 for multi-Regge evolution.

Familiar real collinear singularity

Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

$$\left|\overline{\mathcal{M}}^{\star}\right|^2\left(x_{in}P + k_\perp, k_{\overline{in}}; r, \{p_i\}_{i=1}^n\right) \xrightarrow{r\to x_r P} \frac{2N_c}{P\cdot r}\,\frac{x_{in}^2}{x_r(x_{in}-x_r)^2}\left|\overline{\mathcal{M}}^{\star}\right|^2\left((x_{in}-x_r)P + k_\perp, k_{\overline{in}}; \{p_i\}_{i=1}^n\right)$$

Collinear splitting function with only the $1/z/(1-z)$ part.

Integrate over relevant phase space with restriction

$$\frac{\bar{x}_r}{\bar{x}_{\overline{in}}} < a\,\frac{x_r}{x_{in}} \quad \text{with} \quad a = \frac{|k_\perp + r_\perp|^2}{S x_{in}\bar{x}_{\overline{in}}} \quad \text{and} \quad |r_\perp|^2 = S x_r \bar{x}_r \quad \Rightarrow \quad |r_\perp| < |k_\perp + r_\perp|\,\frac{x_r}{x_{in}}$$

which is the complement of the restriction on the unfamiliar phase space.

$$\int_0^1 \frac{dx_{in}}{x_{in}} \int d^2k_\perp\, F(x_{in}, k_\perp)\, dR^{\star\,\text{fam}}_{\text{coll}}\left(x_{in}P_A + k_\perp, k_{\overline{in}}; \{p_i\}_{i=1}^n\right) = \int_0^1 \frac{dx_{in}}{x_{in}} \int d^2k_\perp\, \tilde{F}(x_{in}, k_\perp)\, dB^{\star}\left(x_{in}, k_\perp, \bar{x}_{\overline{in}}; \{p_i\}_{i=1}^n\right)$$

$$\tilde{F}(x_{in}, k_\perp) = a_\epsilon N_c \left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \left\{\frac{F(x_{in}, k_\perp)}{\epsilon^2} - \frac{2}{\epsilon}\int_{x_{in}}^1 dz \left[\frac{1}{[1-z]_+} + \frac{1}{z}\right] F\left(\frac{x_{in}}{z}, k_\perp\right)\right\} + \mathcal{O}(\epsilon^0)$$

Real contribution to the non-cancelling collinear remnant.

Summary

$$d\sigma^{\mathrm{NLO}} = \int \frac{dx_{\mathrm{in}}}{x_{\mathrm{in}}}\, d^2k_\perp \frac{d\bar{x}_{\overline{\mathrm{in}}}}{\bar{x}_{\overline{\mathrm{in}}}} \Big\{ F(x_{\mathrm{in}}, k_\perp)\, f(\bar{x}_{\overline{\mathrm{in}}}) \Big[dV^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) + dR^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) \Big]_{\text{cancelling}}$$
$$+ \Big[F^{\mathrm{NLO}}(x_{\mathrm{in}}, k_\perp) + F(x_{\mathrm{in}}, k_\perp)\Delta_{\mathrm{unf}}(x_{\mathrm{in}}, k_\perp) + \Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}}, k_\perp) \Big] f(\bar{x}_{\overline{\mathrm{in}}})\, dB^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}})$$
$$+ \Big[f^{\mathrm{NLO}}(\bar{x}_{\overline{\mathrm{in}}}) + \Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) \Big] F(x_{\mathrm{in}}, k_\perp) dB^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) \Big\}$$

$$\Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) = -\frac{a_\epsilon}{\epsilon} \int_{\bar{x}_{\overline{\mathrm{in}}}}^1 dz \left[\mathcal{P}^{\mathrm{reg}}_{\overline{\mathrm{in}}}(z) + \gamma_{\overline{\mathrm{in}}}\delta(1-z) \right] f\left(\frac{\bar{x}_{\overline{\mathrm{in}}}}{z} \right)$$

$$\Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}}, k_\perp) = -\frac{a_\epsilon}{\epsilon} \int_{x_{\mathrm{in}}}^1 dz \left[\frac{2N_c}{[1-z]_+} + \frac{2N_c}{z} + \gamma_g \delta(1-z) \right] F\left(\frac{x_{\mathrm{in}}}{z}, k_\perp \right)$$

$$\Delta_{\mathrm{unf}}(x_{\mathrm{in}}, k_\perp) = \frac{a_\epsilon N_c}{\epsilon} \left(\frac{\mu^2}{|k_\perp|^2} \right)^\epsilon \left[\mathrm{impactFactCorr} + \mathrm{couplingRenorm} - 2 \ln \frac{2P \cdot \bar{P} x_{\mathrm{in}}}{|k_\perp|^2} \right]$$

$$f^{\mathrm{NLO}}(\bar{x}_{\overline{\mathrm{in}}}) + \Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) = \mathrm{finite}$$

$$F^{\mathrm{NLO}}(x_{\mathrm{in}}, k_\perp) + F(x_{\mathrm{in}}, k_\perp)\Delta_{\mathrm{unf}}(x_{\mathrm{in}}, k_\perp) + \Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}}, k_\perp) \stackrel{?}{=} \mathrm{finite}$$