FACTORIZATION AND RESUMMATION AT NEXT-TO-LEADING POWER

Leonardo Vernazza

INFN - University of Torino

RADCOR 2023, 30/05/2023

H2020 MSCA COFUND G.A. 754496

OUTLINE

- **Soft-collinear radiation at NLP**
- **Endpoint divergences at NLP**
- **NLP LLs in Thrust in the two-jet limit**
- **NLP NNLO in Drell-Yan near threshold**

In collaboration with
M. Beneke, A. Broggio, M. Garny, S.Jaskiewicz, R. Szafron, J. Strohm J. Wang,

Based on
JHEP 20 (2020), 078, [arXiv:1912.01585 [hep-ph]],

JHEP 10 (2020), 196, [arXiv:2008.04943 [hep-ph]],

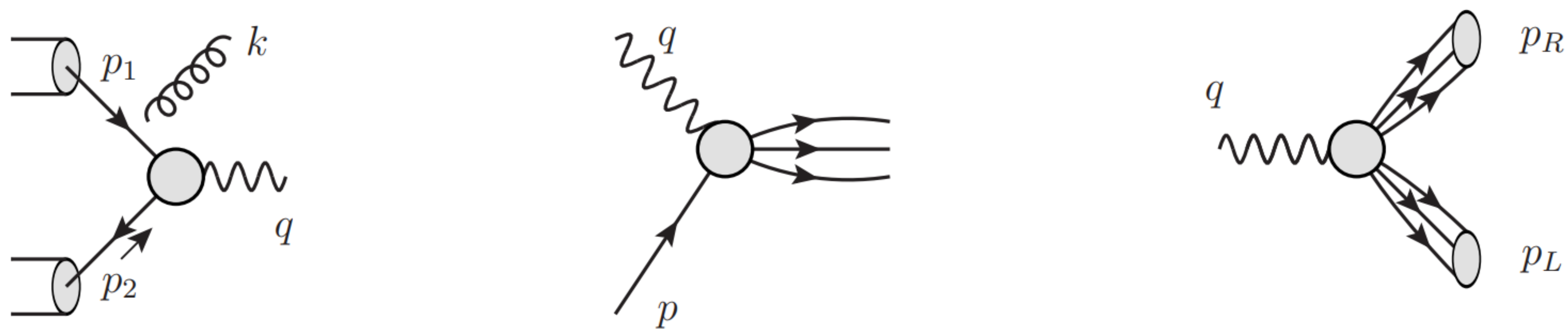
JHEP 10 (2021), 061, [arXiv:2107.07353 [hep-ph]],

JHEP 07 (2022), 144, [arXiv:2205.04479 [hep-ph]],

and to appear.

PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Consider Drell-Yan, DIS near partonic threshold and Thrust in the back-to-back jet limit:



$(p_1 + p_2)^2 \equiv \hat{s}, \quad z \equiv \frac{q^2}{\hat{s}} \to 1$

$Q^2 = -q^2, \quad x \equiv \frac{Q^2}{2p\cdot q} \to 1$

$q^2 \gg p_L^2, p_R^2,$

$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \to 1$

- The partonic cross section has singular expansion

$$\Delta_{ab}(\xi) \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \Bigg[\underbrace{c_n \delta(1-\xi) + \sum_{m=0}^{2n-1} \Bigg(c_{nm} \left[\frac{\ln^m(1-\xi)}{1-\xi}\right]_+}_{\text{LP}} + \underbrace{d_{nm} \ln^m(1-\xi)}_{\text{NLP}} \Bigg) + \dots \Bigg],$$

with $\xi = z$ for DY, $\xi = x$ for DIS, and $\xi = T$ for Thrust.

- Resummation of large logarithms relevant for precision phenomenology.
  - $\rightarrow$ well understood at LP (up to N3LL),
  - $\rightarrow$ progress toward resummation at NLP, yet no systematic approach so far.

PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Subject of intense work in the past few years!
- Within SCET:

  Beneke, Campanario, Mannel, Pecjak, 2004; Larkoski, Neill, Stewart, 2014; Kolodrubetz, Moult, Stewart, 2016; Feige, Kolodrubetz, Moult, Stewart, 2017; Beneke, Garny, Szafron, Wang, 2017-2019; Moult, Rothen, Stewart, Tackmann, Zhu, 2016/17; Boughezal, Liu, Petriello, 2016/17; Moult, Stewart, Vita, Zhu, 2018; Moult, Stewart, Vita, 2019; Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019; Beneke, Broggio, Jaskiewicz, LV, 2019; Broggio, Jaskiewicz, LV, 2021/22; Beneke, Bobeth, Szafron, 2017; Alte, König, Neubert, 2018; Moult et al., 2019; Liu, Neubert, 2019; Wang, 2019; Liu, Mecaj, Neubert, Wang, 2020; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020; Liu, Neubert, Schnubel, Wang, 2021; Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 2018; Moult, Vita, Yan, 2019; Beneke, Hager, Szafron, 2021; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang 2020; Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022 + …

- And "diagrammatic" methods:

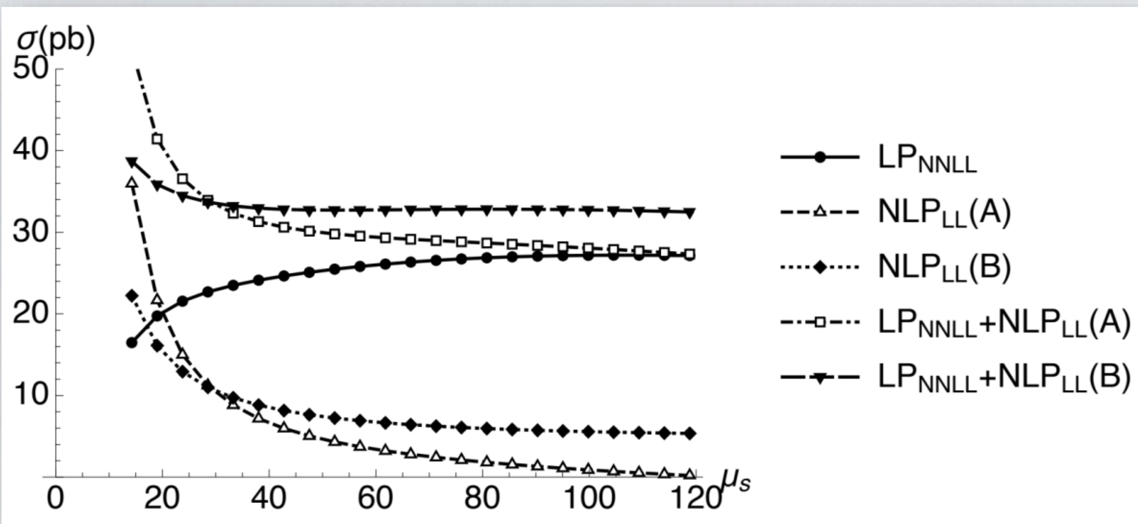
  Del Duca, 1990; Laenen, Magnea, Stavenga, 2008, Laenen, Stavenga, White, 2008; Laenen, Magnea, Stavenga, White, 2010; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016; Bahjat-Abbas, Sinninghe Damsté, LV, White, 2018; Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019; Liu, Penin, 2017/18; Anastasiou, Penin, 2020; Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020; van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021; + …

- Several topics considered:

  LBKD theorem, operator bases, renormalization, N-jettiness subtraction, thrust distribution, Drell-Yan and Higgs production near threshold, DIS for $x \to 1$, QED effects in B decays, New Physics decay, Higgs decay through bottom loops, TMD factorization, energy-energy correlation in $N = 4$ SYM, gravitation, …

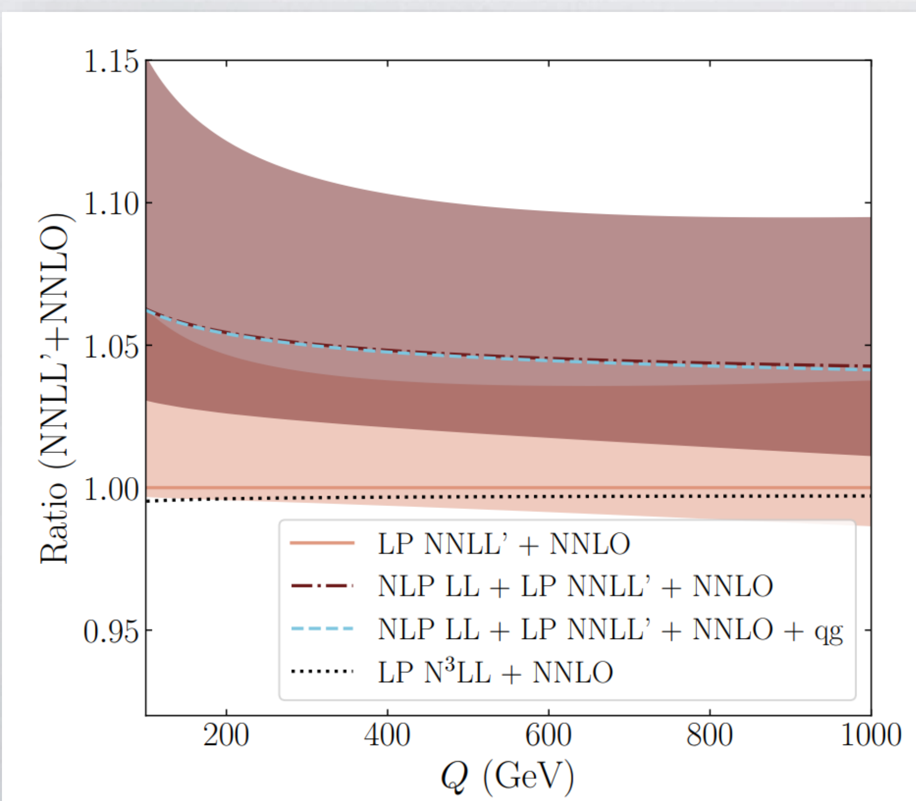
PARTICLE SCATTERING NEAR THRESHOLD

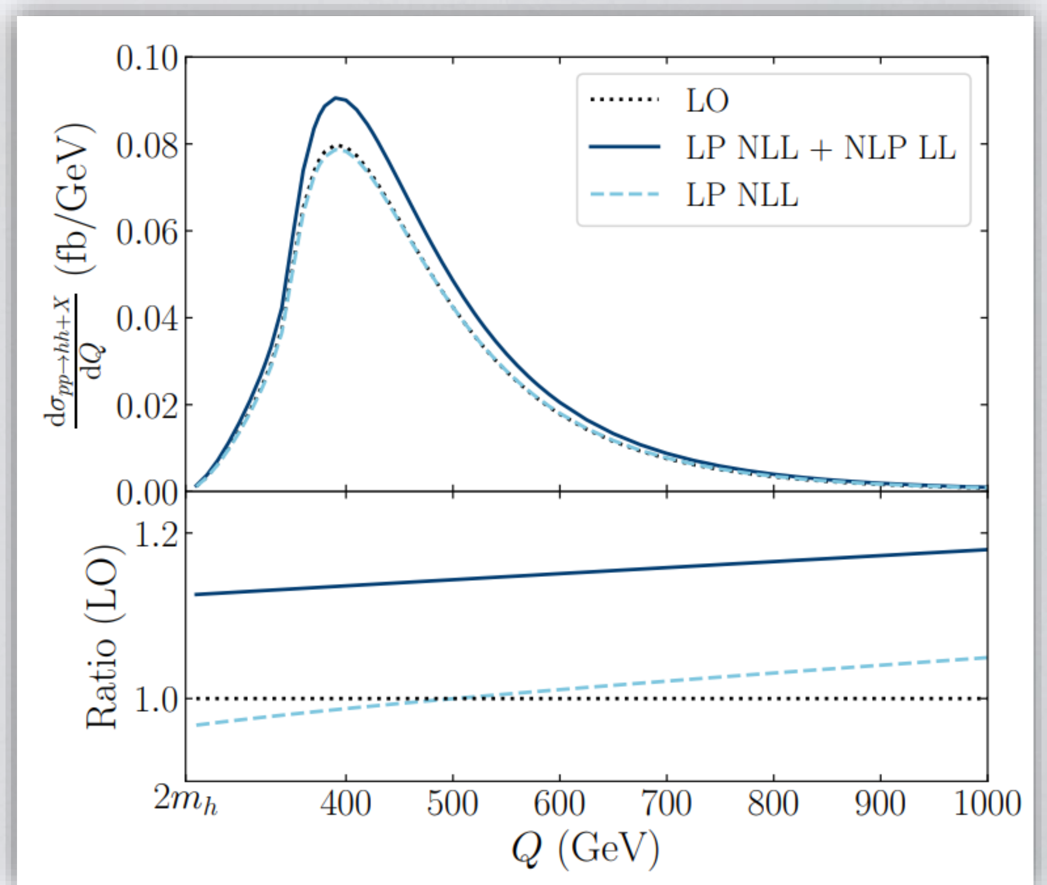
- Phenomenological analyses have shown LLs at NLP to be competitive with NNLLs at LP: relevant for precision physics.



Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019

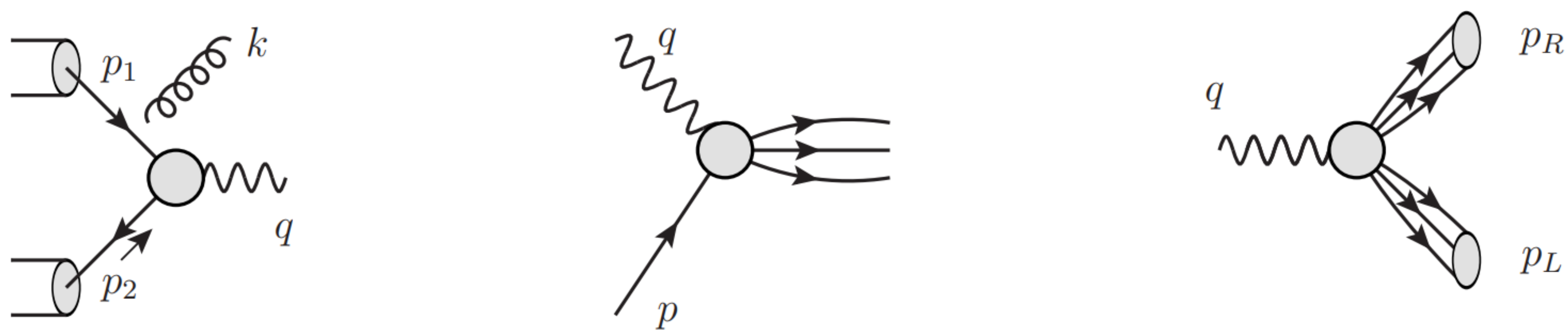
van Beekveld, Laenen, Sinninghe Damsté, LV, 2021.





PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Consider Drell-Yan, DIS near partonic threshold and Thrust in the back-to-back jet limit:



$(p_1 + p_2)^2 \equiv \hat{s}, \quad z \equiv \frac{q^2}{\hat{s}} \to 1$

$Q^2 = -q^2, \quad x \equiv \frac{Q^2}{2p\cdot q} \to 1$

$q^2 \gg p_L^2, p_R^2,$

$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \to 1$

- These limits involve a dynamical enhancement of soft and collinear radiation.
- Factorize soft and collinear radiation from the hard interaction:

  $\rightarrow$ by means of Soft-Collinear Effective Field Theory (SCET);

  $\rightarrow$ by means of a diagrammatic approach in QCD.

- Here we discuss the first method.

SOFT-COLLINEAR EFFECTIVE FIELD THEORY

- Effective Lagrangian and operators made of collinear and soft fields.

$$\mathcal{L}_{\mathrm{SCET}} = \sum_i \mathcal{L}_{c_i} + \mathcal{L}_s,$$

Bauer, Fleming, Pirjol, Stewart, 2000,2001; Beneke, Chapovsky, Diehl, Feldmann, 2002; Hill, Neubert 2002.

$$\mathcal{O}_n = \int dt_1 \dots dt_n \, \mathcal{C}(t_1, \dots, t_n) \, \phi_1(t_1 n_{1+}) \dots \phi_n(t_n n_{n+}).$$

- Constructed to reproduce a scattering process as obtained with the method of regions.
- The cross section factorizes into a hard scattering kernel, and matrix elements of soft and collinear fields.

$$\sigma \sim \mathcal{H} \otimes \mathcal{J}_1 \otimes \dots \otimes \mathcal{J}_n \otimes \mathcal{S}.$$

Hard matching coefficient

Jet functions – matrix elements of collinear fields

Soft function – matrix element of soft fields

- Renormalize UV divergences of EFT operators and obtain renormalization group equations.
- Each function depends on a single scale: solving the RGE resums large logarithms.

See e.g. Becher, Neubert 2006

FACTORIZATION IN SCET: LP

- Factorization theorem at LP are "simple" due to soft-collinear decoupling:
- There is a single eikonal soft-collinear interaction at LP:

Beneke, Chapovsky, Diehl, Feldmann, 2002

$$\mathcal{L}_c^{(0)} = \bar{\xi}\left(in_- D_c + g_s n_- A_s(x_-) + i\not{D}_{\perp c}\frac{1}{in_+ D_c}i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\xi + \mathcal{L}_{c,\mathrm{YM}}^{(0)},$$

$$\text{where} \qquad iD_c = i\partial + g_s A_c, \qquad x_-^\mu = n_+ \cdot x\,\frac{n_-^\mu}{2}.$$

- This can be removed by means of a field redefinition:

Bauer, Pirjol, Stewart, 2001

$$\xi(x) \to Y(x_-)\xi(x), \qquad A_c^\mu(x) \to Y(x_-)A_c^\mu(x)Y^\dagger(x_-), \qquad Y^\dagger in_- D_s Y = in_-\partial,$$

$$\text{with} \qquad Y(x) = \mathcal{P}\exp\left(ig_s\int_{-\infty}^0 ds\, n_- A_s(x+sn_-)\right).$$

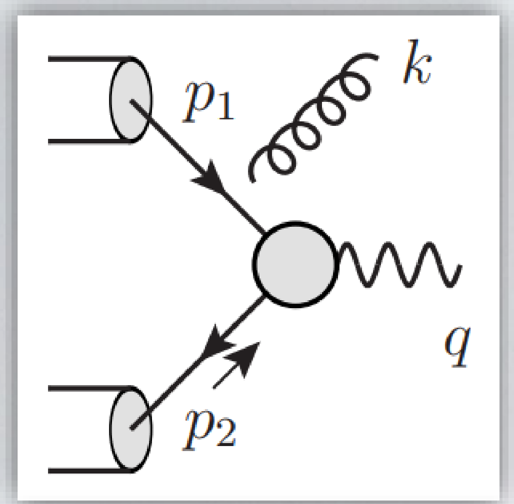
- No soft-collinear interactions are left at LP:

$$\mathcal{L}_c^{(0)} = \bar{\xi}\left(in_- D_c + \cancel{g_s n_- A_s(x_-)} + i\not{D}_{\perp c}\frac{1}{in_+ D_c}i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\xi + \mathcal{L}_{c,\mathrm{YM}}^{(0)}.$$

FACTORIZATION IN SCET: LP

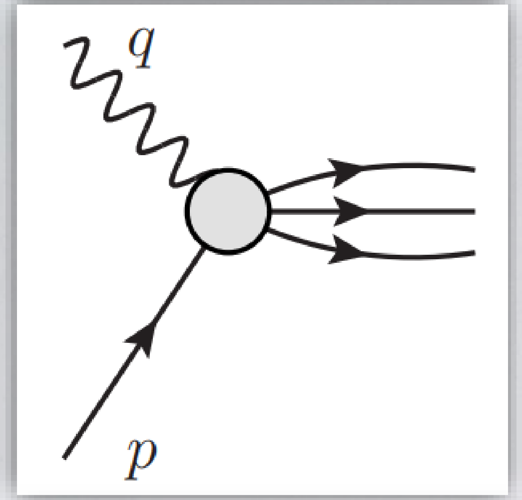
- One obtains "classical" factorization theorem at LP:

Sterman, 1987;
Catani,Trentadue, 1989;
Catani, Turnock, Webber, Trentadue, 1991;
Catani, Trentadue, Turnock, Webber, 1993



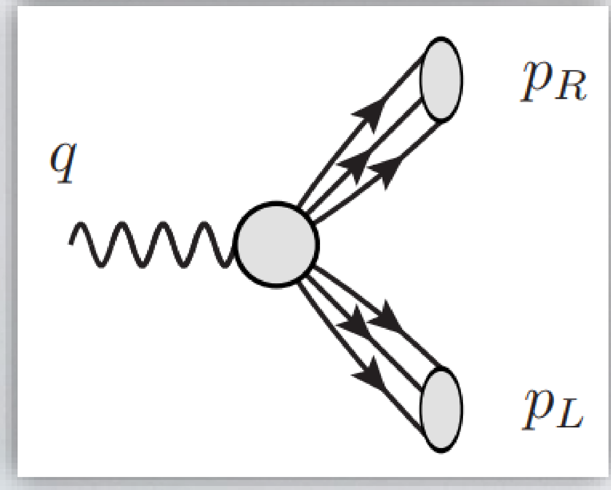
$$\frac{d\sigma}{dQ^2} = |C^{A0}|^2 \times f_{a/A} \otimes f_{b/B} \otimes S_{\rm DY}\big(Q(1-z)\big),$$

Becher, Neubert, Xu 2007;



$$F_2 = |C^{A0}|^2 \times Q^2 \times f_{a/A} \otimes J^{(q)}_{\overline{hc}},$$

Becher, Neubert, Pecjak 2006;



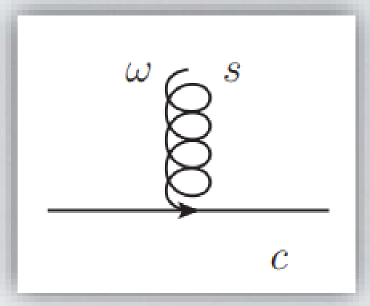
$$\frac{d\sigma}{d\tau} = |C^{A0}|^2 \times J_c^{(q)} \otimes J_{\bar c}^{(\bar q)} \otimes S_{\rm LP}.$$

Becher, Schwartz, 2008

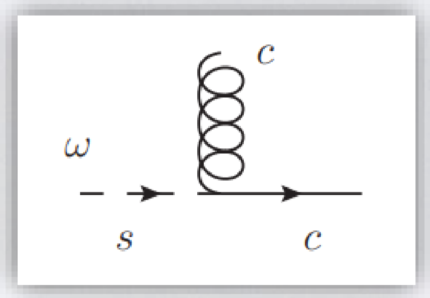
FACTORIZATION IN SCET: NLP

- Soft-collinear interactions are still present at subleading power, and one needs to take into account several effecs: *Beneke, Chapovsky, Diehl, Feldmann, 2002; Beneke, Feldmann, 2002*
- Subleading Lagrangian: $\mathcal{L}_{c_i} = \mathcal{L}_{c_i}^{(0)} + \mathcal{L}_{c_i}^{(1)} + \mathcal{L}_{c_i}^{(2)} + \ldots$ where e.g.

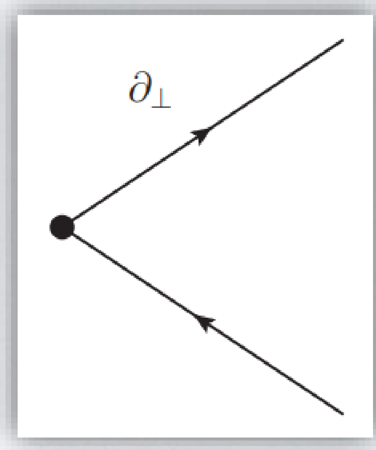
$$\mathcal{L}_c^{(1)\mathrm{gluon}}(x) = \bar{\xi}\left[x_\perp^\mu n_-^\nu W_c g_s F_{\mu\nu}^s(x_-) W_c^\dagger\right]\frac{\not{n}_+}{2}\xi,$$



$$\mathcal{L}_c^{(1)\mathrm{quark}}(x) = \bar{q}(x_-) W_c^\dagger i\not{D}_{\perp c}\xi.$$

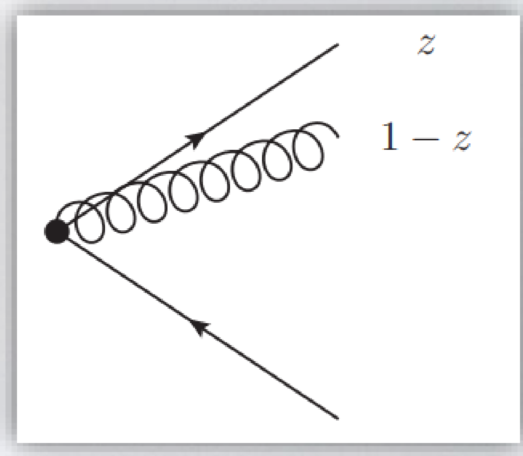


- Power-suppressed operators, e.g.



$$J_\rho^{A0,A1}(t,\bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-) n_{+\rho} i\partial_\perp \chi_c(tn_+),$$

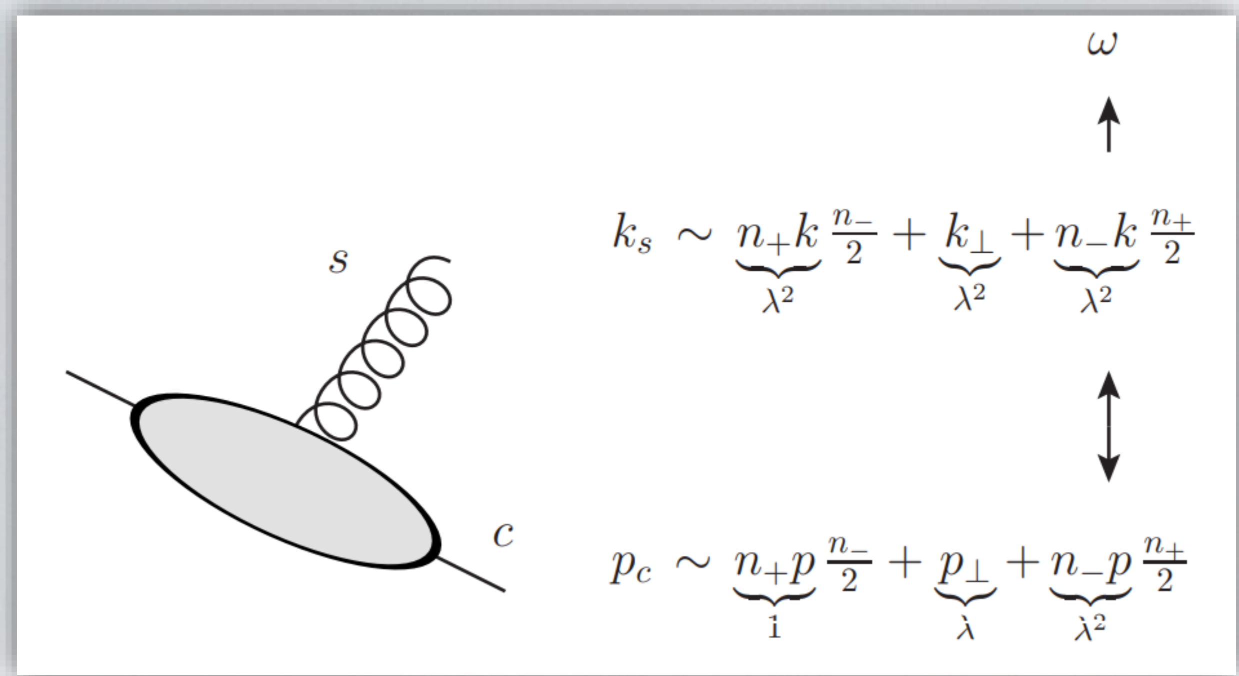
$$J_\rho^{A0,B1}(t_1,t_2,\bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-) n_{\pm\rho} \mathcal{A}_{\perp c}(t_2 n_+)\chi_c(t_1 n_+).$$



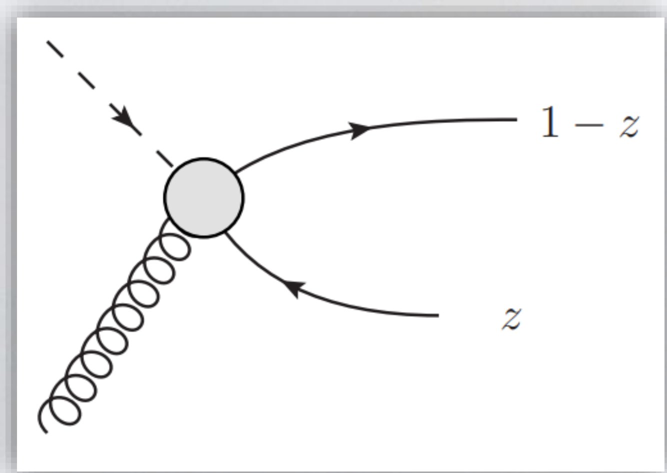
Beneke at Al, 2017-19; see also Stewart at Al., 2014-19

FACTORIZATION IN SCET: NLP

- Matrix elements involve convolutions over momentum fractions/soft momenta components:



$$\longrightarrow \int d\omega\, J(\omega)\, S(\omega),$$



$$\longrightarrow \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z \bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(s_{qg}, z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}\,.$$

FACTORIZATION IN SCET: NLP

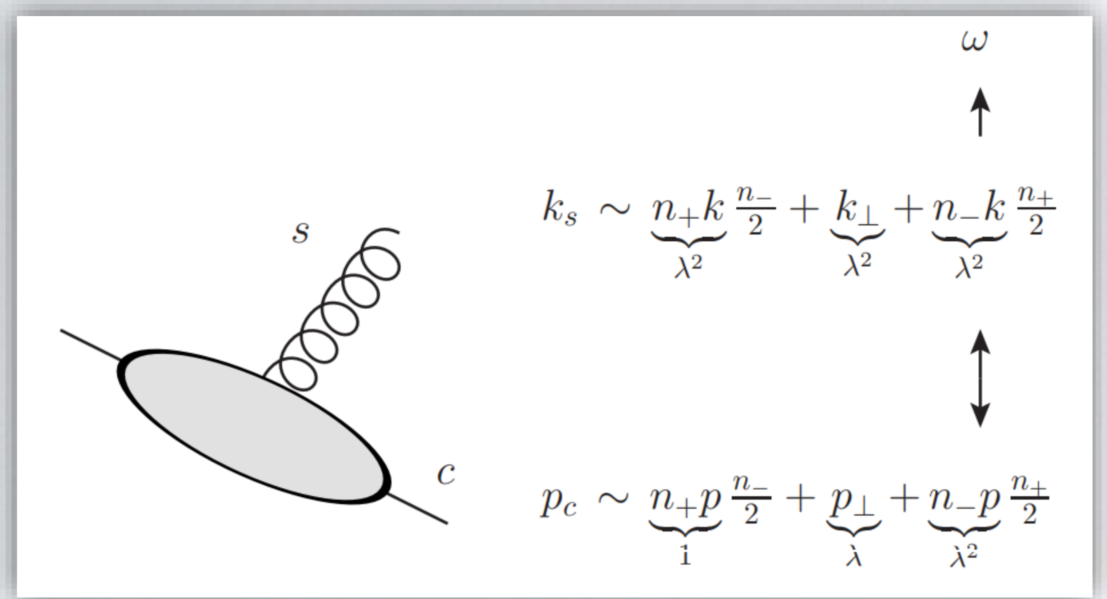
First observed in Beneke, LV 2008;

Liu, Mecaj, Neubert, Wang, 2019-2020;

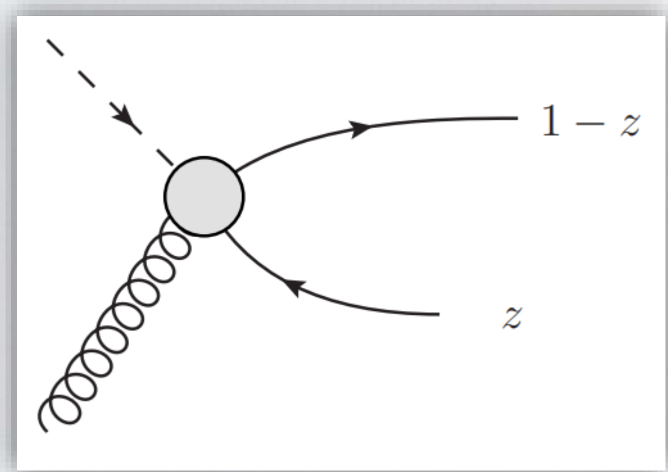
Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018

Beneke, Broggio, Jaskiewicz, LV, 2019

- Convolutions are divergent in $d = 4$!



$$\longrightarrow \int_0^\Omega d\omega \; \underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}} \; \underbrace{\frac{1}{\omega^{1+\epsilon}}\frac{1}{(\Omega-\omega)^\epsilon}}_{\text{soft piece}},$$



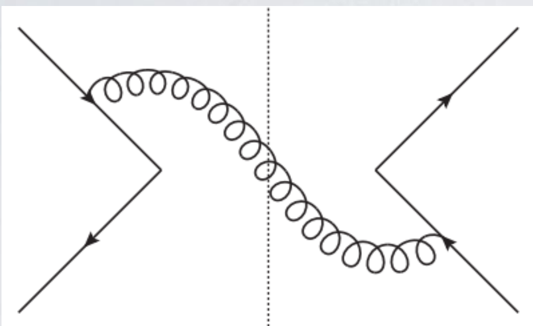
$$\longrightarrow \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^\epsilon \frac{\alpha_s C_F}{2\pi} \frac{(1-z)^2}{z}\Big|_{s_{qg}=Q^2\frac{1-x}{x}}.$$

- Cannot apply the standard RGE methods directly to the collinear and soft functions.

INTERLUDE: DIAGONAL VS OFF-DIAGONAL

- At LP only "diagonal" $q\bar{q}$ channel contributes, endpoint divergences relevant at NLL.
- The "off-diagonal" $g\bar{q},\ qg$ channels start at NLP:

Beneke, Broggio, Jaskiewicz, LV, 2019

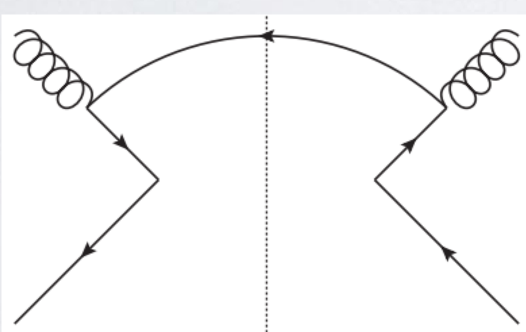


$$\Delta_{q\bar q}(z) = \Delta_{q\bar q}(z)|_{\rm LP} + \underbrace{\Delta_{q\bar q}(z)|_{\rm NLP}}$$

$$\underbrace{\Delta^{kin}_{q\bar q}(z)|_{\rm NLP}}_{\text{starts at NLL}} + \underbrace{\Delta^{dyn}_{q\bar q}(z)|_{\rm NLP}}$$

$$\underbrace{\sum_i \int \{d\omega\} J_i(\{\omega\}) S_i(\{\omega\})}$$

$$\sim \frac{\alpha_s}{4\pi} \sum_i \int \{d\omega\} \Big[\underbrace{S_i^{(n)}(\{\omega\})}_{\text{starts at LL}} + \underbrace{\sum_{m=1}^{n-1} J_i^{(m)}(\{\omega\}) S_i^{(n-m)}(\{\omega\}) \Big]}_{\text{starts at NLL}}$$



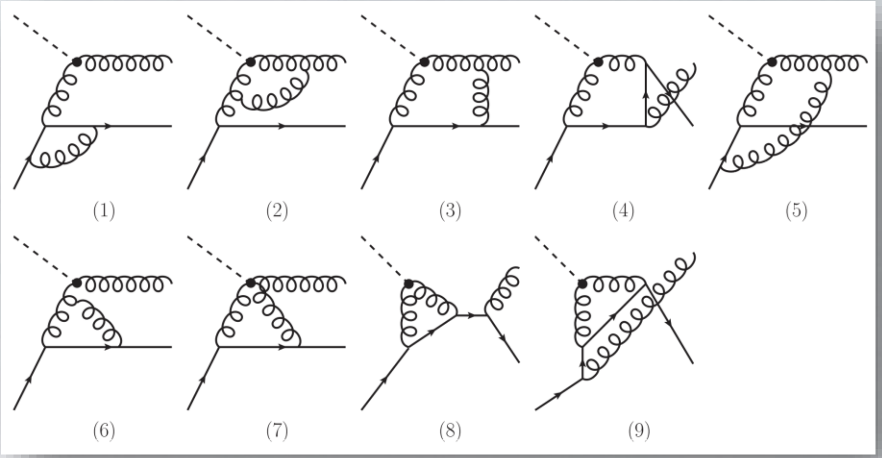
$$\Delta_{g\bar q}(z) = \underbrace{\Delta_{g\bar q}(z)|_{\rm NLP}}$$

$$\underbrace{\Delta^{dyn}_{g\bar q}(z)|_{\rm NLP}}$$

$$\underbrace{\int \{d\omega\} J(\{\omega\}) S(\{\omega\})}_{\text{starts at LL}}$$

- Off-diagonal channels have a simpler structure, but endpoint divergences relevant at LL.

ENDPOINT DIVERGENCES



- Let's investigate the endpoint divergence in off-diagonal gluon DIS in some more detail:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\times\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}+\mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}-\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}+\left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right)+\mathcal{O}(\epsilon^{-1}).$$

- The **T1.T2** term contains a single pole, but: promoted to leading pole after integration!
- Compare exact integration:

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang 2020

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3},$$

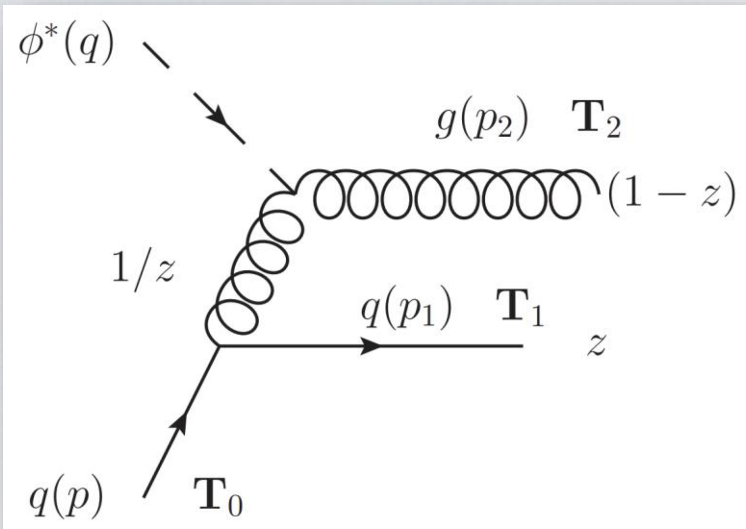
vs integration after expansion:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z-\frac{\epsilon^2}{2!}\ln^2 z+\frac{\epsilon^2}{3!}\ln^3 z+\cdots\right) = -\frac{1}{\epsilon^3}+\frac{1}{\epsilon^3}-\frac{1}{\epsilon^3}+\cdots.$$

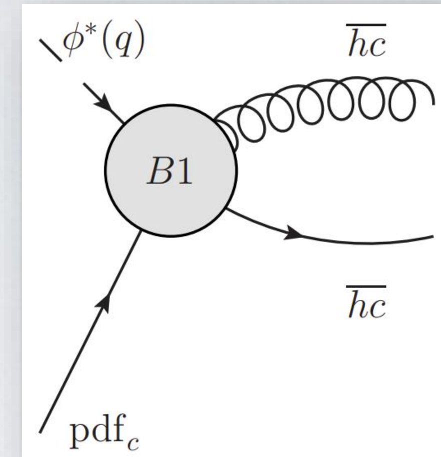
- Expansion in ε not possible before integration! The pole associated to **T1.T2** does not originate from the standard cups anomalous dimension.

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

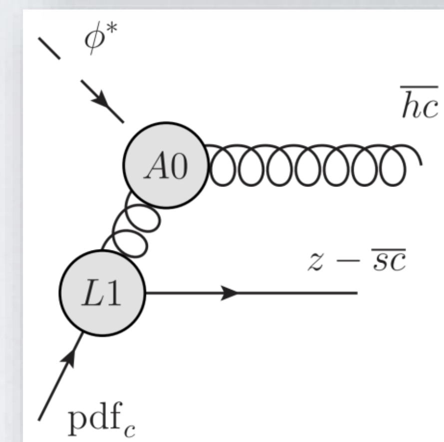
- What happens for $z \to 0$?



For z ~ 1 intermediate propagator is hard



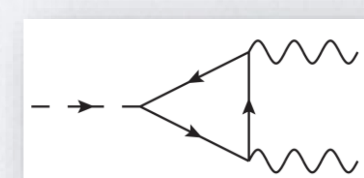
For z « 1 intermediate propagator cannot be integrated out



- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: z-softcollinear.
- Need re-factorization:

$$\underbrace{C^{B1}(Q,z)}_{\text{multi-scale function}}\, J^{B1}(z) \xrightarrow{z\to 0} C^{A0}(Q^2)\int d^4x\,\mathbf{T}\Big[J^{A0},\mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = \underbrace{C^{A0}(Q^2)D^{B1}(zQ^2,\mu^2)}_{\text{single-scale functions}}\, J^{B1}_{z-\overline{sc}}.$$

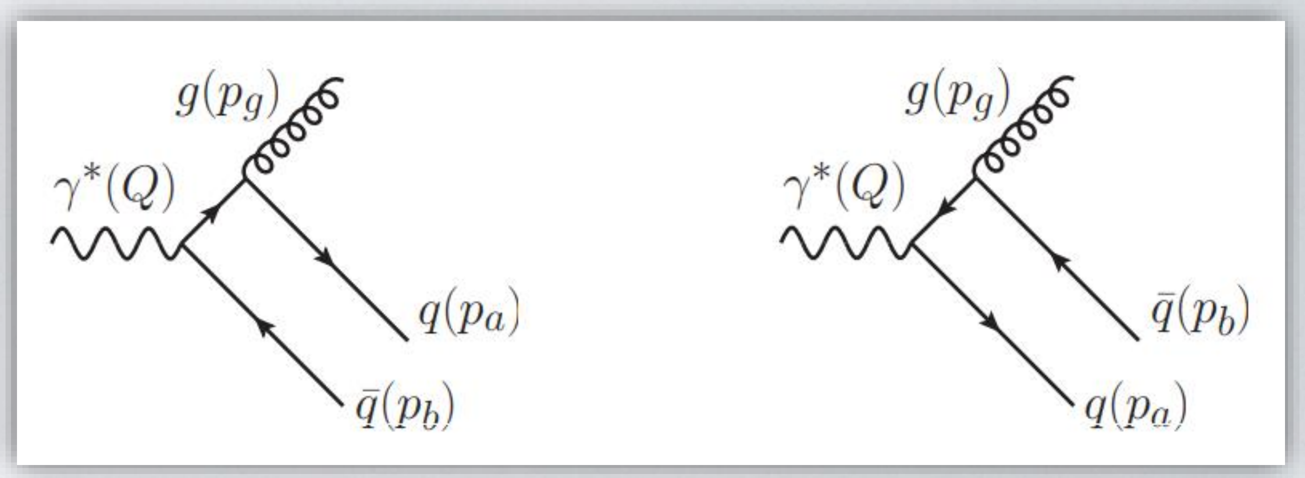
- Similar re-factorization proven in Liu, Mecaj, Neubert, Wang 2020.



OFF-DIAGONAL "GLUON" THRUST

- Consider the power-suppressed contribution to Thrust in the two-jet region:

$$e^+e^- \to \gamma^* \to [g]_c + [q\bar{q}]_{\bar{c}}.$$



- Within SCET one has two contributions:
  "Direct" term (B-type) and time-ordered product soft-quark term (A-type):

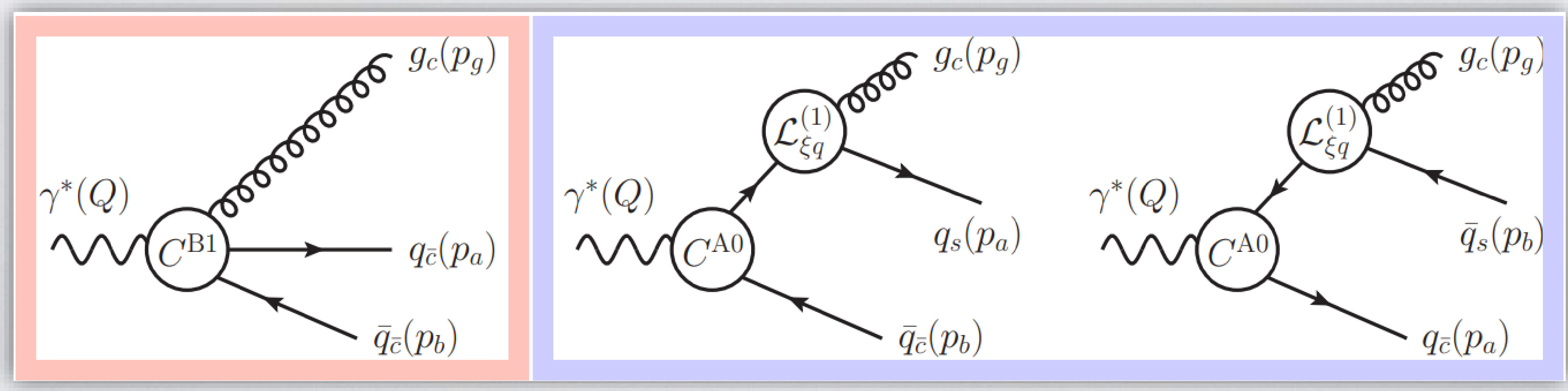
"Soft quark Sudakov" in Moult, Stewart, Vita, Zhu, 2019

$$\bar{\psi}\gamma_\perp^\mu\psi(0) = \int dt d\bar{t}\, \widetilde{C}^{\mathrm{A0}}(t,\bar{t})\, \bar{\chi}_c(tn_+)\gamma_\perp^\mu\chi_{\bar{c}}(\bar{t}n_-) + (c \leftrightarrow \bar{c})$$

$$+ \sum_{i=1,2}\int dt d\bar{t}_1 d\bar{t}_2\, \widetilde{C}_i^{\mathrm{B1}}(t,\bar{t}_1,\bar{t}_2)\, \bar{\chi}_{\bar{c}}(\bar{t}_1 n_-)\Gamma_i^{\mu\nu}\mathcal{A}_{c\perp\nu}(tn_+)\chi_{\bar{c}}(\bar{t}_2 n_-) + \ldots$$

$$\mathcal{L}_{\xi q}(x) = \bar{q}_s(x_-)\not{\mathcal{A}}_{c\perp}(x)\chi_c(x) + \text{h.c.}\,.$$

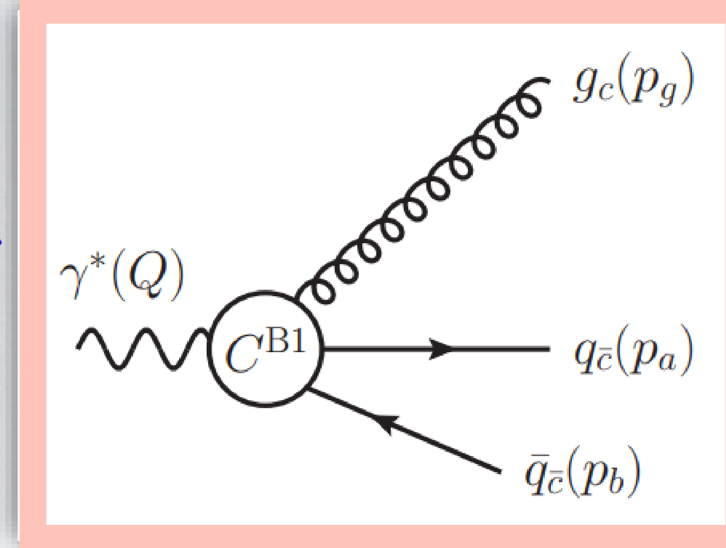
Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022



OFF-DIAGONAL "GLUON" THRUST

- "Direct" B-type term expressed in hard, (anti-)collinear and soft function:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{B}} \sim \int_0^1 dr\, dr'\, C^{B1}(r)\, C^{B1}(r') \times \mathcal{J}_{\bar{c}}^{(q\bar{q})}(r,r') \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}.$$

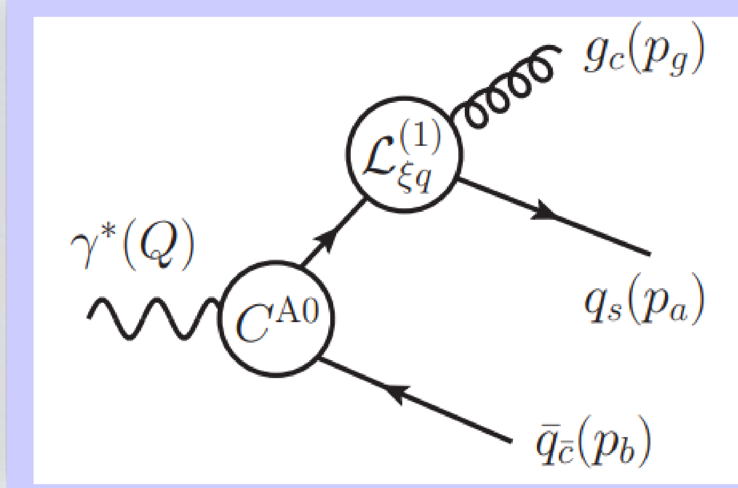


- It develops endpoint divergences when the quark *(r→0)* or anti-quark *(r→1)* become soft:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{B}} \propto \int_0^1 dr\left[\frac{1}{r^{1+\epsilon}} + \frac{1}{(1-r)^{1+\epsilon}}\right].$$

- Time-ordered product A-type term expressed in hard, (anti-)collinear and soft function:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{A}} \sim \int_0^\infty d\omega\, d\omega'\, |C^{A0}|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c(\omega,\omega') \otimes S_{\mathrm{NLP}}(\omega,\omega').$$

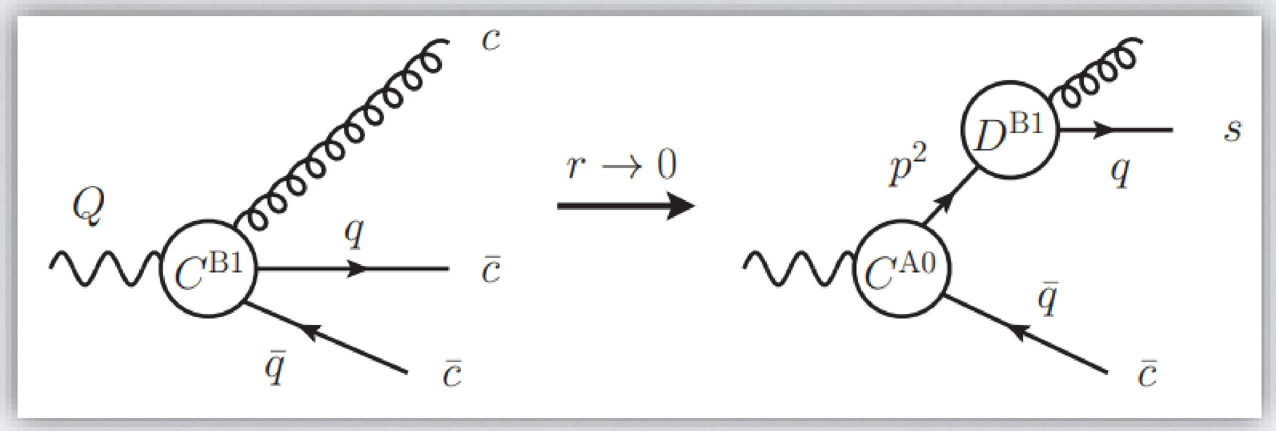


- It develops endpoint divergences when the soft quark or anti-quark become energetic *(ω→∞)*:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{A}} \propto 2\int_{M_R^2/Q}^\infty d\omega\, \frac{1}{\omega^{1+\epsilon}}.$$

OFF-DIAGONAL "GLUON" THRUST

- As for DIS, in the *r→0* (or *r→1*) limit, the B1 coefficient is a two-scale object, which refactorizes, since the intermediate state develops an on-shell pole:

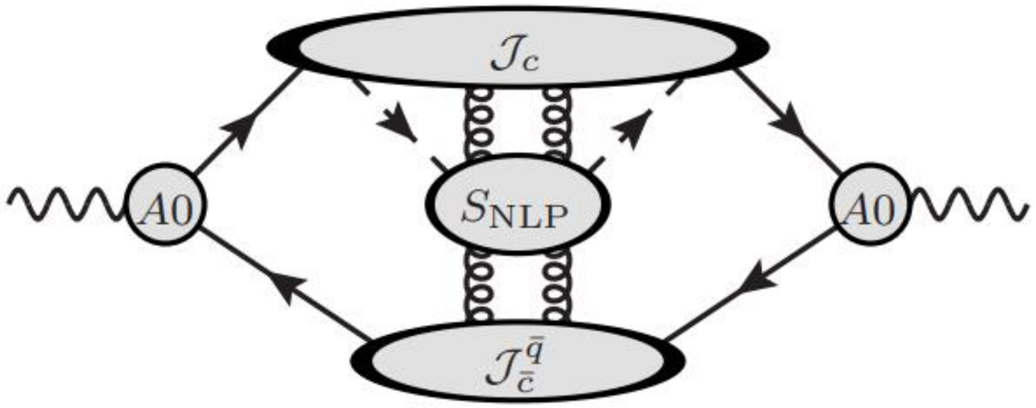


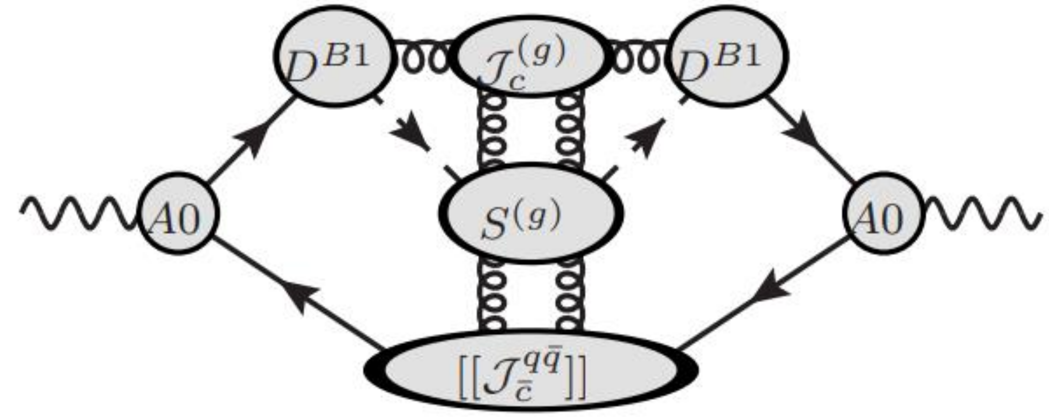
$$C_1^{\mathrm{B1}}(Q^2, r) = C^{\mathrm{A0}}(Q^2) \times \frac{D^{\mathrm{B1}}(rQ^2)}{r} + \mathcal{O}(r^0)\,.$$

- In *d* dimensions the *1/ε* poles from the divergent convolution integrals cancels. The integrands of A and B match in the asymptotic limits *ω,ω′→∞* (A-type) and r,r′→0(1) (B-type).

- This allows a rearrangement between the terms that makes them separately finite, provided two refactorization conditions hold for the soft and jet functions:

OFF-DIAGONAL "GLUON" THRUST





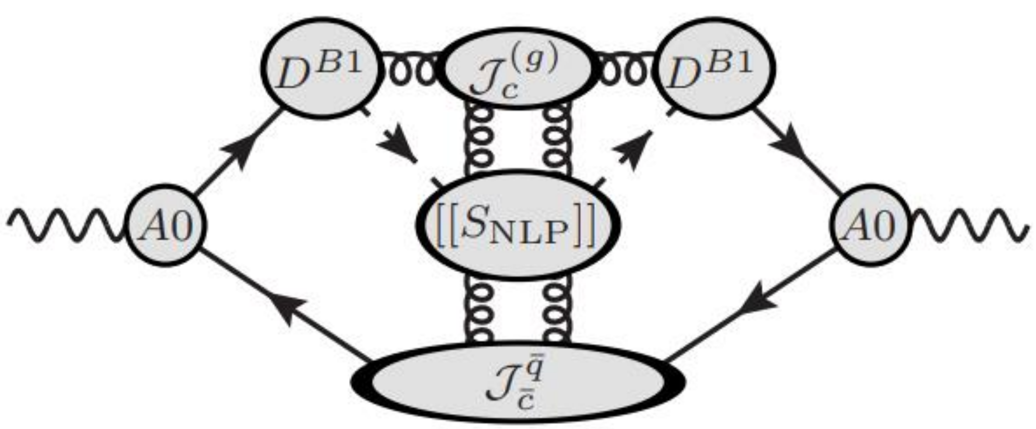
$$\omega, \omega' \to \infty$$

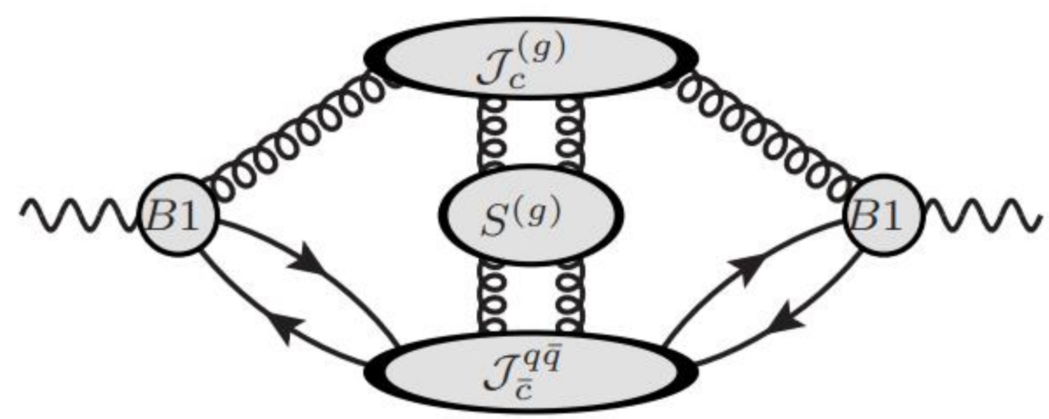
$$[[\mathcal{J}_c\left(p^2, \omega, \omega'\right)]] \to \mathcal{J}_c^{(g)}(p^2)\, \frac{D^{\rm B1}(\omega Q)}{\omega}\, \frac{D^{\rm B1\,*}(\omega' Q)}{\omega'}$$

Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022

$$Q\, \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R)\, [[\widetilde{S}_{\rm NLP}(s_R, s_L, \omega, \omega')]]$$
$$\to [[\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R, r, r')]]\widetilde{S}^{(g)}(s_R, s_L)$$

$$[[C_1^{\rm B1}(Q^2, r)]] \to C^{\rm A0}(Q^2) \times \frac{D^{\rm B1}(rQ^2)}{r}$$
$$r, r' \to 0$$





OFF-DIAGONAL "GLUON" THRUST

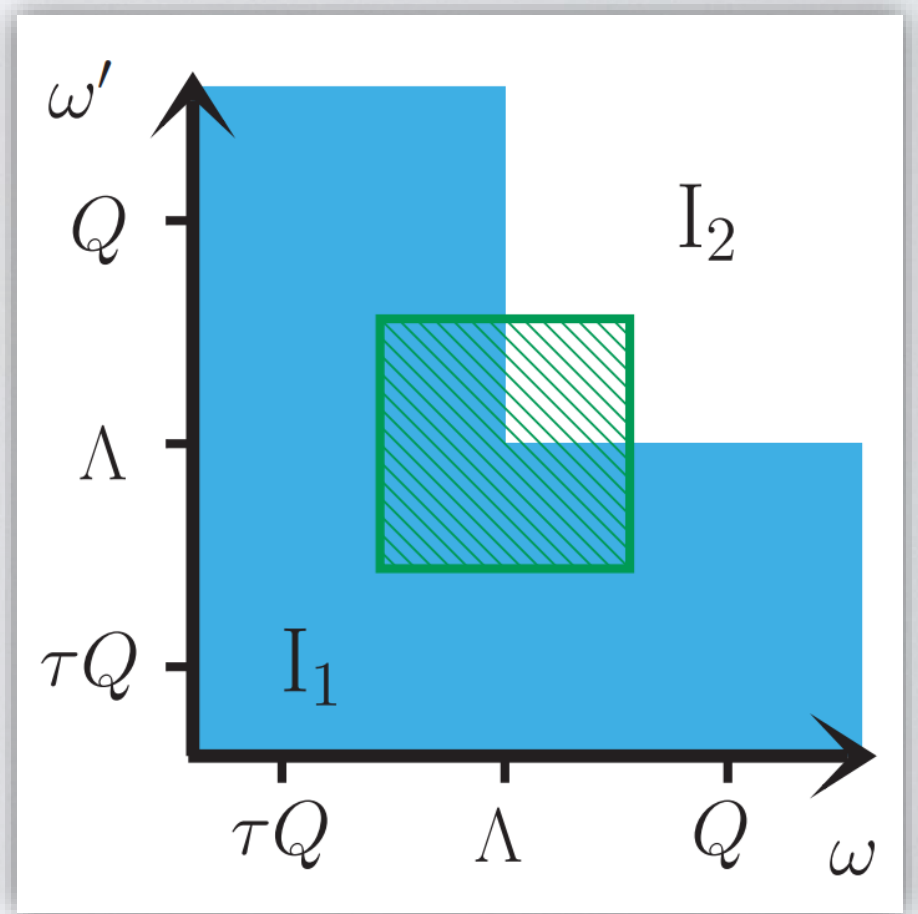
- Refactorization can be achieved as follows: define the asymptotic (scaleless) integral

$$0 = \frac{2C_F}{Q}\, f(\epsilon)\,|C^{\mathrm{A0}}(Q^2)|^2\, \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R)\widetilde{\mathcal{J}}_{c}^{(g)}(s_L)$$
$$\times \int_0^\infty d\omega d\omega' \,\frac{D^{\mathrm{B1}}(\omega Q)}{\omega}\,\frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'}\left[\!\!\left[\widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')\right]\!\!\right].$$

Then split the integral over the two regions

$$0 = I_1 + I_2.$$

by introducing a factorization parameter Λ, and subtract I_1 from the B-type term and I_2 from the A-type term, which makes both endpoint-finite as $d \to 4$.



- One can then proceed with standard resummation methods.

OFF-DIAGONAL "GLUON" THRUST

- Refactorized factorization formula:

Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022

$$
\begin{aligned}
\frac{1}{\sigma_0}\frac{\widetilde{d\sigma}}{ds_R ds_L}|_{\mathrm{A-type}} = \frac{2C_F}{Q}\, f(\epsilon)\, |C^{\mathrm{A0}}(Q^2)|^2\, \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R) \int_0^\infty d\omega d\omega' \\
\times \Big\{ \widetilde{\mathcal{J}}_c(s_L,\omega,\omega')\, \widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega') \\
- \theta(\omega-\Lambda)\theta(\omega'-\Lambda)\, [\![\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')]\!]\, [\![\widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')]\!] \\
+ \widetilde{\widetilde{\mathcal{J}}}_c(s_L,\omega,\omega')\, \widetilde{\widetilde{S}}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega') \Big\},
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\sigma_0}\frac{\widetilde{d\sigma}}{ds_R ds_L}\Big|_{\substack{\mathrm{B-type}\\ \mathrm{i=i'=1}}} = \frac{2C_F}{Q^2}\, f(\epsilon)\, \widetilde{\mathcal{J}}_c^{(g)}(s_L)\, \widetilde{S}^{(g)}(s_R,s_L) \int_0^\infty dr dr' \\
\times \Big[\theta(1-r)\theta(1-r')\, C_1^{\mathrm{B1}*}(Q^2,r') C_1^{\mathrm{B1}}(Q^2,r)\, \widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r') \\
- [1-\theta(r-\Lambda/Q)\theta(r'-\Lambda/Q)] \\
\times\, [\![C_1^{\mathrm{B1}*}(Q^2,r')]\!]_0\, [\![C_1^{\mathrm{B1}}(Q^2,r)]\!]_0\, [\![\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r')]\!]_0 \Big].
\end{aligned}
$$

- *Λ* dependence cancels between the two terms. Each separately independent of *dim reg μ.*
- In principle valid to any log accuracy. At LL only the subtraction terms contribute do to the extra log from large ω/small r.

OFF-DIAGONAL "GLUON" THRUST

- D^{B1} appears as a universal coefficient that renormalizes soft quark emission. Its double logarithms are proportional to the change of colour charge of the collinear particles:

$$\langle g_c^a(p_c) q_{\overline{sc}}(p_{\overline{sc}})| \int d^4x\, T\big\{\bar{\chi}_c(0), \mathcal{L}_{\xi q}(x)\big\}\, |0\rangle = g_s \bar{u}(p_{\overline{sc}}) t^a \not{\epsilon}_{c\perp}(p_c) \frac{i n_+ p_c}{p^2} \frac{\not{n}_-}{2} D^{\mathrm{B1}}(p^2)\,.$$

- It appear in Thrust, DIS, DY, as well as H → gg. Up to one loop:

$$D^{\mathrm{B1}}(p^2) = 1 + \frac{\alpha_s}{4\pi}\,(C_F - C_A)\left(\frac{2}{\epsilon^2} - 1 - \frac{\pi^2}{6}\right)\left(\frac{\mu^2}{-p^2 - i\varepsilon}\right)^{\epsilon} + \mathcal{O}(\alpha_s^2).$$

- D^{B1} has (non-local) anomalous dimension

$$\frac{d}{d\ln\mu} D^{\mathrm{B1}}(p^2) = \int_0^\infty d\hat{p}^2\, \gamma_D(\hat{p}^2, p^2) D^{\mathrm{B1}}(\hat{p}^2)\,,$$

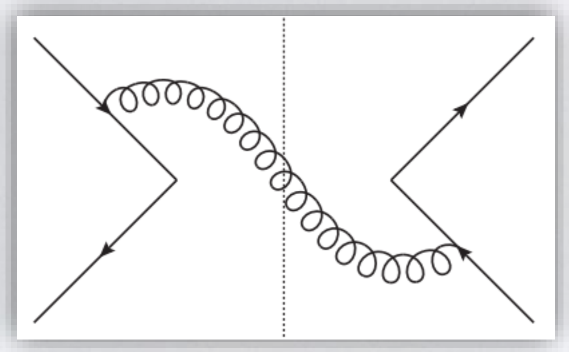
with

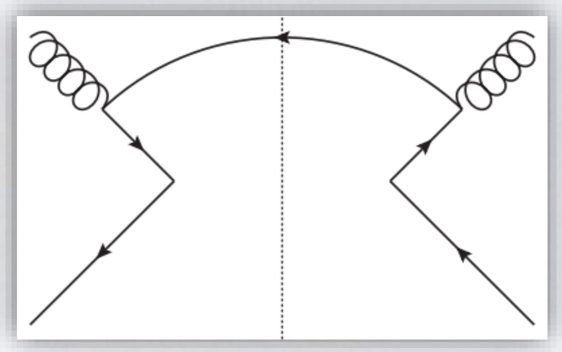
$$\gamma_D(\hat{p}^2, p^2) = \frac{\alpha_s(C_F - C_A)}{\pi}\, \delta(\hat{p}^2 - p^2) \ln\left(\frac{\mu^2}{-p^2 - i\varepsilon}\right) + \frac{\alpha_s}{\pi}\left(\frac{C_A}{2} - C_F\right) p^2 \left[\frac{\theta(\hat{p}^2 - p^2)}{\hat{p}^2(\hat{p}^2 - p^2)} + \frac{\theta(p^2 - \hat{p}^2)}{p^2(p^2 - \hat{p}^2)}\right]_+ \,.$$

Liu, Mecaj, Neubert, Wang 2019-22; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020; Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022.

DRELL YAN AT NLP

- D^{B1} is an example of new universal functions appearing at NLP.
- In general these functions are more involved to compute compared to their LP counterparts (not counting endpoint divergences), as they depend on more variables.
- On the other hands, refactorization conditions imposes additional constraints, as we have seen in case of thrust.
- Important to collect data on these functions. In this respect, in the past few years we have completed the calculation of all terms contributing to Drell-Yan at NLP, up to NNLO: this includes jet functions at NLO, and soft functions at NNLO.





$$\Delta^{dyn}_{q\bar{q},g\bar{q}}(z)|_{\mathrm{NLP}} \sim \sum_i \int \{d\omega\}\, J_i(\{\omega\})\, S_i(\{\omega\}).$$

qqb: J at one loop in
Beneke, Broggio, Jaskiewicz, LV, 2019

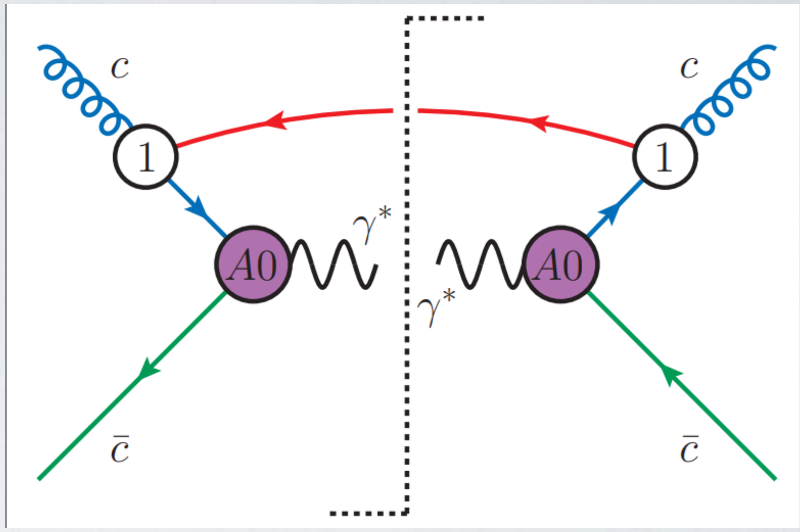
qg: J at one loop in
Broggio, Jaskiewicz, LV, to appear

qqb: S at two loops in
Broggio, Jaskiewicz, LV, 2021

qg: S at otwo loops in
Broggio, Jaskiewicz, LV, to appear

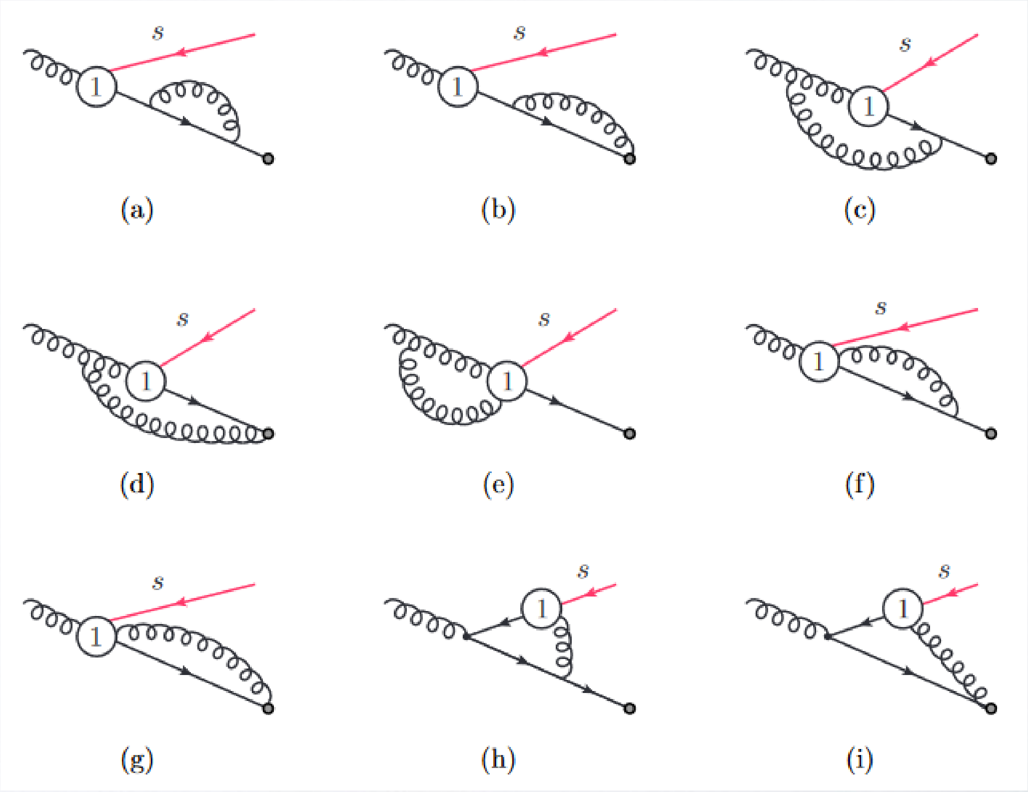
DRELL YAN AT NLP: QG CHANNEL

- Focus on the gqb channel: only a single term contribute, involving the emission of a soft quark:



$$\Delta_{g\bar{q}}|_{\mathrm{NLP}}(z) = 8H(Q^2) \int d\omega\, d\omega'\, G^*_{\xi q}(x_a n_+ p_A;\omega')\, G_{\xi q}(x_a n_+ p_A;\omega)\, S(\Omega,\omega,\omega').$$

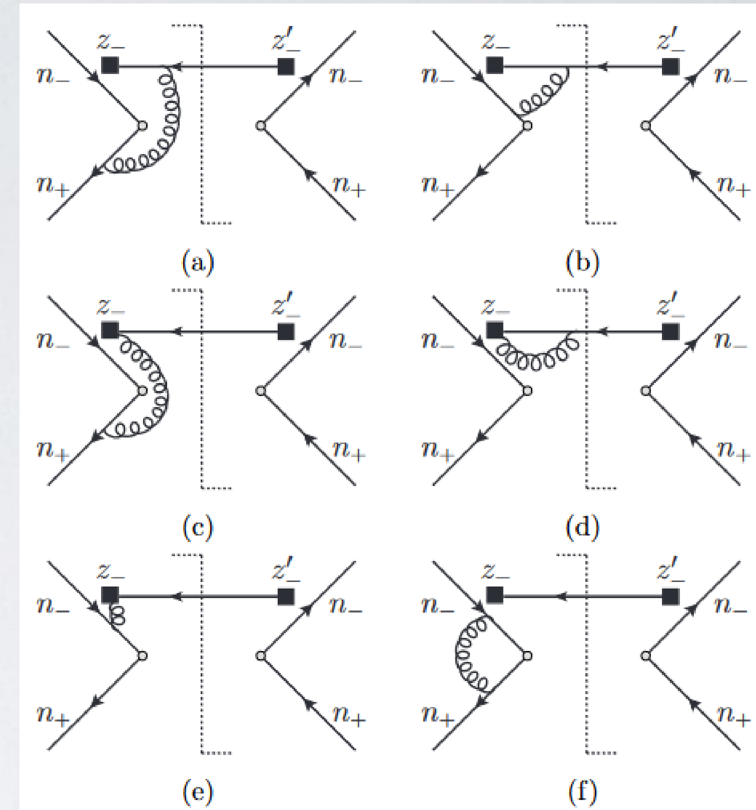
- We have calculated the collinear function at one loop: it corresponds to the coefficient D^{B1}, and it is thus known to two loops.



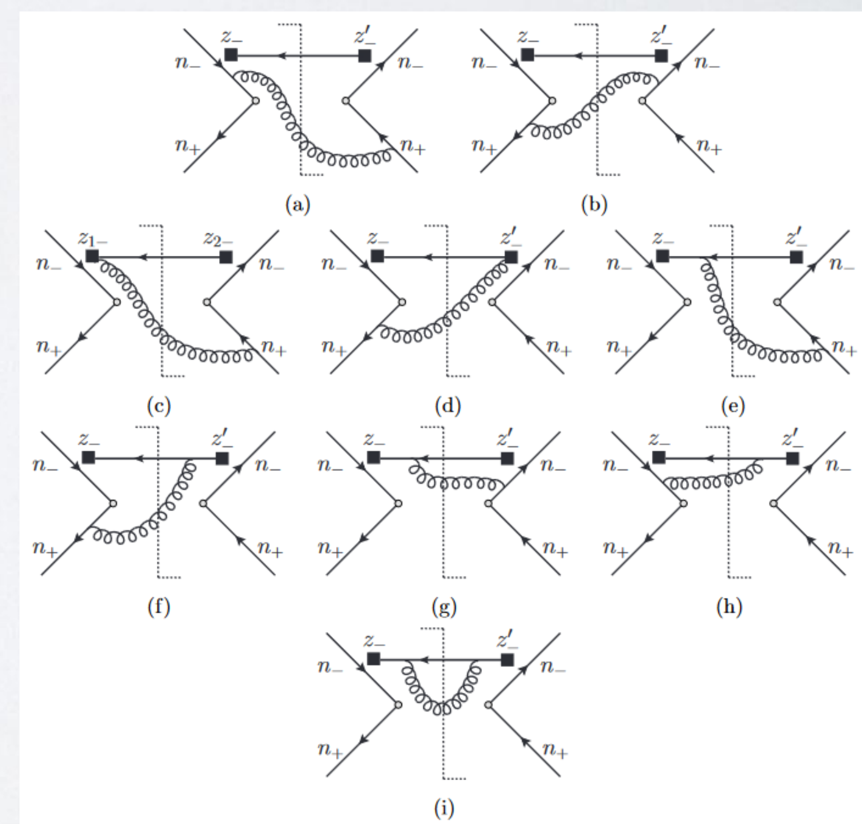
Broggio, Jaskiewicz, LV, to appear

DRELL YAN AT NLP: QG CHANNEL

- The soft function at two loops is more involved: there is a real-virtual contribution and a real-real term, that we calculate by means of standard Feynman parametrization and by means of differential equations:



$$
\begin{aligned}
S^{(2)1r1v}_{g\bar{q}}(\Omega,\omega,\omega') &= \frac{\alpha_s^2\, T_F}{(4\pi)^2}(2C_F - C_A)\frac{e^{2\epsilon\gamma_E}\,\Gamma[1+\epsilon]}{\epsilon\,\Gamma[1-\epsilon]} \\
\times\, \mathrm{Re}\Bigg\{ &\frac{1}{(-\omega)\omega'}\left[\frac{\omega+\omega'}{\omega'}\,{}_2F_1\left(1,1+\epsilon,1-\epsilon,\frac{\omega}{\omega'}\right) - 1\right]\left(\frac{\mu^4}{(-\omega)\omega'(\Omega-\omega')^2}\right)^{\epsilon}\theta(-\omega) \\
&+ \frac{2(\omega+\omega')}{\omega\omega'(\omega'-\omega)}\left(\frac{\mu^4}{(\omega'-\omega)^2(\Omega-\omega')^2}\right)^{\epsilon}\frac{\Gamma[1-\epsilon]^2}{\Gamma[1-2\epsilon]}\theta(\omega'-\omega)\Bigg\}\theta(\omega')\theta(\Omega-\omega'),
\end{aligned}
$$



$$
\begin{aligned}
S^{(2)2r0v}_{g\bar{q}}(\Omega,\omega,\omega') = \frac{\alpha_s^2 T_F}{(4\pi)^2}\Bigg\{ &C_F\frac{e^{2\epsilon\gamma_E}\Gamma[1-\epsilon]}{\epsilon^2}\frac{1}{\omega}\left[\frac{4}{\Gamma[1-3\epsilon]}\left(\frac{\mu^4}{\omega(\Omega-\omega)^3}\right)^{\epsilon}\right. \\
&\left.+ \frac{(4-\epsilon)\Gamma[2-\epsilon]}{(1-2\epsilon)\Gamma[1-2\epsilon]^2}\left(\frac{\mu^4}{\omega^2(\Omega-\omega)^2}\right)^{\epsilon}\right]\delta(\omega-\omega')\theta(\Omega-\omega)\theta(\omega) \\
&+ (C_A - 2C_F)\,\frac{2e^{2\epsilon\gamma_E}}{\epsilon\Gamma[1-2\epsilon]}\,\frac{\omega+\omega'}{\omega\omega'(\omega'-\omega)}\left(\frac{\mu^4}{\omega(\omega'-\omega)(\Omega-\omega')^2}\right)^{\epsilon} \\
&\times\left[{}_2F_1\left(1,-\epsilon,1-\epsilon,\frac{\omega}{\omega-\omega'}\right) - 1\right]\theta(\omega)\theta(\omega')\theta(\omega'-\omega)\theta(\Omega-\omega')\Bigg\}.
\end{aligned}
$$

Broggio, Jaskiewicz, LV, to appear

DRELL YAN AT NLP: QG CHANNEL

- Endpoint divergences: at NLO these arise for *ω→0*:

$$\Delta_{g\bar{q}}^{(1)}(z)|_{\rm NLP} = 2 \int d\omega\, d\omega'\, S^{(1)}(\Omega,\omega,\omega'),$$

  with

$$S_{g\bar{q}}^{(1)}(\Omega,\omega,\omega') = \frac{\alpha_s T_F}{4\pi} \frac{e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]} \frac{1}{\omega} \left(\frac{\mu^2}{\omega\,(\Omega-\omega)}\right)^{\epsilon} \delta(\omega-\omega')\, \theta(\Omega-\omega)\theta(\omega).$$

- For *ω→0* the (threshold-)soft momentum becomes (PDF)-soft-collinear:

$$p \sim Q(\lambda^2,\lambda^2,\lambda^2) \overset{\omega,\omega'\to 0}{\longrightarrow} Q(\lambda^2,\eta\lambda^2,\eta^2\lambda^2), \qquad \lambda^2 \sim 1-z, \qquad \eta \ll \lambda \ll 1.$$

- The endpoint divergence needs to be removed by a corresponding divergent term in the PDF, which needs to be factorized near *x →1*.
- Even in absence of such term, the subtraction procedure is quite similar to the case of gluon thrust: we can formally write a scaleless integral

$$[[\Delta_{g\bar{q}}^{\rm NLP}(z,\mu)]] = 8H(Q^2,\mu) \int d\omega\, d\omega'\, G^*_{\xi q}(Q;\omega';\mu)\, G_{\xi q}(Q;\omega;\mu)\, [[S(\Omega,\omega,\omega',\mu)]]\,,$$

  with

$$[[S_{g\bar{q}}^{(1)}(\omega,\omega')]] = \frac{\alpha_s T_F}{4\pi} \frac{e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]} \frac{1}{\omega} \left(\frac{\mu^2}{\omega\,\Omega}\right)^{\epsilon} \delta(\omega-\omega')\, \theta(\Omega)\theta(\omega).$$

DRELL YAN AT NLP: QG CHANNEL

- We can subtract the asymptotic contribution from both the "partonic cross section" and "PDF" contribution in the renormalized partonic cross section:

$$\Delta^{\rm ren}_{g\bar q}(z) = \Big[\left(\Gamma^{-1}\right)^{\rm LP}_{gg} \otimes \left(\Gamma^{-1}\right)^{\rm LP}_{qq} \otimes \Delta^{\rm bare,\,NLP}_{g\bar q}\Big](z) + \Big[\left(\Gamma^{-1}\right)^{\rm NLP}_{qg} \otimes \left(\Gamma^{-1}\right)^{\rm LP}_{qq} \otimes \Delta^{\rm bare,\,LP}_{q\bar q}\Big](z),$$

to obtain

$$\begin{aligned}\Delta^{\rm ren}_{g\bar q}(z) = H(Q^2)\Big\{ & 8\left(\Gamma^{-1}\right)^{\rm LP}_{gg} \otimes \left(\Gamma^{-1}\right)^{\rm LP}_{qq} \otimes \int_0^\infty d\omega\, d\omega'\, G^*_{\xi q}(\omega')\, G_{\xi q}(\omega) \\ & \times \Big(S(\omega,\omega') - \theta(\delta-\omega)\theta(\delta-\omega')[[S(\omega,\omega')]]\Big) \\ & + \left(\Gamma^{-1}\right)^{\rm NLP}_{qg} \otimes \left(\Gamma^{-1}\right)^{\rm LP}_{qq} \otimes Q\, S^{\rm LP}_{\rm DY} - 8\left(\Gamma^{-1}\right)^{\rm LP}_{gg} \otimes \left(\Gamma^{-1}\right)^{\rm LP}_{qq} \\ & \otimes \int_0^\infty d\omega\, d\omega' \Big[1 - \theta(\delta-\omega)\theta(\delta-\omega')\Big] G^*_{\xi q}(\omega')\, G_{\xi q}(\omega)\, [[S(\omega,\omega')]] \Big\}.\end{aligned}$$

DRELL YAN AT NLP: QG CHANNEL

- It can be shown that the two terms are separately finite, the cutoff dependence cancels between the two terms, and the sum reproduces the PDF-renormalized cross section:
- Partonic cross section contribution:

$$
\begin{aligned}
2\int_0^\infty d\omega\, d\omega' \Big[S^{(1)}(\omega,\omega') - \theta(\delta-\omega)\theta(\delta-\omega')[[S^{(1)}(\omega,\omega')]]\Big] &= 2\int_\delta^\infty d\omega\, \hat{S}^{(1)}(\omega) \\
= \frac{\alpha_s T_F}{4\pi}\bigg\{4\ln(1-z) + 2\ln\left(\frac{\mu^2}{\delta\, Q(1-z)}\right) + \epsilon\bigg[\frac{\pi^2}{3} &- 4\ln^2(1-z) \\
&+ \ln^2\left(\frac{\mu^2}{\delta\, Q(1-z)}\right)\bigg] + \mathcal{O}(\epsilon^2)\bigg\}.
\end{aligned}
$$

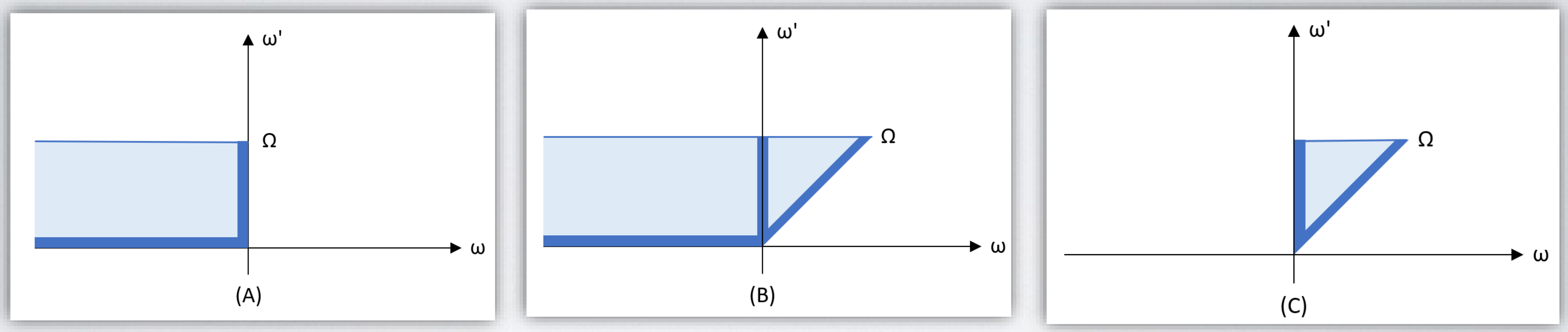
- PDF contribution:

$$
\begin{aligned}
&\frac{\alpha_s}{4\pi}\frac{P_{qg}^{(0),\mathrm{NLP}}(z)}{\epsilon} - 2\int_0^\infty d\omega\, d\omega' \Big[1 - \theta(\delta-\omega)\theta(\delta-\omega')\Big]\,[[S^{(1)}(\omega,\omega')]] \\
&= \frac{\alpha_s}{4\pi}\frac{P_{qg}^{(0),\mathrm{NLP}}(z)}{\epsilon} - 2\int_\delta^\infty d\omega\,[[\hat{S}^{(1)}(\omega)]] \\
&= \frac{\alpha_s T_F}{4\pi}\bigg\{-2\ln\left(\frac{\mu^2}{\delta\, Q(1-z)}\right) + \epsilon\bigg[\frac{\pi^2}{6} - \ln^2\left(\frac{\mu^2}{\delta\, Q(1-z)}\right)\bigg] + \mathcal{O}(\epsilon^2)\bigg\}.
\end{aligned}
$$

DRELL YAN AT NLP: QG CHANNEL

- At NNLO the structure of endpoint divergences appear more involved:

$$
\begin{aligned}
S^{(2)}(\omega,\omega') = {} & \hat{S}^{(2)}(\omega)\,\delta(\omega-\omega')\,\theta(\Omega-\omega)\theta(\omega) \\
& + S^{(2A)}(\omega,\omega')\,\theta(-\omega)\theta(\omega')\theta(\Omega-\omega') \\
& + S^{(2B)}(\omega,\omega')\,\theta(\omega'-\omega)\theta(\omega')\theta(\Omega-\omega') \\
& + S^{(2C)}(\omega,\omega')\,\theta(\omega)\theta(\omega')\theta(\omega'-\omega)\theta(\Omega-\omega').
\end{aligned}
$$



- A formally finite soft function can still be defined, but we need consistency conditions with PDF factorization for validation.

CONCLUSION

- In the past few years, a lot of work has been devoted to understand the structure of large logarithms at next-to-leading power.

- With some optimism, we have now frameworks (SCET and diagrammatic approach) which allows us to resum LLs at NLP, at least for color singlet processes.

- The next task is to understand whether this framework can be applied straightforwardly beyond NLP LLs.