Precision Standard Model Phenomenology at N3LO and Beyond

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RADCOR 2023 - Crieff, 30 May 2023

Based on:

"Collinear expansion for color singlet cross sections"
M.Ebert, B.Mistlberger, **GV** [2006.03055]

"TMD PDFs at N3LO"
M.Ebert, B.Mistlberger, **GV** [2006.05329]

"N-jettiness beam functions at N3LO"
M.Ebert, B.Mistlberger, **GV** [2006.03056]

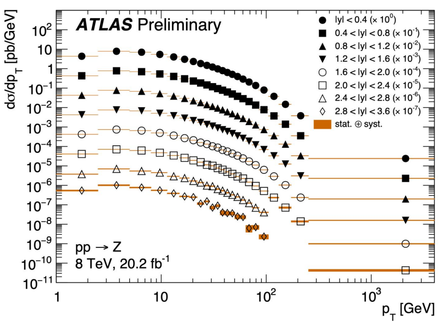
"The Four-Loop Rapidity Anomalous Dimension and Event Shapes to 4th Logarithmic Order"
C.Duhr, B.Mistlberger, **GV** [2205.02242]

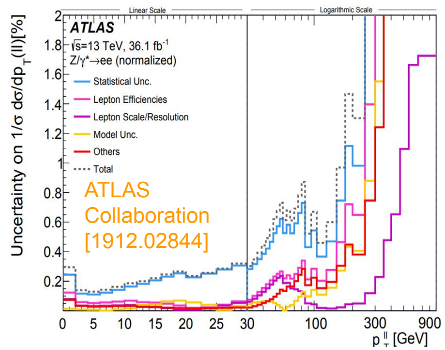
B.Mistlberger, **GV** [to appear]

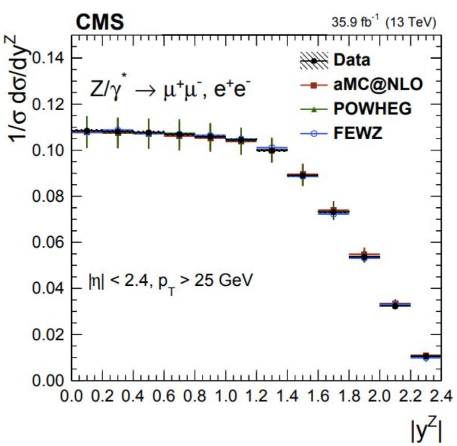
Testing the SM at Percent Level Accuracy

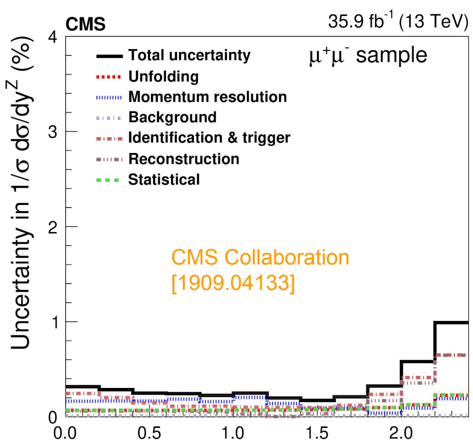
Astonishing level of precision in experimental measurements of key benchmark processes.

Example: normalized differential distributions in Drell-Yan measured with few **per-mille** level accuracy



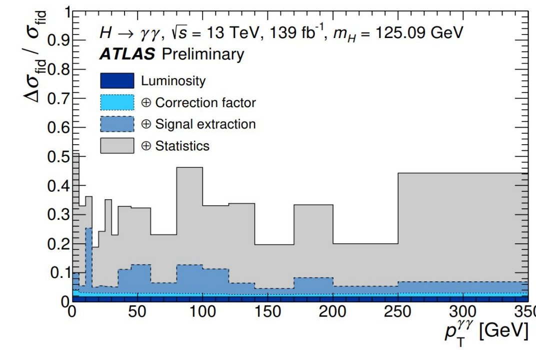




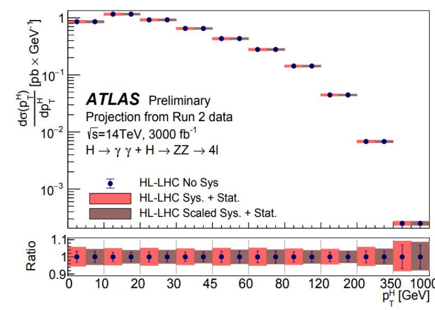


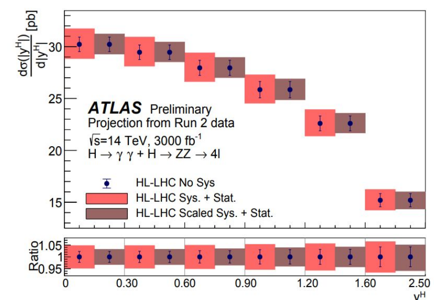
...and plethora of very precise differential distributions from LEP, future EIC measurements, possible future colliders, etc...

Higgs measurements at the moment are limited by statistics

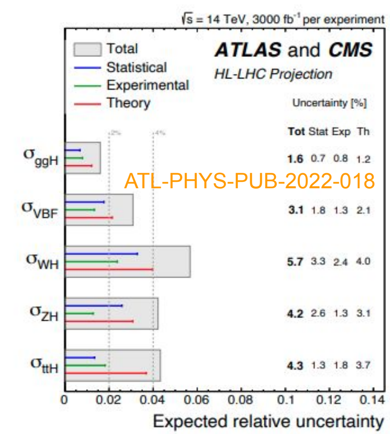


...but statistics **will improve dramatically** with HL LHC...





With **percent level measurement** of Higgs distributions, theory errors are projected to be a major limiting factor for Higgs precision program



Standard Model Phenomenology at percent level

We should aim at comparable precision from the theory side!

Non Perturbative   Perturbative

$$\sigma_{pp\to X} \sim \int f_a(x_1) f_b(x_2) \otimes \hat{\sigma}_{ab\to X}$$

$$\hat{\sigma}_{ab\to X} = \underbrace{\sigma_0}_{\text{LO}} + \underbrace{\alpha_s\sigma_1}_{\text{NLO}} + \underbrace{\alpha_s^2\sigma_2}_{\text{NNLO}} + \underbrace{\alpha_s^3\sigma_3}_{\text{N}^3\text{LO}} + \dots$$

N3LO corrections (or at least *good* estimates of them) will be **necessary for percent level** phenomenology

"The Path Forward to N3LO"
Snowmass Whitepaper
[Caola, Chen, Duhr, Liu, Mistlberger, Petriello, GV, Weinzierl]

CAVEAT!
Often times convergence turns out to be slower than naive estimate
=> **N3LO gives few <u>percent</u> (not per-mille) shift**

	Q [GeV]	$\delta\sigma^{\text{N}^3\text{LO}}$	$\delta\sigma^{\text{NNLO}}$
$gg \to$ Higgs	m_H	3.5%	30%
$b\bar{b} \to$ Higgs	m_H	-2.3%	2.1%
NCDY	30	-4.8%	−0.34%
	100	-2.1%	−2.3%
CCDY(W^+)	30	-4.7%	−0.1%
	150	-2.0%	−0.1%
CCDY(W^-)	30	-5.0%	−0.1%
	150	-2.1%	−0.6%

n3loxs [Baglio, Duhr, Mistlberger, Szafron '22]

Predictions for Differential Cross Sections: IR singularities

$$\sigma = f_1 \circ f_2 \circ \boxed{\int d\Phi} |M|^2$$

- Cross sections require **integration over phase space**
- Complexity of infrared singularities grows with loop order
- Extremely challenging to systematize their treatment order by order
- Use EFT methods to systematize study of collinear and soft radiation at the cross section level
- Obvious applications: building universal counterterms (e.g. EFT-based subtractions) and improve resummation

Singular Region of LHC Observables

- For several observables, **singular region** understood at all orders using cross section level hard, collinear and soft functions. Classical example:

Leading power factorization for __Transverse-Momentum Distributions__ in pp

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}^2\vec{q}_T} = \sigma_0 \sum_{i,j} \underbrace{\boxed{H_{ij}(Q^2,\mu)}}_{\text{Hard Function}} \int \mathrm{d}^2\vec{b}_T\, e^{\mathrm{i}\,\vec{q}_T\cdot\vec{b}_T}\, \tilde{B}_i\Big(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\Big)\, \tilde{B}_j\Big(x_2^B, b_T, \mu, \frac{\nu}{\omega_b}\Big)\, \underbrace{\boxed{\tilde{S}(b_T,\mu,\nu)}}_{\text{Soft Function}}$$

q_T **Beam Functions**

- At each order in perturbation theory:
  - Log-enhanced terms (predicted by RGE/anomalous dims. and lower order results)
  - **Fixed order terms** (non-log enhanced terms, boundary values of RG equations, need explicit f.o. calculation)

Fixed order terms for Hard and Soft functions (for color singlet processes) are **constants.** => Several results at N3LO since a long time [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]

[Li, v.Manteuffel, Schabinger, Zhu '14] [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger '14]

[Li, Zhu '16] and more…

This talk

Beam function fixed order terms are **full functions** (of the collinear splitting variable):

- Significantly more complicated objects
- Recent progress at N3LO

[Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, **GV** '20] [Mistlberger, **GV** to appear]

[Ebert, Mistlberger, **GV** '20] [Baranowski, Behring, Melnikov, Tancredi, Wever '22]

- Enabled many of the LHC N3LO distributions

Beam Functions

- **Beam Functions** can be understood as **generalization** of **Parton Distribution Functions** (PDFs)

Longitudinal momentum fraction

PDF: $$f_q(x) = \langle p_n | \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \left[\delta(p^- - \bar{n}\cdot\mathcal{P}) \chi_n \right] | p_n \rangle$$

Longitudinal momentum fraction

Beam Function: $$B_q(x, q_T) = \langle p_n | \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \left[\delta(p^- - \bar{n}\cdot\hat{\mathcal{P}}) \delta(q_T - \hat{k}_T) \chi_n \right] | p_n \rangle$$

Additional observable (q_T, beam thrust, etc...)

- Beam functions are **non-perturbative** objects!

  However, in **perturbative regime** of the observable $\mathcal{T} \gg \Lambda_{\rm QCD}$, they can be matched perturbatively onto PDF, via an **observable dependent matching kernel** $\mathcal{I}_{ij}(x, \mathcal{T}, \mu)$

$$B_i(x, \mathcal{T}, \mu) = \sum_j \mathcal{I}_{ij}(x, \mathcal{T}, \mu) \otimes_x f_j(x, \mu) \times \left[1 + \mathcal{O}(\Lambda_{\rm QCD}/\mathcal{T}) \right]$$

Beam Functions

- **Beam Functions** can be understood as **generalization** of **Parton Distribution Functions** (PDFs)

Bare **matching kernel** can be calculated using **collinear expansion of differential partonic cross sections for LHC processes!**

$$\mathcal{I}_{ij}^{\mathrm{bare}}(z,\mathcal{T}) = \int_0^1 \mathrm{d}x \int_0^\infty \mathrm{d}w_1 \mathrm{d}w_2\, \delta\left[z-(1-w_1)\right] \times \lim_{\text{strict } n-\text{coll.}} \left\{ \delta\left[\mathcal{T}-\mathcal{T}(Q,Y,w_1,w_2,x)\right] \frac{\mathrm{d}\eta_{j\bar{i}}}{\mathrm{d}Q^2 \mathrm{d}w_1 \mathrm{d}w_2 \mathrm{d}x} \right\}$$

$$w_1 = -\frac{\bar{n}\cdot k}{\bar{n}\cdot p_1}, \qquad w_2 = -\frac{n\cdot k}{n\cdot p_2} \qquad x = \frac{k^2}{(\bar{n}\cdot k)(n\cdot k)} = 1 - \frac{\vec{k}_\perp^2}{(\bar{n}\cdot k)(n\cdot k)}$$

"Collinear expansion for color singlet cross sections"
[2006.03055]
Ebert, Mistlberger, **GV**

- Beam functions are **non-perturbative** objects!

However, in **perturbative regime** of the observable $\mathcal{T} \gg \Lambda_{\mathrm{QCD}}$, they can be matched perturbatively onto PDF, via an **observable dependent matching kernel** $\mathcal{I}_{ij}(x,\mathcal{T},\mu)$

$$B_i(x,\mathcal{T},\mu) = \sum_j \mathcal{I}_{ij}(x,\mathcal{T},\mu) \otimes_x f_j(x,\mu) \times \left[1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/\mathcal{T})\right]$$

Beam Functions at N3LO

Collinear expansion of the partonic cross section for Drell Yan and Higgs at N3LO **<u>differential</u> in (Q_T, $\boldsymbol{\tau}$, z)**

project to $\boldsymbol{\tau}$

project to $\mathbf{q_T}$

$$B_a(t_a, x_1^B, \mu)$$

$$\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right)$$

"N-Jettiness Beam Functions at N3LO"

M.Ebert, B.Mistlberger, **GV**
[2006.03056]

- Quark $\boldsymbol{\tau}$ **beam functions**
  (Quark N-Jettiness Beam Function)
- Gluon $\boldsymbol{\tau}$ **beam functions**
  (Gluon N-Jettiness Beam Function)

"Transverse Momentum Dependent PDFs at N3LO"

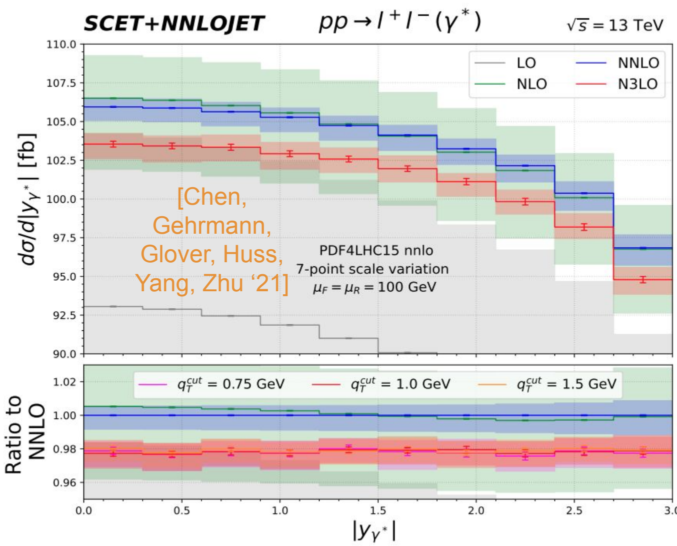
M.Ebert, B.Mistlberger, **GV**
[2006.05329]

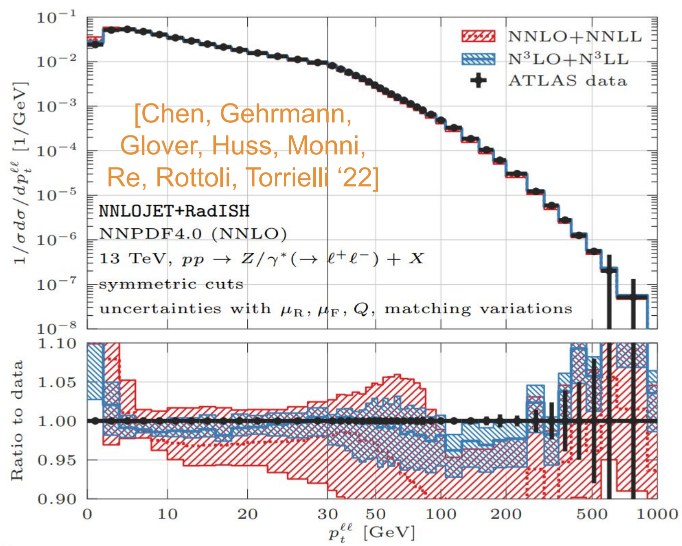
- Quark **TMDPDF**
  (Quark q_T Beam Function)
- Unpolarized **Gluon TMDPDF**
  (Gluon q_T Beam Function)

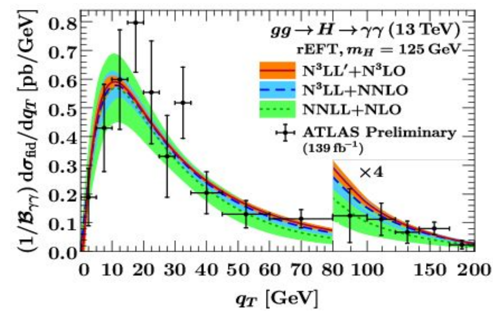
See also
[Luo, Yang, Zhu, Zhu '19]

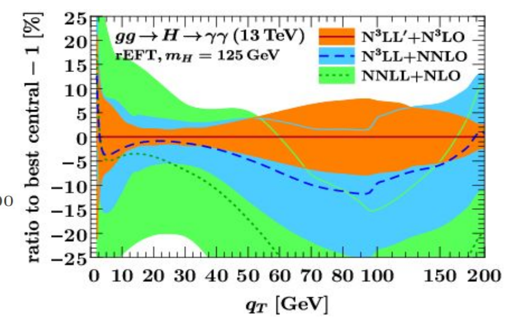
Slicing at N3LO

- **q_T beam functions** at N3LO were **last missing ingredient** for:
  - q_T subtraction for differential and fiducial Drell-Yan and Higgs production at N3LO
  - q_T resummation at N3LL`
- Many new exciting phenomenological results at N3LO employing them!

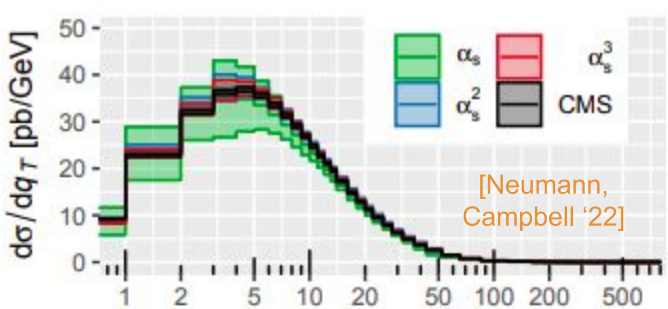








[Billis, Dehnadi, Ebert, Michel, Tackmann '21]



And many more:

[Ju, Schönherr '21]

[Camarda, Cieri, Ferrera '21]

[Re, Rottoli, Torrielli '21]

…

Approximations for LHC Cross Sections at N3LO using Collinear Expansions

B.Mistlberger, GV

[to appear]

See Bernhard's talk

Singular Limit of Rapidity Distributions

Fixed order rapidity distribution hadronic cross section at the LHC

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = \int \frac{\mathrm{d}z_a}{z_a}\frac{\mathrm{d}z_b}{z_b}\, \hat{\sigma}_{ij}(z_a, z_b)\, f_i\left(\frac{x_a}{z_a}\right) f_j\left(\frac{x_b}{z_b}\right)$$

$$x_a = \frac{Q}{E_{\mathrm{cm}}} e^{+Y}, \quad x_b = \frac{Q}{E_{\mathrm{cm}}} e^{-Y}, \quad \tau = x_a x_b$$

$x_a \to 1$

and

$x_b \to 1$

Threshold Expansion

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = H_{ij}(Q^2) \int \mathrm{d}k^- \,\mathrm{d}k^+ S(k^-, k^+) \times f_i^{\mathrm{thr}}\left[x_a\left(1 + \frac{k^-}{Qe^{+Y}}\right)\right] f_j^{\mathrm{thr}}\left[x_b\left(1 + \frac{k^+}{Qe^{-Y}}\right)\right]$$

$x_a \to 1$

or

$x_b \to 1$

Collinear Expansion

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1 \mathrm{d}x_2} = H_{ij}(Q^2, \mu) \boxed{B_i^Y(Q^2 \bar{x}_2, x_1, \mu)} \underset{x_1;\bar{x}_2}{\otimes} S(Q^2 \bar{x}_1 \bar{x}_2, \mu) \underset{x_2;\bar{x}_1}{\otimes} \boxed{B_j^Y(Q^2 \bar{x}_1, x_2, \mu)}$$

[Mistlberger, **GV**] (to appear)

Rapidity Beam Functions

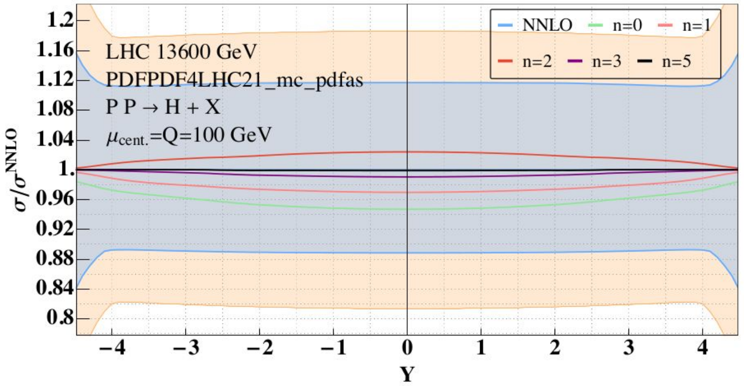
See also: $$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = H_{ij}\left[f_i^{\mathrm{thr}} \otimes \boxed{\tilde{B}_j} + \boxed{\tilde{B}_i} \otimes f_j^{\mathrm{thr}} - S \otimes f_i^{\mathrm{thr}} f_j^{\mathrm{thr}}\right]$$

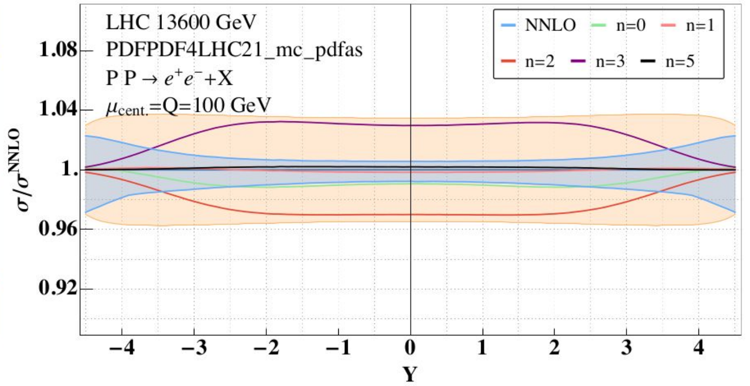
"Generalized Threshold Factorization"

[Lustermans, Michel, Tackmann '19]

Approximation of Rapidity Distributions

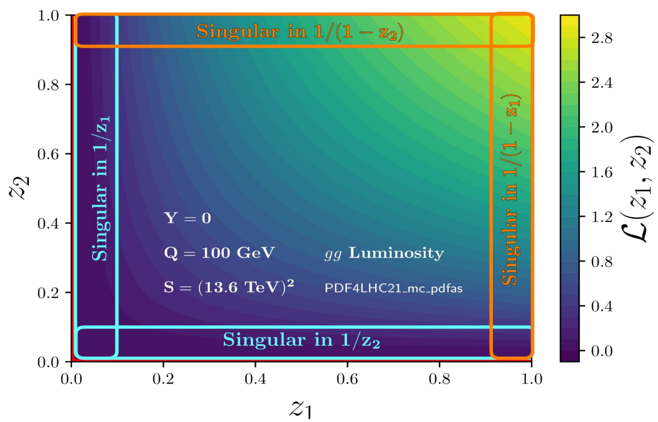
- Tests at **NNLO**: Very good approximation both for Higgs and DY

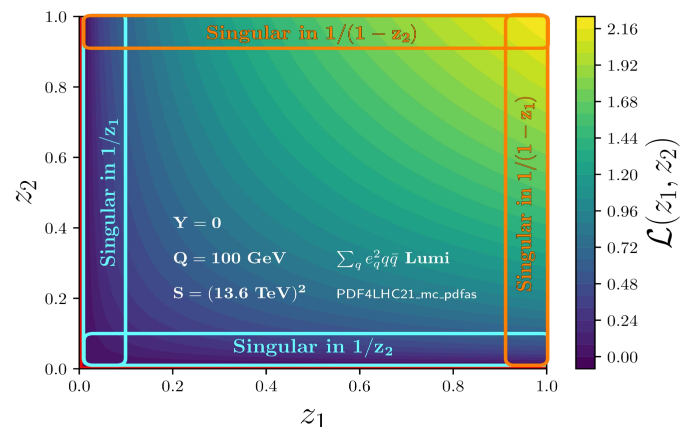




- Significant improvement over threshold thanks to capturing all collinear distributions

- Convolution structure between two rapidity beam functions allow description of power suppressed channels





- Great convergence away from singular limit thanks to PDFs suppression of other singular regions

$$\mathcal{L}_{\text{Higgs}}(z_1, z_2) \sim \frac{1}{z_1}\frac{1}{z_2} f_g\left(\frac{\xi_1}{z_1}\right) f_g\left(\frac{\xi_2}{z_2}\right) \qquad \mathcal{L}_{\text{DY}}(z_1, z_2) \sim \frac{1}{z_1}\frac{1}{z_2}\sum_{i\neq q} f_i\left(\frac{\xi_1}{z_1}\right) f_{-i}\left(\frac{\xi_2}{z_2}\right)$$

Singular Limit of Rapidity Distributions

$x_a \to 1$

or

$x_b \to 1$

Collinear Expansion

Hard Function

Rapidity Beam Functions

$$\frac{d\sigma}{dx_1 dx_2} = \boxed{H_{ij}(Q^2,\mu)} \boxed{B_i^Y\left(Q^2\bar{x}_2, x_1, \mu\right)} \underset{x_1;\bar{x}_2}{\otimes} S\left(Q^2\bar{x}_1\bar{x}_2,\mu\right) \underset{x_2;\bar{x}_1}{\otimes} \boxed{B_j^Y\left(Q^2\bar{x}_1, x_2, \mu\right)}$$

[Mistlberger, **GV**] (to appear)

Process dependence enters only through hard function (fully virtual corrections)

=> **Universality of collinear and soft objects for ALL color singlet processes**

=> **GOAL:** create approximation/estimate of N3LO corrections to distributions for processes where full kinematic calculation is currently out of reach (e.g. di-boson)

Singular Limit of Rapidity Distributions

$x_a \to 1$

or

$x_b \to 1$

Collinear Expansion

[Mistlberger, **GV**] (to appear)

Hard Function

Rapidity Beam Functions

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– What is known? –

Soft function:

- Closely related to inclusive threshold SF
- **Known to N3LO** [Anastasiou, Dulat, Duhr, Mistlberger '13] [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger '14] [Li, v.Manteuffel, Schabinger, Zhu '14]

Rapidity Beam Functions:

- NNLO known [Lustermans, Michel, Tackmann '19] [Gaunt, Stahlhofen '20]
- Significant more complicated than TMD and N-Jettiness BFs: GPLs already at NNLO
- Extracted from double differential BF [Gaunt, Stahlhofen '14] [Gaunt, Stahlhofen '20]

Singular Limit of Rapidity Distributions

$x_a \to 1$

or

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Collinear Expansion

[Mistlberger, GV] (to appear)

Hard Function

Rapidity Beam Functions

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– What is known?

Soft function:

•

•

[Li, v.Manteuffel, Schabinger, Zhu '14]

Push formalism to N3LO calculating **Rapidity BF to N3LO for quark and gluon** induced processes by using method of **collinear expansion of cross sections!**

Rapidity Beam Functions:

• N ...mans, Michel, Tackmann '19] [Gaunt, Stahlhofen '20]

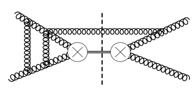
$$I_{ij}^Y(\xi_2;\xi_1) = \int_0^1 dx \int_0^\infty dw_1 dw_2\, \delta\left[\xi_2 - (1-w_2)\right] \times \lim_{\text{strict } n-\text{coll.}} \left\{ \delta\left[\bar{\xi}_1 - \frac{2-w_2-w_2 x}{2(1-w_2)} w_1\right] \frac{d\eta_{j\bar{i}}}{dQ^2 dw_1 dw_2 dx} \right\}$$

Fully differential calculation in collinear limit @N3LO

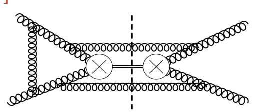
- We calculated the **collinear expansion of the partonic cross section** for DY and Higgs @N3LO **differential in** z_a, z_b

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = \int \frac{\mathrm{d}z_a}{z_a} \frac{\mathrm{d}z_b}{z_b} \, \hat{\sigma}_{ij}(z_a, z_b) \, f_i\left(\frac{x_a}{z_a}\right) f_j\left(\frac{x_b}{z_b}\right)$$

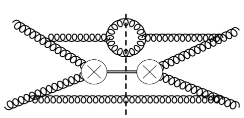
  - O(100k) Feynman diagrams
  - Use reverse unitarity for phase space
  - Collinear expansion applied before IBP reduction → get basis of Collinear MIs
  - RVV: known in full kinematics [Duhr, Gehrmann] [Duhr, Gehrmann, Jaquier] [Dulat, Mistlberger]



  - RRV: **270** Collinear MIs



  - RRR: **410** Collinear MIs



  - **1 scale problem** (dependence on small scale in collinear limit trivializes), **algebraic dependence** on variable and **elliptic letters**, e.g. $\sqrt{z_a^4 + 2z_a^3 + 7z_a^2 + 6z_a + 1}$

  - Obtained analytic solution by deriving **differential equations** for MIs and putting system in **canonical form.**
  - Constructed **dLog integrands** on different cut surfaces for sectors involving algebraic and elliptic letters
  - Boundaries from inclusive N3LO soft integrals and double soft behaviour [Anastasiou, Duhr, Dulat, Mistlberger]
  - From this we obtain bare results. SCET I observable so no rapidity divergences. Standard Laplace renormalization as for N-Jettiness BF

Going Beyond 3 Loops: Resummation at N3LL’ and N4LL

Resummation for Rapidity Distributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = H_{ij}(Q^2,\mu)B_i^Y(Q^2\bar{x}_2, x_1,\mu) \underset{x_1;\bar{x}_2}{\otimes} S(Q^2\bar{x}_1\bar{x}_2,\mu) \underset{x_2;\bar{x}_1}{\otimes} B_j^Y(Q^2\bar{x}_1, x_2,\mu)$$

RG Equations of each object

Related to Collinear Anomalous Dimension from form factors

Hard Function Evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln H_i(Q,\mu) = \gamma_H^i(Q,\mu)$$

0-Jettiness/Thrust Jet anomalous dimension

Rapidity Beam Function Evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln \tilde{B}_i^Y(\kappa_2, x_1,\mu) = \gamma_B^i(\kappa_2,\mu)$$

Threshold anomalous dimension

Soft Function Evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln \tilde{S}(Q^2\kappa_1\kappa_2,\mu) = -\gamma_{th}^i(Q^2\kappa_1\kappa_2,\mu)$$

$$\boxed{\gamma_i(t,\mu) \sim \Gamma_{\text{cusp}}[\alpha_s(\mu)]\log\frac{t}{\mu} + \gamma_i[\alpha_s(\mu)]}$$

Resummation for Rapidity Distributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = H_{ij}(Q^2,\mu) B_i^Y(Q^2\bar{x}_2, x_1, \mu) \otimes_{x_1;\bar{x}_2} S(Q^2\bar{x}_1\bar{x}_2, \mu) \otimes_{x_2;\bar{x}_1} B_j^Y(Q^2\bar{x}_1, x_2, \mu)$$

Boundary functions: known to N3LO
(I've just shown their calculation)

Hard Function Evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln H_i(Q,\mu) = \gamma_H^i(Q,\mu)$$

Related to Collinear Anomalous Dimension from form factors
Known at 4 loops
[Manteuffel, Panzer, Schabinger '21]

Rapidity Beam Function Evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln \tilde{B}_i^Y(\kappa_2, x_1, \mu) = \gamma_B^i(\kappa_2,\mu)$$

0-Jettiness/Thrust Jet anomalous dimension
Known at 4 loops
[Duhr, Mistlberger, **GV** '22]
[Moult, Zhu, Zhu '22]

Soft Function Evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln \tilde{S}(Q^2\kappa_1\kappa_2, \mu) = -\gamma_{th}^i(Q^2\kappa_1\kappa_2,\mu)$$

Threshold anomalous dimension
Known at 4 loops
[Duhr, Mistlberger, **GV** '22]
[Moult, Zhu, Zhu '22]
[Das, Moch, Vogt '19]

$$\boxed{\gamma_i(t,\mu) \sim \Gamma_{\text{cusp}}[\alpha_s(\mu)] \log\frac{t}{\mu} + \gamma_i[\alpha_s(\mu)]}$$

Resummation for Rapidity Distributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = H_{ij}(Q^2,\mu)\boxed{B_i^Y(Q^2\bar{x}_2,x_1,\mu)} \otimes_{x_1;\bar{x}_2} \boxed{S(Q^2\bar{x}_1\bar{x}_2,\mu)} \otimes_{x_2;\bar{x}_1} \boxed{B_j^Y(Q^2\bar{x}_1,x_2,\mu)}$$

Boundary functions: known to N3LO
(I've just shown their calculation)

Log accuracy dictated by perturbative order of ingredients

Accuracy	Boundaries	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_i(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	1-loop
NLL	Tree level	2-loop	1-loop	2-loop
NLL$'$	1-loop	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop
NNLL$'$	2-loop	3-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	4-loop
N^{3}LL$'$	3-loop	4-loop	3-loop	4-loop
N^{4}LL	3-loop	5-loop	4-loop	5-loop

$$_I(Q,\mu)$$

Related to Collinear Anomalous Dimension from form factors
Known at 4 loops
[Manteuffel, Panzer, Schabinger '21]

$$) = \gamma_B^i(\kappa_2,\mu)$$

0-Jettiness/Thrust Jet anomalous dimension
Known at 4 loops
[Duhr, Mistlberger, **GV** '22]
[Moult, Zhu, Zhu '22]

$$-\gamma_{th}^i(Q^2\kappa_1\kappa_2,\mu)$$

Threshold anomalous dimension
Known at 4 loops
[Duhr, Mistlberger, **GV** '22]
[Moult, Zhu, Zhu '22]
[Das, Moch, Vogt '19]

Resummation for Rapidity Distributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = H_{ij}(Q^2,\mu)\boxed{B_i^Y(Q^2\bar{x}_2,x_1,\mu)}\otimes_{x_1;\bar{x}_2}\boxed{S(Q^2\bar{x}_1\bar{x}_2,\mu)}\otimes_{x_2;\bar{x}_1}\boxed{B_j^Y(Q^2\bar{x}_1,x_2,\mu)}$$

Boundary functions: known to N3LO
(I've just shown their calculation)

Log accuracy dictated by perturbative order of ingredients

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NLL′	1-loop	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop
NNLL′	2-loop	3-loop	2-loop	3-loop
N^3LL	2-loop	4-loop	3-loop	4-loop
N^3LL′	3-loop	4-loop	3-loop	4-loop
N^4LL	3-loop	5-loop	4-loop	5-loop

All ingredients known for resummation at **N3LL'**

For N4LL, all anomalous dimensions known, but **need N3LO DGLAP** for consistent perturbative matching to PDF.

(Note: 5-loop cusp is numerically irrelevant. Approx of [Herzog, Moch, Ruijl,Ueda, Vermaseren, Vogt '18] enough for any pheno)

Resummation at N4LL for event shapes

Unrelated to rapidity distributions…

We achieved **N4LL accuracy** for an event shapes observable in electron-positron annihilation, the **Energy-Energy Correlation (EEC) in the back-to-back limit.**

Singular structure **identical to $\mathbf{q_T}$**, but no PDFs, so no need for DGLAP.

Need Rapidity Anomalous Dimension to N4LO [Duhr, Mistlberger, **GV** '22] [Moult, Zhu, Zhu '22]

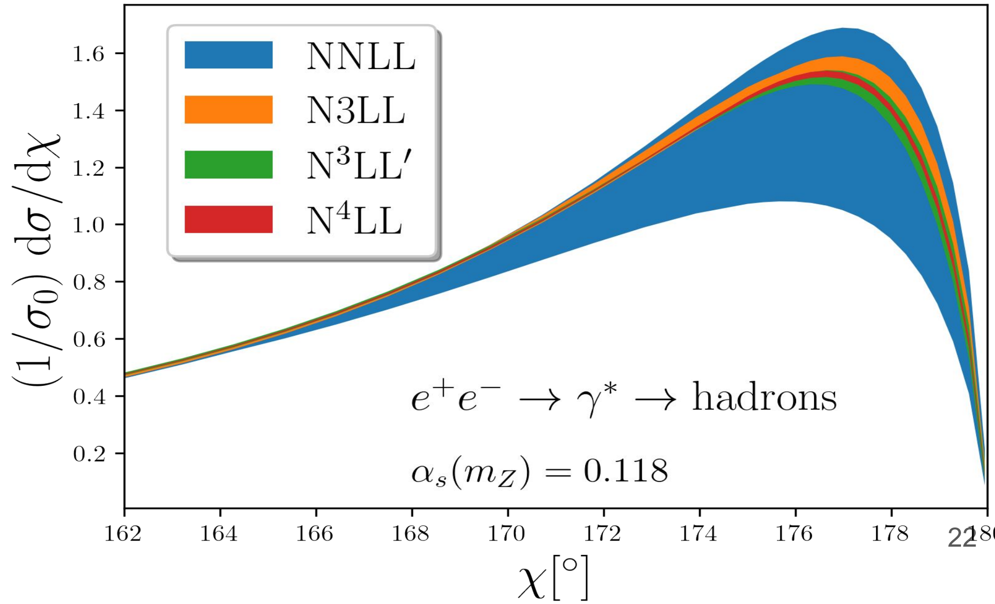
First resummation for an event shape at this accuracy!

"The Four-Loop Rapidity Anomalous Dimension and Event Shapes to Fourth Logarithmic Order"

C.Duhr, B.Mistlberger, GV [2205.02242]

Resummed EEC cross section to all orders (at LP in z → 1)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{\hat{\sigma}_0}{8}\int_0^\infty \mathrm{d}(b_T Q)^2\, J_0\left(b_T Q\sqrt{1-z}\right) H_{q\bar{q}}(Q,\mu_H)$$
$$\times\, \mathcal{J}_q\left(b_T,\mu_J,\frac{Qb_T}{\upsilon_n}\right)\mathcal{J}_{\bar{q}}\left(b_T,\mu_J,Qb_T\upsilon_{\bar{n}}\right)\left(\frac{\upsilon_n}{\upsilon_{\bar{n}}}\right)^{\boxed{\frac{1}{2}\gamma_r^q(b_T,\mu_J)}}$$
$$\times\, \exp\left[4\int_{\mu_J}^{\mu_H}\frac{\mathrm{d}\mu'}{\mu'}\Gamma^q_{\mathrm{cusp}}[\alpha_s(\mu')]\ln\frac{\mu'}{Q} - \gamma^q_H[\alpha_s(\mu')]\right]$$



Conclusion

- ➢ Motivated need for theoretical predictions at N3LO for percent level pheno

- ➢ Introduced **Beam Functions** to describe singular structure of infrared observables to **N3LO**

Longitudinal momentum fraction

PDF: $f_q(x) = \langle p_n | \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \left[\delta(p^- - \bar{n}\cdot\mathcal{P}) \chi_n \right] | p_n \rangle$

Longitudinal momentum fraction

Beam Function: $B_q(x, q_T) = \langle p_n | \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \left[\delta(p^- - \bar{n}\cdot\hat{\mathcal{P}}) \delta(q_T - \hat{k}_T) \chi_n \right] | p_n \rangle$

Additional observable (q_T, beam thrust, etc...)

$$B_i(x, \mathcal{T}, \mu) = \sum_j \mathcal{I}_{ij}(x, \mathcal{T}, \mu) \otimes_x f_j(x, \mu) \times \left[1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/\mathcal{T})\right]$$

Collinear expansion of the partonic cross section for Drell Yan and Higgs at N3LO differential in (Q_T, τ, z)

project to τ

project to q_T

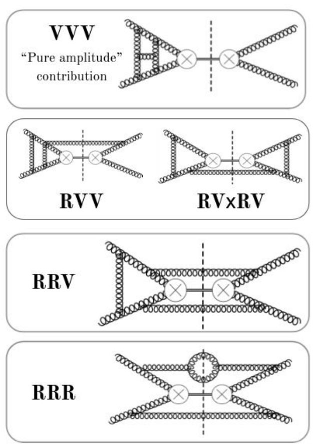
$B_a(t_a, x_1^B, \mu)$

"N-Jettiness Beam Functions at N3LO"
M.Ebert, B.Mistlberger, **GV**
[2006.03056]

$\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right)$

"Transverse Momentum Dependent PDFs at N3LO"
M.Ebert, B.Mistlberger, **GV**
[2006.05329]

- ➢ Discussed **collinear expansion** of rapidity distributions to **N3LO**



Hard Function

$$\frac{d\sigma}{dx_1 dx_2} = H_{ij}(Q^2, \mu) B_i^Y(Q^2 \bar{x}_2, x_1, \mu) \underset{x_1;\bar{x}_2}{\otimes} S(Q^2 \bar{x}_1 \bar{x}_2, \mu) \underset{x_2;\bar{x}_1}{\otimes} B_j^Y(Q^2 \bar{x}_1, x_2, \mu)$$

Rapidity Beam Functions

[Mistlberger, **GV**] (to appear)

- ➢ Explained ingredients and formalism for **resummation to N4LL**

Accuracy	Boundaries	$\Gamma_{\mathrm{cusp}}(\alpha_s)$	$\gamma_i(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	1-loop
NLL	Tree level	2-loop	1-loop	2-loop
NLL′	1-loop	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop
NNLL′	2-loop	3-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	4-loop
N^{3}LL′	3-loop	4-loop	3-loop	4-loop
N^{4}LL	3-loop	5-loop	4-loop	5-loop

$$\frac{d\sigma}{dx_1 dx_2} = H_{ij}(Q^2, \mu) B_i^Y(Q^2 \bar{x}_2, x_1, \mu) \underset{x_1;\bar{x}_2}{\otimes} S(Q^2 \bar{x}_1 \bar{x}_2, \mu) \underset{x_2;\bar{x}_1}{\otimes} B_j^Y(Q^2 \bar{x}_1, x_2, \mu)$$

Boundary functions: known to N3LO
(I've just shown their calculation)

Hard Function Evolution

$$\frac{d}{d\ln\mu} \ln H_i(Q, \mu) = \gamma_H^i(Q, \mu)$$

Related to Collinear Anomalous Dimension from form factors
Known at 4 loops
[Manteuffel, Panzer, Schabinger '21]

Rapidity Beam Function Evolution

$$\frac{d}{d\ln\mu} \ln \tilde{B}_i^Y(\kappa_2, x_1, \mu) = \gamma_B^i(\kappa_2, \mu)$$

0-Jettiness/Thrust Jet anomalous dimension
Known at 4 loops
[Duhr, Mistlberger, **GV** '22]
[Moult, Zhu, Zhu '22]

Soft Function Evolution

$$\frac{d}{d\ln\mu} \ln \tilde{S}(Q^2 \kappa_1 \kappa_2, \mu) = -\gamma_{th}^i(Q^2 \kappa_1 \kappa_2, \mu)$$

Threshold anomalous dimension
Known at 4 loops
[Duhr, Mistlberger, **GV** '22]
[Moult, Zhu, Zhu '22]
[Das, Moch, Vogt '19]

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Backup

Resummation Pulling out PDFs

$$B_i(\xi_2,\mu^2;\bar{\xi}_1) = \sum_j I_{ij}(\xi_2,\mu^2;\bar{\xi}_1) \otimes_{\xi_2} f_j(\xi_2,\mu^2)$$

Take Factorization Formula and pull out PDFs from Beam Functions

$$\frac{\mathrm{d}\sigma_{PP\to h+X}}{\mathrm{d}Y}(\xi_1,\xi_2) = \tau \sum_{i,j} f_i(\xi_1) \otimes_{\xi_1} \eta_{ij}^{Y_{\text{approx.}}}(\xi_1,\xi_2) \otimes_{\xi_2} f_j(\xi_2) + \mathcal{O}\left(\bar{\xi}_1^0, \bar{\xi}_2^0\right),$$

Get purely perturbative object in terms of Beam Function matching kernels

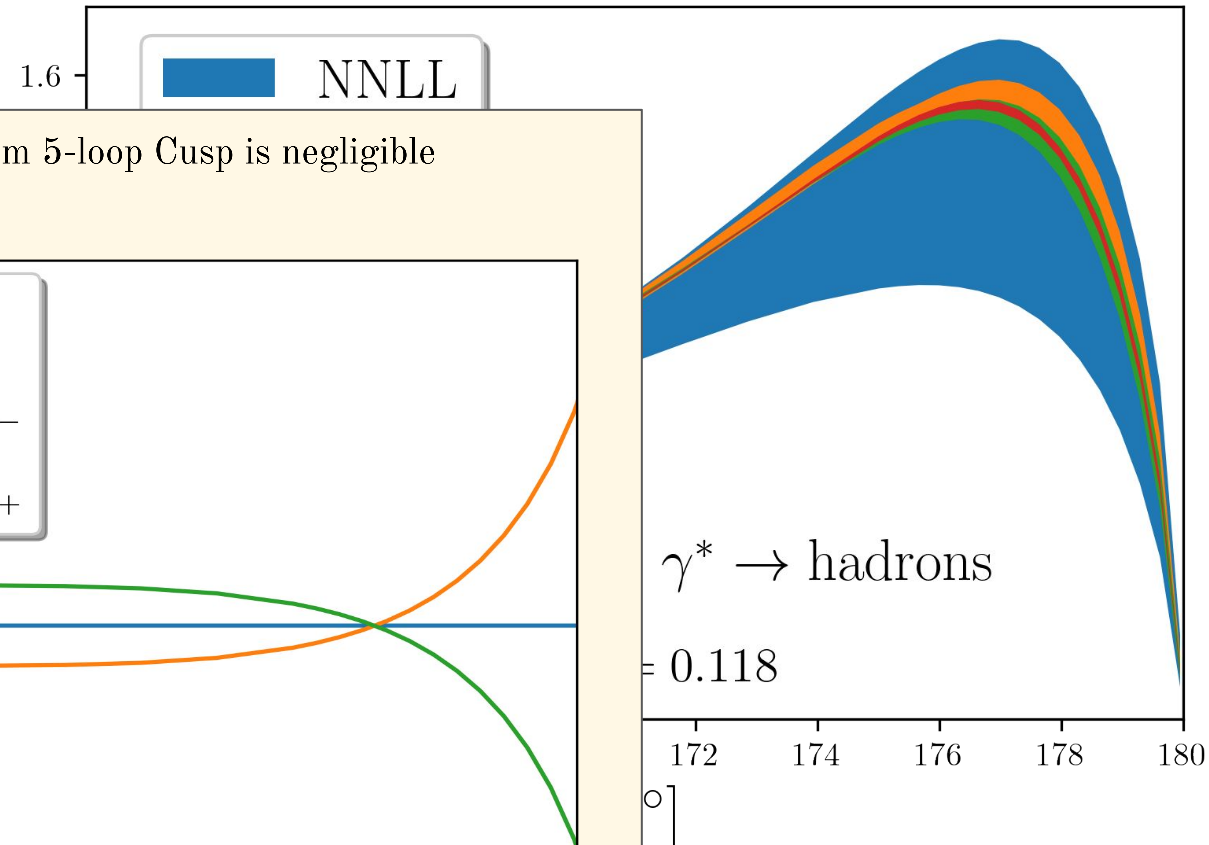
$$\eta_{ij}^{Y_{\text{approx.}}}(\xi_1,\xi_2,\mu^2) = \sum_{k.l} H_{kl}(\mu^2) I_{ik}(\xi_1,\mu^2;\bar{\xi}_2) \otimes_{\xi_1;\xi_2} S^{r_{kl}}(\mu^2;\bar{\xi}_1\bar{\xi}_2) \otimes_{\xi_2;\xi_1} I_{lj}(\xi_2,\mu^2;\bar{\xi}_1).$$

RGE of Rapidity Beam Function **matching kernel**

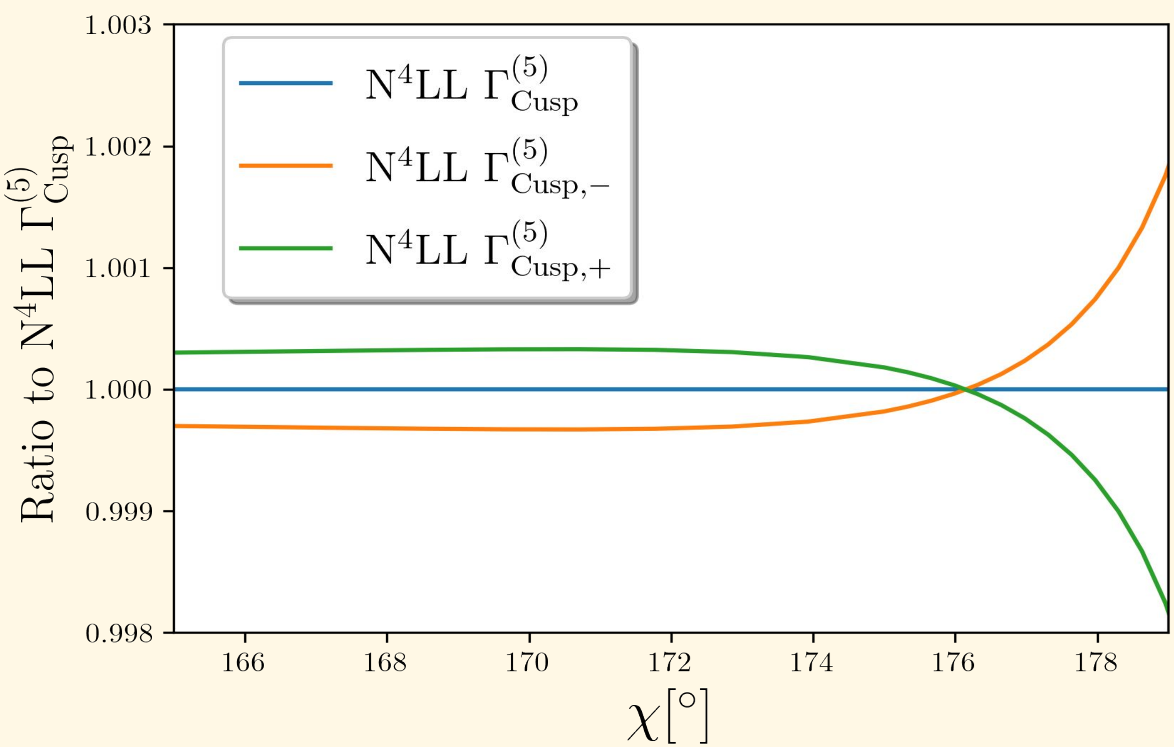
$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2}\tilde{I}_{ij}^{Y}\left(\log\frac{\kappa_1\mu^2}{Q^2},\xi_2,\mu^2\right) = -\sum_k \tilde{I}_{ik}^{Y}\left(\log\frac{\kappa_1\mu^2}{Q^2},\xi_2,\mu^2\right) \otimes_{\xi_2} P_{kj}(\xi_2,\mu^2)$$
$$+\left[\Gamma_{\mathrm{B}}^{r}(\alpha_S(\mu^2))\log\left(\frac{\kappa_1\mu^2}{Q^2}\right) + \frac{1}{2}\gamma^{B,r}(\alpha_S(\mu^2))\right]\tilde{I}_{ij}^{Y}\left(\log\frac{\kappa_1\mu^2}{Q^2},\xi_2,\mu^2\right).$$

EEC in the back to back limit to N4LL

- Implemented the resummation of this event shape at **N4LL** in



Impact of uncertainty from 5-loop Cusp is negligible

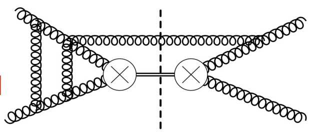


Beam Functions calculation at N3LO

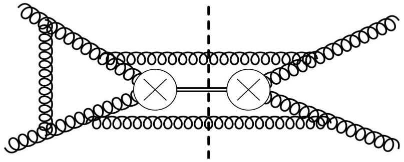
(Ebert, Mistlberger, **GV**)
[2006.05329], [2006.03056]

We calculated the **collinear expansion of the partonic cross section** for Drell-Yan and Higgs @N3LO **differential in (Q_T, τ, z)**

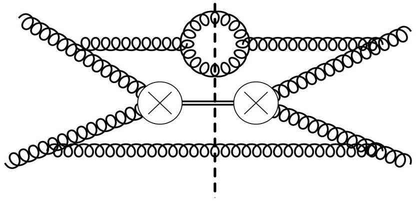
- O(100k) Feynman diagrams
- Collinear expansion applied before IBP reduction.
- **RVV**: known in full kinematics
[Duhr, Gehrmann] [Duhr, Gehrmann, Jaquier] [Dulat, Mistlberger]



- **RRV**: 170 Collinear Master Integrals



- **RRR**: 320 Collinear Master Integrals



- Problem has 2 non trivial scales with algebraic dependence on the variables

- Derived system of Differential Equations for Master Integrals
- Analytic solution by going to canonical form obtained by constructing **dLog integrands** on different cut surfaces for sectors involving algebraic functions
- Boundaries fixed by inclusive soft integrals and constraints on singular behavior [Anastasiou, Duhr, Dulat, Mistlberger] [Dulat, Mistlberger, **GV**]
- From this we obtain bare double differential results. Several steps to get to final result: SCET renormalization, handling of rapidity divergences, DGLAP poles removal, ...

Bare Beam Functions and Renormalization

N-Jettiness Beam Function

q_T Beam Function

$B_a(t_a, x_1^B, \mu)$ ← project to $\boldsymbol{\tau}$ — **Collinear expansion of the partonic cross section** for Drell Yan and Higgs at N3LO **differential in (Q_T, τ, z)** — project to $\mathbf{q_T}$ → $\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right)$

Bare Results

N-Jettiness Beam Function:

- Poles in dimensional regularization (up to $1/\varepsilon^6$)
- Logs/Plus Distributions in $\boldsymbol{\tau}$
- Iterated Integrals up to weight 5, with alphabet

$$\mathcal{A} = \left\{\frac{1}{z}, \frac{1}{1-z}, \frac{1}{2-z}, \frac{1}{1+z}, \frac{1}{z}, \frac{1}{\sqrt{4-z}\sqrt{z}}\right\}$$

- Constants to weight 6

q_T Beam Function:

- Poles in dimensional regularization
- Rapidity divergences regulated by exponential regulator
- Logs/Plus Distributions in $\mathbf{b_T/q_T}$
- HPLs in z up to weight 5
- Constants to weight 6

Renormalization

N-Jettiness Beam Function:

- Coupling renormalization
- SCET_{I} renormalization
- IR poles subtracted via NNLO PDF counterterms

q_T Beam Function:

- Coupling renormalization
- Zero-bin subtraction via calculation of bare q_T Soft Function at N3LO
- SCET_{II} renormalization
- IR poles subtracted via NNLO PDF counterterms

Checks

N-Jettiness Beam Function

q_T Beam Function

$B_a(t_a, x_1^B, \mu)$ $\xleftarrow{\text{project to } \boldsymbol{\tau}}$ **Collinear expansion of the partonic cross section** for Drell Yan and Higgs at N3LO **differential in (Q_T, τ, z)** $\xrightarrow{\text{project to } \mathbf{q_T}}$ $\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right)$

Checks

- 6 orders of poles cancel in all channels
- Terms involving $\mathcal{L}_n\left(\frac{t}{\mu^2}\right)$ $n = 0, \ldots, 5$ vs RGE prediction
- Eikonal limit vs threshold consistency [Billis, Ebert, Michel and Tackmann]
- Generalized leading color approx [Behring, Melnikov, Rietkerk, Tancredi, Wever]

Confirmation of our results
in later independent calculation

(Baranowski, Behring, Melnikov, Tancredi, Wever) [2211.05722]

- All rapidity divergences regulated
- 3 orders of ε poles cancel for all channels
- Log terms vs RGE prediction [Billis, Ebert, Michel and Tackmann]
- Eikonal limit vs threshold consistency
- Quark channels vs [Luo, Yang, Zhu, Zhu 1912.05778] (found small discrepancy)

Confirmation of our results
in later independent calculation

(Luo, Yang, Zhu, Zhu) [2012.03256]

More things towards percent level predictions…

$$\sigma = f_1 \circ f_2 \circ \int d\Phi |M|^2 + \mathcal{O}(\Lambda^2/Q^2)$$

1. **Accessibility and User Friendliness:** Creating frameworks that make N^3LO (and NNLO) predictions easily accessible for comparison to experimental data.

2. **Corrections beyond QCD:** EWK and masses.

3. **Factorisation Violation at N^3LO:** tops, PDFs.

4. **Parton Showers:** Consistent combination of parton showers with fixed order perturbative computations at N^3LO.

5. **Resummation:** Complementing N^3LO computations and resummation techniques for infrared sensitive observables.

6. **Uncertainties:** Deriving / defining reliable uncertainty estimates for theoretical computations at the percent level.

7. **Beyond Leading Power Factorisation:** Exploring the limitations of leading power perturbative descriptions of hadron collision cross sections.

Collinear Expansion for Matrix Elements

- Kinematic limit $\longrightarrow$ expansion of Feynman integrands appearing in the calculation of **partonic cross sections**

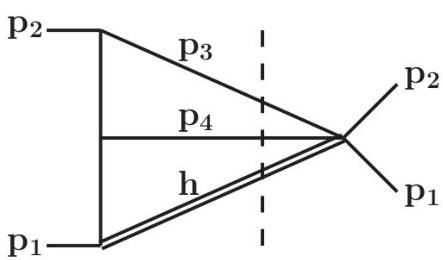
General idea has long history, see e.g.
Expansion by region [Beneke, Smirnov '97]

- Take for example double real emission (RR) scalar integral

$$w_1 = -\frac{\bar{n}\cdot k}{\bar{n}\cdot p_1}, \quad w_2 = -\frac{n\cdot k}{n\cdot p_2},$$

$$x = \frac{k^2}{(\bar{n}\cdot k)(n\cdot k)} = 1 - \frac{\vec{k}_\perp^2}{(\bar{n}\cdot k)(n\cdot k)}$$

$I_{\mathrm{RR}} =$



$$= \int \frac{\mathrm{d}\Phi_{h+2}}{\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x} \frac{1}{(p_2+p_3)^2(p_2+p_3+p_4)^2}$$

- ○ Differential double real particle phase space scales homogeneously

In the **collinear** limit:

$$\int \frac{\mathrm{d}\Phi_{h+2}}{\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x} \to \lambda^{2-4\epsilon} \int \frac{\mathrm{d}\Phi_{h+2}}{\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x}$$

- ○ Propagators can be **expanded** easily

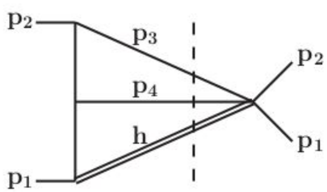
$$\frac{1}{(p_2+p_3+p_4)^2} \xrightarrow{\text{coll}} \frac{1}{2p_2\cdot(p_3+p_4) + \lambda^2\, 2p_3\cdot p_4} = \sum_{n=0}^{\infty} (\lambda^2)^n \frac{(-2p_3\cdot p_4)^n}{\left[p_2^+(p_3^- + p_4^-)\right]^{n+1}}$$

Collinear Expansion for double real graphs

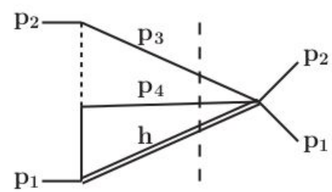
- We can perform a **collinear expansion** of the **integrand**

$$I_{\mathrm{RR}} \xrightarrow{\text{coll}} \lambda^{2-4\epsilon} \int \frac{\mathrm{d}\Phi_{h+2}}{\mathrm{d}w_1 \mathrm{d}w_2 \mathrm{d}x} \left[\frac{1}{(p_2+p_3)^2 \left[p_2^+(p_3^- + p_4^-)\right]} + \lambda^2 \frac{(-2p_3 \cdot p_4)}{(p_2+p_3)^2 \left[p_2^+(p_3^- + p_4^-)\right]^2} + \mathcal{O}(\lambda^3) \right]$$

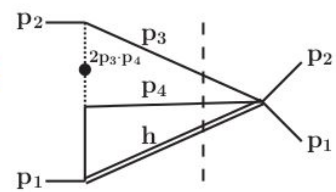
- Collinear expansion admits **diagrammatic** representation!



$\rightarrow \lambda^{2-4\epsilon} \Bigg[$



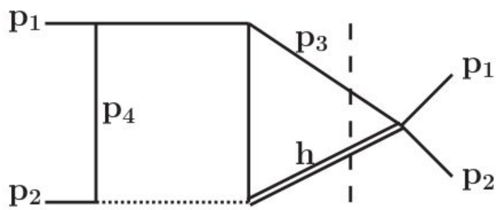
$-\lambda^2$



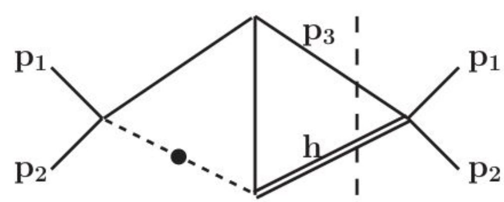
$+\mathcal{O}(\lambda^3) \Bigg]$

- Same procedure can be applied for mixed loop/radiation integrals
  (like RV integrals at NNLO)

$I_{\mathrm{RV}} \xrightarrow{\text{coll}} \lambda^{-2-4\epsilon} \Bigg[$



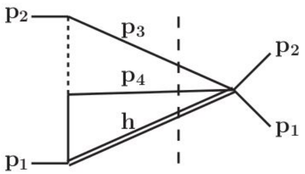
$-\lambda^2$



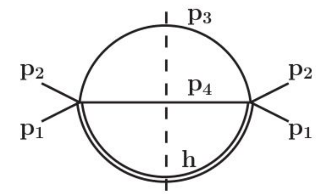
$+\mathcal{O}(\lambda^3) \Bigg]$

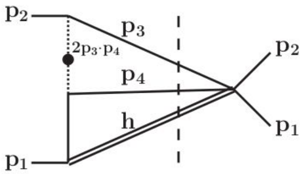
Collinear Expansion and IBPs

Key Point!
Expanded diagrams admit (simplified) integration by parts (**IBPs**) identities

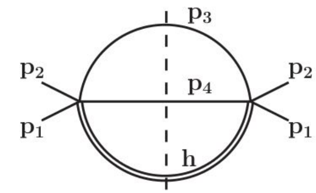


$$= -\frac{1-2\epsilon}{\epsilon(p_2^+ k^-)^2} \times$$





$$= -\frac{k^+ x}{p_2^+}\frac{1-2\epsilon}{\epsilon(p_2^+ k^-)^2} \times$$



- We can make use of modern technology for multiloop calculations with simplified kinematic dependence!

IBPs

Canonical Differential Equations

Reverse Unitarity

- **Simplifications w.r.t. full kinematics are huge** and enter at each step:
  - IBPs (smaller set of MI, smaller coefficients)
  - System of DE (e.g. ~ 10 MB for differential N3LO in collinear limit vs ~ 10 GB in full kinematics)
  - Space of functions (e.g. @N3LO: Elliptic functions for inclusive color singlet production in full kinematics vs only HPL for q_T distributions in collinear limit)