

Three-loop four-particle QCD amplitudes with Caola, Chakraborty, Gambuti, Manteuffel, Tancredi

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Presentation plan

1 Overview

2 Structures

3 Tensors

4 Integrals

5 Results

Presentation plan

1 Overview

2 Structures

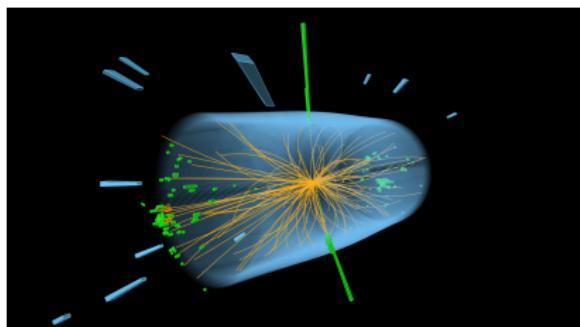
3 Tensors

4 Integrals

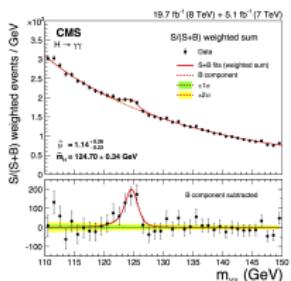
5 Results

Motivation

precision **LHC** measurements



Higgs analysis [*talk by Buccioni*]



multiloop scattering amplitudes



Towards 3-loop revolution

3-loop amplitude milestones

- ⌚ 1→1 QCD [[Tarasov et al. PRLB 1980](#)]
- ⌚ 2→1 QCD [[Moch et al. arXiv:0508055](#)]
- ⌚ 2→2 SYM [[Henn, Mistlberger arXiv:1608.00850](#)]

first 3-loop 2→2 QCD results

- ⌚ $q\bar{q} \rightarrow \gamma\gamma$ [[Caola, Manteuffel, Tancredi arXiv:2011.13946](#)]
- ⌚ $q\bar{q} \rightarrow q\bar{q}$ [[Caola, Chakraborty, Gambuti, Manteuffel, Tancredi arXiv:2108.00055](#)]
- ⌚ $gg \rightarrow \gamma\gamma$ [[PB, Caola, Manteuffel, Tancredi arXiv:2111.13595](#)]
- ⌚ $gg \rightarrow gg$ [[Caola, Chakraborty, Gambuti, Manteuffel, Tancredi arXiv:2112.11097](#)]
- ⌚ $pp \rightarrow j\gamma$ [[PB, Chakraborty, Gambuti arXiv:2212.14069](#)]

challenge = complexity

# diagrams	0L	1L	2L	3L
$q\bar{q} \rightarrow \gamma\gamma$	2	10	143	2922
$q\bar{q} \rightarrow q\bar{q}$	1	9	158	3584
$gg \rightarrow \gamma\gamma$	0	6	138	3299
$gg \rightarrow gg$	4	81	1771	48723
$q\bar{q} \rightarrow g\gamma$	2	13	229	5334
$gg \rightarrow g\gamma$	0	12	264	7356

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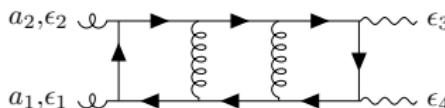
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Amplitude structure

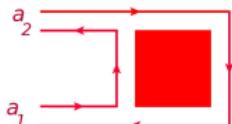
consider example 3-loop diagram for

$$g(p_1) + g(p_2) \rightarrow \gamma(-p_3) + \gamma(-p_4)$$

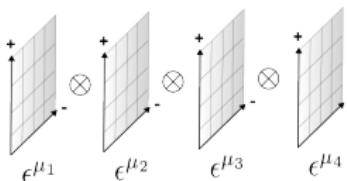


$$\begin{aligned}
 &= g_s^6 e^2 n_f^{(V_2)} C_f^2 \delta^{a_1, a_2} \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{d^d k_3}{(2\pi)^d} \\
 &\times \frac{\text{tr} \left[\not{\epsilon}_1(\not{k}_1) \not{\epsilon}_2(\not{k}_1 + \not{p}_2) \gamma^\mu (\not{k}_{13} + \not{p}_2) \gamma^\nu (\not{k}_2 + \not{p}_4) \not{\epsilon}_4(\not{k}_2) \not{\epsilon}_3(\not{k}_2 - \not{p}_3) \gamma_\nu (\not{k}_{13} - \not{p}_1) \gamma_\mu (\not{k}_1 - \not{p}_1) \right]}{(k_1)^2 (k_1 + p_2)^2 (k_{13} + p_2)^2 (k_2 + p_4)^2 (k_2)^2 (k_2 - p_3)^2 (k_{13} - p_1)^2 (k_1 - p_1)^2 (k_3)^2 (k_{123} - p_{13})^2
 \end{aligned}$$

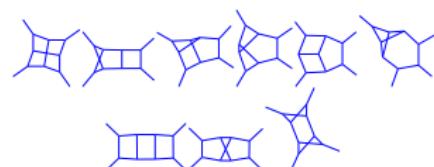
color



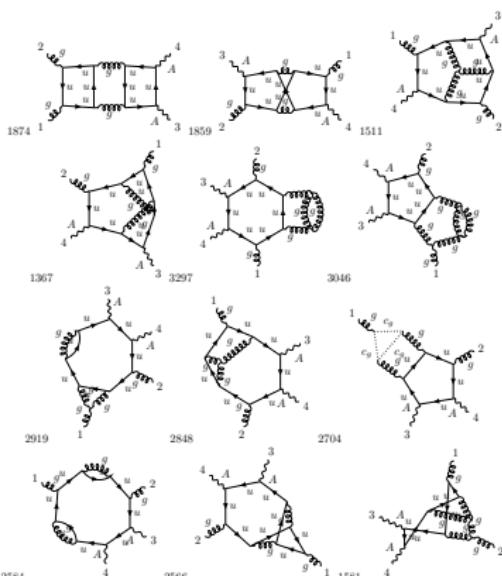
tensors



integrals



$$\mathcal{A}^{\vec{a}, \vec{\lambda}} = \sum_{c,t,i} \mathcal{C}_c^{\vec{a}} T_t^{\vec{\lambda}} \mathcal{I}_i r_{c,t,i}$$

3-loop $gg \rightarrow \gamma\gamma$ 

$\times 275$ pages = 3299 diagrams

	1L	2L	3L
# diagrams	6	138	3299
# integral families	1	2	3
# integrals	209	20935	4370070
# MIs	6	39	486
Qgraf result [kB]	4	90	2820
amplitudes before IBPs [kB]	276	54364	19734644
amplitudes after IBPs [kB]	12	562	304409
expanded amplitudes [kB]	136	380	1195

complexity summary

Overview of the computation

overcome complexity with recently proposed ideas

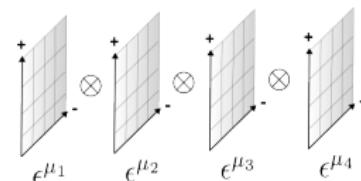
- **simple color structure**

color algebra closes



- managing tensor structures

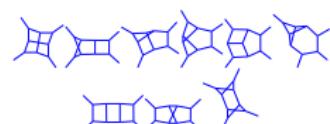
independent structures = # helicity amplitudes
& project out unphysical $(d - 4)$ -dim structures
 $\Rightarrow 138 \rightarrow 8$ physical tensors



- **reducing integrals to a minimal set**

finite field reconstruction
& syzygy constraints

$\Rightarrow 4370070 \xrightarrow{IBP} 486$ integrals



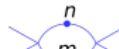
- **evaluating integrals**

differential equations in canonical Master Basis
& regularity requirement for boundary conditions

$\Rightarrow 486 \rightarrow 1$ integral



$\Downarrow \Downarrow \Downarrow$



physical constraints \Rightarrow only

by direct integration

Kinematics and color

kinematics : 4-point process \Rightarrow 1 dimensionless variable

$$(\text{Regge, forward}) \quad 0 < x = -\frac{t}{s} < 1 \quad (\text{backward})$$

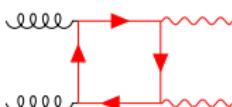
$$\mathcal{A}^{\vec{a}, \vec{\lambda}} = \sum_{c,t} \mathcal{C}_c^{\vec{a}} T_t^{\vec{\lambda}} \mathcal{F}_{c,t}(x, d)$$

simple **color** structure : $\mathcal{A}^{\vec{a}} = \delta^{a_1 a_2} \mathcal{C}_c A_c$

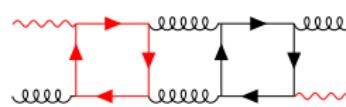
where \mathcal{C}_c is a degree=3 monomial in $\{C_A, C_F, n_f, n_f^V, n_f^{V_2}\}$

3 closed fermion loop types

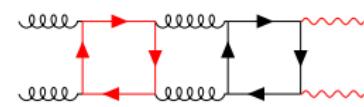
$$n_f^{V_2} = \sum_f Q_f^2$$



$$n_f^V = \sum_f Q_f$$



$$n_f = \sum_f 1$$



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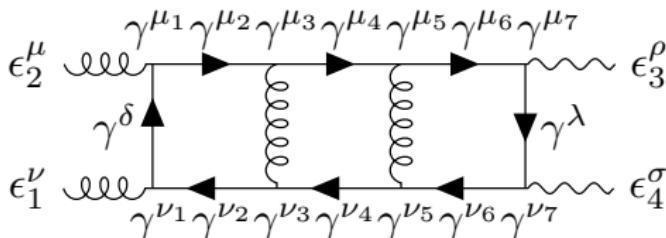
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Tensors in $d=4-2\epsilon$ dimensions

$$A_{(c)} = \sum_i T_i \mathcal{F}_t$$



$\sum_{\text{diagrams}} \# \text{ Lorentz indices} > \# \text{ all invariant structures}$

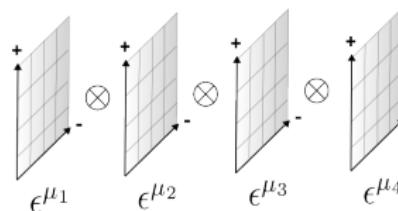
$$\begin{aligned} \# T_i &= 138 \text{ (Lorentz tensors)} - 81 \text{ (by transversality } \epsilon_i \cdot p_i = 0) \\ &\quad - 47 \text{ (fixing ref. momenta } \epsilon_i \cdot p_{i+1} = 0) \\ &= 10 \text{ (independent in d dimensions)} \end{aligned}$$

$$\begin{aligned} T_i = & (p_1 \cdot \epsilon_2 p_1 \cdot \epsilon_3 p_2 \cdot \epsilon_4 p_3 \cdot \epsilon_1, \quad \epsilon_3 \cdot \epsilon_4 p_1 \cdot \epsilon_2 p_3 \cdot \epsilon_1 \quad \epsilon_2 \cdot \epsilon_4 p_1 \cdot \epsilon_3 p_3 \cdot \epsilon_1, \\ & \epsilon_2 \cdot \epsilon_3 p_2 \cdot \epsilon_4 p_3 \cdot \epsilon_1, \quad \epsilon_1 \cdot \epsilon_4 p_1 \cdot \epsilon_2 p_1 \cdot \epsilon_3, \quad \epsilon_1 \cdot \epsilon_3 p_1 \cdot \epsilon_2 p_2 \cdot \epsilon_4, \\ & \epsilon_1 \cdot \epsilon_2 p_1 \cdot \epsilon_3 p_2 \cdot \epsilon_4, \quad \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4, \quad \epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3, \quad \epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4) \end{aligned}$$

Tensors in 4 dimensions

recent loop-universal **claim** in the 'tHV scheme [Peraro, Tancredi [arXiv:2012.00820](#)] :
 # tensors indpt in 4-dim = # indpt helicity states (here = $2^4/2 = 8$)

$$A = \sum_{i=1}^{10} \mathcal{F}_i T_i = \sum_{i=1}^8 \overline{\mathcal{F}}_i \overline{T}_i$$



orthogonalization : projects out $\overline{T}_9, \overline{T}_{10}$ from the **physical** 4-dim subspace

$$\sum_{pol} \overline{T}_i^\dagger \overline{T}_j = \left(\begin{array}{c|c} 8 \times 8 \text{ (4-dim)} & 0 \\ \hline 0 & 2 \times 2 \text{ (-2e-dim)} \end{array} \right)$$

gain : 1-1 **correspondence** between form factors and helicity amplitudes

$$\overline{\mathcal{F}}_i \iff A_{\vec{\lambda}_i}$$

Tensor projectors

resulting tensors live purely in the **unphysical** -2ϵ -dim subspace

$$\bar{T}_i = T_i - \sum_{j=1}^8 (\mathcal{P}_j T_i) \bar{T}_j, \quad i = 9, 10, \quad \sum_{\text{pol}} \mathcal{P}_i \bar{T}_j = \delta_{ij}$$

$$\begin{aligned}\bar{T}_9 &= T_9 - \frac{1}{3} \left(-\frac{2\bar{T}_1}{su} - \frac{\bar{T}_6}{s} - \frac{\bar{T}_2 + \bar{T}_3 + 2\bar{T}_4 - 2\bar{T}_5 - \bar{T}_6 - \bar{T}_7}{t} + \frac{\bar{T}_3}{u} + \bar{T}_8 \right) \\ \bar{T}_{10} &= T_{10} - \frac{1}{3} \left(\frac{4\bar{T}_1}{su} + \frac{2\bar{T}_6}{s} - \frac{\bar{T}_2 - \bar{T}_4 - 2\bar{T}_3 + 2\bar{T}_6 + \bar{T}_5 - \bar{T}_7}{t} - \frac{2\bar{T}_3}{u} + \bar{T}_8 \right)\end{aligned}$$

and they **vanish** ($\forall \epsilon$) for each fixed helicity configuration

Helicity amplitudes

evaluate tensors at fixed helicity configuration
spinor weights

$$A_{\vec{\lambda}} = \sum_i \bar{\mathcal{F}}_i \bar{T}_{\vec{\lambda}} = \mathcal{S}_{\vec{\lambda}} f_{\vec{\lambda}}$$

$$\begin{aligned} \mathcal{S}_{++++} &= \frac{\langle 12 \rangle [34]}{\langle 12 \rangle \langle 34 \rangle}, & \mathcal{S}_{-+++} &= \frac{\langle 12 \rangle \langle 14 \rangle [24]}{\langle 34 \rangle \langle 23 \rangle \langle 24 \rangle}, & \mathcal{S}_{+-++} &= \frac{\langle 21 \rangle \langle 24 \rangle [14]}{\langle 34 \rangle \langle 13 \rangle \langle 14 \rangle}, & \mathcal{S}_{++-+} &= \frac{\langle 32 \rangle \langle 34 \rangle [24]}{\langle 14 \rangle \langle 21 \rangle \langle 24 \rangle}, \\ \mathcal{S}_{+++-} &= \frac{\langle 42 \rangle \langle 43 \rangle [23]}{\langle 13 \rangle \langle 21 \rangle \langle 23 \rangle}, & \mathcal{S}_{--++} &= \frac{\langle 12 \rangle [34]}{\langle 12 \rangle \langle 34 \rangle}, & \mathcal{S}_{-+-+} &= \frac{\langle 13 \rangle [24]}{\langle 13 \rangle \langle 24 \rangle}, & \mathcal{S}_{+---} &= \frac{\langle 23 \rangle [14]}{\langle 23 \rangle \langle 14 \rangle} \end{aligned}$$

little group scalars

$$f_{++++} = \frac{t^2}{4} \left(\frac{2\bar{\mathcal{F}}_6}{u} - \frac{2\bar{\mathcal{F}}_3}{s} - \bar{\mathcal{F}}_1 \right) + \bar{\mathcal{F}}_8 \left(\frac{s}{u} + \frac{u}{s} + 4 \right) + \frac{t}{2} (\bar{\mathcal{F}}_2 - \bar{\mathcal{F}}_4 + \bar{\mathcal{F}}_5 - \bar{\mathcal{F}}_7),$$

$$f_{-+++} = \frac{t^2}{4} \left(\frac{2\bar{\mathcal{F}}_3}{s} + \bar{\mathcal{F}}_1 \right) + t \left(\frac{\bar{\mathcal{F}}_8}{s} + \frac{1}{2} (\bar{\mathcal{F}}_4 + \bar{\mathcal{F}}_6 - \bar{\mathcal{F}}_2) \right),$$

$$f_{+-++} = -\frac{t^2}{4} \left(\frac{2\bar{\mathcal{F}}_6}{u} - \bar{\mathcal{F}}_1 \right) + t \left(\frac{\bar{\mathcal{F}}_8}{u} - \frac{1}{2} (\bar{\mathcal{F}}_2 + \bar{\mathcal{F}}_3 + \bar{\mathcal{F}}_5) \right),$$

$$f_{++-+} = \frac{t^2}{4} \left(\frac{2\bar{\mathcal{F}}_3}{s} + \bar{\mathcal{F}}_1 \right) + t \left(\frac{\bar{\mathcal{F}}_8}{s} + \frac{1}{2} (\bar{\mathcal{F}}_6 + \bar{\mathcal{F}}_7 - \bar{\mathcal{F}}_5) \right),$$

$$f_{+++-} = -\frac{t^2}{4} \left(\frac{2\bar{\mathcal{F}}_6}{u} - \bar{\mathcal{F}}_1 \right) + t \left(\frac{\bar{\mathcal{F}}_8}{u} + \frac{1}{2} (\bar{\mathcal{F}}_4 + \bar{\mathcal{F}}_7 - \bar{\mathcal{F}}_3) \right),$$

$$f_{--++} = -\frac{t^2}{4} \bar{\mathcal{F}}_1 + \frac{1}{2} t (\bar{\mathcal{F}}_2 + \bar{\mathcal{F}}_3 - \bar{\mathcal{F}}_6 - \bar{\mathcal{F}}_7) + 2\bar{\mathcal{F}}_8,$$

$$f_{-+-+} = t^2 \left(\frac{\bar{\mathcal{F}}_8}{su} - \frac{\bar{\mathcal{F}}_3}{2s} + \frac{\bar{\mathcal{F}}_6}{2u} - \frac{\bar{\mathcal{F}}_1}{4} \right),$$

$$f_{+---} = -\frac{t^2}{4} \bar{\mathcal{F}}_1 + \frac{1}{2} t (\bar{\mathcal{F}}_3 - \bar{\mathcal{F}}_4 + \bar{\mathcal{F}}_5 - \bar{\mathcal{F}}_6) + 2\bar{\mathcal{F}}_8$$

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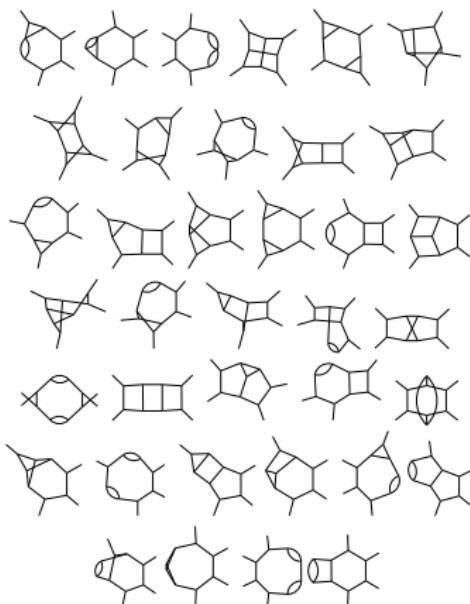
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3-loop 4-point massless integrals



$$f_{(c, \vec{\lambda})} = \sum_i \mathcal{I}_i r_i$$

$$\mathcal{I}_{(\text{topo}), \vec{n}} = \int \left(\prod_{l=1}^3 \frac{d^d k_l}{(2\pi)^d} \right) \frac{\mathcal{D}_{11}^{-n_{11}} \cdots \mathcal{D}_{15}^{-n_{15}}}{\mathcal{D}_1^{n_1} \cdots \mathcal{D}_{10}^{n_{10}}}$$

$$\mathcal{D}_i \sim (k_l + p_i)^2$$

- $r \leq 10$ different denominators
- $s \leq 6$ irreducible scalar products
- $d \leq 2$ squared denominators

$\times 118110$ pages = 4370070 integrals

Integral reduction

simplifying 4×10^6 integrals **before** the Integration by Parts (IBP) reduction

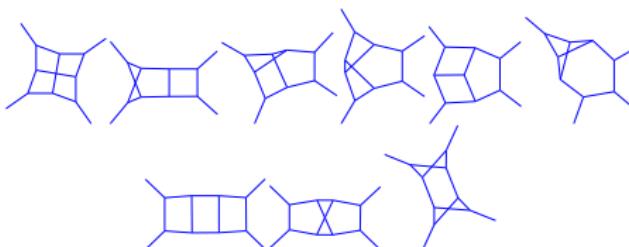
- map redundant crossings onto the 6 independent ones
& remove scaleless integrals $\rightarrow \mathcal{O}(10)$ reduction
- map equivalent sectors into common family $\rightarrow \mathcal{O}(2)$ reduction

$$f_{(c,\vec{\lambda})}(x,d) = \sum_{i=1}^{2 \times 10^5} \tilde{r}_i(x,d) \mathcal{I}_i(x,d) \stackrel{IBP}{=} \sum_{i=1}^{486} r_i(x,d) M_i(x,d)$$

taming the complexity of the **IBP** reduction

- solve IBP system with Laporta algorithm [[Laporta arXiv:0102033](#)]
 $\int_d \frac{\partial}{\partial l_j^\mu} \frac{v_j^\mu(l,p)}{D_1^{n_1} \dots D_m^{n_m}} = 0 \quad \text{and} \quad \sum_{i \in \text{ext}} (p_i^\mu \frac{\partial}{\partial p_i^\nu} - p_i^\nu \frac{\partial}{\partial p_i^\mu}) \mathcal{I}_{\{n_j\}} = 0$
- use \mathbb{F}_p finite field arithmetic
[\[Peraro arXiv:1905.08019\]](#) [\[Manteuffel, Schabinger arXiv:1406.4513\]](#)
 $r(x,d) = \sum_j \#_j g_j(x,d), \quad \text{where} \quad g \sim \frac{\mathcal{N}(x,d)}{(d-\#)^p (x-\#)^q}$
- impose syzygy constraints
[\[Gluza et al. arXiv:1009.0472\]](#) [\[Agarwal, Jones, Manteuffel arXiv:2011.15113\]](#)
 $v_j^\mu \frac{\partial}{\partial l_j^\mu} \mathcal{D}_i + b_i \mathcal{D}_i = 0$
- partial fraction in d and x

Master Integrals



- differential equation [[Henn, Mistlberger et al. arXiv:2002.09492](#)]

$$\partial_x \vec{M}(\epsilon; x) = \epsilon \left(\frac{a_0}{x} + \frac{a_1}{1-x} \right) \vec{M}(\epsilon; x),$$

- solved perturbatively in ϵ up to boundary Master Integrals

$$\vec{M}(\epsilon; x) = \mathbb{P} e^{\epsilon \int_{x_0}^x dx' \left(\frac{a_0}{x'} + \frac{a_1}{1-x'} \right)} \vec{M}(\epsilon; x_0)$$

- solution in terms of Harmonic Polylogarithms (**HPLs**)

$$G(\alpha_n, \dots, \alpha_1; x) = \int_0^x \frac{dz}{z - \alpha_n} G(\alpha_{n-1}, \dots, \alpha_1; z), \quad G(\underbrace{0, \dots, 0}_n; x) \equiv \frac{\ln^n x}{n!}, \quad \alpha_i \in \{0, 1\}$$

Boundary Master Integrals

claim : can relate **all** boundary Master Integrals $M_i(\epsilon; x_0) = \sum_{n=0} \epsilon^n c_{i,n}$
 to a **single** overall normalization (3L sunrise ) if

$$\text{require } \lim_{s_{ij} \rightarrow 0} \vec{M}(\epsilon; x) \rightarrow s^{a_{s_{ij}}} \epsilon \vec{M}_{0,s_{ij}} \quad \text{regular}$$

- 1 general solution
 for the canonical differential equation in terms of HPLs(x)
- 6 crossings of the general solution in $x \rightarrow \left\{x, 1-x, \frac{1}{1-x}, \frac{x-1}{x}, \frac{x}{x-1}, \frac{1}{x}\right\}$
 careful analytic continuation $s_{ij} + i\epsilon$: cross only 1 branch cut at a time
 e.g. $x - i\epsilon \rightarrow \frac{1}{x} \pm? i\epsilon \Rightarrow x \rightarrow -x \rightarrow -\frac{1}{x}$
- 3 regularity conditions
 $\lim_{x \rightarrow 0, 1, \infty} M_i(\epsilon; x)$ regular \Rightarrow linear relations between constants $c_{i,n}$
- 7 orders in ϵ
 required to fix all but 1-2 boundary Master Integrals at ϵ^6 per top sector
- 9 top sector remnants mapped onto 1 overall 3L sunrise
 e.g. $I_{2,0,0,0,0,0,0,2,0,0,0,2,0,1}^{(\text{PL})} = I_{2,0,0,0,0,0,0,0,2,0,0,0,2,1,0}^{(\text{NPL})} = \text{sunrise diagram}$

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All-plus helicity amplitude

$\overline{\text{MS}}$ renormalized and IR regularized [[Catani arXiv:9802439](#)]
 finite part in the **simplest** helicity configuration yields

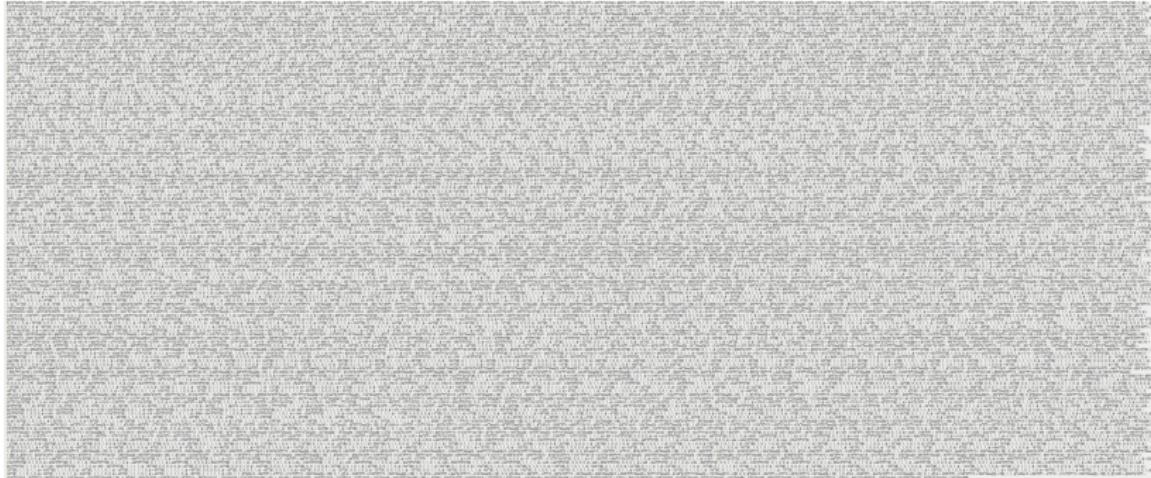
$$\begin{aligned}
 f_{+++}^{(3,\text{fin})} &= \Delta_1(x) n_f^{V_2} C_A^2 + \Delta_2(x) n_f^{V_2} C_A C_F + \Delta_3(x) n_f n_f^{V_2} C_A + \Delta_4(x) (n_f^V)^2 C_A + \Delta_5(x) n_f^{V_2} C_F^2 + \Delta_6(x) (n_f^V)^2 C_F + \Delta_7(x) n_f n_f^{V_2} C_F + \Delta_8(x) n_f^2 n_f^{V_2} \\
 &\quad + \{(x) \leftrightarrow (1-x)\}, \\
 \Delta_1(x) &= -\frac{23L_1(L_1+2i\pi)}{9x^2} + \frac{32L_1(L_1+2i\pi)-46(L_1+i\pi)}{9x} - \frac{17}{36}L_0^2 - \frac{19}{36}L_0L_1 + \frac{1}{9}L_0 - 2i\pi L_0 + \frac{1}{288}\pi^4 \\
 &\quad - \frac{373}{72}\zeta_3 - \frac{185}{72}\pi^2 + \frac{4519}{324} + \frac{1}{2}i\pi\zeta_3 + \frac{11}{144}i\pi^3 + \frac{157}{12}i\pi + \frac{43}{9}L_0x - \frac{7}{9}x^2 ((L_0-L_1)^2 + \pi^2), \\
 \Delta_2(x) &= \frac{8L_1(L_1+2i\pi)}{3x^2} + \frac{16(L_1+i\pi)-8L_1(L_1+2i\pi)}{3x} - \frac{1}{3}L_0^2 + \frac{5}{6}L_0L_1 + \frac{17}{3}L_0 + i\pi L_0 - \frac{5}{12}\pi^2 - \frac{199}{6} - 8i\pi - \frac{16}{3}L_0x + \frac{4}{3}x^2 ((L_0-L_1)^2 + \pi^2), \\
 \Delta_3(x) &= \frac{L_1(L_1+2i\pi)}{18x^2} + \frac{2(L_1+i\pi)-L_1(L_1+2i\pi)}{18x} - \frac{1}{36}L_0^2 + \frac{1}{36}L_0L_1 - \frac{1}{9}L_0 - \frac{61}{36}\zeta_3 + \frac{475}{432}\pi^2 - \frac{925}{324} - \frac{1}{72}i\pi^3 - \frac{175}{54}i\pi + \frac{2}{9}L_0x + \frac{1}{36}x^2 ((L_0-L_1)^2 + \pi^2), \\
 \Delta_4(x) &= -\frac{5L_1(L_1+2i\pi)}{4x^2} + \frac{L_1(L_1+2i\pi)-8(L_1+i\pi)}{2x} + \frac{1}{4}L_0^2 - \frac{1}{4}L_0L_1 - 2L_0 - 6\zeta_3 + \frac{1}{8}\pi^2 - \frac{1}{2} + 4L_0x - x^2 ((L_0-L_1)^2 + \pi^2), \\
 \Delta_5(x) &= -\frac{L_1(L_1+2i\pi)}{x^2} + \frac{L_1(L_1+2i\pi)-2(L_1+i\pi)}{x} - \frac{1}{2}L_0^2 - i\pi L_0 + \frac{39}{4} + i\pi + 2L_0x - \frac{1}{2}x^2 ((L_0-L_1)^2 + \pi^2), \\
 \Delta_6(x) &= \frac{10L_1(L_1+2i\pi)}{3x^2} + \frac{32(L_1+i\pi)-4L_1(L_1+2i\pi)}{3x} - \frac{2}{3}L_0^2 + \frac{2}{3}L_0L_1 + \frac{16}{3}L_0 + 16\zeta_3 - \frac{1}{3}\pi^2 + \frac{4}{3} - \frac{32}{3}L_0x + \frac{8}{3}x^2 ((L_0-L_1)^2 + \pi^2), \\
 \Delta_7(x) &= \frac{5L_1(L_1+2i\pi)}{3x^2} + \frac{10(L_1+i\pi)-8L_1(L_1+2i\pi)}{3x} + \frac{2}{3}L_0^2 + \frac{1}{3}L_0L_1 - \frac{10}{3}L_0 + 2i\pi L_0 + 4\zeta_3 - \frac{\pi^2}{6} + 5 - 3i\pi - \frac{10}{3}L_0x + \frac{1}{3}x^2 ((L_0-L_1)^2 + \pi^2), \\
 \Delta_8(x) &= -\frac{23}{216}\pi^2 + \frac{5}{27}i\pi,
 \end{aligned}$$

where $L_0 = \ln(x)$, $L_1 = \ln(1-x)$.

--++ helicity amplitude

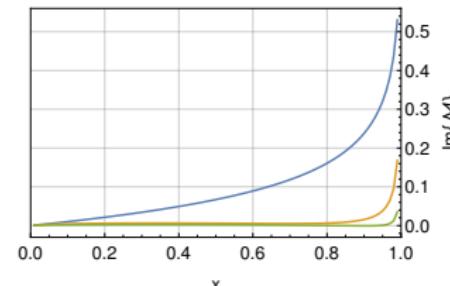
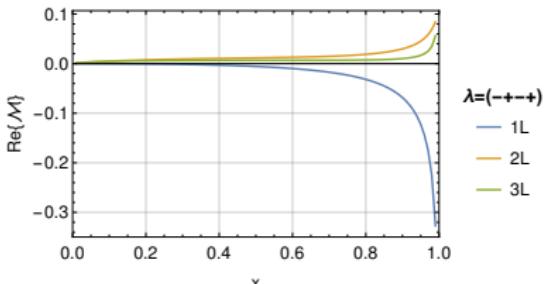
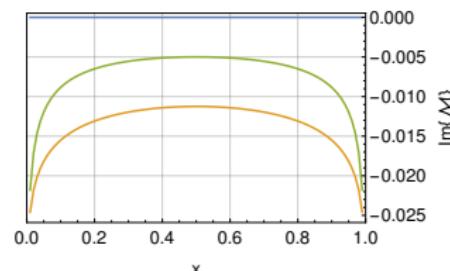
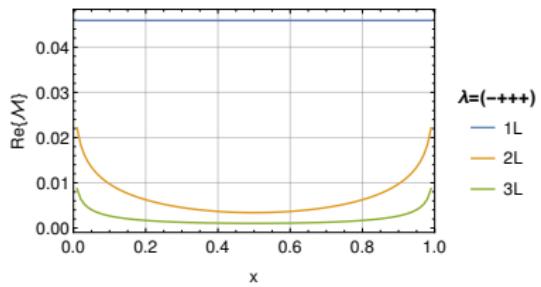
compact result even for the most complicated helicity configuration

$$f_{--++}^{(3,\text{fin})} =$$



Kinematic dependence

$$0 < x = -\frac{t}{s} < 1$$



results can be evaluated numerically in $[\mu\text{s}]$

Application to the Higgs width

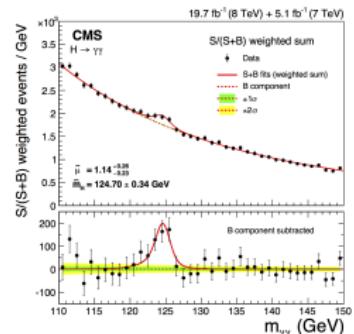
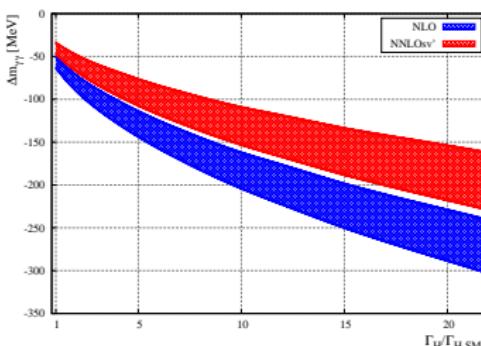
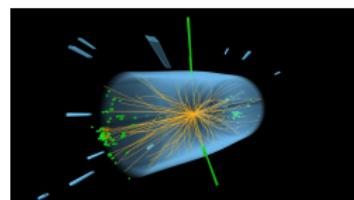
Signal-background interference effects in Higgs-mediated diphoton production beyond NLO

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[*talk by Buccioni*]

Outlook

phenomenological

formal

- interesting **background** for Higgs production
- full-NNLO interference to constrain the **Higgs width**
- why is only **1** boundary integral enough ?
- can we unveil **hidden** amplitude **structure** ?

THANK YOU

Presentation plan

6 Appendix

Tensor projectors

$$\mathcal{P}_i = \sum_{k=1}^{10} (M^{-1})_{ik} \bar{T}_k^\dagger \quad M_{ij} = \sum_{pol} \bar{T}_i^\dagger \bar{T}_j \quad \sum_{pol} \epsilon_i^\mu \epsilon_i^{*\nu} = -g^{\mu\nu} + \frac{p_i^\mu q_i^\nu + q_i^\mu p_i^\nu}{p_i \cdot q_i}$$

$$M^{-1} = \begin{pmatrix} \frac{X^{(0)} + dX^{(1)}}{3(d-1)(d-3)t^2} & 0 & 0 \\ 0 & \frac{2}{(d-4)(d-3)} & \frac{1}{(d-4)(d-3)} \\ 0 & \frac{1}{(d-4)(d-3)} & \frac{2}{(d-4)(d-3)} \end{pmatrix},$$

$$X^{(0)} = \begin{pmatrix} -\frac{8}{s^2} + \frac{32}{su} - \frac{8}{u^2} & -\frac{2}{s} - \frac{2}{u} & \frac{4s}{u^2} - \frac{4}{s} - \frac{12}{u} & \frac{2}{s} + \frac{2}{u} & -\frac{2}{s} - \frac{2}{u} & -\frac{4u}{s^2} + \frac{12}{s} + \frac{4}{u} & \frac{2}{s} + \frac{2}{u} & -\frac{2s}{u} - \frac{2u}{s} - 7 \\ -\frac{2}{s} - \frac{2}{u} & -2 & \frac{s}{u^2} + 2 & -1 & 1 & -\frac{u}{s} - 2 & -1 & -t \\ \frac{4s}{u^2} - \frac{4}{s} - \frac{12}{u} & \frac{s}{u} + 2 & -\frac{2s^2}{u^2} + \frac{4s}{u} + 4 & -\frac{s}{u} - 2 & \frac{s}{u} + 2 & -\frac{2s}{u} - \frac{2u}{s} - 5 & -\frac{s}{u} - 2 & \frac{s^2}{u} + 3s + 2u \\ \frac{2}{s} + \frac{2}{u} & -1 & -\frac{s}{u} - 2 & -2 & -1 & \frac{u}{s} + 2 & 1 & t \\ -\frac{2}{s} - \frac{2}{u} & 1 & \frac{s}{u} + 2 & -1 & -2 & -\frac{u}{s} - 2 & -1 & -t \\ -\frac{4u}{s^2} + \frac{12}{su} + \frac{4}{u} & -\frac{u}{s} - 2 & -\frac{2s}{u} - \frac{2u}{s} - 5 & \frac{u}{s} + 2 & -\frac{u}{s} - 2 & -\frac{2u^2}{s^2} + \frac{4u}{s} + 4 & \frac{u}{s} + 2 & -\frac{u^2}{s} - 2s - 3u \\ \frac{2}{s} + \frac{2}{u} & -1 & -\frac{s}{u} - 2 & 1 & -1 & \frac{u}{s} + 2 & -2 & t \\ -\frac{2s}{u} - \frac{2u}{s} - 7 & -t & \frac{s^2}{u} + 3s + 2u & t & -t & -\frac{u^2}{s} - 2s - 3u & t & t^2 \end{pmatrix},$$

$$X^{(1)} = \begin{pmatrix} \frac{12}{s^2} - \frac{12}{su} + \frac{3(2+d)}{t^2} + \frac{12}{u^2} & \frac{3}{t} & -\frac{6s}{u^2} + \frac{3}{t} + \frac{3}{u} & -\frac{3}{t} & \frac{3}{t} & \frac{6u}{s^2} - \frac{3}{s} - \frac{3}{t} & -\frac{3}{t} & 0 \\ \frac{2}{t} & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{6s}{u^2} + \frac{3}{t} + \frac{3}{u} & 0 & \frac{3s^2}{u^2} & 0 & 0 & 0 & 0 & 0 \\ -\frac{3}{t} & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ \frac{3}{t} & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ \frac{6u}{s^2} - \frac{3}{s} - \frac{3}{t} & 0 & 0 & 0 & 0 & \frac{3u^2}{s^2} & 0 & 0 \\ -\frac{3}{t} & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$