MIXED QCD-EW CORRECTIONS TO DILEPTON PRODUCTION AT THE LHC IN THE HIGH INVARIANT MASS REGION

In collaboration with: F. Buccioni, F. Caola, H. Chawdhry, M. Heller, A. von Manteuffel, K. Melnikov, R. Röntsch, C.Signorile-Signorile

Based on: arXiv 2203.11237, *JHEP* 06 (2022) 022

OUTLINE

- Motivation: why do we care about mixed QCD-EW corrections @ high invariant mass?
- How to tackle such a computation?
- What do we learn in the end?

LARGE INVARIANT MASS REGION

NEW PHYSICS SEARCHES

- New resonances with dilepton decays, distortions in kinematic distributions
- Constrain NP in model-independent way using SMEFT operators
- Flavour anomalies

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High precision theoretical predictions within the SM needed!

MIXED CORRECTIONS IN THE LARGE INVARIANT MASS REGION

SUDAKOV FACTORIZATION

- $\alpha_s \sim 0.1, \alpha_{ew} \sim 0.01$ → QCDxEW $\sim 0.1\,\%$

At high energies spoiled by Sudakov logs!

- Physical cut-off given by W/Z mass, large single and double logs at high energies from virtual corrections

- W and Z boson can be detected as "real" particles!

$$\frac{\alpha_{ew}}{4\pi \sin^2\theta_W} \log^2\left(\frac{s}{m_W^2}\right) \sim 6\,\% \,, \quad \frac{\alpha_{ew}}{4\pi \sin^2\theta_W} \log\left(\frac{s}{m_W^2}\right) \sim 1\,\% \,, \quad \sqrt{s} = 1\text{TeV}$$

[Kuhn, Penin, Smirnov '00][Ciafaloni, Ciafaloni, Comelli '01]
[Denner, Pozzorini '01]

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MIXED CORRECTIONS IN THE LARGE INVARIANT MASS REGION

How well does it work in TeV region?

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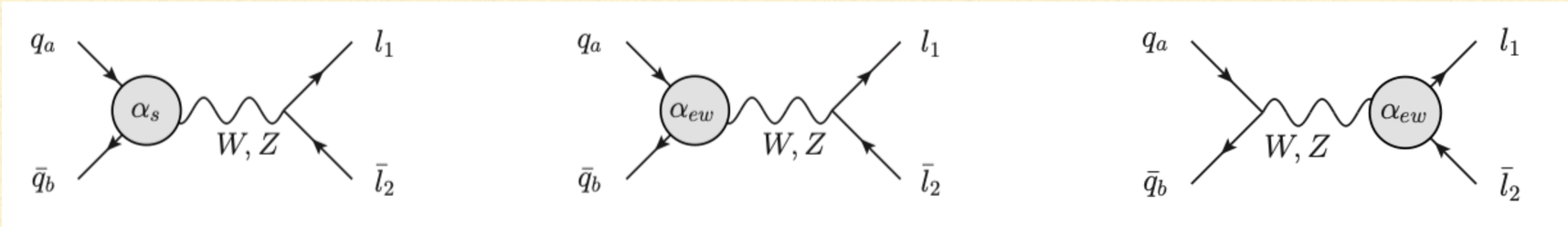
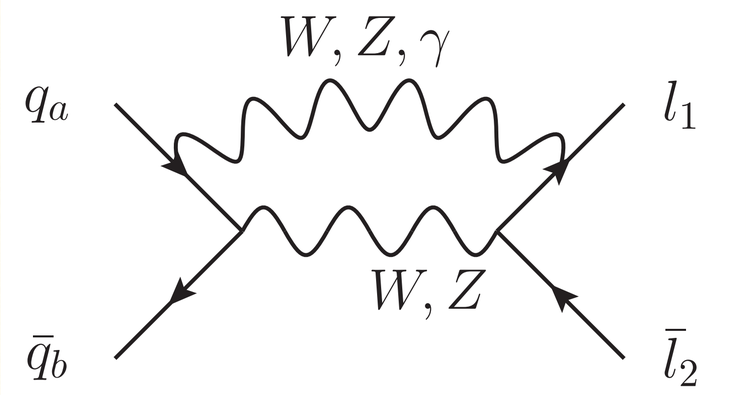
ON-SHELL VS OFF-SHELL



[Dittmaier, Huss, Schwinn '15] [Delto, Jaquier, Melnikov, Röntsch '20] [Buccioni et al. '20] [Bonciani, Buccioni, Rana, Vicini '20] [Behring et al. '21] [Bonciani, Buccioni, Rana, Vicini '21]

Non-factorizable contributions suppressed in the resonance region but not in far off-shell region

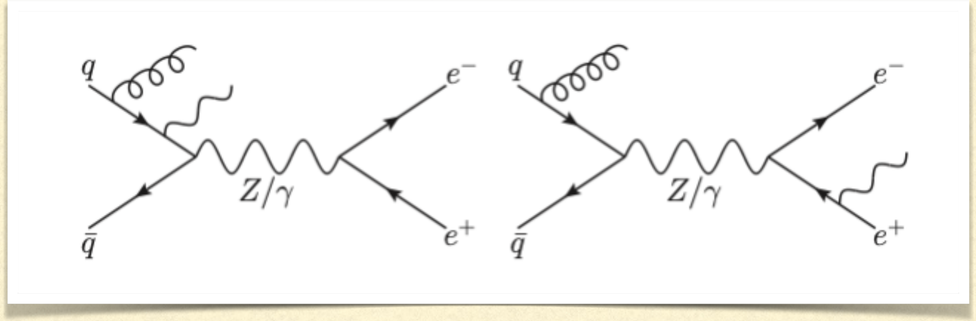


[Buonocore, Grazzini, Kallweit, Savoini, Tramontano 2102.12539]

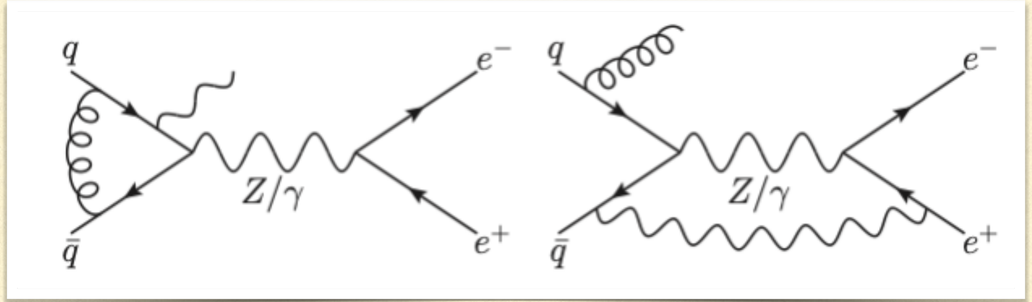
[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini 2106.11953]

*[Buccioni, Caola, Chawdhry, **FD**, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile]*

$$d\hat{\sigma}_{ij} = d\hat{\sigma}_{ij,\,\mathrm{LO}}\left(1+\alpha_s\,\Delta^{QCD}_{ij,\,\mathrm{NLO}}+\alpha_{ew}\,\Delta^{EW}_{ij,\,\mathrm{NLO}}+\alpha_s^2\,\Delta^{QCD}_{ij,\,\mathrm{NNLO}}+\alpha_s\,\alpha_{ew}\,\Delta^{QCD\otimes EW}_{ij,\,\mathrm{NNLO}}+\alpha_s^3\,\Delta^{QCD}_{ij,\,\mathrm{N3LO}}+\ldots\right)$$



Double real contributions



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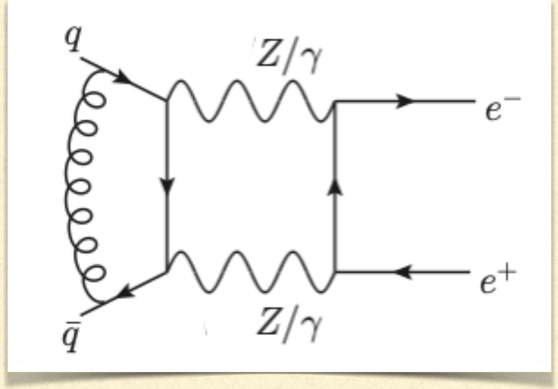

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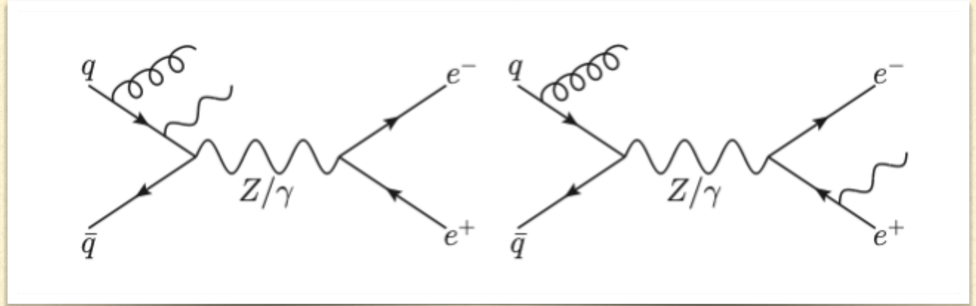

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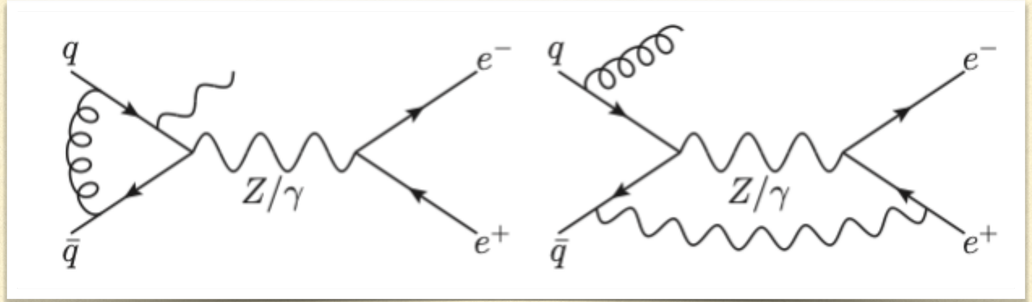

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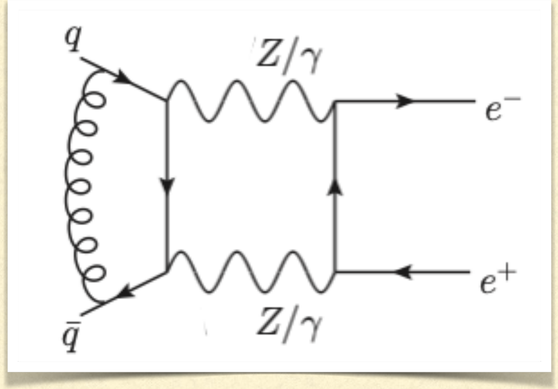

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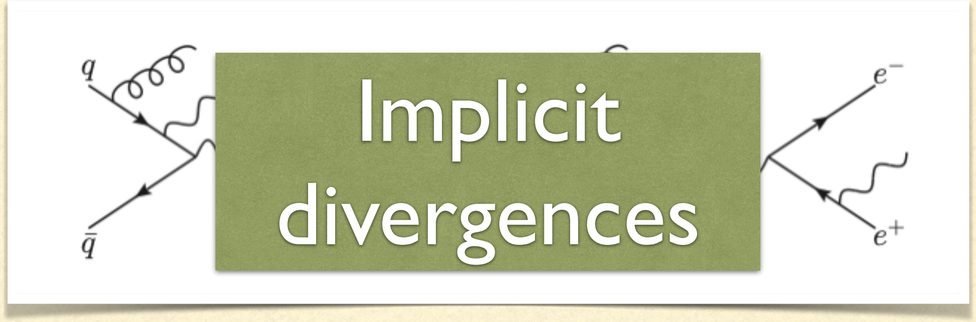
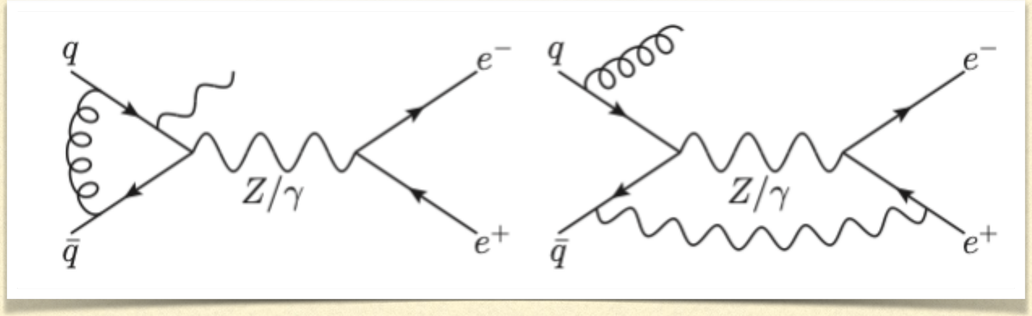


Implicit divergences

Double real contributions



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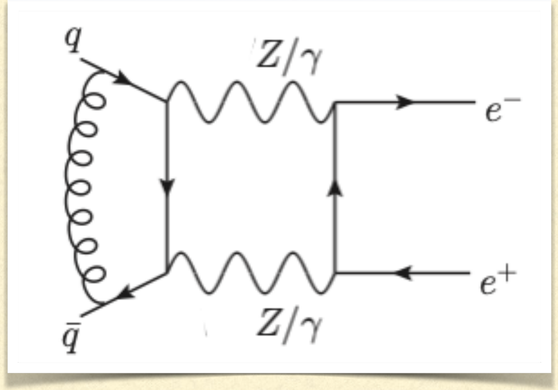

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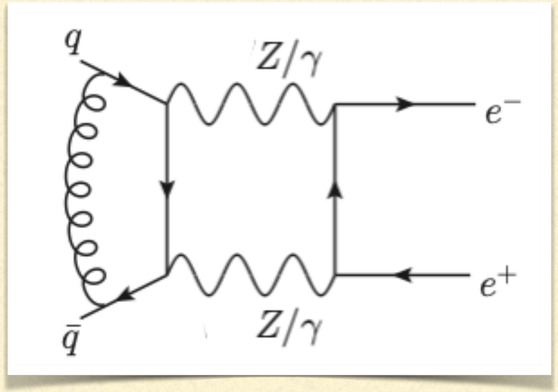

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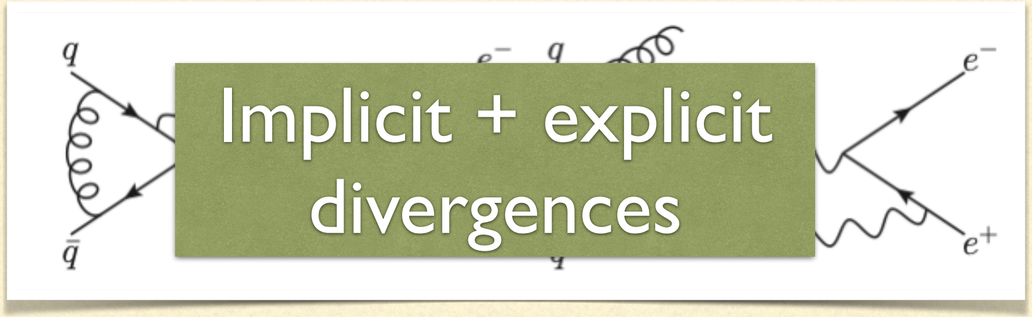

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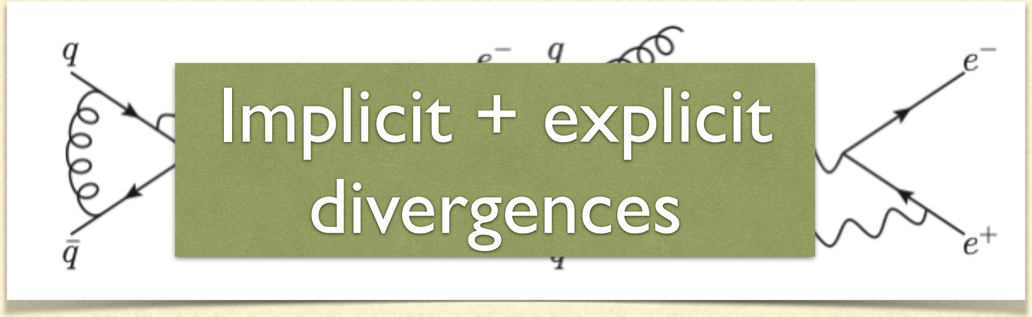

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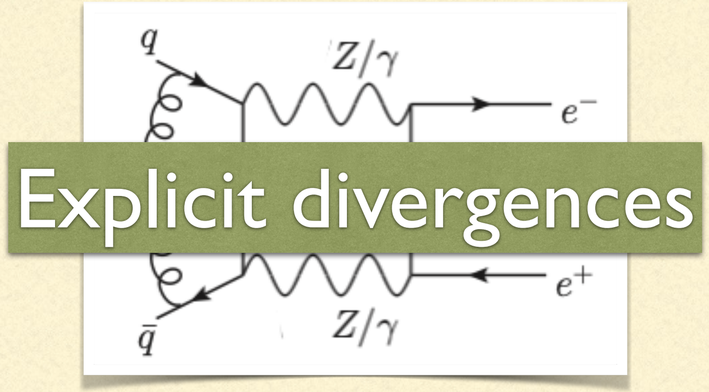


Double virtual contributions

Scattering amplitudes

Two-loop integrals involving several energy scales

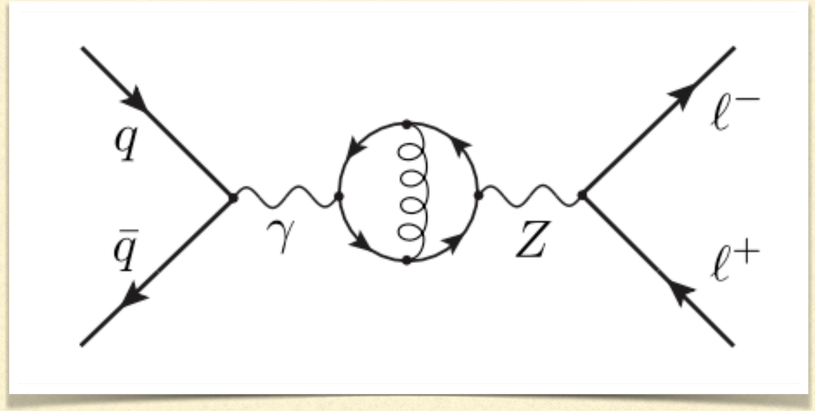
- *[Heller, von Manteuffel, Schabinger '20] [Heller, von Manteuffel, Schabinge, Spiesberger '21]*

Extension of nested soft-collinear subtraction scheme to deal with mixed corrections

*[Buccioni, Caola, Chawdhry, **FD**, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile]*

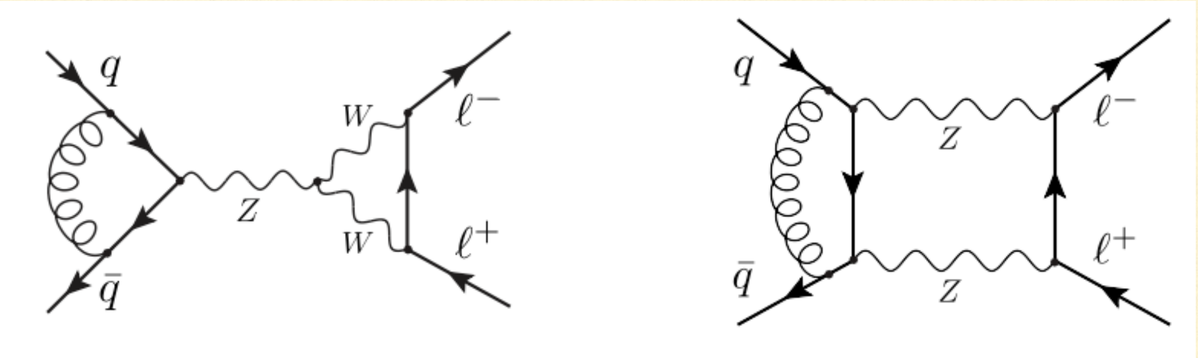
Regularisation of infrared divergences

TWO-LOOP AMPLITUDES



Fermionic contributions

Bosonic contributions



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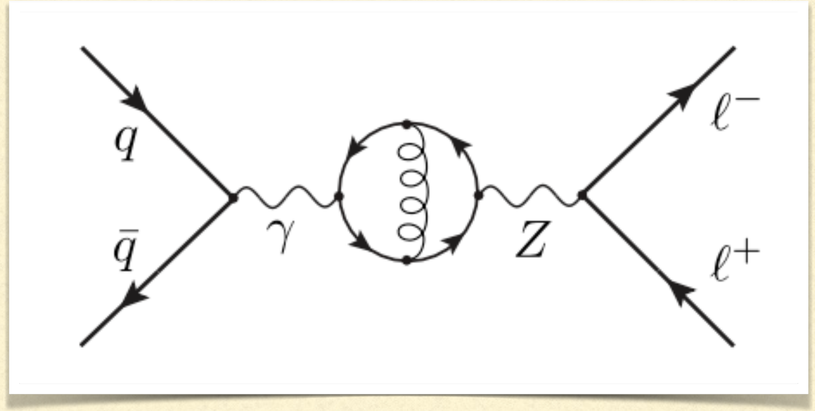


Fermionic contributions

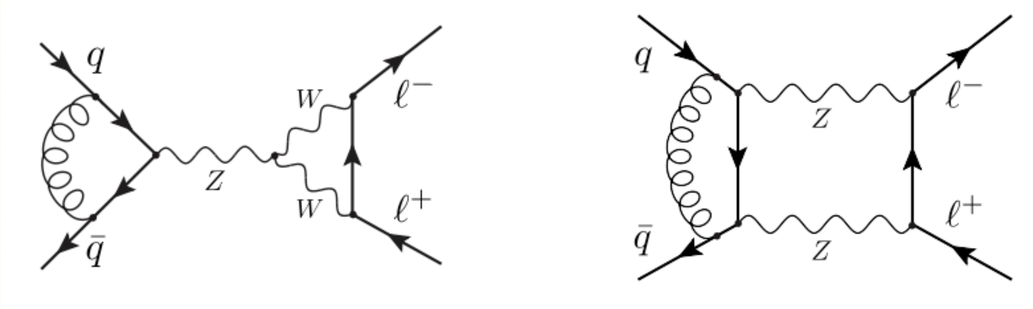
2-point function: not so hard to compute…

Bulk of renormalization @ $\mathcal{O}(\alpha\,\alpha_s)$

Contains large logs at high invariant mass

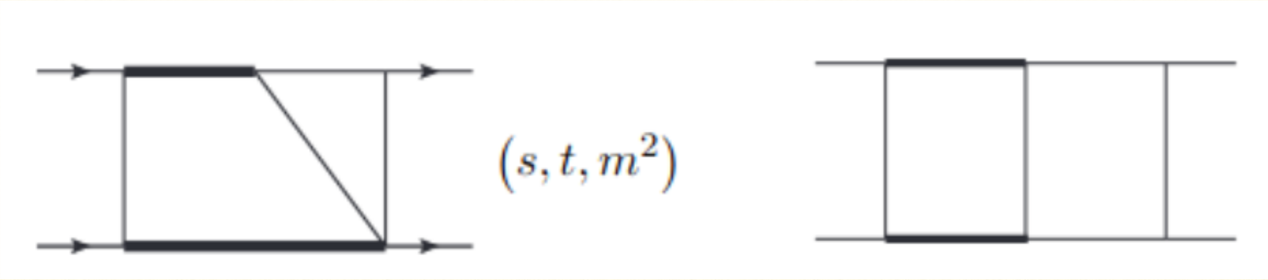
$$\log^2 \frac{m_t^2}{s}$$

TWO-LOOP AMPLITUDES



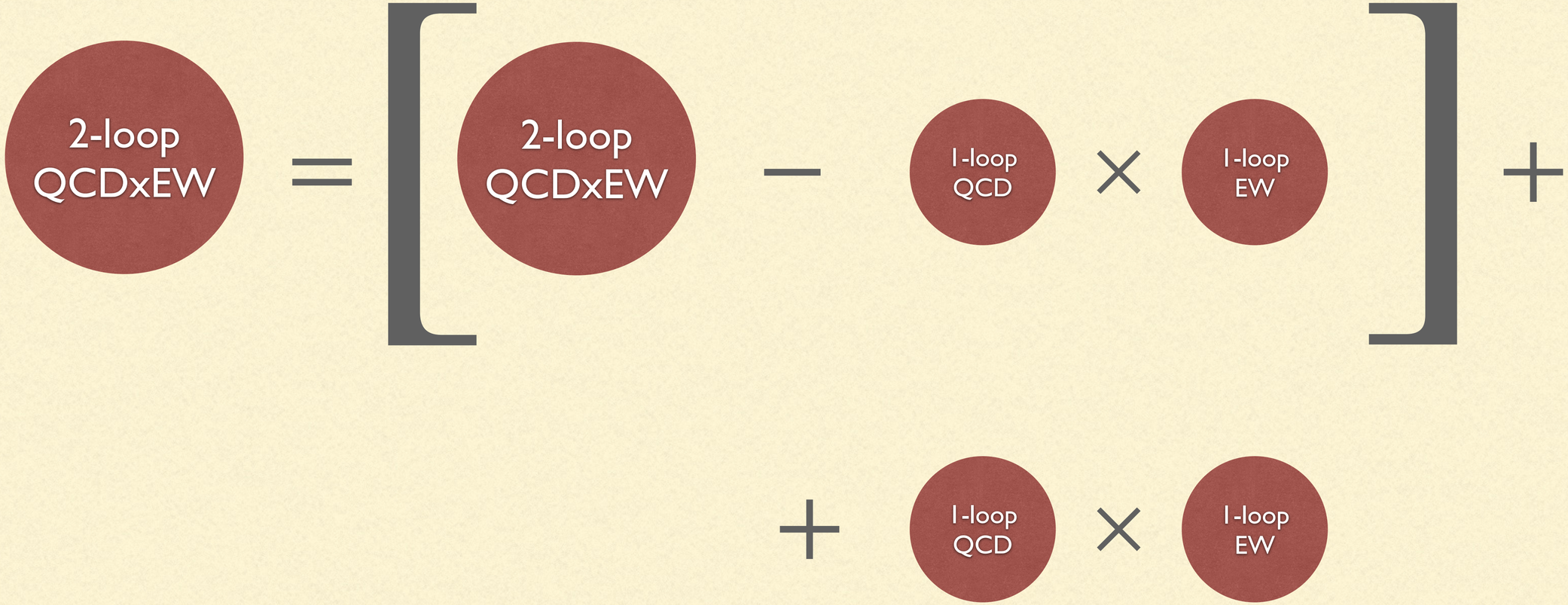
Bosonic contributions

Complicated two-loop master integrals with up to two internal masses and non-rationalizable square roots



Fully analytic computation of two-loop helicity amplitudes for dilepton production

[Heller, von Manteuffel, Schabinger, Spiesberger 1907.00491,2012.05918]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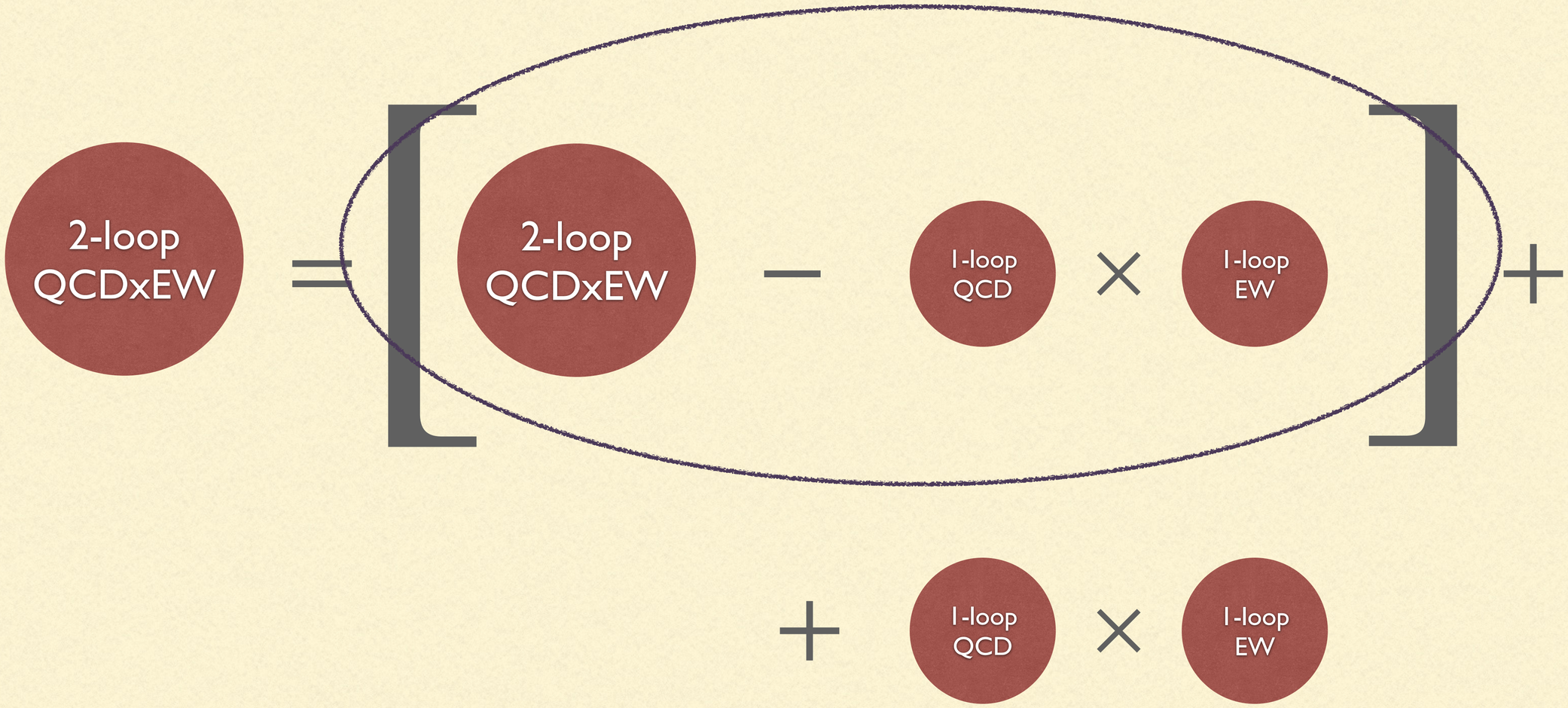
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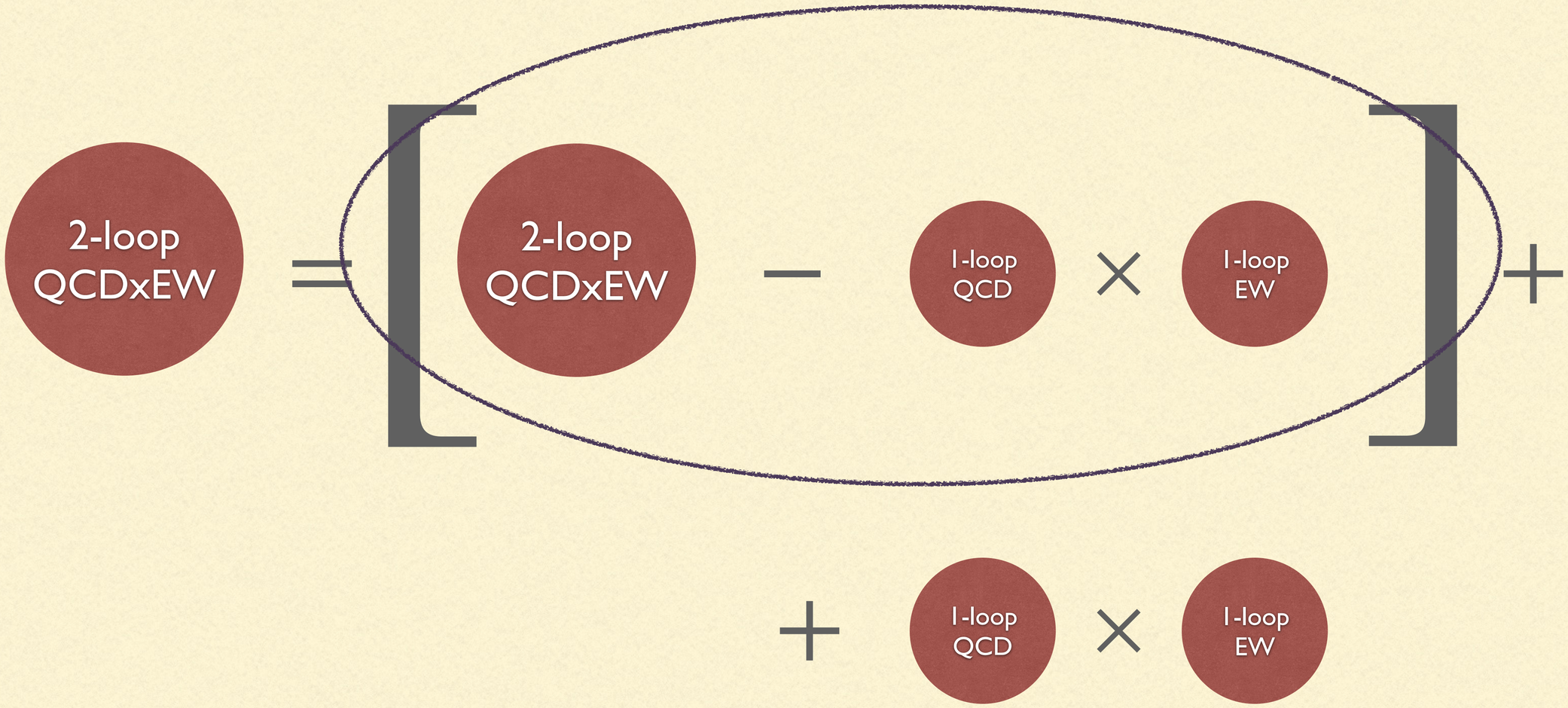
“Non-factorizable” contribution



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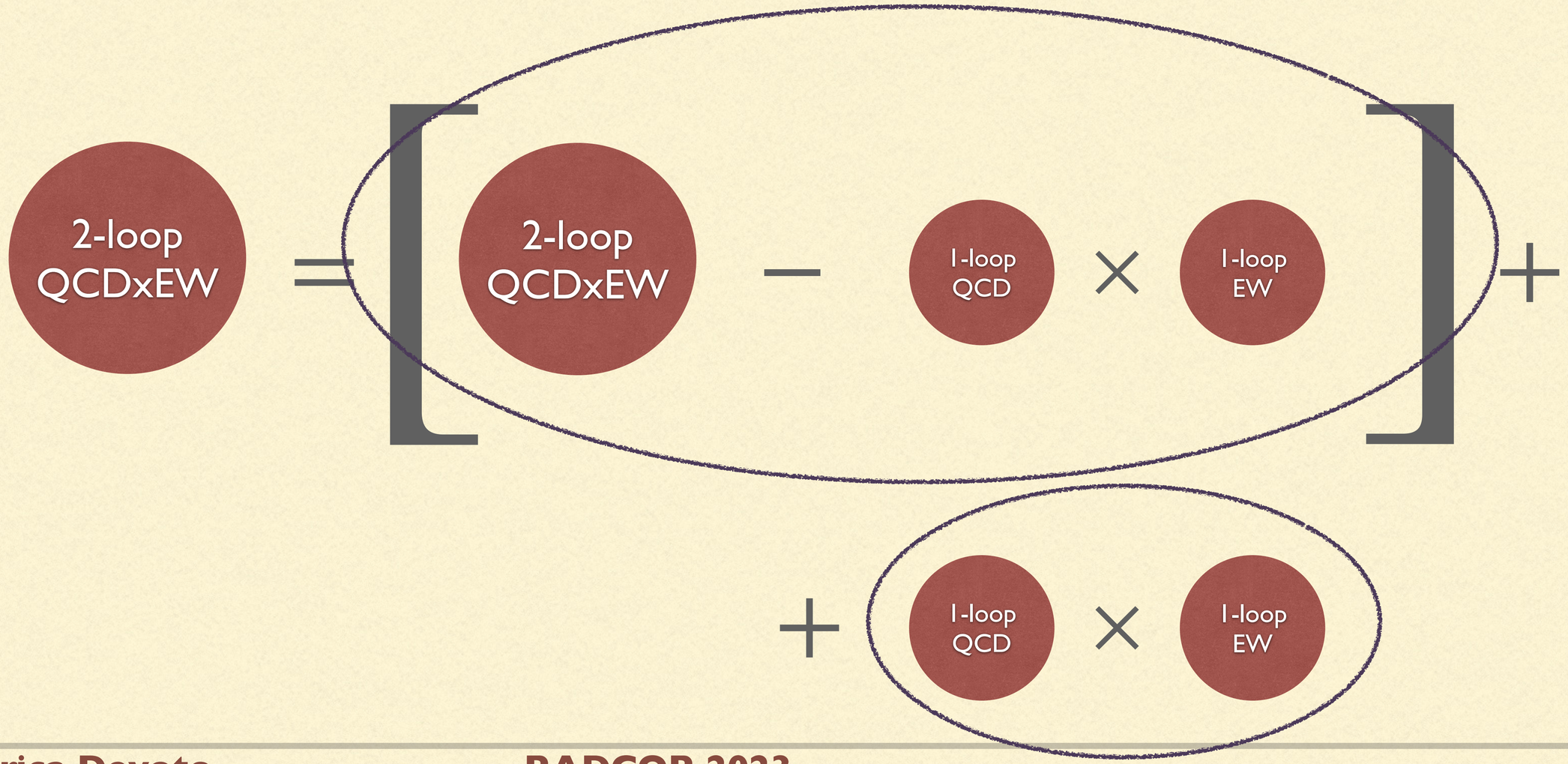

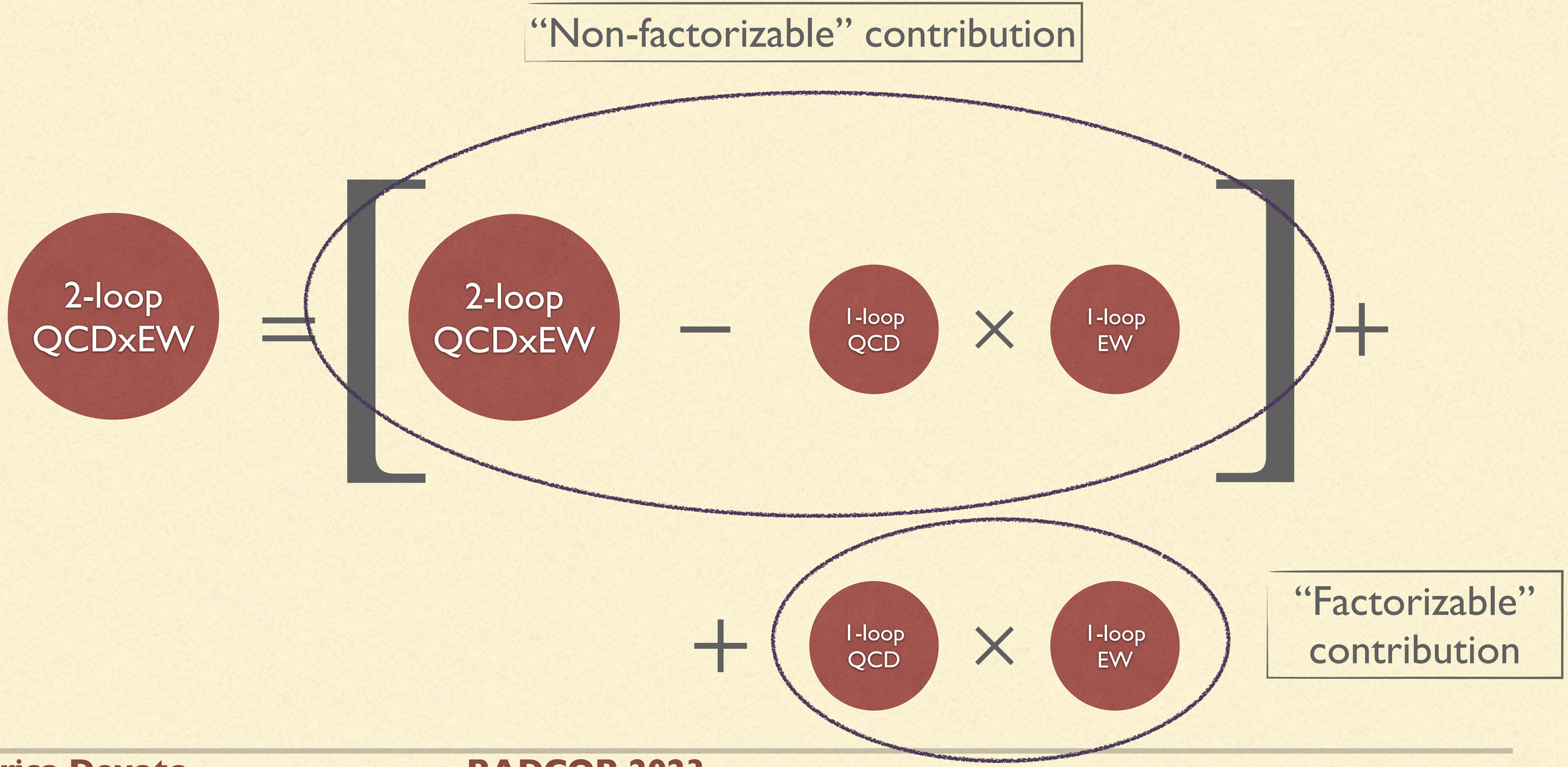
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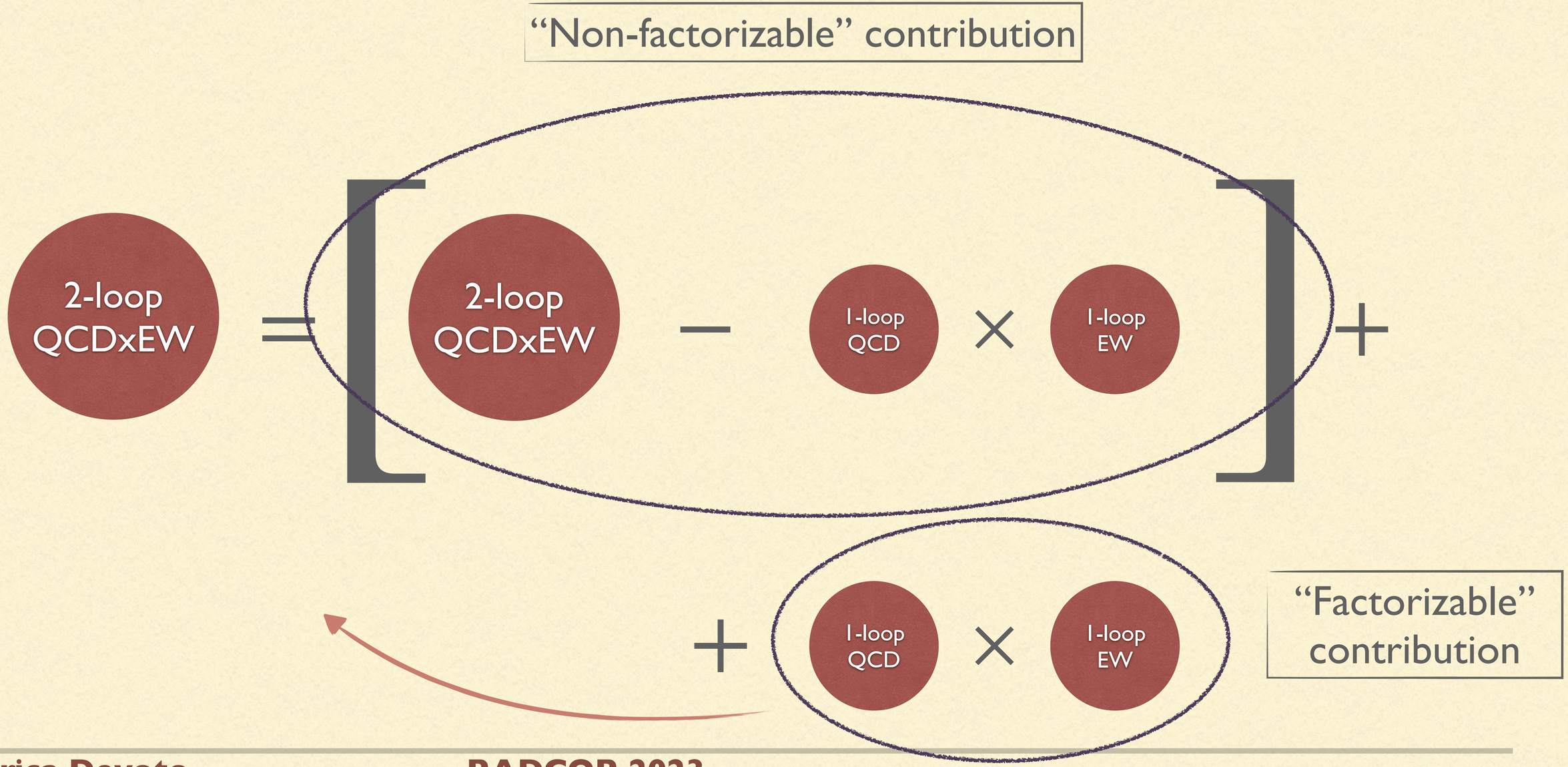
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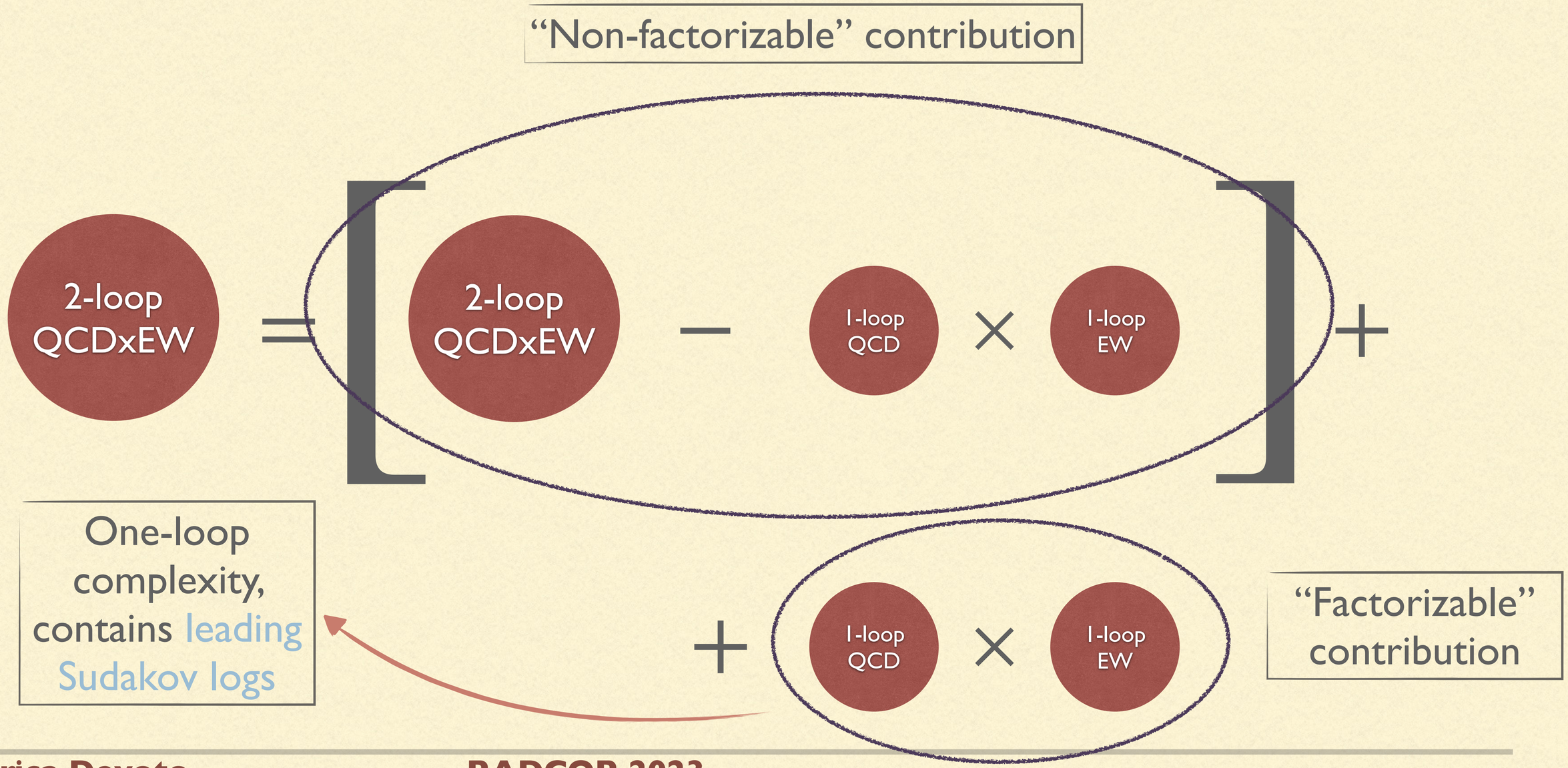
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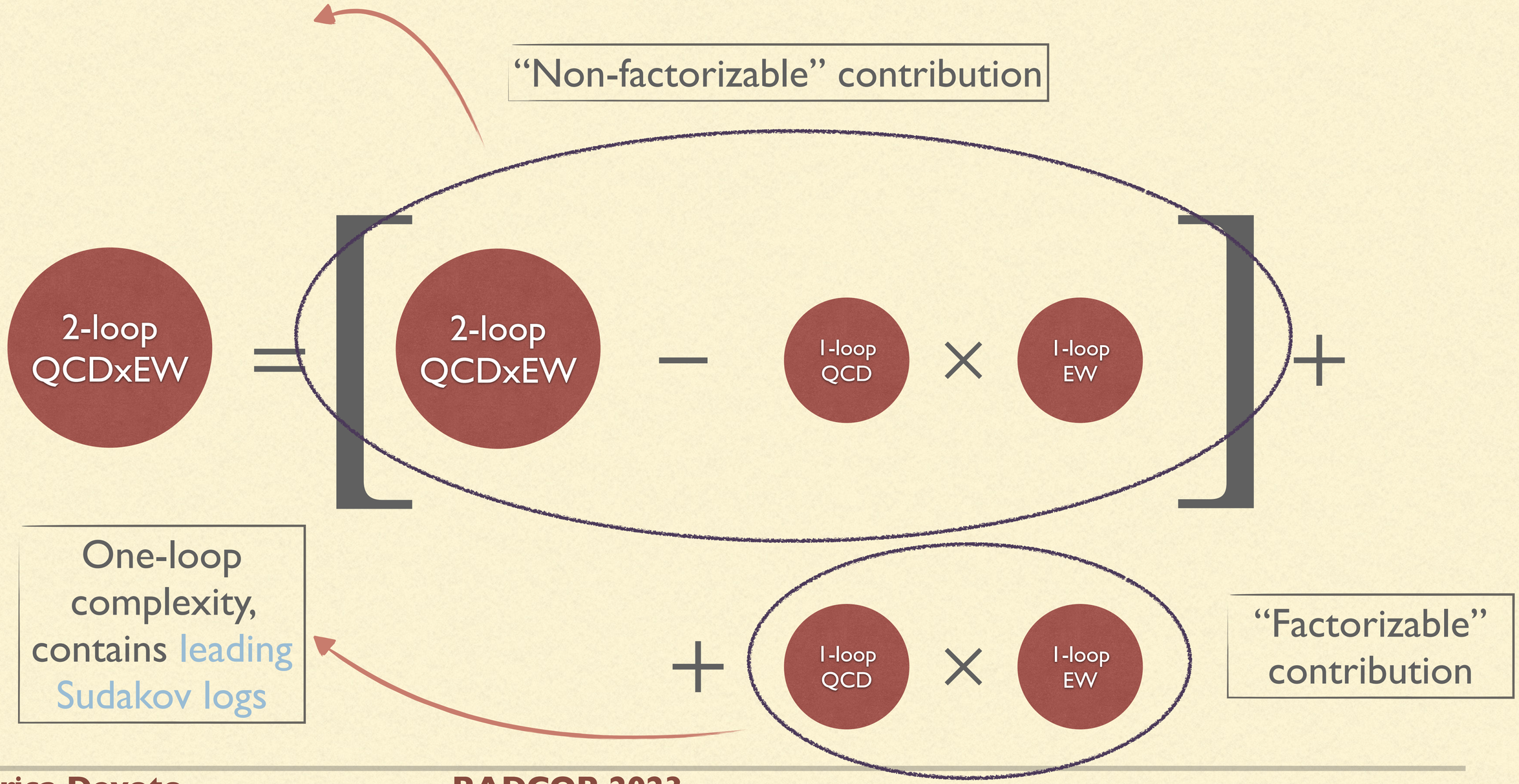
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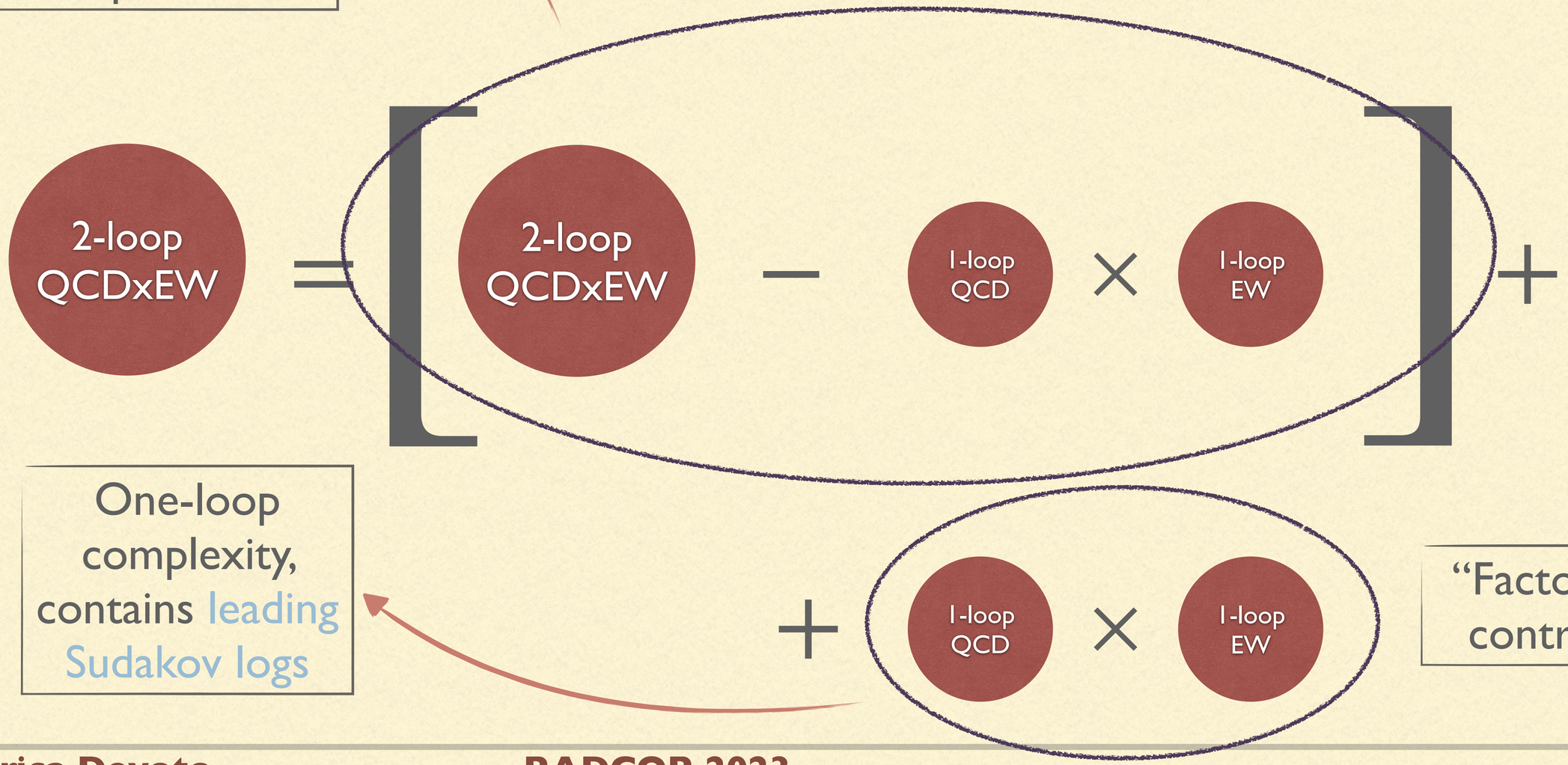
Genuine two-loop, much harder to compute

"Non-factorizable" contribution



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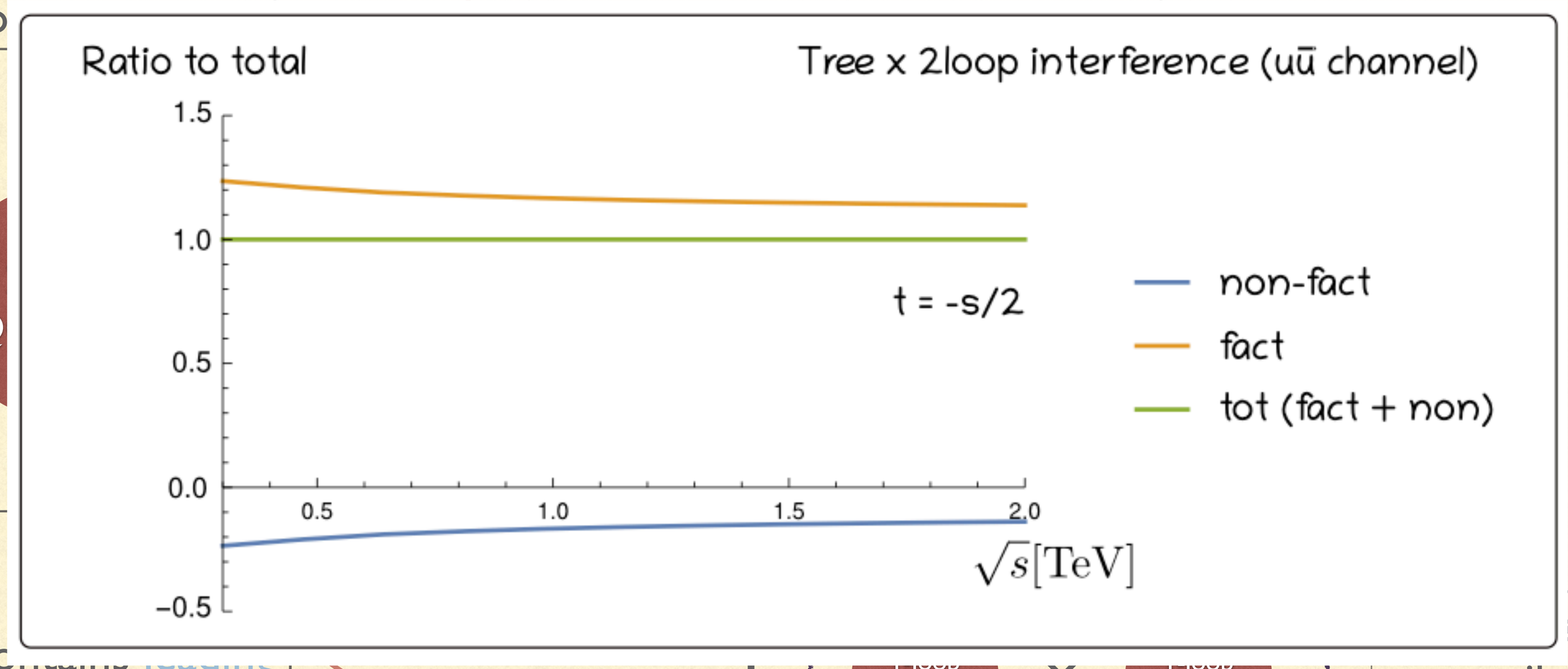

Q

contains leading Sudakov logs

+

QCD

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EW

zable" contribution

INFRARED REGULARISATION

- Nested soft-collinear subtraction scheme
  [Caola, Melnikov, Röntsch 1702.01352]

Extension of FKS subtraction to NNLO

Fully local and analytic

Exploit color coherence, e.g. soft and collinear limits commute

Initially developed for pure NNLO QCD processes

INFRARED REGULARISATION

How do we extend the scheme to mixed QCD-EW corrections?

- Nested soft-collinear sub

[Caola, Melnikov, Röntsch 1702

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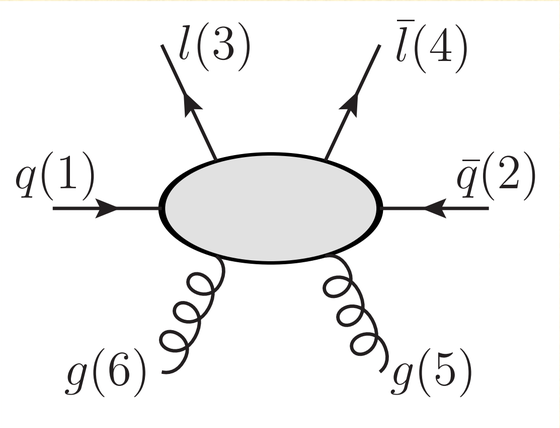
SOFT LIMITS

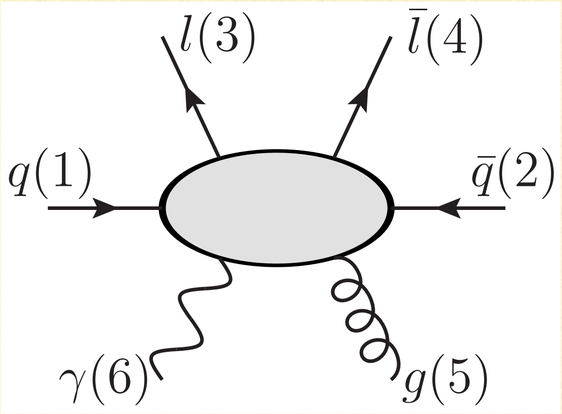
NNLO QCD



Non-trivial structure of double-soft eikonal, need energy ordering

$$1 = \theta\left(E_{g_1} - E_{g_2}\right) + \theta\left(E_{g_2} - E_{g_1}\right)$$

Mixed QCDxEW



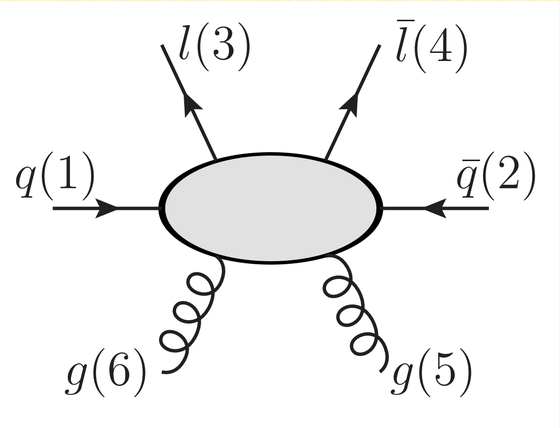
Double-soft limit factorizes into product of two "NLO"-eikonals

$$\lim_{E_g, E_\gamma \to 0} |\mathcal{M}_{RR}|^2 = g_s^2 \, \mathrm{Eik}(p_1, p_2; p_5) \; e^2 \sum_{i,j} Q_i Q_j \, \mathrm{Eik}(p_i, p_j; p_6) \, |\mathcal{M}_B|^2$$

Non-trivial structures to integrate over phase-space

COLLINEAR LIMITS

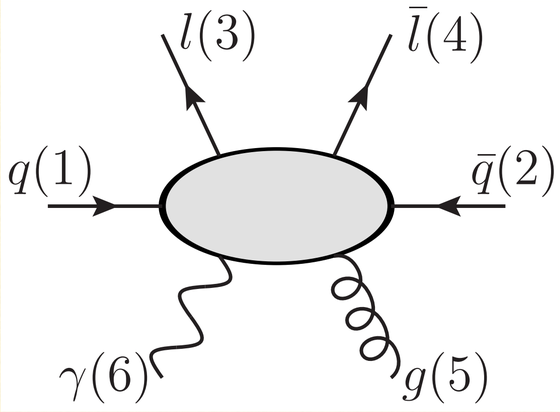
NNLO QCD



Initial-state radiation only

Strongly ordered limits to disentangle in triple-collinear sectors

Mixed QCDxEW

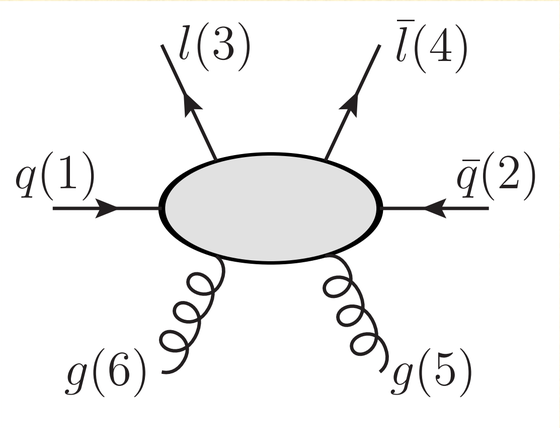


Final state QED radiation, more sectors needed

No photon-gluon collinear limit!

COLLINEAR LIMITS

NNLO QCD



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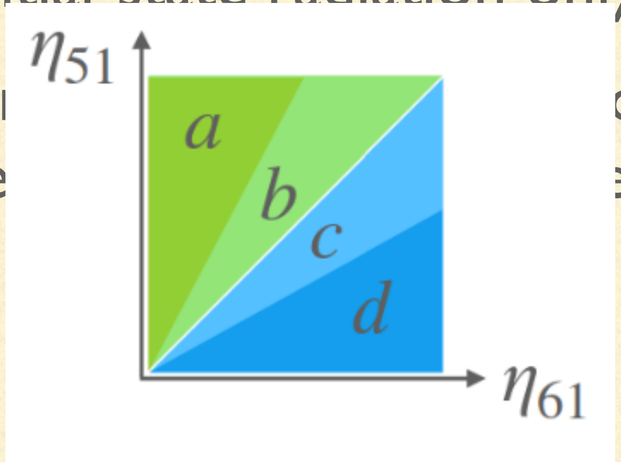

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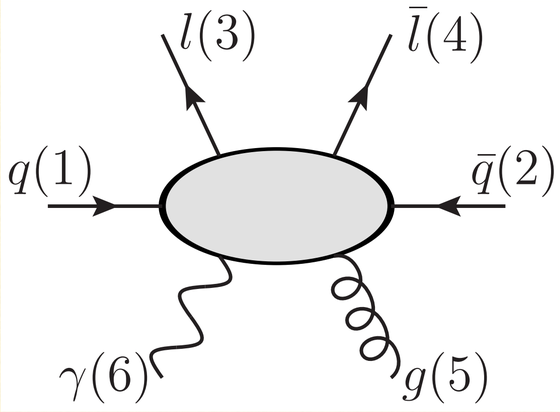

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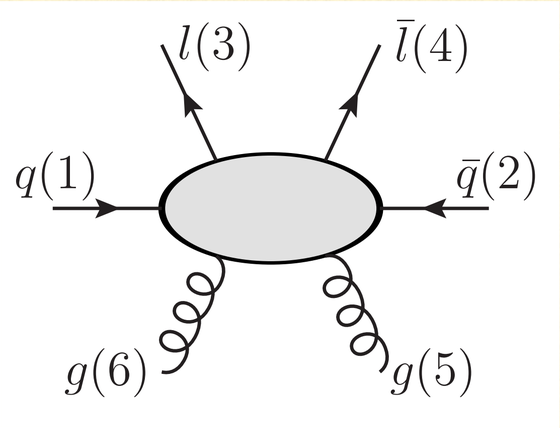
COLLINEAR LIMITS

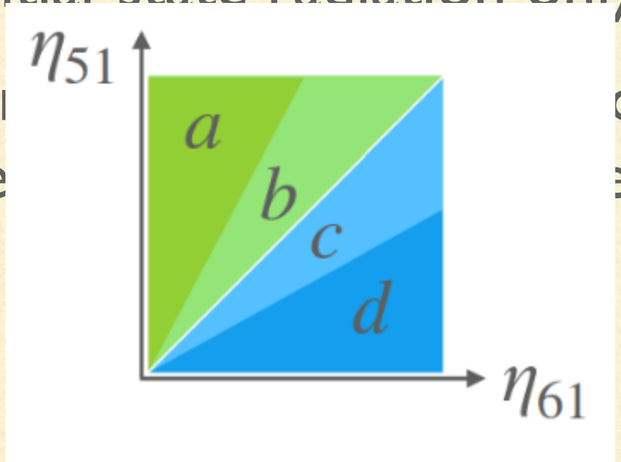
NNLO QCD



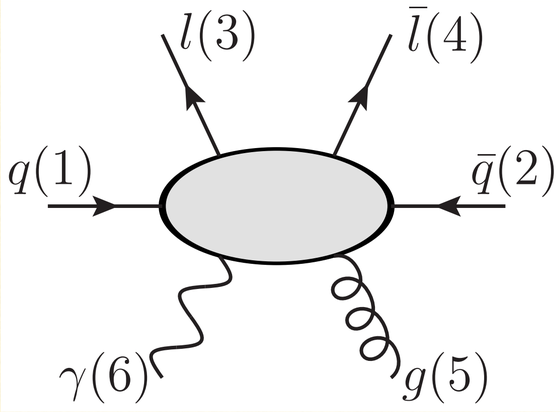
Initial-state radiation only

Str o

dise ear



Mixed QCDxEW

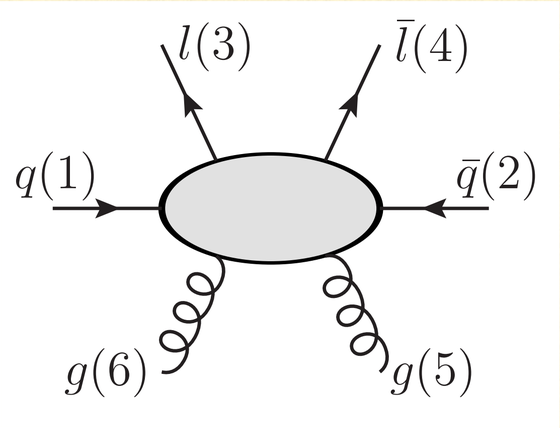


Final state QED radiation, more sectors needed

No photon-gluon collinear limit!

COLLINEAR LIMITS

NNLO QCD



Initial-state radiation only

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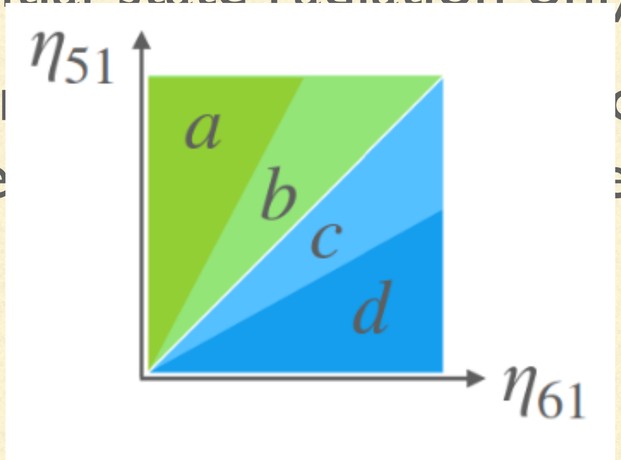
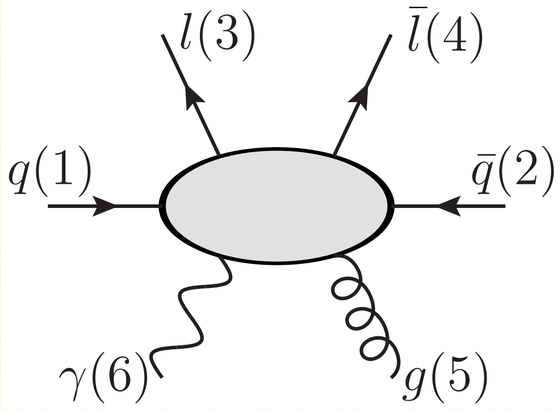
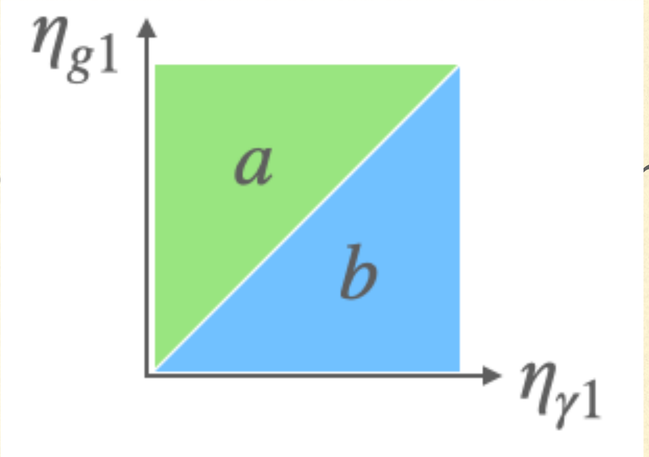


Mixed QCDxEW



Final state QED radiation, more

No p mit!



$$d\hat{\sigma}^{q\bar{q}}_{\rm mix} = d\hat{\sigma}^{q\bar{q}}_{\rm vv} + d\hat{\sigma}^{q\bar{q}}_{{\rm rv},\gamma} + d\hat{\sigma}^{q\bar{q}}_{{\rm rv},g} + d\hat{\sigma}^{q\bar{q}}_{{\rm rr},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{{\rm rr},q\bar{q}} + d\hat{\sigma}^{q\bar{q}}_{\rm pdf}$$

Sum of divergent pieces

Fully analytic

Explicit poles cancellation

Fully differential wrt unresolved particles

$$d\hat{\sigma}^{q\bar{q}}_{{\rm mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{{\rm el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{{\rm bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\rm nlo},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{{\rm reg},g\gamma}$$

Sum of finite pieces

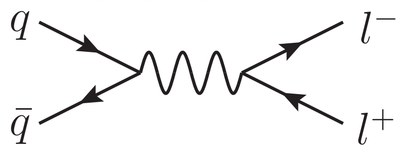
Final result written in terms of simple recurring structures!

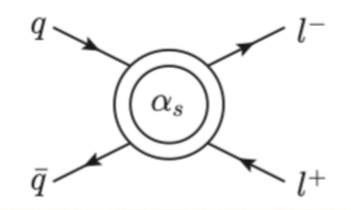
Stable numerical evaluation

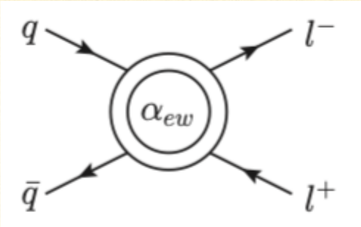
See Chiara's talk on Thursday for advances in nested soft-collinear subtraction!

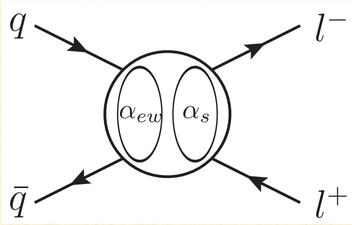
$$d\hat{\sigma}^{q\bar{q}}_{\mathrm{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\mathrm{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\mathrm{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{reg},g\gamma}$$

"Elastic" contribution

$f(s,\eta_{ij}) \times$ 

$+\ g(s,\eta_{ij}) \times$ 

$+\ h(s) \times$ 

$+$ 

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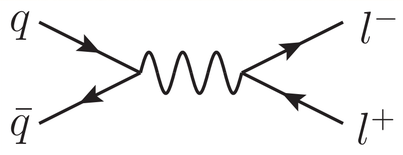
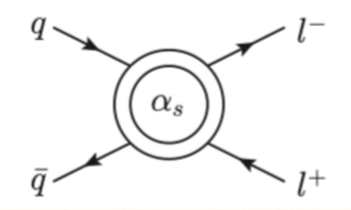
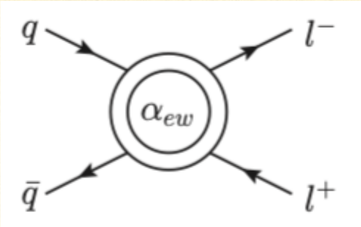
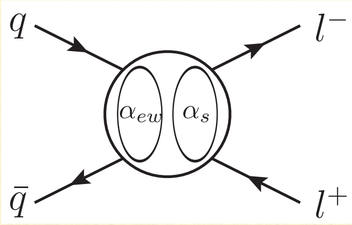
$f(s,\eta_{ij}) \times$ 

$+\ g(s,\eta_{ij}) \times$ 

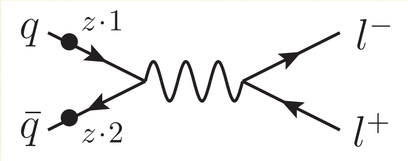
$+\ h(s) \times$ 

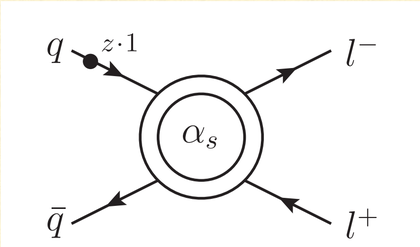
$+$ 

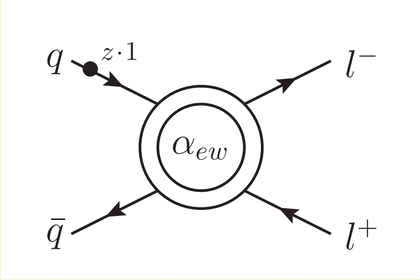
$$
\begin{aligned}
2s \cdot d\hat{\sigma}^{q\bar{q}}_{\mathrm{el},g\gamma} = {} & \left\langle F^{(\mathrm{QCD}\times\mathrm{EW}),\,\mathrm{fin}}_{\mathrm{LVV+LV^2}}(1_q, 2_{\bar{q}}, 3, 4)\right\rangle \\
& + [\alpha] \left\langle \left[\mathcal{G}_{\mathrm{EW}} + 3Q_q^2 \log\left(\frac{s}{\mu^2}\right)\right] F^{(\mathrm{QCD}),\,\mathrm{fin}}_{\mathrm{LV}}(1_q, 2_{\bar{q}}, 3, 4)\right\rangle \\
& + [\alpha_s]\, C_F \left[\frac{2}{3}\pi^2 + 3\log\left(\frac{s}{\mu^2}\right)\right] \left\langle F^{(\mathrm{EW}),\,\mathrm{fin}}_{\mathrm{LV}}(1_q, 2_{\bar{q}}, 3, 4)\right\rangle \qquad (2 \\
& + [\alpha]\,[\alpha_s]\, C_F \left\langle \left\{ Q_q^2 \left[-\frac{4\pi^4}{45} + \left(2\pi^2 + 32\zeta_3\right) \log\left(\frac{s}{\mu^2}\right)\right.\right.\right. \\
& \left.\left.\left. + \left(9 - \frac{4\pi^2}{3}\right) \log^2\left(\frac{s}{\mu^2}\right)\right] + \mathcal{G}_{\mathrm{EW}} \left(\frac{2\pi^2}{3} + 3\log\left(\frac{s}{\mu^2}\right)\right)\right\} F_{\mathrm{LM}}(1_q, 2_{\bar{q}}, 3, 4)\right\rangle
\end{aligned}
$$

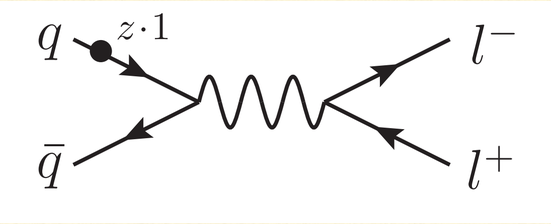
$$d\hat{\sigma}^{q\bar{q}}_{\mathrm{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\mathrm{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\mathrm{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{reg},g\gamma}$$

"Boosted" contribution

$f(s, \eta_{ij}, z_1, z_2) \times$ 

$+ \quad g(s, z) \times$ 

$+ \quad h(s, z) \times$ 

$+ \quad k(s, z, \eta_{ij}) \times$ 

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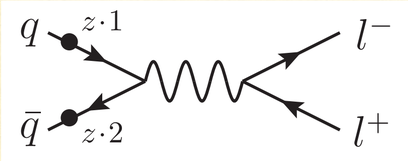
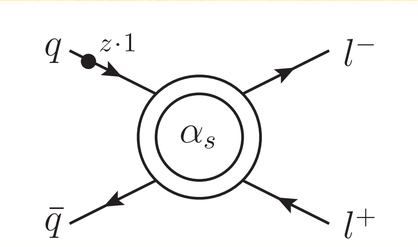
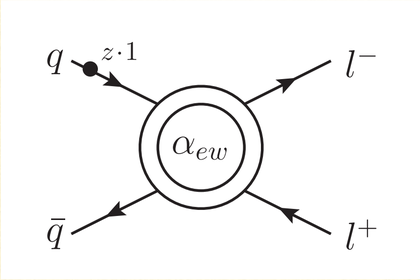
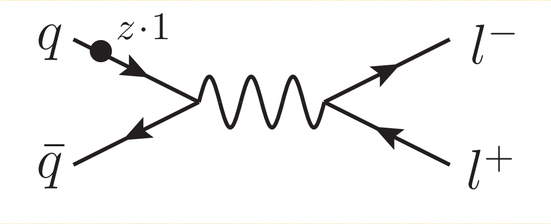
$f(s, \eta_{ij}, z_1, z_2) \times$ 

$+ \quad g(s, z) \times$ 

$+ \quad h(s, z) \times$ 

$+ \quad k(s, z, \eta_{ij}) \times$ 

$$
\begin{aligned}
2s \cdot d\sigma^{q\bar{q}}_{\mathrm{bt},g\gamma} = {} & [\alpha]\,[\alpha_s]\,2C_F\,Q_q^2 \int_0^1 dz_1 dz_2\,\tilde{P}^{\mathrm{NLO}}_{qq}(z_1, E_c) \left\langle \frac{F_{\mathrm{LM}}(z_1 \cdot 1, z_2 \cdot 2, 3, 4)}{z_1\, z_2} \right\rangle \tilde{P}^{\mathrm{NLO}}_{qq}(z_2, E_c) \\
& + \sum_{i=1}^{2} \int_0^1 dz\,\tilde{P}^{\mathrm{NLO}}_{qq}(z, E_c) \Big[[\alpha]\,Q_q^2 \left\langle F^{(i),(\mathrm{QCD}),\,\mathrm{fin}}_{\mathrm{LV}}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle \\
& \qquad + [\alpha_s]\,C_F \left\langle F^{(i),(\mathrm{EW}),\,\mathrm{fin}}_{\mathrm{LV}}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle \Big] \\
& + [\alpha]\,[\alpha_s]\,C_F \sum_{i=1}^{2} \int_0^1 dz \Big\langle \Big\{ Q_q^2\,P^{\mathrm{NNLO}}_{qq}(z, E_c) + \tilde{P}^{\mathrm{NLO}}_{qq}(z, E_c) \\
& \qquad \times \Big[Q_e^2\,G_{e^2} + 2Q_q Q_e \Big(G^{(1,2)}_{eq} + (-1)^i \log\Big(\frac{s_{i3}}{s_{i4}}\Big) \log(z) \Big) \Big] \Big\}\, F^{(i)}_{\mathrm{LM}}(1_q, 2_{\bar{q}}, 3, 4; z) \Big\rangle
\end{aligned}
$$

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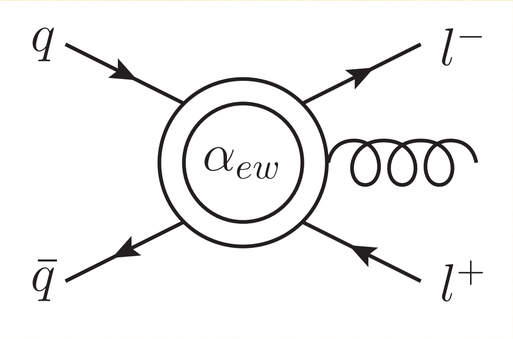
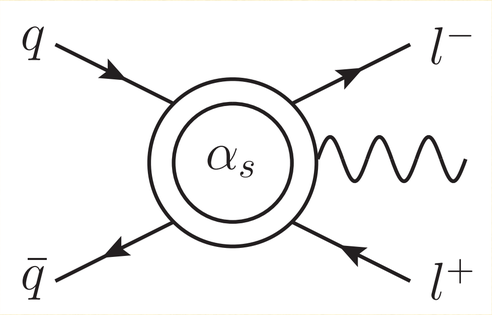
"ONLO" contribution





NLO-like contributions with extra resolved particle

$$d\hat{\sigma}^{q\bar{q}}_{\mathrm{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\mathrm{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\mathrm{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{reg},g\gamma}$$

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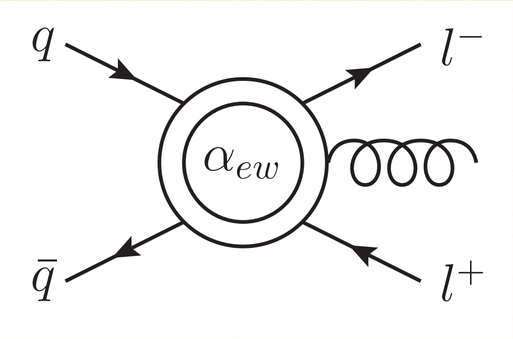

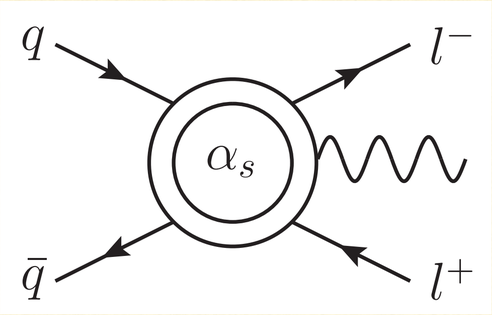

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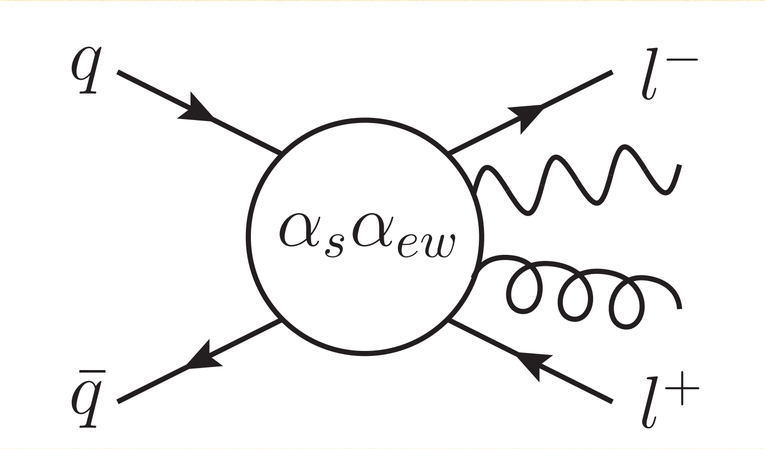
Fully regulated contribution!



Two extra resolved particles, but finite in d=4! All singular contributions have been regulated

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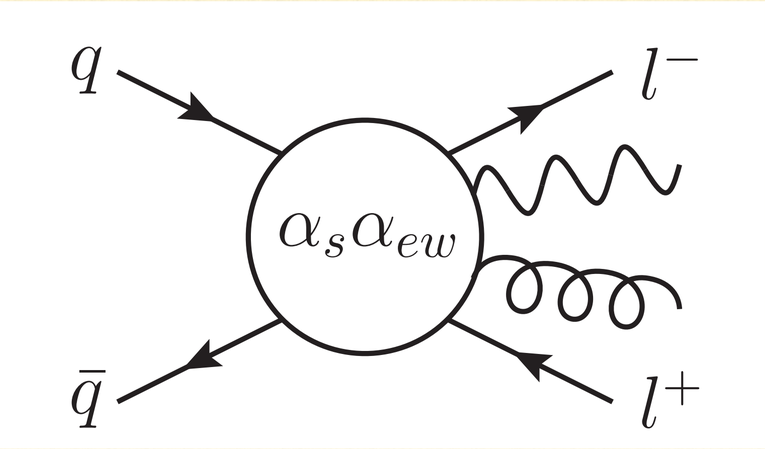

Two extra resolved particles, but finite in d=4! All singular contributions have been regulated

… WHAT DO WE LEARN?

PHENOMENOLOGICAL RESULTS

Setup:

$\sqrt{s} = 13.6\,\text{TeV}$
$m_l = 0$

$m_{ll} > 200\,\text{GeV}$
$R_{l\gamma} = 0.1$ (dressed leptons)
$p_\perp^l > 30\,\text{GeV}$

$\sqrt{p_\perp^{l^+} p_\perp^{l^-}} > 35\,\text{GeV}$
$|y_l| < 2.5$

QCD-bit: $\overline{MS}$ renormalisation $\quad \mu_F = \mu_R = m_{ll}/2$

EW-bit: on-shell renormalisation $\quad G_\mu$ scheme vs $\alpha(m_Z)$ scheme

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \ldots$$

LO NLO QCD NLO EW NNLO QCD NNLO QCDxEW

σ[fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	−49.907	44.60	−16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		−32.66	1.03
$q\gamma$			−0.305		−0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	−47.046	13.88	−15.71

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20% NLO-QCD

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20% NLO-QCD

-3% NLO-EW

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LO NLO QCD NLO EW NNLO QCD NNLO QCDxEW

σ[fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	−49.907	44.60	−16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		−32.66	1.03
$q\gamma$			−0.305		−0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	−47.046	13.88	−15.71

20% NLO-QCD

-3% NLO-EW

<1% NNLO-QCD

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LO NLO QCD NLO EW NNLO QCD NNLO QCDxEW

-1% Mixed QCD-EW

σ[fb]			$\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1		49.907	44.60	−16.80
$\gamma\gamma$			3.166		
qg				−32.66	1.03
$q\gamma$			−0.305		−0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	−47.046	13.88	−15.71

20% NLO-QCD

-3% NLO-EW

<1% NNLO-QCD

THEORY UNCERTAINTIES

QCD

Scale variations

EW

G_μ scheme vs $\alpha(m_Z)$ scheme

Prediction up to NNLO-QCD

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%}\ \text{fb}.$$

Mixed corrections push down theory uncertainty below 1%!

Prediction including mixed QCD-EW

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%}\ \text{fb}$$

THEORY UNCERTAINTIES

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THEORY UNCERTAINTIES

QCD → Scale variations

EW → G_μ scheme vs $\alpha(m_Z)$ scheme

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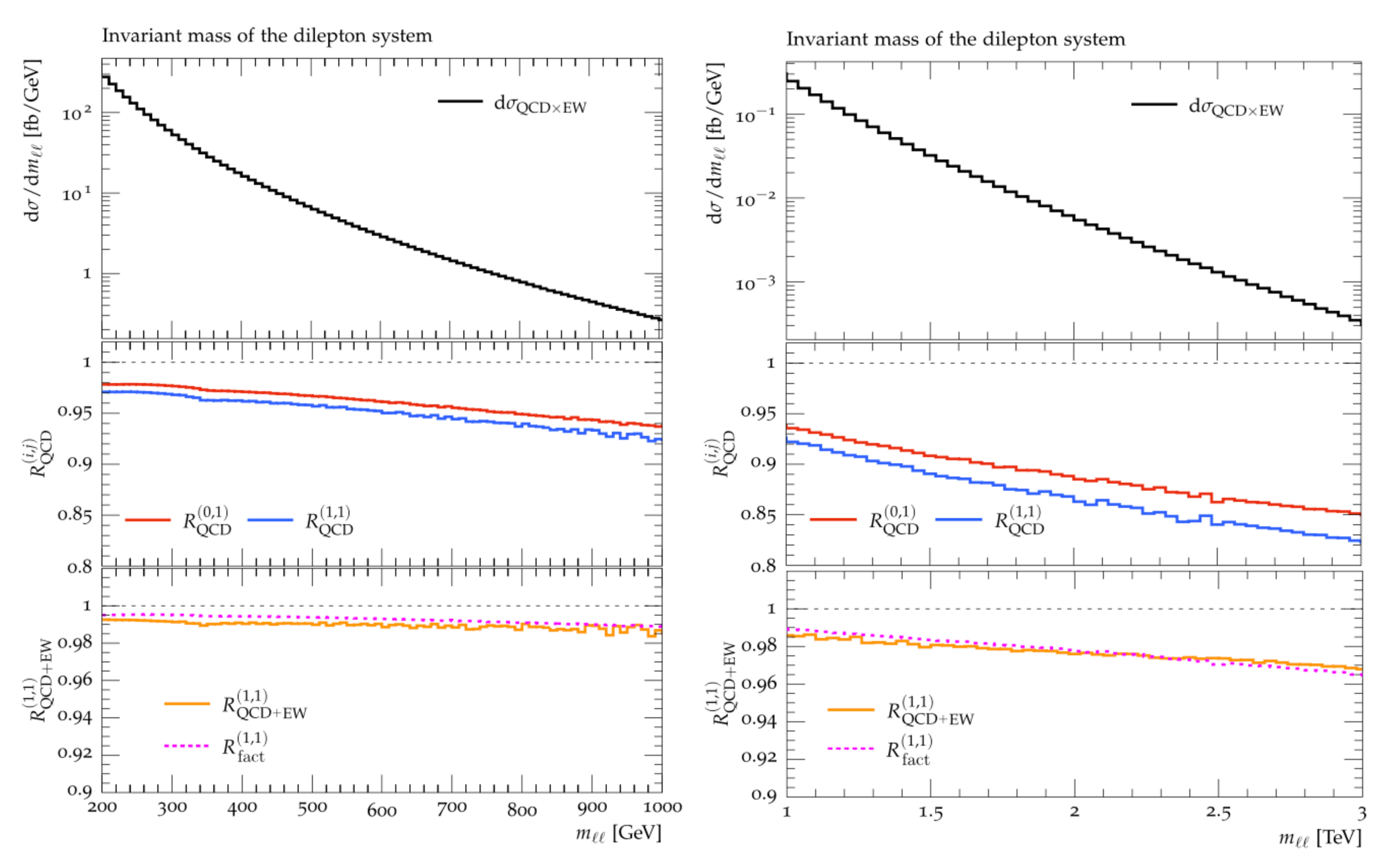
Invariant mass of the dilepton system
$d\sigma/dm_{\ell\ell}$ [fb/GeV]
$d\sigma_{QCD\times EW}$
$R_{QCD}^{(i,j)}$
$R_{QCD}^{(0,1)}$
$R_{QCD}^{(1,1)}$
$R_{QCD+EW}^{(1,1)}$
$R_{QCD+EW}^{(1,1)}$
$R_{fact}^{(1,1)}$
$m_{\ell\ell}$ [GeV]
Invariant mass of the dilepton system
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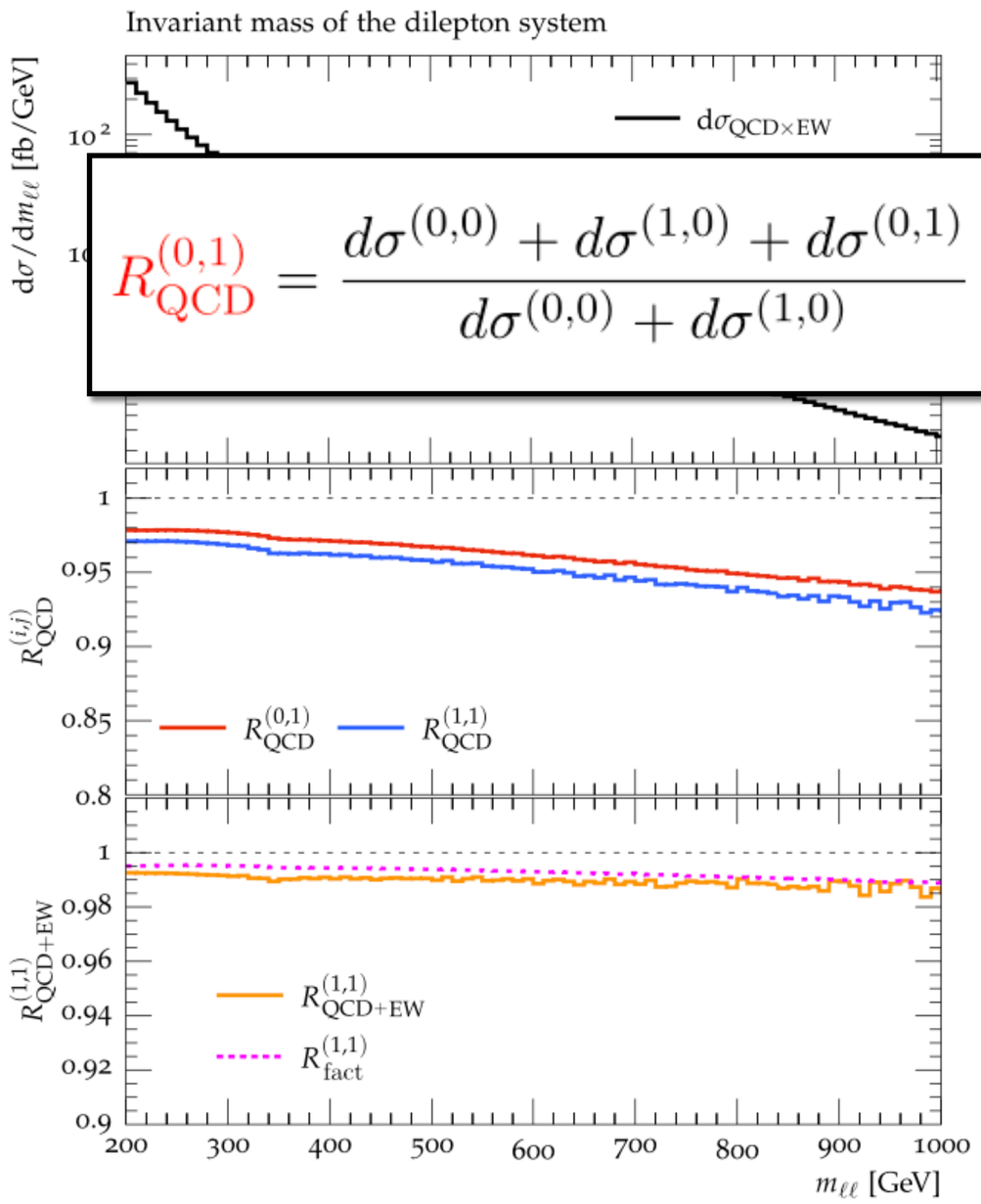
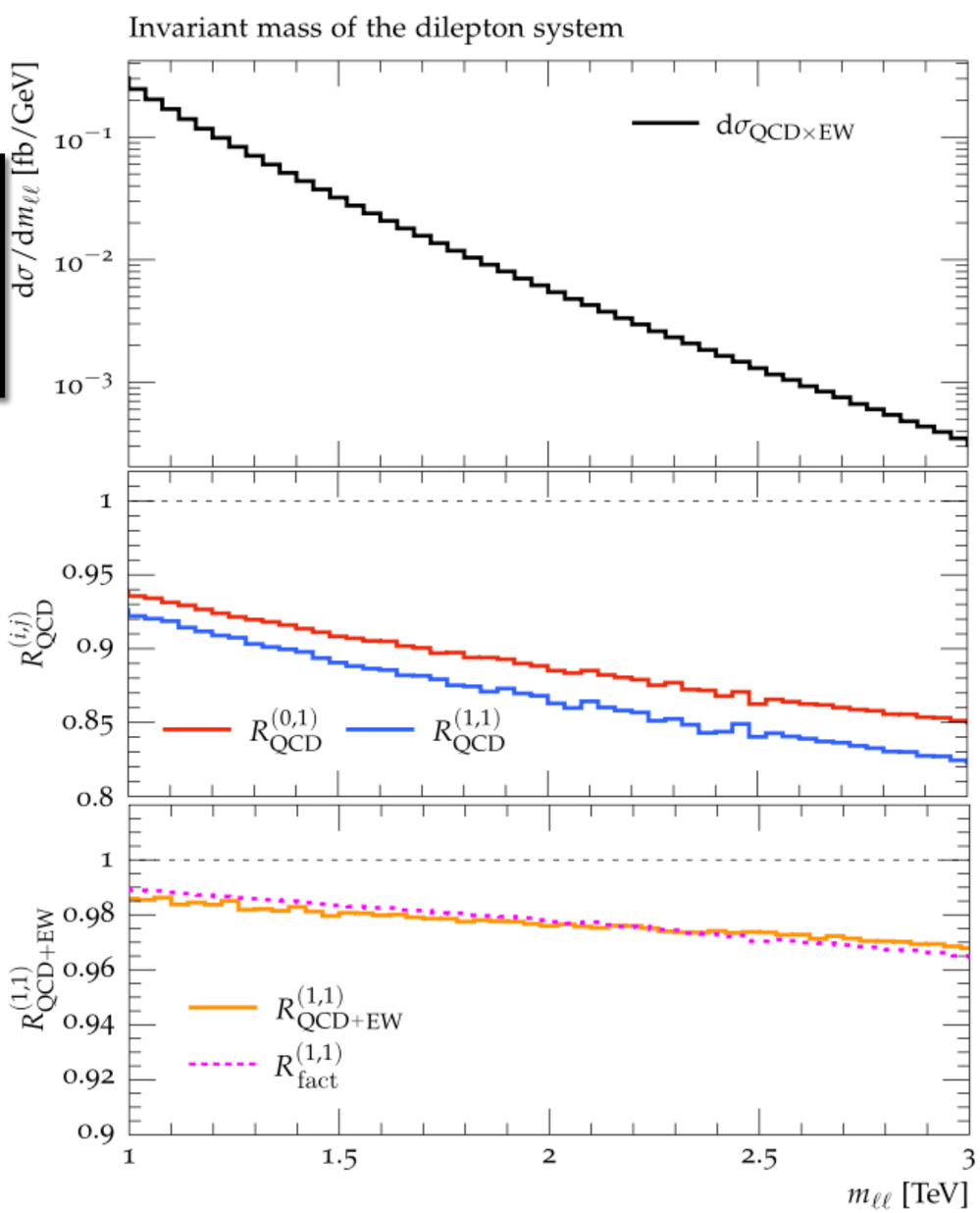
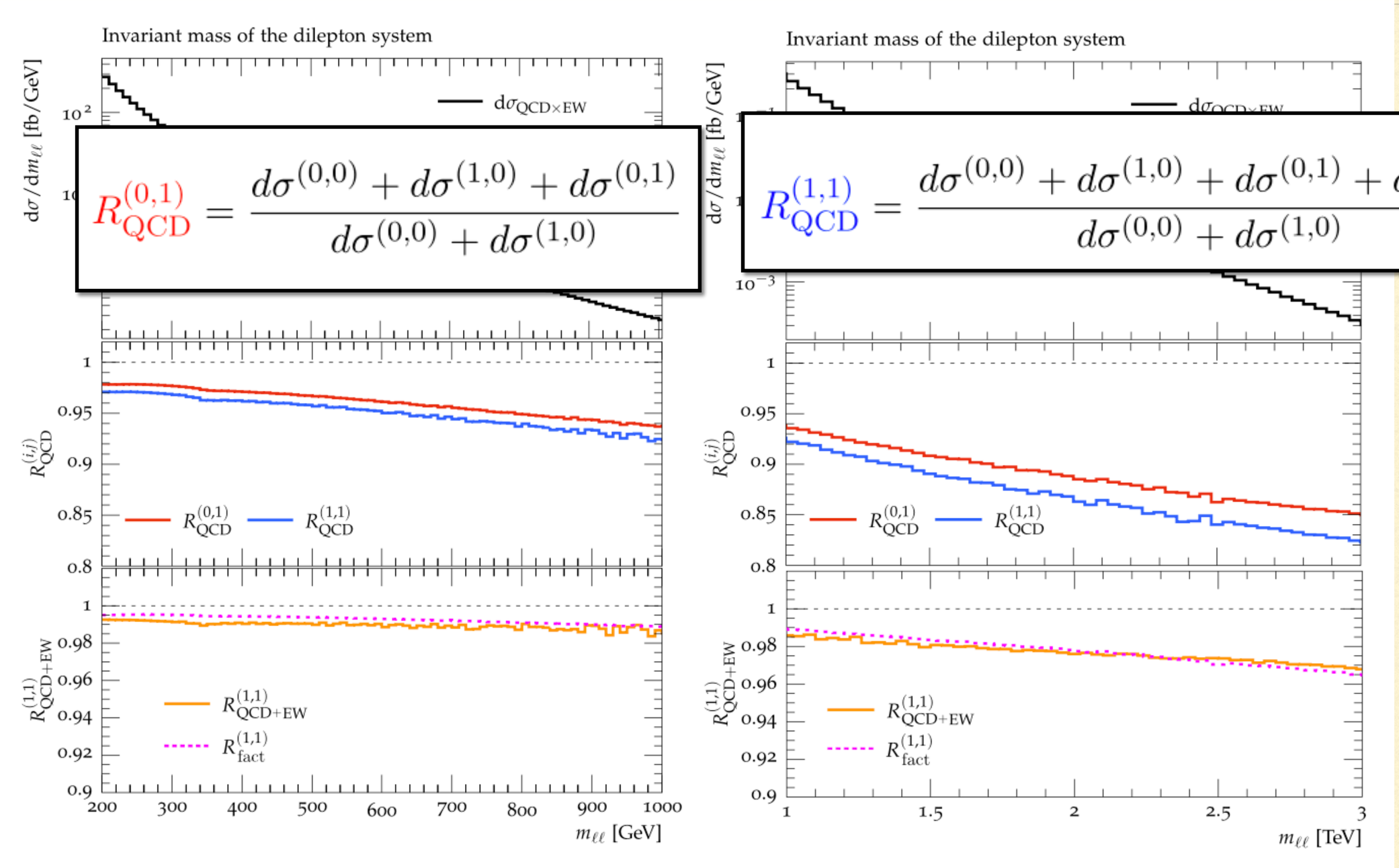
$R_{QCD}^{(1,1)}$
$R_{QCD+EW}^{(1,1)}$
$R_{QCD+EW}^{(1,1)}$
$R_{fact}^{(1,1)}$
$m_{\ell\ell}$ [TeV]

$$R_{\mathrm{QCD}}^{(0,1)} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$







$$R^{(0,1)}_{QCD} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

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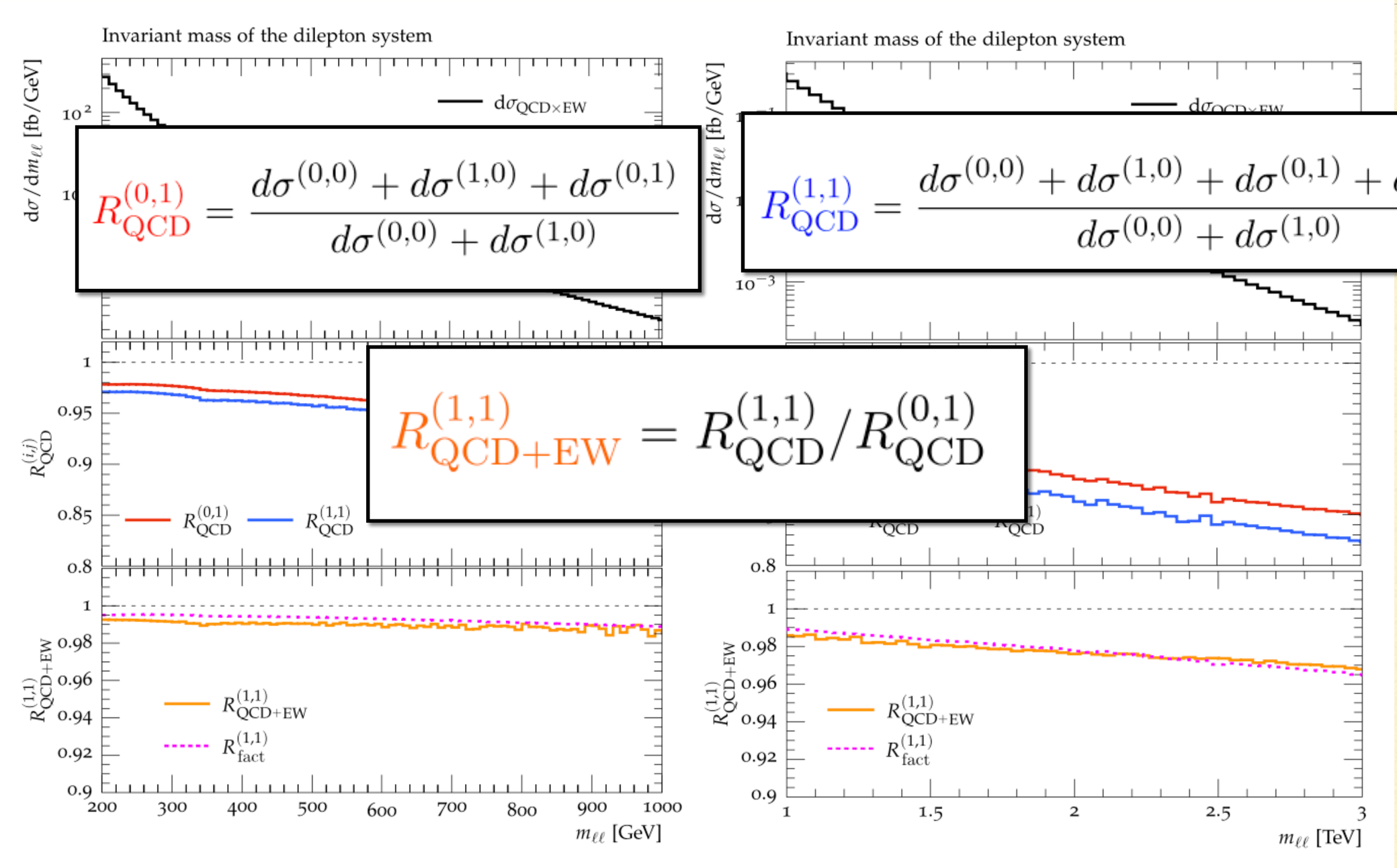

$$R^{(0,1)}_{QCD} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

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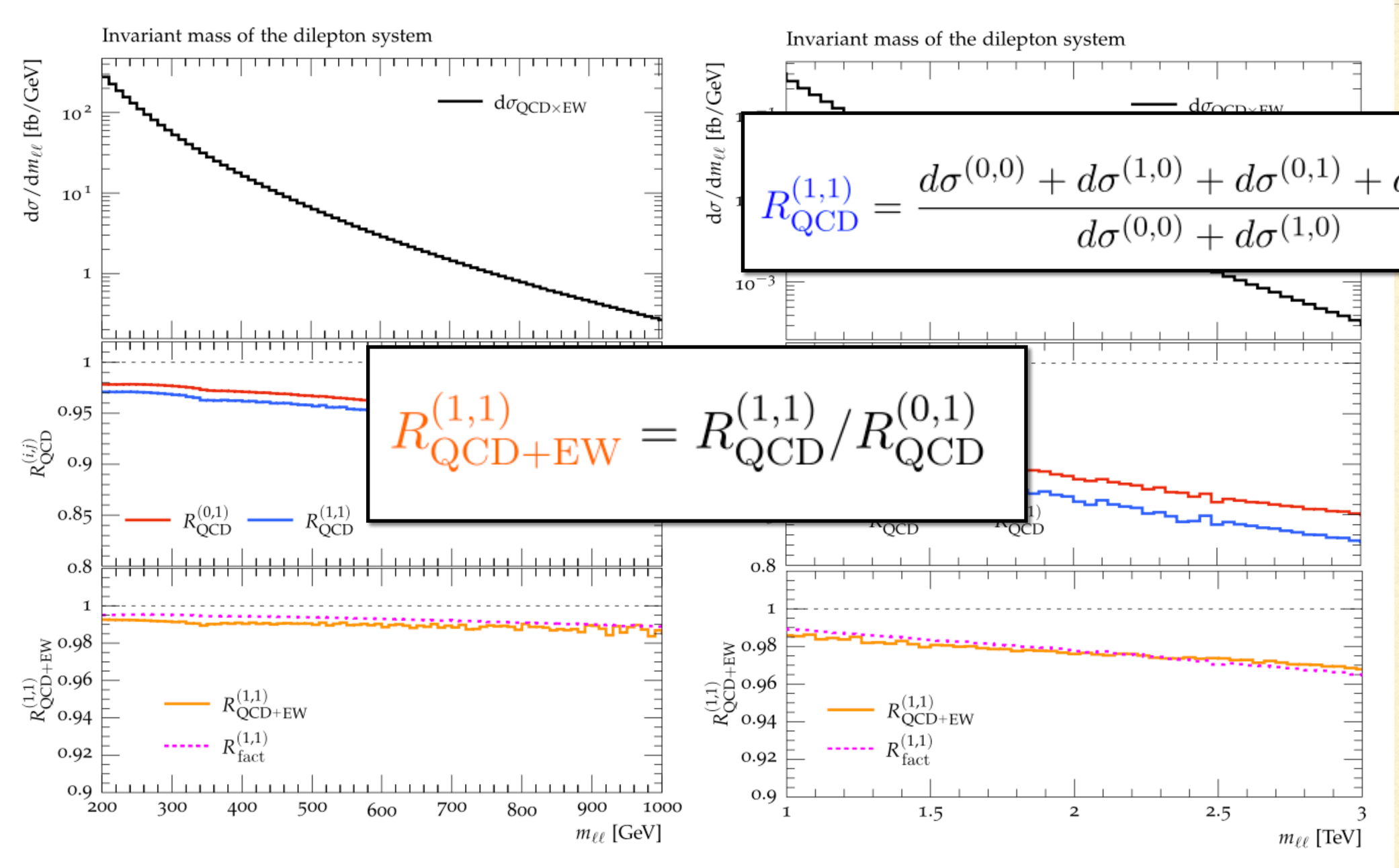
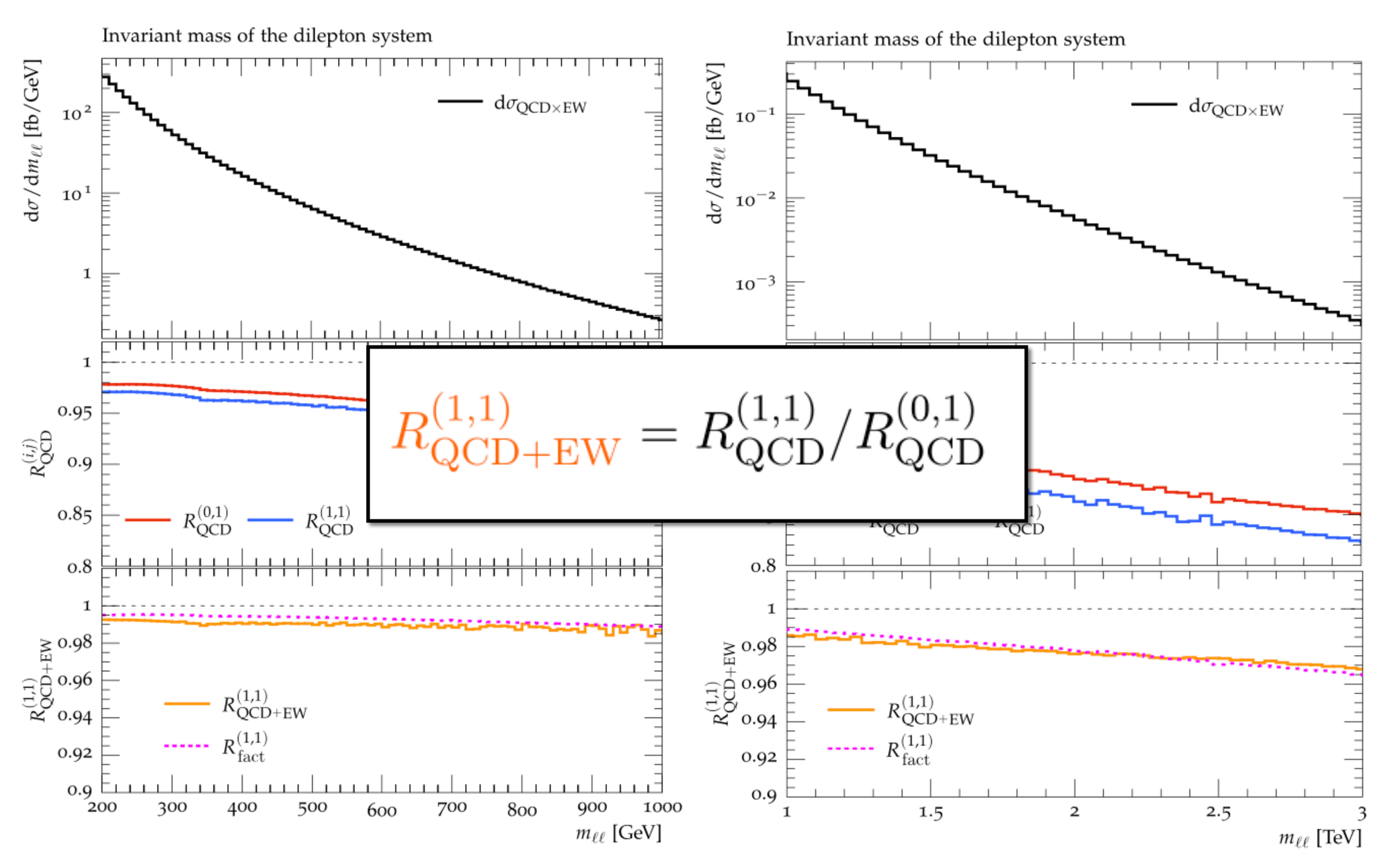
$$R^{(1,1)}_{QCD+EW} = R^{(1,1)}_{QCD} / R^{(0,1)}_{QCD}$$



$$R^{(1,1)}_{\mathrm{QCD}} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(1,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

$$R^{(1,1)}_{\mathrm{QCD+EW}} = R^{(1,1)}_{\mathrm{QCD}} / R^{(0,1)}_{\mathrm{QCD}}$$



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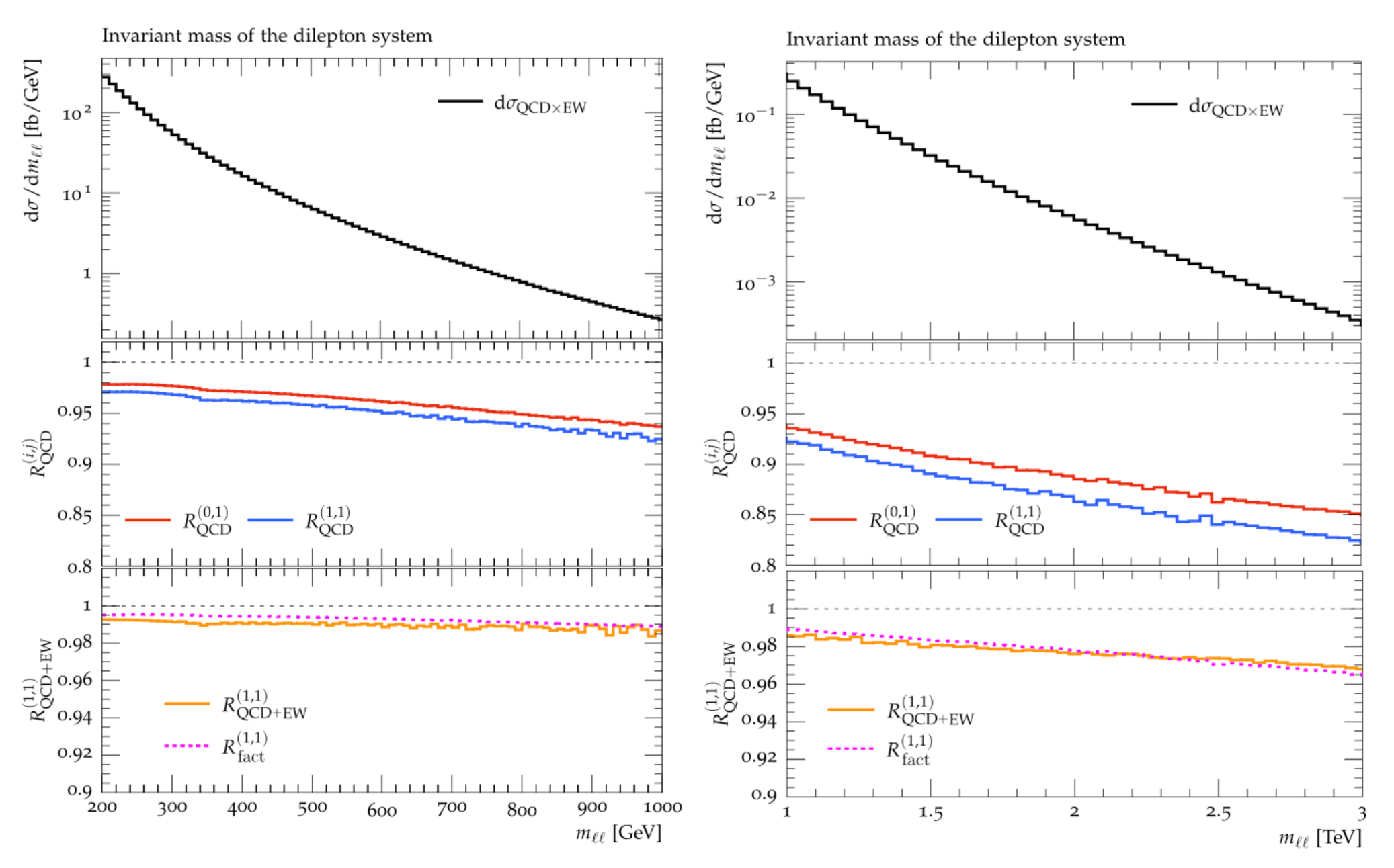
Invariant mass of the dilepton system
$d\sigma/dm_{\ell\ell}$ [fb/GeV]
$d\sigma_{QCD\times EW}$
$R_{QCD}^{(i,j)}$
$R_{QCD}^{(0,1)}$
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$R_{QCD+EW}^{(1,1)}$
$R_{QCD+EW}^{(1,1)}$
$R_{fact}^{(1,1)}$
$m_{\ell\ell}$ [GeV]
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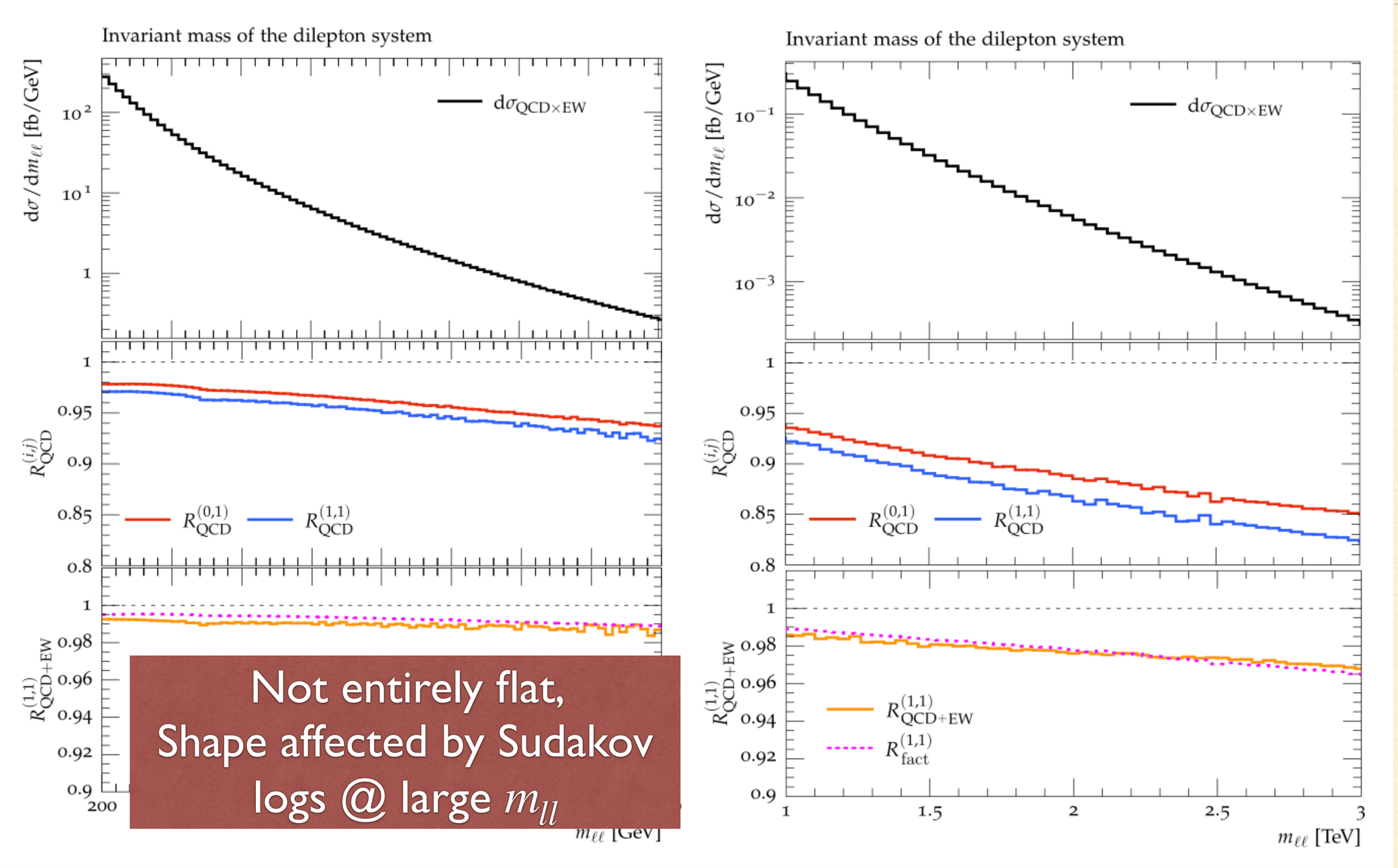
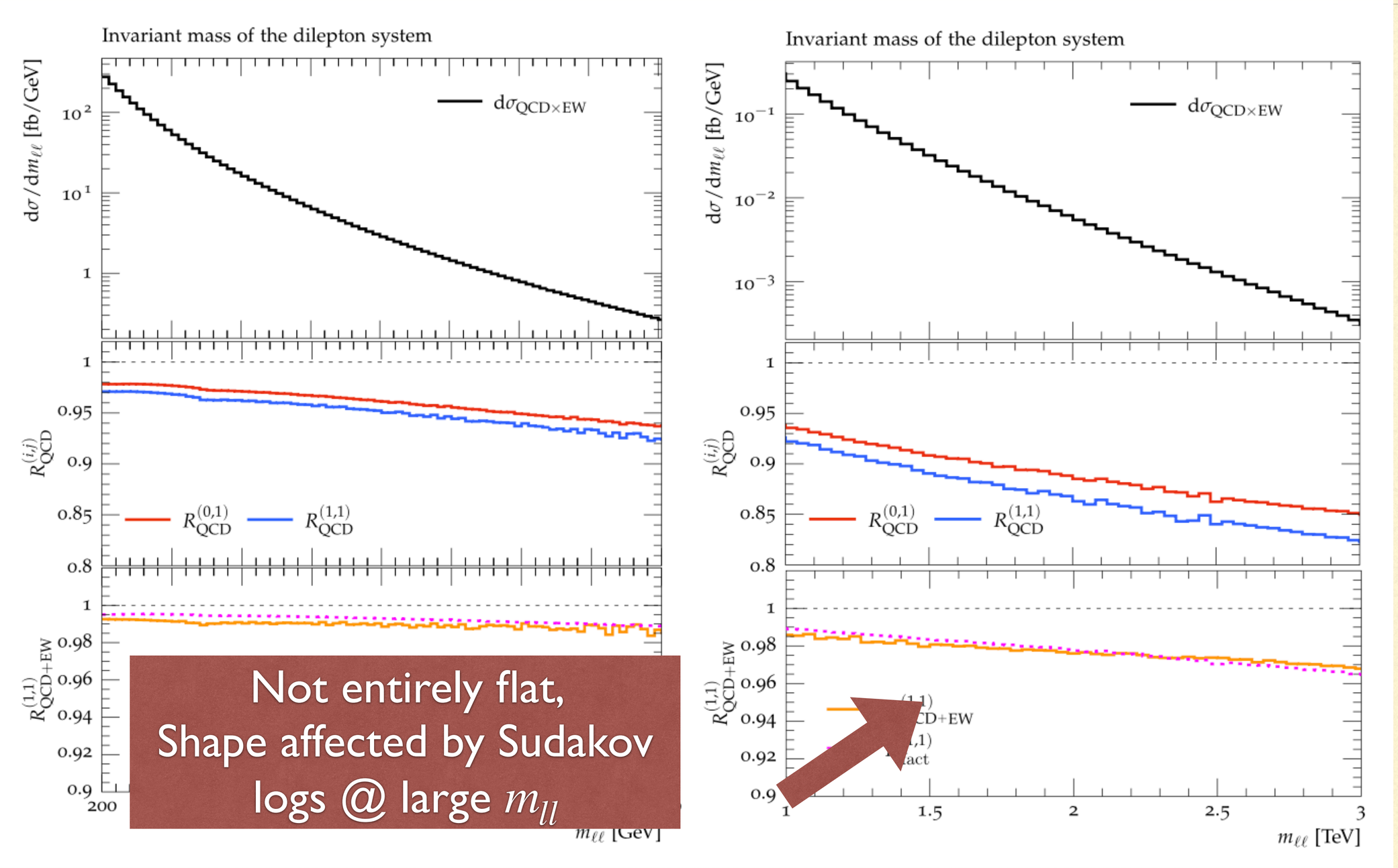


Not entirely flat,
Shape affected by Sudakov
logs @ large m_{ll}



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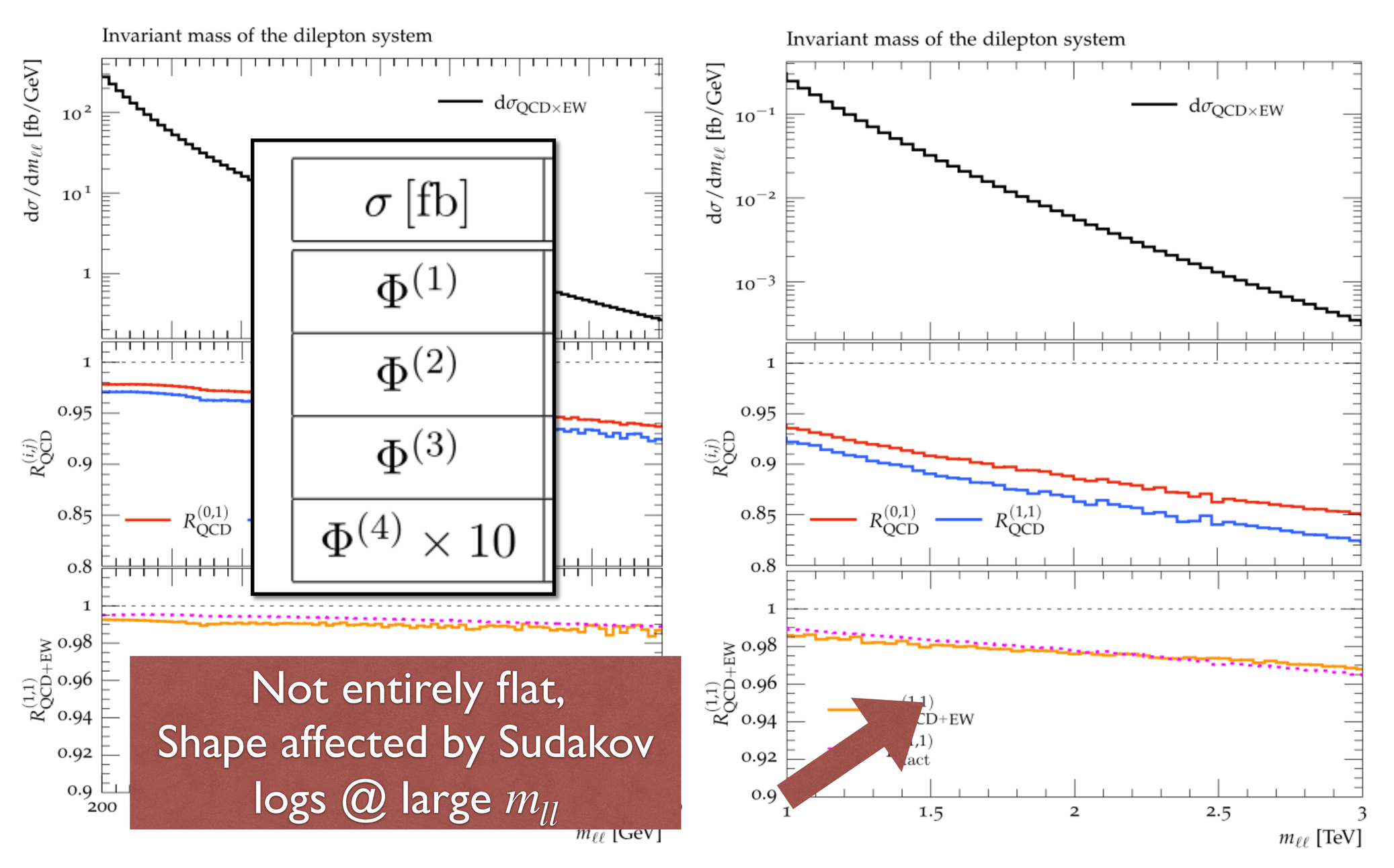
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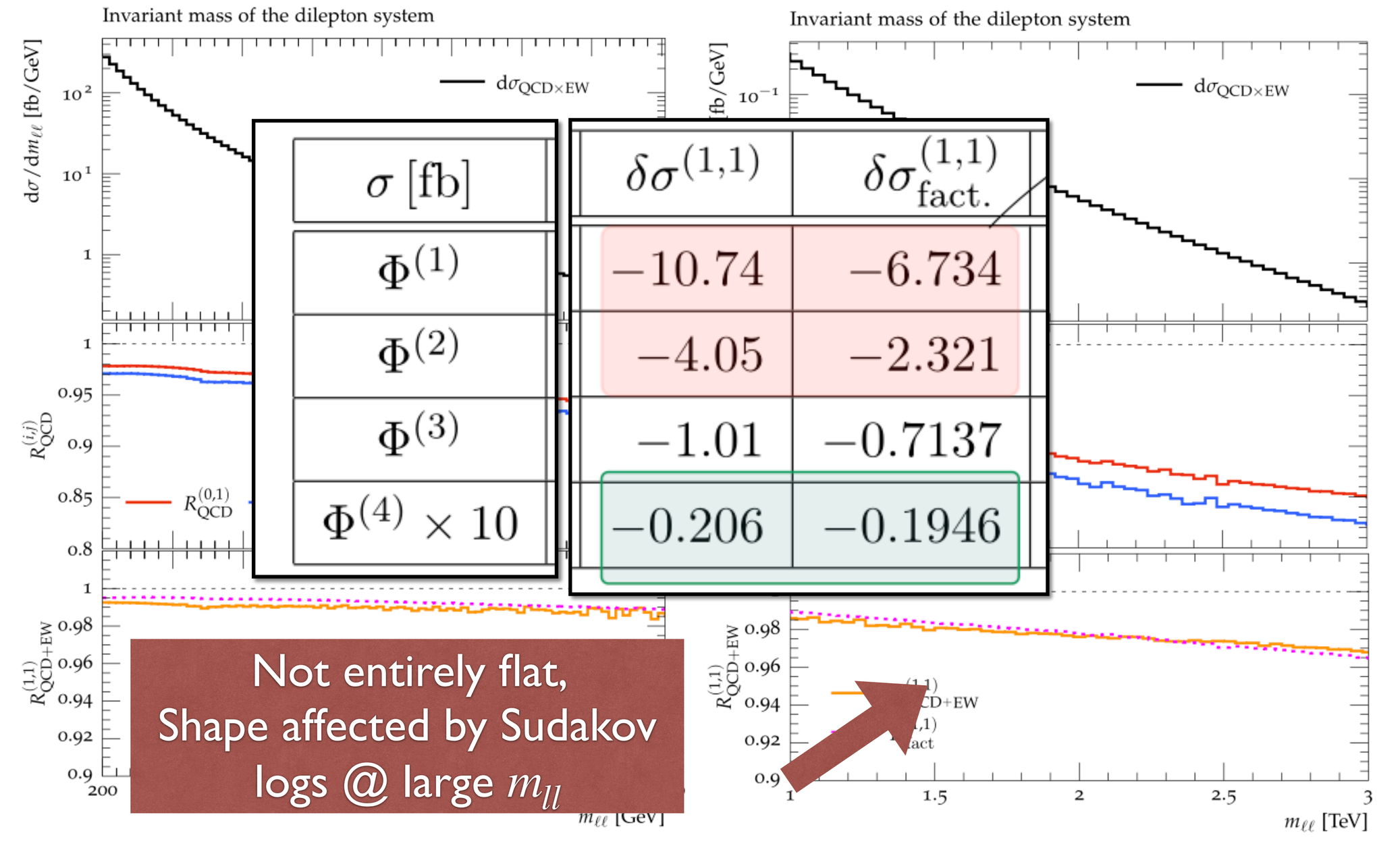
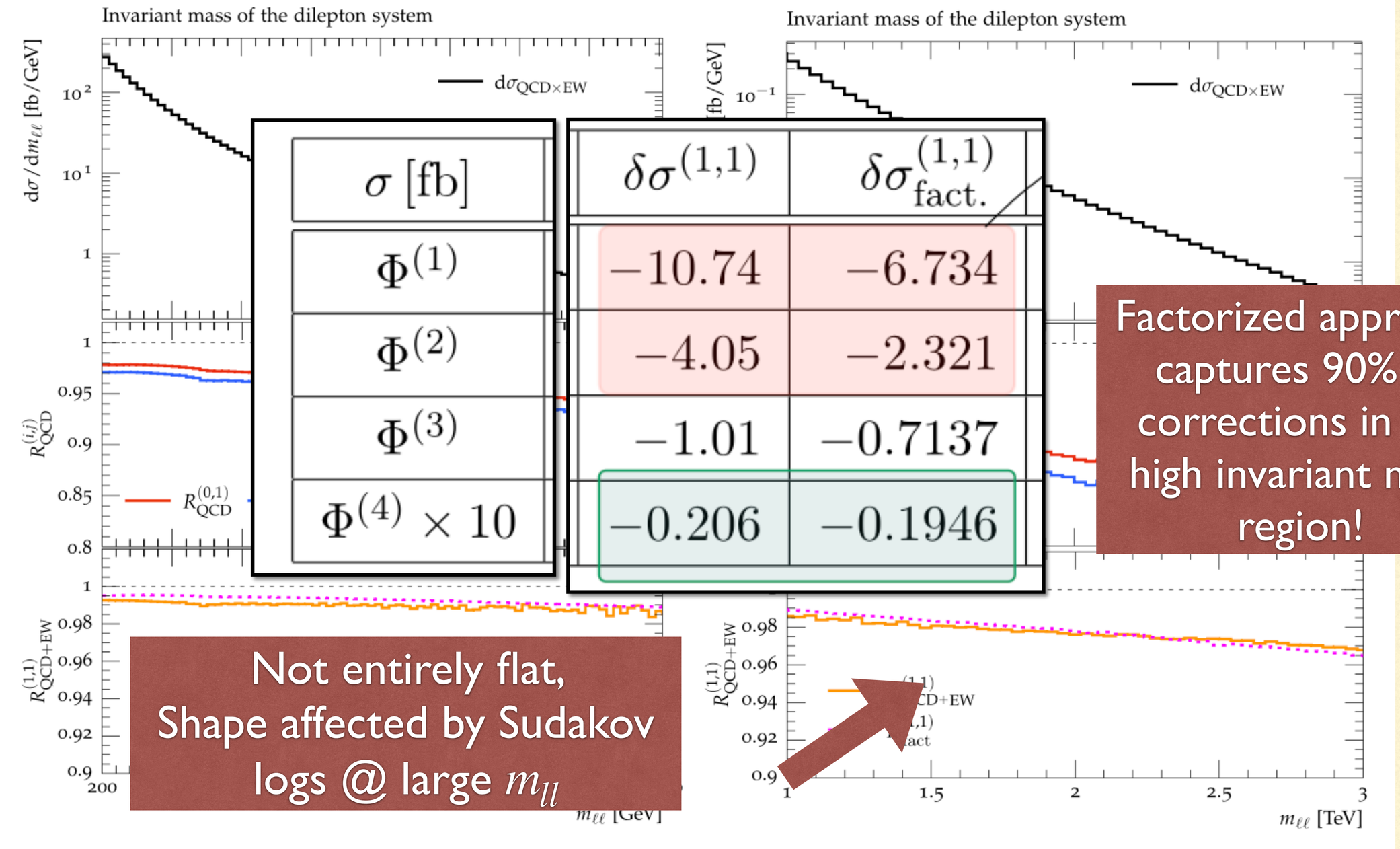
$d\sigma_{QCD\times EW}$
$R^{(i,j)}_{QCD}$
$R^{(0,1)}_{QCD}$
$R^{(1,1)}_{QCD}$
$R^{(1,1)}_{QCD+EW}$
$m_{\ell\ell}$ [GeV]
$m_{\ell\ell}$ [TeV]
σ [fb]
$\Phi^{(1)}$
$\Phi^{(2)}$
$\Phi^{(3)}$
$\Phi^{(4)} \times 10$
Not entirely flat,
Shape affected by Sudakov
logs @ large m_{ll}



σ [fb]	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$
$\Phi^{(1)}$	-10.74	-6.734
$\Phi^{(2)}$	-4.05	-2.321
$\Phi^{(3)}$	-1.01	-0.7137
$\Phi^{(4)} \times 10$	-0.206	-0.1946

Not entirely flat,
Shape affected by Sudakov
logs @ large m_{ll}



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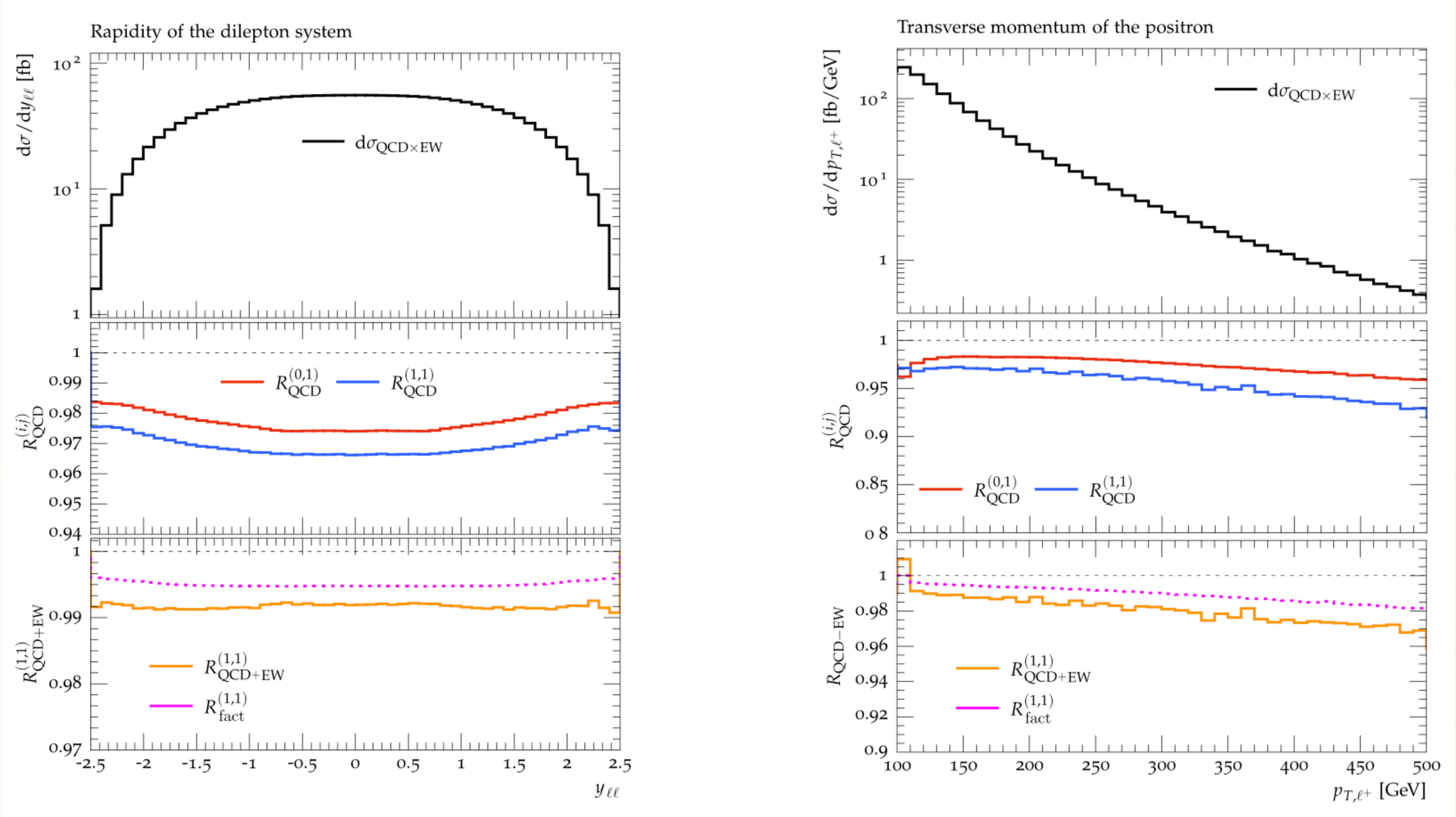
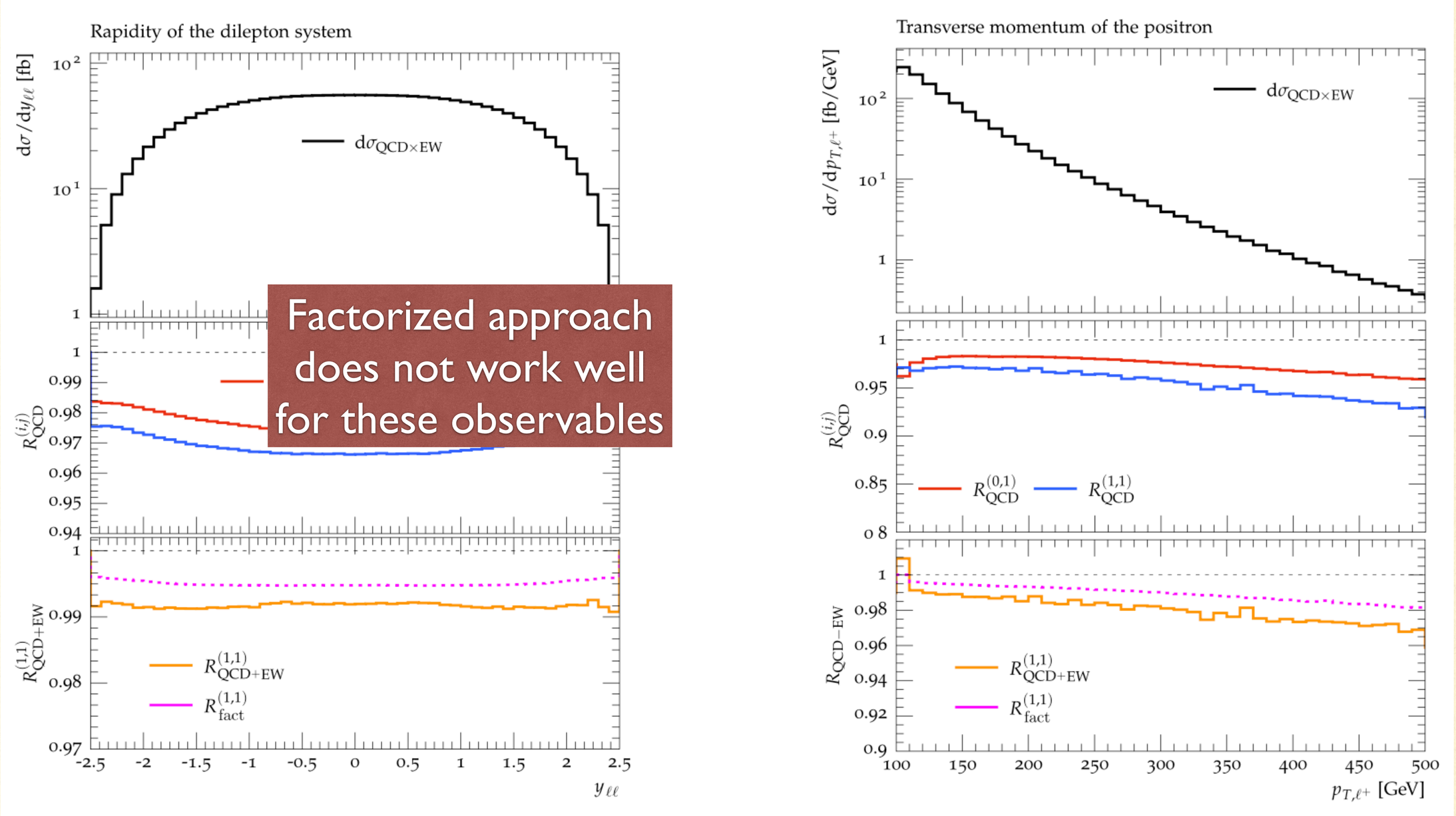
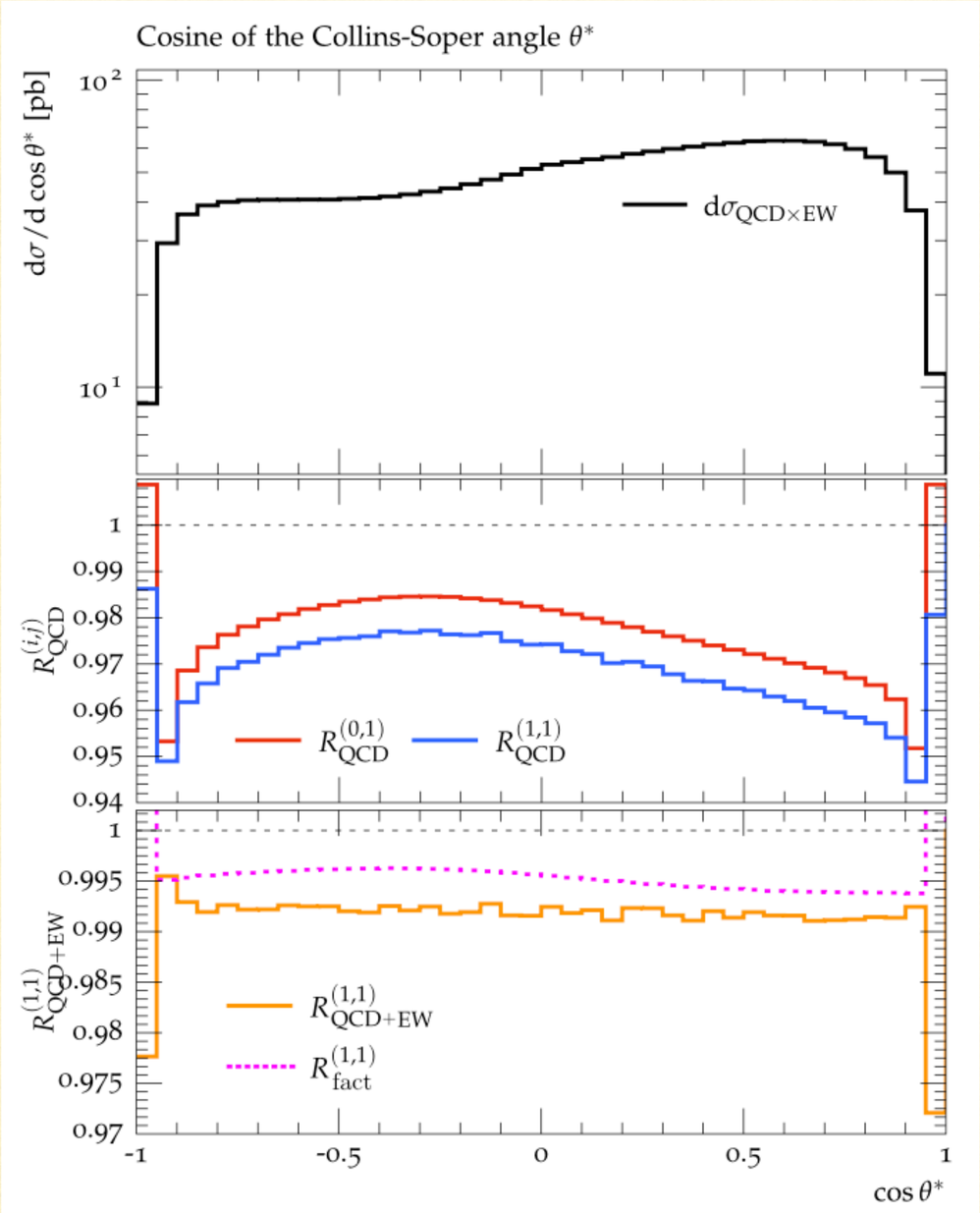
Factorized approach captures 90% of corrections in the high invariant mass region!

Not entirely flat,
Shape affected by Sudakov logs @ large m_{ll}


Rapidity of the dilepton system
$d\sigma/dy_{\ell\ell}$ [fb]
$d\sigma_{QCD\times EW}$
$R_{QCD}^{(i,j)}$
$R_{QCD}^{(0,1)}$
$R_{QCD}^{(1,1)}$
$R_{QCD+EW}^{(1,1)}$
$R_{fact}^{(1,1)}$
$y_{\ell\ell}$
Transverse momentum of the positron
$d\sigma/dp_{T,\ell^+}$ [fb/GeV]
$d\sigma_{QCD\times EW}$
$R_{QCD}^{(i,j)}$
$R_{QCD}^{(0,1)}$
$R_{QCD}^{(1,1)}$
R_{QCD-EW}
$R_{QCD+EW}^{(1,1)}$
$R_{fact}^{(1,1)}$
p_{T,ℓ^+} [GeV]



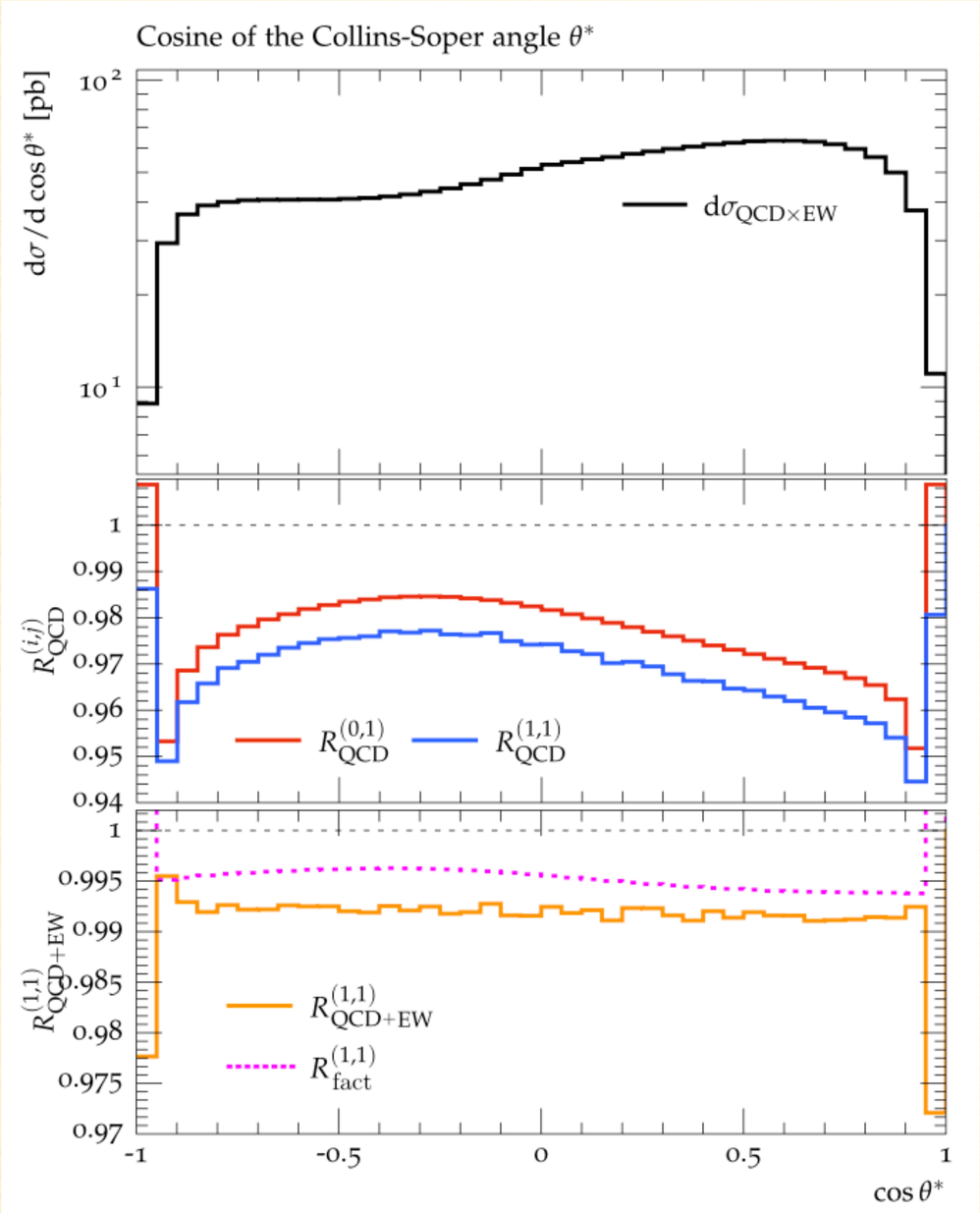
Factorized approach does not work well for these observables



$$\cos\theta^* = \frac{p^+_{\ell^-}p^-_{\ell^+} - p^-_{\ell^-}p^+_{\ell^+}}{m_{\ell\ell}\sqrt{m^2_{\ell\ell} + p^2_{\ell\ell,\perp}}} \times \mathrm{sgn}(p_{\ell\ell,z})$$

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \qquad \sigma_F = \int_0^1 d\cos\theta^* \frac{d\sigma(pp\to\ell^-\ell^+)}{d\cos\theta^*} \qquad \sigma_B = \int_{-1}^0 d\cos\theta^* \frac{d\sigma(pp\to\ell^-\ell^+)}{d\cos\theta^*}$$

	$\tilde{A}_{\mathrm{FB}}$	A_{FB}
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
$\Phi^{(2)}$	$0.1852^{+0.08\%}_{-0.40\%}$	$0.1847^{+0.10\%}_{-0.19\%}$
$\Phi^{(3)}$	$0.2401^{+0.13\%}_{-0.64\%}$	$0.2388^{+0.06\%}_{-0.47\%}$
$\Phi^{(4)}$	$0.3070^{+0.49\%}_{-1.5\%}$	$0.3031^{+0.19\%}_{-1.2\%}$



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Without mixed QCDxEW

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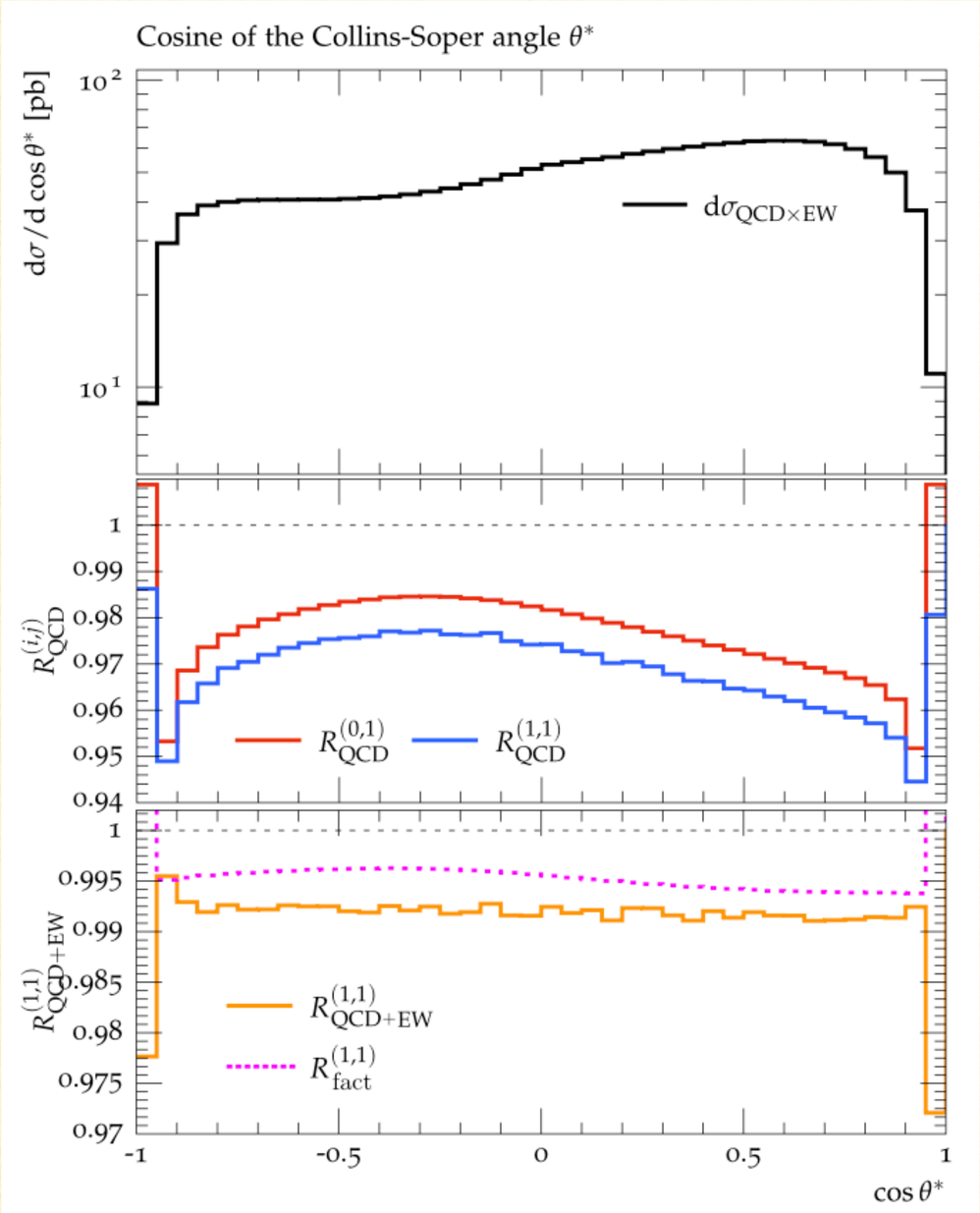

$$\cos\theta^* = \frac{p^+_{\ell^-}p^-_{\ell^+} - p^-_{\ell^-}p^+_{\ell^+}}{m_{\ell\ell}\sqrt{m^2_{\ell\ell} + p^2_{\ell\ell,\perp}}} \times \mathrm{sgn}(p_{\ell\ell,z})$$

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \qquad \sigma_F = \int_0^1 d\cos\theta^* \frac{d\sigma(pp\to\ell^-\ell^+)}{d\cos\theta^*} \qquad \sigma_B = \int_{-1}^0 d\cos\theta^* \frac{d\sigma(pp\to\ell^-\ell^+)}{d\cos\theta^*}$$

	Without mixed QCDxEW	With mixed QCDxEW
	$\tilde{A}_{\mathrm{FB}}$	A_{FB}
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
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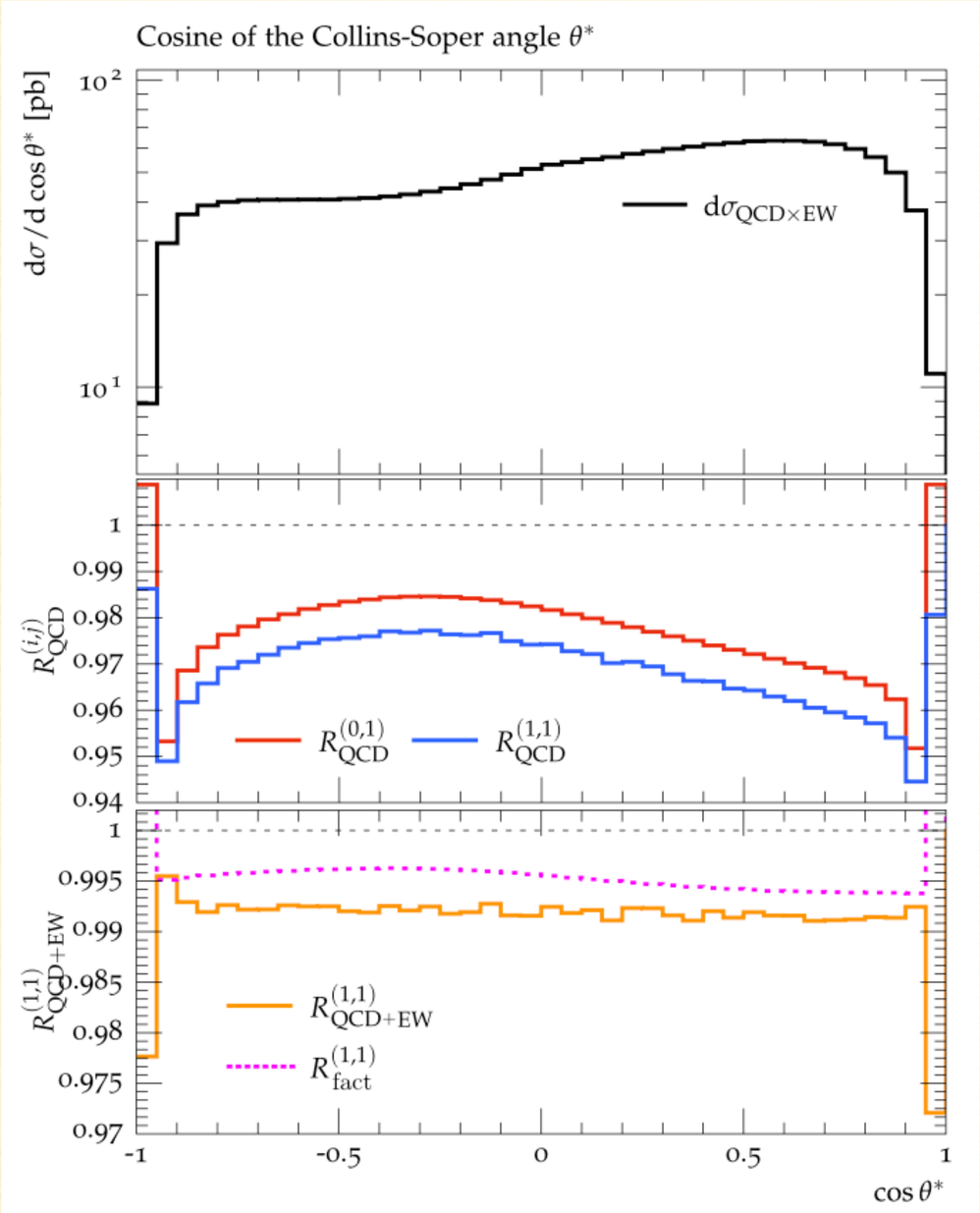

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	$\tilde{A}_{\mathrm{FB}}$	A_{FB}
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
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$\Phi^{(4)}$	$0.3070^{+0.49\%}_{-1.5\%}$	$0.3031^{+0.19\%}_{-1.2\%}$

1% correction from mixed corrections in TeV region!

CONCLUSIONS

- For % level accuracy on tail of distributions as well as NP searches mixed QCD-EW effects have to be taken into account
- Our computation makes use of fully analytic amplitudes and subtraction scheme, allowing for efficient and extensive phenomenological studies
- Mixed corrections amount to 1% even in non Sudakov-dominated regime, at extreme energies Sudakov factorisation works very well

THANK YOU FOR THE ATTENTION!