Precision QCD calculations for vector boson observables

Thomas Gehrmann (Universität Zürich)

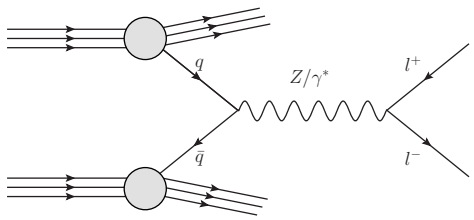
RADCOR 2023, Crieff, 29.5.-2.6.2023

Swiss National Science Foundation

European Research Council
Established by the European Commission

based on: 2107.09085, 2203.01565, 2205.11426, 2211.13596, 2301.11827

Drell-Yan process



- Drell-Yan lepton pair (neutral-current or charged-current) production
  - Benchmark observable: multi-differential measurements
  - Precision measurements of EW parameters and parton distributions
- Standard Model theory well understood
  - NLO EW [C.Carloni Calame, G.Motagna, A.Nicrosini, A.Vicini; S.Dittmaier, M.Huber]
  - NNLO QCD and NNLO QCD+EW (+ total cross section to N3LO QCD)
    [K.Melnikov, F.Petriello; S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini; C.Duhr, F.Dulat, B.Mistlberger; R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, A.Vicini; F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile] (talks of G.Vita, F.Devoto, N.Rana)
  - transverse momentum resummation to N3LL QCD
    [W.Bizon, P.F.Monni, E.Re, L.Rottoli, P.Torrielli]
- Precision Tools: FEWZ, DYNNLO, DYturbo, POWHEG, ...

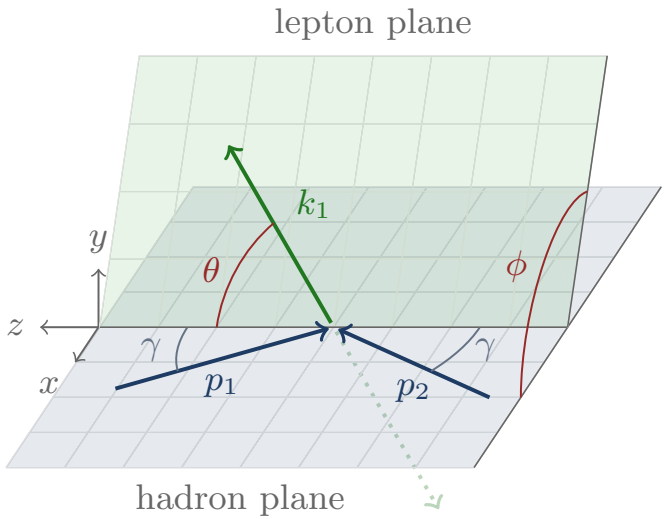
Triple-differential Drell-Yan cross section

- Lepton pair production: EW precision observable

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}m_{ll}\mathrm{d}y_{ll}\mathrm{d}\cos\theta^*} = \frac{\pi\alpha^2}{3m_{ll}s}\sum_q P_q(\cos\theta^*)\left[f_q(x_1,Q^2)f_{\bar{q}}(x_2,Q^2) + (q \leftrightarrow \bar{q})\right]$$

- ATLAS 8 TeV measurement [1710.05167]

Observable	Central-Central	Central-Forward
m_{ll} [GeV]	[46,66,80,91,102,116,150,200]	[66,80,91,102,116,150]
$\lvert y_{ll}\rvert$	[0,0.2,0.4,0.6,0.8,1,1.2, 1.4,1.6,1.8,2,2.2,2.4]	[1.2,1.6,2,2.4,2.8,3.6]
$\cos\theta^*$	[-1,-0.7,-0.4,0,0.4,0.7,1]	[-1,-0.7,-0.4,0,0.4,0.7,1]
Total Bin Count:	504	150



Triple-differential Drell-Yan cross section

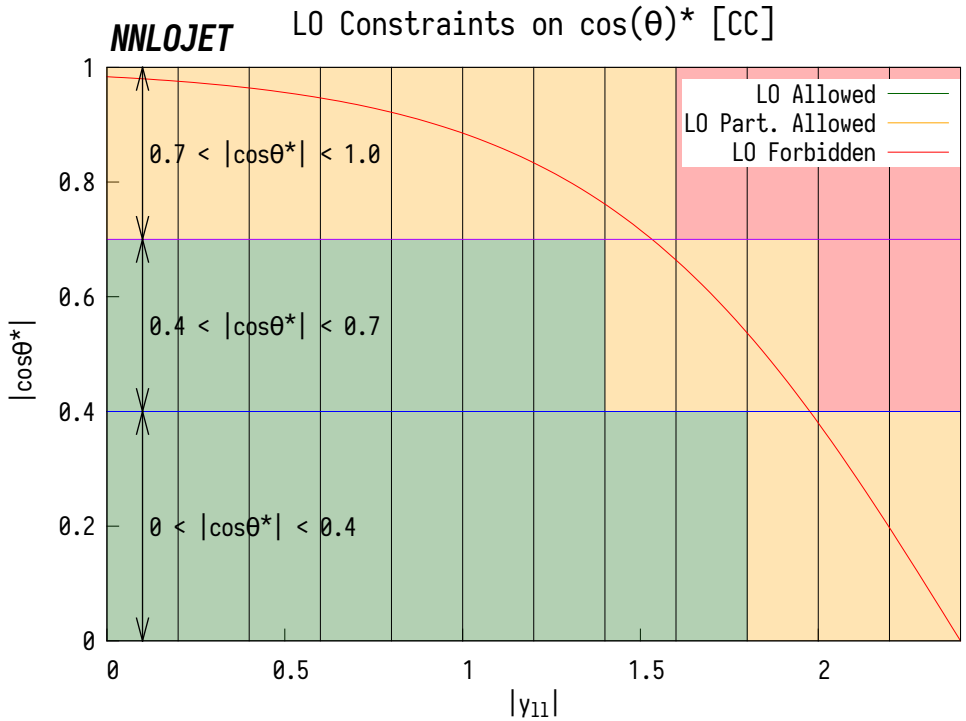
- Measured with fiducial event selection cuts (on single leptons)

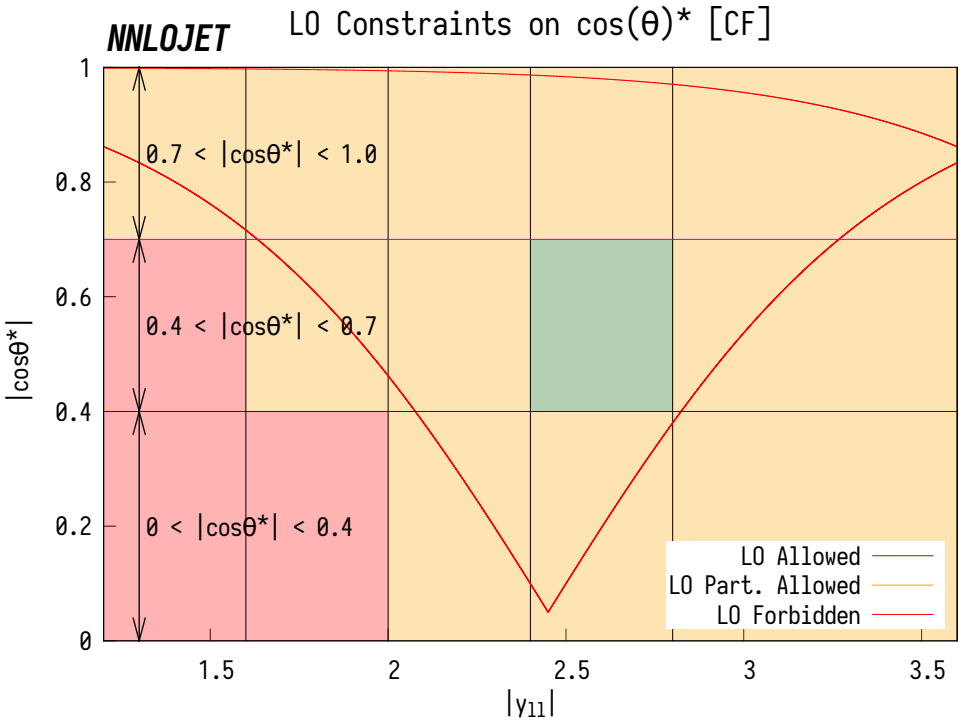
Central-Central	Central-Forward	
$p_T^l > 20$ GeV	$p_{T,F}^l > 20$ GeV	$p_{T,C}^l > 25$ GeV
$\lvert y^l \rvert < 2.4$	$2.5 < \lvert y_F^l \rvert < 4.9$	$\lvert y_C^l \rvert < 2.4$
$46 \text{ GeV} < m_{ll} < 200 \text{ GeV}$	$66 \text{ GeV} < m_{ll} < 150 \text{ GeV}$	

- Fiducial cuts influence acceptances in triple-differential bins

Triple-differential Drell-Yan cross section

- Leading order: fiducial cuts intersect bin definitions
  [A.Gehrmann-De Ridder, E.W.N.Glover, A.Huss, C.Preuss, D.Walker, TG: 2301.11827]

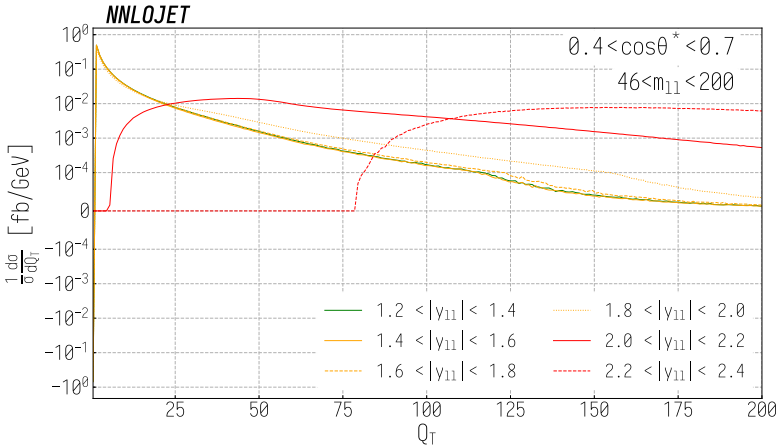


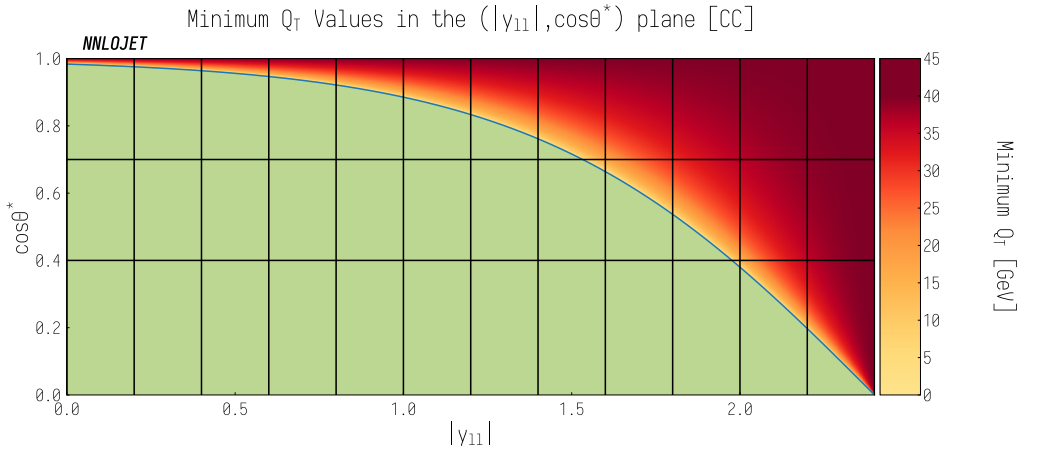


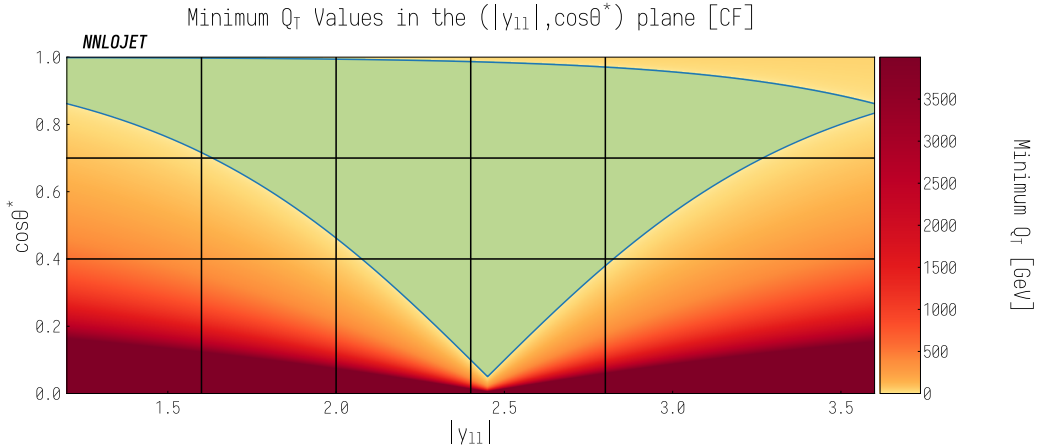
Triple-differential Drell-Yan cross section

- Leading-order forbidden bins
  - require finite Q_T of lepton pair
  - shown here: symmetric lepton pair

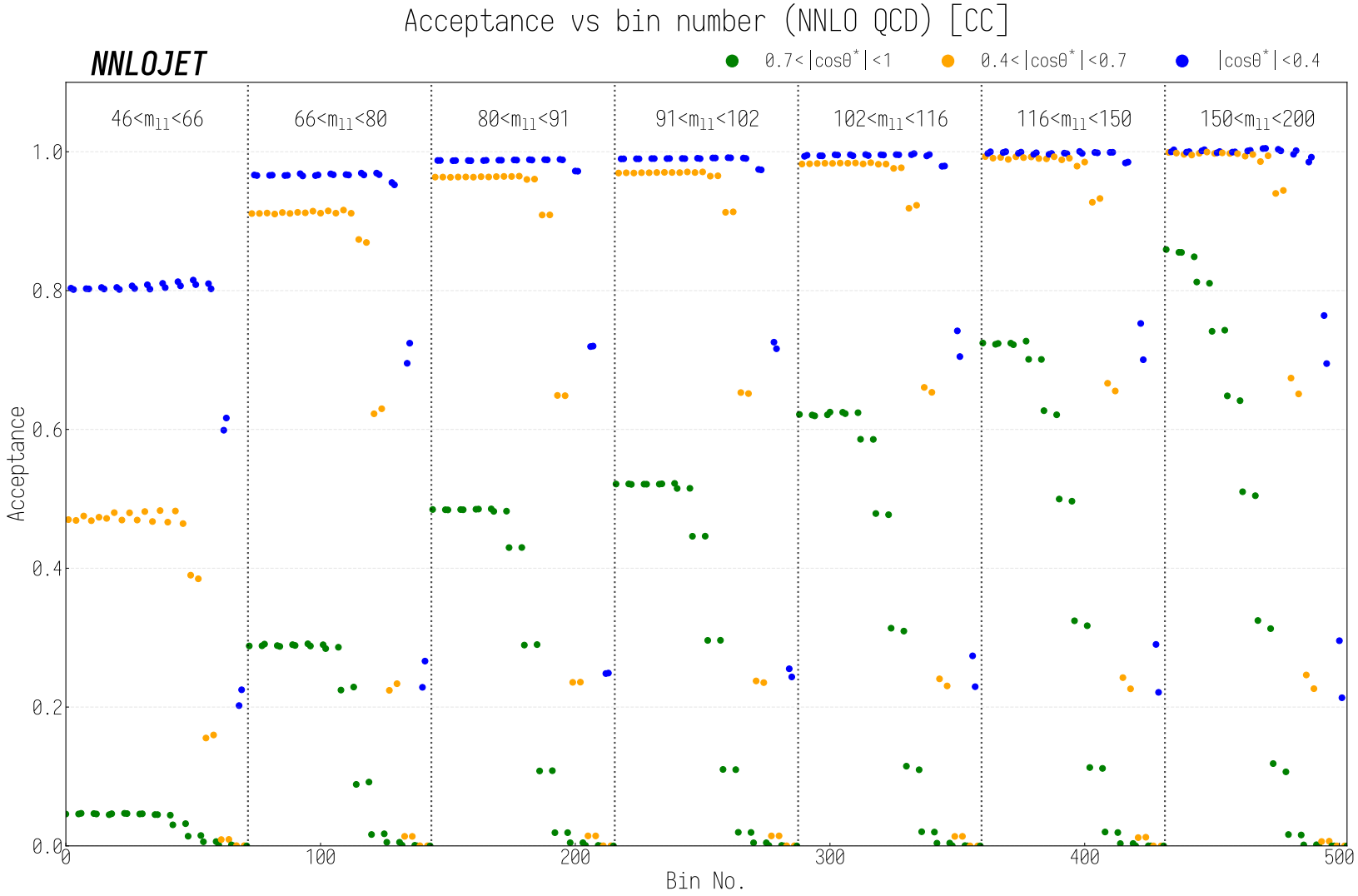
→ prediction starts only at NLO
  - lower accuracy
  - potential perturbative instabilities







Triple-differential Drell-Yan cross section



Triple-differential Drell-Yan cross section

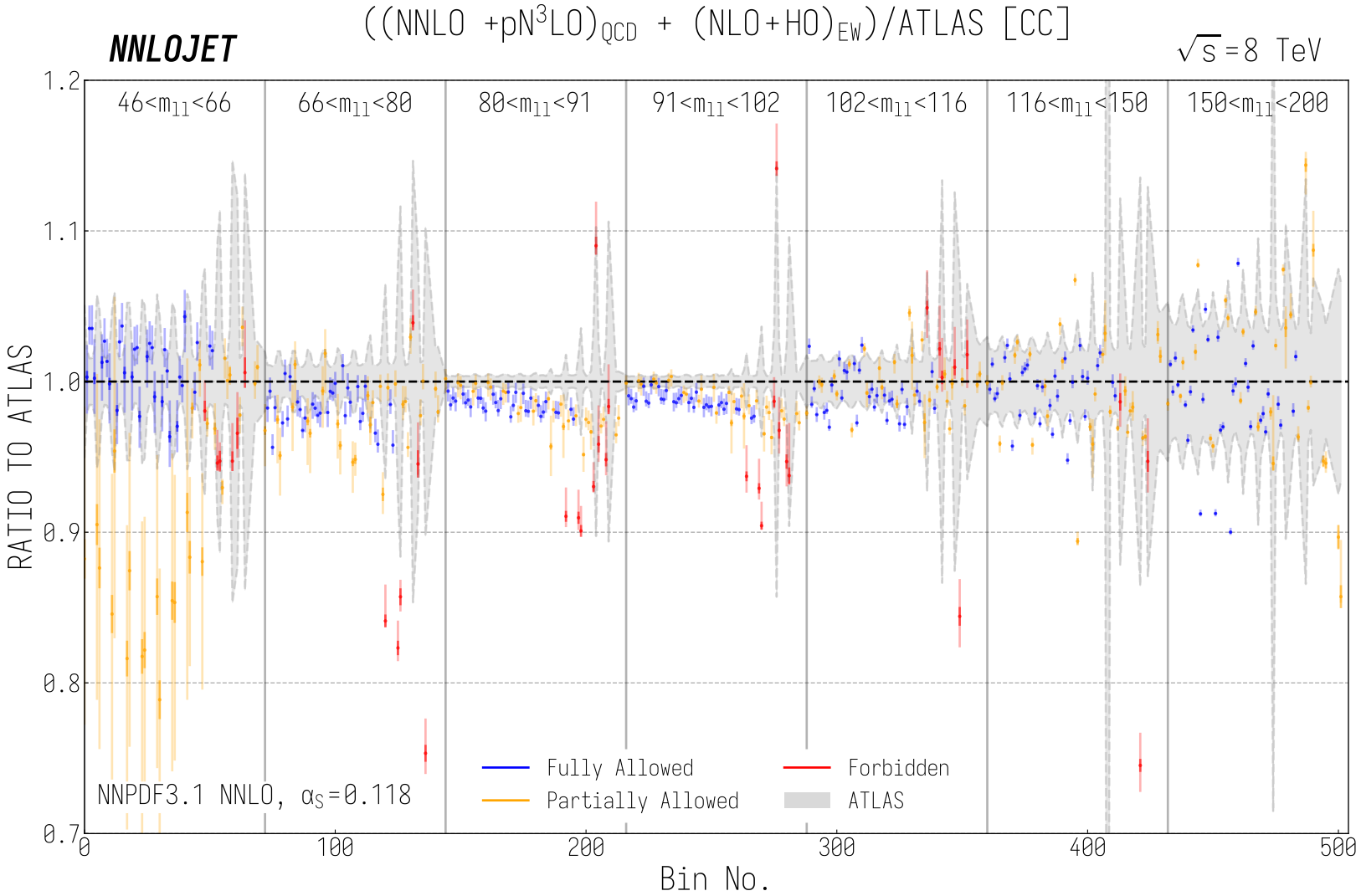
Forbidden bins at leading order

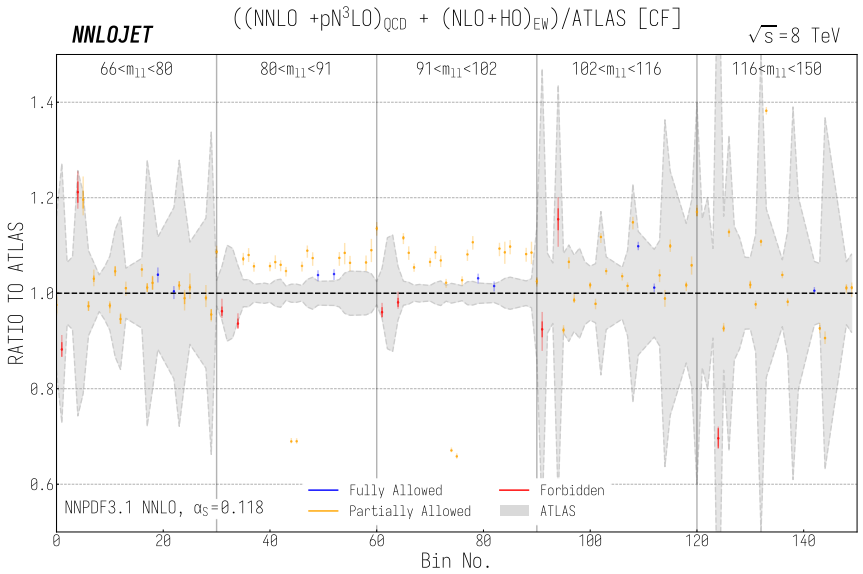
- large theory uncertainty, poor agreement with data
- $O(\alpha_s^3)$ corrections (Drell-Yan N^3LO) obtained from V+jet at NNLO
  [MCFM: T.Neumann, J.Campbell; NNLOJET: A.Gehrmann-De Ridder, N.Glover, A.Huss, T.Morgan, D.Walker, TG]
  - implemented in NNLOJET using antenna subtraction (talks of M.Marcoli, O.Braun-White)
  - replace jet requirement by (small) Q_T cut
  - numerical convergence at small Q_T challenging

State-of-the-art theory prediction

- QCD NNLO (α_s^2) plus N3LO (α_s^3) in LO-forbidden bins
- combined with (NLO+HO) EW corrections [C.Carloni Calame, G.Motagna, A.Nicrosini, A.Vicini]

Triple-differential Drell-Yan cross section





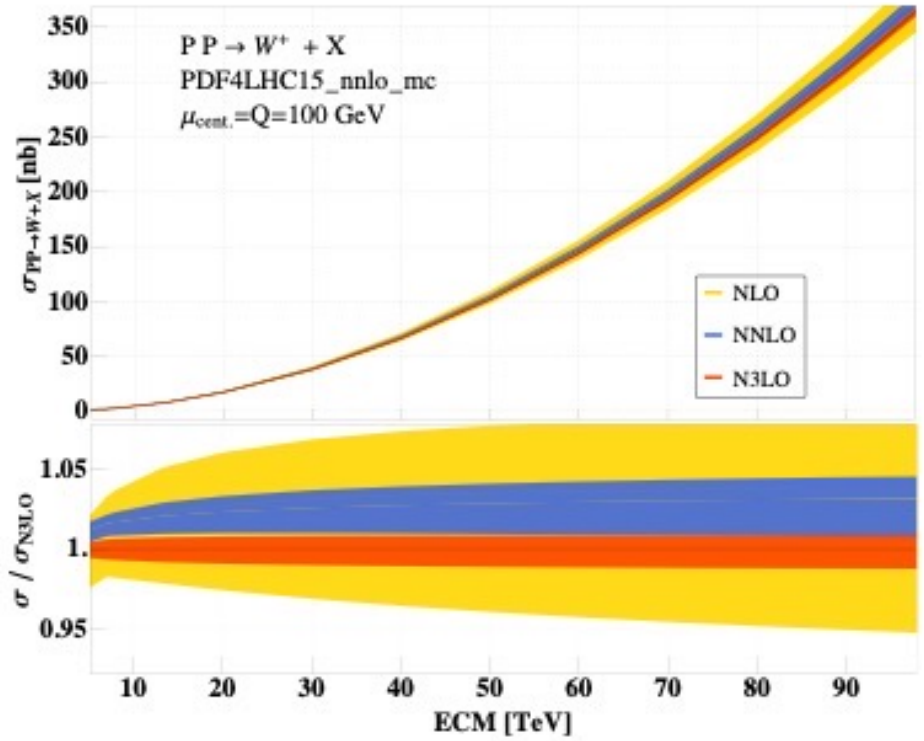
Future applications

- measurement of $\sin^2\Theta w$
- determination of parton distributions

Towards full N3LO in Drell-Yan observables

Inclusive coefficient functions (total cross section) at N3LO

- computed analytically
  - three-loop form factors
  - inclusive phase space up to triple emission
  - 100s of loop and phase-space master integrals
- Results
  - virtual photon exchange [C.Duhr, F.Dulat, B.Mistlberger]
  - charged-current Drell-Yan: $W^{\pm}$ production [C.Duhr, F.Dulat, B.Mistlberger]
  - neutral-current Drell-Yan: γ^*/Z^0 production [C.Duhr, B.Mistlberger]
  - associated VH production [n3loxs: J.Baglio, C.Duhr, B.Mistlberger, R.Szafron]



Towards full N3LO in Drell-Yan observables

Differential distributions at N3LO

- parton-level implementation of all V+jet processes at NNLO
- combined with three-loop virtual corrections (form factor)
- subtraction scheme for handling of infrared-singular contributions

Subtraction methods applicable at N3LO

- Projection to Born [M.Cacciari, F.Dreyer, A.Karlberg, G.Salam, G.Zanderighi]

$$\frac{d\sigma_X^{N3LO}}{dO} = \frac{d\sigma_{X+j}^{NNLO}}{dO} - \frac{d\sigma_{X+j}^{NNLO}}{dO_B} + \frac{d\sigma_X^{N3LO,\,incl}}{dO_B}$$

- q_T subtraction [S.Catani, M.Grazzini]

$$\frac{d\sigma_X^{N3LO}}{dO} = \mathcal{H}_{N3LO} \otimes \frac{d\sigma_X^{LO}}{dO} + \left[\int_{q_{T,X}} \frac{d\sigma_{X+j}^{NNLO}}{dO} - \frac{d\sigma_{X,CT}^{NNLO}}{dO}(q_T) \right]$$

Towards full N3LO in Drell-Yan observables

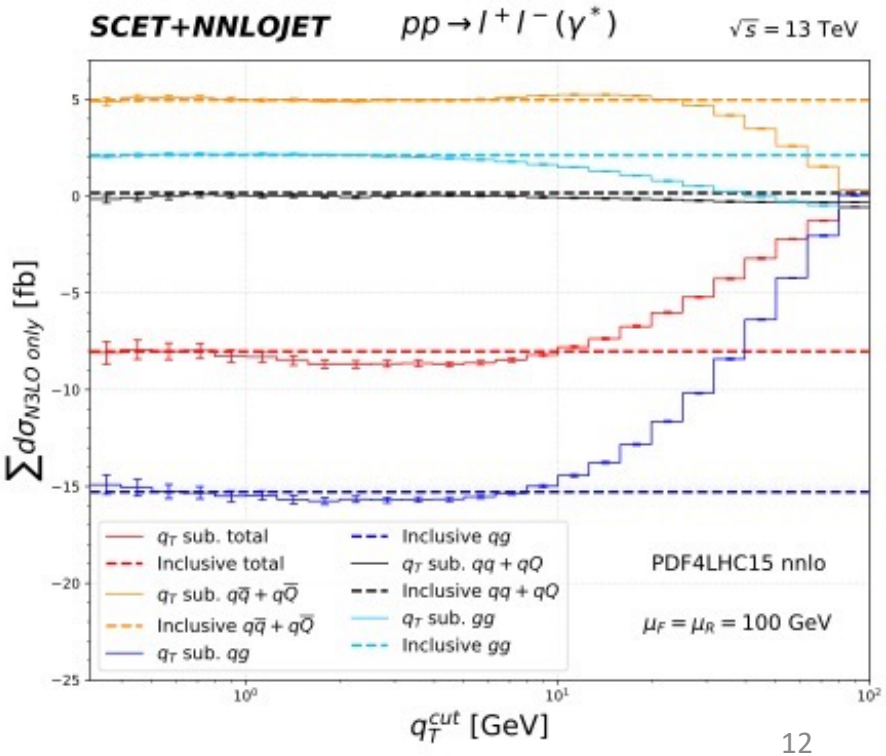
NNLOJET implementation of Drell-Yan processes at N3LO

[X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG: 2107.09085]

- based on V+jet at NNLO
- using antenna subtraction for infrared subtraction at NNLO

Genuine N3LO singularities: q_T subtraction

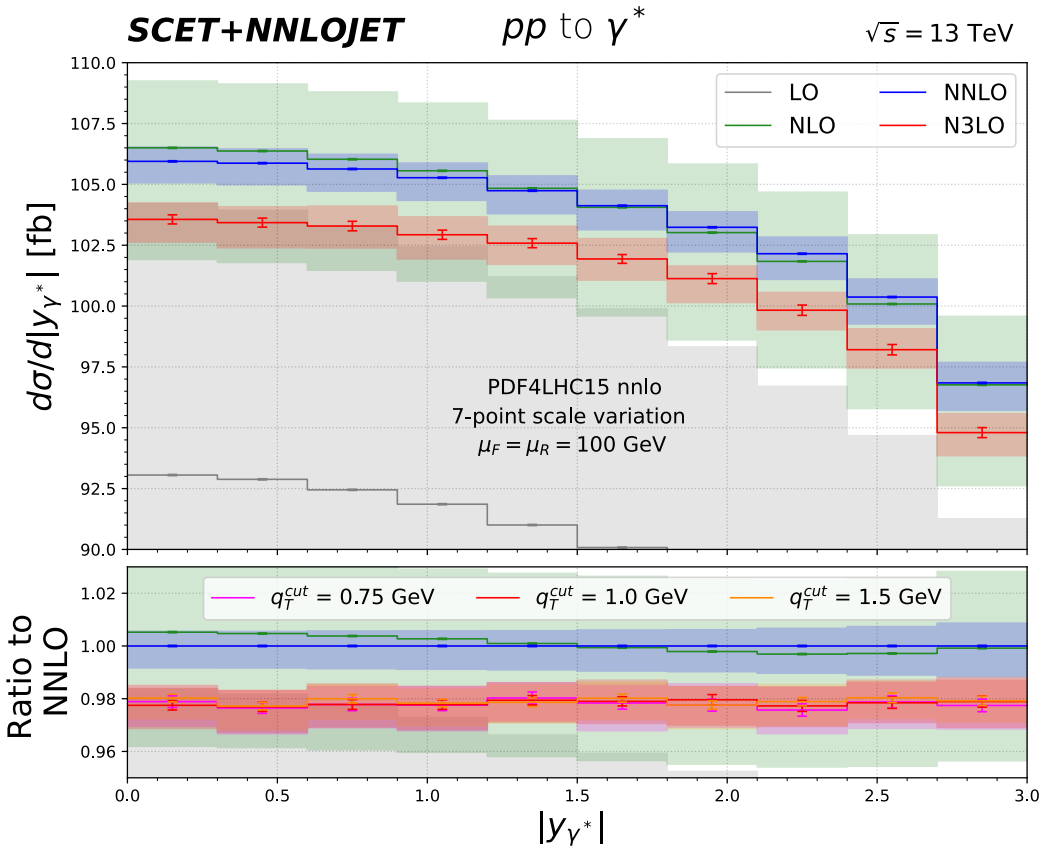
- obtain q_T counterterm from expansion of N3LL q_T resummation to $O(\alpha_s^3)$
- ingredients: three-loop soft and beam functions
  [Y.Li, H.X.Zhu; M.Ebert, B.Mistlberger, G.Vita; M.X.Luo, H.X.Zhu, T.Z.Yang, Y.J.Zhu] (talk of G.Vita)
- check: independence on $q_{T,cut}$ slicing parameter
- check: reproduce inclusive coefficient functions (no ingredients or methodology in common!)



N3LO in Drell-Yan observables

Rapidity distribution of lepton pair

- N3LO corrections uniform in y
- same size as inclusive N3LO K-factor
- N3LO outside NNLO scale uncertainty
- scale uncertainty remains at 1% level
- still: inclusive in lepton kinematics

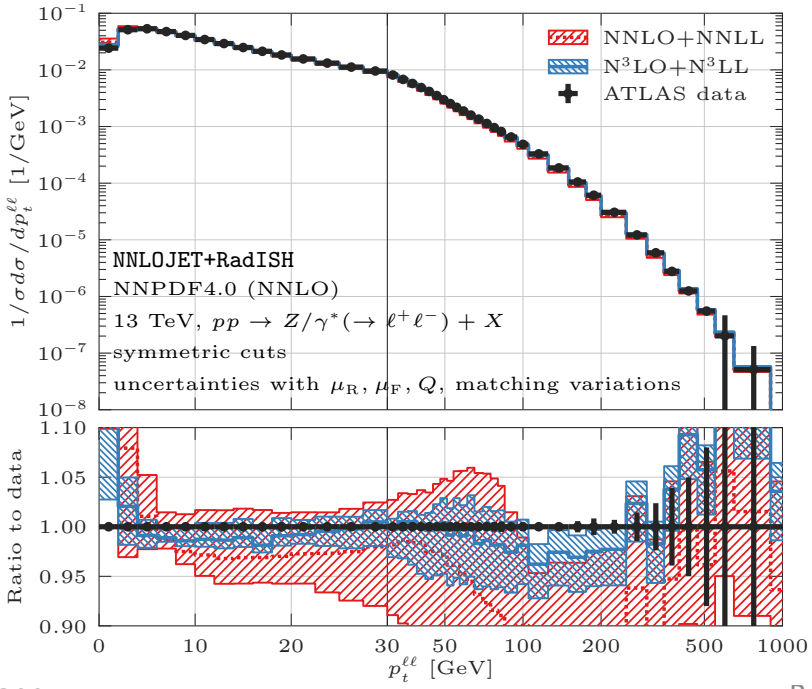


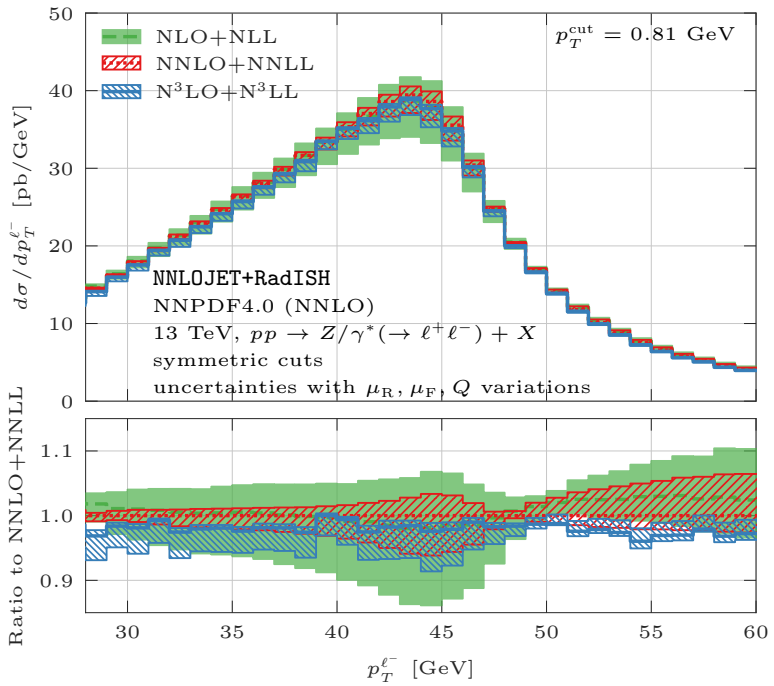
N3LO in Drell-Yan observables

Matching of N3LO with N3LL resummation

[X.Chen, E.W.N.Glover, A.Huss, P.F.Monni, E.Re, L.Rottoli, P.Torrielli, TG: 2203.01565]

- resummation in momentum space (RadISH)
- fiducial cross sections: lepton pair and single lepton distributions in NC Drell-Yan process
- improved perturbative convergence: uncertainty on NNLO+NNLL larger than NNLO-only





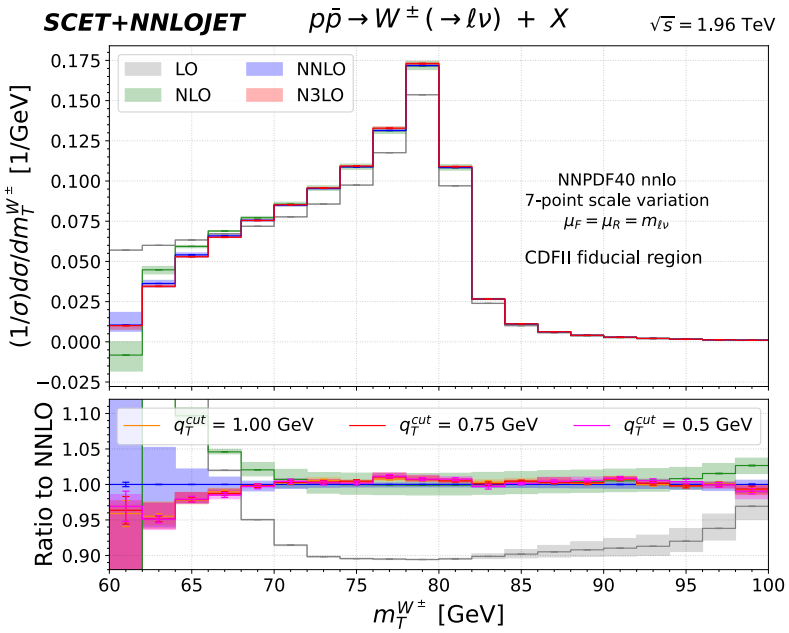
N3LO in Drell-Yan observables

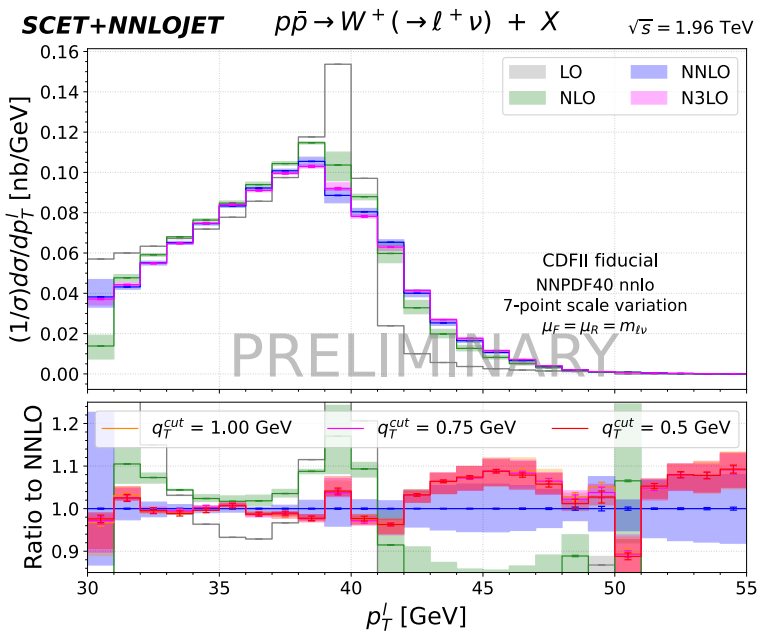
[G.Fontana]

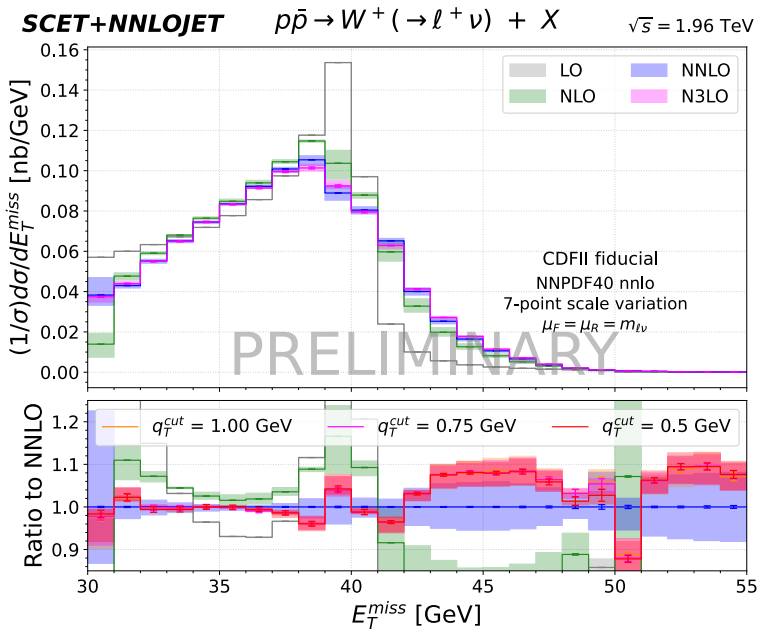
Normalized fiducial distributions in W production

[X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG: 2205.11426 and work in progress]

- relevant for W mass extraction (CDF II, future LHC measurements) (talk of P.Torrielli)
- N3LO corrections for CDFII kinematics flat in m_T, but non-trivial shape in p_T^l, E_T^{miss}



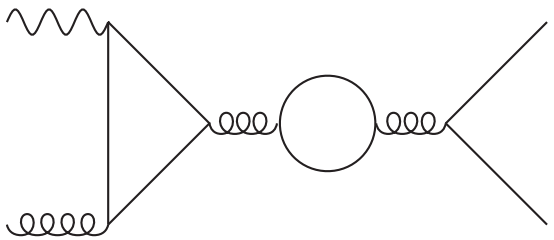




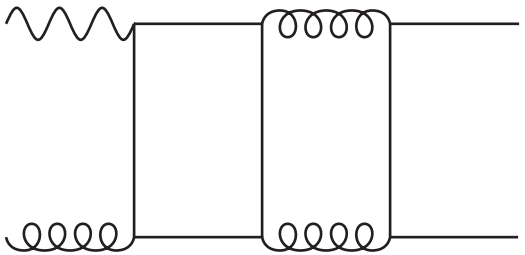
Quark singlet contributions

Vector boson coupling to internal quark loop

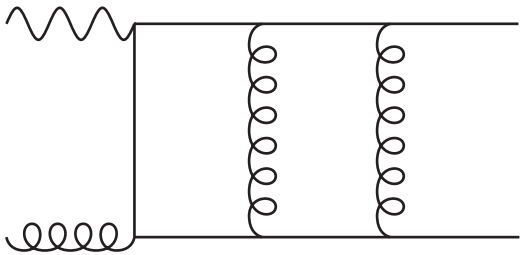
- contributions not included in differential Drell-Yan calculations at $O(\alpha_s^3)$
- but: included in inclusive N3LO coefficient function [C.Duhr, B.Mistlberger]
- amplitudes with vector and axial-vector couplings
  - three-loop quark form factors [A.Primo, TG; L.Chen, M.Czakon, M.Niggetiedt]
  - two-loop V+qqg [T.Peraro, L.Tancredi, TG: 2211.13596]
  - one-loop V+qqgg, V+qqq'q' [Z.Bern, L.Dixon, D.Kosower]



quark singlet



quark singlet



quark non-singlet

Two-loop axial-vector V+qqg amplitudes

Non-singlet V+qqg two-loop amplitudes well known

- computed with vector coupling, axial vector follows from chirality conservation
  [L.Garland, N.Glover, A.Koukoutsakis, E.Remiddi, TG]

Singlet amplitudes: basis decomposition [T.Peraro, L.Tancredi; L.Chen]

- four-dimensional basis vectors: $p_1^\mu; p_2^\mu; p_3^\mu; \quad v_A^\mu = \varepsilon^{\alpha\beta\gamma\mu} p_{1\alpha} p_{2\beta} p_{3\gamma}$
- with: $v_A \cdot v_A = (d-3) s_{12} s_{13} s_{23}/4$
- amplitude decomposition into form factors (F_i: vector, G_i: axial-vector)

$$\mathcal{M} = -i\sqrt{4\pi\bar{\alpha}_s}\, \mathrm{T}^a_{ij}\, \varepsilon_{4,\mu}\varepsilon_{3,\nu} A^{\mu\nu}$$

$$\begin{aligned} A^{\mu\nu} &= \bar{u}(p_2)\not{p}_3 u(p_1)\Big[F_1 p_1^\mu p_1^\nu + F_2 p_2^\mu p_1^\nu + F_3 g^{\mu\nu} + G_1 p_1^\mu v_A^\nu + G_2 p_2^\mu v_A^\nu + G_3 v_A^\mu p_1^\nu\Big] \\ &+ \bar{u}(p_2)\gamma^\nu u(p_1)\Big[F_4 p_1^\mu + F_5 p_2^\mu\Big] + \bar{u}(p_2)\gamma^\mu u(p_1) F_6 p_1^\nu \\ &+ \bar{u}(p_2)\not{v}_A u(p_1)\Big[G_4 p_1^\mu p_1^\nu + G_5 p_2^\mu p_1^\nu\Big] + G_6\Big[\bar{u}(p_2)\gamma^\mu u(p_1) v_A^\nu + \bar{u}(p_2)\gamma^\nu u(p_1) v_A^\mu\Big] \end{aligned}$$

Two-loop axial-vector V+qqg amplitudes

Computation of form factors using projectors

- four-dimensional basis, projections performed in *d* dimensions
- using Larin γ_5 in axial-vector coupling $\gamma^\mu\gamma^5 \to \frac{1}{2}\left(\gamma^\mu\gamma^5 - \gamma^5\gamma^\mu\right) = \frac{1}{6}\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_\rho\gamma_\sigma$
- renormalization of axial vector non-singlet and singlet currents in QCD
- coupling constant renormalization

Resulting form factors are IR-divergent at one-loop and two-loop

- define finite remainders using IR subtraction
  [S.Catani; L.Dixon, E.Gardi, L.Magnea; T.Becher, M.Neubert]
- results four-dimensional, justify absence of evanescent form factors and projectors

$$F^{(1)}_{i,\text{fin}} = F^{(1)}_i - I_1(\epsilon)F^{(0)}_i \qquad G^{(1)}_{i,\text{fin}} = G^{(1)}_i - I_1(\epsilon)G^{(0)}_i$$

$$F^{(2)}_{i,\text{fin}} = F^{(2)}_i - I_1(\epsilon)F^{(1)}_i - I_2(\epsilon)F^{(0)}_i \qquad G^{(2)}_{i,\text{fin}} = G^{(2)}_i - I_1(\epsilon)G^{(1)}_i - I_2(\epsilon)G^{(0)}_i$$

Two-loop axial-vector V+qqg amplitudes

Helicity amplitudes, including vector boson decay to leptons, e.g.

$$\mathrm{M}^v_{L+L} = \frac{1}{\sqrt{2}}\left[\langle 12\rangle[13]^2\Big(\alpha_1\langle 536] + \alpha_2\langle 526]\Big) + \alpha_3\langle 25\rangle[13][36]\right]$$

$$\mathrm{M}^a_{L+L} = \frac{1}{\sqrt{2}}\left[\langle 12\rangle[13]^2\Big(\beta_1\langle 536] + \beta_2\langle 526]\Big) + \beta_3\langle 25\rangle[13][36]\right]$$

- coefficients expressed in terms of form factors
- non-singlet: $\alpha_i = -\beta_i$, after IR subtraction (chirality conservation)
- dressed with SU(2)$\otimes$U(1) couplings and gauge boson propagator factors

New result: singlet axial-vector amplitudes at two loops

- anomalous for single quark flavour, vanish for mass-degenerate quark isospin doublets
- finite remainder for top-bottom mass splitting (large-m_t expansion, $m_b = 0$)

Summary

- Drell-Yan process enables broad range of precision studies
- Complex interplay between observable definitions and fiducial cuts
- Demands ultimate per-cent level precision on fiducial distributions
  - N3LO fixed-order, matched on N3LL resummation
  - combined with higher-order electroweak corrections
- First results, enabled by important computational advances
  - N3LO corrections uniform in inclusive observables
  - non-trivial shape deformations for some fiducial distributions
  - quark-singlet type loop amplitudes ready to be included
- Preparing for LHC phenomenology at ultimate precision