Power corrections to EEC meets conformal bootstrap

Hua Xing Zhu
Zhejiang University

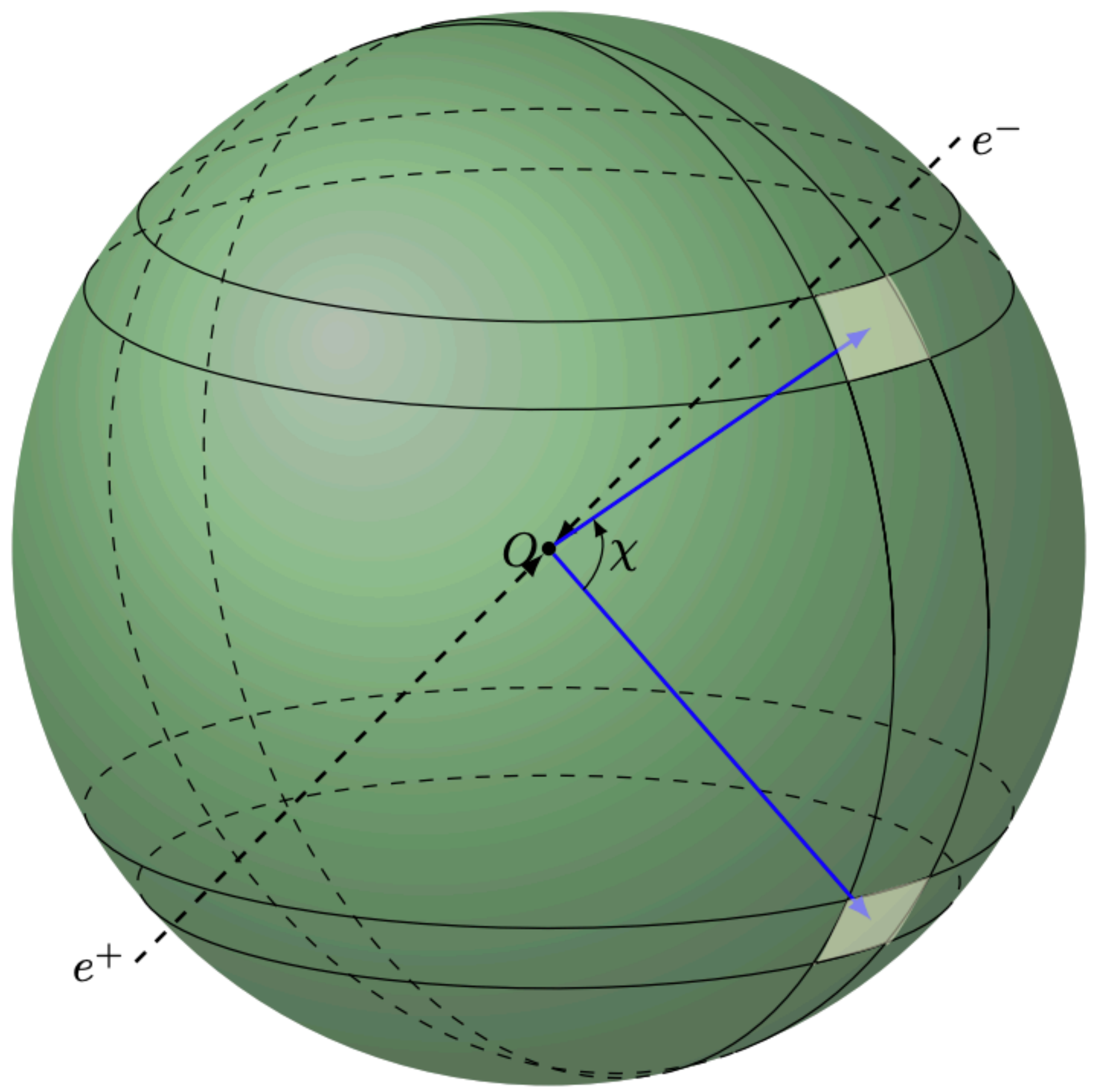
with Hao Chen, Xinan Zhou, 2301.03616
with Hao Chen, in preparation

RADCOR 2023
University of Edinburgh
May 28th to June 2nd, 2023

Energy-Energy Correlation functions

- EEC is the energy weighting two-particle angular correlation in e^+e^-.

Basham, Brown, Ellis, Love, 1978



$$\frac{d^2\Sigma}{d\Omega\, d\Omega'} = \sum_{N=2}^{\infty} \int \prod_{a=1}^{N} E_a^{-1} d^3p_a \frac{d^N\sigma}{E_1^{-1} d^3p_1 \cdots E_N^{-1} d^3p_N} S_N \left[\sum_{b,c=1}^{N} \frac{E_b E_c}{W^2} \delta(\Omega_b - \Omega) \delta(\Omega_c - \Omega') \right]$$

- Energy weighting is the unique way to make the measurement IRC safe.

Field theory definition

Hofman, Maldacena, 2008; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2013

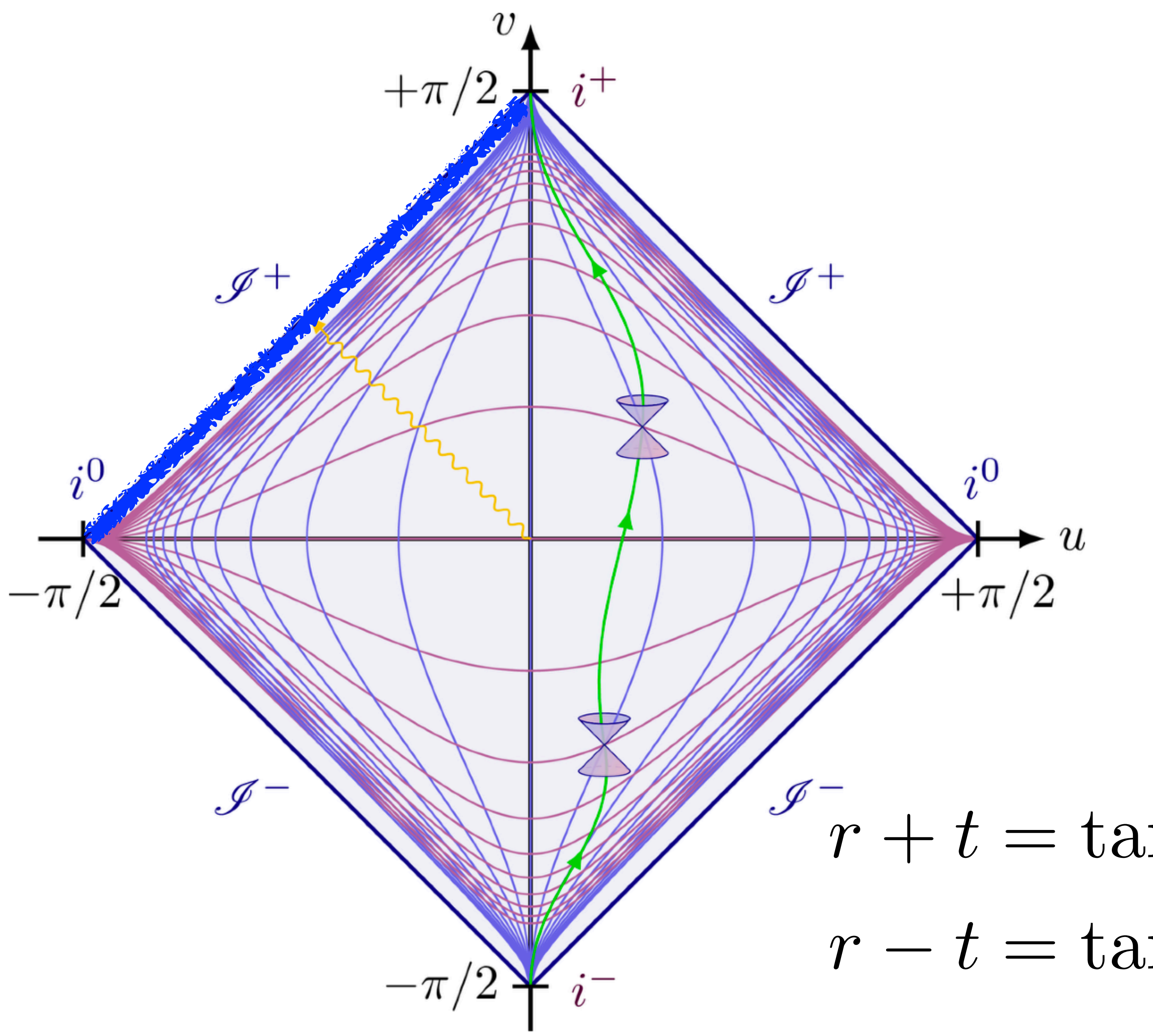
- EEC can be defined as Wightman correlator of sources, sink, and energy flow operators

$$J^\mu(x) = \psi(x)\gamma^\mu\psi(x)$$

$$\text{EEC}(\chi) = \int d^4x e^{-ix\cdot q}\langle\Omega|J^\mu(x)\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)J_\mu(0)|\Omega\rangle$$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ \vec{n}_i T^{0i}(t, r\vec{n})$$

Tkachov, 1995



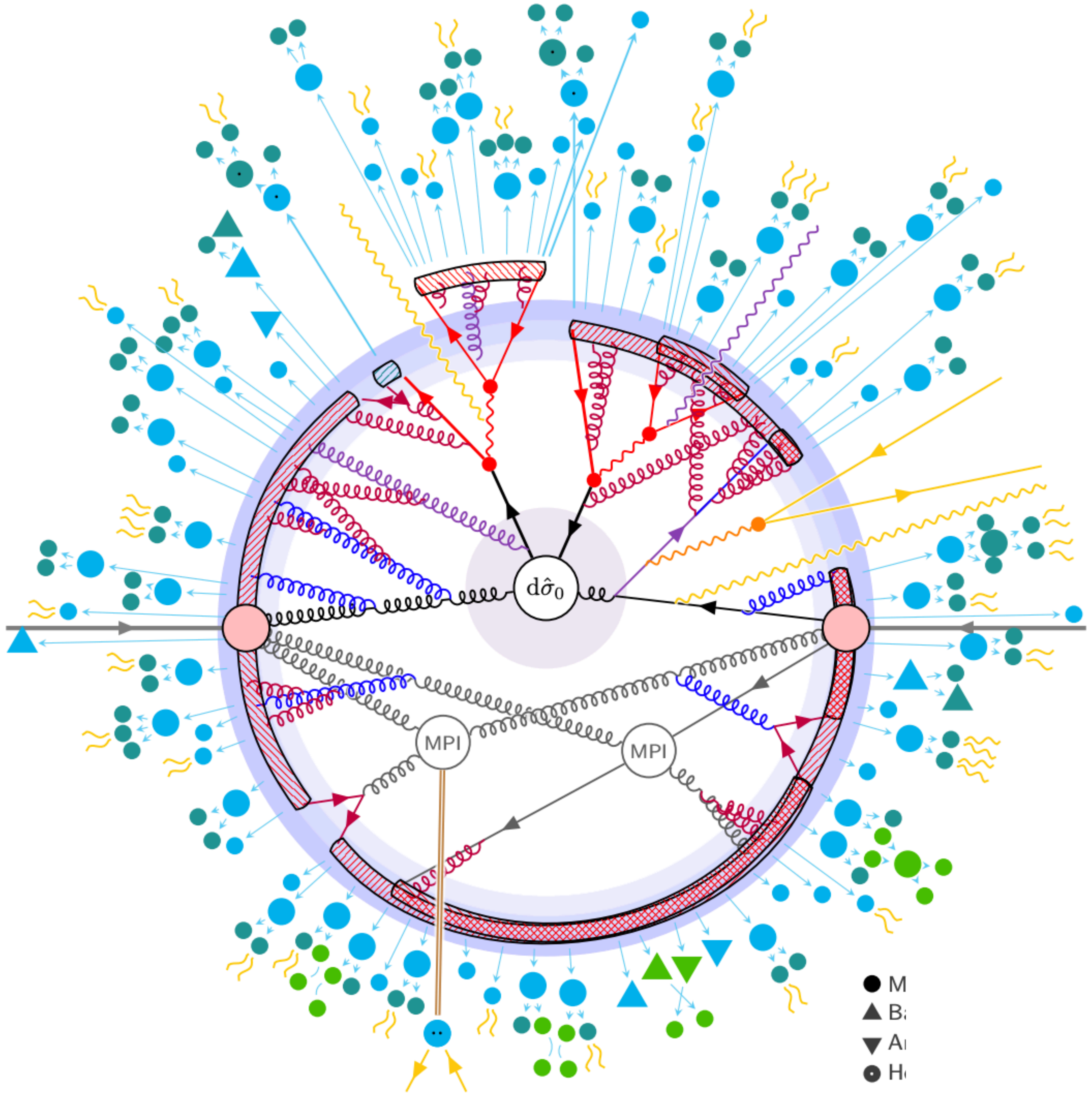
$$r + t = \tan(u+v)$$
$$r - t = \tan(u-v)$$

https://tikz.net/relativity_penrose_diagram/

Modern phenomenological application

- Multi-point projected energy correlators
  2004.11381
- Spin correlation in gluon jet
  2011.02492
- Applications to track based observables
  2108.01674, 2201.05166
- Visualization of the parton fragmentation evolution
  2201.07800
- Top quark mass measurement
  2201.08393
- Non-Gaussiantities in collider energy flux
  2205.02857
- Decay density matrix for weak gauge boson
  2207.03511
- Nuclear energy correlator and gluon saturation
  2209.02080, 2301.01788
- Dead cone effects for massive quark jets
  2210.09311
- QGP and medium modification
  2209.11236, 2303.03413, 2303.08143

Visualizing time evolution of parton fragmentation

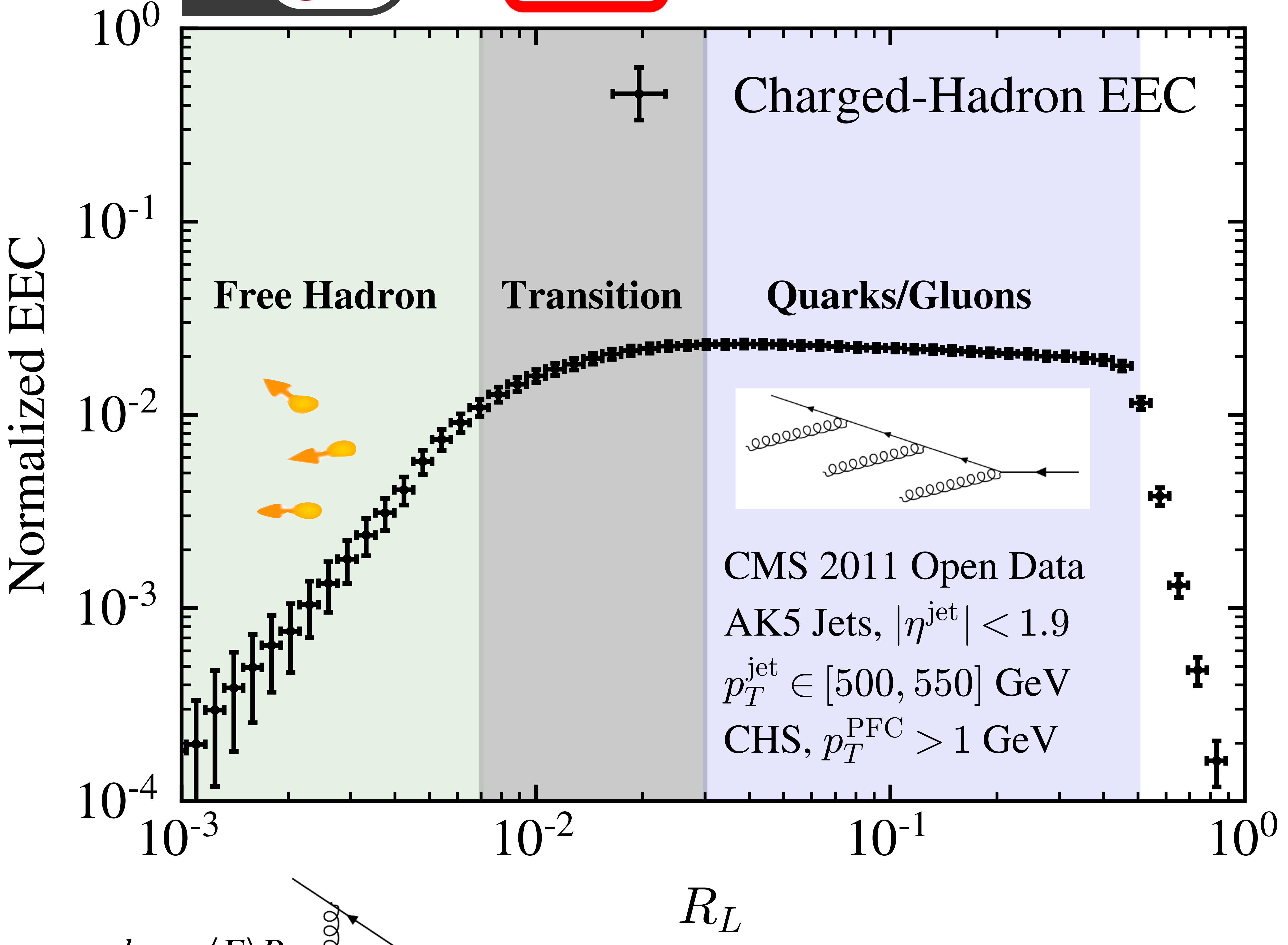


Pythia manual

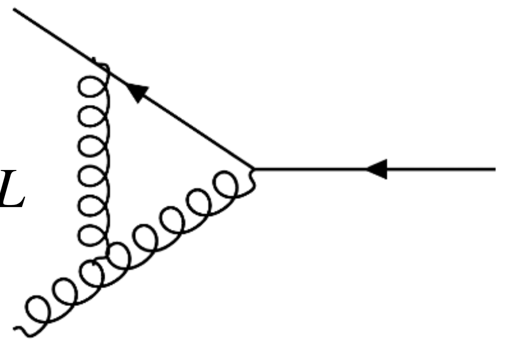
MOD

WARNING

NOT REAL MEASUREMENT



$k_\perp = \langle E \rangle R_L$



Komiske, Moult, Thaler, HXZ, 2022

There are a number of pQCD-related stories I have left untold.

Why did it take almost 20 years for the inclusive energy–energy correlation in $e^+e^- \to h_1 h_2 X$, believed to be the most reliable IRCS pQCD prediction, to agree with the experimental data?

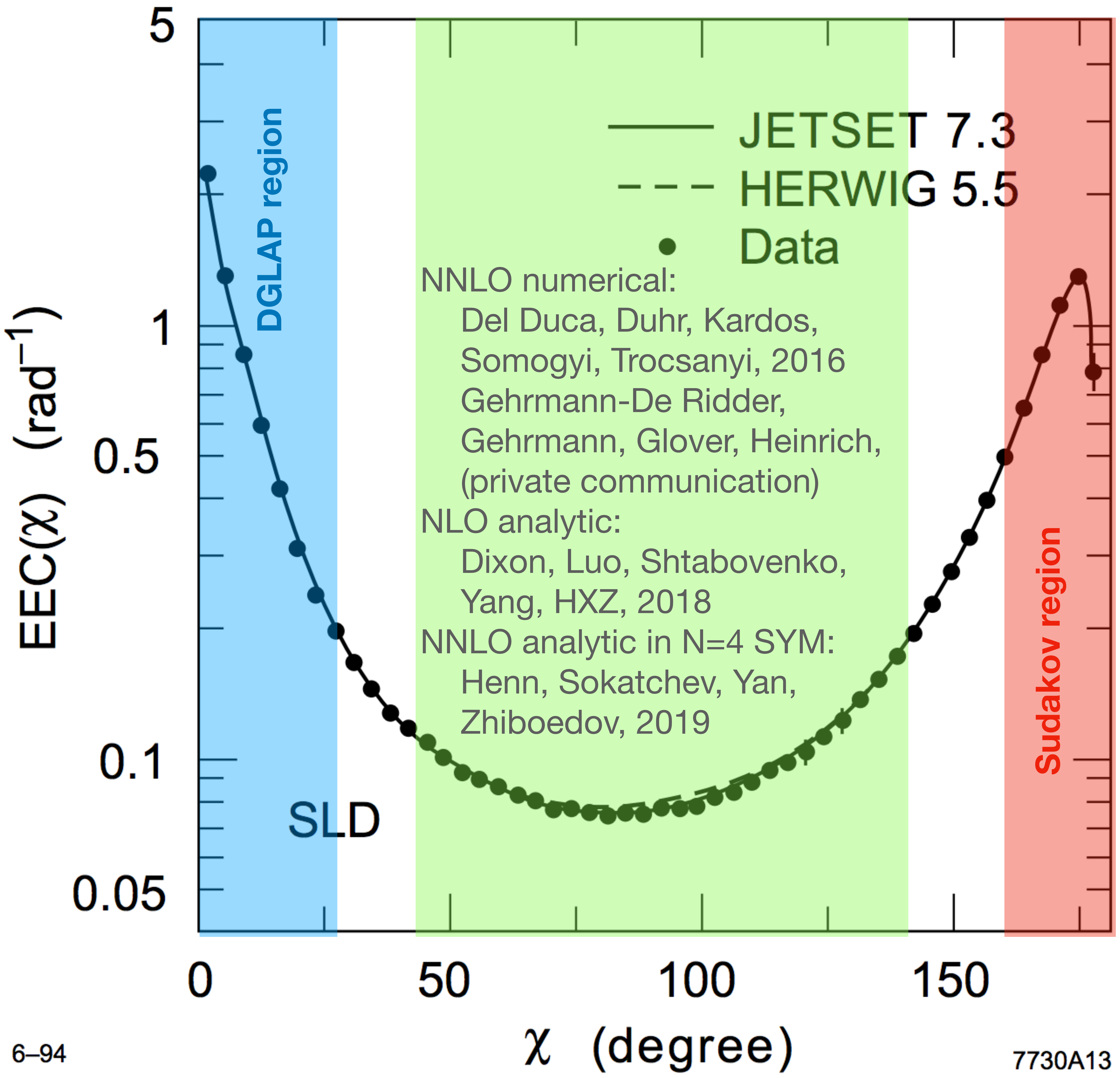
Dokshitzer, in 《50 Years of Quantum Chromodynamics》 2022

collinear divergences

Leading Log:
- Konishi, Ukawa, Veneziano, 1978
- Richards, Stirling, Ellis, 1982

NNLL and beyond:
- Dixon, Moult, HXZ, 2019

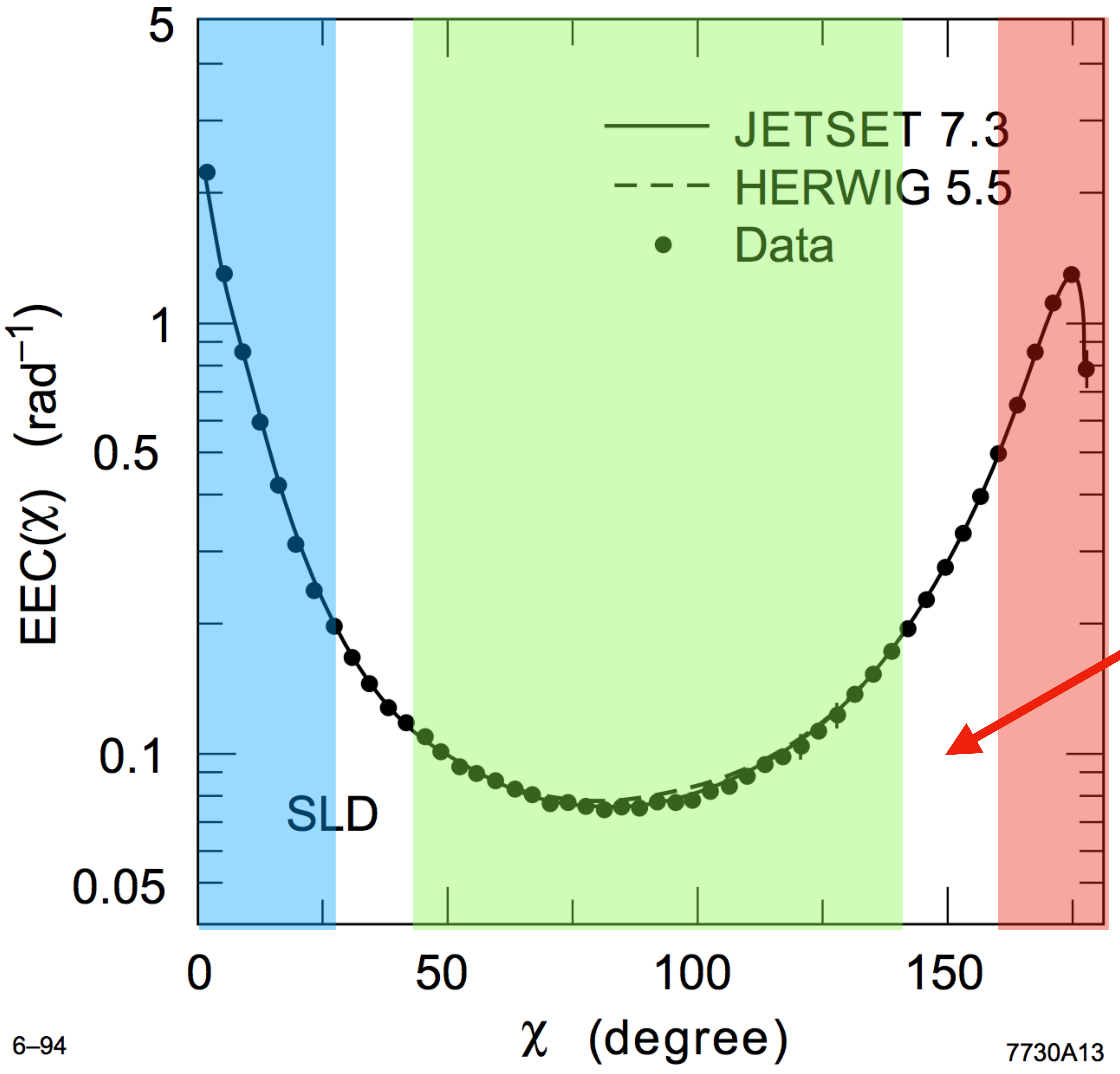
CFT:
- Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019
- Korchemsky, 2019



Soft + collinear divergences

CSS:
- Collins, Soper, 1982
- Dokshitzer, Marchesini, Webber, 1999
- de Florian, Grazzini, 2004
- Kardos, Kluth, Somogyi, Tulipant, Verbytskyi, 2018

SCET:
- Moult, HXZ, 2018
- Duhr, Mistlberger, Vita, 2022



Power corrections to Sudakov logarithms

This talk

The main goal of this talk is to introduce
a new method of resumming Sudakov logarithms in EEC,
by exploiting its position space definition,
and using techniques from conformal bootstrap program.

Resummation in momentum space

$$\text{EEC}(\chi) = \int d^4x e^{-ix\cdot q} \langle \Omega | J^\mu(x) \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \sum_i |X_i\rangle\langle X_i| J_\mu(0)|\Omega\rangle$$

Insertion of complete state

$$\sum_i |X_i\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}gg\rangle + |q\bar{q}q'\bar{q}'\rangle + \cdots$$

Amplitudes, Loops and phase space integrals

$$\text{EEC}(\chi) = \int \text{LIPS} \sum_i |\mathcal{M}_{e^+e^-\to X_i}|^2 E_{\hat{n}_1} E_{\hat{n}_2}$$

Degenerate states: soft, collinear, rapidity

Regulator + factorization ⇒ resummation by evolution equation

🙂 On-shell amplitudes easy to calculate; particle physics intuition

🙁 Requires cancellation of unphysical parameters; (conformal, crossing) symmetry not manifest

Position space calculation

$$\text{EEC}(\chi) = \int d^4x e^{-ix\cdot q} \langle \Omega | J^\mu(x) \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) J_\mu(0) | \Omega \rangle$$

$$\text{EEC}(\chi) = \int d^4x\, e^{-ixq} \int dt_1 \int dt_2$$

$$\lim_{r_1 \to \infty} r_1^2 \lim_{r_2 \to \infty} r_2^2 \langle \Omega | J^\mu(x) T^{0\hat{n}_1}(t_1, r_1\hat{n}_1) T^{0\hat{n}_2}(t_2, r_2\hat{n}_2) J_\mu(0) | \Omega \rangle$$

A local, infrared finite four-point Wightman correlator in Minkowskian signature

Where does the Sudakov logarithms come from?

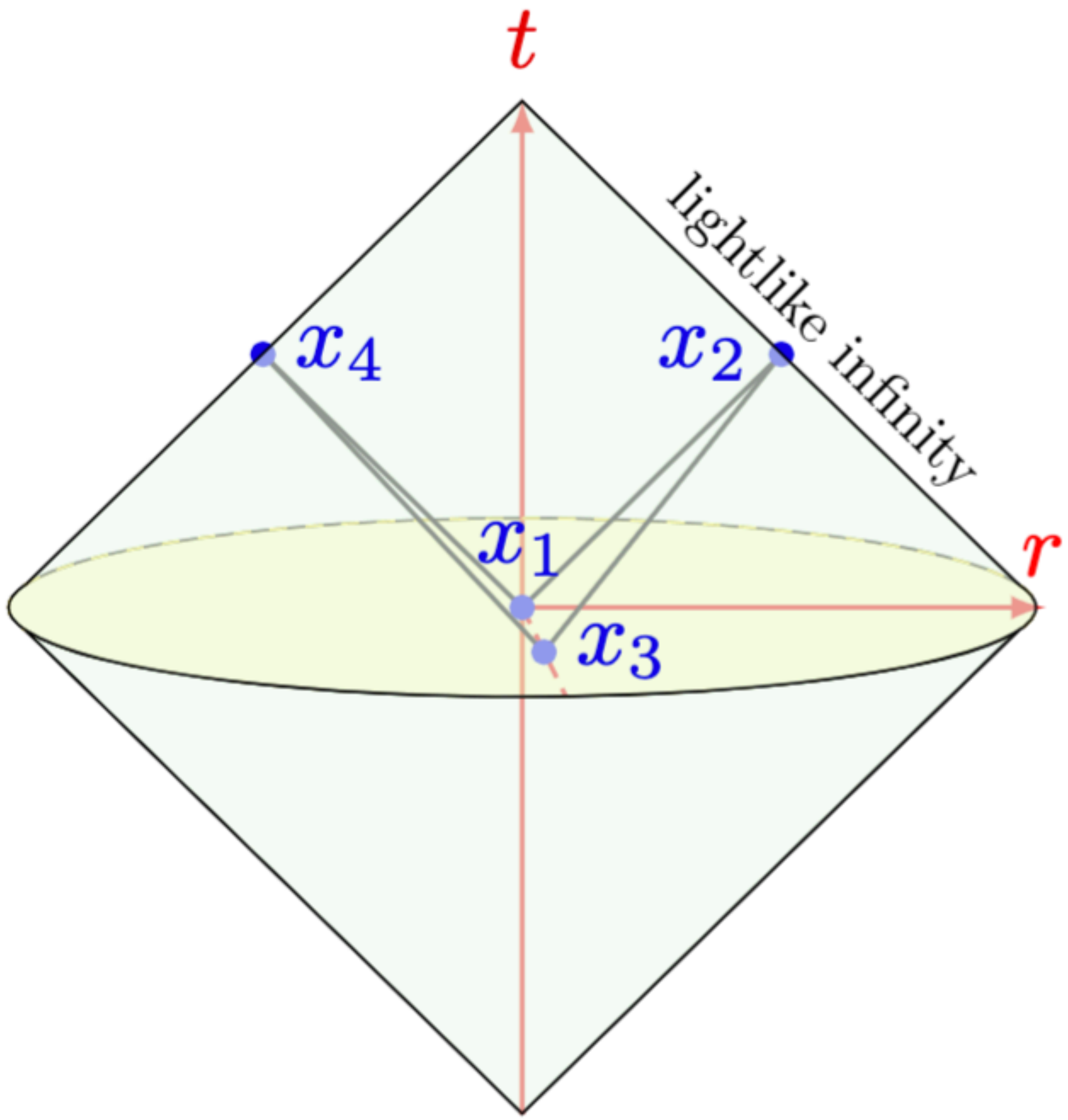
$$\int d^4x_{13} e^{-ix_{13}q} \quad \langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3) \rangle \qquad x_1 = 0, \quad x_2 = (t_2, r\vec{n}_2), \quad x_4 = (t_4, r\vec{n}_4)$$

Choose a frame where detectors are exactly back-to-back $n_2 = \bar{n}_4$

$$\frac{q_\perp^2}{q^2} \sim 0 \Rightarrow \frac{x_{13}^+ x_{13}^-}{x_{13}^2} \sim 0$$

$$|x_{13,\perp}|^2 \gg x_{13}^+ x_{13}^-$$

- x_2 is integrated over a null line at infinity
- Its dominated contribution comes from the region where $(x_1 - x_2)^2/r^2 \sim 0$
- x_3 is transverse separated from $x_1 \Rightarrow (x_2 - x_3)^2/r^2 \sim 0$

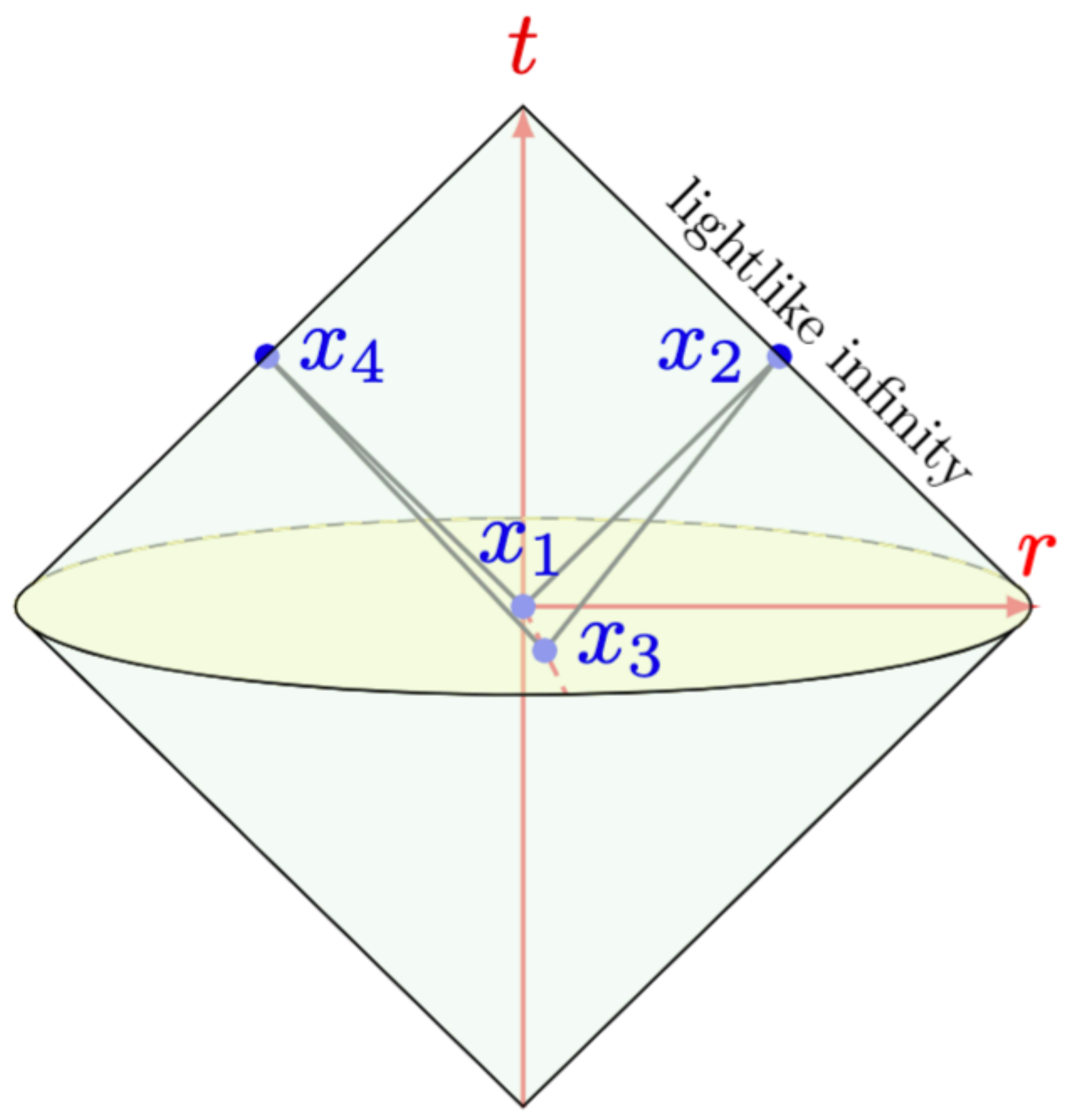


Double lightcone limit

- E.g. a 4-pt scalar correlator in N=4 SYM:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \frac{1}{(2\pi)^4}\frac{x_{13}^4 x_{24}^4}{(x_{12}^2 x_{34}^2)^4}\mathcal{F}(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}\,, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$



- Double lightcone limit: $u \to 0\,, v \to 0\ (z \to 0\,, \bar{z} \to 1)$

$$\text{one loop} = \left[-\frac{1}{4}\log u \log v + 0\cdot\log(uv) + \cdots\right] - \left[\frac{1}{4}(u+v)\log u\log v + \frac{1}{2}(u\log u + v\log v) + \cdots\right] + \cdots\,,$$

$$\text{two loop} = \left[\frac{1}{16}\log^2 u\log^2 v + 0\cdot\log u\log v\log(uv) + \cdots\right] + \left[\frac{1}{8}(u+v)\log^2 u\log^2 v + \frac{3}{16}\log u\log v(u\log u + v\log v) + \frac{1}{8}\log u\log v(v\log u + u\log v) + \cdots\right] + \cdots\,,$$

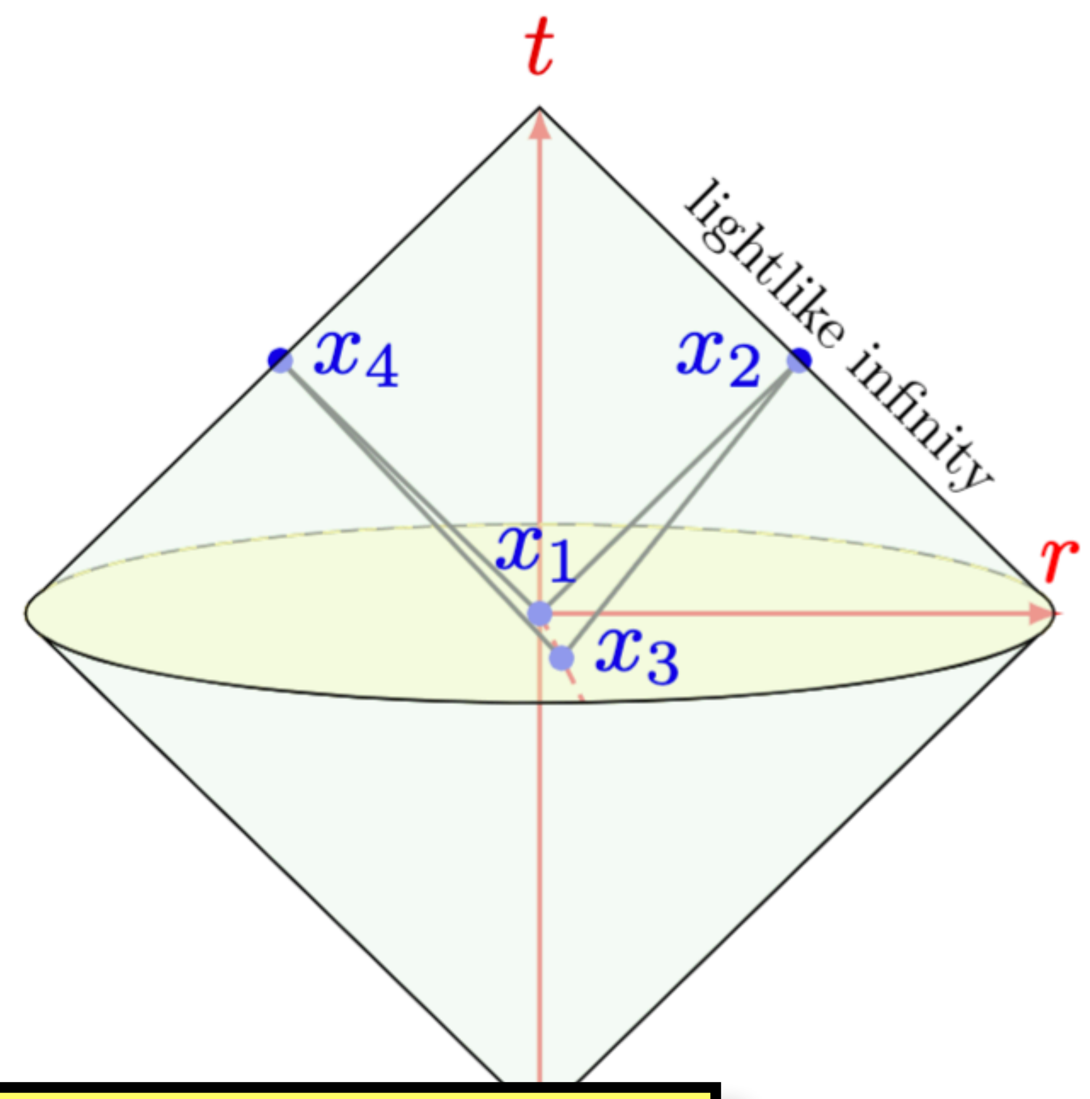
$$\text{three loop} = \left[-\frac{1}{96}\log^3 u\log^3 v + 0\cdot\log^2 u\log^2 v\log(uv) + \cdots\right] - \left[\frac{1}{48}(u+v)\log^3 u\log^3 v\right.$$

Drummond, Duhr, Eden, Heslop, Pennington, Smirnov, 2013

- E.g. a 4-pt scalar correlator in N=4 SYM:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \frac{1}{(2\pi)^4}\frac{x_{13}^4 x_{24}^4}{(x_{12}^2 x_{34}^2)^4}\mathcal{F}(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}\,, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$



- D

Message #1: The Sudakov limit of EEC is corresponding to the double lightcone limit of a local 4-point Minkowskian correlator

one $\cdots$,

$$\text{two loop} = \left[\frac{1}{16}\log^2 u \log^2 v + 0\cdot \log u \log v \log(uv) + \cdots\right] + \left[\frac{1}{8}(u+v)\log^2 u \log^2 v\right.$$

$$\left. + \frac{3}{16}\log u \log v(u\log u + v \log v) + \frac{1}{8}\log u \log v(v \log u + u \log v) + \cdots\right] + \cdots,$$

$$\text{three loop} = \left[-\frac{1}{96}\log^3 u \log^3 v + 0\cdot \log^2 u \log^2 v \log(uv) + \cdots\right] - \left[\frac{1}{48}(u+v)\log^3 u \log^3 v\right.$$

Drummond, Duhr, Eden, Heslop, Pennington, Smirnov, 2013

- Where does the double logarithms come from in a local 4-pt correlator?

- Lightcone OPE of

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} \lambda_{\mathcal{O}} C_{\mathcal{O}}(x_{12}, \partial_{x_2})\langle \mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle$$

OPE of 1, 2

$$\mathcal{C}_{\mathcal{O}}(x_{12}, \partial_{x_2}) = \frac{1}{|x_{12}|^{2\delta - \Delta_{\mathcal{O}}}}\left[1 + \frac{1}{2}x_2^\mu \partial_{2,\mu} + +\alpha x_2^\mu x_2^\nu \partial_{2,\mu}\partial_{2,\nu} + \cdots\right]$$

twist expansion: lightcone singularity of x_{12}

- Where does the double logarithms come from in a local 4-pt correlator?

- Lightcone OPE of

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} \lambda_{\mathcal{O}} C_{\mathcal{O}}(x_{12}, \partial_{x_2}) \langle \mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle$$

OPE of 1, 2

$$\mathcal{C}_{\mathcal{O}}(x_{12}, \partial_{x_2}) = \frac{1}{|x_{12}|^{2\delta - \Delta_{\mathcal{O}}}} \left[1 + \frac{1}{2} x_2^{\mu} \partial_{2,\mu} + +\alpha x_2^{\mu} x_2^{\nu} \partial_{2,\mu} \partial_{2,\nu} + \cdots \right]$$

sum over infinite descendent operators

twist expansion: lightcone singularity of x_{12}

- Where does the double logarithms come from in a local 4-pt correlator?

- Lightcone OPE of

$$\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} \lambda_{\mathcal{O}} C_{\mathcal{O}}(x_{12}, \partial_{x_2})\langle\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle$$

OPE of 1, 2

sum over infinite primary operators

$$\mathcal{C}_{\mathcal{O}}(x_{12}, \partial_{x_2}) = \frac{1}{|x_{12}|^{2\delta-\Delta_{\mathcal{O}}}}\left[1 + \frac{1}{2}x_2^{\mu}\partial_{2,\mu} + +\alpha x_2^{\mu}x_2^{\nu}\partial_{2,\mu}\partial_{2,\nu} + \cdots\right]$$

sum over infinite descendent operators

twist expansion: lightcone singularity of x_{12}

- Where does the double logarithms come from in a local 4-pt correlator?

- Lightcone OPE of

$$\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} \lambda_{\mathcal{O}} C_{\mathcal{O}}(x_{12}, \partial_{x_2})\langle\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle$$

OPE of 1, 2

sum over infinite primary operators

$$\mathcal{C}_{\mathcal{O}}(x_{12}, \partial_{x_2}) = \frac{1}{|x_{12}|^{2\delta-\Delta_{\mathcal{O}}}}\left[1 + \frac{1}{2}x_2^{\mu}\partial_{2,\mu} + +\alpha x_2^{\mu}x_2^{\nu}\partial_{2,\mu}\partial_{2,\nu} + \cdots\right]$$

sum over infinite descendent operators

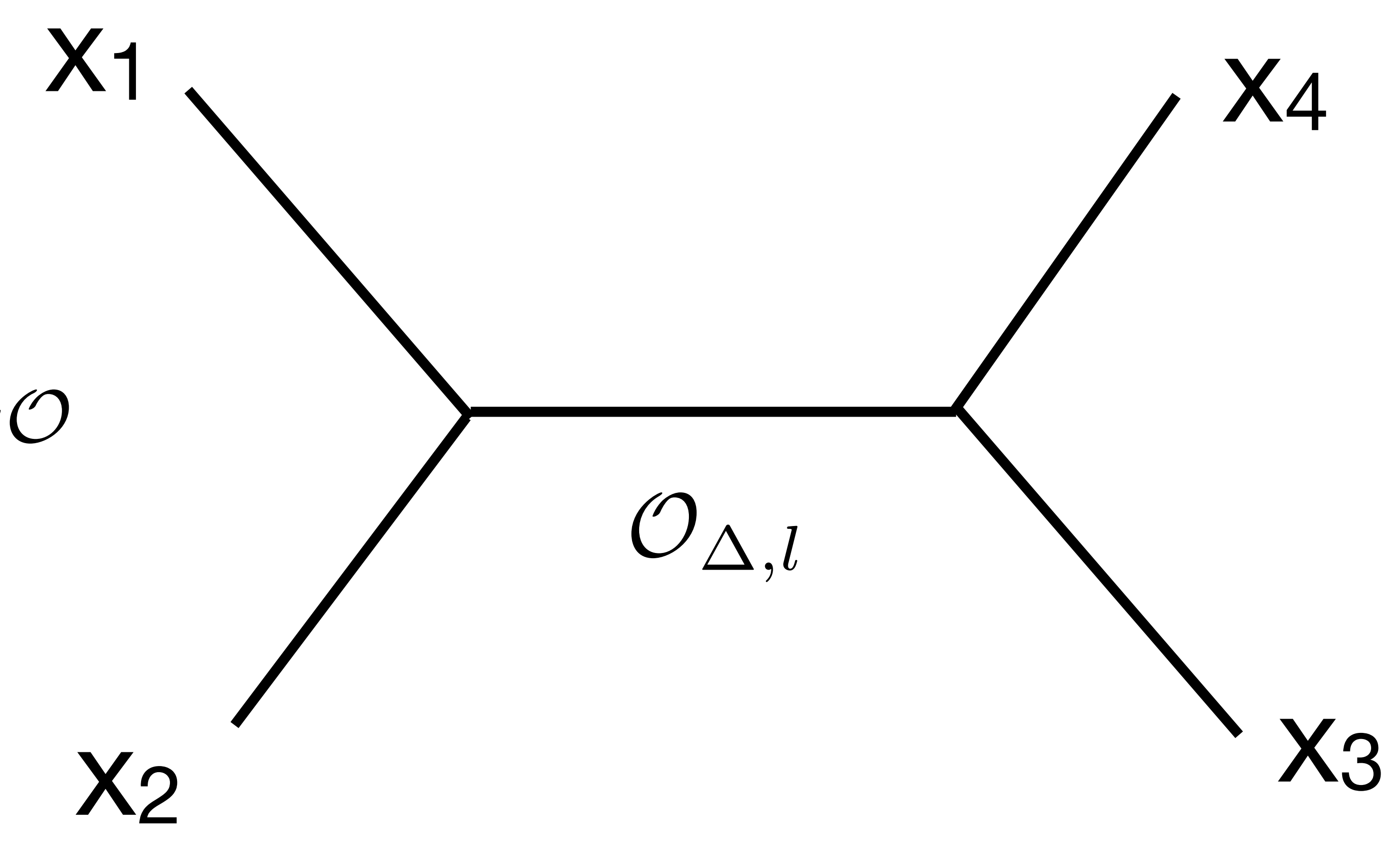
twist expansion: lightcone singularity of x_{12}

- Singularity in x_{23} from sum over infinite operators (descendant of primary)

- Conformal block expansion of 4-pt correlator:

$$\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} \lambda_{\mathcal{O}} C_{\mathcal{O}}(x_{12}, \partial_{x_2})\langle\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle$$

$$= \sum_{\mathcal{O}} a_{\mathcal{O}}$$



conformal block

$$= \text{prefactor} \times \sum_{\mathcal{O}} a_{\mathcal{O}} G_{\Delta,l}(u,v)$$

Dolan, Osborn, 2001

$$G_{\Delta,\ell}(u,v) = \frac{z\bar{z}}{\bar{z}-z}\left[k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})\right]$$

$$k_\beta(x) = x^{\beta/2}\,{}_2F_1(\beta/2, \beta/2, \beta; x)$$

- Double lightcone limit ($z \to 0$, $\bar{z} \to 1$): $G_{\Delta,l}(u,v) \to z^{\tau/2}\left[\log(1-\bar{z}) + \cdots\right]$

log div. from infinite descendant

- Conformal block expansion of 4-pt correlator:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} \lambda_{\mathcal{O}} C_{\mathcal{O}}(x_{12}, \partial_{x_2})\langle \mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle$$

conformal block

Message #2: Instead of expanding in fixed-multiplicity amplitudes, we can expand in local operator exchange. Double logarithms from lightcone singularity + infinite operator summation.

2001

$\bar{z})]$

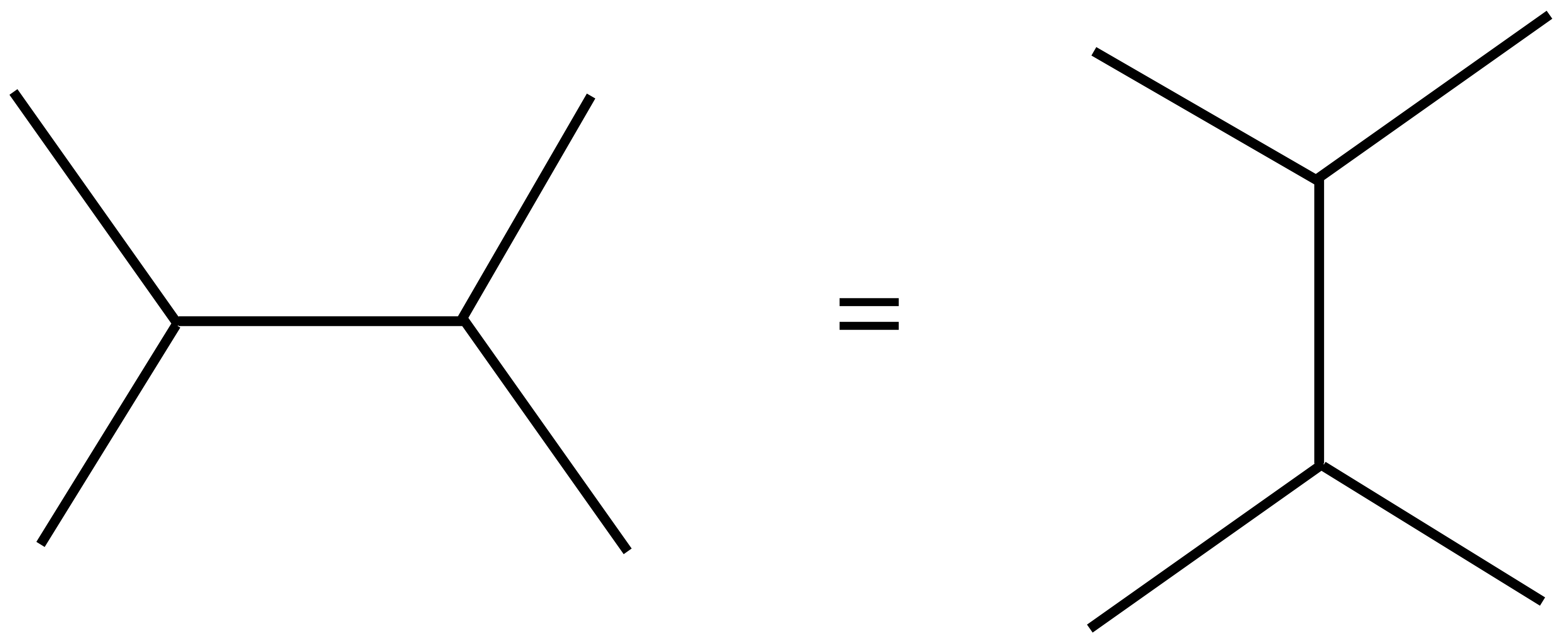
$$k_\beta(x) = x^{\beta/2}\,{}_2F_1(\beta/2, \beta/2, \beta; x)$$

- Double lightcone limit ($z \to 0,\ \bar{z} \to 1$): $G_{\Delta,l}(u,v) \to z^{\tau/2}\left[\log(1-\bar{z}) + \cdots\right]$

log div. from infinite descendant

Crossing symmetry

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \langle \mathcal{O}(x_3)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_1)\rangle$$



Leading Power — Next-to-Leading Power

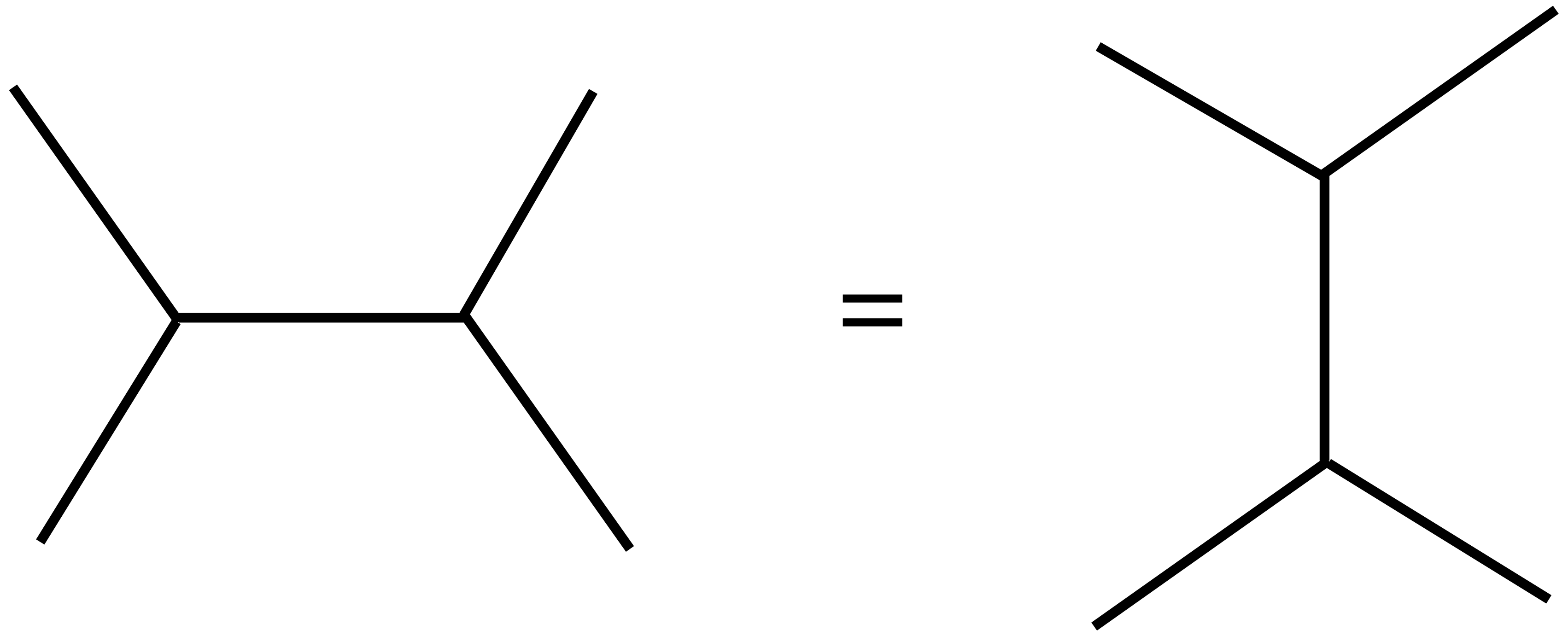
$$\text{one loop} = \left[-\frac{1}{4}\log u \log v + 0\cdot \log(uv) + \cdots\right] - \left[\frac{1}{4}(u+v)\log u \log v + \frac{1}{2}(u \log u + v \log v) + \cdots\right] + \cdots,$$

$$\text{two loop} = \left[\frac{1}{16}\log^2 u \log^2 v + 0\cdot \log u \log v \log(uv) + \cdots\right] + \left[\frac{1}{8}(u+v)\log^2 u \log^2 v + \frac{3}{16}\log u \log v (u \log u + v \log v) + \frac{1}{8}\log u \log v (v \log u + u \log v) + \cdots\right] + \cdots,$$

$$\text{three loop} = \left[-\frac{1}{96}\log^3 u \log^3 v + 0\cdot \log^2 u \log^2 v \log(uv) + \cdots\right] - \left[\frac{1}{48}(u+v)\log^3 u \log^3 v\right.$$

Crossing symmetry

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \langle \mathcal{O}(x_3)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_1)\rangle$$



Message #3: Crossing symmetry relates twist expansion and large spin summation.

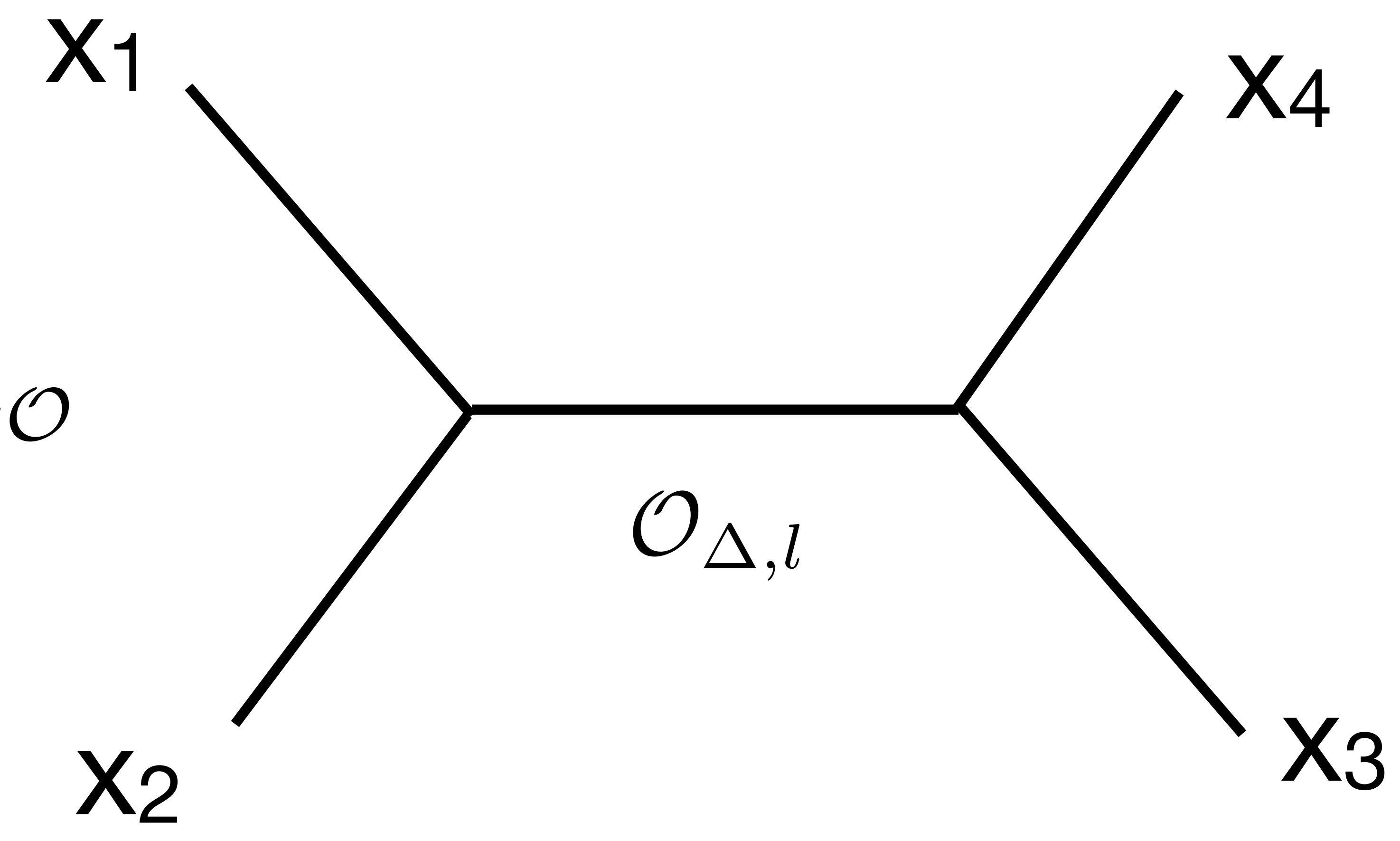
one ... $+\cdots,$

$$\text{two loop} = \left[\frac{1}{16}\log^2 u\log^2 v + 0\cdot\log u\log v\log(uv) + \cdots\right] + \left[\frac{1}{8}(u+v)\log^2 u\log^2 v + \frac{3}{16}\log u\log v(u\log u + v\log v) + \frac{1}{8}\log u\log v(v\log u + u\log v) + \cdots\right] + \cdots,$$

$$\text{three loop} = \left[-\frac{1}{96}\log^3 u\log^3 v + 0\cdot\log^2 u\log^2 v\log(uv) + \cdots\right] - \left[\frac{1}{48}(u+v)\log^3 u\log^3 v\right.$$

Analyticity in spin

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} a_{\mathcal{O}}$$



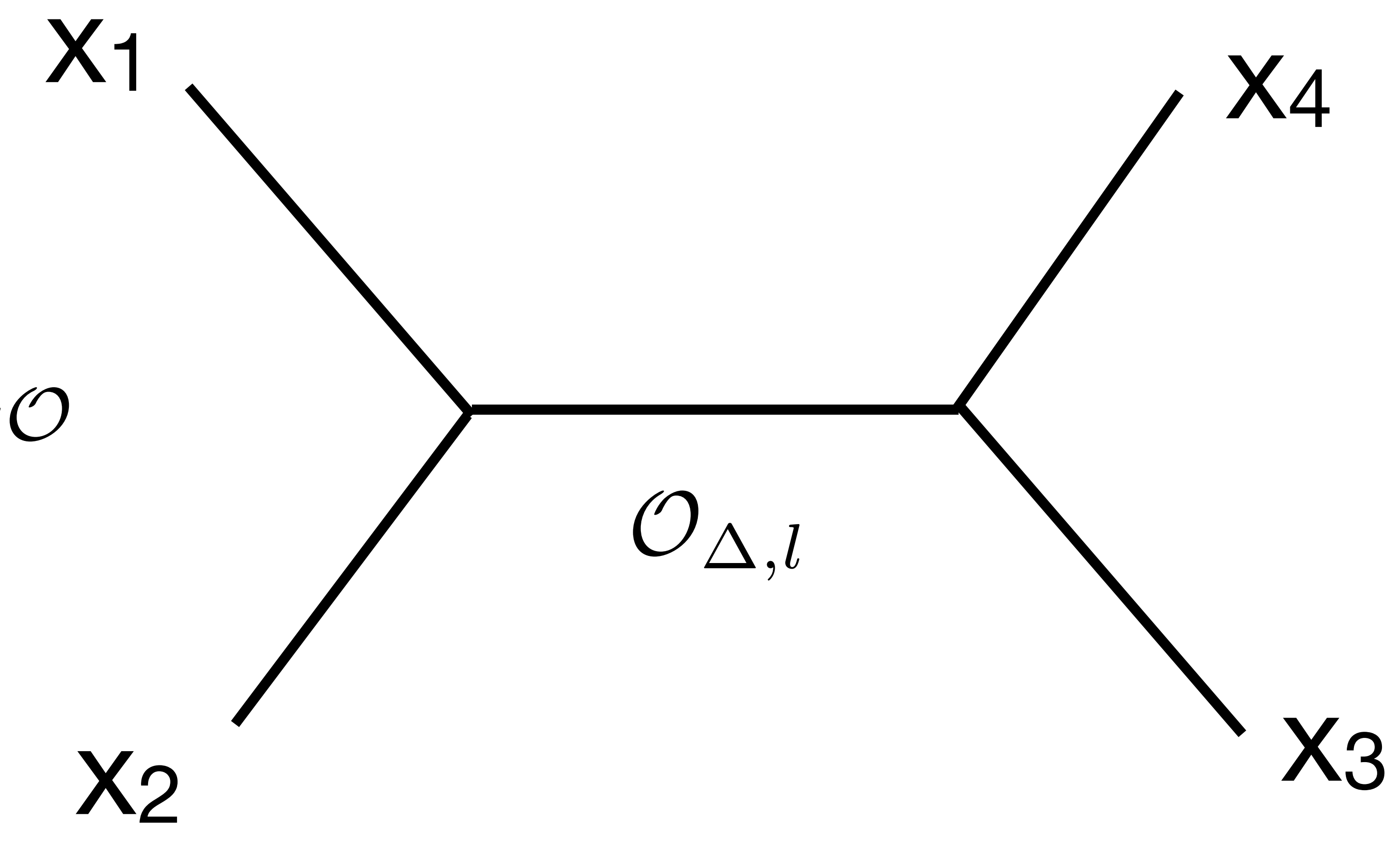
- Twist operators: $\bar{\psi}\gamma^+(D^+)^{l-1}\psi$

$$\begin{aligned}\gamma_{qq}(l) =& 2C_F\left[\frac{3}{2}+\frac{1}{l(l+1)}-2\Psi(l+1)-2\gamma_E\right]\\ =& \gamma_{\text{cusp}}\log l + B + \frac{1}{l} + \dots\end{aligned}$$

Kotikov, Lipatov, 2002
Brower, Polchinski, Strassler, Tan, 2006
Kravchuk, Simmons-Duffin, 2018

Analyticity in spin

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_4)\mathcal{O}(x_3)\rangle = \sum_{\mathcal{O}} a_{\mathcal{O}}$$



- Twist operators: $\bar{\psi}\gamma^+(D^+)^{l-1}\psi$

$$\gamma_{qq}(l) = 2C_F\left[\frac{3}{2} + \frac{1}{l(l+1)} - 2\Psi(l+1) - 2\gamma_E\right]$$
$$= \gamma_{\text{cusp}}\log l + B + \frac{1}{l} + \ldots$$

Kotikov, Lipatov, 2002
Brower, Polchinski, Strassler, Tan, 2006
Kravchuk, Simmons-Duffin, 2018

Message #4: Analyticity in spin allows Laurent expansion in 1/spin.

Conformal block expansion

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

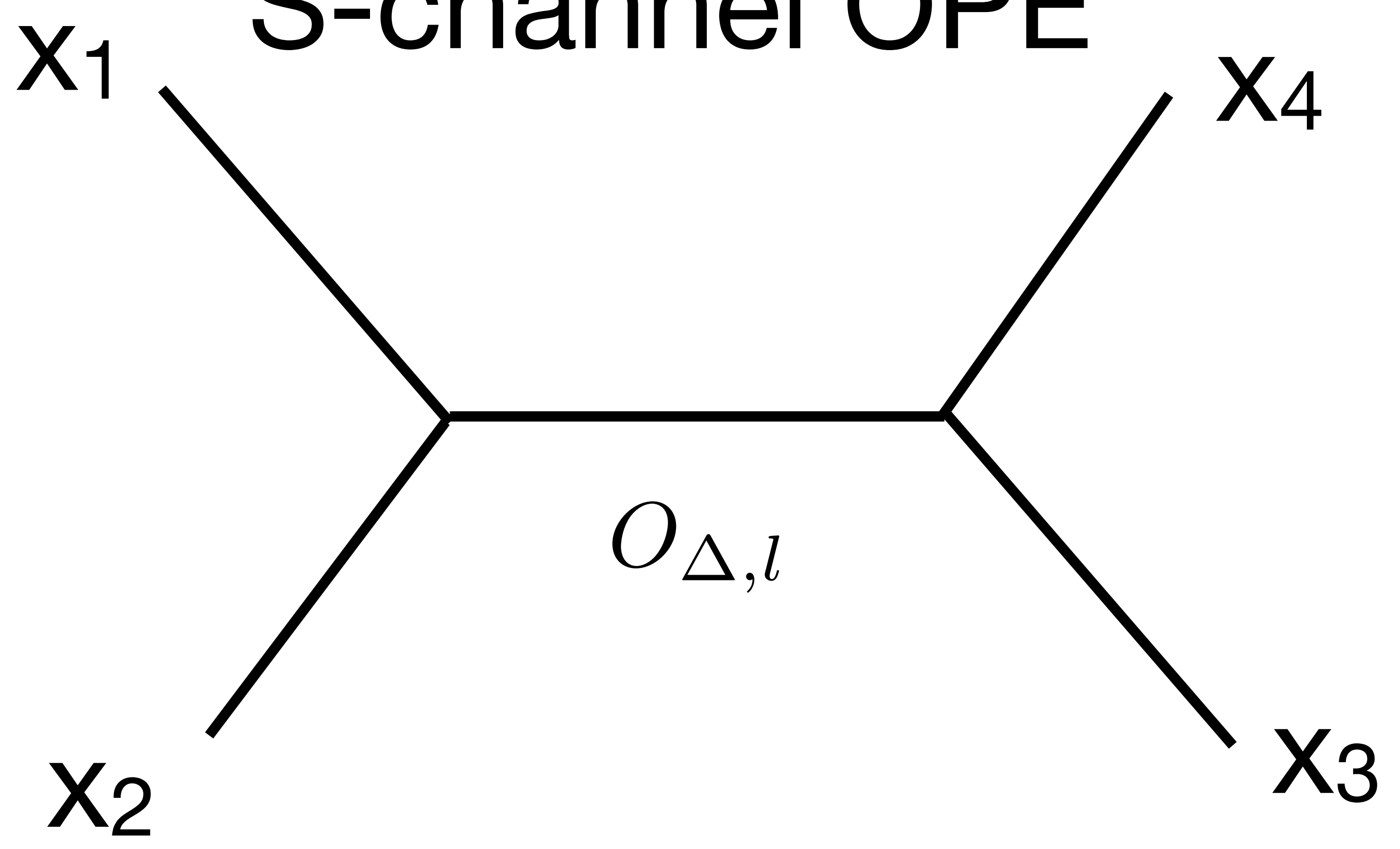
eigenfunction of conformal Casimir

Dolan, Osborn, 2001

$$G_{\Delta,\ell}(u,v) = \frac{z\bar{z}}{\bar{z}-z}\left[k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})\right]$$

S-channel OPE

$$\sum_{O_{\Delta,l}}$$



$$G_{\Delta,l}(u,v) \to z^{\tau/2} \times \left[\log(1-\bar{z}) + \cdots\right]$$

Conformal block expansion

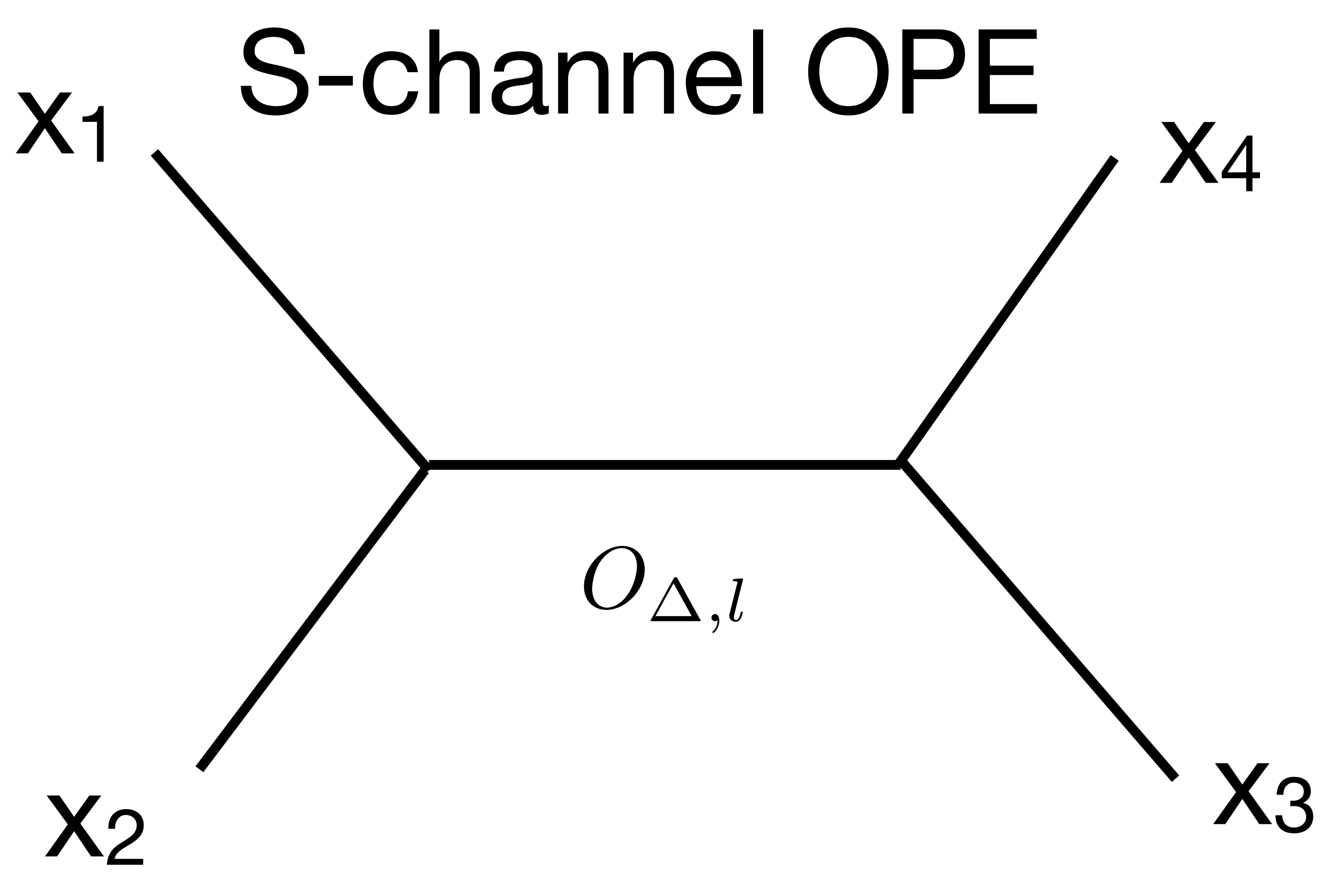
$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

eigenfunction of conformal Casimir

Dolan, Osborn, 2001

$$G_{\Delta,\ell}(u,v) = \frac{z\bar{z}}{\bar{z}-z}\left[k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})\right]$$

$$\sum_{O_{\Delta,l}}$$



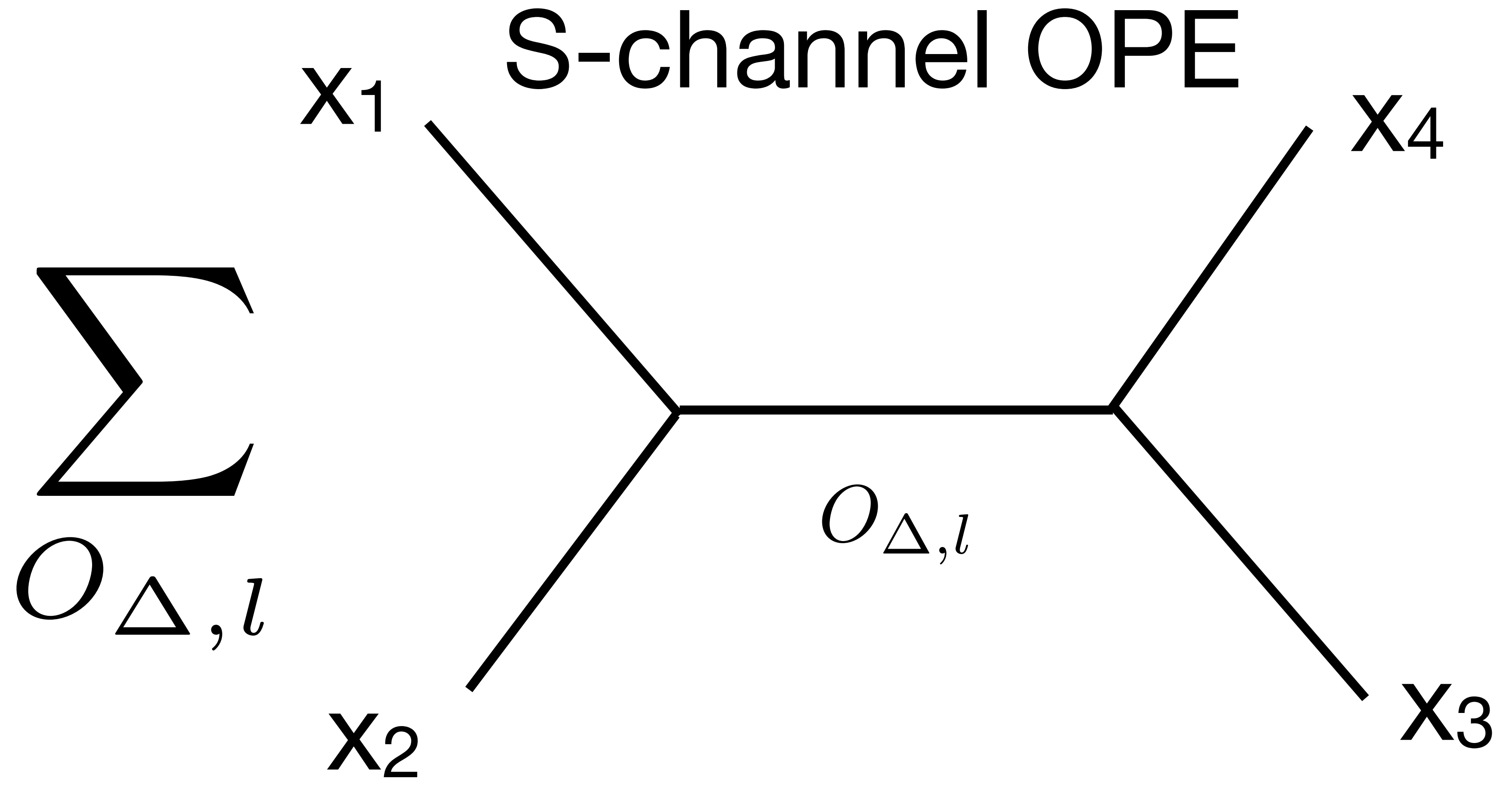
Leading twist expansion u->0 (z->0): $L_z = \log z$ $\quad G_{\Delta,l}(u,v) \to z^{\tau/2} \times \left[\log(1-\bar{z}) + \cdots\right]$

$$\mathcal{F}^{(n)} = z^3 \sum_{\text{even } \ell} a_{2,\ell}^{(0)} \left\{ \frac{\left(\gamma_{2,\ell}^{(1)}\right)^n}{2^n n!} L_z^n + \frac{\left(\gamma_{2,\ell}^{(1)}\right)^{n-1} L_z^{n-1}}{2^{n-1}(n-1)!} \times \left[\frac{a_{2,\ell}^{(1)}}{a_{2,\ell}^{(0)}} + (n-1)\frac{\gamma_{2,\ell}^{(2)}}{\gamma_{2,\ell}^{(1)}} + \frac{\gamma_{2,\ell}^{(1)}\partial_\ell}{2}\right]\right\} k_{2\ell+6}(\bar{z}) + \cdots$$

Leading in u but including infinite powers in v (fixed by conformal symmetry)
Resumming large logarithms in v requires sum over infinite spin
Resumming Power corrections in v requires systematic expansion over large spin

Twist conformal block

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$



twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

Leading twist expansion u->0 (z->0): $L_z = \log z$

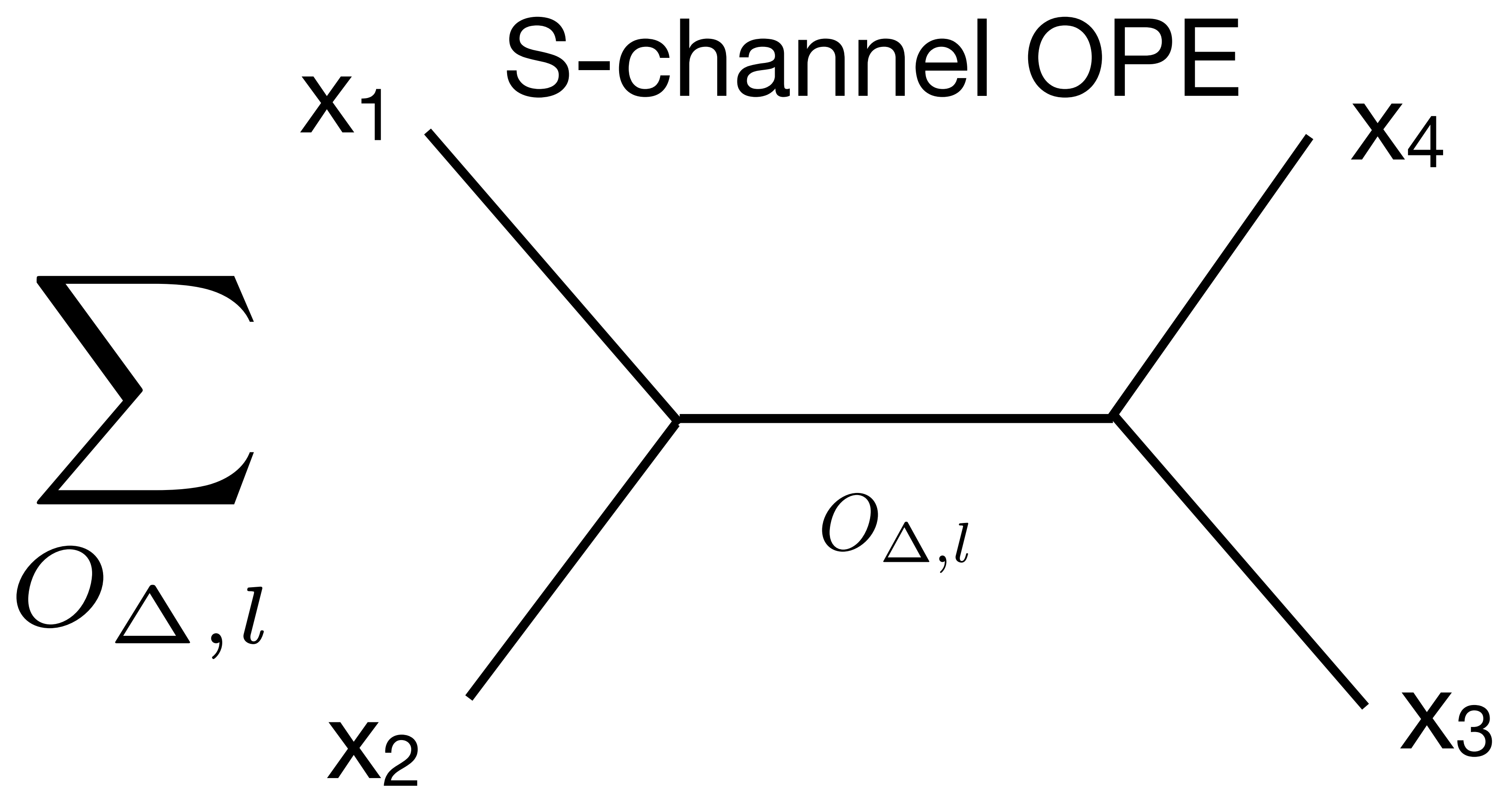
$$k_\beta(x) = x^{\beta/2}\, {}_2F_1(\beta/2, \beta/2, \beta; x)$$

LL

$$\mathcal{F}^{(n)} = z^3 \sum_{\text{even } \ell} a^{(0)}_{2,\ell} \left\{ \frac{\left(\gamma^{(1)}_{2,\ell}\right)^n}{2^n n!} L_z^n + \frac{\left(\gamma^{(1)}_{2,\ell}\right)^{n-1} L_z^{n-1}}{2^{n-1}(n-1)!} \times \left[\frac{a^{(1)}_{2,\ell}}{a^{(0)}_{2,\ell}} + (n-1)\frac{\gamma^{(2)}_{2,\ell}}{\gamma^{(1)}_{2,\ell}} + \frac{\gamma^{(1)}_{2,\ell}\partial_\ell}{2} \right] \right\} k_{2\ell+6}(\bar{z}) + \cdots$$

Twist conformal block

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$



twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

Leading twist expansion u->0 (z->0): $L_z = \log z$

$$k_\beta(x) = x^{\beta/2} {}_2F_1(\beta/2, \beta/2, \beta; x)$$

LL

$$\mathcal{F}^{(n)} = z^3 \sum_{\text{even } \ell} a^{(0)}_{2,\ell} \left\{ \frac{\left(\gamma^{(1)}_{2,\ell}\right)^n}{2^n n!} L_z^n + \frac{\left(\gamma^{(1)}_{2,\ell}\right)^{n-1} L_z^{n-1}}{2^{n-1}(n-1)!} \times \left[\frac{a^{(1)}_{2,\ell}}{a^{(0)}_{2,\ell}} + (n-1)\frac{\gamma^{(2)}_{2,\ell}}{\gamma^{(1)}_{2,\ell}} + \frac{\gamma^{(1)}_{2,\ell}\partial_\ell}{2} \right] \right\} k_{2\ell+6}(\bar{z}) + \cdots$$

$$= \frac{L_z^n}{n!} \sum_i \binom{n}{i} \left[\frac{\gamma_E^{n-i}}{2^i} H_2^{(0,i)} + \frac{n-i}{3} \frac{\gamma_E^{n-1-i}}{2^{i+1}} H_2^{(1,i)} \right] + \cdots$$

$$a^{(0)}_{2,\ell} = \frac{\Gamma(\ell+3)^2}{\Gamma(2\ell+5)},$$

$$\gamma^{(1)}_{2,\ell} = \log J^2_{6,\ell} + 2\gamma_E + \frac{1}{3J^2_{6,\ell}} + \mathcal{O}(J^{-4}_{6,\ell})$$

Resummation by analytic continuation

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

Resummation by analytic continuation

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4,\ell}(z,\bar{z}) = J^2_{\tilde{\tau}_0,\ell} G_{\Delta+4,\ell}(z,\bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0} = \sum_{l} a^{(0)}_{\tau_0,l} \frac{\log^i J^2_{\tilde{\tau}_0,l}}{J^{2(m-1)}_{\tilde{\tau}_0,l}} G_{\Delta+4,l}$$

Resummation by analytic continuation

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4,\ell}(z,\bar{z}) = J^2_{\tilde{\tau}_0,\ell} G_{\Delta+4,\ell}(z,\bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0} = \sum_{l} a^{(0)}_{\tau_0,l} \frac{\log^i J^2_{\tilde{\tau}_0,l}}{J^{2(m-1)}_{\tilde{\tau}_0,l}} G_{\Delta+4,l}$$

$$C_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0}(z,\bar{z}) = H^{(m-1,i)}_{\tau_0}(z,\bar{z})$$

Resummation by analytic continuation

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4,\ell}(z,\bar{z}) = J^2_{\tilde{\tau}_0,\ell} G_{\Delta+4,\ell}(z,\bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0} = \sum_{l} a^{(0)}_{\tau_0,l} \frac{\log^i J^2_{\tilde{\tau}_0,l}}{J^{2(m-1)}_{\tilde{\tau}_0,l}} G_{\Delta+4,l}$$

$$C_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0}(z,\bar{z}) = H^{(m-1,i)}_{\tau_0}(z,\bar{z})$$

$$H^{(0,0)}_{\tau_0}(z,\bar{z})$$

$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z}\log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1 - \bar{z} = \epsilon$$

Resummation by analytic continuation

$$\mathcal{F}(u,v)=\sum_{\Delta}\sum_{\text{even }\ell} a_{\tau,\ell}G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2}=\sum_{\text{even }\ell} a^{(0)}_{\tau,\ell}G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar z)=\sum_{\ell} a^{(0)}_{\tau_0,\ell}\frac{\log^i J^2_{\tilde\tau_0,\ell}}{J^{2m}_{\tilde\tau_0,\ell}}G_{\Delta+4,\ell}(z,\bar z)$$

$$J^2_{\tau,\ell}=(\ell+\frac{\tau}{2})(\ell+\frac{\tau}{2}-1)$$

$$\tilde\tau_0=\tau+4$$

$$\mathcal{C}_{\tilde\tau_0}G_{\Delta+4,\ell}(z,\bar z)=J^2_{\tilde\tau_0,\ell}G_{\Delta+4,\ell}(z,\bar z)$$

$$\mathcal{C}_{\tilde\tau_0}H^{(m,i)}_{\tau_0}=\sum_{l} a^{(0)}_{\tau_0,l}\frac{\log^i J^2_{\tilde\tau_0,l}}{J^{2(m-1)}_{\tilde\tau_0,l}}G_{\Delta+4,l}$$

$$C_{\tilde\tau_0}H^{(m,i)}_{\tau_0}(z,\bar z)=H^{(m-1,i)}_{\tau_0}(z,\bar z)$$

$$H^{(-1,0)}_{\tau_0}(z,\bar z)\xleftarrow{\ \mathcal{C}_{\tilde\tau_0}\ }H^{(0,0)}_{\tau_0}(z,\bar z)$$

$$\frac{\bar z^2(2-\bar z)}{2(1-\bar z)}+\bar z\log(1-\bar z)=\frac{1}{2\epsilon}+\text{regular terms}$$

$$1-\bar z=\epsilon$$

Resummation by analytic continuation

$$\mathcal{F}(u,v)=\sum_{\Delta}\sum_{\text{even }\ell} a_{\tau,\ell}G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2}=\sum_{\text{even }\ell} a^{(0)}_{\tau,\ell}G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar z)=\sum_{\ell} a^{(0)}_{\tau_0,\ell}\frac{\log^i J^2_{\tilde\tau_0,\ell}}{J^{2m}_{\tilde\tau_0,\ell}}G_{\Delta+4,\ell}(z,\bar z)$$

$$J^2_{\tau,\ell}=(\ell+\frac{\tau}{2})(\ell+\frac{\tau}{2}-1)$$

$$\tilde\tau_0=\tau+4$$

$$\mathcal{C}_{\tilde\tau_0}G_{\Delta+4,\ell}(z,\bar z)=J^2_{\tilde\tau_0,\ell}G_{\Delta+4,\ell}(z,\bar z)$$

$$\mathcal{C}_{\tilde\tau_0}H^{(m,i)}_{\tau_0}=\sum_{l} a^{(0)}_{\tau_0,l}\frac{\log^i J^2_{\tilde\tau_0,l}}{J^{2(m-1)}_{\tilde\tau_0,l}}G_{\Delta+4,l}$$

$$C_{\tilde\tau_0}H^{(m,i)}_{\tau_0}(z,\bar z)=H^{(m-1,i)}_{\tau_0}(z,\bar z)$$

$$H^{(-2,0)}_{\tau_0}(z,\bar z)\xleftarrow{\mathcal{C}_{\tilde\tau_0}}H^{(-1,0)}_{\tau_0}(z,\bar z)\xleftarrow{\mathcal{C}_{\tilde\tau_0}}H^{(0,0)}_{\tau_0}(z,\bar z)$$

$$\frac{\bar z^2(2-\bar z)}{2(1-\bar z)}+\bar z\log(1-\bar z)=\frac{1}{2\epsilon}+\text{regular terms}$$

$$1-\bar z=\epsilon$$

Resummation by analytic continuation

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4,\ell}(z,\bar{z}) = J^2_{\tilde{\tau}_0,\ell} G_{\Delta+4,\ell}(z,\bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0} = \sum_{l} a^{(0)}_{\tau_0,l} \frac{\log^i J^2_{\tilde{\tau}_0,l}}{J^{2(m-1)}_{\tilde{\tau}_0,l}} G_{\Delta+4,l}$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \tfrac{\tau}{2})(\ell + \tfrac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$C_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0}(z,\bar{z}) = H^{(m-1,i)}_{\tau_0}(z,\bar{z})$$

$$H^{(m,0)}_{\tau_0}(z,\bar{z}) \quad \cdots \quad H^{(-2,0)}_{\tau_0}(z,\bar{z}) \xleftarrow{\mathcal{C}_{\tilde{\tau}_0}} H^{(-1,0)}_{\tau_0}(z,\bar{z}) \xleftarrow{\mathcal{C}_{\tilde{\tau}_0}} H^{(0,0)}_{\tau_0}(z,\bar{z})$$

$$H^{(m,0)}_{\tau_0}(z,\bar{z}) = \frac{1}{2}\epsilon^{m-1}\Gamma(1-m)^2 + \cdots$$

$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z}\log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1-\bar{z} = \epsilon$$

Resummation by analytic continuation

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$J^2_{\tau,\ell} = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4,\ell}(z,\bar{z}) = J^2_{\tilde{\tau}_0,\ell} G_{\Delta+4,\ell}(z,\bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0} = \sum_{l} a^{(0)}_{\tau_0,l} \frac{\log^i J^2_{\tilde{\tau}_0,l}}{J^{2(m-1)}_{\tilde{\tau}_0,l}} G_{\Delta+4,l}$$

$$C_{\tilde{\tau}_0} H^{(m,i)}_{\tau_0}(z,\bar{z}) = H^{(m-1,i)}_{\tau_0}(z,\bar{z})$$

$H^{(m,0)}_{\tau_0}(z,\bar{z})$ $\cdots$ $H^{(-2,0)}_{\tau_0}(z,\bar{z})$ $\xleftarrow{\mathcal{C}_{\tilde{\tau}_0}}$ $H^{(-1,0)}_{\tau_0}(z,\bar{z})$ $\xleftarrow{\mathcal{C}_{\tilde{\tau}_0}}$ $H^{(0,0)}_{\tau_0}(z,\bar{z})$ $H^{(0,i)}_{\tau_0}(z,\bar{z})$ $H^{(1,i)}_{\tau_0}(z,\bar{z})$

$$H^{(m,0)}_{\tau_0}(z,\bar{z}) = \frac{1}{2}\epsilon^{m-1}\Gamma(1-m)^2 + \cdots$$

$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z}\log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1 - \bar{z} = \epsilon$$

Resummation by analytic continuation

$$\mathcal{F}(u,v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau,\ell} G_{\Delta+4,\ell}(u,v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a^{(0)}_{\tau,\ell} G_{\ell+6,\ell}(u,v)$

Alday, 2016

$$H^{(m,i)}_{\tau_0}(z,\bar z) = \sum_{\ell} a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde\tau_0,\ell}}{J^{2m}_{\tilde\tau_0,\ell}} G_{\Delta+4,\ell}(z,\bar z)$$

$$J^2_{\tau,\ell} = (\ell+\frac{\tau}{2})(\ell+\frac{\tau}{2}-1)$$

$$\tilde\tau_0 = \tau+4$$

$$\mathcal{C}_{\tilde\tau_0} G_{\Delta+4,\ell}(z,\bar z) = J^2_{\tilde\tau_0,\ell} G_{\Delta+4,\ell}(z,\bar z)$$

$$\mathcal{C}_{\tilde\tau_0} H^{(m,i)}_{\tau_0} = \sum_{l} a^{(0)}_{\tau_0,l} \frac{\log^i J^2_{\tilde\tau_0,l}}{J^{2(m-1)}_{\tilde\tau_0,l}} G_{\Delta+4,l}$$

$$C_{\tilde\tau_0} H^{(m,i)}_{\tau_0}(z,\bar z) = H^{(m-1,i)}_{\tau_0}(z,\bar z)$$

analytic continuation in m

$H^{(m,0)}_{\tau_0}(z,\bar z)$ $\cdots$ $H^{(-2,0)}_{\tau_0}(z,\bar z)$ $\xleftarrow{\mathcal{C}_{\tilde\tau_0}}$ $H^{(-1,0)}_{\tau_0}(z,\bar z)$ $\xleftarrow{\mathcal{C}_{\tilde\tau_0}}$ $H^{(0,0)}_{\tau_0}(z,\bar z)$ $H^{(0,i)}_{\tau_0}(z,\bar z)$ $H^{(1,i)}_{\tau_0}(z,\bar z)$

$$H^{(m,0)}_{\tau_0}(z,\bar z) = \frac{1}{2}\epsilon^{m-1}\Gamma(1-m)^2 + \cdots$$

$$\frac{\bar z^2(2-\bar z)}{2(1-\bar z)} + \bar z\log(1-\bar z) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1-\bar z = \epsilon$$

Alday, 2016
Henriksson, Lukowski, 2017

Examples:

- $\mathcal{N}$=4 SYM
- QCD charge-charge correlator QQC

Explicit example for a toy model: N=4 SYM

$$\mathcal{F}^{(n)}(z,\bar z) = z^3 \Bigg\{ \frac{1}{n!}\log^n z \left[\frac{1}{\epsilon}\left(\frac{(-1)^n}{2^{n+1}}\log^n \epsilon + \cdots \right) + \left(\frac{(-1)^{n+1}}{2^{n+1}}\log^n\epsilon + \frac{(-1)^n n}{3\times 2^n}\log^{n-1}\epsilon + \cdots \right) + \cdots \right]$$

$$+ \frac{\log^{n-1} z}{(n-1)!}\Bigg[\frac{1}{\epsilon}\left(\frac{(-1)^n}{2^{n+1}}(n+1)\zeta_2\log^{n-1}\epsilon - \frac{(-1)^n}{2^{n+1}}3(n-1)\zeta_3\log^{n-2}\epsilon + \cdots \right)$$

$$+ \left(\frac{(-1)^n}{2^{n+1}n}\log^n\epsilon + \frac{(-1)^{n+1}}{2^{n+1}}(n+1)\zeta_2\log^{n-1}\epsilon + \cdots \right) \Bigg] + \cdots \Bigg\} + \mathcal{O}(z^4)$$

Agree with fixed-oder expansion (up to terms not enhanced by large spin)

$$\text{one loop} = \left[-\frac{1}{4}\log u \log v + 0\cdot \log(uv) + \cdots \right] - \left[\frac{1}{4}(u+v)\log u \log v + \frac{1}{2}(u\log u + v \log v) + \cdots \right] + \cdots,$$

$$\text{two loop} = \left[\frac{1}{16}\log^2 u \log^2 v + 0\cdot \log u \log v \log(uv) + \cdots \right] + \Bigg[\frac{1}{8}(u+v)\log^2 u \log^2 v$$

$$+ \frac{3}{16}\log u \log v(u\log u + v\log v) + \frac{1}{8}\log u \log v(v\log u + u \log v) + \cdots \Bigg] + \cdots,$$

$$\text{three loop} = \left[-\frac{1}{96}\log^3 u \log^3 v + 0\cdot \log^2 u \log^2 v \log(uv) + \cdots \right] - \left[\frac{1}{48}(u+v)\log^3 u \log^3 v \right.$$

Resummation of EEC in N=4 SYM

$$\mathrm{EEC}(y) \sim \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \left(c_{n,m} \frac{\log^m y}{y} + d_{n,m} \log^m y \right)$$

"NLL": $m \geq 2n-2$

	Power Corrections		Perturbative Corrections	
	twist	large spin	LL	"NLL"
LP	2	$\mathcal{O}(\ell^0)$	$a_{2,\ell}^{(0)}, \gamma_{2,\ell}^{(1)}$	$a_{2,\ell}^{(1)}, \gamma_{2,\ell}^{(2)}$
NLP	2	$\mathcal{O}(\ell^{-2})$	$a_{2,\ell}^{(0)}, \gamma_{2,\ell}^{(1)}$	$a_{2,\ell}^{(1)}, \gamma_{2,\ell}^{(2)}$
	4	$\mathcal{O}(\ell^0)$	$a_{4,\ell}^{(0)}, \gamma_{4,\ell}^{(1)}$	$a_{4,\ell}^{(1)}, \gamma_{4,\ell}^{(2)}$

$$\mathrm{EEC}(y) = -\frac{aL_y e^{-\frac{aL_y^2}{2}}}{4y} - \frac{1}{4}\left[\sqrt{\frac{\pi}{2}}\sqrt{a}\,\mathrm{erf}\left(\sqrt{\frac{a}{2}}L_y\right) + aL_y e^{-\frac{aL_y^2}{2}}\right] + \frac{a}{48}(7aL_y^2 - 4)e^{-\frac{aL_y^2}{2}} + \frac{a}{12} + \cdots$$

$$y = \frac{1+\cos\chi}{2}$$

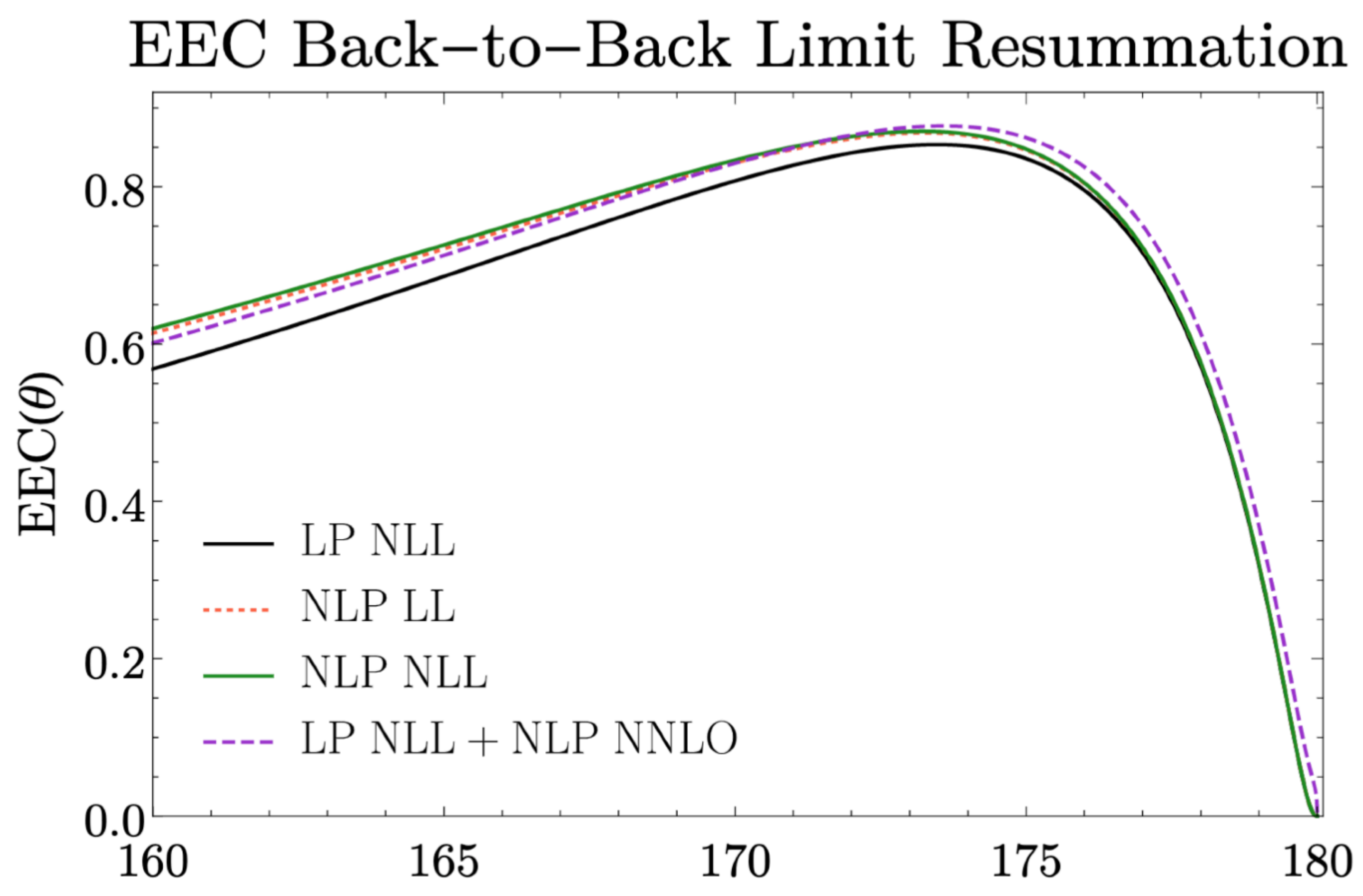
$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

$$\mathrm{EEC}^{(1)} = -\frac{1}{4y}\log y - \frac{1}{2}\log y + 0 \cdot y^0 + \mathcal{O}(y)\,,$$

$$\mathrm{EEC}^{(2)} = \frac{1}{y}\left(\frac{\log^3 y}{8} + 0 \cdot \log^2 y + \cdots\right) + \left(\frac{\log^3 y}{6} + \frac{3}{16}\log^2 y + \cdots\right) + \mathcal{O}(y)$$

$$\mathrm{EEC}^{(3)} = \frac{1}{y}\left(-\frac{\log^5 y}{32} + 0 \cdot \log^4 y + \cdots\right) + \left(-\frac{3\log^5 y}{80} - \frac{\log^4 y}{12} + \cdots\right) + \mathcal{O}(y)\,.$$

Henn, Sokatchev, Yan, Zhiboedov, 2019



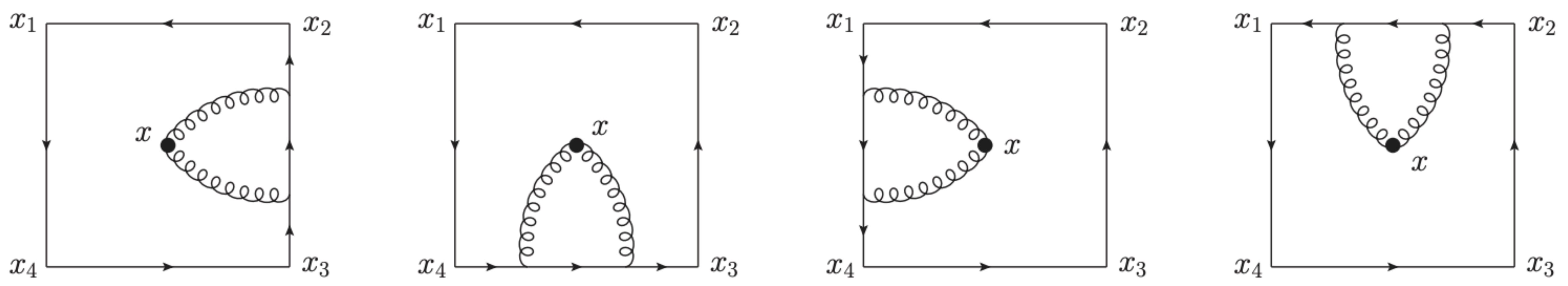
Local charge-charge correlator in QCD

$$\gamma = \frac{x_{13}^+ x_{13}^-}{x_{13}^2}$$

$$\lim_{r_2\to\infty} r_2^2 \lim_{r_4\to\infty} r_4^2 \langle J^\mu(x_1) J^+(x_2) J^+(x_4) J_\mu(x_3)\rangle = \text{prefactor} \times \mathcal{G}(u,v,\gamma)$$

$$\mathcal{G}^{(1)}(u,v,\gamma) = \frac{\log(z)}{z^2}\left[\left(\frac{1-2\log(1-\bar{z})}{(1-\bar{z})^2} + \frac{2-4\log(1-\bar{z})}{(1-\bar{z})} + \cdots\right) + \gamma\left(-\frac{2}{(1-\bar{z})^2} + \cdots\right) + \cdots\right]$$

Chicherin, Henn, Sokatchev, Yan, 2020



twist 3 operators

$$\tilde{O}_{m,1}^{[\tau=3]} = \sum_{k=0}^{m} \frac{(-1)^k}{\Gamma(k+1)^2\Gamma(m-k+1)\Gamma(m-k+2)} \left((iD_{1\dot{1}}^\dagger)^k \bar{\psi}_{\dot{2}}\right)\left((iD_{1\dot{1}})^{m-k}\psi_1\right)$$

$$\tilde{O}_{m,2}^{[\tau=3]} = \sum_{k=0}^{m-1} \frac{(-1)^k}{\Gamma(k+1)\Gamma(k+2)\Gamma(m-k)\Gamma(m-k+2)} \left((iD_{1\dot{1}}^\dagger)^k \bar{\psi}_{\dot{1}}\right)\left((iD_{1\dot{1}})^{m-k-1}(iD_{1\dot{2}})\psi_1\right)$$

Chen, HXZ, in preparation

Summary

- We have proposed a new method to resum Sudakov logarithms in EEC based on double lightcone OPE
  - Power corrections from twist expansion and infinite spin expansion
  - Simplify by crossing symmetry
  - Resummation by RG + large spin perturbation via twist conformal block
- Towards QCD (work in progress):
  - Spinning conformal block
  - Running coupling effects (only appear at NLL and beyond)
  - Generalization to more observables

Analytic continuation of twist conformal block

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_\ell a^{(0)}_{\tau_0,\ell} \frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}} G_{\Delta+4,\ell}(z,\bar{z})$$

$$H_{\tau_0}(m,i) = (-1)^i \frac{d^i}{d^i m} H_{\tau_0}(m,0)$$

Generic formula at negative m:

$$\begin{aligned}\widetilde{H}^{(m)}_{\tilde{\tau}=6}(\epsilon) =& \frac{1}{2}\epsilon^{m-1}\Gamma(1-m)^2 + \frac{1}{6}m\left(2m^2-6m+1\right)\epsilon^m\Gamma(-m)^2 \\ &+ \frac{1}{180}(m-1)m(m+1)\left(20m^3-54m^2-35m+36\right)\epsilon^{m+1}\Gamma(-m-1)^2 + \cdots\end{aligned}$$

Expand at m=0, 1:

$$\begin{aligned}\widetilde{H}^{(0,\log^n)}_{\tilde{\tau}=6}(\epsilon) =& \frac{(-1)^n}{\epsilon}\left[\frac{1}{2}\log^n\epsilon + n\gamma_E\log^{n-1}\epsilon + \frac{n(n-1)}{12}(12\gamma_E^2+\pi^2)\log^{n-2}\epsilon + \cdots\right] \\ &+ (-1)^n\left[\frac{1}{6(n+1)}\log^{n+1}\epsilon + \frac{\gamma_E-3}{3}\log^n\epsilon + \frac{\pi^2+12\gamma_E^2-72\gamma_E+12}{36}n\log^{n-1}\epsilon + \ldots\right] + \cdots\end{aligned}$$

higher power of log than we expected at NLP, but it cancels with that in $\widetilde{H}^{(1,\log^{n-1})}_{\tilde{\tau}=6}(\epsilon)$

$$\widetilde{H}^{(1,\log^n)}_{\tilde{\tau}=6}(\epsilon) = (-1)^n\left[\frac{1}{2(n+1)(n+2)}\log^{n+2}\epsilon + \frac{\gamma_E}{n+1}\log^{n+1}\epsilon + \frac{12\gamma_E^2+\pi^2}{12}\log^n\epsilon + \cdots\right] + \cdots$$