Towards N3LL jet veto

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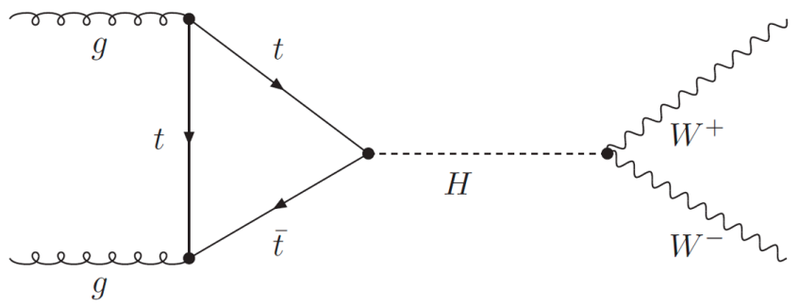
JET VETO

Let's say we want to measure Higgs decay into WW pair

Signal



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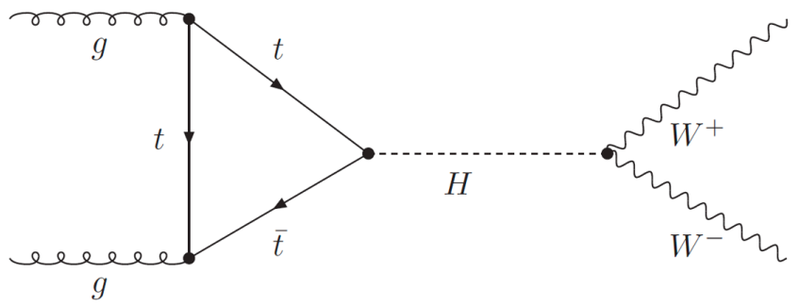
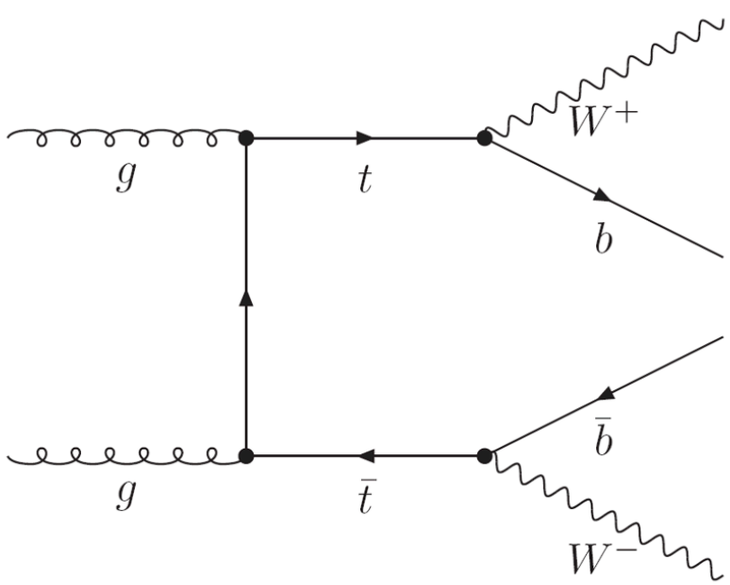


There is significant background from top production



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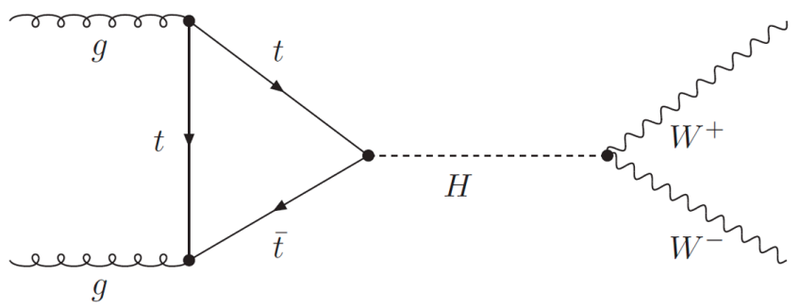

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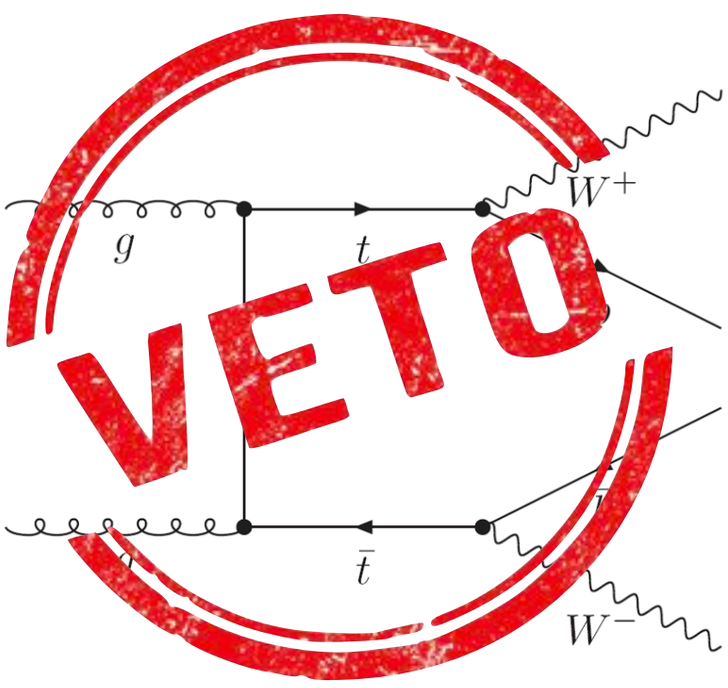

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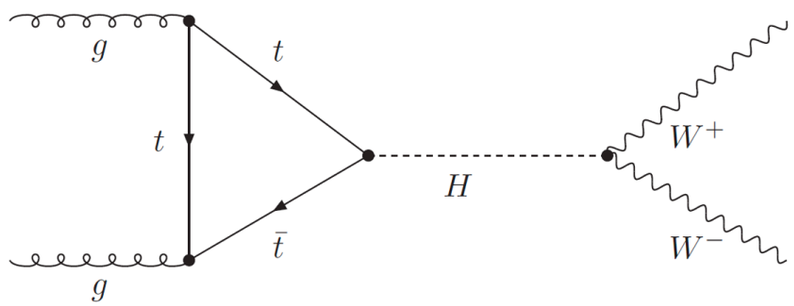

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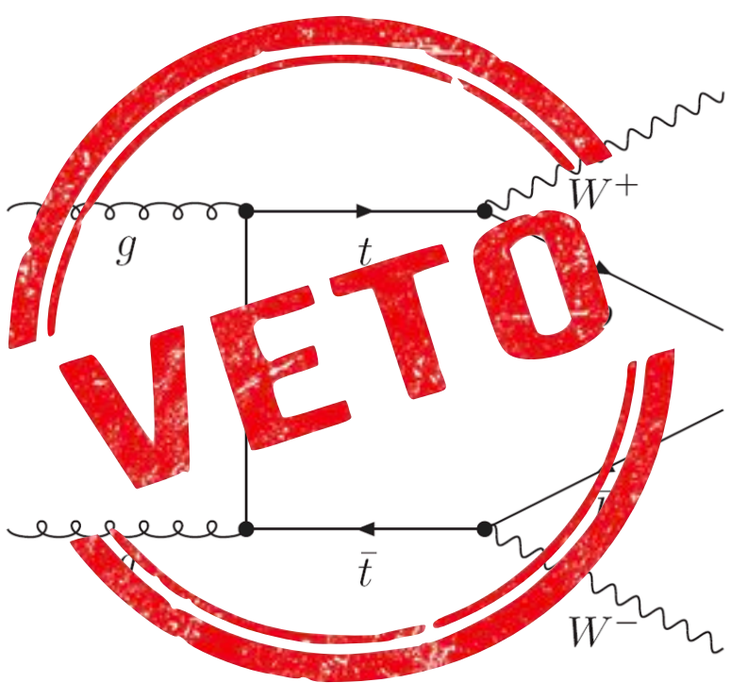

Jet veto

Jet vetos are applied in many electroweak and Higgs measurements

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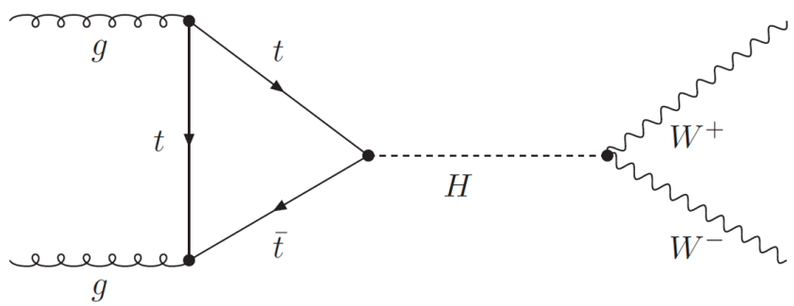

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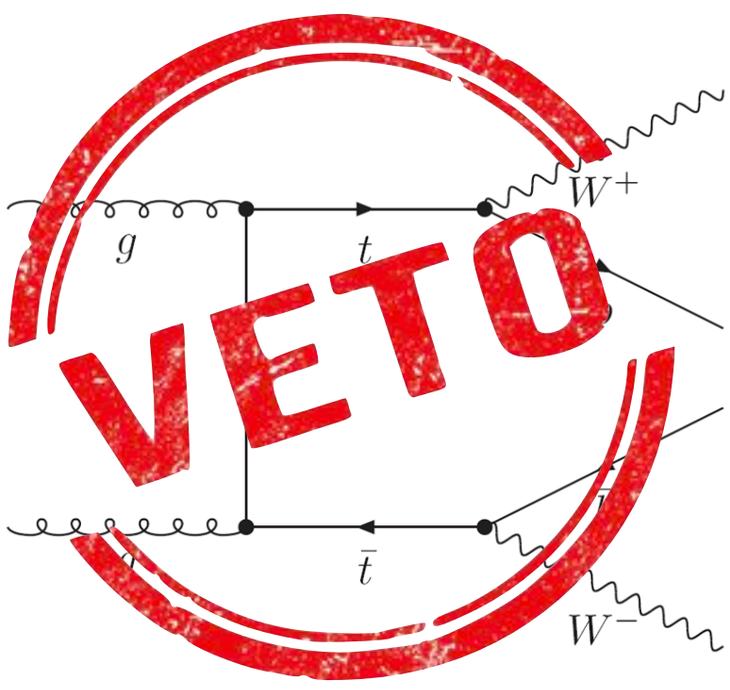
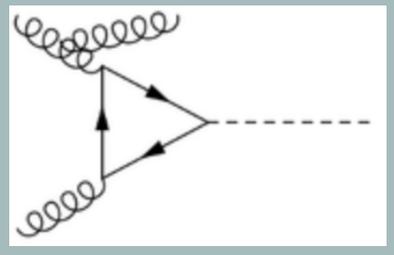


Jet veto

Jet vetos are applied in many electroweak and Higgs measurements

But the signal is also modified, so we need accurate predictions



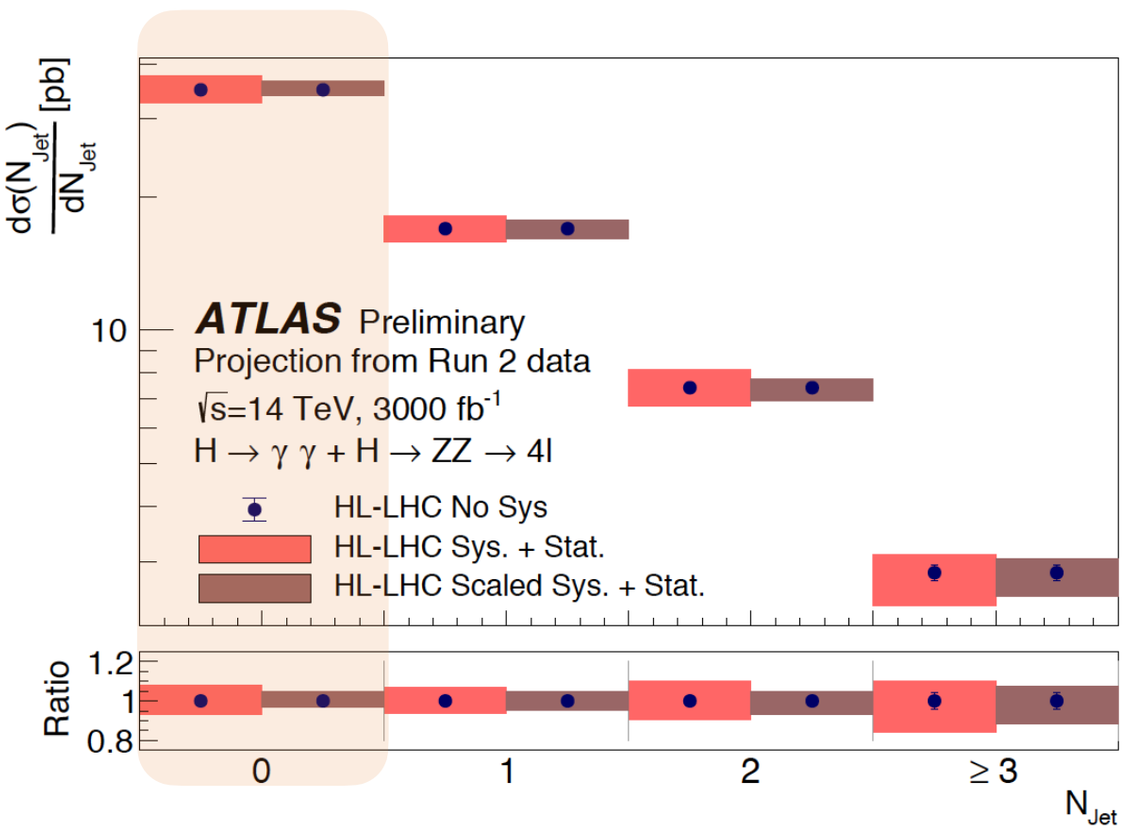
Jet vetos generate large logarithms

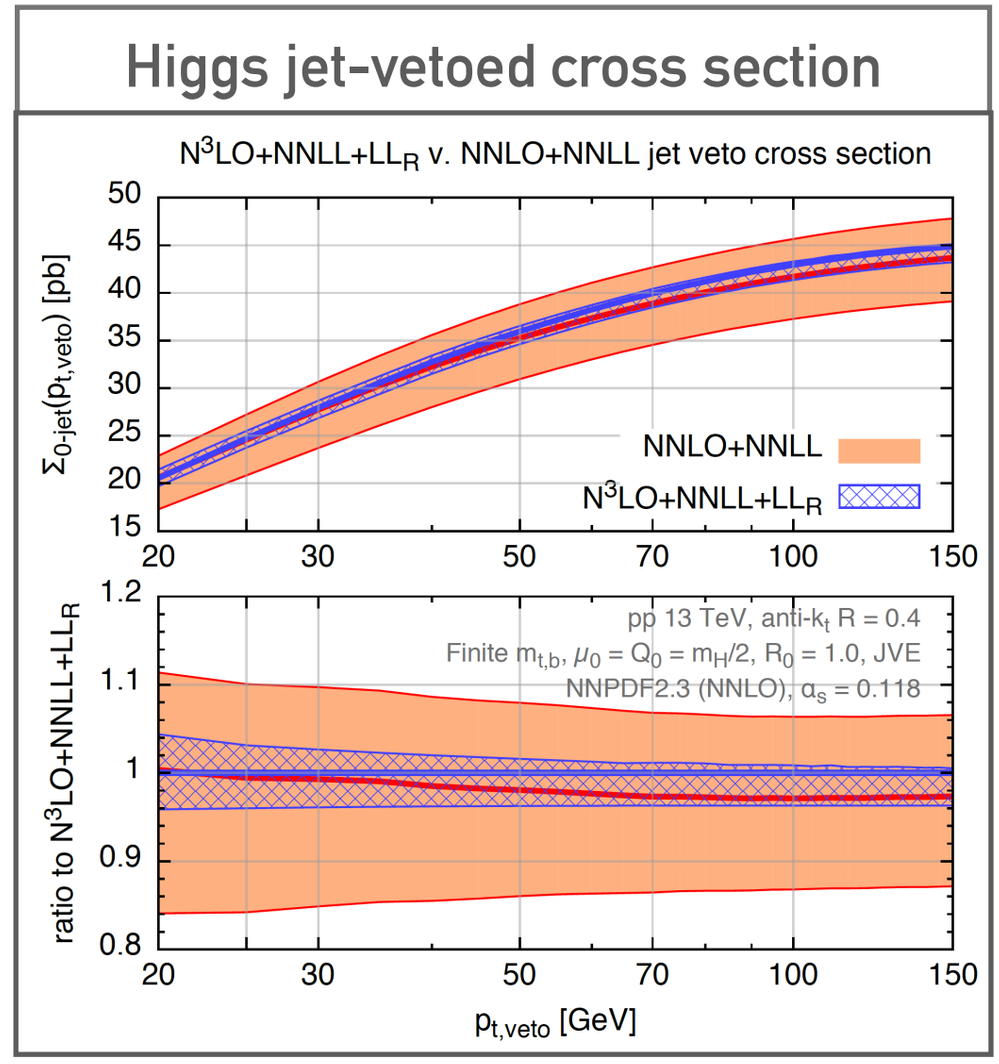
$$\log(p_T^{\text{veto}}/Q)$$

MOTIVATION

Jet-vetoed cross sections: reduce backgrounds

e.g. $H \to WW$ vs $t\bar{t}$





Formalism for resummation

Banfi, Monni, Salam, Zanderighi 12; Becher, Neubert 12; Becher, Neubert, Rothen 13; Stewart, Tackmann, Walsh, Zuberi 13

State of the art: **N3LO+NNLL** [Banfi et al. 16]

Goal: improve resummation to N3LL

JET VETO FACTORIZATION

- Color-singlet production with a jet veto $p_T^{\rm jet} < p_T^{\rm veto}$
- In the limit $p_T^{\rm veto} \ll Q$:

$$\frac{d\sigma(p_T^{\rm veto})}{d\Phi_{\rm Born}} = |A^F_{\rm Born}|^2\, \mathscr{H}(Q;\mu)\, \mathscr{B}_n(x_1, Q, p_T^{\rm veto}, R^2; \mu, \nu)\, \mathscr{B}_{\bar{n}}(x_2, Q, p_T^{\rm veto}, R^2; \mu, \nu)\, \mathscr{S}(p_T^{\rm veto}, R^2; \mu, \nu)$$

- Hard function: $\mathscr{H}(Q;\mu)$
- Beam functions: $\mathscr{B}_{n,\bar{n}}(x, Q, p_T^{\rm veto}, R^2; \mu, \nu)$
- Soft function: $\mathscr{S}(p_T^{\rm veto}, R^2; \mu, \nu)$

QCD on-shell form-factor

Subject of this talk

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Subject of this talk

- R: jet radius defined with jet algorithms:

$$d_{ij} = \min\{k_{\perp i}^{2p}, k_{\perp j}^{2p}\} \frac{\left[(\Delta\eta_{ij})^2 + (\Delta\phi_{ij})^2\right]}{R^2}, \quad d_{iB} = k_{\perp i}^{2p}\,.$$

anti-k_T (p = –1); Cambridge-Aachen (p = 0); the k_T (p = 1)

RG EVOLUTION EQUATIONS

- Hard function $\mathscr{H}(Q;\mu) = |\mathscr{C}(Q;\mu)|^2$

$$\frac{d}{d\ln\mu}\ln\mathscr{C}(Q;\mu) = \Gamma_{\text{cusp}}(\alpha_s(\mu))\ln\frac{-Q^2}{\mu^2} + \gamma_H(\alpha_s(\mu))$$

$$2\gamma_H + \gamma_S + 2\gamma_B = 0$$

We use the rapidity renormalization group, ν is the rapidity scale

- Soft function

$$\frac{d}{d\ln\mu}\ln\mathscr{S}(p_T^{\text{veto}}, R^2;\mu,\nu) = 4\,\Gamma_{\text{cusp}}(\alpha_s(\mu))\ln\frac{\mu}{\nu} + \gamma_S(\alpha_s(\mu))$$

$$\frac{d}{d\ln\nu}\ln\mathscr{S}(p_T^{\text{veto}}, R^2;\mu,\nu) = -4\int_{p_T^{\text{veto}}}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\alpha_s(\mu')) + \gamma_\nu(p_T^{\text{veto}}, R^2)$$

Rapidity anomalus dimension

- Beam functions

$$\frac{d}{d\ln\mu}\ln\mathscr{B}_n(x, Q, p_T^{\text{veto}}, R^2;\mu,\nu) = 2\,\Gamma_{\text{cusp}}(\alpha_s(\mu))\ln\frac{\nu}{Q} + \gamma_B(\alpha_s(\mu))$$

$$\frac{d}{d\ln\nu}\ln\mathscr{B}_n(x, Q, p_T^{\text{veto}}, R^2;\mu,\nu) = 2\int_{p_T^{\text{veto}}}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\alpha_s(\mu')) - \frac{1}{2}\gamma_\nu(p_T^{\text{veto}}, R^2)$$

INGREDIENTS FOR N3LL

	LL	NLL	NLL'	N2LL	N2LL'	N3LL	N3LL'
LO	1						
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s				
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2		
N3LO	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$\alpha_s^3 L^2$	$\alpha_s^3 L$	α_s^3
N4LO	$\alpha_s^4 L^8$	$\alpha_s^4 L^7$	$\alpha_s^4 L^6$	$\alpha_s^4 L^5$	$\alpha_s^4 L^4$	$\alpha_s^4 L^3$	$\alpha_s^4 L^2$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

	Γ_{cusp}	γ	$\mathcal{S}, \mathcal{B}, \mathcal{C}$
LL	1 loop	tree	tree
NLL	2 loops	1 loop	tree
N2LL	3 loops	2 loops	1 loop
N3LL	4 loops	3 loops	2 loop

To reach N3LL, we need:

Anomalous dimensions:

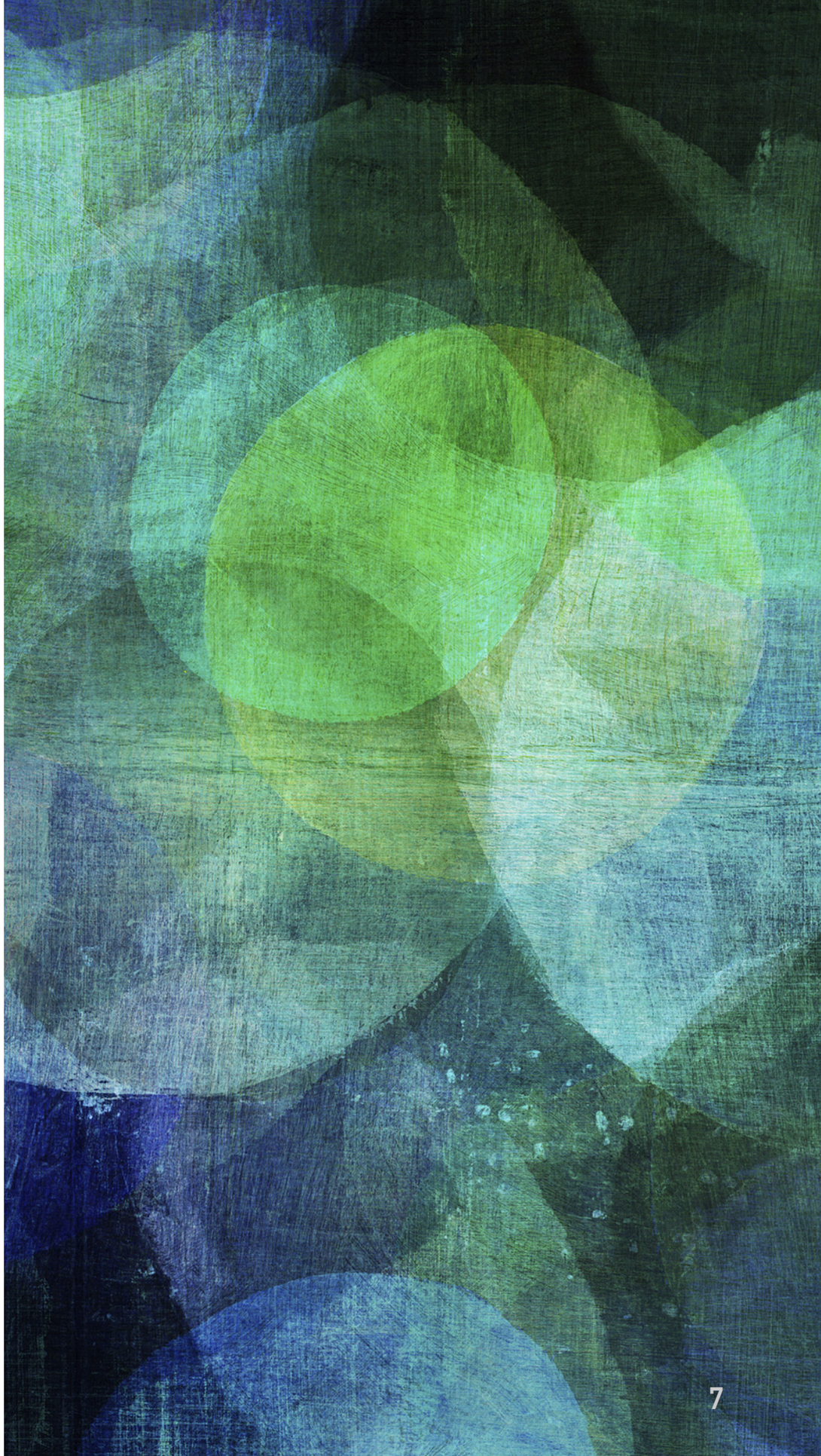
- 4 loop Γ_{cusp} — Henn, Korchemsky, Mistlberger 19
- 3 loop γ_s, γ_B, γ_H — Moch, Vermaseren, Vogt 04; Gehrmann et. al. 10; Li, von Manteuffel, Schabinger, Zhu 14
- 3 loop γ_ν, — Work in progress

Initial conditions:

- 2 loop hard function $\mathcal{C}$ — Matsuura, van Neerven 88; Gehrmann, Huber, Maitre 05
- 2 loop soft and beam functions $\mathcal{S}$, $\mathcal{B}_n$

Soft Function

Calculation and results



THE SOFT FUNCTION

$$\mathcal{S}(p_T^{\rm veto}, R^2; \mu, \nu) = \frac{1}{d_F} \sum_{X_s} \text{Tr} \left\{ \mathcal{M}(p_T^{\rm veto}, R^2) \langle 0 | Y_n^\dagger Y_{\bar{n}} | X_s \rangle \langle X_s | Y_{\bar{n}}^\dagger Y_n | 0 \rangle \right\}$$

- Soft Wilson lines: $Y_{n,\bar{n}}$ $\qquad Y_n = P \exp\left[ig \int ds\, n \cdot A_s(ns) \right]$
- Measurement function: $\mathcal{M}(p_T^{\rm veto}, R^2)$

$$\mathcal{M}(p_T^{\rm veto}, R^2) = \Theta(p_T^{\rm veto} - \max\{p_T^{\rm jet_i}\}) \Theta_{\rm cluster}(R^2)$$

- Regularization of divergences:
  - UV/IR/Coll. divergences: dimensional regularization
  - Rapidity divergences: exponential regulator

$$\prod_i d^d k_i \delta(k_i^2) \theta(k_i^0) \to \prod_i d^d k_i \delta(k_i^2) \theta(k_i^0) \exp\left[\frac{-e^{-\gamma_E}}{\nu} (n \cdot k_i + \bar{n} \cdot k_i) \right]$$

$\nu \to \infty$

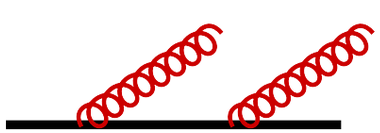
Li, Neill, Zhu 16

REFERENCE OBSERVABLE

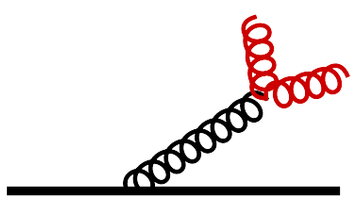
$$\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu) = \mathcal{S}_\perp(p_T^{\text{veto}}, \mu, \nu) + \Delta\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu)$$

e.g.:Banfi, Salam, Zanderighi 12; Gangal, Gaunt, Stahlhofen, Tackmann 16; Bauer, Manohar, Monni 20

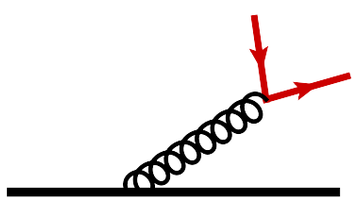
Jet algorithms starts to play a role when there are two or more real emissions



★ $\mathcal{S}_\perp(p_T^{\text{veto}}, \mu, \nu)$: soft function for p_T resummation



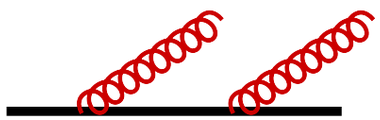
with the same regulator: Li, Neill, Zhu 16; Li, Zhu 16



★ $\Delta\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu)$: remainder defined with the measurement function:

$$\Delta\mathcal{M}(p_T^{\text{veto}}, R^2) \equiv \Theta(p_T^{\text{veto}} - \max\{p_T^{\text{jet}_i}\})\Theta_{\text{cluster}}(R^2) - \Theta\left(p_T^{\text{veto}} - \left|\sum_{X_s} p_T^{\text{jet}_i}\right|\right)$$

REFERENCE OBSERVABLE



- $\Delta\mathcal{S}(p_T^{\text{veto}}, R^2; \mu, \nu)$: remainder

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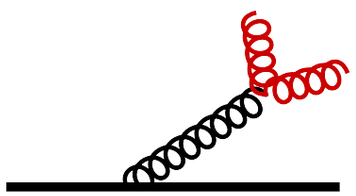
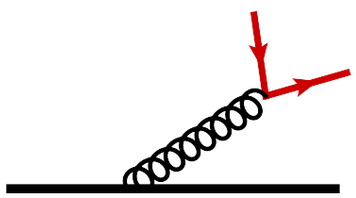


- Double-real emission is required to for the jet algorithm to have a non-trivial effect



$\Delta\mathcal{S}^{(2)}(p_T^{\text{veto}}, R^2; \mu, \nu)$: double-real diagrams with two soft gluons or a soft quark-antiquark pair

- Remainder: only rapidity divergences present, we work in four dimensions

➡ Two-loop correlated and uncorrelated contributions

$$\Delta\mathcal{S}^{(2)}(p_T, R^2; \mu, \nu) = \Delta S^{\text{corr.}}(p_T, R^2; \mu, \nu) + \Delta S^{\text{uncorr.}}(p_T, R^2; \mu, \nu)$$

- $\Delta S^{\text{uncorr.}}(p_T, R^2; \mu, \nu)$: two emissions are widely separated in rapidity
- $\Delta S^{\text{corr.}}(p_T, R^2; \mu, \nu)$: $\to 0$ when two emissions are widely separated

CALCULATION: SETUP

- Phase-space parametrization:

$$k_i = k_{i\perp}\left(\cosh\eta_i\,, \cos\phi_i\,, \sin\phi_i, \sinh\eta_i\right), \quad i = 1,2$$

$$\left\{k_{2\perp}, \eta_2, \phi_2\right\} \to \left\{\zeta \equiv k_{2\perp}/k_{1\perp}, \eta \equiv \eta_1 - \eta_2, \phi \equiv \phi_1 - \phi_2\right\}$$

- Squared amplitudes:

$$\mathscr{A}^{\text{cor./uncor.}}(k_1, k_2) = \frac{1}{k_{1\perp}^4}\frac{1}{\zeta^2}\mathscr{D}^{\text{cor./uncor.}}(\zeta, \eta, \phi)$$

- Integrals to compute:

$$\boxed{\int \frac{dk_{1\perp}}{k_{1\perp}}\, d\eta_1\, \frac{d\zeta}{\zeta}\, d\eta\, \frac{d\phi}{2\pi}\, e^{-2k_{1\perp}\frac{e^{-\gamma_E}}{\nu}[\cosh(\eta_1)+\zeta\cosh(\eta-\eta_1)]}\, \mathscr{D}(\zeta, \eta, \phi)\, \Delta\mathscr{M}(p_T^{\text{veto}}, R^2)}$$

- Measurement function, after some manipulation

$$\boxed{\Delta\mathscr{M}(p_T^{\text{veto}}, R^2) \equiv \left[\Theta(p_T^{\text{veto}} - k_{1\perp}\max\{1,\zeta\}) - \Theta\left(p_T^{\text{veto}} - k_{1\perp}\sqrt{1+\zeta^2+2\zeta\cos\phi}\right)\right]\Theta(\eta^2+\phi^2-R^2)}$$

- Goal: $\Delta\mathcal{S}^{(2)}(p_T, R^2; \mu, \nu)$ as a series in powers of R^2

CALCULATION: CORRELATED CONTRIBUTION

- Rapidity divergences:

$$\mathscr{D}^{\text{cor.}}(\zeta, \eta, \phi) \to 0 \text{ for } \eta = \eta_1 - \eta_2 \to \infty \quad \Rightarrow$$

Only η_1 integral needs rapidity regulation

- Integrate over η_1 (easy!), keep terms that survive when $\nu \to \infty$

$$I\left(p_T^{\text{veto}}/\nu, R^2\right) = \int \frac{dk_{1\perp}}{k_{1\perp}} \frac{d\zeta}{\zeta} d\eta \frac{d\phi}{2\pi} \Omega\left(\frac{k_{1\perp}}{\nu}, \zeta, \eta\right) \mathscr{D}^{\text{cor.}}(\zeta, \eta, \phi) \, \Delta\mathscr{M}(p_T^{\text{veto}}, R^2)$$

$$\Omega\left(\frac{k_{1\perp}}{\nu}, \zeta, \eta\right) = \eta + 2 \ln \frac{\nu}{k_{1\perp}} - \ln\left(1 + \zeta e^{\eta}\right) - \ln\left(\zeta + e^{\eta}\right)$$

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- Measurement function: $\Theta\left(\eta^2 - R^2 + \phi^2\right) = \underbrace{\Theta\left(\phi^2 - R^2\right)}_{\text{part } A} + \underbrace{\Theta\left(R^2 - \phi^2\right)\Theta\left(\eta^2 - R^2 + \phi^2\right)}_{\text{part } B}$
  - $I_A\left(p_T^{\text{veto}}/\nu, R^2\right)$: full R^2 dependance, expand in powers of R^2
  - $I_B\left(p_T^{\text{veto}}/\nu, R^2\right)$: regular at R^2=0 – collinear divergence in part A
    - At $\mathscr{O}(R^2)$ there is contribution from two regions
    - Instead, we compute $\frac{\partial}{\partial R^2} I_B$
    - Solve differential equation order by order in R^2

HypExp: Huber, Maitre 05
PolyLogTools: Duhr, Dulat 19

CALCULATION: UNCORRELATED CONTRIBUTION

- Rapidity divergences: both on η and η_1!

$$\mathscr{D}^{\text{uncor.}}(\zeta,\eta,\phi) = 16C_R^2$$

$$\int \frac{dk_{1\perp}}{k_{1\perp}}\, d\eta_1\, \frac{d\zeta}{\zeta}\, d\eta\, \frac{d\phi}{2\pi}\, e^{-2k_{1\perp}\frac{e^{-\gamma_E}}{\nu}[\cosh(\eta_1)+\zeta\cosh(\eta-\eta_1)]}\, \Delta\mathscr{M}(p_T^{\text{veto}}, R^2)$$

- More subtle η and η_1 integration of exponential regulator
  - ✓ Set $w = e^{\eta}$, $x = e^{\eta_1}$, take Laplace transform
  - ✓ In Laplace space, expand exponential regulator in distributions
  - ✓ Take inverse Laplace transform, keep terms that survive when $\nu \to \infty$

$$\int \frac{dx}{x\,w} e^{-k_{1\perp}\frac{e^{-\gamma_E}}{\nu x}[1+w\zeta+\frac{x^2}{w}(w+\zeta)]} \to 4\delta(w)\ln\left(\frac{k_{1\perp}}{\nu}\right)\ln\left(\frac{\zeta\, k_{1\perp}}{\nu}\right) + \left[\frac{1}{w}\right]_+ \ln\left(\frac{\nu^2 w}{k_{1\perp}^2(w+\zeta)(1+\zeta w)}\right) + \mathscr{O}\left(\frac{1}{\nu^2}\right)$$

- Continue as for the correlated contributions for remaining integrals

NUMERICAL CALCULATION, FULL R DEPENDENCE

Exponential regulator: as in analytic calculation

- Correlated contributions
  - Variables: $\phi, \quad \eta, \quad \eta_t = \frac{1}{2}\left(\eta_1 + \eta_2\right), \quad z = \frac{k_{1\perp}^2}{k_{1\perp}^2 + k_{2\perp}^2},$

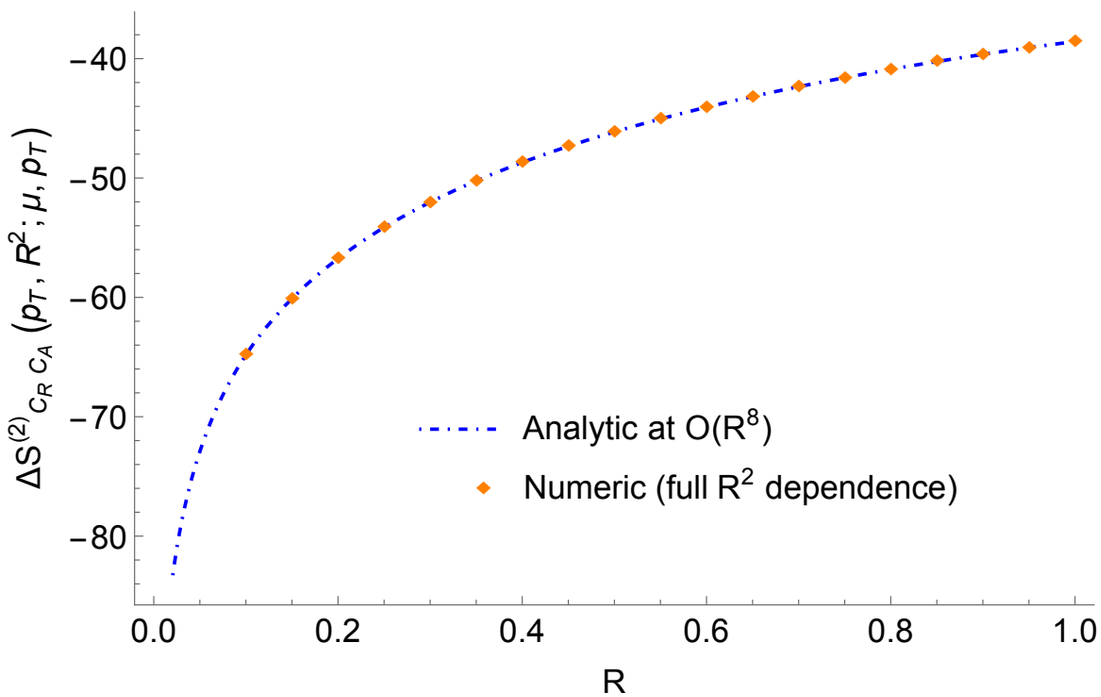
    $\mathscr{K}_T^2 = k_{1\perp}^2 + k_{2\perp}^2$
  - Analytic integrations: $\eta_t\,, \quad \mathscr{K}_T^2$
  - Numerical integrations: $\eta, \quad \phi, \quad z$
- Uncorrelated corrections
  - Variables: $\phi, \quad \eta_1, \quad \eta_2, \quad z = \frac{k_{1\perp}^2}{k_{1\perp}^2 + k_{2\perp}^2}, \quad \mathscr{K}_T^2 = k_{1\perp}^2 + k_{2\perp}^2$
  - Analytic integrations: $\eta_1, \quad \eta_2\,, \quad \mathscr{K}_T^2$
  - Numerical integrations: $\phi, \quad z$

Full R dependence, per mille precision in numerical evaluation

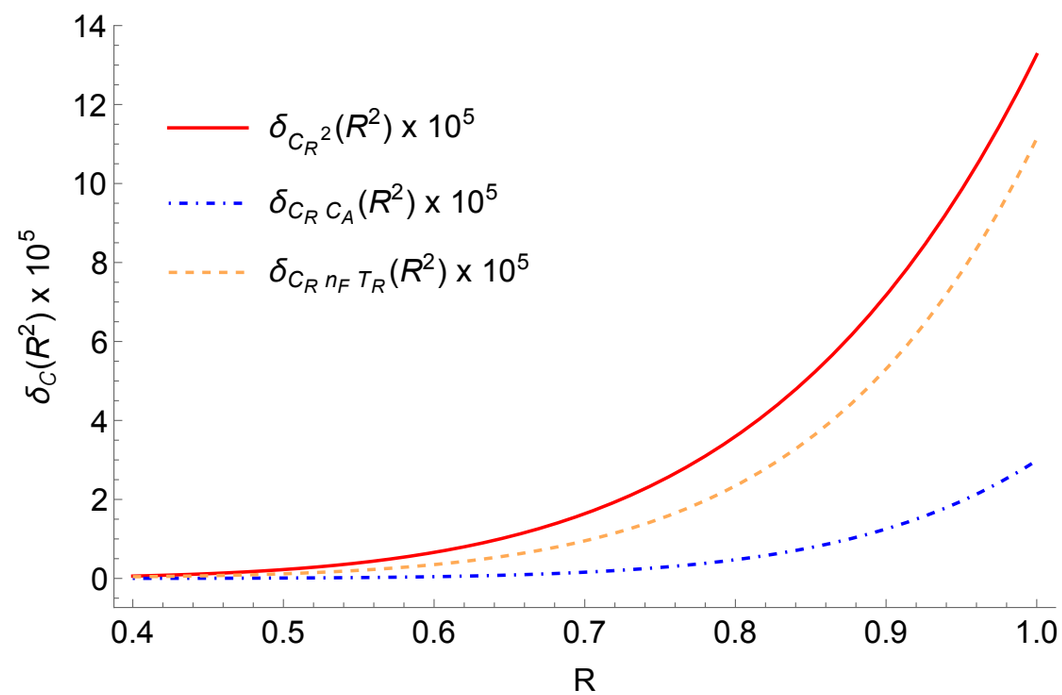
RESULTS AND CHECKS

- Analytic results for $\Delta\mathcal{S}^{(2)}(p_T^{\rm veto}, R^2; \mu, \nu)$ to $\mathcal{O}(R^8)$ (also $\mathcal{S}_\perp^{(2)}(p_T^{\rm veto}, R^2; \mu, \nu)$)
- Reproduce known two-loop rapidity anomalous dimension
- Verify suitability of R^2 expansion for $0 < R < 1$

Banfi, Monni, Salam, Zanderighi 12;
Becher, Neubert, Rothen 13;
Stewart, Tackmann, Walsh, Zuberi 13



Comparison with numerics, with full R^2 dependence



Check $\mathcal{O}(R^8)$ corrections are negligible

$$\delta_{\mathscr{C}}(R) = \left| 1 - \frac{\Delta\mathcal{S}_{\mathscr{C}}^{(2)}(p_T, R^2; \mu, p_T)|_{R^6}}{\Delta\mathcal{S}_{\mathscr{C}}^{(2)}(p_T, R^2; \mu, p_T)|_{R^8}} \right|$$

Beam Function

Calculation and results

THE BEAM FUNCTIONS

- Operatorial definition:

Quarks: $$\mathscr{B}_q(x, Q, p_T^{\text{veto}}, R^2; \mu, \nu) = \frac{1}{2\pi} \sum\!\!\!\!\!\!\int_{X_C} dt e^{-ixt\bar{n}\cdot p} \mathscr{M}(p_T^{\text{veto}}, R^2) \left\langle P(p) \middle| \bar{\chi}_n(t\bar{n}) \frac{\not{\bar{n}}}{2} \middle| X_C \right\rangle \left\langle X_C \middle| \chi_n(0) \middle| P(p) \right\rangle$$

Gluons: $$\mathscr{B}_g(x, Q, p_T^{\text{veto}}, R^2; \mu, \nu) = -\frac{x\bar{n}\cdot p}{2\pi} \sum\!\!\!\!\!\!\int_{X_C} dt e^{-ixt\bar{n}\cdot p} \mathscr{M}(p_T^{\text{veto}}, R^2) \left\langle P(p) \middle| \mathscr{A}_\perp^{\mu,a}(t\bar{n}) \middle| X_C \right\rangle \left\langle X_C \middle| \mathscr{A}_{\perp,\mu}^{a}(0) \middle| P(p) \right\rangle$$

  - χ_n, $\mathscr{A}_\perp^{\mu,a}$: collinear gauge invariant collinear fields
  - Measurement function $\mathscr{M}$: same as for Soft function

- For $p_T^{\text{veto}} \gg \Lambda_{\text{QCD}}$ can be perturbatively matched to PDFs

$$\mathscr{B}_F(x, Q, p_T^{\text{veto}}, R^2; \mu, \nu) = \sum_{F'} \int_x^1 \frac{dz}{z} I_{FF'}(z, Q, p_T^{\text{veto}}, R^2; \mu, \nu) f_{F'/P}(x/z, \mu) + \mathscr{O}(\Lambda_{\text{QCD}}/p_T^{\text{veto}})$$

- Regularization of divergences: as for Soft function

Exponential regulator:

$$\prod_i d^d k_i \delta(k_i^2)\theta(k_i^0) \to \prod_i d^d k_i \delta(k_i^2)\theta(k_i^0) \exp\left[\frac{-e^{-\gamma_E}}{\nu}(n\cdot k_i + \bar{n}\cdot k_i)\right]$$

BEAM FUNCTION CALCULATION

- Reference observable:

$$\mathscr{B}(x, p_T^{\text{veto}}, R^2; \mu, \nu) = \mathscr{B}_\perp(x, p_T^{\text{veto}}, \mu, \nu) + \Delta\mathscr{B}(x, p_T^{\text{veto}}, R^2; \mu, \nu)$$

- Same approach as Soft function: decompose into different contributions
  - ✓ Several channels and several colour factors

$$\int \frac{dk_{1\perp}}{k_{1\perp}} d\eta_1 \frac{d\zeta}{\zeta} d\eta \frac{d\phi}{2\pi} e^{-2k_{1\perp}\frac{e^{-\gamma_E}}{\nu}[\cosh(\eta_1)+\zeta\cosh(\eta-\eta_1)]} \Delta\mathscr{M}(p_T^{\text{veto}}, R^2) \mathscr{D}_{1\to 3}(\zeta, \eta, \phi)\, \delta(k_1^\pm + k_2^\pm - (1-x)p^\pm)$$

Catani, Grazzini 99

- Structure of our results

$$\Delta\mathscr{B}^{(2)}(x, R^2) = \delta(1-x)\, f_1(R^2) + \left[\frac{1}{1-x}\right]_+ \Big(f_2(x, R^2) + f_3(x) \Big)$$

Series in R^2, up to $\mathscr{O}(R^8)$, analytic

Numerical grid at $R = 0$, 3-fold integral, per mille precision

- We also performed numerical calculation with full R dependence
- Calculation in Mellin space/different scheme also recently available

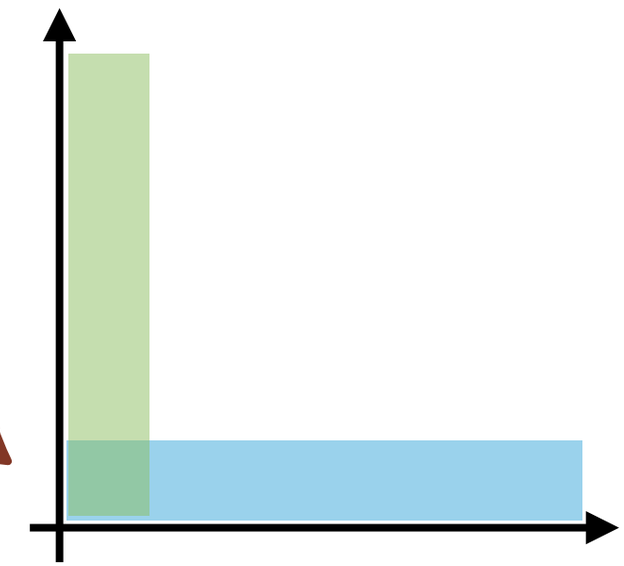
Bell, Brune, Das, Wald 22

COMMENT ON POTENTIAL FACTORIZATION BREAKING

- Zero bin subtraction: remove soft modes from beam functions

$$\mathscr{B}_{cc} = \mathscr{B} - \mathscr{B}_{sc} - \mathscr{B}_{cs} + \mathscr{B}_{ss}$$

Overlap contributions exist in amplitudes and measurement function



- Can this be consistently done in SCET?
  - If not, SCET factorization is broken by soft-collinear mixing terms, i.e. when collinear and soft modes are clustered together
  - For jet veto: OK at NLO, contested beyond

    Becher, Neubert, Rothen 13, Tackmann, Walsh, Zuberi 12, Stewart, Tackmann, Walsh, Zuberi 13

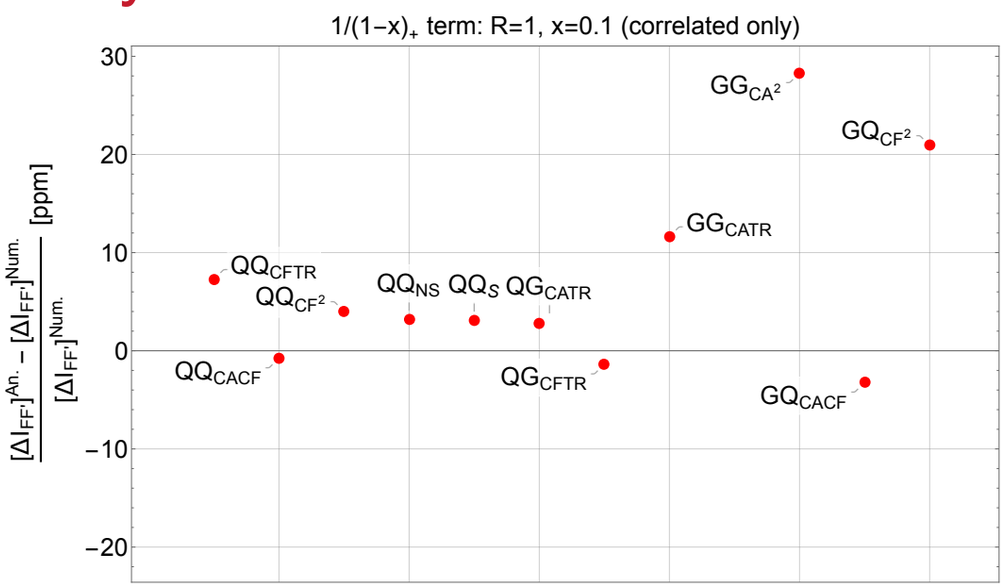
  - Our explicit computation shows that the two-loop mixing terms are absent after consistent expansions of amplitudes and measurement functions

SCET factorization theorem holds at NNLO and reproduces QCD

RESULTS AND CHECKS

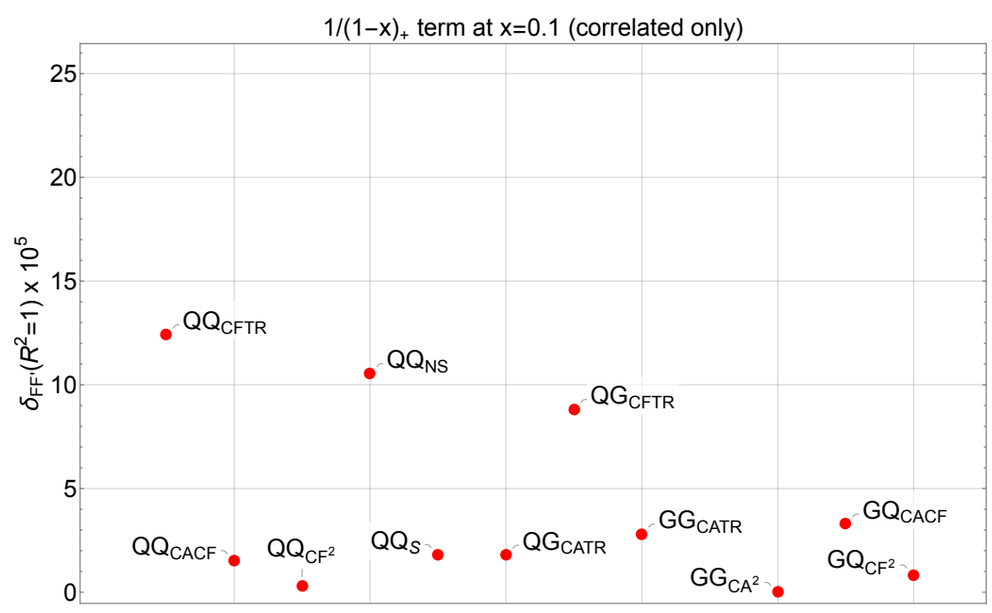
- Reproduce known two-loop rapidity anomalous dimension
- Analytic vs Numerical calculation

Banfi, Monni, Salam, Zanderighi 12;
Becher, Neubert, Rothen 13;
Stewart, Tackmann, Walsh, Zuberi 13



$$\Delta\mathscr{B}^{(2)}(x,R^2) = \delta(1-x)\, f_1(R^2) + \left[\frac{1}{1-x}\right]_+ \Big(f_2(x,R^2) + f_3(x) \Big)$$

- Convergence of R^2 expansion



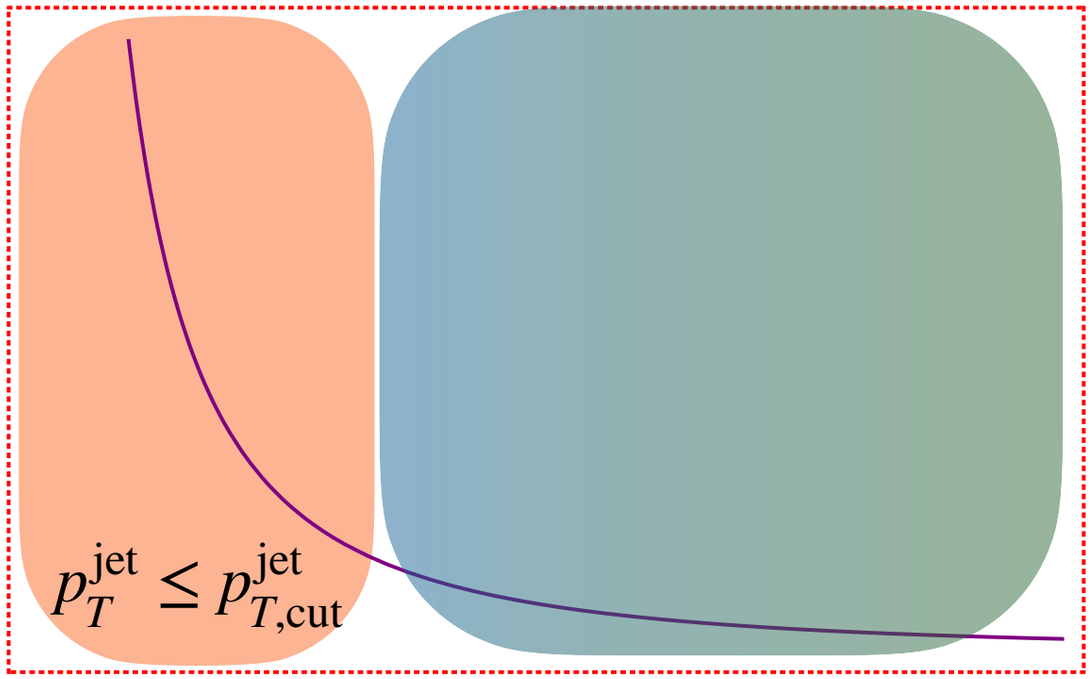
$$\delta_{FF'}(R^2) = \left| 1 - \frac{\Delta I^{(2)}_{FF'}|_{R^6}}{\Delta I^{(2)}_{FF'}|_{R^8}} \right|$$

Applications and more checks

Leading-jet p_T slicing

LEADING-JET p_T SLICING

We have now all ingredients for NNLO slicing with $p_T^{\rm jet}$ for color singlet production



$$\mathrm{d}\sigma^F_{\rm NNLO} = \mathbb{H}^{\rm NNLO}_{\rm veto} \otimes \mathrm{d}\sigma_{\rm Born} + \lim_{p_{T,\rm cut}^{\rm jet}\to 0} \int_{p_{T,\rm cut}^{\rm jet}}^{+\infty} \mathrm{d}p_T^{\rm jet} \left(\frac{\mathrm{d}^2\sigma^{F+\rm jet}_{\rm NLO}}{\mathrm{d}p_T^{\rm jet}} - \frac{\mathrm{d}^2\sigma(p_T^{\rm veto})}{\mathrm{d}p_T^{\rm veto}}\bigg|^{(\alpha_s^2)}_{p_T^{\rm veto}=p_T^{\rm jet}} \right)$$

Finite terms in two-loop soft and beam functions

From e.g. MCFM

Two-loop anomalous dimensions

LEADING-JET p_T SLICING AS A CHECK

$$\mathrm{d}\sigma^{F}_{\mathrm{NNLO}} = \mathbb{H}^{\mathrm{NNLO}}_{\mathrm{veto}} \otimes \mathrm{d}\sigma_{\mathrm{Born}} + \lim_{p_{T,\mathrm{cut}}^{\mathrm{jet}} \to 0} \int_{p_{T,\mathrm{cut}}^{\mathrm{jet}}}^{+\infty} \mathrm{d}p_T^{\mathrm{jet}} \left(\frac{\mathrm{d}^2\sigma^{F+\mathrm{jet}}_{\mathrm{NLO}}}{\mathrm{d}p_T^{\mathrm{jet}}} - \left. \frac{\mathrm{d}^2\sigma(p_T^{\mathrm{veto}})}{\mathrm{d}p_T^{\mathrm{veto}}} \right|^{(\alpha_s^2)}_{p_T^{\mathrm{veto}}=p_T^{\mathrm{jet}}} \right)$$

n3loxs

Finite terms in two-loop soft and beam functions

Baglio, Duhr, Mistleberger, Szafron 19

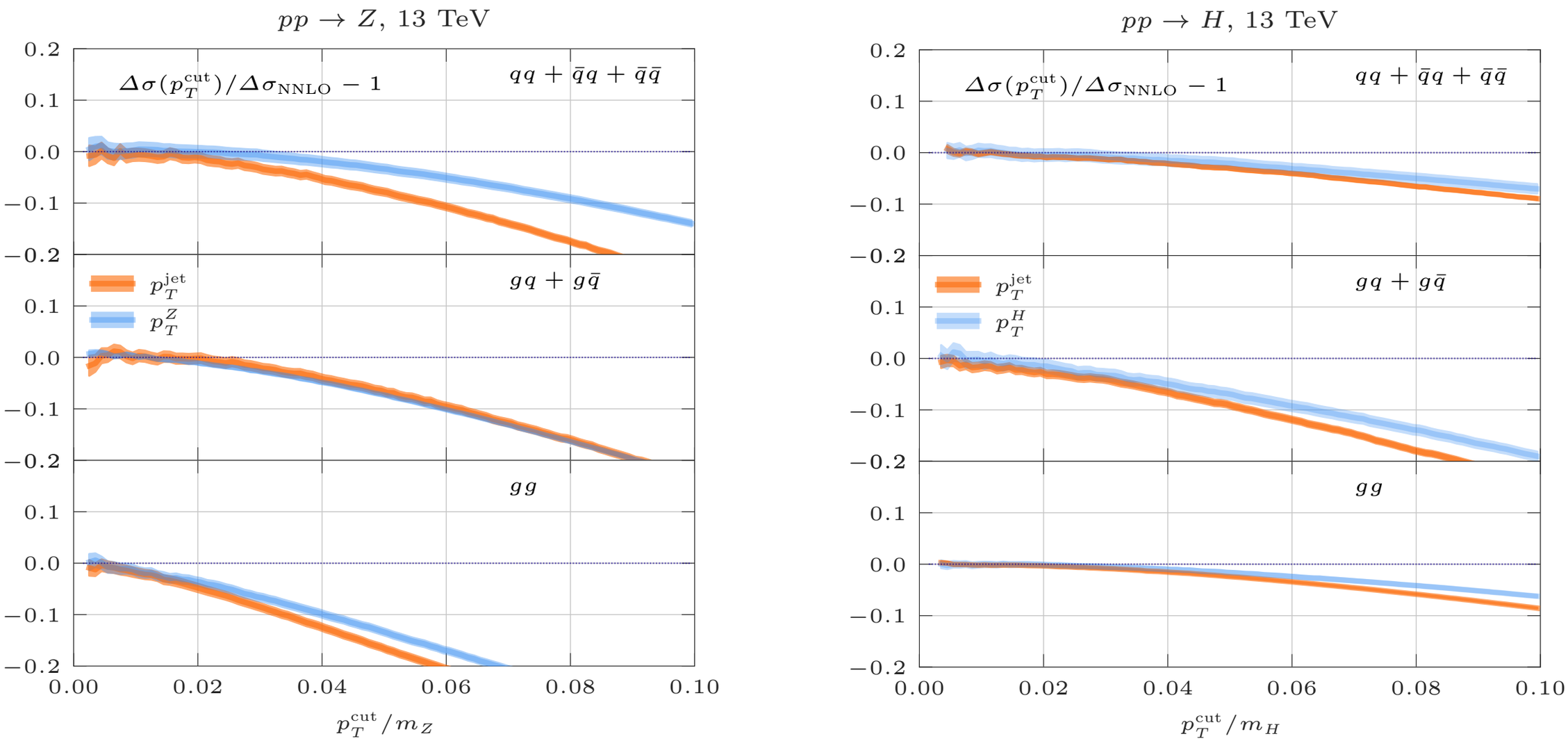
From e.g. MCFM

Campbell, Neumann 19

Two-loop anomalous dimensions

- Implemented in RadISH
- We reproduced know NNLO cross-section: very strong check
- Residual dependence on p_T^{jet}: determined by power corrections

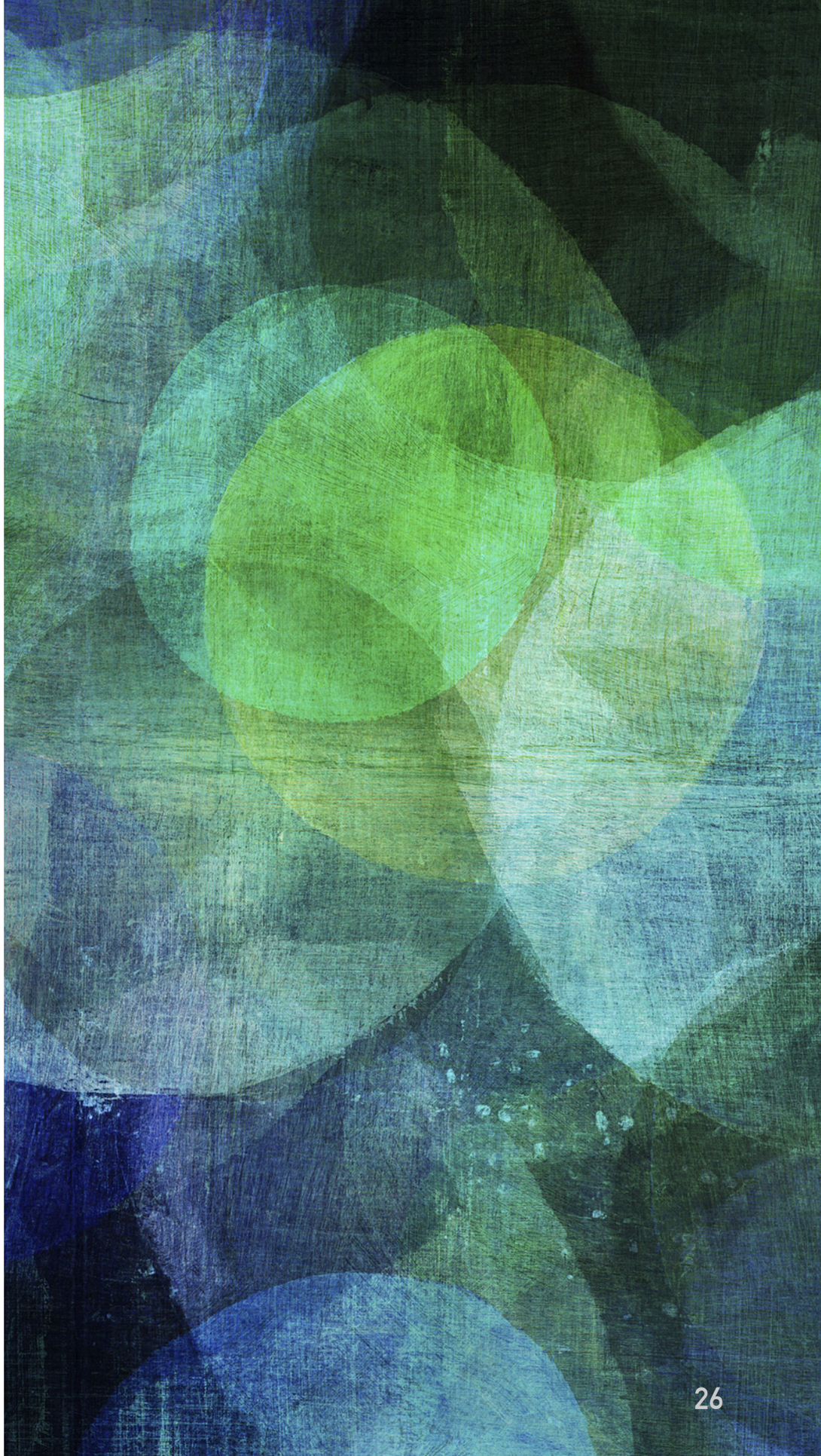
LEADING-JET p_T SLICING: CHANNEL DECOMPOSITION



- Convergence at low $p_T^{\rm cut}$ — correct two-loop anomalous dimensions
- Convergence to 0 — correct finite terms in two-loop soft and beam functions
- Shape of curves: form of power corrections

Leading-jet p_T slicing is competitive with q_T slicing

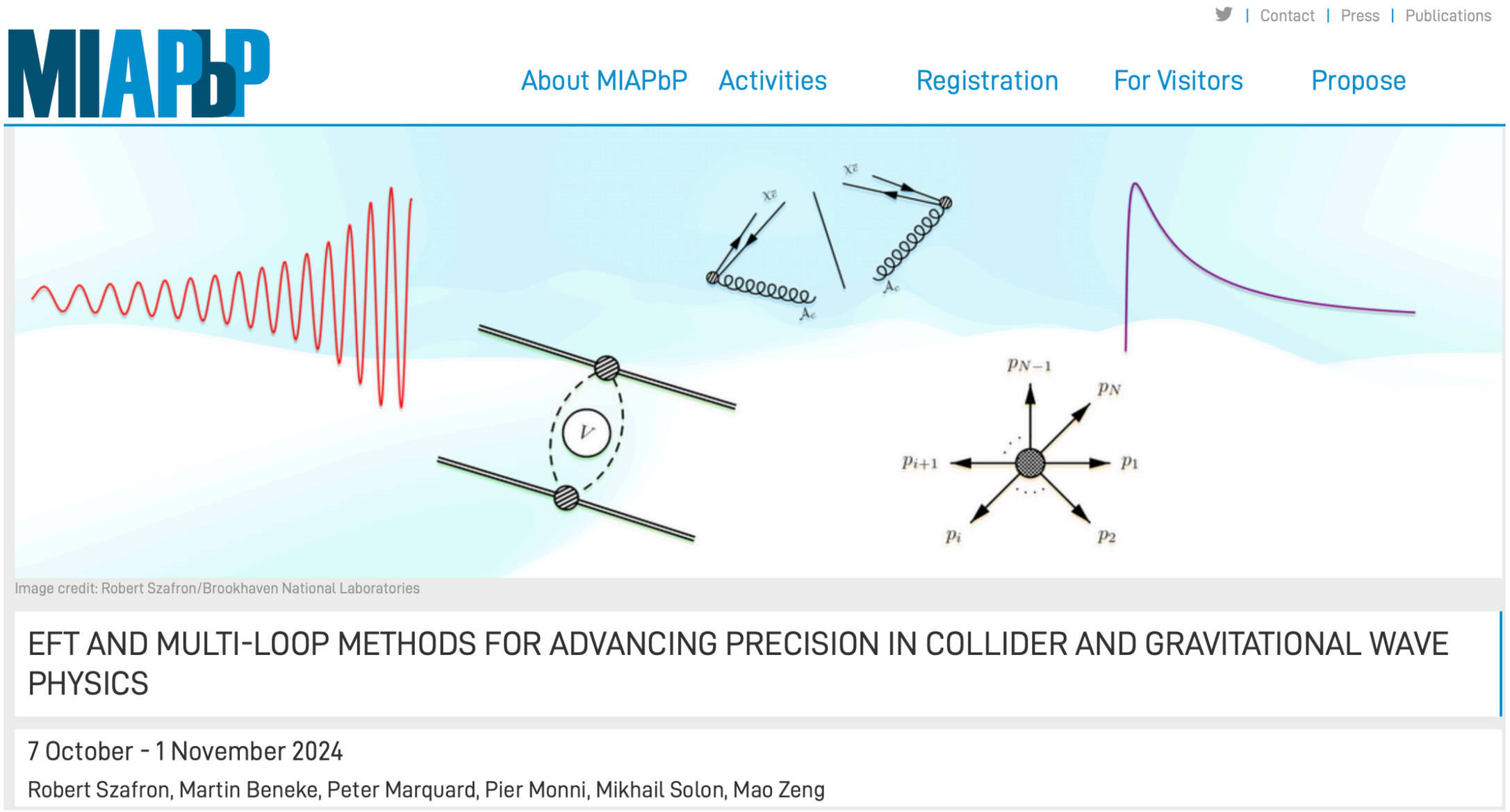
Conclusion and outlook



CONCLUSION AND OUTLOOK

- ✓ Analytic two-loop soft function for jet-cross-sections
- ✓ Analytic two-loop beam functions (up to boundary condition at $R = 0$)
- ✓ Developed and applied leading-jet p_T slicing
- ✓ Validated finite terms of soft/beam functions by reproducing NNLO DY and Higgs cross-sections
- ➡ Work in progress: Calculate the three-loop γ_ν
- ➡ Work in progress: complete N^3LL resummation of jet-vetoed cross-section

THANK YOU!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EFT AND MULTI-LOOP METHODS FOR ADVANCING PRECISION IN COLLIDER AND GRAVITATIONAL WAVE PHYSICS
7 October - 1 November 2024
Robert Szafron, Martin Beneke, Peter Marquard, Pier Monni, Mikhail Solon, Mao Zeng