Jet and Photon Physics at High Perturbative Orders in QCD

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RADCOR 2023, Crieff, Scotland, 31st May 2023

On the menu today

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet

Determination of the strong coupling constant from transverse energy–energy correlations in multijet events at $\sqrt{s}$ = 13 TeV with the ATLAS detector

The ATLAS Collaboration

Isolated photon production in association with a jet pair through next-to-next-to-leading order in QCD
Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia

STRIPPER

The framework

Czakon
“A novel subtraction scheme for double-real radiation at NNLO”
Phys. Lett. B 693 (2010) 259

Czakon and Heymes
“Four-dimensional formulation of the sector-improved residue subtraction scheme”
Nucl. Phys. B 890 (2014) 152

Czakon, van Hameren, Mitov and Poncelet
“Single-jet inclusive rates with exact color at $\mathcal{O}(\alpha_s^4)$”
JHEP 10 (2019) 262

High-multiplicity processes at NNLO

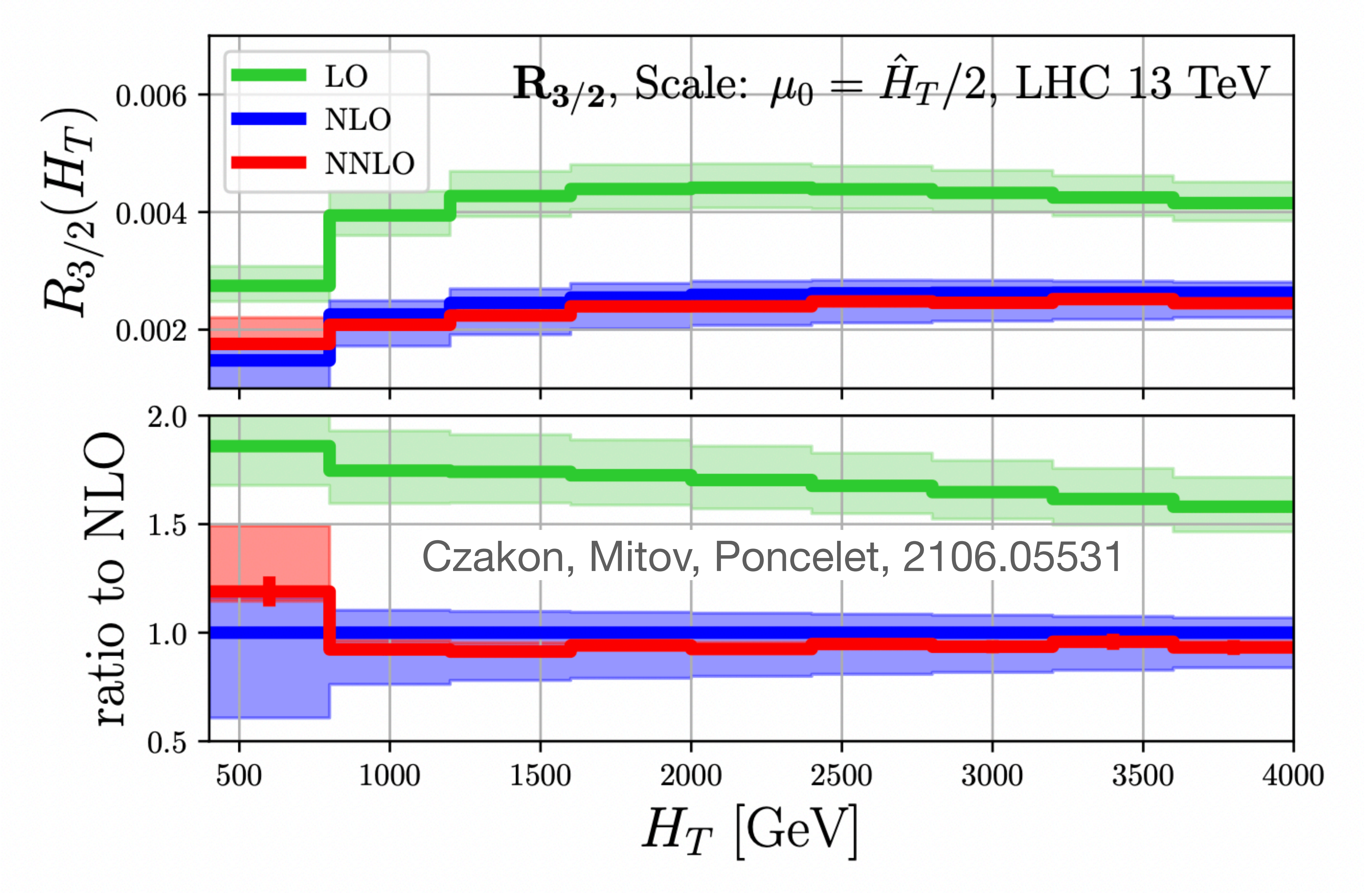
From last year's Loops & Legs talk:

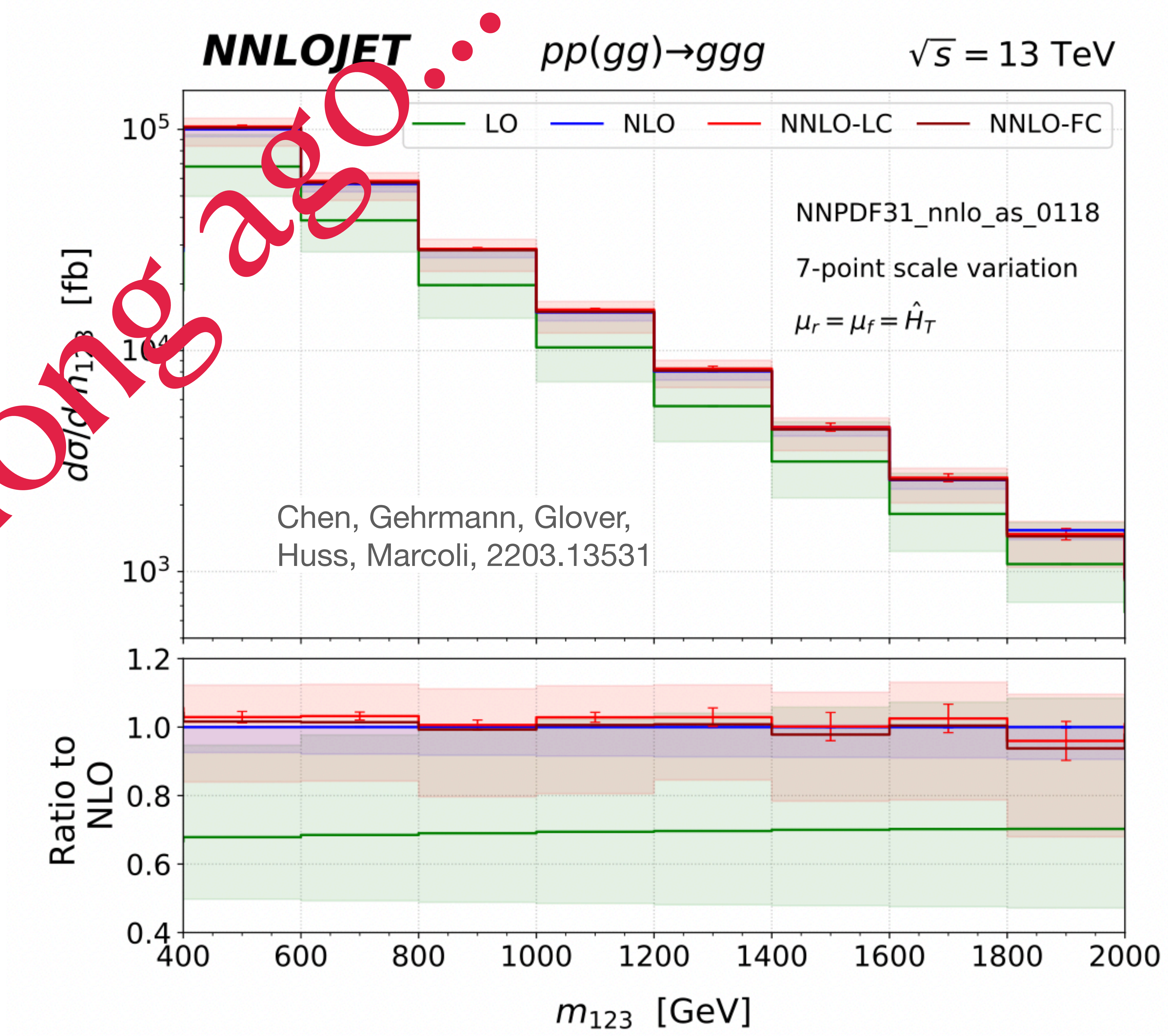
- An impressive number of 2 to 3 processes calculated at NNLO in the past few years:
  - $pp \to 3$ jets   Czakon, Mitov, Poncelet; Chen, Gehrmann, Glover, Huss, Marcoli
  - $pp \to \gamma\gamma+$ jet   Chawdry, Czakon, Mitov, Poncelet; Badger, Gehrmann, Marcoli, Moddie
  - $pp \to \gamma\gamma\gamma$ Chawdry, Czakon, Mitov, Poncelet; Kallweit, Sotnikov, Wiesemann
  - $pp \to Wb\bar{b}$ Hartanto, Poncelet, Popescu, Zoia
- Remarkable progress in the past few years thanks to break-through calculations of two-loop five point amplitudes: master integral calculation (analytic or numerical) and efficient integral reduction. Very impressive community efforts!

Abreu, Agawal, Badger, Bern, Bohm, Bonetti, Bonnum-Hansen, Buccioni, Canko, Chawdry, Chicherin, Czakon, de Laurentis, Dixon, Dormans, Duhr, Febres Cordero, Gehrmann, Georgoudis, Georgoudis, Gluza, Guan, Hartanto, Heinrich, Heller, Henn, Hermann, Hidding, Ita, Jones, Kadja, Kardos, Klinkert, Kosher, Kraus, Kreer, Krys, Larsen, Liu, Lo Presti, Ma, Maitre, Marcoli, Mitov, Moddie, Moriello, Page, Panzer, Papadopoulos, Pascual, Peraro, Poncelet, Ruf, Schabinger, Schulze, Smirnov, Sotnikov, Syrrakos, Tancredi, Tommasini, Tschernow, von Manteuffel, Wang, Wasser, Weinzierl, Wever, Zeng, Zhang, Zoia, ...

Three-jet production

Three-jet production at leading color





Not that long ago...

- Different subtraction schemes: sector-improved residue subtraction (left) v.s. antenna (right)
- Main bottleneck to full color is virtual two-loop amplitudes
- 3/2 jet ratio is important for α_s measurement
- NNLO stabilizes the ratio and smaller scale uncertainties
  How to better estimate scale uncertainties?
- Small corrections to triple jet invariant mass at NNLO

Three-jet production at leading color

Shout out to these guys for the excellent work on amplitudes:

Chicherin, Gehrmann, Henn, Wasser, Zhang and Zoia
"All Master Integrals for Three-Jet Production at Next-to-Next-to-Leading Order"
Phys. Rev. Lett. 123 (2019) 041603

Chicherin and Sotnikov
"Pentagon Functions for Scattering of Five Massless Particles"
JHEP 20 (2020) 167

Abreu, Febres Cordero, Ita, Page and Sotnikov
"Leading-color two-loop QCD corrections for three-jet production at hadron colliders"
JHEP 07 (2021) 095

Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang et al.
"OpenLoops 2"
Eur. Phys. J. C 79 (2019) 866

Bury and van Hameren
"Numerical evaluation of multi-gluon amplitudes for High Energy Factorization"
Comput. Phys. Commun. 196 (2015) 592

NNLO QCD corrections to event shapes at the LHC

Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet, JHEP 03 (2023) 129

- Collection of event shapes:

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,, \quad \text{and} \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}$$

Transverse trust and its minor component

$$\mathcal{M}_{xyz} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{1}{|\vec{p}_i|} \begin{pmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 \end{pmatrix}$$

linearised sphericity tensor

$$A = \frac{3}{2}\lambda_3\,, \quad C = 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)\,, \quad D = 27\lambda_1\lambda_2\lambda_3\,.$$

aplanarity

$$\mathcal{M}_{xy} = \frac{1}{\sum_i |\vec{p}_{T,i}|} \sum_i \frac{1}{|\vec{p}_{T,i}|} \begin{pmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 \end{pmatrix}$$

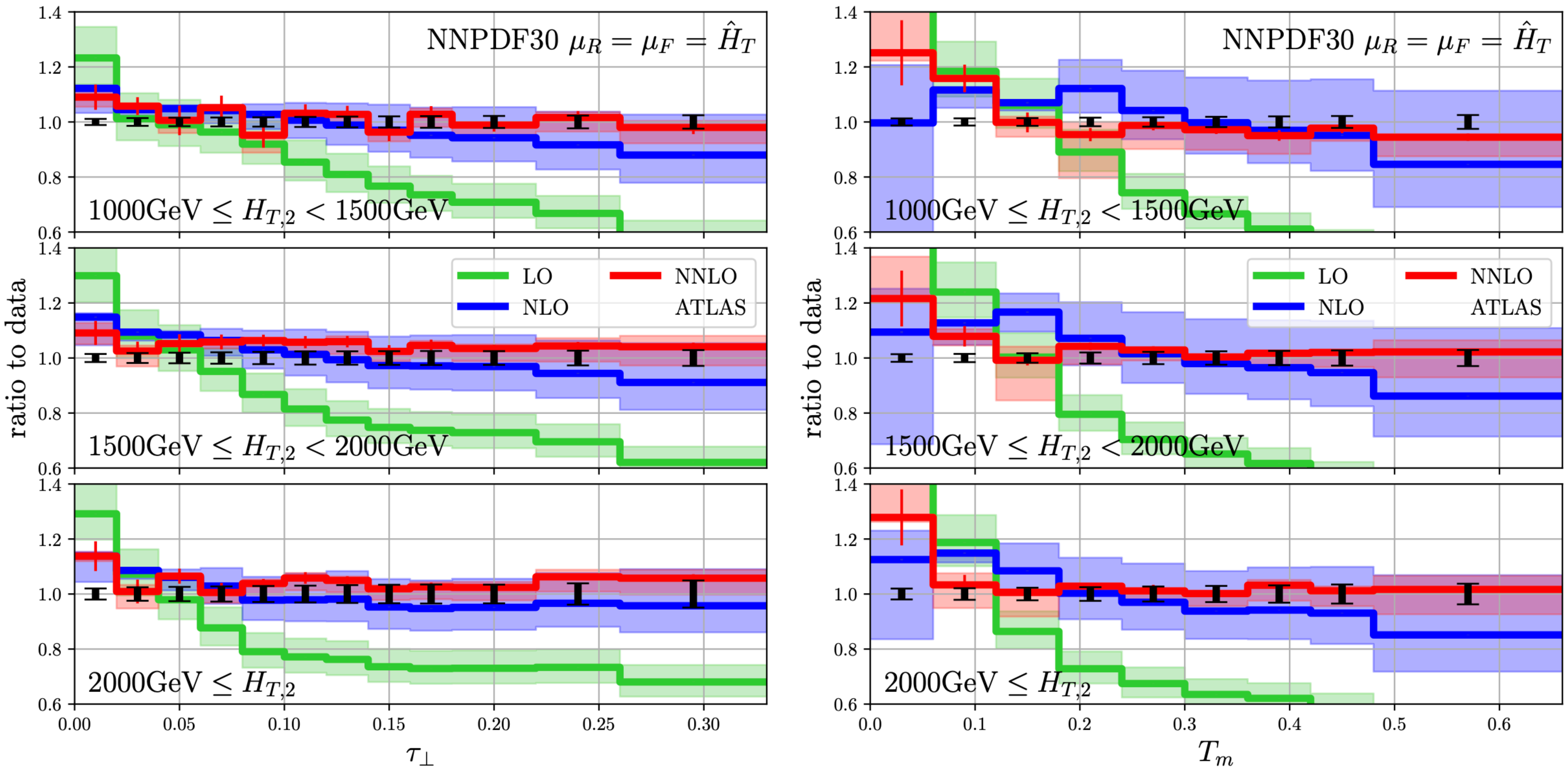
transverse linearised sphericity tensor

$$S_\perp = \frac{\mu_2}{\mu_1 + \mu_2}$$

transverse sphericity

NNLO QCD corrections to event shapes at the LHC

Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet, JHEP 03 (2023) 129



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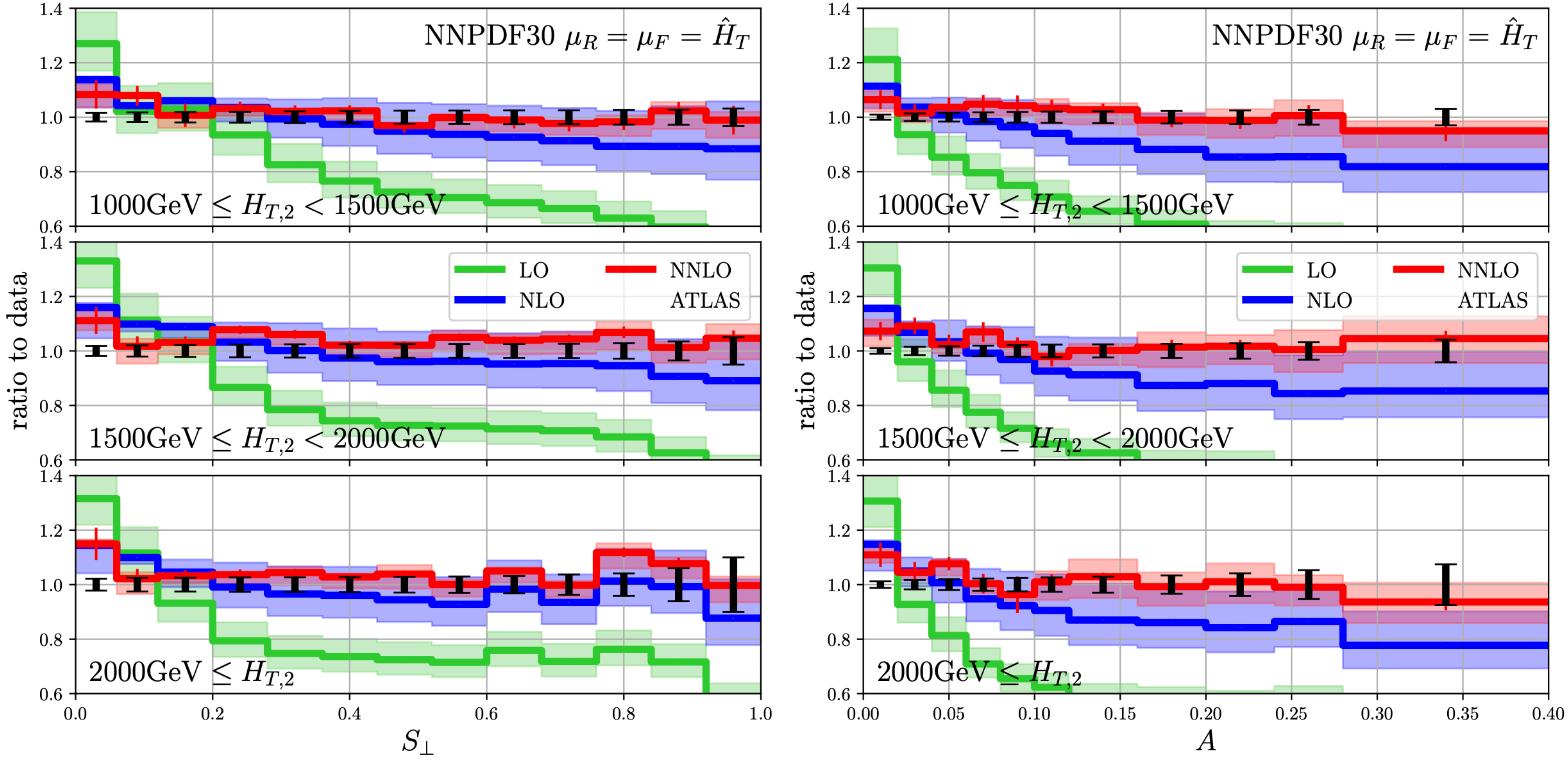


Transverse Energy-Energy Correlations in ATLAS

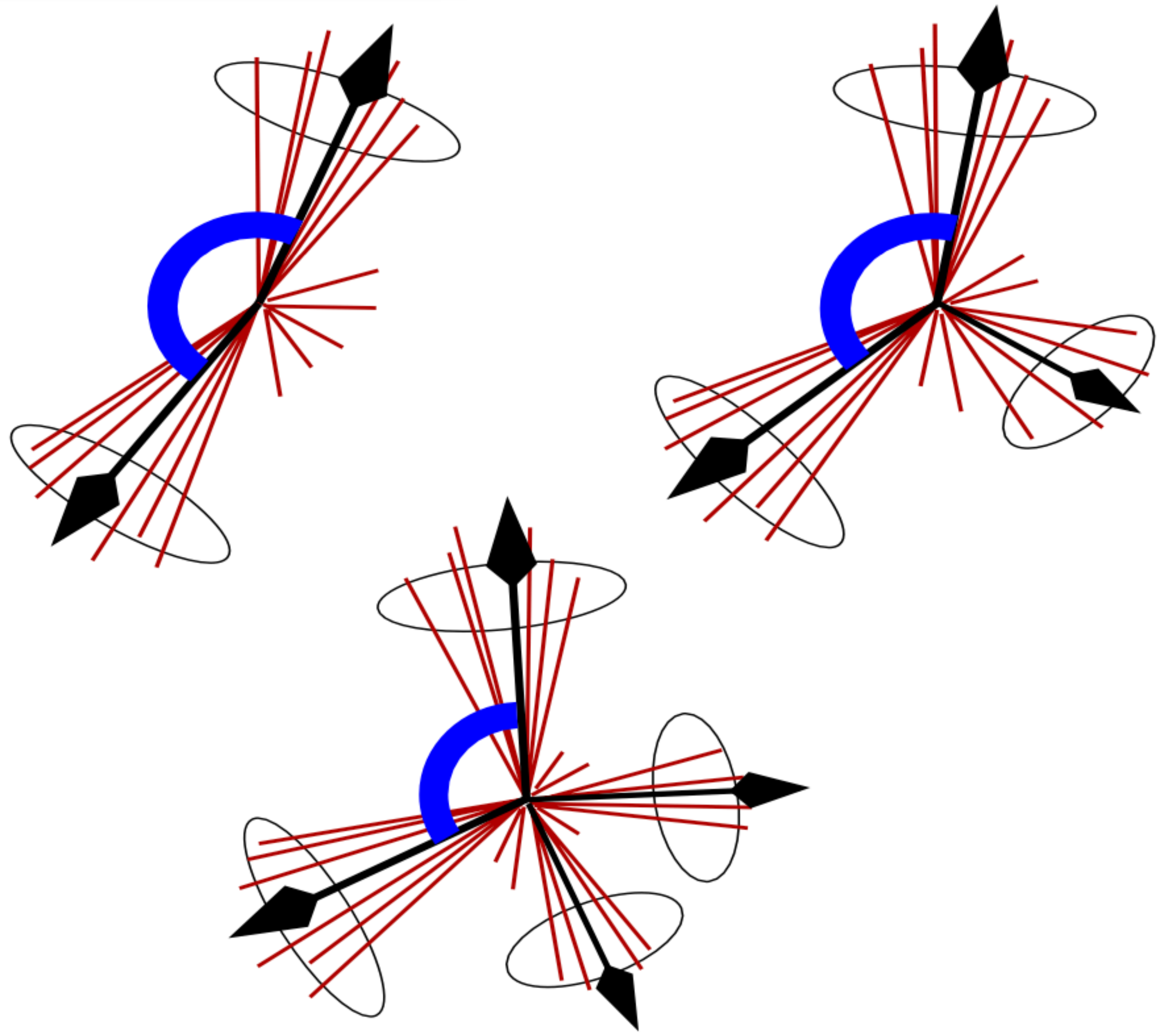
arXiv:2301.09351

- TEEC : transverse-energy-weighted distribution of the azimuthal differences between jet pairs in the final state:

$$\frac{1}{\sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi} \equiv \frac{1}{\sigma}\sum_{ij}\int\frac{\mathrm{d}\sigma}{\mathrm{d}x_{\mathrm{T}i}\mathrm{d}x_{\mathrm{T}j}\mathrm{d}\cos\phi}x_{\mathrm{T}i}x_{\mathrm{T}j}\mathrm{d}x_{\mathrm{T}i}\mathrm{d}x_{\mathrm{T}j} = \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E_{\mathrm{T}i}^{A}E_{\mathrm{T}j}^{A}}{\left(\sum_k E_{\mathrm{T}k}^{A}\right)^2}\delta(\cos\phi-\cos\varphi_{ij})$$

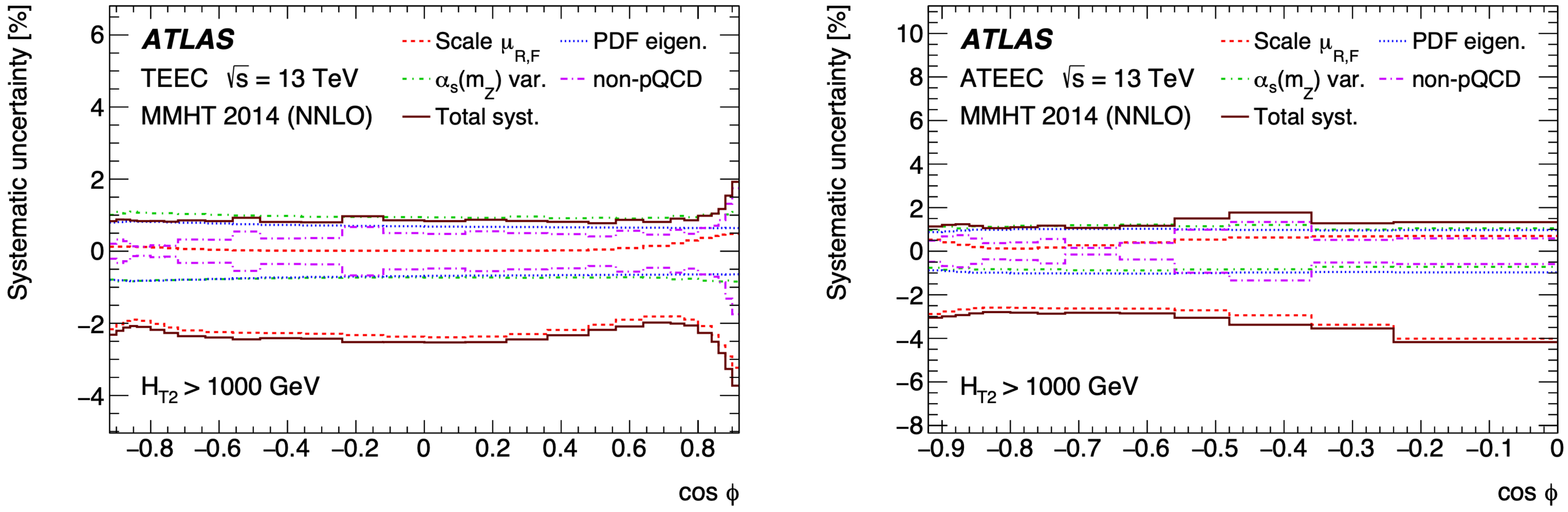
- ATEEC : difference between the forward ($\cos\phi > 0$) and the backward ($\cos\phi < 0$) part of the TEEC function

$$\frac{1}{\sigma}\frac{\mathrm{d}\Sigma^{\mathrm{asym}}}{\mathrm{d}\cos\phi} = \frac{1}{\sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi}\bigg|_{\phi} - \frac{1}{\sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi}\bigg|_{\pi-\phi}$$



Transverse Energy-Energy Correlations in ATLAS

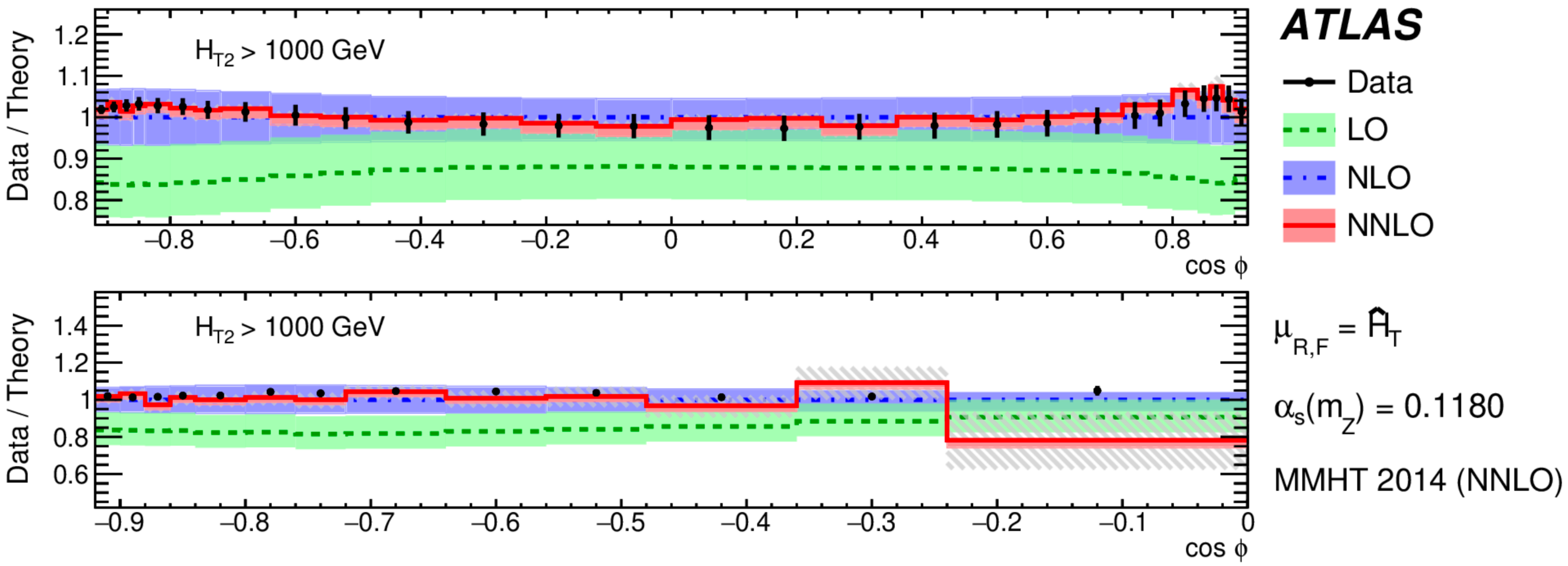
- Predictions avoid back-to-back and collinear regions ⇒ $|\cos\phi| < 0.92$ [arXiv:2301.09351](https://arxiv.org/abs/2301.09351)
- Theoretical uncertainties on the predictions are estimated on:
    1. Renormalisation and factorisation scales μ_R , μ_F variations by a restricted factor of 2
    2. Variations of the PDF parameters (eigenvectors / replicas)
    3. Non-perturbative corrections with different MC tunes
    4. Variations of the strong coupling α_s (m_Z) by 0.0001
- Total uncertainties of O(2%) (3%) for TEEC (ATEEC)



Transverse Energy-Energy Correlations in ATLAS

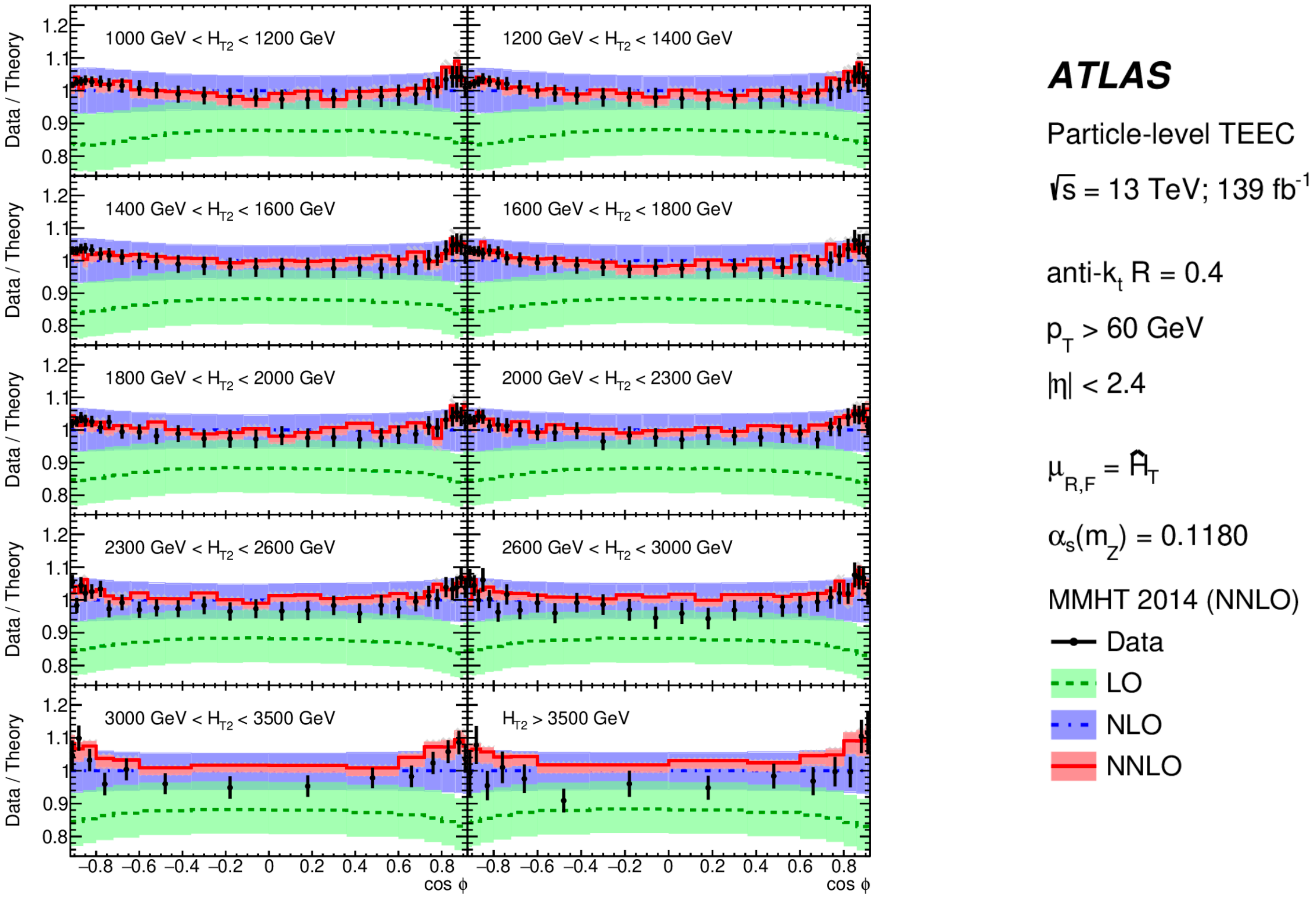
- Intensive use of computing grid (over 100M CPU hours ~ 11K years!)
- Excellent description of collinear and back-to-back regions
- Important reduction of theoretical uncertainties on QCD scales

arXiv:2301.09351



Transverse Energy-Energy Correlations in ATLAS

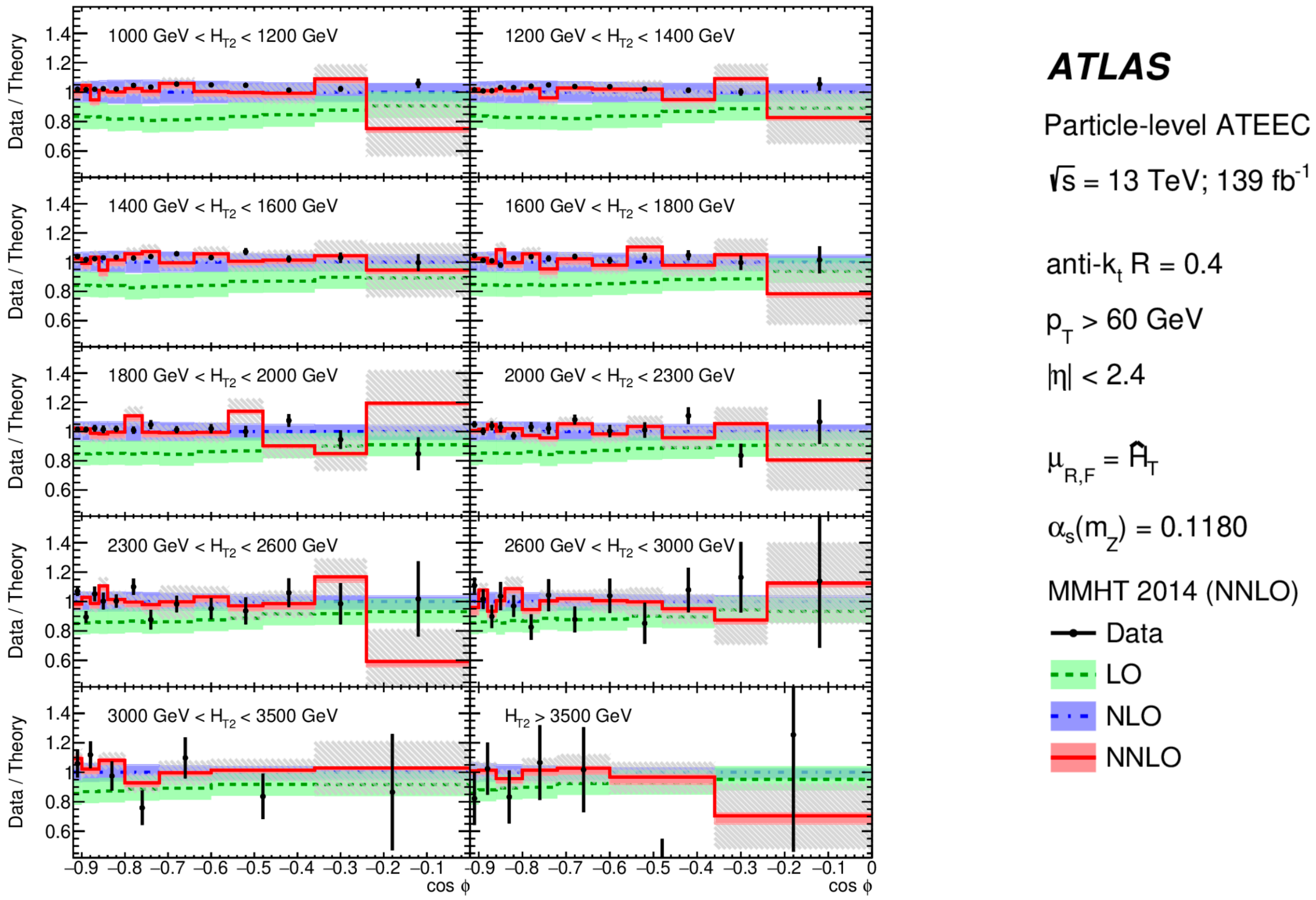
- Good overall description, with theory slightly above the data for high H_{T2} bins



arXiv:2301.09351

Transverse Energy-Energy Correlations in ATLAS

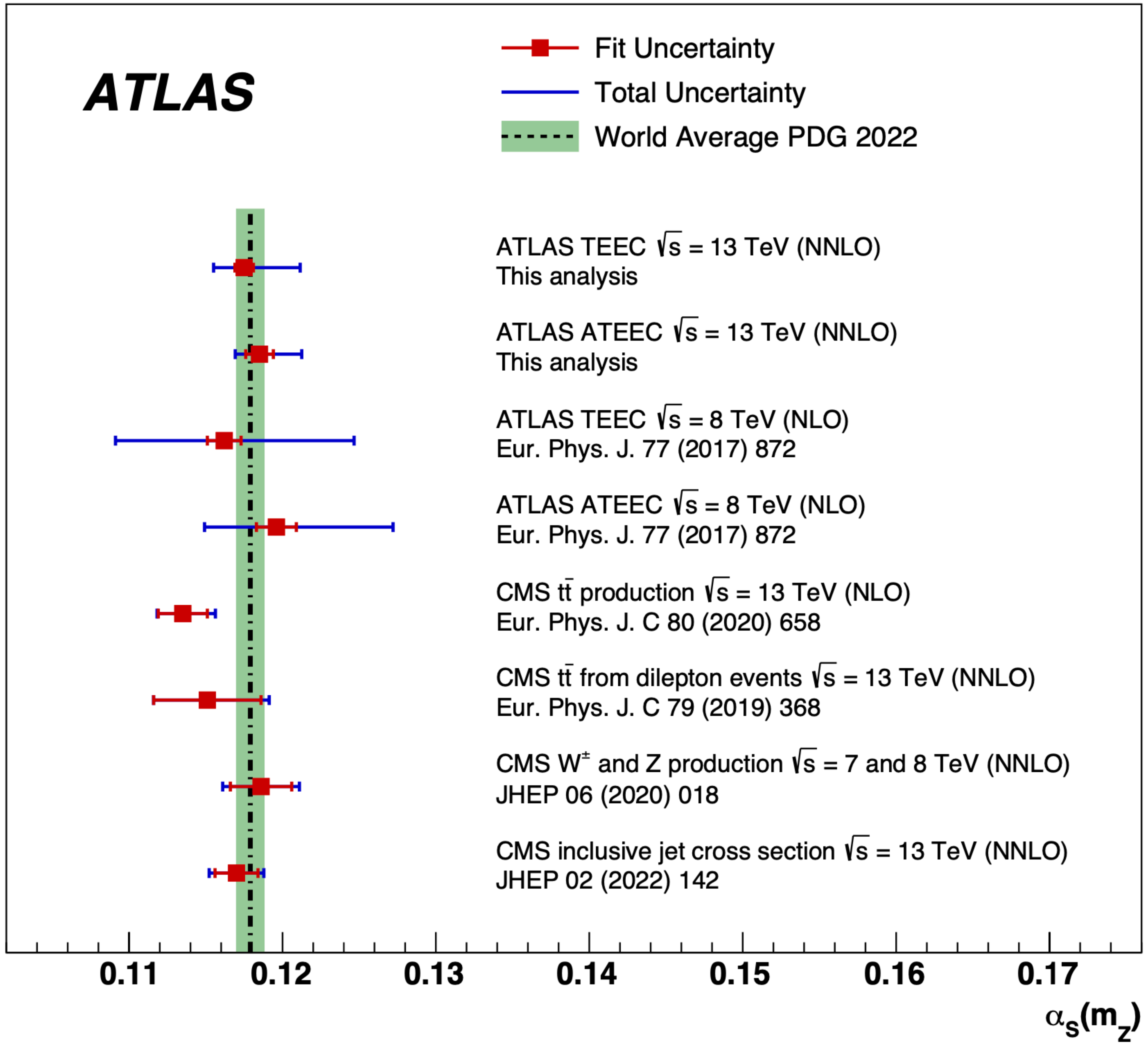
- Good overall description, for ATEEC in all H_{T2} bins



arXiv:2301.09351

Transverse Energy-Energy Correlations in ATLAS

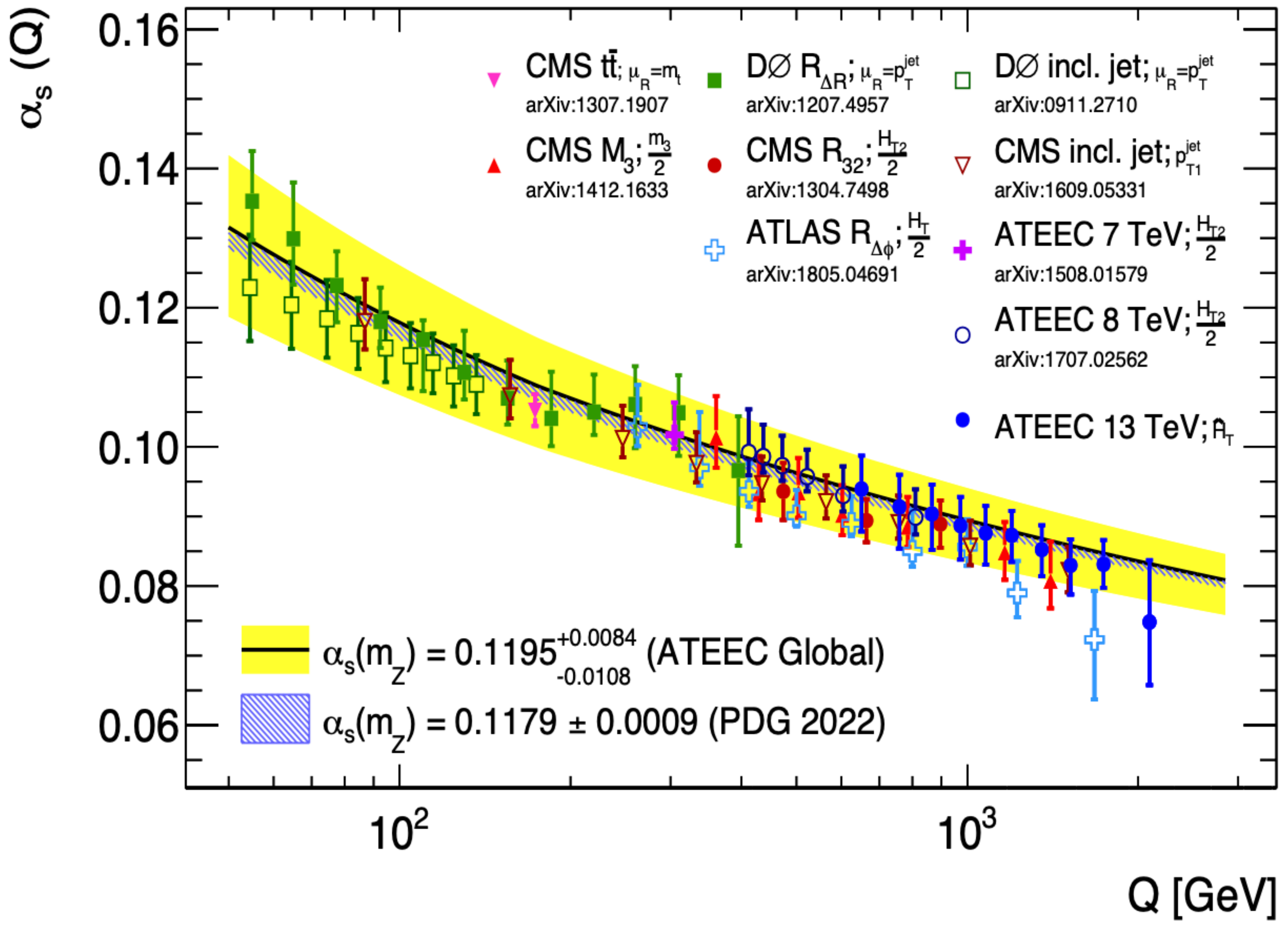
- TEEC and ATEEC used to extract $\alpha_s(M_Z)$
- Value of $\alpha_s(M_Z)$ evolved to $\alpha_s(Q)$ for each H_{T2} bin using RGE
- Value $Q = \langle \hat{H}_T \rangle / 2$ chosen for comparison with other analyses
- $\langle \hat{H}_T \rangle$ obtained for each H_{T2} bin using NNLO predictions
- Improves theory uncertainties by factor of 3 w.r.t. NLO
- Good agreement with world average and previous measurements
- Renormalisation Group Equation probed at highest scales to date
- Provides highest precision points beyond TeV scale to date



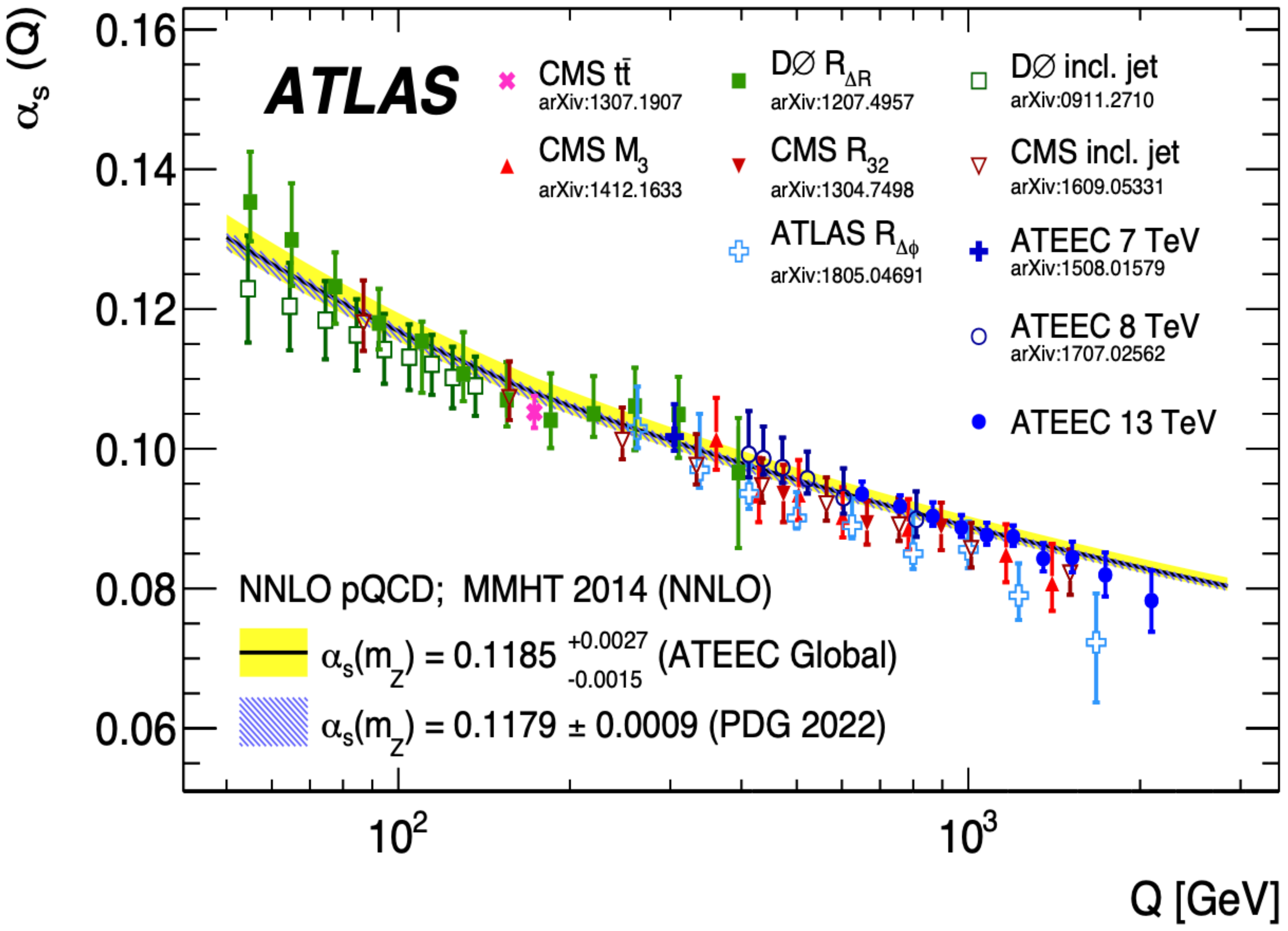
arXiv:2301.09351

Transverse Energy-Energy Correlations in ATLAS

[ATLAS-CONF-2020-025] (NLO)



[arXiv:2301.09351] (NNLO)



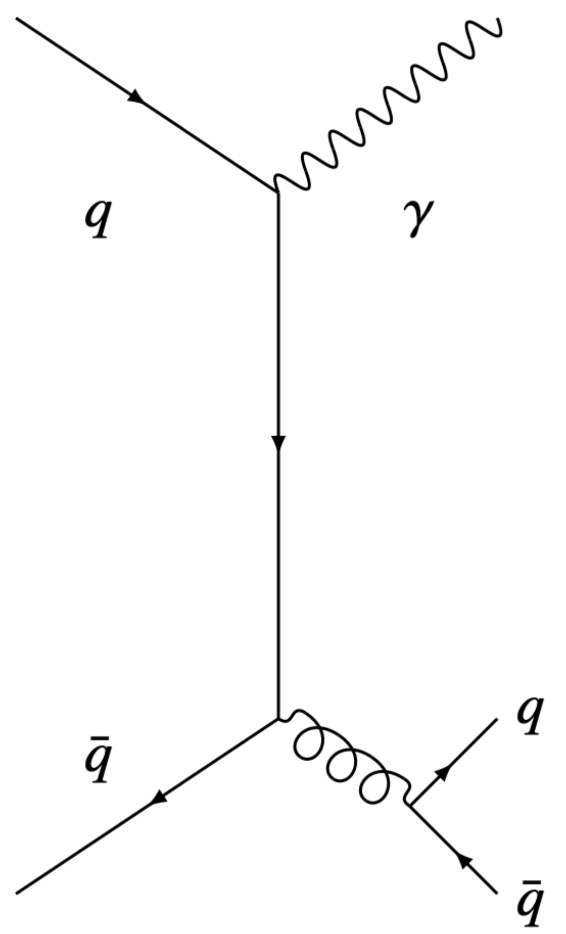
Photon+di-jet production

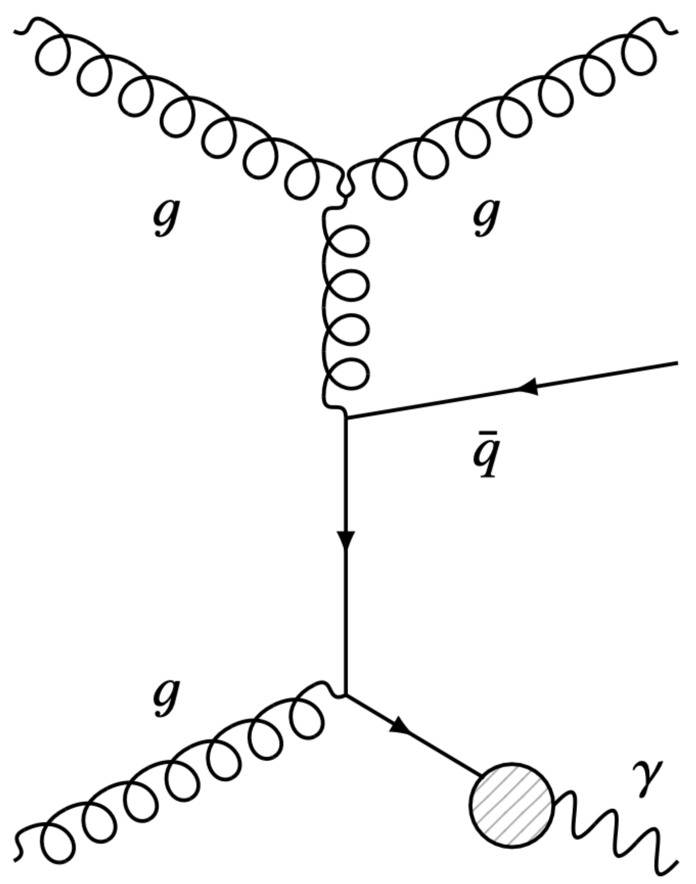
Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

→ Simone Zoia's talk on Tuesday discussed technical details of the evaluation of the virtual amplitudes

- Highlight: first complete 2 → 3 process with full-color virtual amplitudes
- Analysis matching ATLAS measurements from JHEP 03 (2020) 179
- Photon production through direct emission and fragmentation

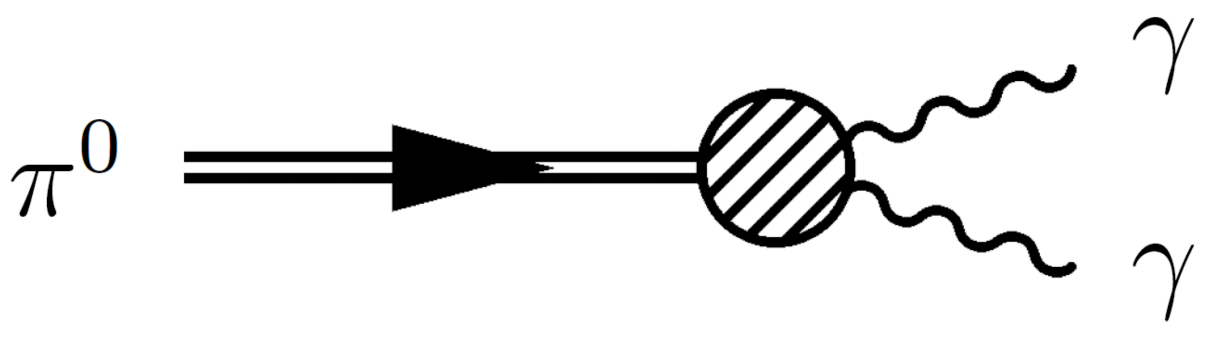




Requirements on photon	$E_T^\gamma > 150\,\text{GeV}$, $\lvert\eta^\gamma\rvert < 2.37$ (excluding $1.37 < \lvert\eta^\gamma\rvert < 1.56$) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level)		
Requirements on jets	at least two jets using anti-k_t algorithm with $R = 0.4$ $p_T^{\text{jet}} > 100\,\text{GeV}$, $\lvert y^{\text{jet}}\rvert < 2.5$, $\Delta R^{\gamma\text{-jet}} > 0.8$		
Phase space	**total**	**fragmentation enriched** $E_T^\gamma < p_T^{\text{jet2}}$	**direct enriched** $E_T^\gamma > p_T^{\text{jet1}}$
Number of events	755 270	111 666	386 846

- Main source however: hadron decays

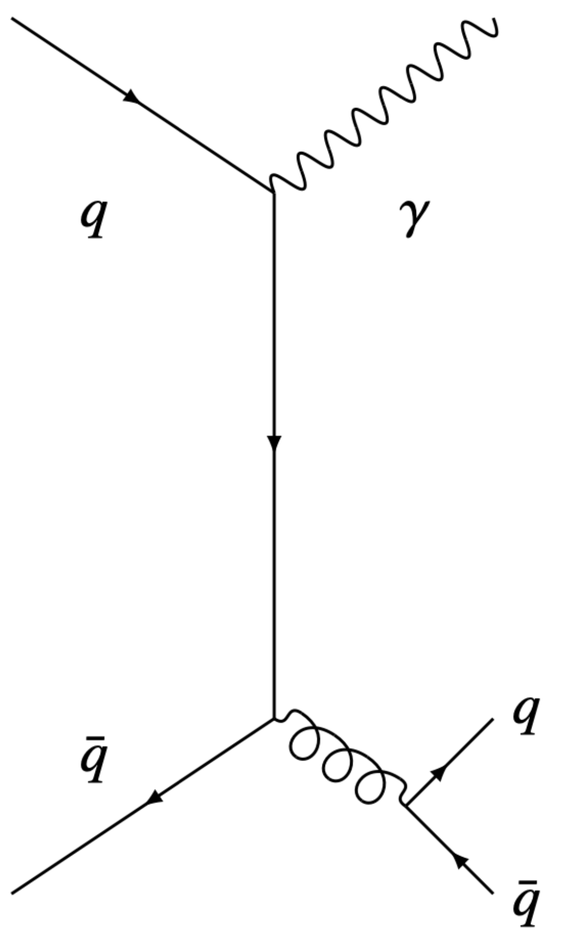
  ⇒ remove by judicious cuts

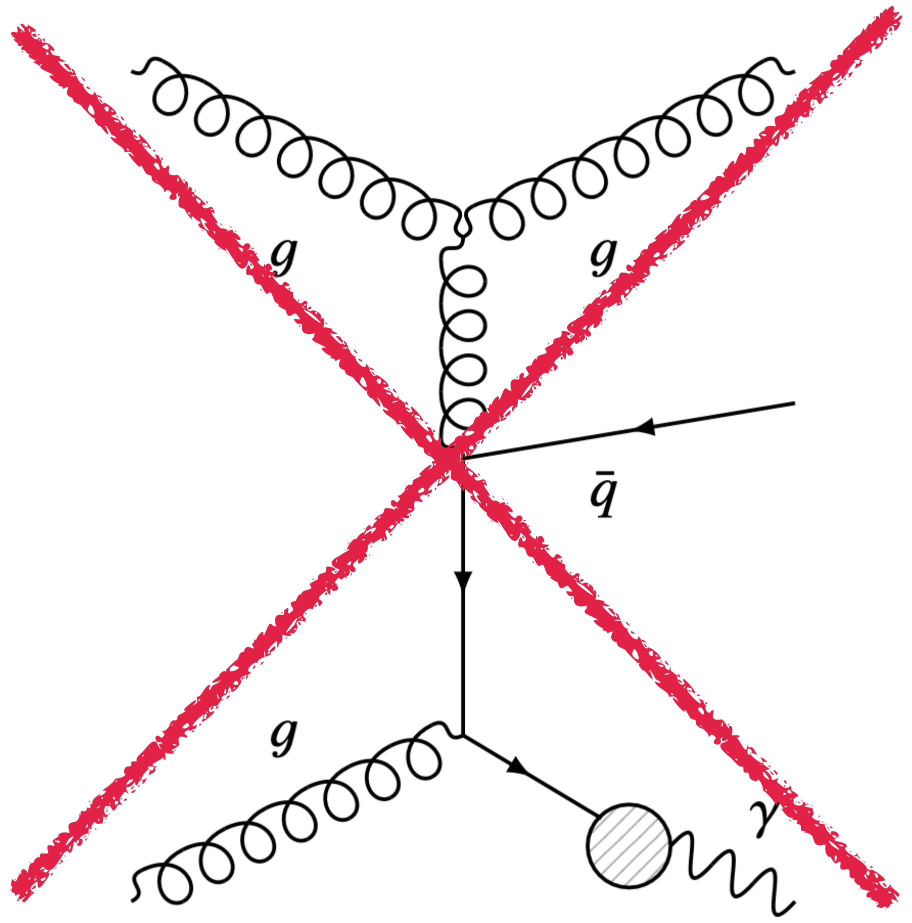


Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

- Fragmentation missing in our computation





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- Infrared divergences kept under control with Frixione's isolation prescription ⇒ hybrid photon isolation

$$E_\perp(r) \leq E_{\perp\max}(r) = 0.1\, E_\perp(\gamma) \left(\frac{1-\cos(r)}{1-\cos(R_{\max})} \right)^2 \quad \text{for} \quad r \leq R_{\max} = 0.1$$

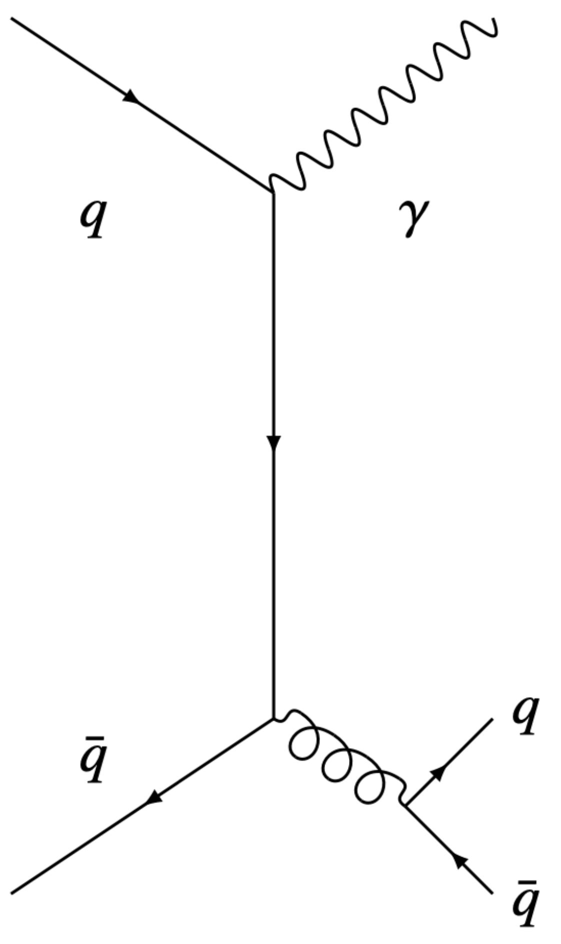
together with

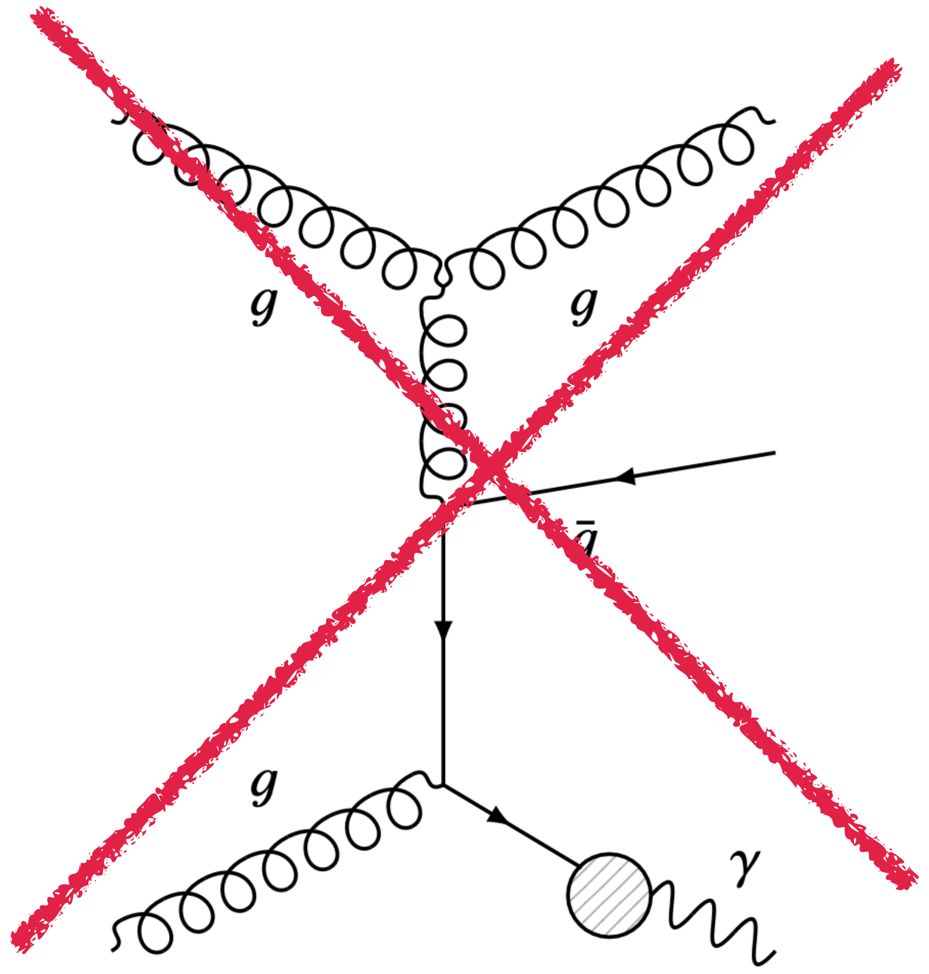
$$E_\perp(r) \leq E_{\perp\max} = 0.0042\, E_\perp(\gamma) + 10\ \text{GeV} \quad \text{for} \quad r \leq R_{\max} = 0.4$$

Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

- Fragmentation missing in our computation





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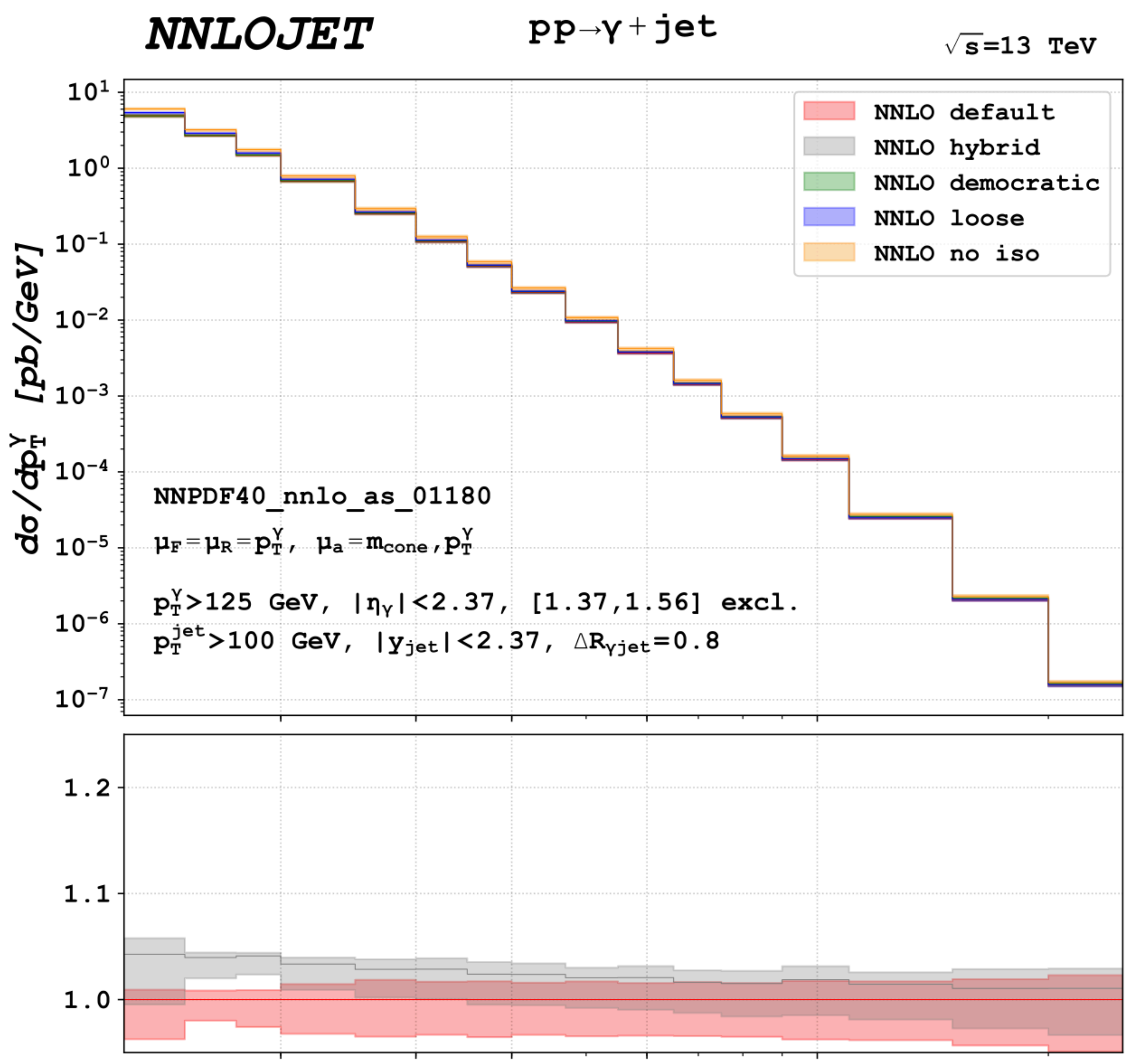


- Difference between calculation with fragmentation and hybrid isolation estimated at < 5% based on

Chen, Gehrmann, Glover, Höfer, Huss, Schürmann,
"Single photon production at hadron colliders at NNLO QCD with realistic photon isolation"
JHEP 08 (2022) 094

Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

- Scale choice:

$$\mu_R = \mu_F = H_T = E_\perp(\gamma) + p_T(j_1) + p_T(j_2) \quad \text{and}$$
$$\mu_R = \mu_F = E_\perp(\gamma)\ ,$$

- Very interesting comparison: ATLAS used SHERPA predictions for reference
  - NLO-matched QCD parton-shower merged with LO photon+four-jet samples
  - In principle this corresponds to the double-real radiation contributions in the NNLO QCD predictions
  - $E_\perp(\gamma)$ used for renormalisation and factorisation scale

- Why is this process interesting otherwise?
  - Non-back-to-back Born configurations

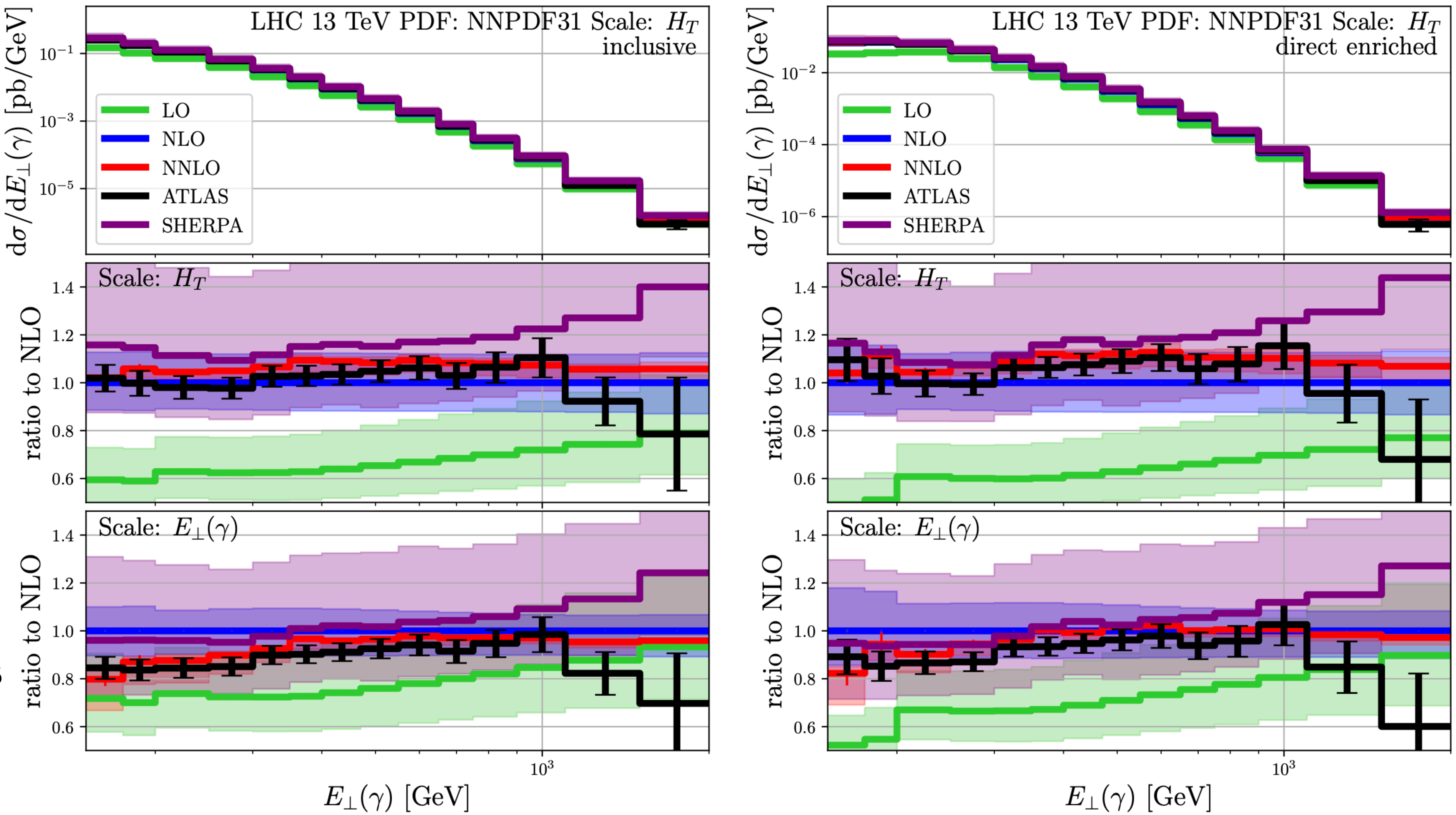
    $\rightarrow$ access to angular correlations between the photon and jets
  - Access to different kinematic regimes through distinguishable photon

    $\rightarrow$ enhance direct, high- or low-z fragmentation
  - Background process for BSM: $pp \rightarrow \gamma + Y(\rightarrow jj)$

Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

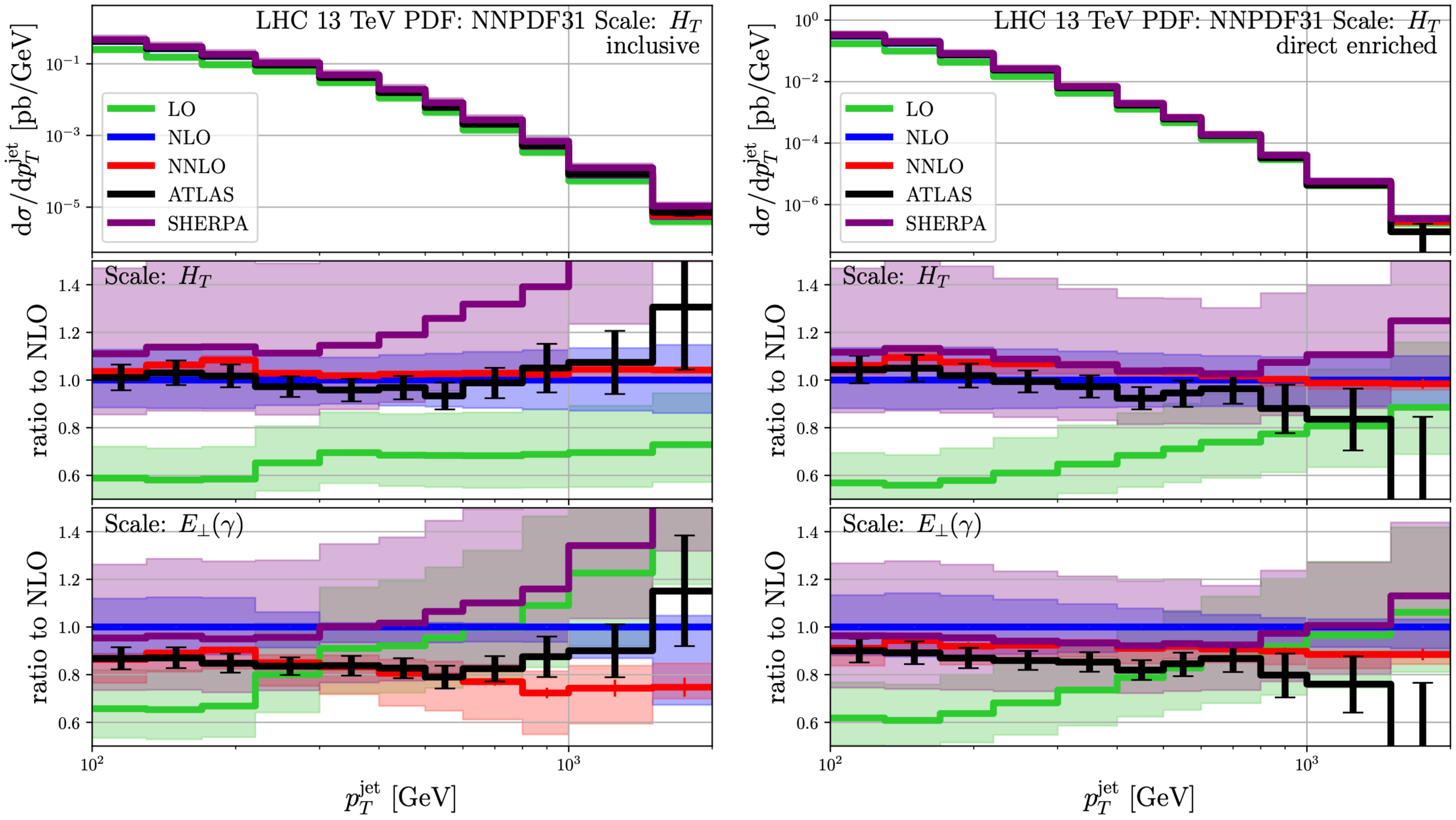
- Transverse photon energy
  1. NNLO corrections from 1% to 10%
  2. Improved description of data up to 1 TeV
  3. Larger scale uncertainties with the $E_\perp(\gamma)$ scale in the inclusive phase space
  4. Large experimental uncertainties beyond 1 TeV
  5. Default SHERPA predictions with merging are a poor description with large uncertainties



Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

- Transverse jet momentum
  1. Similar features to the photon transverse energy
  2. Particularly large scale uncertainties with the $E_\perp(\gamma)$ scale in the inclusive phase space
  3. SHERPA barely in agreement with data in the inclusive phase space



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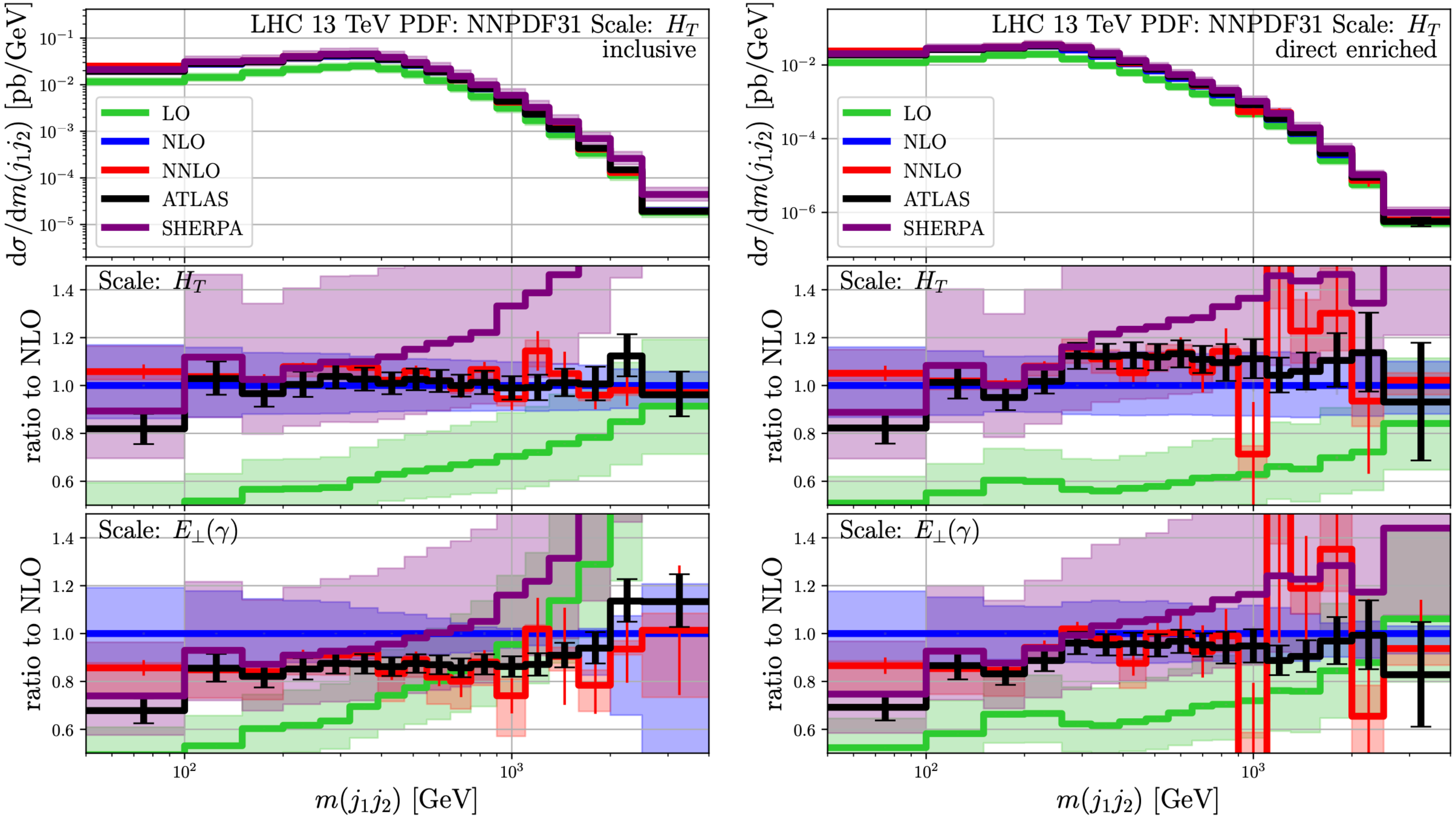
Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

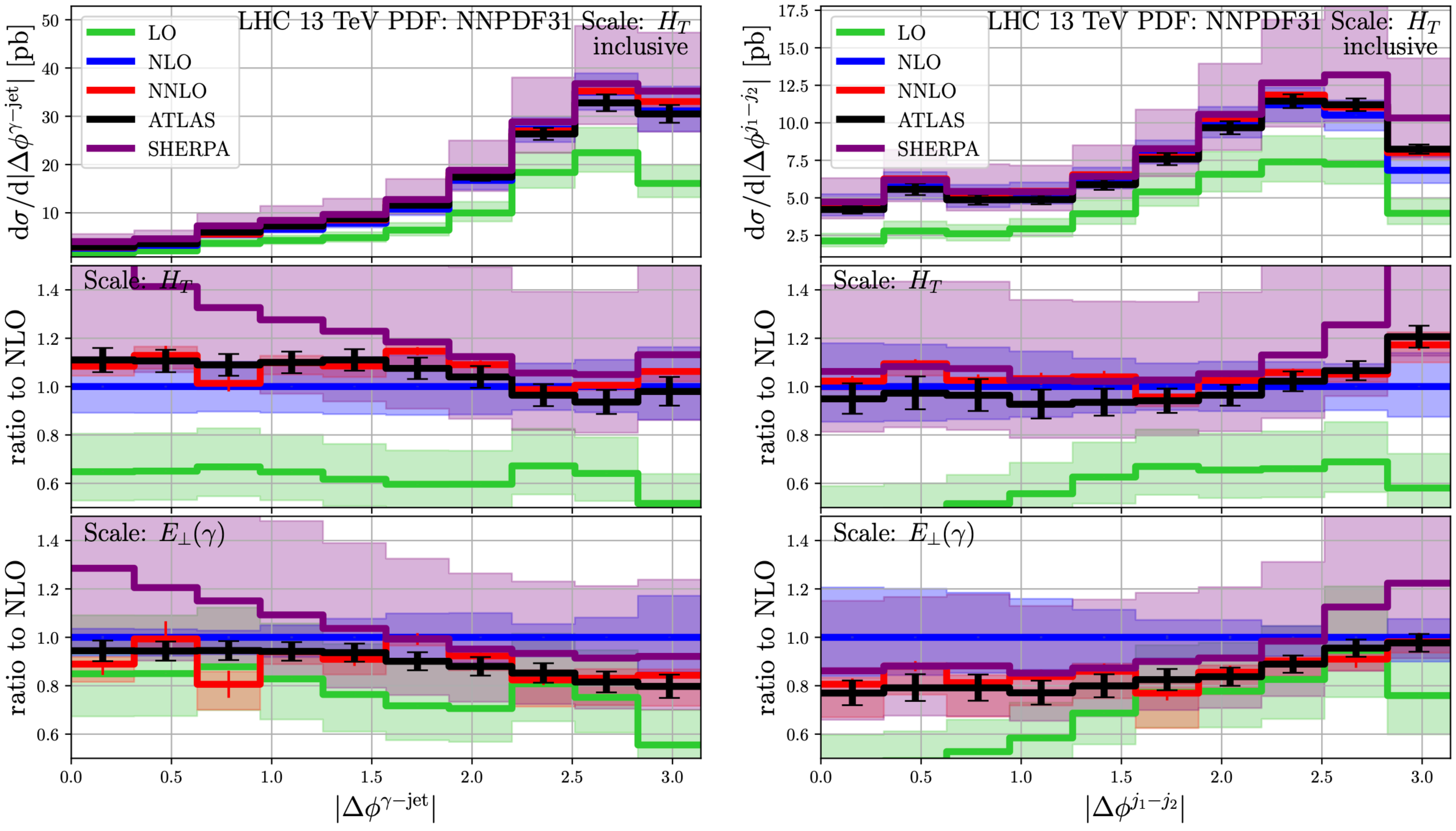
- di-jet invariant mass
  1. Difficulties in the first bin: resummation?
  2. Particularly poor description with the $E_{\perp}(\gamma)$ scale in the inclusive phase space
  3. Still large MC errors beyond 1 TeV in the direct enriched phase space
  4. Very poor description with SHERPA



Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

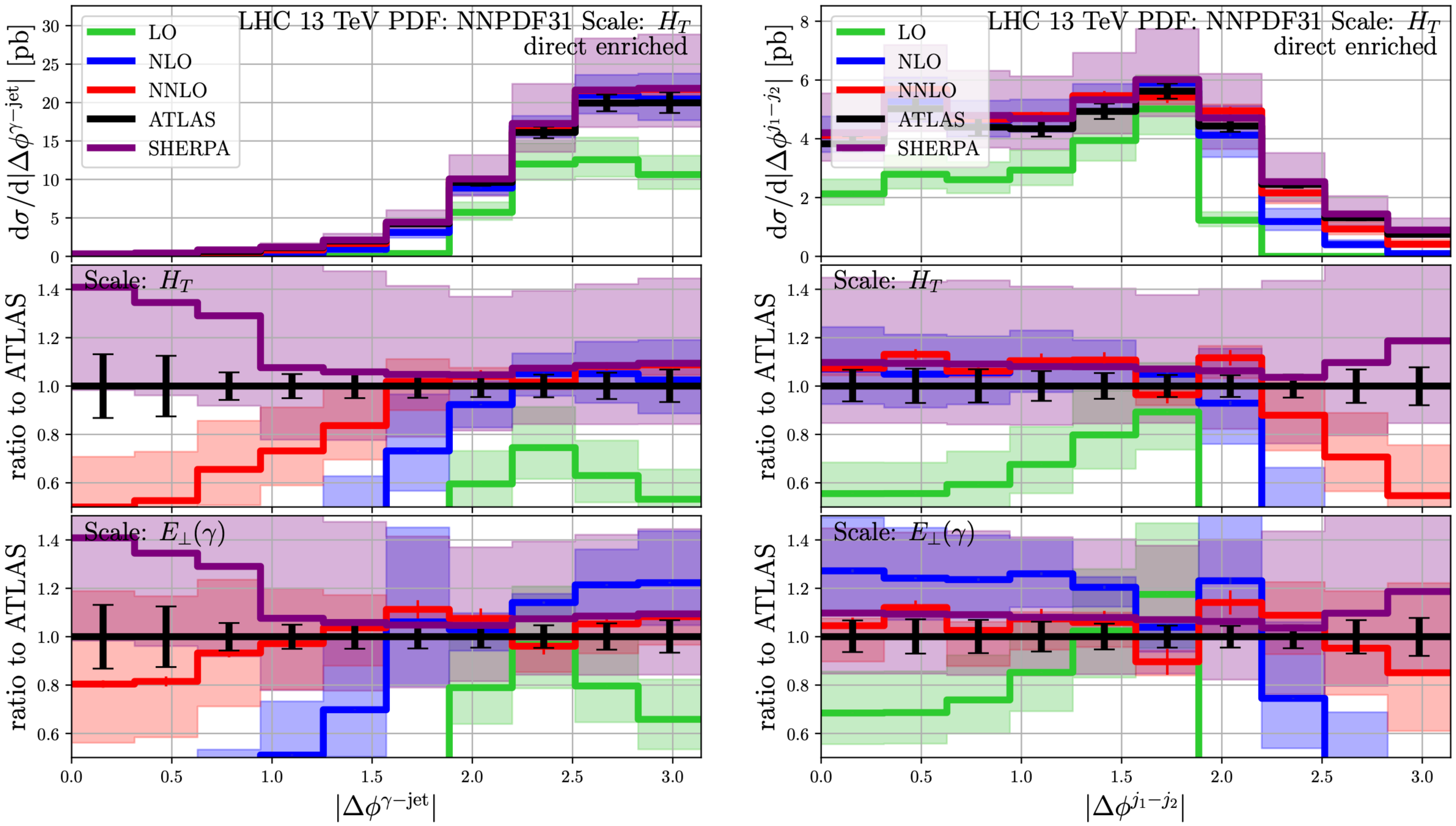
- azimuthal separations
    1. not much different in the inclusive phase space



Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, arXiv:2304.06682

- azimuthal separations
  1. Very poor convergence in the direct enriched phase space
  2. Effect due to kinematical edges induced by the cuts



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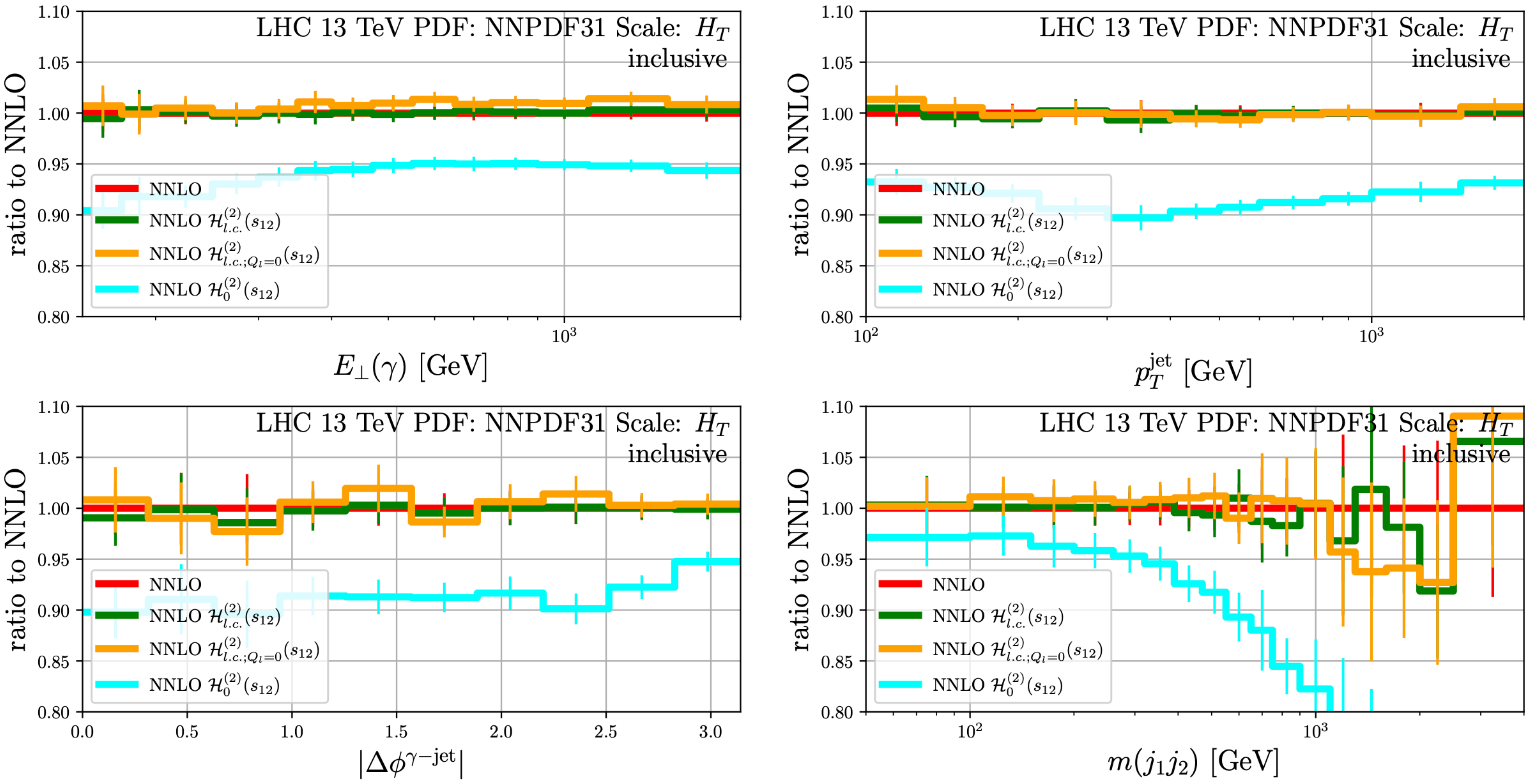
- Negligible size of the subleading color corrections



Conclusions

Conclusions

- All photon and jet processes in the $2 \to 3$ class completed with STRIPPER
- Although available, full-color amplitudes not yet used for $\gamma\gamma j$ and $\gamma\gamma\gamma$
- Full-color amplitudes for three jet production would be great to have...

- Do you have two-loop amplitudes for a pheno-relevant process ???

  $\rightarrow$ Contact us !!! We'll make a complete analysis using our general framework...