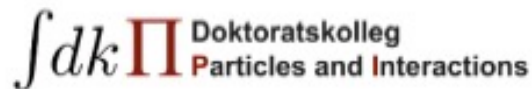

Top Quark Mass Calibration for MC Event Generators - An Update -

In collaboration with Bahman Dehnadi, Oliver Jin,
Vicent Mateu and Simon Plätzer
work in progress

André H. Hoang

University of Vienna



Main Top Mass Measurements Methods

LHC+Tevatron: Direct Reconstruction

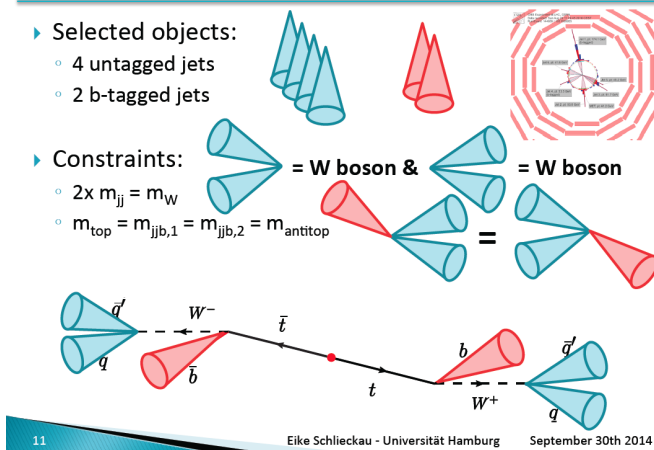
Kinematic Fit

Selected objects:

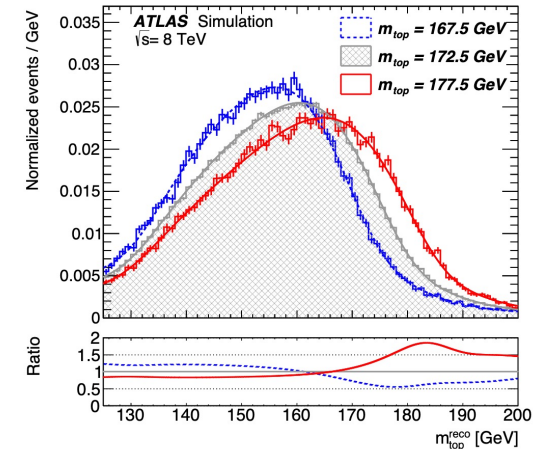
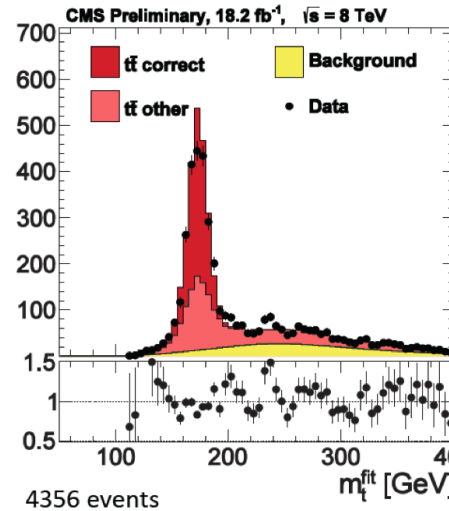
- 4 untagged jets
- 2 b-tagged jets

Constraints:

- $2 \times m_{jj} = m_W$
- $m_{top} = m_{jjb,1} = m_{jjb,2} = m_{antitop}$



CMS-PAS-TOP-14-002



$$m_t^{\text{MC}} = 171.77 \pm 0.37 \text{ GeV}$$

CMS collaboration. arXiv: 2302.01967

⊕ High top mass sensitivity

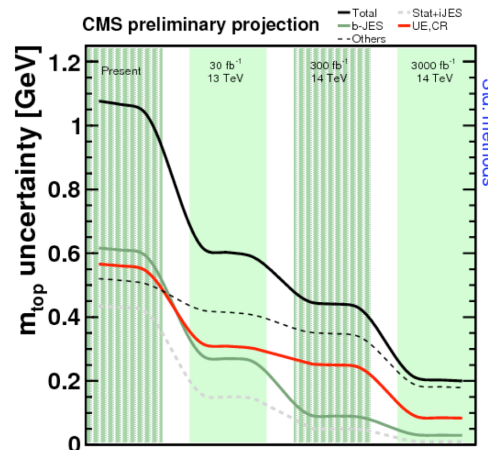
⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

← $\Delta m_t \sim 200 \text{ MeV}$ (projection)

kinematic mass determination

Determination of the best-fit value of the Monte-Carlo top quark mass parameter



Status of m_t^{MC} Interpretation

Issue is not yet resolved, but a number of important insights have been gained.

First principles insights:

- “Scheme” of m_t^{MC} in principle determined by the precision of parton shower (PS)
 - For perfect NLL PS, we could control m_t^{MC} at NLO
 - Problem: PS have different precision depending on the observable (tested for inclusive observables)
 - Shower cut Q_0 represents IR resolution parameter: $m_t^{\text{MC}} = m_t^{\text{MC}}(Q_0)$, linear dependence
 - Realistic observables have many other sources of linear IR cutoff dependence
 - Q_0 dependence of m_t^{MC} is not related to any nonperturbative effect
- Hadronization model influences m_t^{MC} indirectly as it corrects for PS imperfections
 - LHC Initial state effects (MPI, UE) not at all understood systematically from QCD: big problem for LHC direct top mass measurement interpretation, no clear path how to address this without models
 - Consistency with systematic QCD methods (e.g. OPE) largely untested
- Coherent branching + massive eventshapes (e^+e^-):
$$m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3} \alpha_s(Q_0) Q_0 + \dots$$

AHH, Plätzer, Samitz 2018
- m_t^{pole} has $O(\Lambda_{\text{QCD}})$ renormalon $\rightarrow$ more precision with MSR mass $m_t^{\text{MSR}}(R)$
 - m_t^{pole} ambiguity: 110 MeV – 250 MeV, but unrelated to m_t^{MC} interpretation
 - $m_t^{\text{CB}}(Q_0) = m_t^{\text{MSR}}(Q_0) - 0.24 \alpha_s(Q_0) Q_0$

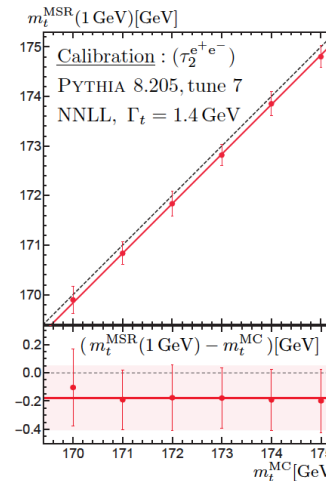
Beneke etal 2016; AHH etal 2017

Status of m_t^{MC} Interpretation

Numerical insights:

- Combined LHC direct + total cross section analysis: $|m_t^{\text{MC}} - m_t^{\text{pole}}| < 2 \text{ GeV}$
Kieseler etal 1511.00841
- Top mass calibration: N²LL+NLO e⁺e⁻ 2-jettiness fitted to Pythia 8.205 pseudo data
 - Strongly mass-sensitive hadron level (as closely as possible related to direct measurement observables) Butenschön etal 1608.01318
 - Accurate hadron level QCD predictions at $\geq \text{NLL/NLO}$ with full control over the quark mass scheme dependence.
 - QCD masses as function of m_t^{MC} from fits of theory to MC samples.
 - Cross check observable independence

	order	central	perturb.	incomp.
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	N ² LL	172.82	0.19	0.11
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	NLL	172.80	0.26	0.14
m_t^{pole}	N ² LL	172.43	0.18	0.22
m_t^{pole}	NLL	172.10	0.34	0.16
Ω_1	N ² LL	0.42	0.07	0.03
Ω_1	NLL	0.41	0.07	0.02



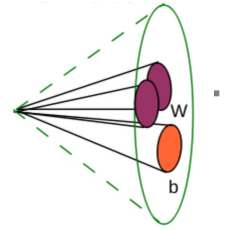
NLL groomed jet mass:

→ ATL-PHYS-PUB-2021-034

Outline

- Introduction: direct top mass determinations and status of the interpretation of m_t^{MC}
 - Top mass calibration
 - Calibration of the Pythia Monte Carlo top mass parameter:
 $e^+e^- \rightarrow t\bar{t}$ (2-jettiness)
Butenschön, Dehnadi, Mateu, Preisser, Stewart, AH 1608.01318
 - Updates:
 - ▶ more observables,
 - ▶ more soft function renormalon subtractions
 - ▶ $(m_t/Q)^2$ power corrections
 - Summary, future plans
- All results shown are preliminary !

Shape Observables

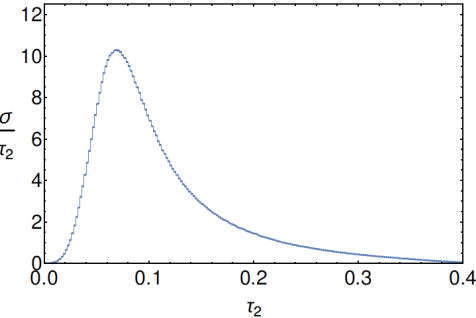


Shape observables: e^+e^- for $Q = 2p_T \gg m_t$ (boosted tops)

$$\tau_2 = \frac{1}{Q} \min_{\vec{n}_t} \sum_i (E_i - |\vec{n}_t \cdot \vec{p}_i|) \quad (2\text{-jettiness})$$

$$\tau_s = \rho_a + \rho_b, \quad \rho_{a,b} = \frac{1}{Q^2} \left(\sum_{i \in a,b} p_i \right) \quad (\text{sum of jet masses, sJM})$$

$$\tau_m = \tau_s + \frac{1}{2} \tau_s^2 \quad (\text{modified jet masses, mJM})$$



Excellent mass sensitivity:

$$\tau_{2,\min} = 1 - \sqrt{1 - 4\hat{m}^2} \quad (\text{tree level})$$

$$\tau_{s,\min} = 2\hat{m}^2$$

$$\tau_{m,\min} = 2\hat{m}^2 + 2\hat{m}^4$$

$$\hat{m}^2 \equiv \frac{m_t^2}{Q^2}$$

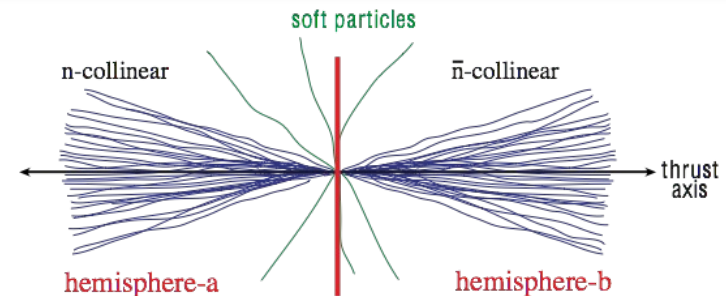
All observables agree at leading power!
Differ by $\hat{m}^2$ power corrections

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q, \mu) \int d\ell J_0(Q\ell, \mu) S_0(Q\tau - \ell, \mu)$$

Scale hierarchy and sequence of EFTs:

$$Q \gg m_t \gg \Gamma_t \gg \Lambda_{\text{QCD}}$$

$$\text{QCD} \rightarrow \text{SCET} \rightarrow \text{bHQET}$$

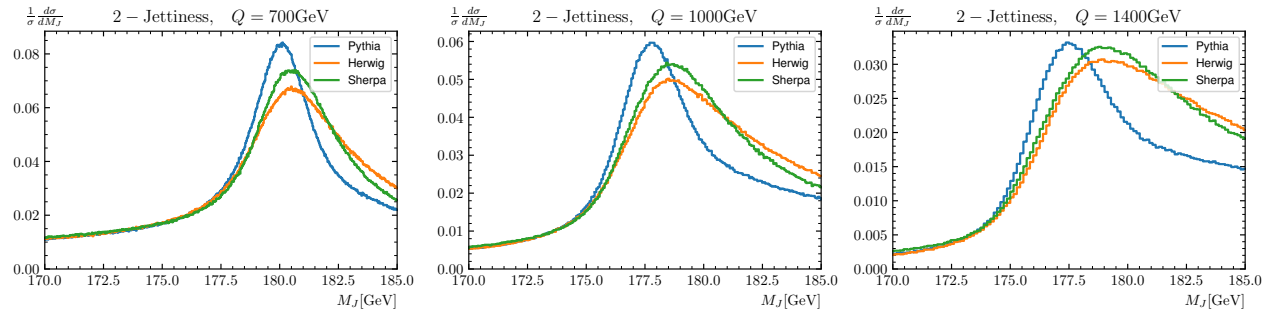


Shape Observables

$$m_t^{\text{MC}} = 173 \text{ GeV}$$

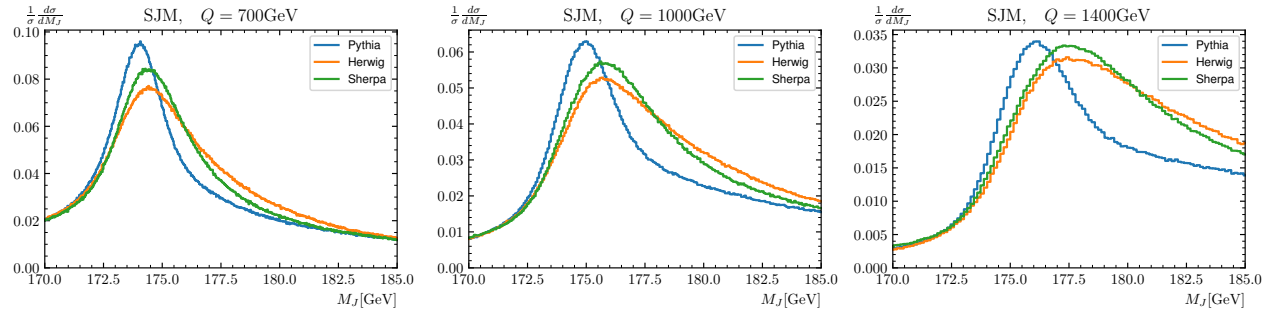
2-jettiness

τ_2



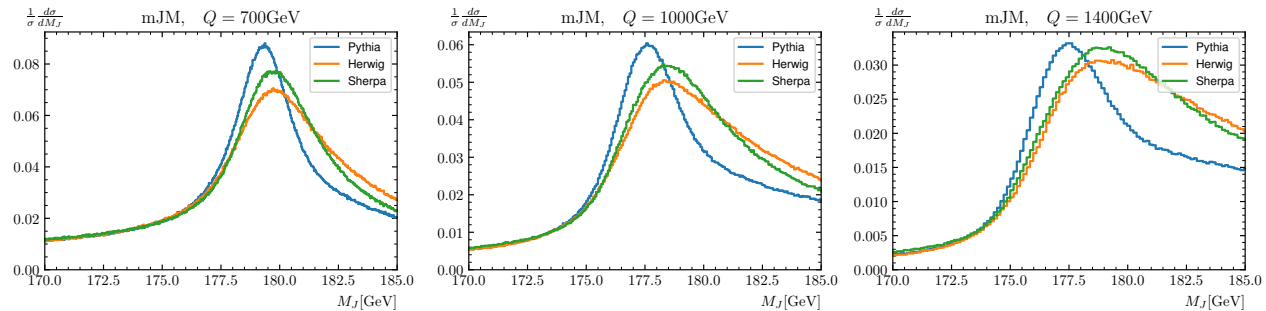
sJM

τ_s



mJM

τ_m



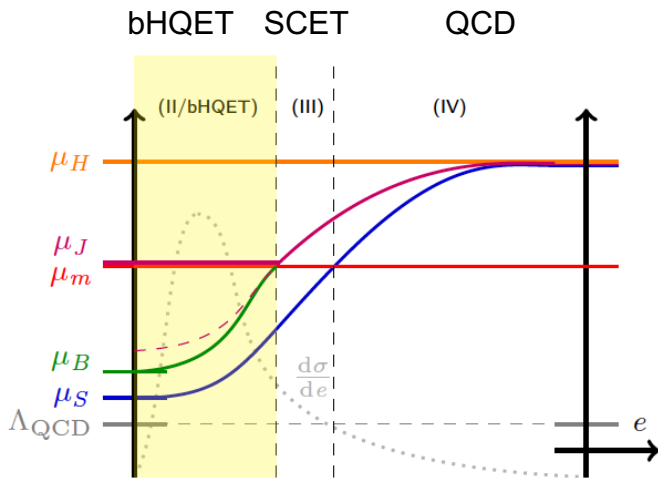
$$M_j \equiv \frac{Q^2}{2m_t} \tau$$

- Substantial differences between the shape observables
- Substantial differences between Monte Carlo event generators
- Substantial $\hat{m}^2 \sim \frac{1}{12}$ power corrections

N²LL + NLO matched distributions

$$\frac{d\sigma_{\text{full}}(\tau)}{d\tau} = \frac{d\sigma_{\text{bHQET}}(\tau)}{d\tau} + \frac{d\sigma_{\text{SCET}_{\text{ns}}}(\tau)}{d\tau} + \frac{d\sigma_{\text{QCD}_{\text{ns}}}(\tau)}{d\tau}$$

Fleming, AH, Mantry, Stewart 2007
Dehnadi PhD-Thesis (2018)



$$\left. \frac{1}{\sigma_0^C} \frac{d\sigma_{\text{bHQET}}^C}{d\tau} \right|_{\text{strict}}$$

$$= m_t Q^2 H_Q^{(6)}(Q, \mu_H) U_{H_Q}^{(6)}(Q, \mu_H, \mu_m) H_m^{(6)}(m_t, \varrho, \mu_m) U_v^{(5)}(\varrho, \mu_m, \mu) \\ \times \int d\ell d\hat{s} \delta[\hat{s}_\tau - \hat{s} - \varrho\ell] \\ \times \int d\hat{s}' U_B^{(5)}(\hat{s} - \hat{s}', \mu, \mu_B) J_{B,\tau}^{(5)}(\hat{s}', \Gamma_t, \delta m, \mu_B) \\ \times \int d\ell' dk U_S^{(5)}(\ell - \ell', \mu, \mu_S) \hat{S}_\tau^{(5)}(\ell' - k, \bar{\delta}, \mu_S) F(k - 2\bar{\Delta})$$

measurement function

ultra-collinear
bHQET jet function

large-angle soft
function

non-perturbative
soft function

QCD singular coefficients cannot
be determined from EFT due to
 $\hat{m}^2$ power corrections

Plus distribution
coefficient B_{plus}
universal!

$$\frac{1}{\sigma_0^C} \frac{d\sigma_{\text{QCD}}^C}{d\tau} = R_0^C(\hat{m}) \left\{ \delta(\tau - \tau_{\min}) + \frac{C_F \alpha_s}{4\pi} A_\tau^C(\hat{m}) \delta(\tau - \tau_{\min}) \right. \\ \left. + \frac{C_F \alpha_s}{4\pi} B_{\text{plus}}(\hat{m}) \left[\frac{1}{\tau - \tau_{\min}} \right]_+ \right\} + \frac{C_F \alpha_s}{4\pi} F_\tau^{\text{NS,C}}(\tau, \hat{m}) + \mathcal{O}(\alpha_s^2)$$

$$\hat{s}_\tau \equiv \frac{Q^2(\tau - \tau_{\min})}{m_t}$$

exact tree-level
expression!

N²LL + NLO matched distributions

MSR mass

Jain, Mateu, Preisser, AHH 2017

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(R) = R \frac{C_F \alpha_s^{(5)}(R)}{\pi} + \dots$$

- Renormalon subtraction through change of mass scheme
- We use $m_t^{\text{MSR}}(1 \text{ GeV})$ as the reference input mass
- R set equal to the μ_B (jet function renormalization scale)
- 2-loop RGE (R-Evolver library)
- Can be related to any other short-distance mass with $O(10 \text{ MeV})$ precision

Lepenik, Mateu, AHH 2022

Shape function

Ligeti, Stewart, Tackmann 2008

$$F(k; \lambda, \{c_i\}, N) \equiv \frac{1}{\lambda} \left[\sum_{n=0}^N c_n f_n \left(\frac{k}{\lambda} \right) \right]^2,$$

$$f_n(z) = 8 \sqrt{\frac{2z^3(2n+1)}{3}} e^{-2z} P_n(g(z)),$$

$$g(z) = \frac{2}{3} [3 - e^{-4z} (3 + 12z + 24z^2 + 32z^3)] - 1$$

- We need $N=3$ to have sufficient flexibility to fit the shape function
- The first moment Ω_1 is the most relevant nonperturbative parameter
 $\rightarrow$ Peak position modified by nonperturbative corrections $\delta M_J = \frac{Q}{m_t} \Omega_1 \sim 2 \text{ GeV}$ for $\Omega_1 \sim 0.5 \text{ GeV}$
- Need data from several different Q values to lift m_t - Q degeneracy

$$\Omega_1(\lambda, \Delta, N) = \frac{1}{2} \int_0^\infty k F(k - 2\Delta; \lambda, \{c_i\}, N) dk$$

Gap parameter

$$\begin{aligned} \Omega_1(\lambda, \Delta, 3) = & \Delta + \lambda(0.5c_0^2 + 0.47360764c_0c_1 + 0.10067713c_0c_2 + 0.094954074c_0c_3 \\ & + 0.54502418c_1^2 + 0.50700667c_1c_2 + 0.12507929c_1c_3 \\ & + 0.55015667c_2^2 + 0.50982331c_2c_3 + 0.55170015c_3^2), \end{aligned}$$

Fit parameters: c_0, c_1, c_2

N²LL + NLO matched distributions

Gap subtraction

Stewart, AHH 2007

$$S(\ell, \mu_S) = \int dk \hat{S}_\tau^{(5)}(\ell - k, \bar{\delta} = 0, \mu_S) F(k - 2\Delta)$$

$$= \int dk \hat{S}_\tau^{(5)}(\ell - 2\bar{\delta}(R_s, \mu_S) - k, 0, \mu_S) F(k - 2\bar{\Delta}(R_s, \mu_S))$$

$$\Delta = \bar{\Delta}(R_s, \mu_S) + \bar{\delta}(R_s, \mu_S)$$

Bachu, Mateu, Pathak, Stewart, AHH 2021

- Renormalon subtraction through redefinition of the gap parameter

$$\bar{\delta}(R_s, \mu_S; A, n, \xi) \equiv \begin{cases} \frac{R_s}{2\xi} \frac{d^n}{d \ln(iy)^n} \log \left[\tilde{S}_\tau(y, \mu_S) \right]_{iy=\frac{\xi}{R_s}} & \text{if } A=\text{on} \\ \frac{R_s}{2\xi} \frac{d^n}{d \ln(iy)^n} \log \left[\tilde{S}_\tau(y, R_s) \right]_{iy=\frac{\xi}{R_s}} & \text{if } A=\text{off} \end{cases}$$

$$= R_s \sum_{i=1} d_i(R_s, \mu_S) \left[\frac{\alpha_s^{(5)}(\mu_S)}{4\pi} \right]^i$$

- if A=on $\leftarrow$ with soft function anomalous dimension
- if A=off $\leftarrow$ w/o soft function anomalous dimension

- 3 gap schemes with completely different character
- We use 2-loop RGEs and $\bar{\Omega}_1(2 \text{ GeV}, 2 \text{ GeV})$ as reference input

$$\bar{\delta}^{(1)}(R_s, \mu_S) \equiv \bar{\delta}(R_s, \mu_S; \text{on}, 1, e^{-\gamma_E})$$

$$d_1^{(1)}(R_s, \mu_S) = -19.0 \ln \left(\frac{\mu_S}{R_s} \right)$$

$$\bar{\delta}^{(2)}(R_s, \mu_S) \equiv \bar{\delta}(R_s, \mu_S; \text{off}, 0, e^{5\gamma_E})$$

$$d_1^{(2)}(R_s, \mu_S) = -3.9$$

$$\bar{\delta}^{(3)}(R_s, \mu_S) \equiv \bar{\delta}(R_s, \mu_S; \text{off}, 0, 1)$$

$$d_1^{(3)}(R_s, \mu_S) = -8.4$$

Gap scheme of original 2016 calibration analysis and for strong coupling event shape fits

Fit Procedure Details

- Code $\frac{d\sigma}{d\tau} = f(m_t^{\text{MSR}}, \alpha_s(M_Z), \Gamma_t, c_0, c_1, c_2, \mu_H, \mu_m, \mu_B, \mu_S, R, R_s, \text{gap})$
 - fit parameters
 - renormalization scales
 - ▶ Grids in fit and renormalization scale profile function parameters (very fine bins)
- MC samples at Q= 600, 700, 800, ... , 1400 GeV
 - ▶ **update:** Pythia 8.205 → **Pythia 8.305** (Monash Tune 7), **Herwig 7.2.2**, **Sherpa 2.2.11**
 - ▶ masses: $m_t^{\text{MC}} = 170, 171, 172, 173, 175, 175 \text{ GeV}$
 - ▶ width $\Gamma_t = 1.4 \text{ GeV}, \Delta = 0.05 \text{ GeV}$
 - ▶ Statistics: 10^7 events for each parameter set
 - ▶ **update:** HepMC, Rivet, Yoda classes
- Fitting procedure
 - ▶ Take $\alpha_s(M_Z)$ as input from world average. (Sensitivity to strong coupling very weak.)
 - ▶ χ^2 analysis standard [only relative values for χ^2_{min} matter]
 - ▶ 500 sets of profile functions (τ -dependent renormalization scales)
 - ▶ different Q sets: 6 sets of energies between 600 – 1400 GeV
 - ▶ different fit ranges: 3 choices of fit ranges in peak region

}

perturbative uncertainty

18 fit setups:

compatibility uncertainty

We exactly reproduced the calibration results from 2016 for Pythia

Revisiting the 2016 Analysis

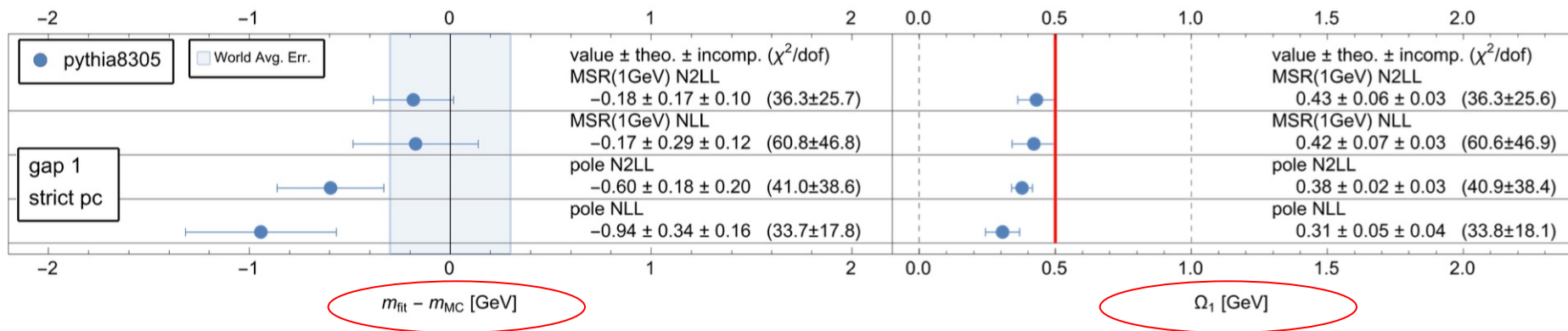
2016, Pythia 8.205, $m_{MC} = 173$ GeV

update, Pythia 8.305

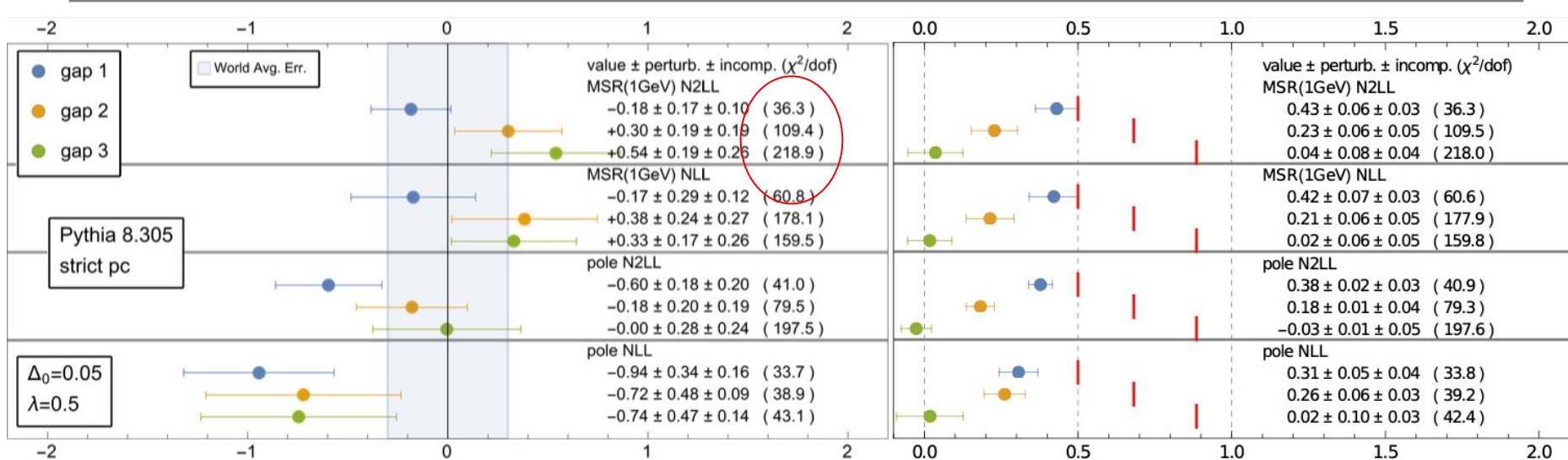
	order	central	perturb.	incomp.	central	perturb.	incomp.
$m_{t,1\text{GeV}}^{\text{MSR}}$	N ² LL	172.82	0.19	0.11	172.82	0.17	0.10
$m_{t,1\text{GeV}}^{\text{MSR}}$	NLL	172.80	0.26	0.14	172.83	0.29	0.12
m_t^{pole}	N ² LL	172.43	0.18	0.22	172.40	0.18	0.20
m_t^{pole}	NLL	172.10	0.34	0.16	172.06	0.34	0.16
Ω_1	N ² LL	0.42	0.07	0.03	0.43	0.06	0.03
Ω_1	NLL	0.41	0.07	0.02	0.42	0.07	0.03

(in GeV)

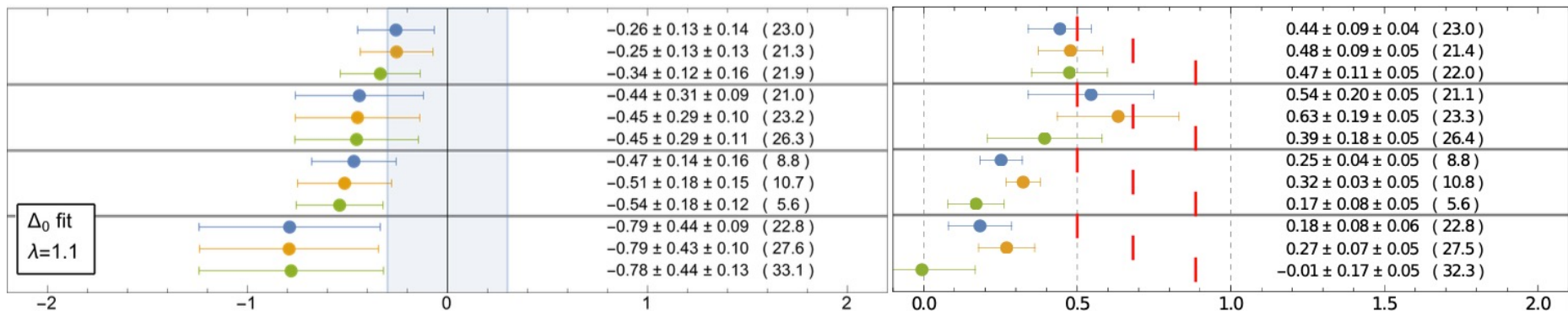
► Agreement within 30 MeV



Gap Scheme Independence

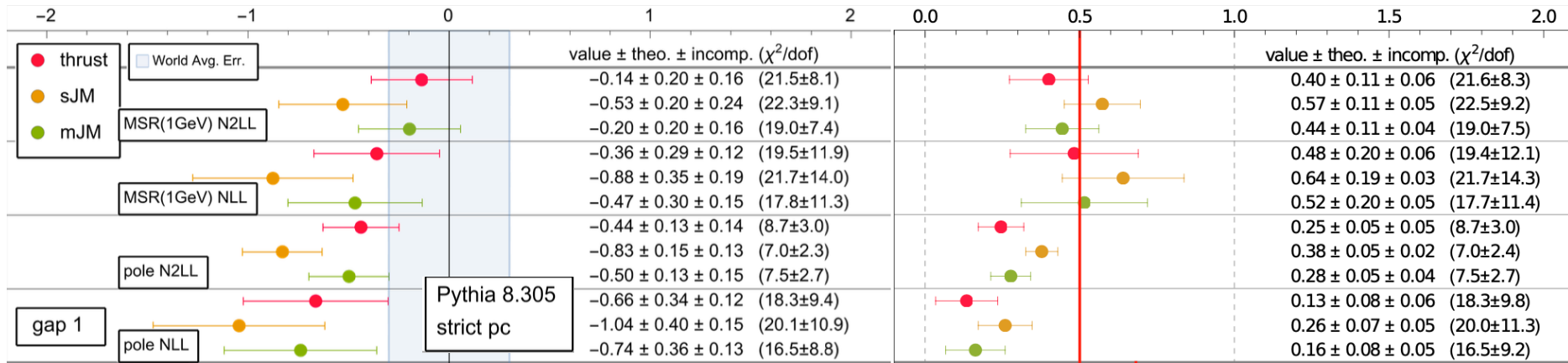


➡ Gap term Δ in shape function needs to be fitted as well



➡ Gap scheme independence if there is sufficient flexibility in shape function parametrization (gap scheme 2 for final results)

Observable Independence



→ Observable independence not achieved with strict power counting approach and the exact expression for $\tau_{\min}$ in the bHQET factorization formula

Soft $(m_t/Q)^2$ power corrections in the measurement function can lead to ~ 250 MeV effects in the top mass:

$$\delta[\hat{s}_\tau - \hat{s} - r_{\tau,s}(\hat{m})\varrho\ell] \quad r_{\tau,s}(\hat{m}) = 1 + \mathcal{O}(\hat{m}^2)$$

$$\rightarrow \delta M_J \sim 2 \frac{m_t}{Q} \Omega_1 \sim 250 \text{ MeV}$$

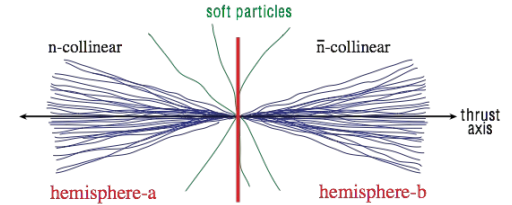
Observable Independence

Consider soft momenta in the two hemispheres addition to tree-level top quark momenta

(soft momentum k_s contains soft top recoil)

$$p_n = m_t v_+ + k_s$$

$$p_{\bar{n}} = m_t v_- + k_{\bar{s}}$$



2-jettiness: $\tau_2 = \tau_{\min} + \frac{k_s^+ + k_{\bar{s}}^-}{Q} \rightarrow r_{\tau_2, s}(\hat{m}) = 1$

sum of jet mass: $\tau_s = \frac{1}{Q^2} (p_n^+ p_n^- + p_{\bar{n}}^- p_{\bar{n}}^+)$ Modified jet mass: $\tau_m = \tau_s + \frac{1}{2} \tau_s^2$

Energy conservation for the two hemispheres: $\Delta E \sim k_s$ cancels between hemispheres

$$p_n^- + p_n^+ = Q + \Delta E$$

$$p_{\bar{n}}^- + p_{\bar{n}}^+ = Q - \Delta E$$

$$\tau_s = \tau_{2, \min} + \frac{\sqrt{1 - 4\hat{m}^2}}{Q} (k_s^+ + k_{\bar{s}}^-) + \mathcal{O}(k_{s, \bar{s}})^2$$

Cross check: soft power correction absent in modified jet mass!

$$\rightarrow r_{\tau_s, s}(\hat{m}) = \sqrt{1 - 4\hat{m}^2} = 1 - 2\hat{m}^2 + \mathcal{O}(\hat{m}^4)$$

$$r_{\tau_m, s}(\hat{m}) = 1 + \mathcal{O}(\hat{m}^4)$$

Observable Independence

→ Improved version of the bHQET factorization formula

$$\left. \frac{1}{\sigma_0^C} \frac{d\sigma_{\text{bHQET}}^C}{d\tau} \right|_{\text{pow 1}} = R_0^C(\hat{m}) m_t Q^2 H_Q^{(6)}(Q, \mu_H) U_{H_Q}^{(6)}(Q, \mu_H, \mu_m) H_m^{(6)}(m, \varrho, \mu_m) U_v^{(5)}(\varrho, \mu_m, \mu) \\ \times \int d\ell d\hat{s} \delta[\hat{s}_\tau - \hat{s} - r_{\tau,s}(\hat{m}) \varrho \ell] \\ \times \int d\hat{s}' U_B^{(5)}(\hat{s} - \hat{s}', \mu, \mu_B) J_{B,\tau}^{(5)}(\hat{s}', \Gamma_t = 0, \delta m = 0, \mu_B) \\ \times \int d\ell' U_S^{(5)}(\ell - \ell', \mu, \mu_S) \hat{S}_\tau^{(5)}(\ell', \bar{\delta} = 0, \mu_S),$$

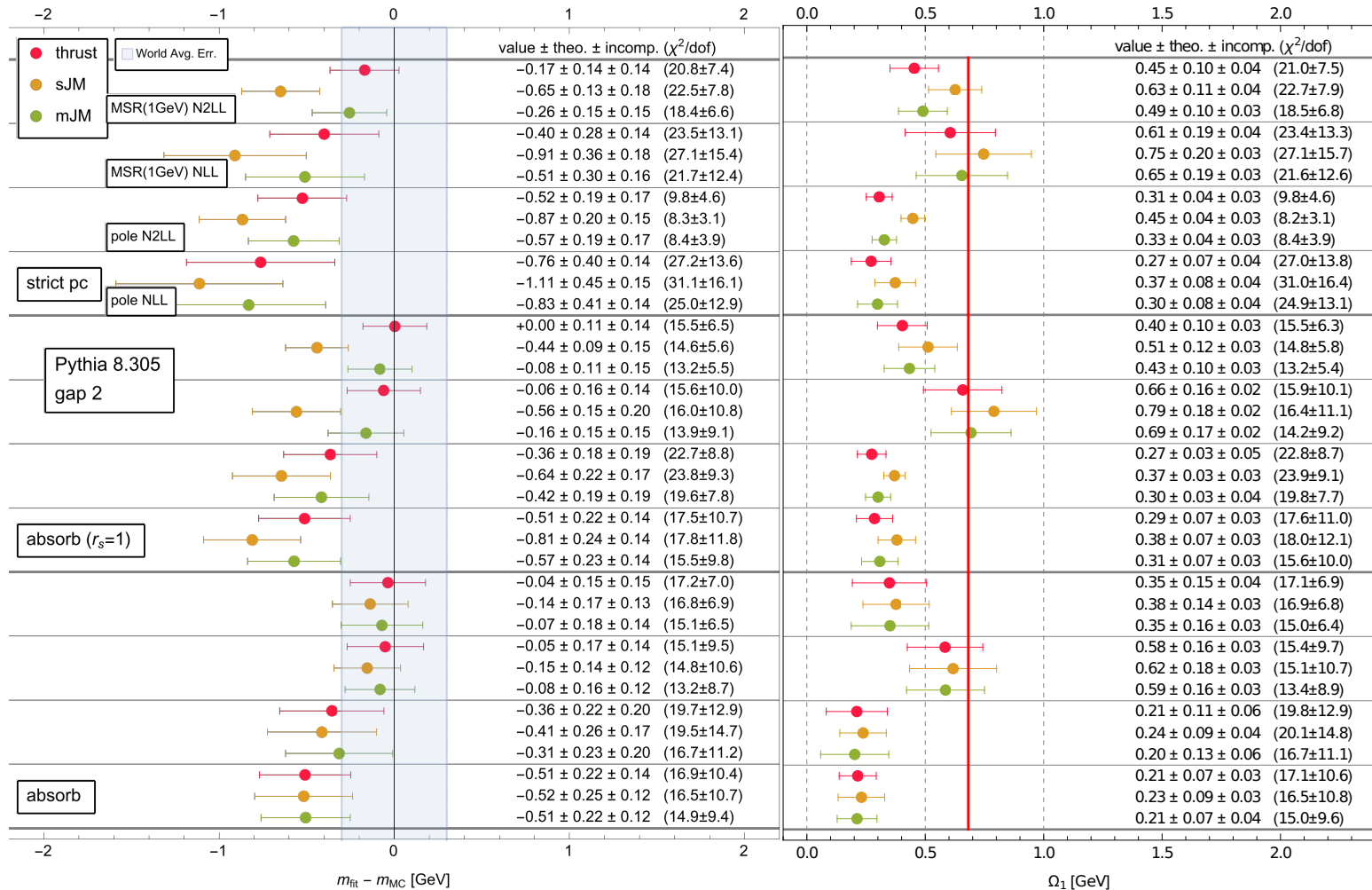
$$\frac{d\tilde{\sigma}_{\text{bHQET}}^C}{d\tau} = \left. \frac{d\sigma_{\text{bHQET}}^C}{d\tau} \right|_{\text{pow 1}, \{H_Q \rightarrow \tilde{H}_Q, H_m \rightarrow \tilde{H}_m, J_{B,\tau}^{(5)} \rightarrow \tilde{J}_{B,\tau}^{(5)}\}}$$

Estimates the uncertainty in the treatment of soft power corrections

$$\tilde{H}_Q(\mu_H) = H_Q(\mu_H) + \frac{C_F \alpha_s(\mu_H)}{4\pi} (1 - \xi_J - \xi_B) \xi_{A1} h_\tau^C(\hat{m}) \\ \tilde{H}_m(\mu_m) = H_m(\mu_m) + \frac{C_F \alpha_s(\mu_m)}{4\pi} \xi_J \xi_{A1} h_\tau^C(\hat{m}) \\ m_t^2 \tilde{J}_{B,\tau}^{(5)}(\hat{s}, \mu_B) = m_t^2 J_{B,\tau}^{(5)}(\hat{s}, \mu_B) + \frac{C_F \alpha_s(\mu_B)}{4\pi} \left\{ \xi_B \xi_{A1} h_\tau^C(\hat{m}) \delta(\hat{s}) + \xi_{B1} b(\hat{m}) \frac{1}{m} \left[\frac{1}{\hat{s}/m} \right]_+ \right\}$$

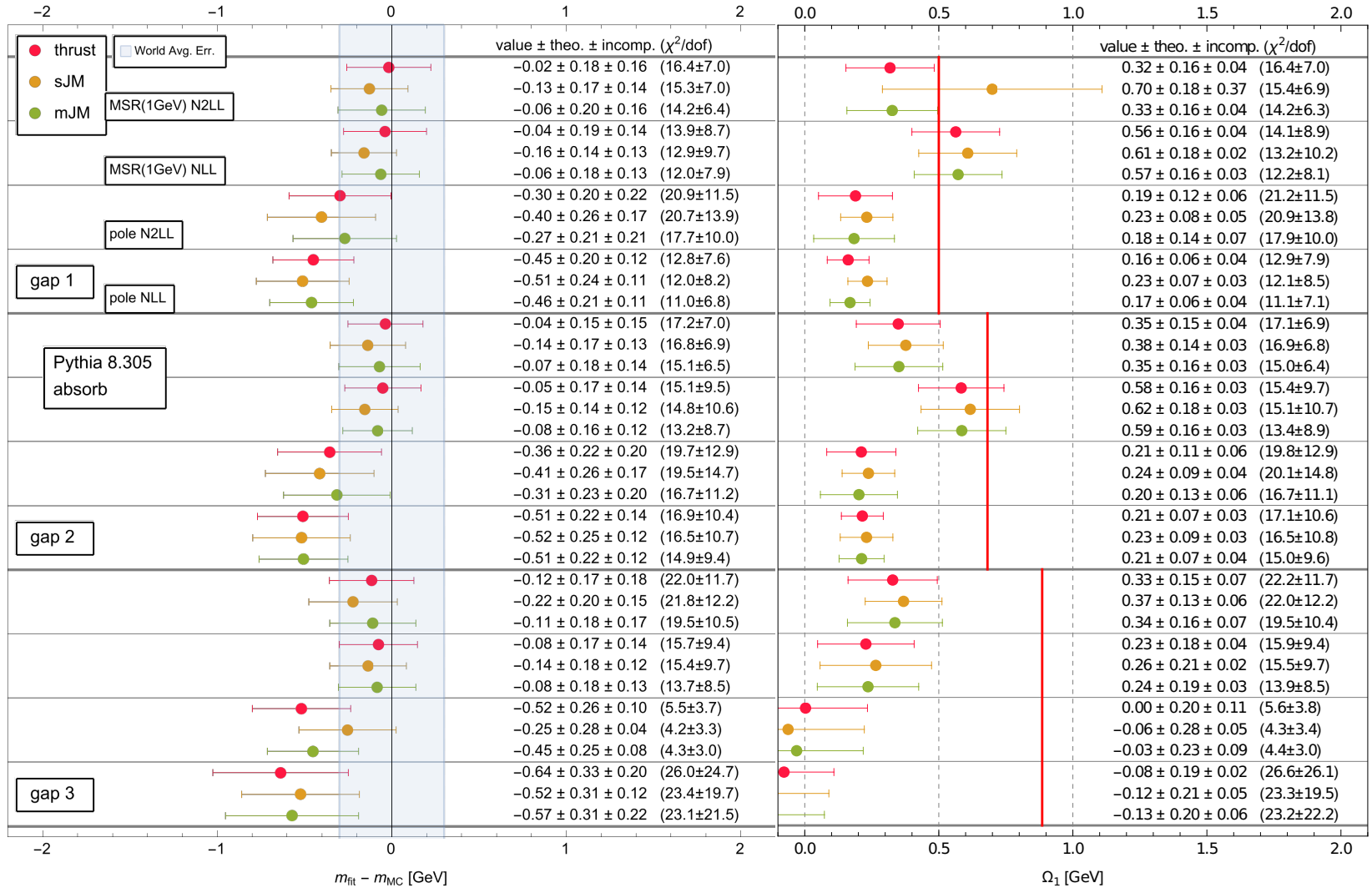
- ▶ soft power correction in measurement delta function
- ▶ global tree-level phase space factor
- ▶ absorb and vary the remaining distributional terms of the full QCD result

Observable Independence

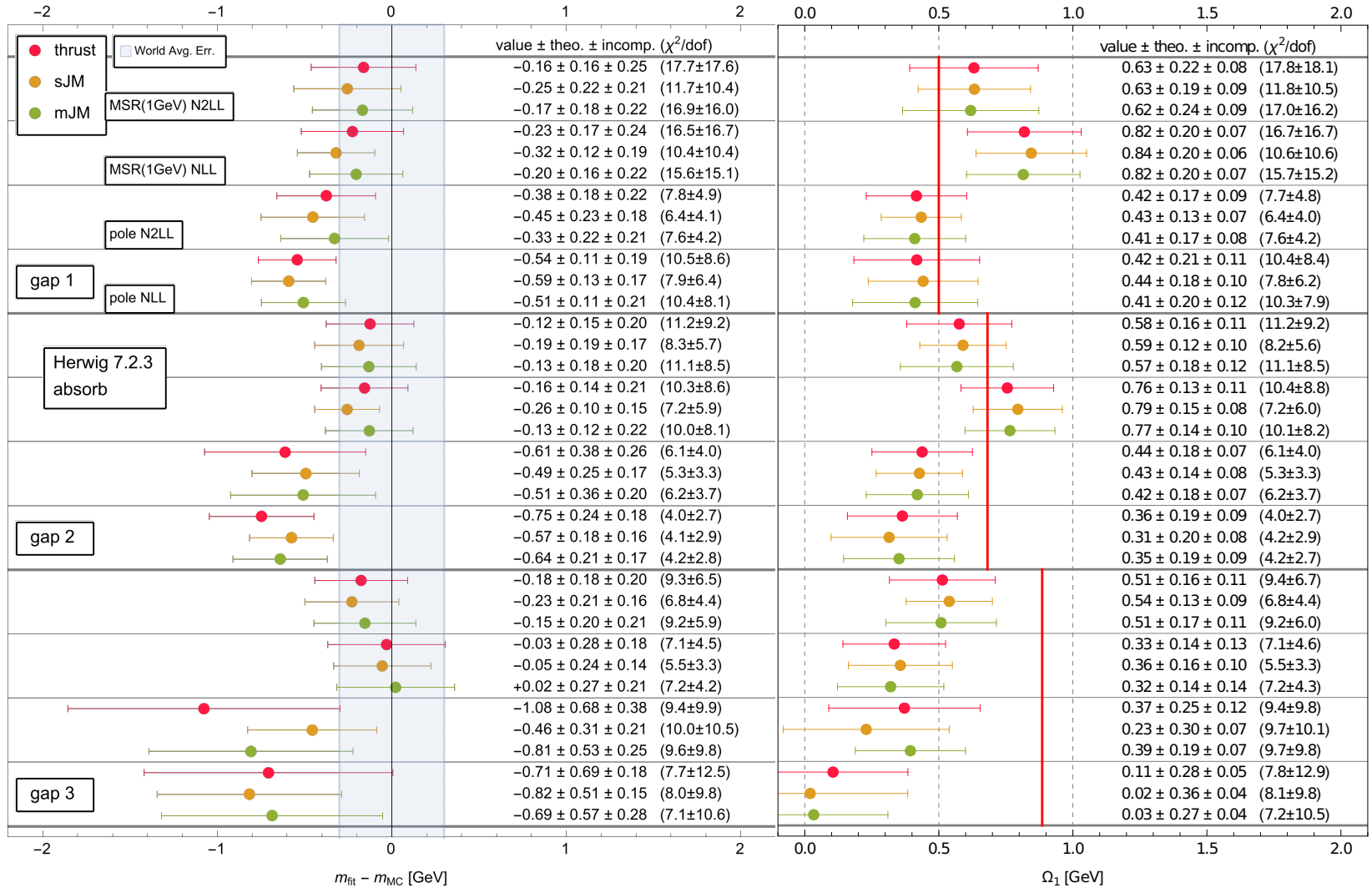


→ Observable independent results obtained for top mass and Ω_1

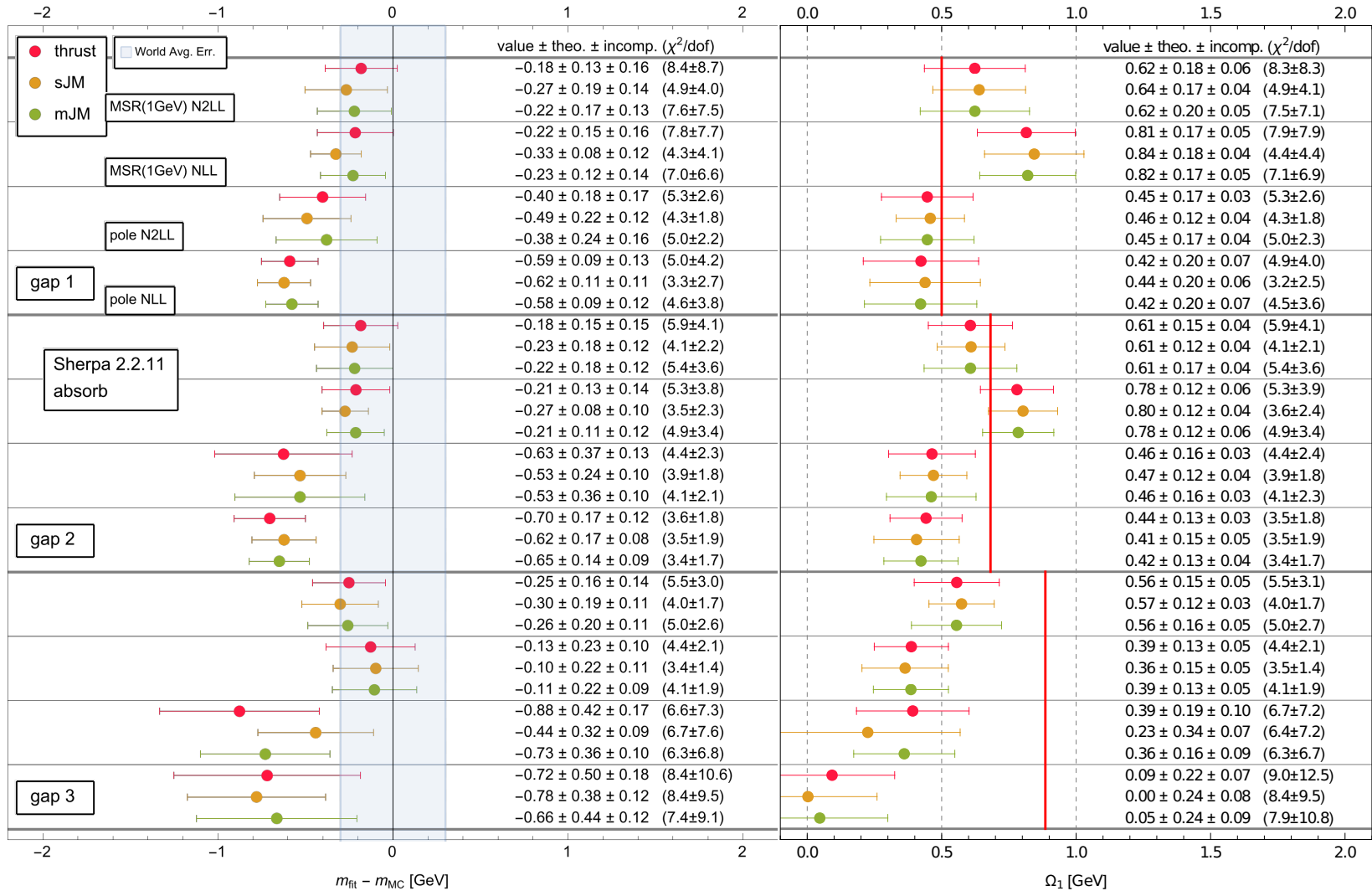
Final Results (Pythia)



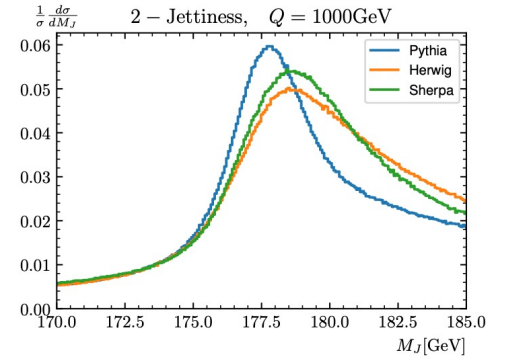
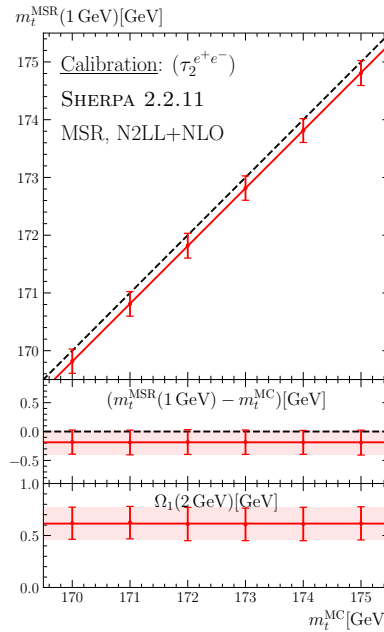
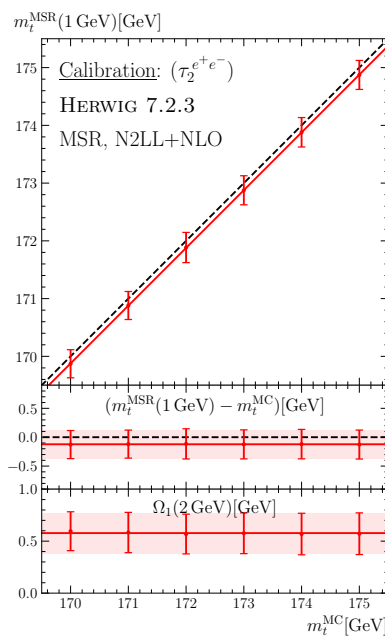
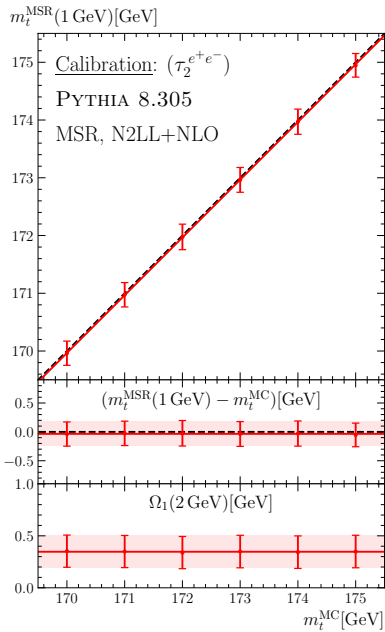
Final Results (Herwig)



Final Results (Sherpa)



Final Results (all)



$$m_t^{\text{PYTHIA}} = m_t^{\text{MSR}}(1 \text{ GeV}) + (0.03 \pm 0.21) \text{ GeV}$$

$$m_t^{\text{HERWIG}} = m_t^{\text{MSR}}(1 \text{ GeV}) + (0.12 \pm 0.25) \text{ GeV}$$

$$m_t^{\text{SHERPA}} = m_t^{\text{MSR}}(1 \text{ GeV}) + (0.19 \pm 0.21) \text{ GeV}$$

$$m_t^{\text{PYTHIA}} = m_t^{\text{pole}} + (0.35 \pm 0.30) \text{ GeV}$$

$$m_t^{\text{HERWIG}} = m_t^{\text{pole}} + (0.61 \pm 0.46) \text{ GeV}$$

$$m_t^{\text{SHERPA}} = m_t^{\text{pole}} + (0.63 \pm 0.39) \text{ GeV}$$

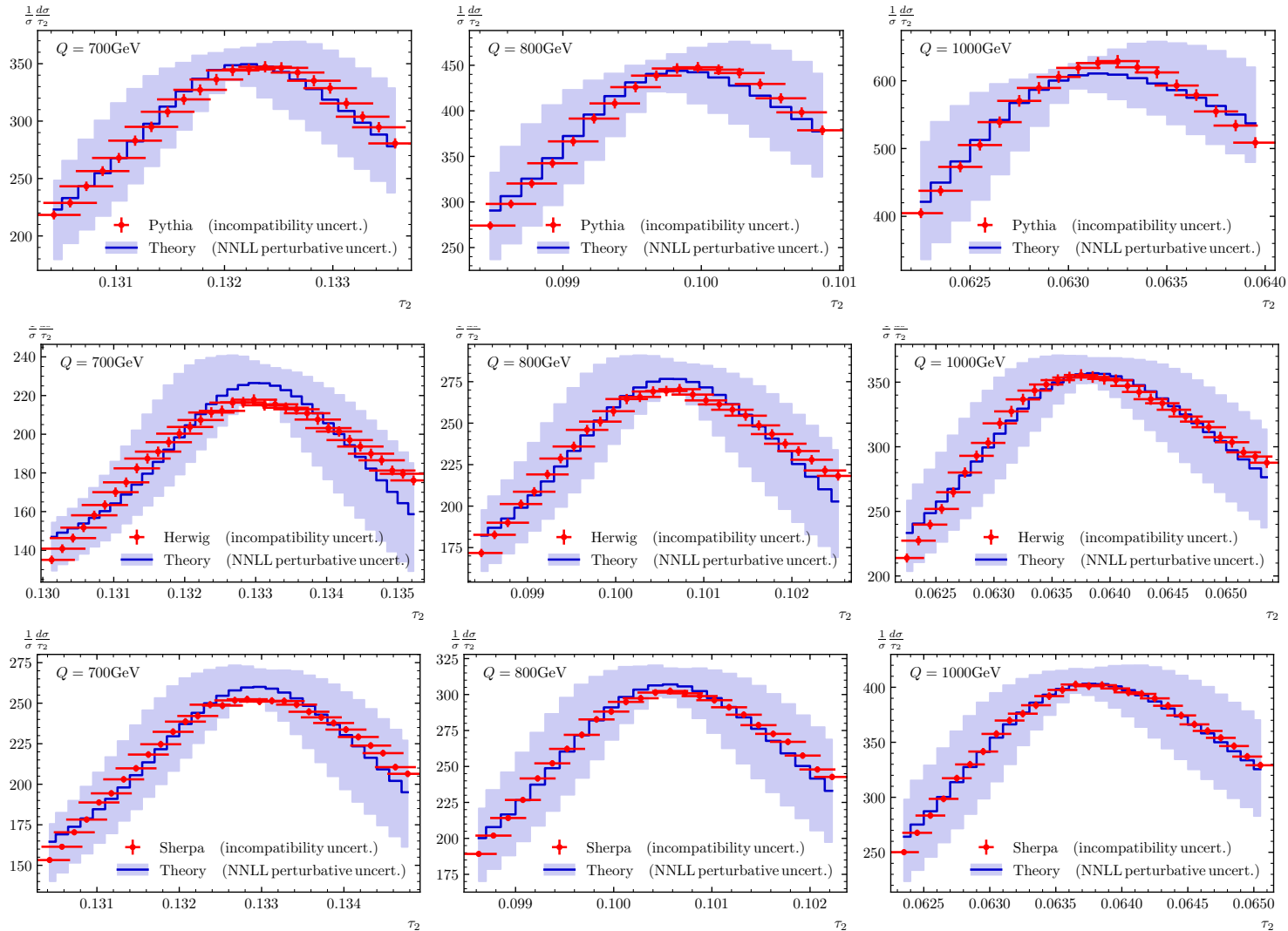
- ▶ Final results: 2-jettiness, gap 2
- ▶ m_t^{MC} for all MCs consistent
- ▶ calibration to pole mass has larger uncertainties
- ▶ soft nonperturbative effects: $\Omega_1^{\text{Pythia}} < \Omega_1^{\text{Herwig}}, \Omega_1^{\text{Sherpa}}$

Outlook on upcoming results

- Dynamical hadronization model in Herwig
 - Old default Herwig hadronization model interferes with meaning of m_t^{MC}
 - New dynamical hadronization model does not
 - Shower cutoff Q_0 as an IR factorization scale (invariance under Q_0 variations)
 - Control over m_t^{MC} through tuning
 - Calibration as diagnostic tool for consistency
- Analytic shower cut dependence of top decay differential distributions for boosted top production
 - Generalization of calibration for inclusive shape observables to top decay differential observables
- LHC: MPI and UE still need to be better understood from the QCD perspective

Backup Slides

Final Results (all)



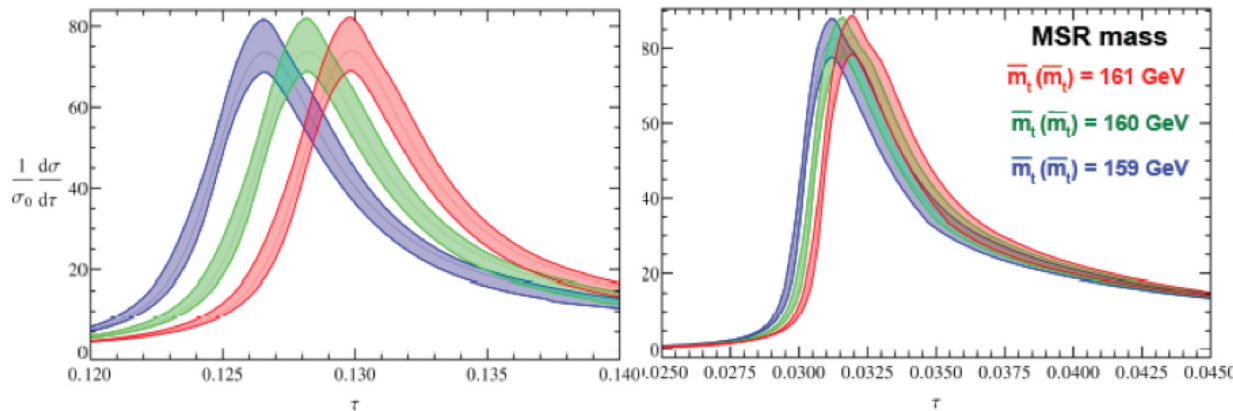
2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = \underbrace{f(m_t^{\text{MSR}}(R), \alpha_s(M_Z), \Omega_1, \Omega_2, \dots)}_{\text{any scheme possible}} \underbrace{\mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t}_{\text{renorm. scales}} \underbrace{\Gamma_t}_{\text{finite lifetime}}$$

any scheme possible Non-perturbative renorm. scales finite lifetime

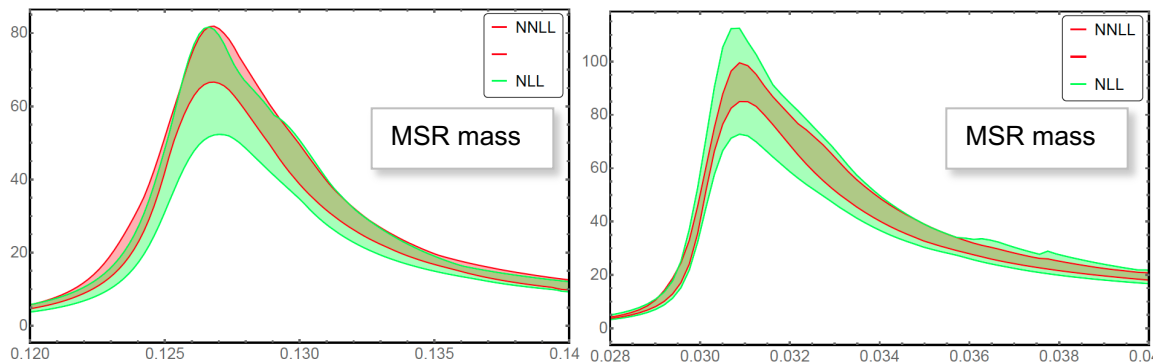
Q=700 GeV ($p_T = 350$ GeV)

Q=1400 GeV ($p_T = 700$ GeV)



Q=700 GeV

Q=1400 GeV



- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: $\Omega_{1,2,\dots}$
- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution