Matching and parton-shower NNDL accuracy

Ludovic Scyboz

(with K. Hamilton, A. Karlberg, G. Salam and R. Verheyen)

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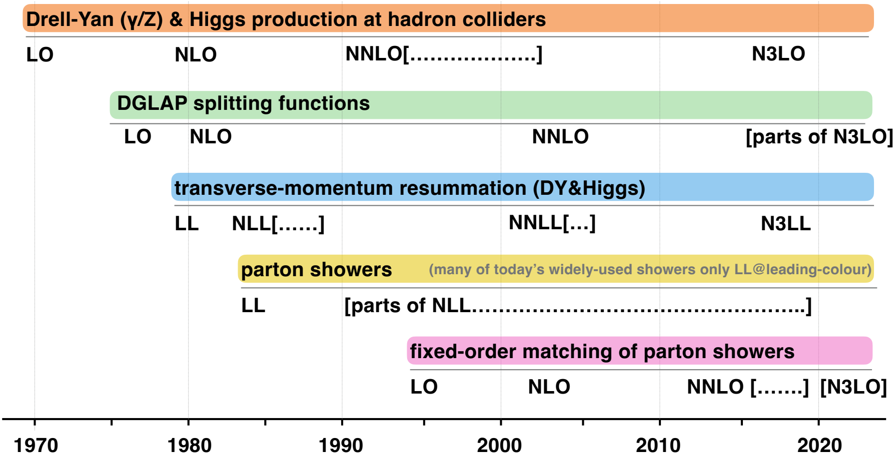
June 1$^{\text{st}}$ 2023

A history of accuracy

selected collider-QCD accuracy milestones



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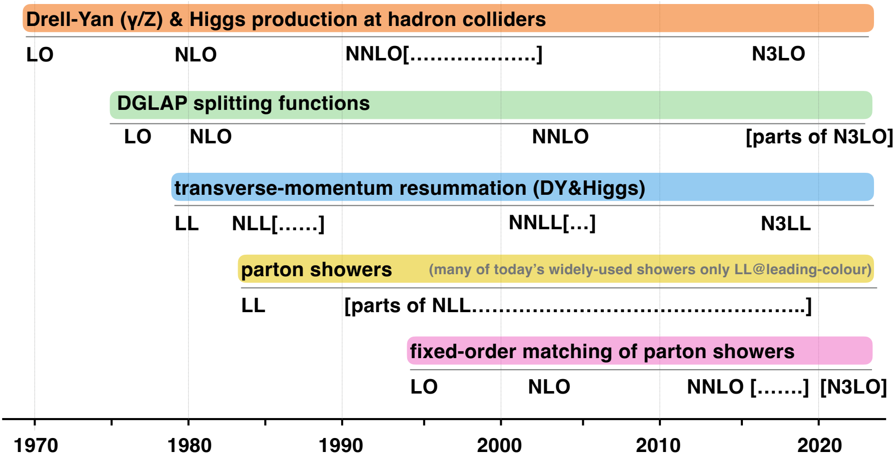
selected collider-QCD accuracy milestones

→ see Silvia's talk



slide stolen from G. Salam

- For a specific observable (e.g. event-shape V): probability $\Sigma\left(V < e^{-|L|}\right)$

$$\Sigma(\alpha_s, L) = \exp\left(\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \cdots\right)$$

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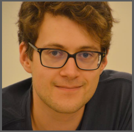
- Double-logarithmic: e.g. subjet multiplicity

$$\Sigma(\alpha_s, L) = \underbrace{h_1(\alpha_s L^2)}_{\text{DL}} + \underbrace{\sqrt{\alpha_s} h_2(\alpha_s L^2)}_{\text{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\text{NNDL}} + \ldots$$

PanScales: A project to bring logarithmic understanding and accuracy to parton showers

Mrinal Dasgupta
Manchester/CERN



Frédéric Dreyer
Oxford

Keith Hamilton
UCL

Pier Monni
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Gavin Salam
Oxford

Grégory Soyez
IPhT Saclay/CERN

since 2017

Emma Slade
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2018-2020

Basem El-Menoufi
Manchester

Alexander Karlberg
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Ludovic Scyboz
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since 2019

Rob Verheyen
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2019-2022

Melissa van Beekveld
Oxford

Silvia Ferrario Ravasio
CERN

Alba Soto-Ontoso
CERN

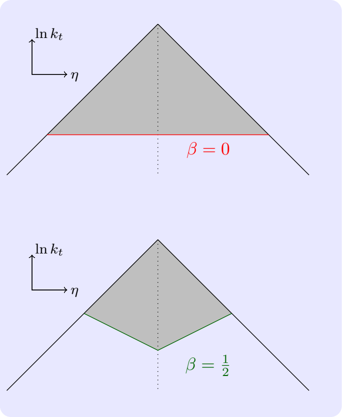
since 2020

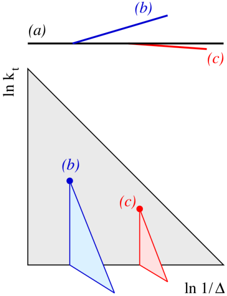
Jack Helliwell
Oxford

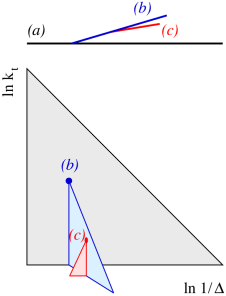
since 2022

PanScales showers: NLL-accurate (i.e. reproduce $g_1(\alpha_s L)$, $g_2(\alpha_s L)$)

- Ordering variable $v \sim k_t e^{-\beta|\eta|}$, $0 \leq \beta < 1$
- $\rightarrow$ see Silvia's talk on Friday for details







NNLL & matching

- On the way to NNLL (i.e. terms $\alpha_s^n L^{n-1}$): we need NLO matching to get e.g. the term α_s
- NLO matching was solved 20 years ago:
  - MC@NLO [Frixione, Webber '02]
  - POWHEG [Nason '04],[Nason, Oleari '07]
  - . . . & many others since then
- All appeared at a time where showers were LL-accurate
- ... so focus on their interplay with NLL-accurate showers
- Take the simple example of $Z/\gamma^* \to q\bar{q}$ and $H \to gg$

- **Strategy**: Correct hardest emission @ NLO (MEC, MC@NLO, POWHEG)

Problem solved 20 years ago

Matching & NNDL accuracy

- **Strategy**: Correct hardest emission @ NLO (MEC, MC@NLO, POWHEG)
- Gets correct C_1 coefficient

$$\Sigma(\alpha_s, L) = \left(1 + C_1 \frac{\alpha_s}{\pi} + \cdots\right) \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots]$$

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- In analytic resummation: C_1 (+NLL) suffices for NNDL event-shape observables (terms $\sim \alpha_s^n L^{2n-2}$)

$$\Sigma(\alpha_s, L) = \underbrace{h_1(\alpha_s L^2)}_{\text{DL}} + \sqrt{\alpha_s}\, \underbrace{h_2(\alpha_s L^2)}_{\text{NDL}} + \alpha_s\, \underbrace{h_3(\alpha_s L^2)}_{\text{NNDL}} + \cdots$$

Matching & NNDL accuracy

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Is this true for NLO matching of NLL parton showers?

- The POWHEG "formula": (HEG $\equiv$ hardest emission generator)

$$\mathrm{d}\sigma = \bar{B}(\Phi_B) S_{\mathrm{HEG}}(v_\Phi^{\mathrm{HEG}}, \Phi_B) \times \frac{R_{\mathrm{HEG}}(\Phi)}{B_0(\Phi_B)} \mathrm{d}\Phi \times I_{\mathrm{PS}}(v_\Phi^{\mathrm{HEG}}, \Phi)$$

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1st emission: exact matrix element

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Sudakov down to $v = v_\Phi^{\mathrm{HEG}}$

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Iterated shower starting from $v = v_\Phi^{\mathrm{HEG}}$

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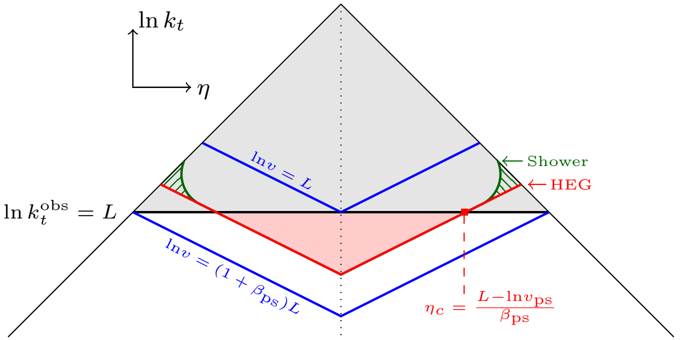
With PanScales:

- NLL: for instance PanLocal requires $\beta_{\mathrm{PS}} > 0$
- We work with a "generalised" POWHEG [Frixione, Kunszt, Signer '96] map: POWHEG_{β}

- Assumption: $\beta = \beta_{\mathrm{PS}}$ (contours align in the soft-collinear)

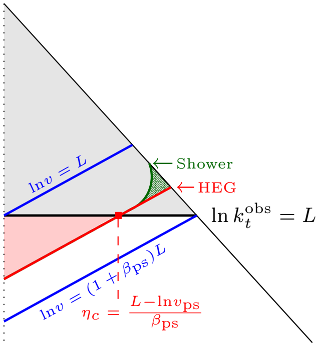


- Need to veto shower emissions (i.e. no holes or double-counting)

  [Nason '02], [Corke, Sjöstrand '10]

- In POWHEG+Pythia: $p_{\perp,\mathrm{evol}} \neq p_{\perp,\mathrm{POWHEG}}$

POWHEG: kinematic contour mismatch



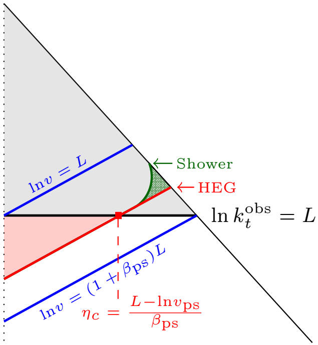
- Shower start scale: $v_{\text{PS}} = v_{\text{HEG}}^{\Phi}$
- Correct result ($\bar{\alpha} = \frac{2C_F\alpha_s}{\pi}$):

$$\Sigma = e^{-\bar{\alpha}L^2}$$

- If kinematic contours match everywhere:

$$\Sigma_{\text{HEG+PS}} = e^{-\bar{\alpha}L^2}$$

POWHEG: kinematic contour mismatch



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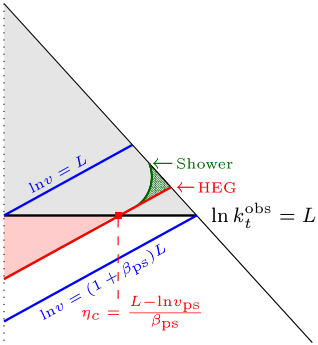
- If contours do not match in hard-collinear:

  $$\Sigma_{\rm HEG+PS} = e^{-\bar\alpha L^2}\left[1 + (e^{-\bar\alpha\beta_{\rm PS}L^2} - 1)\bar\alpha\Delta_{\rm kin}\right]$$

  $$(\bar\alpha\Delta_{\rm kin} = \frac{2\alpha_s C_F}{\pi}\frac{4\pi^2 - 15}{24} \text{ for } \gamma^* \to q\bar q)$$

POWHEG: kinematic contour mismatch



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  ($\bar{\alpha}\Delta_{\mathrm{kin}} = \frac{2\alpha_s C_F}{\pi}\frac{4\pi^2-15}{24}$ for $\gamma^* \to q\bar{q}$)

Breaks exponentiation: $\ln\Sigma = -\bar{\alpha}L^2 - \sum_{n=2}^{\infty}\frac{2\beta_{\mathrm{PS}}^{n-1}\Delta_{\mathrm{kin}}}{(n-1)!}\bar{\alpha}^n L^{2n-2}$

POWHEG: desymmetrisation mismatch

- Showers (and POWHEG) de-symmetrise the gluon splitting functions (divergence only when radiation = soft, $\zeta \to 1$)

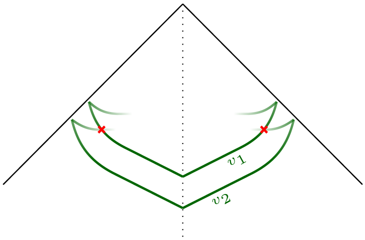
  In PanScales:

  de-symm. parameter

  $$\frac{1}{2!} P_{gg}^{\text{asym}}(\zeta) = C_A \left[\frac{1+\zeta^3}{1-\zeta} + (2\zeta - 1) w_{gg} \right]$$

  - ... and $P_{gg}^{\text{asym}}(\zeta) + P_{gg}^{\text{asym}}(1-\zeta) = 2P_{gg}(\zeta)$

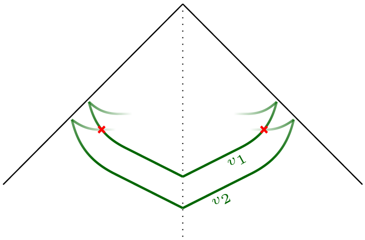
- NNDL issues (similar to previous issue) arise if HEG and shower de-symmetrisation do not align (“$w^{\text{HEG}} \neq w^{\text{PS}}$”)



$$v_1(\zeta > \tfrac{1}{2}):\ \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Phi} \sim f_{X,1}(w)$$

$$v_2(\zeta < \tfrac{1}{2}):\ \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Phi} \sim f_{X,2}(w)$$

$$f_{X,1}(w) + f_{X,2}(w) = 1$$



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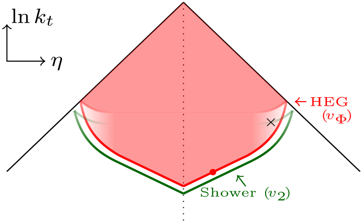

$$f_{X,1}(w^{\mathrm{HEG}}) + f_{X,2}(w^{\mathrm{PS}}) \neq 1$$

$$\Sigma_{\mathrm{mismatch}} \simeq e^{-\bar{\alpha}L^2}\left[1 + (e^{-\bar{\alpha}\beta_{\mathrm{PS}}L^2} - 1)\bar{\alpha}\Delta\right]$$

$$\bar{\alpha}\Delta \propto (C_A - T_R n_f)(w^{\mathrm{HEG}} - w^{\mathrm{PS}})$$

NNDL accuracy tests: pure showers

$$\Sigma = h_1(\alpha_s L^2) + \sqrt{\alpha_s} h_2(\alpha_s L^2) + \alpha_s h_3(\alpha_s L^2) + \dots$$

- Fix $\xi = \alpha_s L^2$, and take the small-α_s limit

$$\delta_{\text{NNDL}} = \lim_{\alpha_s \to 0} \frac{\Sigma_{\text{PS}} - \Sigma_{\text{NNDL}}}{\alpha_s \Sigma_{\text{DL}}}$$

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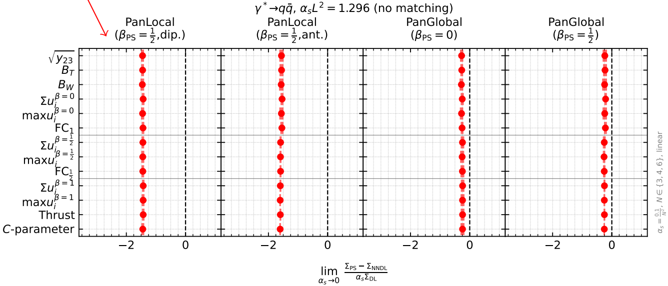
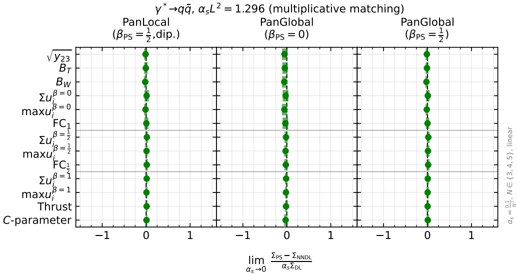
- Fix $\xi = \alpha_s L^2$, and take the small-α_s limit

$\delta_{\mathrm{NNDL}} \sim -2$

$$\delta_{\mathrm{NNDL}} = \lim_{\alpha_s \to 0} \frac{\Sigma_{\mathrm{PS}} - \Sigma_{\mathrm{NNDL}}}{\alpha_s \Sigma_{\mathrm{DL}}}$$



NNDL tests: multiplicative/MC@NLO matching



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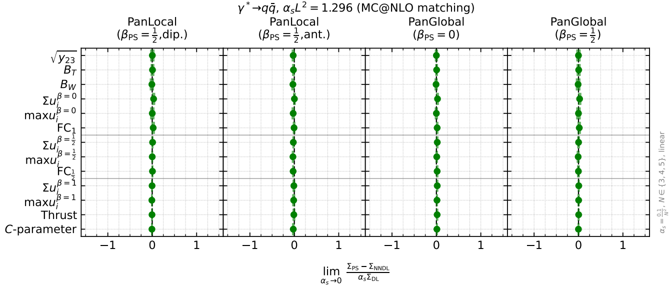

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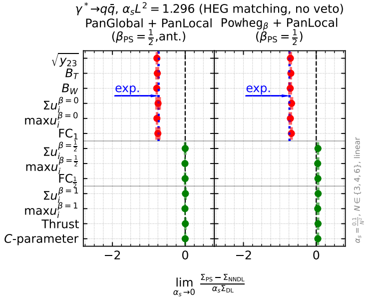
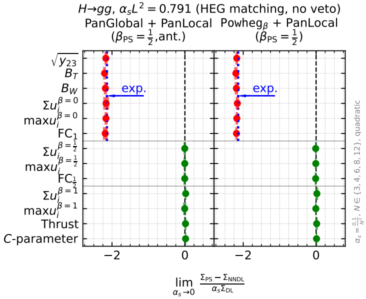
NNDL tests: HEG without veto

- Start $v_{\mathrm{PS}} = v^{\Phi}_{\mathrm{HEG}}$ without veto
- For $\beta_{\mathrm{obs}} < \beta_{\mathrm{PS}}$: expected discrepancy (exp.)

$$\Sigma_{\mathrm{mismatch}} \simeq e^{-\bar{\alpha}L^2}\left[1 + (e^{-\bar{\alpha}\beta_{\mathrm{PS}}L^2} - 1)\bar{\alpha}\Delta_{\mathrm{kin}}\right]$$



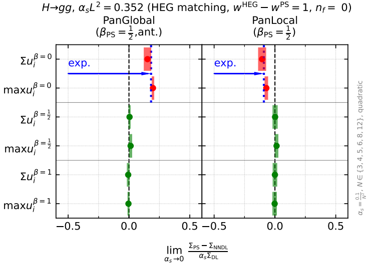


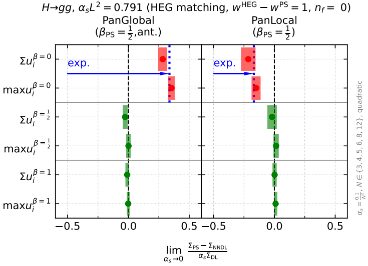
NNDL tests: HEG with $w^{\mathrm{HEG}} \neq w^{\mathrm{PS}}$

- Start $v_{\mathrm{PS}} = v^{\Phi}_{\mathrm{HEG}}$ with aligned contours (HEG $\equiv$ shower) but misaligned de-symmetrisation parameters, $w^{\mathrm{HEG}} \neq w^{\mathrm{PS}}$
- For $\beta_{\mathrm{obs}} < \beta_{\mathrm{PS}}$: expected discrepancy (exp.)

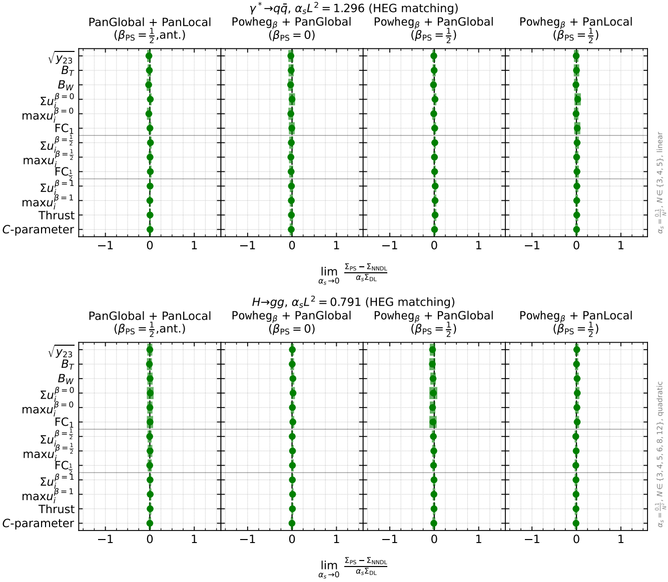
$$\Sigma_{\mathrm{mismatch}} \simeq e^{-\bar{\alpha}L^2}\left[1 + (e^{-\bar{\alpha}\beta_{\mathrm{PS}}L^2} - 1)\bar{\alpha}\Delta_w\right]$$

$$\bar{\alpha}\Delta_w \propto (C_A - T_R n_f)(w^{\mathrm{HEG}} - w^{\mathrm{PS}})$$





NNDL tests: POWHEG (properly)



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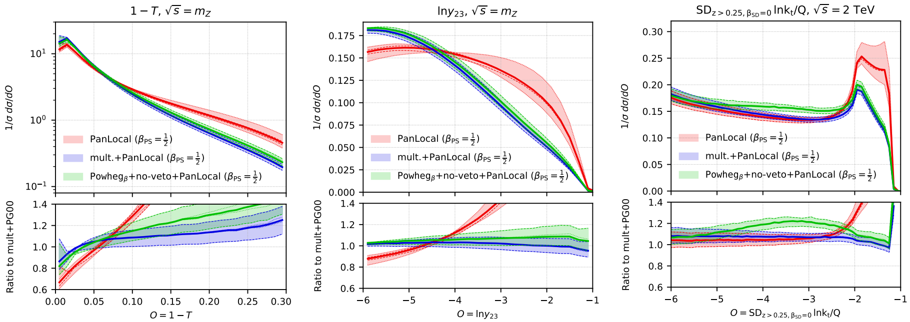
Phenomenological results



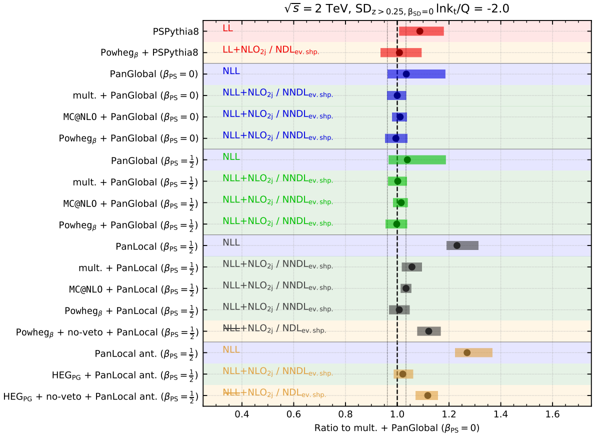
- Scale compensation (inspired my [Mrenna,Skands '16])

$$\alpha_s(\mu_R)\left(1+\frac{K\alpha_s(\mu_R)}{2\pi}+\frac{2(1-z)\beta_0\alpha_s(\mu_R)}{2\pi}\ln(x_R)\right),\ \mu_R = x_R\mu_R^{\text{central}}$$

- Hard uncertainty: $P(x_{\text{hard}}) = P^{(\text{default})}\times\left[1+(x_{\text{hard}}-1)\min\left(\frac{4k_\perp^2}{Q^2},1\right)\right]$

Phenomenological results

- Good agreement among (correctly) matched showers
- Reduction of scale uncertainties from LL $\rightarrow$ NLL
- Visible effect of NLL exponentiation breaking ($\cancel{\text{NLL}}$)



Summary & Outlook

- Showers with higher logarithmic accuracy are making an appearance

  [PanScales],[Forshaw,Holguin,Plätzer],[Nagy,Soper],[Herren,Höche,Krauss,Reichelt,Schönherr]
- Good control of logarithmic terms allows us to place meaningful shower uncertainties

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- First steps to gauge interplay of matching with higher-log accurate showers

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- Working on the next steps: full NNLL, quark masses, hadronisation/MPI & tuning
- Getting in shape to have a usable code ready

In general: for an observable $\Sigma(\alpha_s, L)$

- N^kLL accuracy ($\lambda := \alpha_s L$)

$$\Sigma = \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right]$$

$$\delta\Sigma_{\mathrm{N}^k\mathrm{LL}} = \lim_{\substack{\alpha_s \to 0 \\ \lambda \text{ fixed}}} \left(\frac{\ln \Sigma_{\mathrm{PS}}(\alpha_s, \lambda/\alpha_s) - \ln \Sigma_{\mathrm{N}^k\mathrm{LL}}(\alpha_s, \lambda/\alpha_s)}{\alpha_s^{k-1}} \right)$$

$\delta\Sigma_{\mathrm{N}^k\mathrm{LL}} = 0$ if
shower is N^kLL accurate

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- N^kDL accuracy ($\xi := \alpha_s L^2$)

$$\Sigma = h_1(\alpha_s L^2) + \sqrt{\alpha_s} h_2(\alpha_s L^2) + \alpha_s h_3(\alpha_s L^2) + \ldots$$

$$\delta\Sigma_{\mathrm{N}^k\mathrm{DL}} = \lim_{\substack{\alpha_s \to 0 \\ \xi \text{ fixed}}} \left(\frac{\Sigma_{\mathrm{PS}}(\alpha_s, -\sqrt{\xi/\alpha_s}) - \Sigma_{\mathrm{N}^k\mathrm{DL}}(\alpha_s, -\sqrt{\xi/\alpha_s})}{\alpha_s^{k/2}} \right)$$

- NLL tests (with full colour) for global event shapes: ✓