

NLL/NLO* STUDIES ON HIGGS + JET PRODUCTION

WITH POWHEG + JETHAD

Francesco Giovanni Celiberto

UAH Madrid



In collaboration with: **Luigi Delle Rose, Michael Fucilla, Gabriele Gatto, Alessandro Papa**

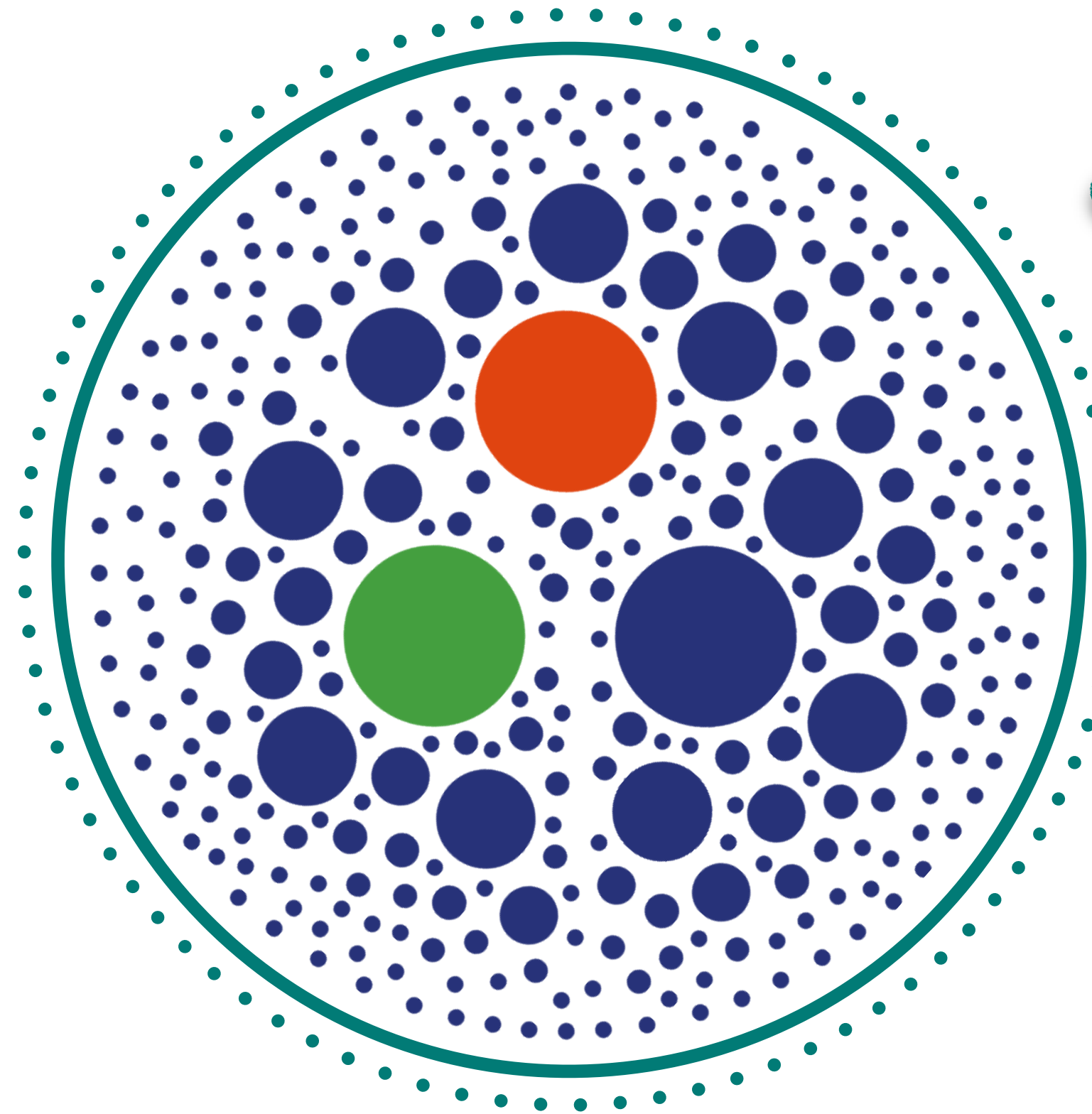
RADCOR 2023 - CRIEFF (SCOTLAND) - 1ST JUNE 2023



QCD & Resummations

LHC & new-gen. colliders:

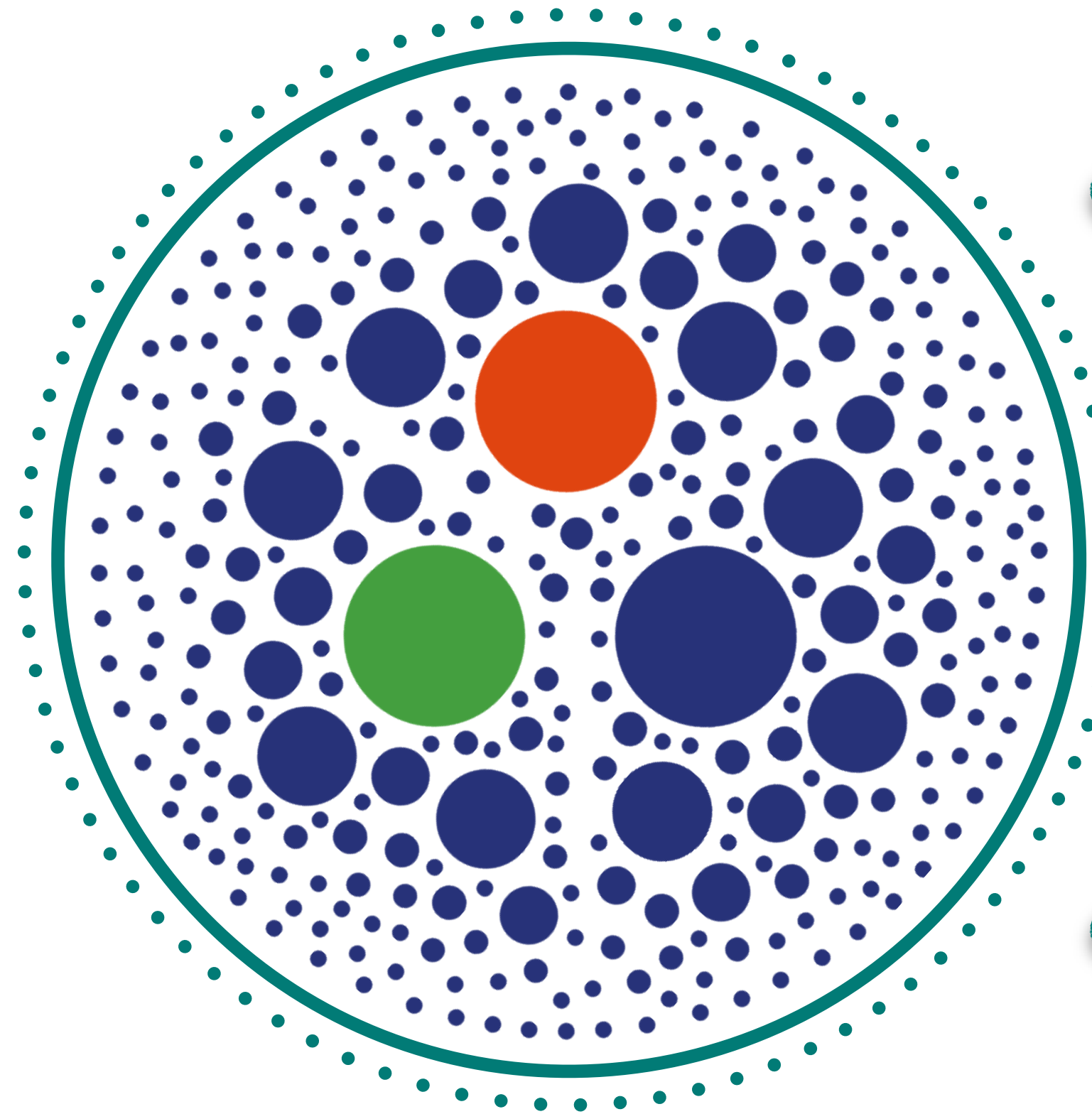
A new era
for particle physics



QCD & Resummations

LHC & new-gen. colliders:

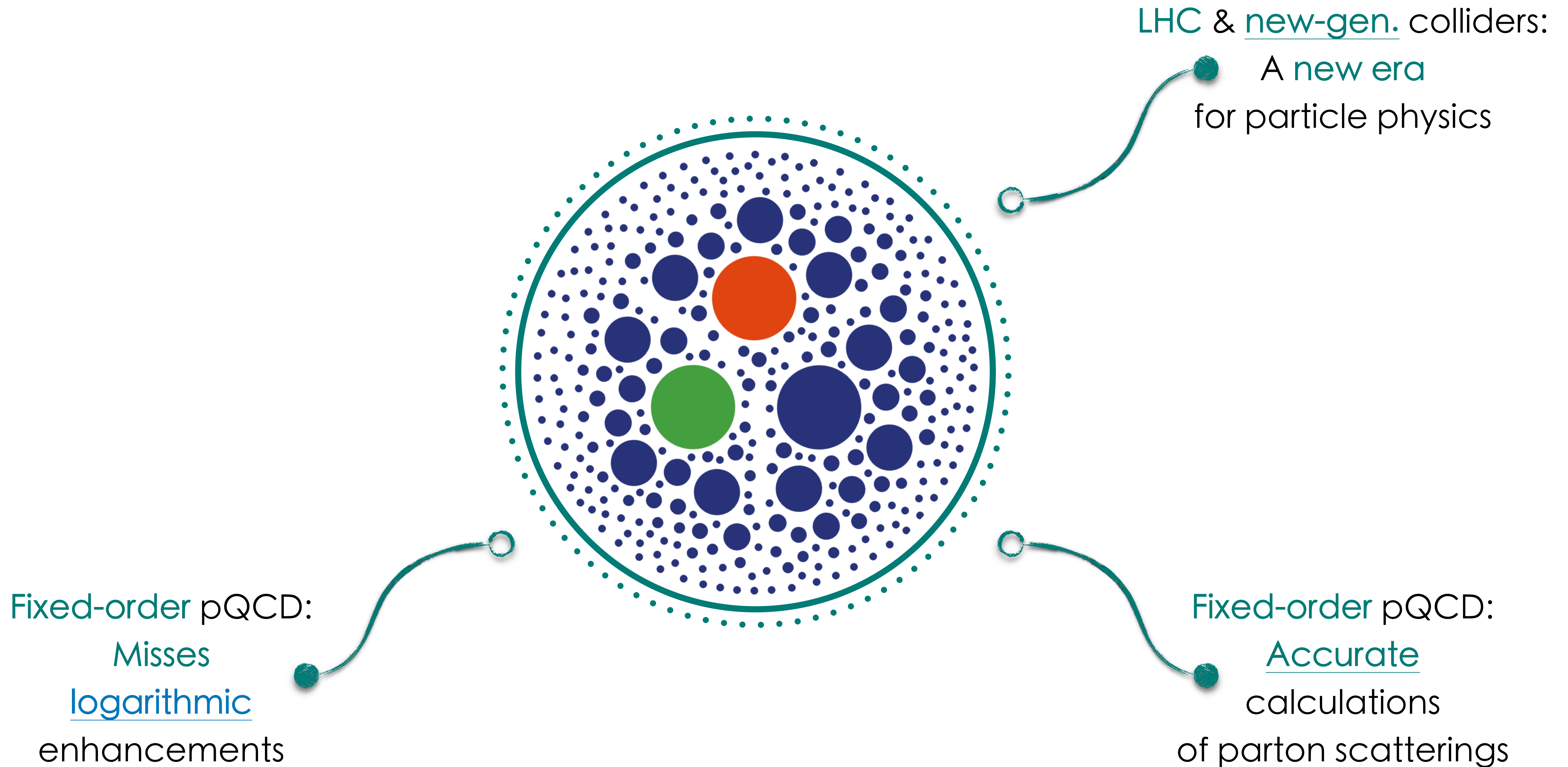
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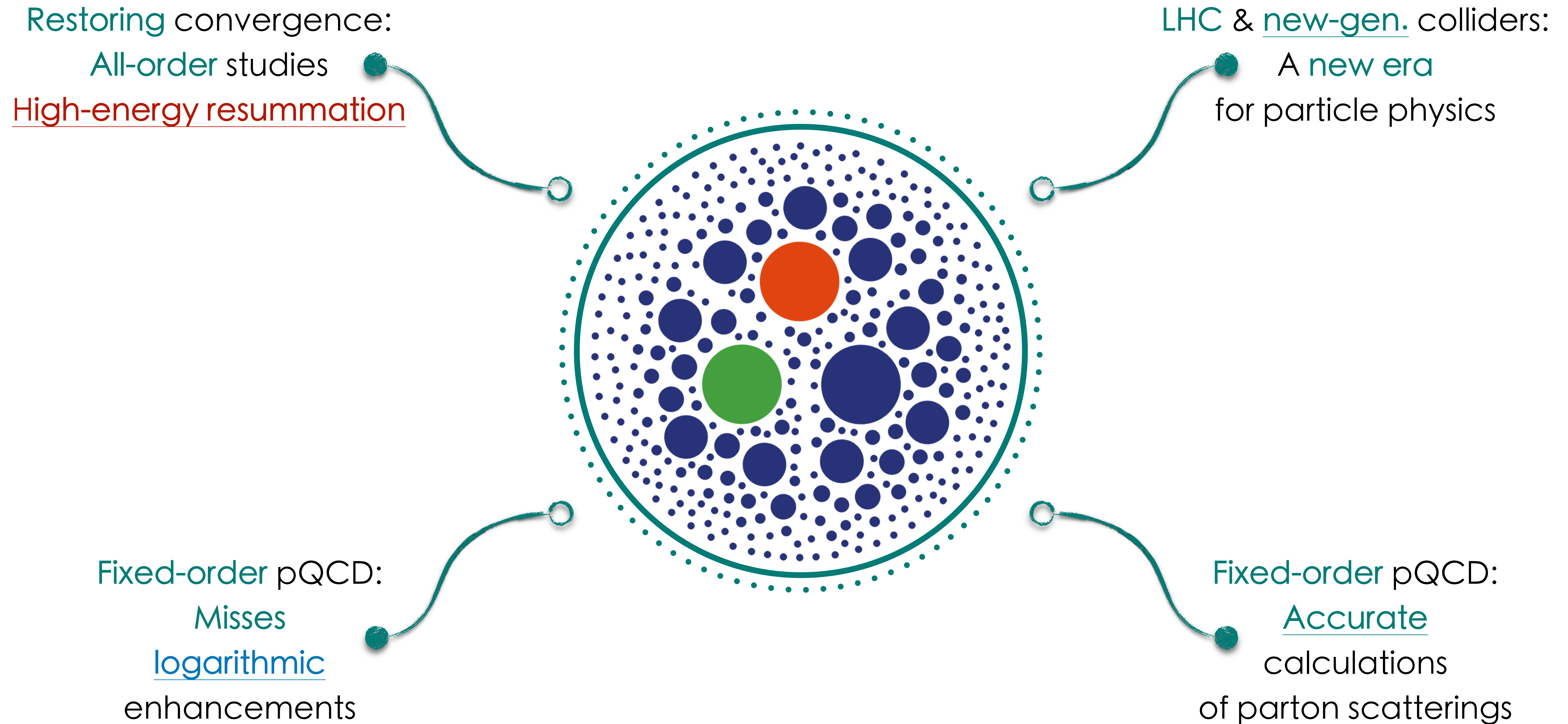
Fixed-order pQCD:

Accurate
calculations
of parton scatterings

QCD & Resummations

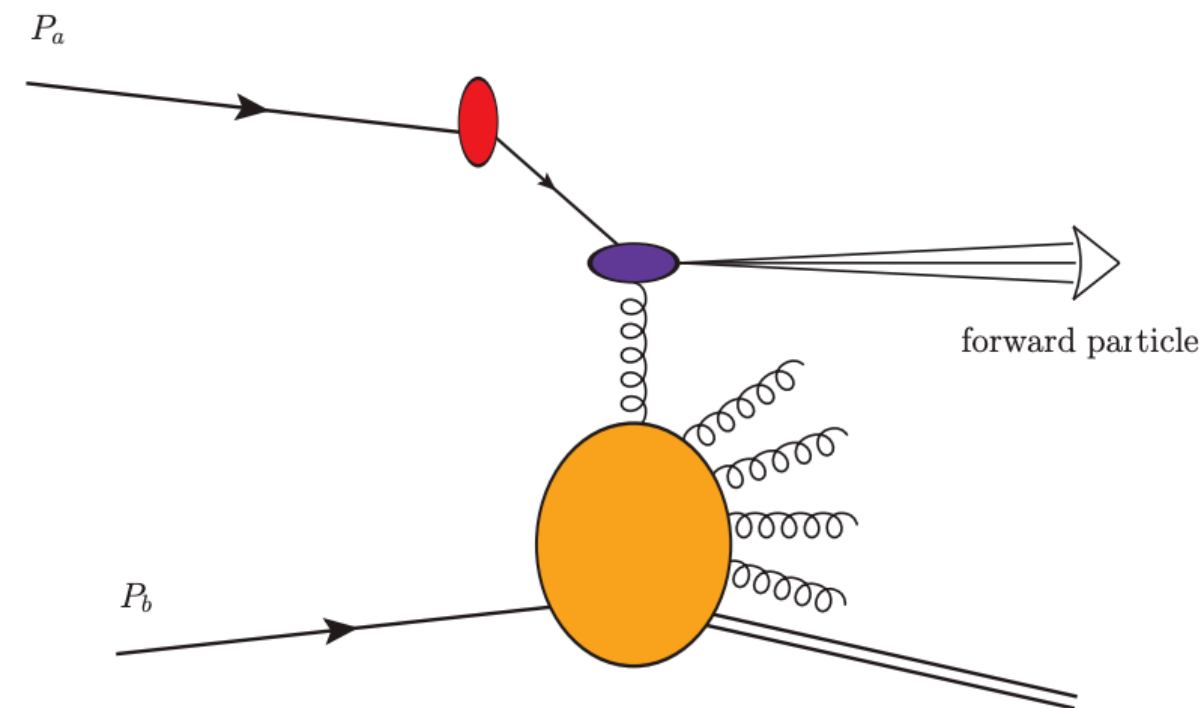


QCD & Resummations

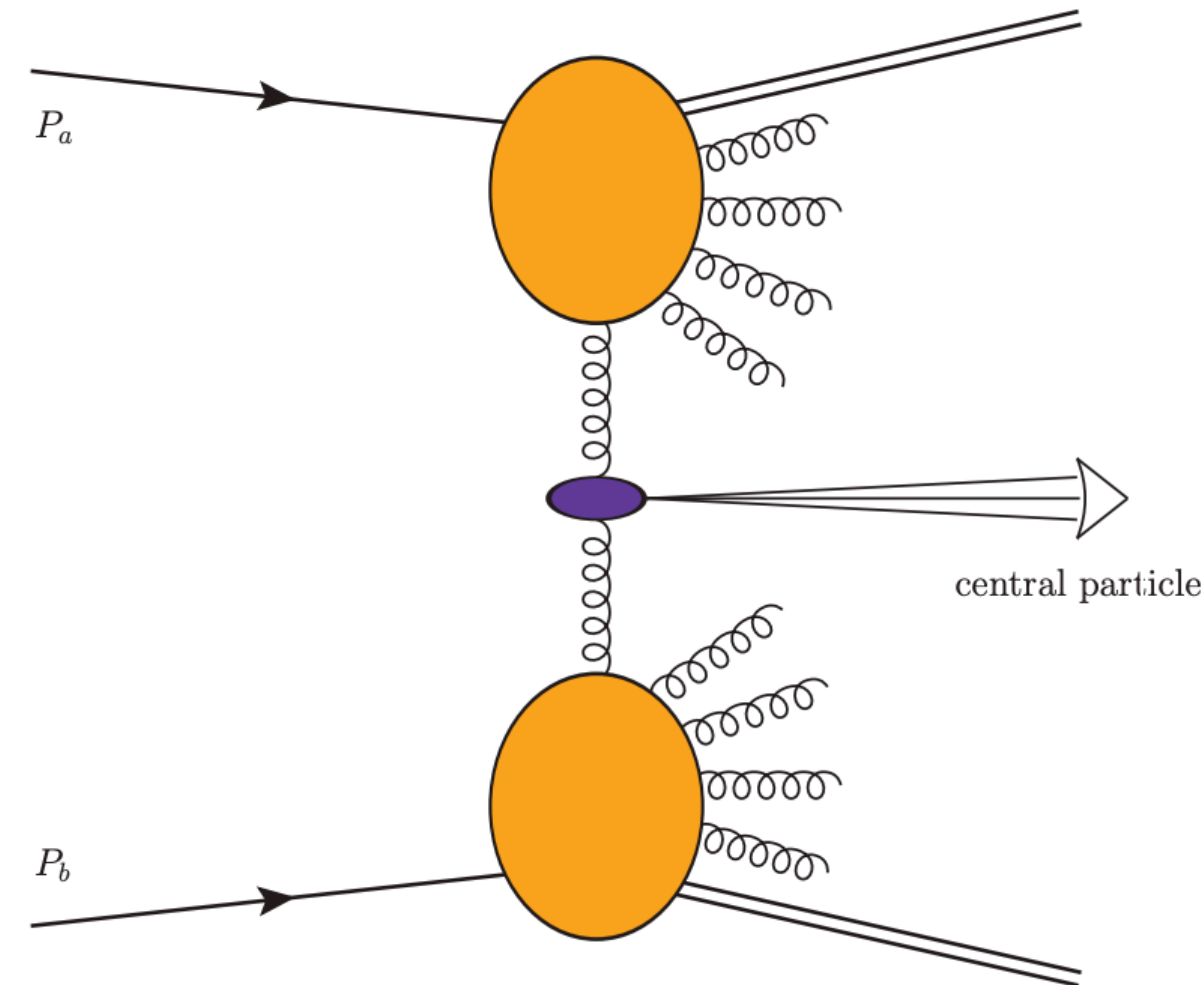


High-energy factorization at a glance

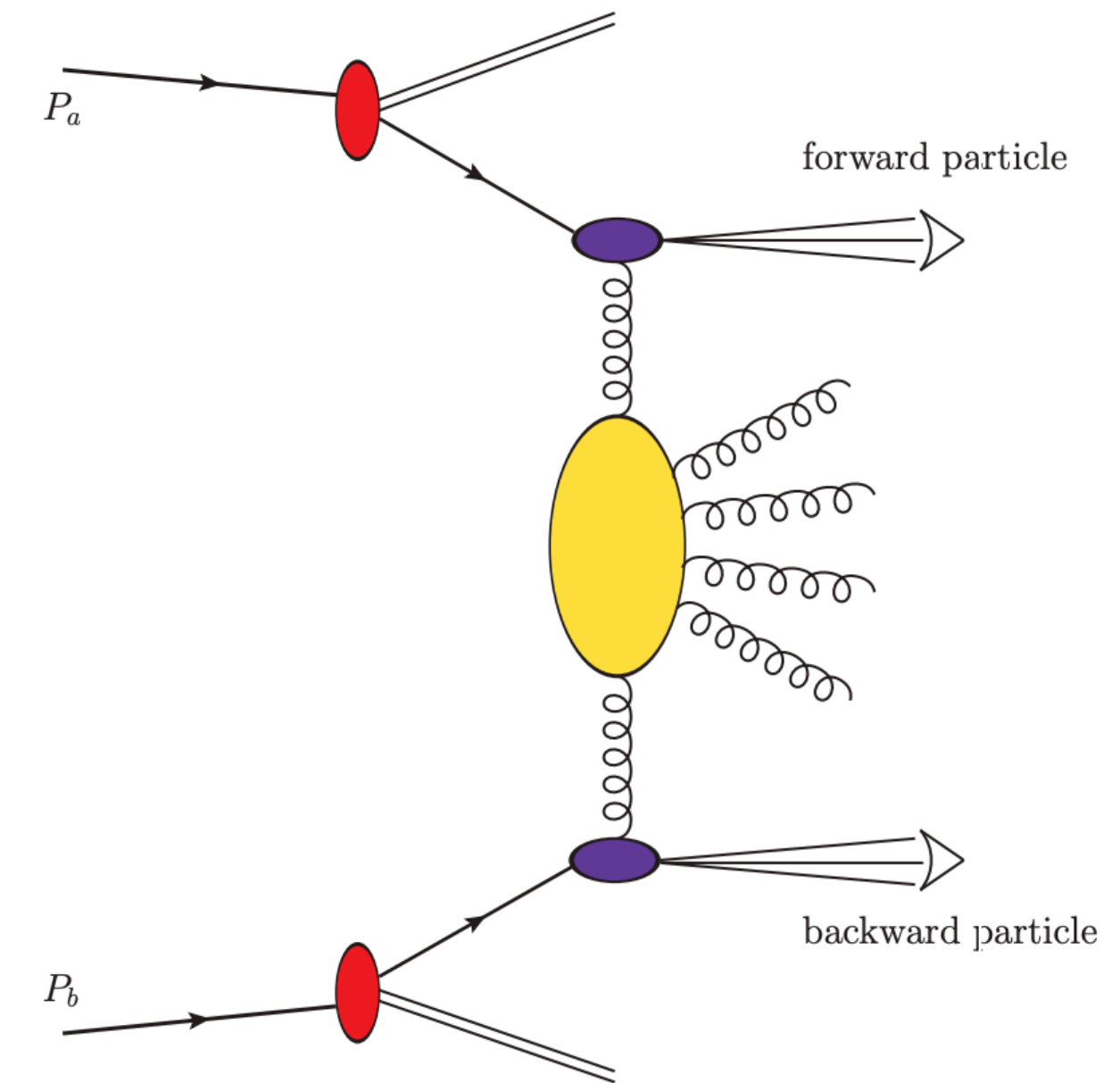
Singly/double off-shell coefficient functions
Forward/central production impact factors



(a) Single forward



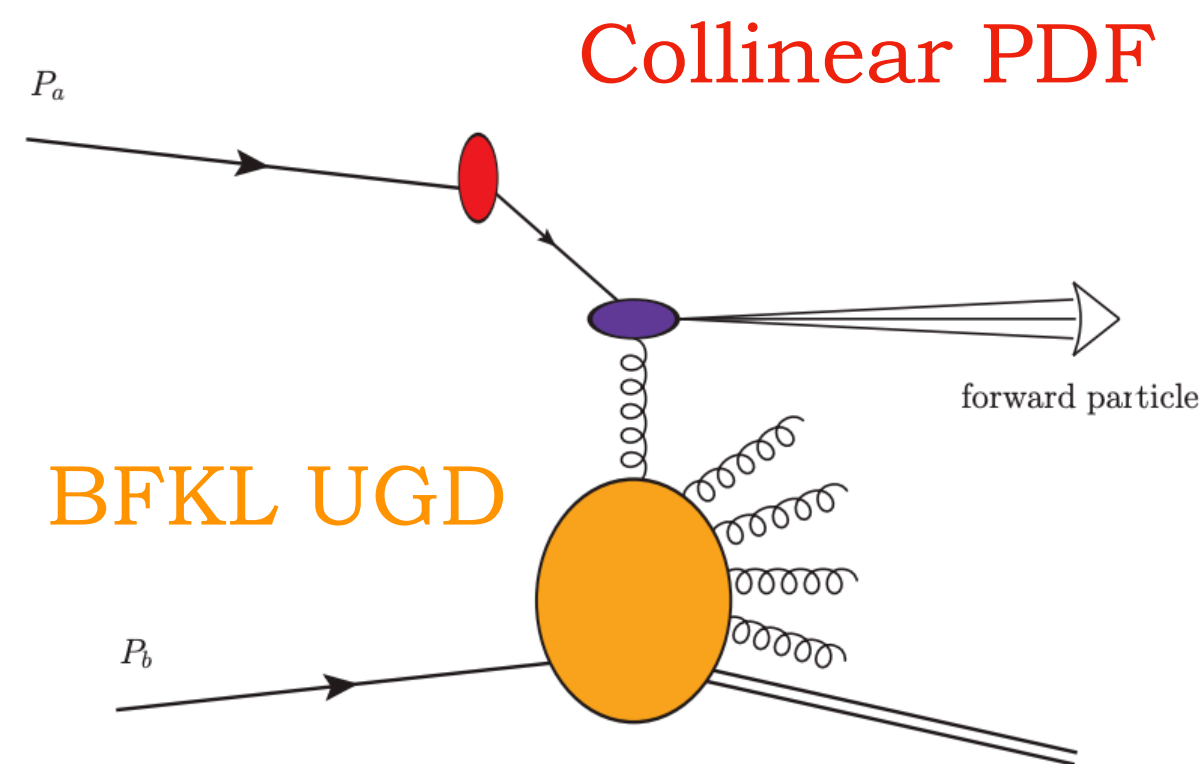
(b) Single central



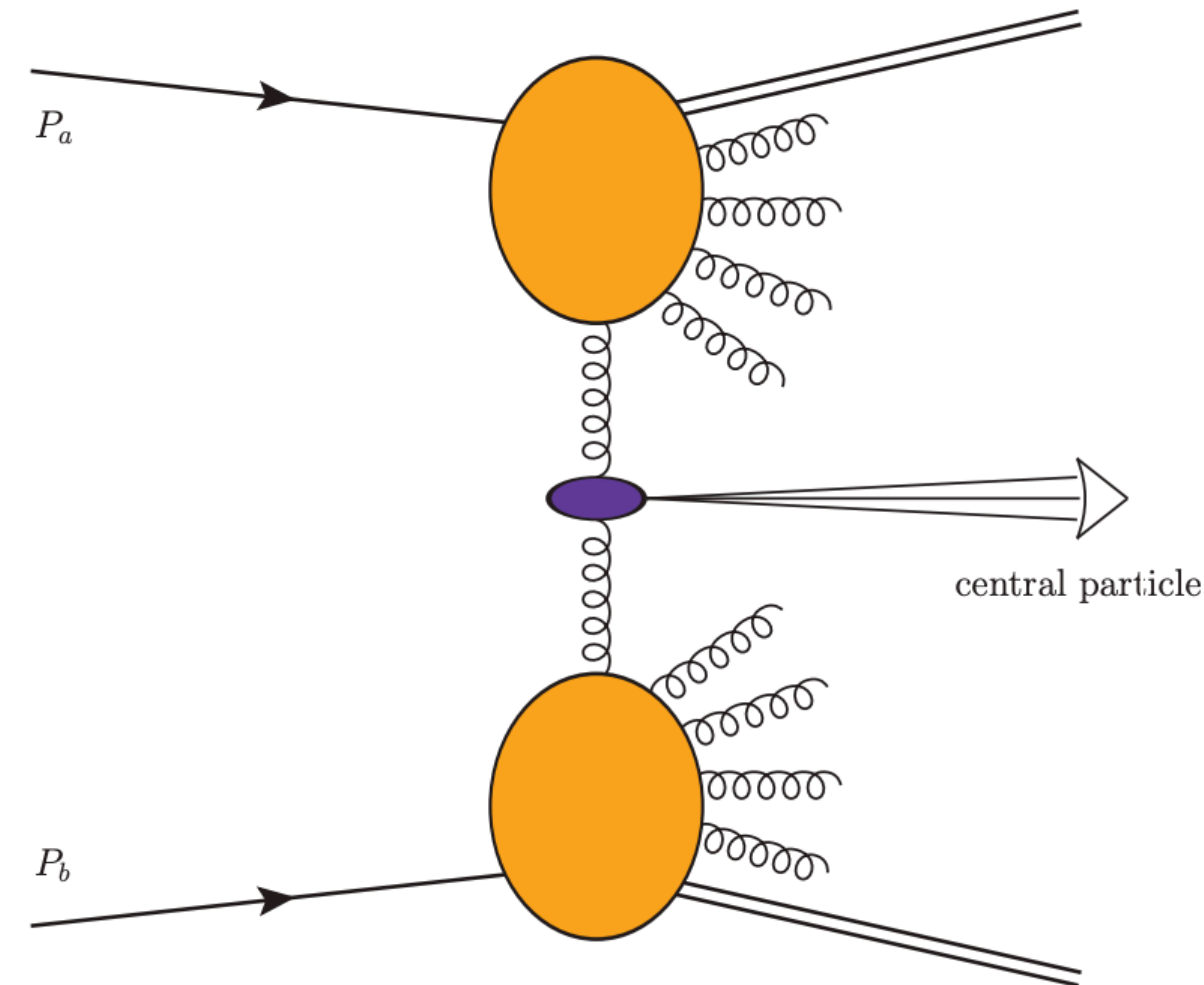
(c) Forward-backward

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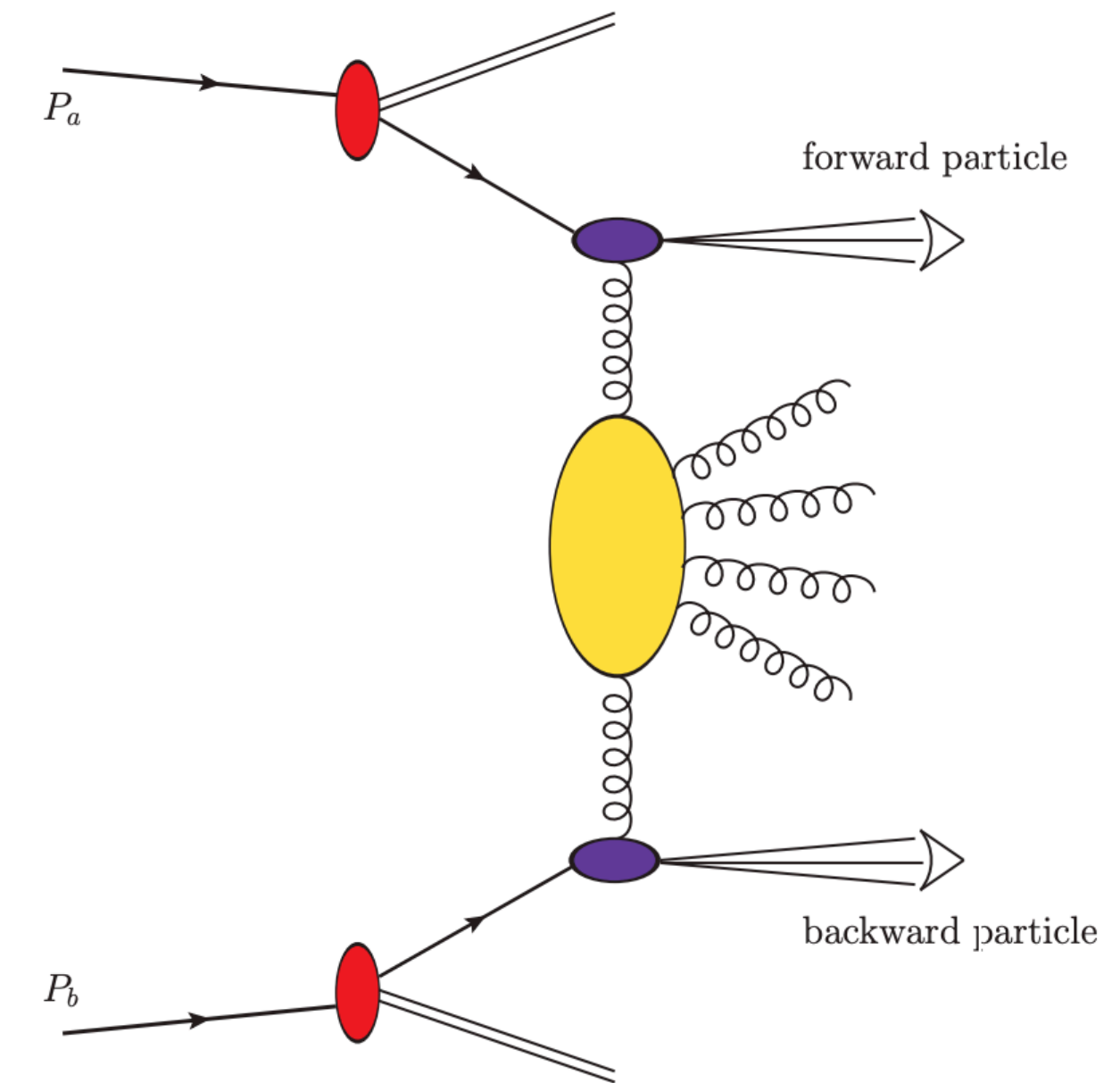
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(c) Forward-backward

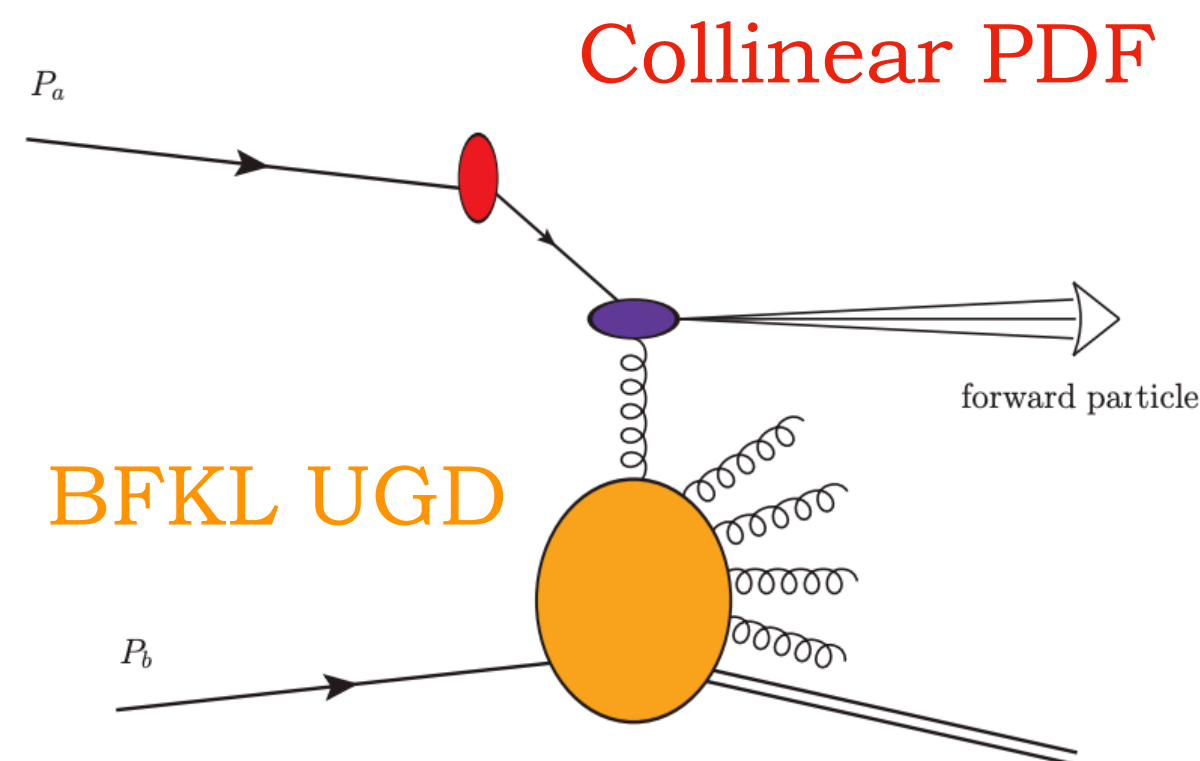
Fast q/g + small- x g

Hybrid factorization

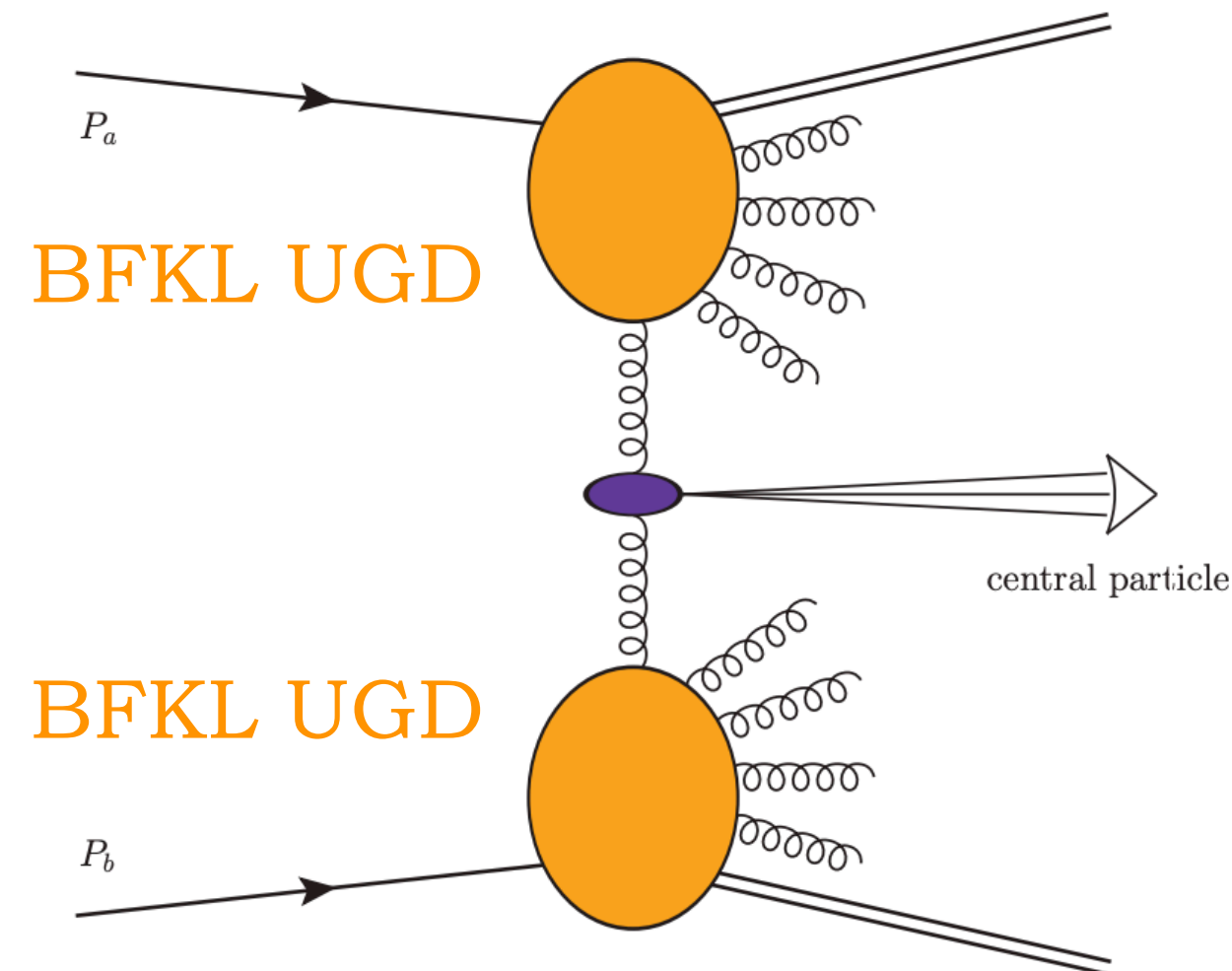
BFKL + Threshold

High-energy factorization at a glance

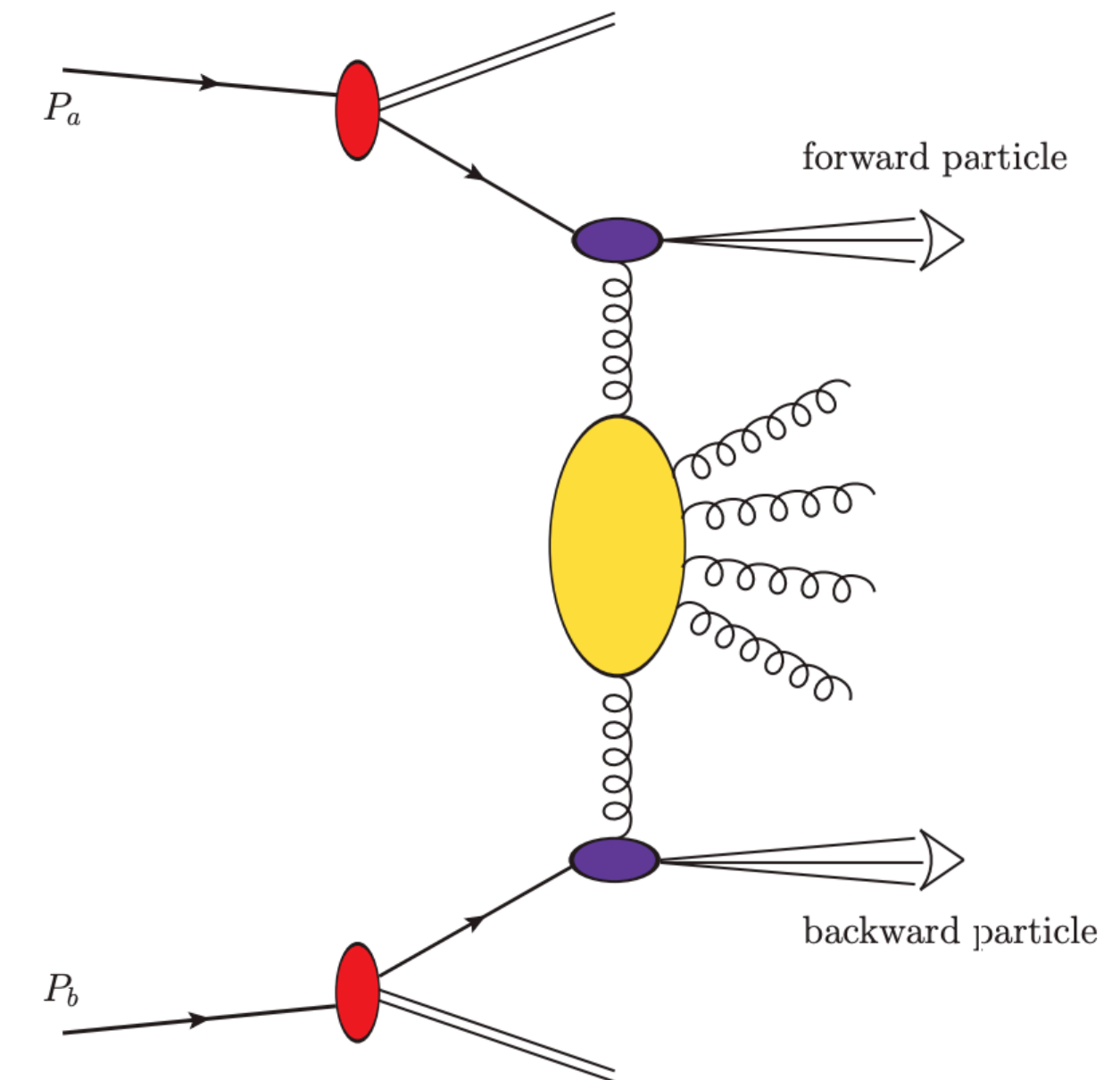
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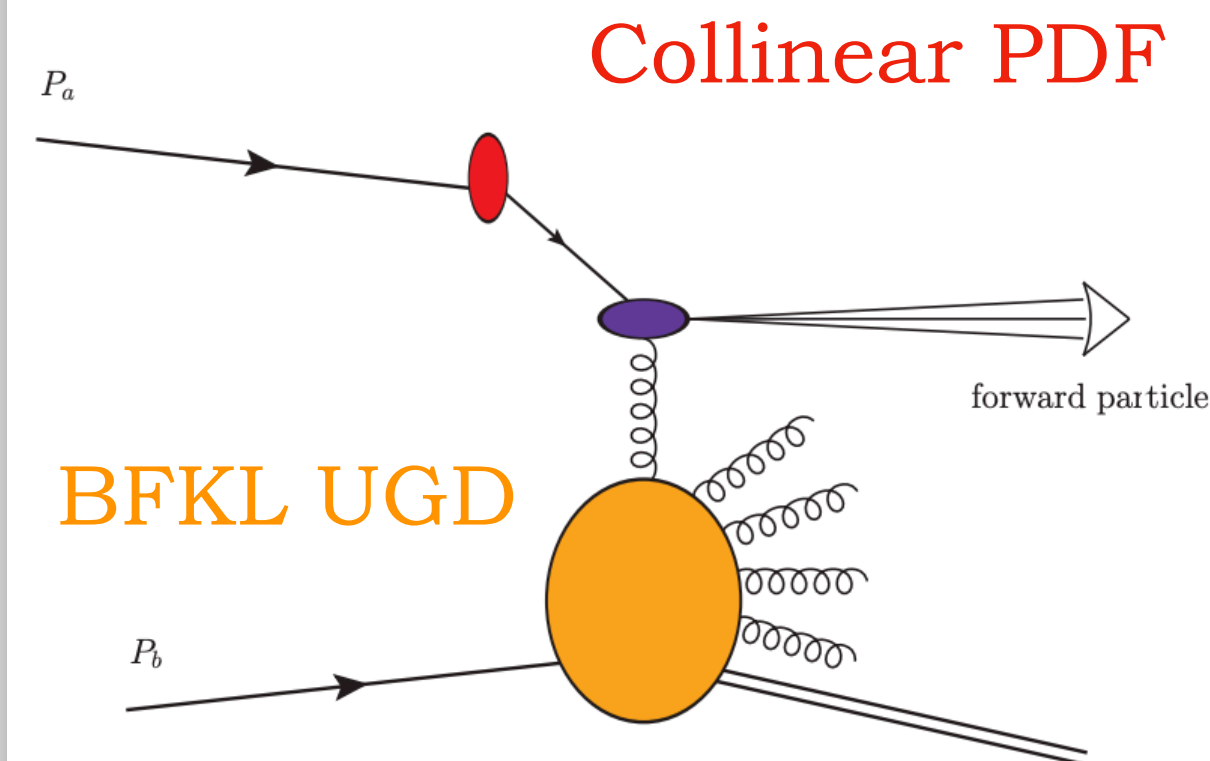
gg induced

High-energy factorization

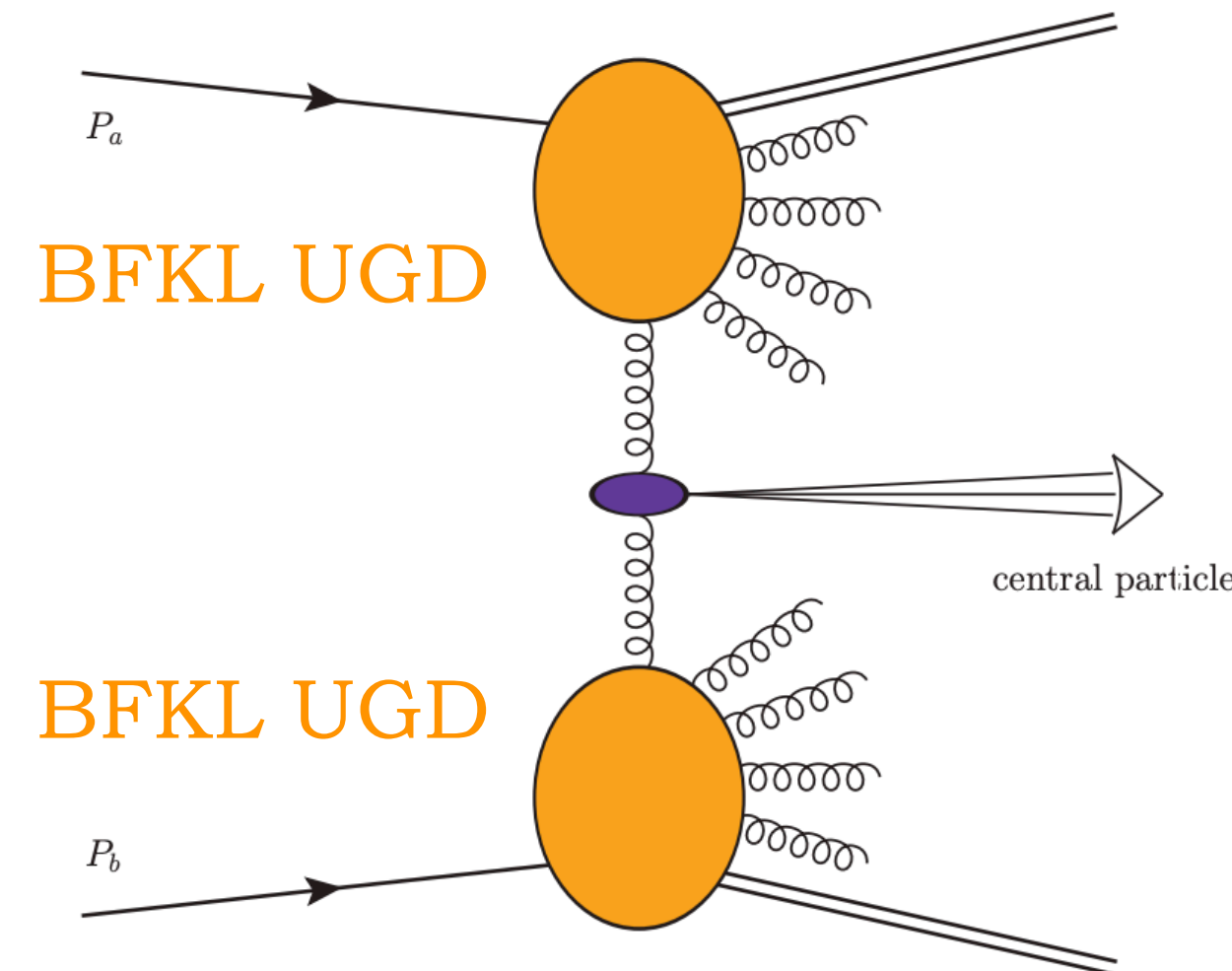
BFKL or small- x improved PDFs

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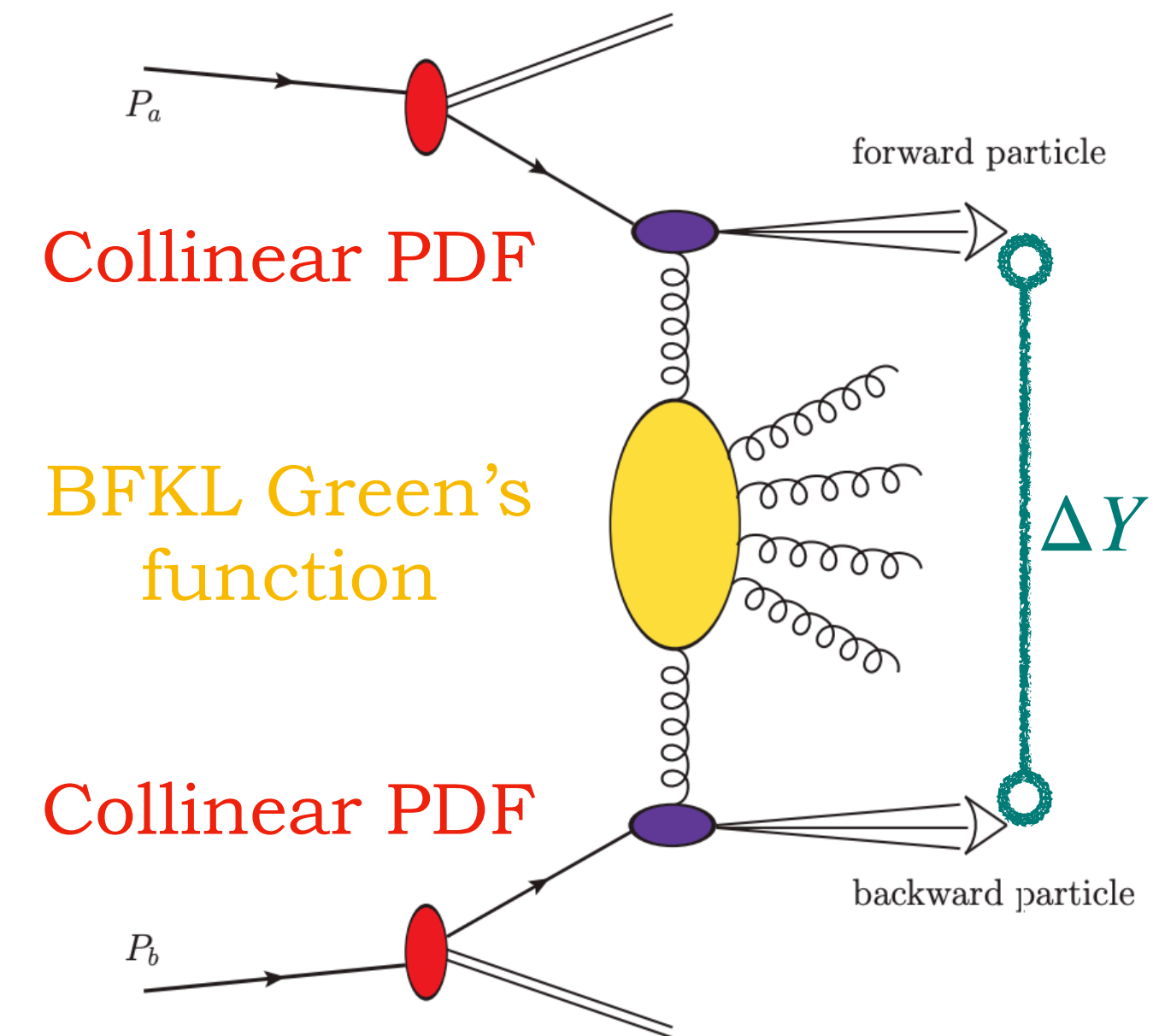
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(a) Single forward



(b) Single central



(c) Forward-backward

Fast q/g + small- x g

Hybrid factorization

BFKL + Threshold

gg induced

High-energy factorization

BFKL or small- x improved PDFs

🔗 [Tuesday's talk by Andreas]

Large rapidity distances, $\Delta Y \gg 1$

High energies, moderate x

PDFs + t-channel BFKL (NLL/NLO HyF)

Imbalance logs $\leftarrow$ back-to-back

Higgs production from LHC to FCC

PHYSICAL REVIEW LETTERS **120**, 202003 (2018)

Double **Resummation** for Higgs Production

Marco Bonvini^{1,*} and Simone Marzani^{2,†}

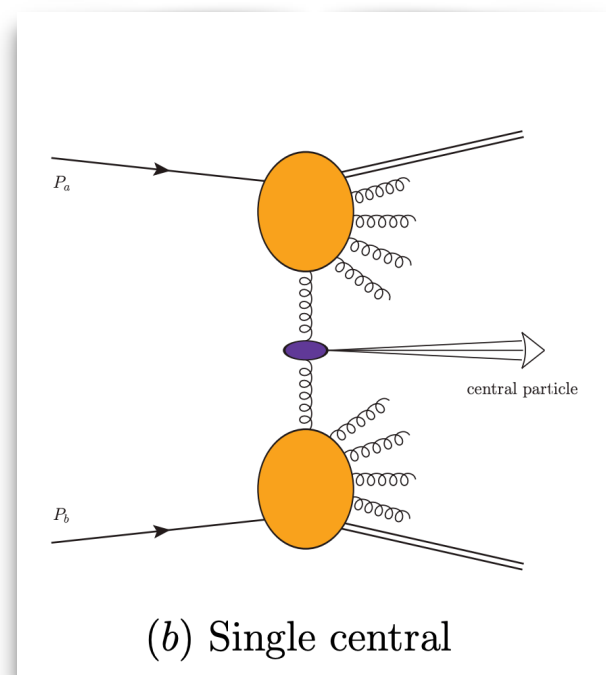
¹*INFN, Sezione di Roma 1, Piazzale Aldo Moro 5, 00185 Roma, Italy*

²*Dipartimento di Fisica, Università di Genova and INFN, Sezione di Genova, Via Dodecaneso 33, I-16146 Genova, Italy*

 (Received 26 February 2018; published 16 May 2018)

We present the first double-resummed prediction of the inclusive cross section for the main Higgs production channel in proton-proton collisions, namely, gluon fusion. Our calculation incorporates to all orders in perturbation theory two distinct towers of logarithmic corrections which are enhanced, respectively, at threshold, i.e., large x , and in the high-energy limit, i.e., small x . Large- x logarithms are resummed to next-to-next-to-next-to-leading logarithmic accuracy, while small- x ones to leading logarithmic accuracy. The double-resummed cross section is furthermore matched to the state-of-the-art fixed-order prediction at next-to-next-to-next-to-leading accuracy. We find that double resummation corrects the Higgs production rate by 2% at the currently explored center-of-mass energy of 13 TeV and its impact reaches 10% at future circular colliders at 100 TeV.

DOI: [10.1103/PhysRevLett.120.202003](https://doi.org/10.1103/PhysRevLett.120.202003)



High-energy resummation (BFKL) $\Rightarrow$ PDFs at small- x 

Altarelli-Ball-Forte  to stabilize the NLL_{sx} BFKL kernel 

$N^3LL_{lx}/LL_{sx}/N^3LO$ rapidity-inclusive coefficient functions

$$C_{ij}(x, \alpha_s) = \boxed{C_{ij}^{fo}}(x, \alpha_s) + \boxed{\Delta C_{ij}^{lx}}(x, \alpha_s) + \boxed{\Delta C_{ij}^{sx}}(x, \alpha_s)$$

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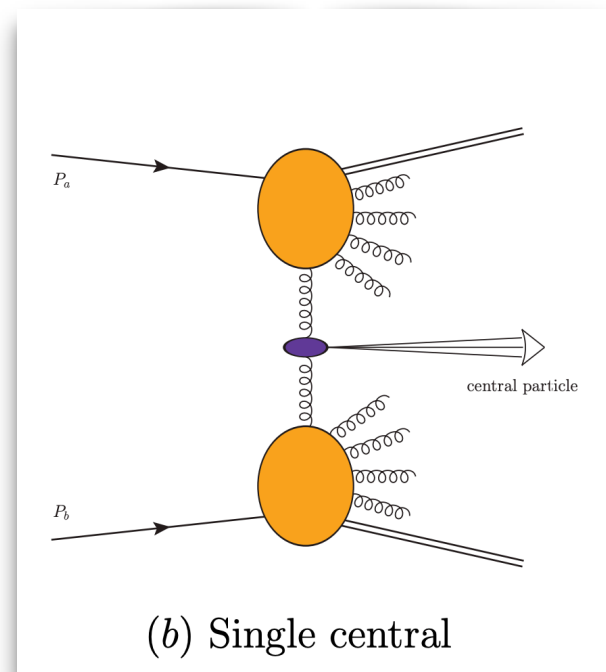
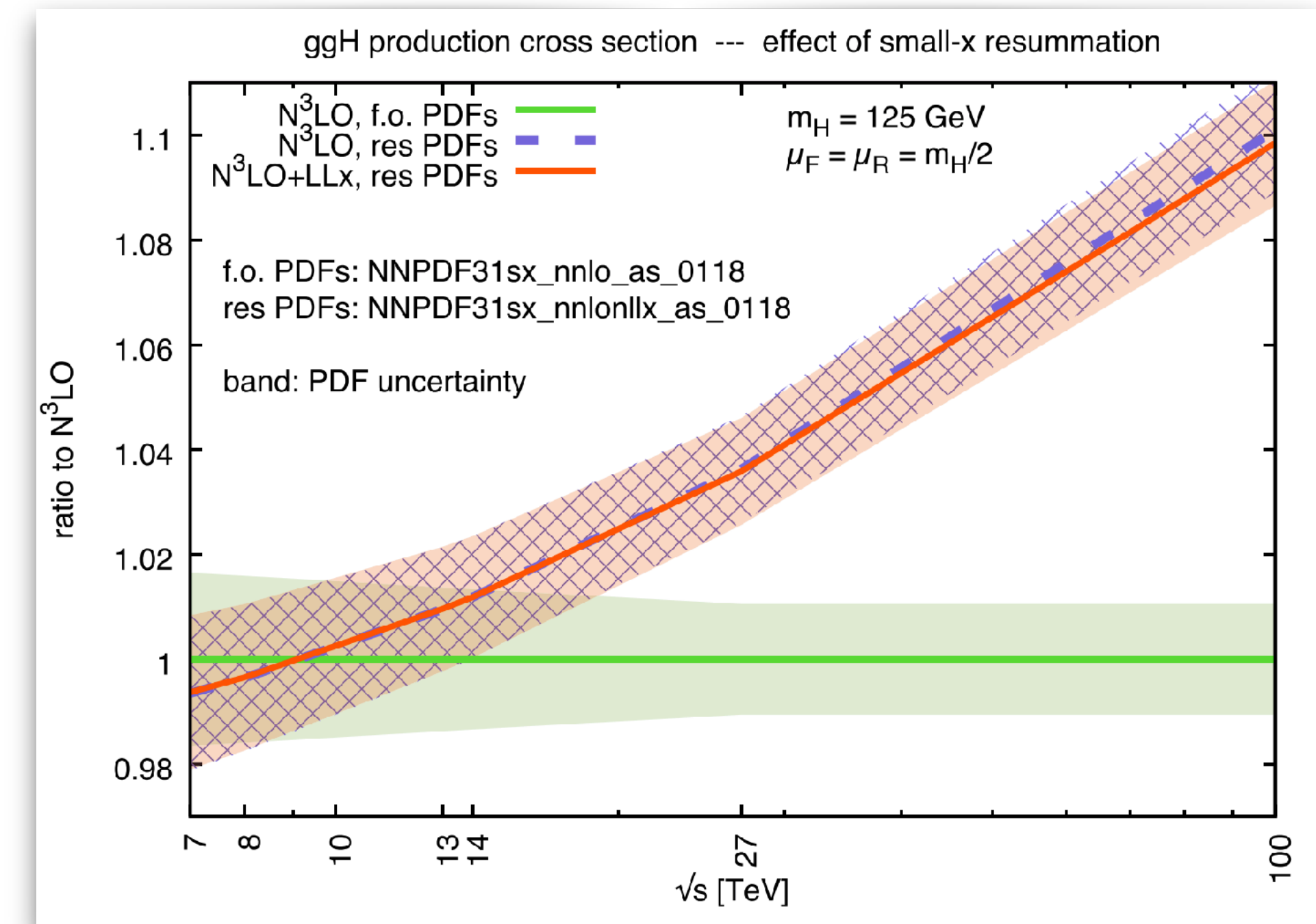
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$N^3LL_{lx}/LL_{sx}/N^3LO$ **rapidity-inclusive** coefficient functions

(i!) 100 TeV electroweak physics is **small- x** physics!

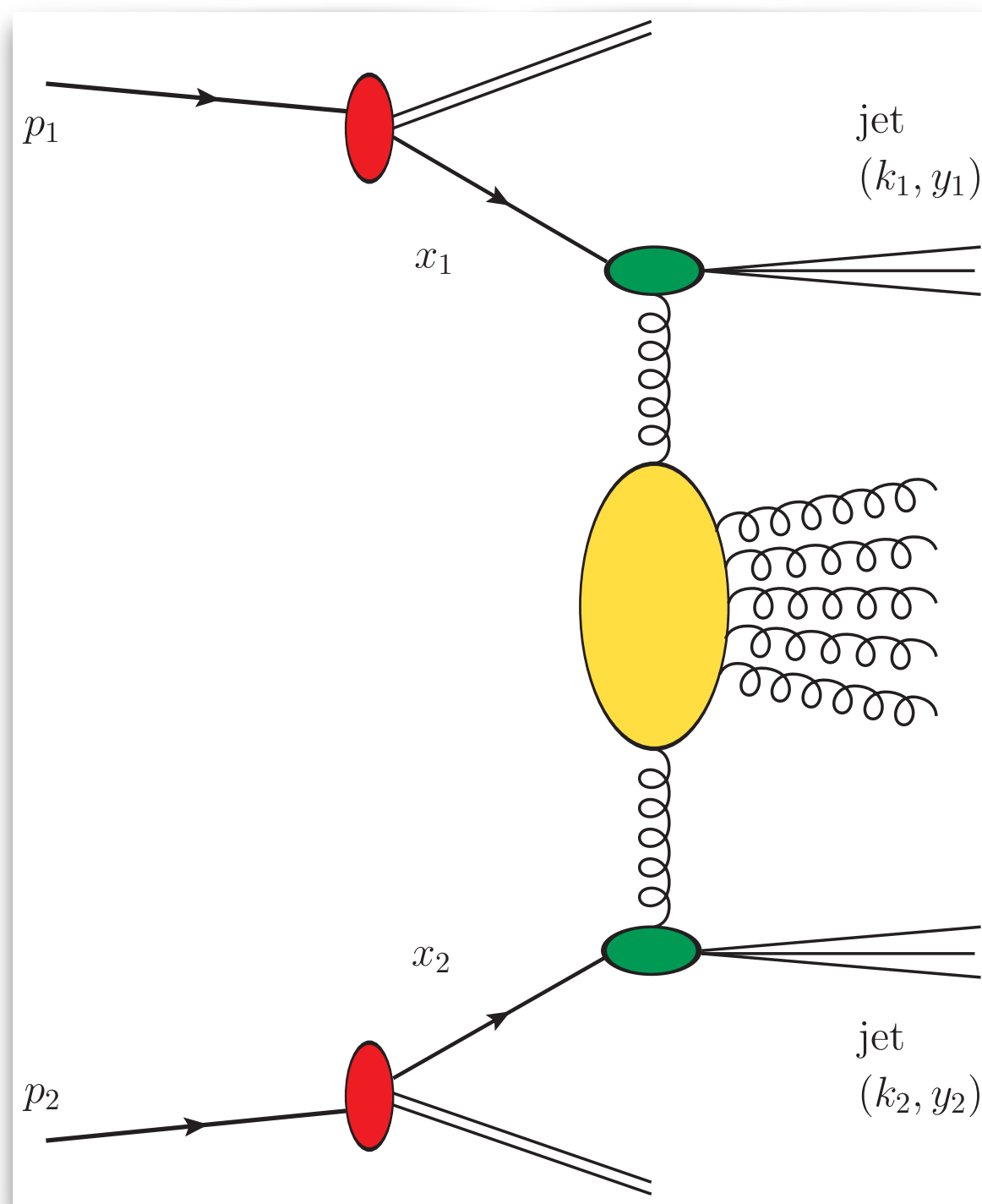
(e?) Can LHC physics be **BFKL** physics?

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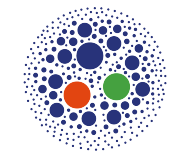
Mueller-Navelet jets @LHC & resummation instabilities

- Inclusive hadroproduction of two jets with high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) $\Rightarrow$ hybrid factorization (HyF)

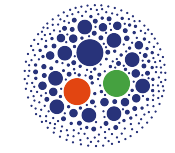
$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2 f_r(x_1, \mu_F) f_s(x_2, \mu_F) \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$



Mueller-Navelet jets @LHC & resummation instabilities



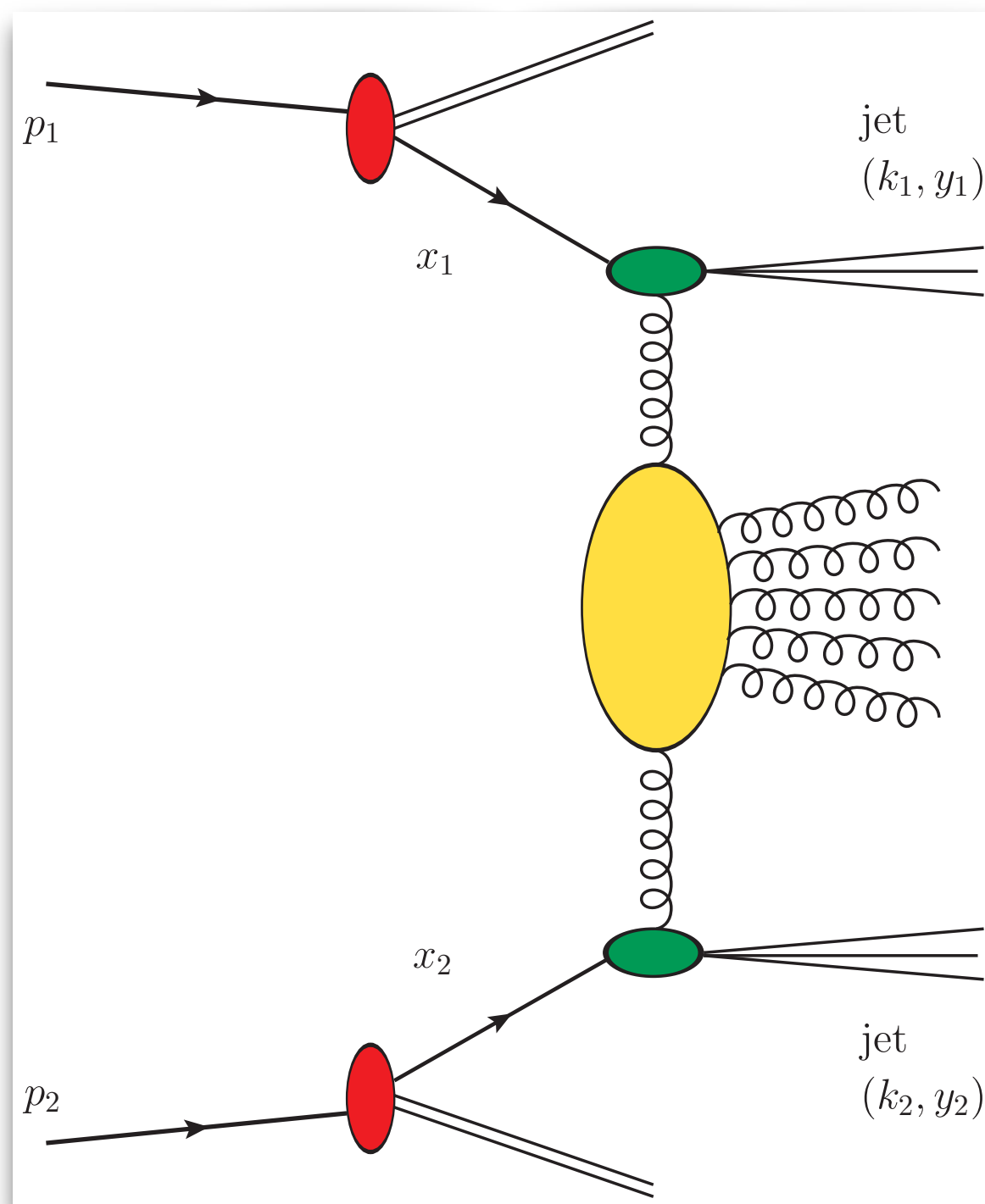
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jet vertices
(off-shell coefficient functions)



NLO(+)

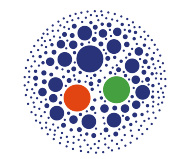
NLL

NLO(+)

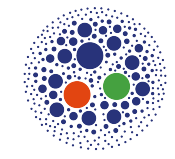
$$\begin{aligned} \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} &= \frac{1}{(2\pi)^2} \\ &\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^2} \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1) \\ &\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0} \right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2) \\ &\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^2} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{k}_2) \end{aligned}$$

BFKL Green's function

Mueller-Navelet jets @LHC & resummation instabilities



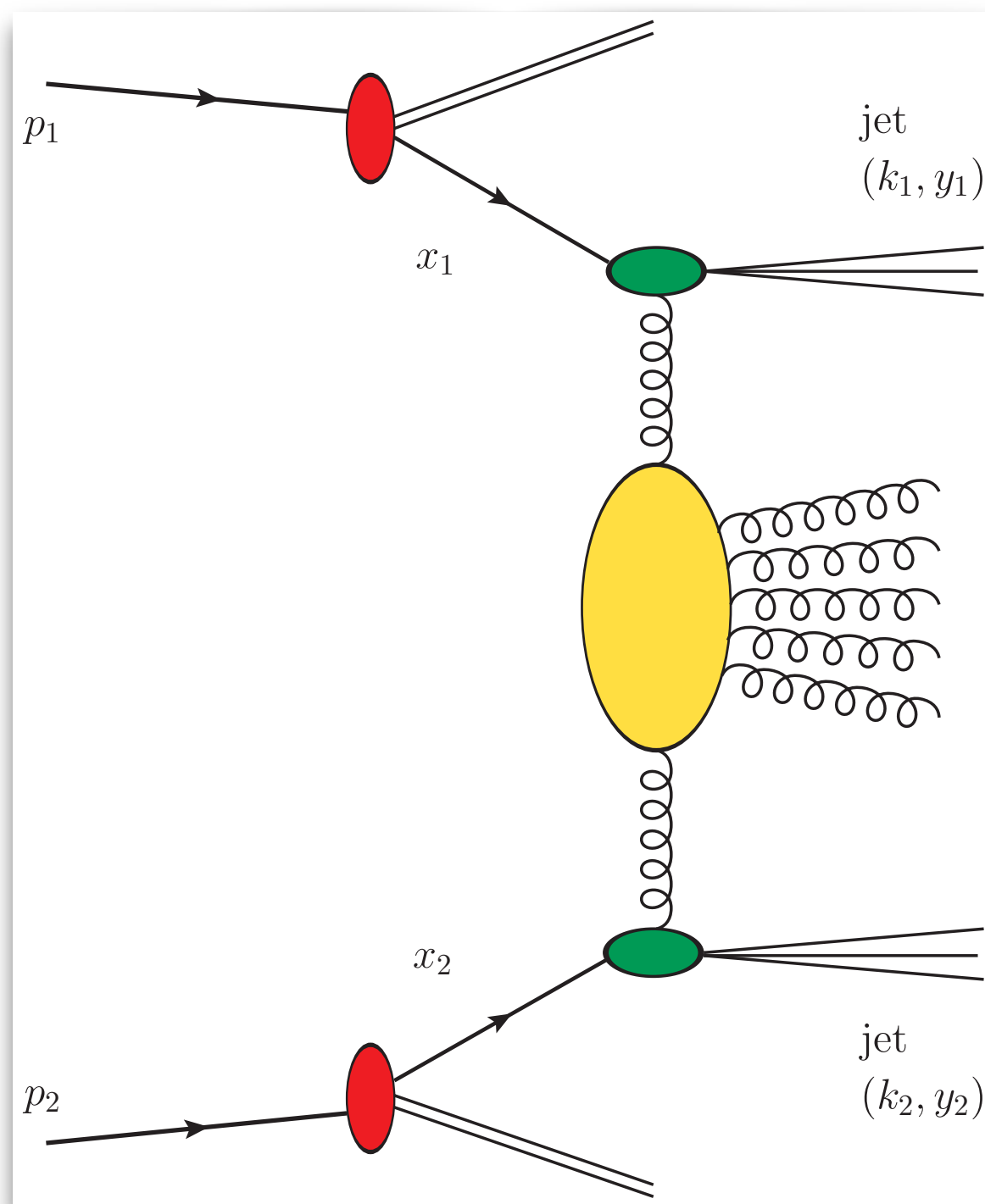
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NLO(+)

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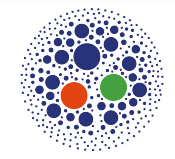
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BFKL Green's function



NLL/LL instabilities + NLO missing threshold $\Rightarrow$ precision studies hampered

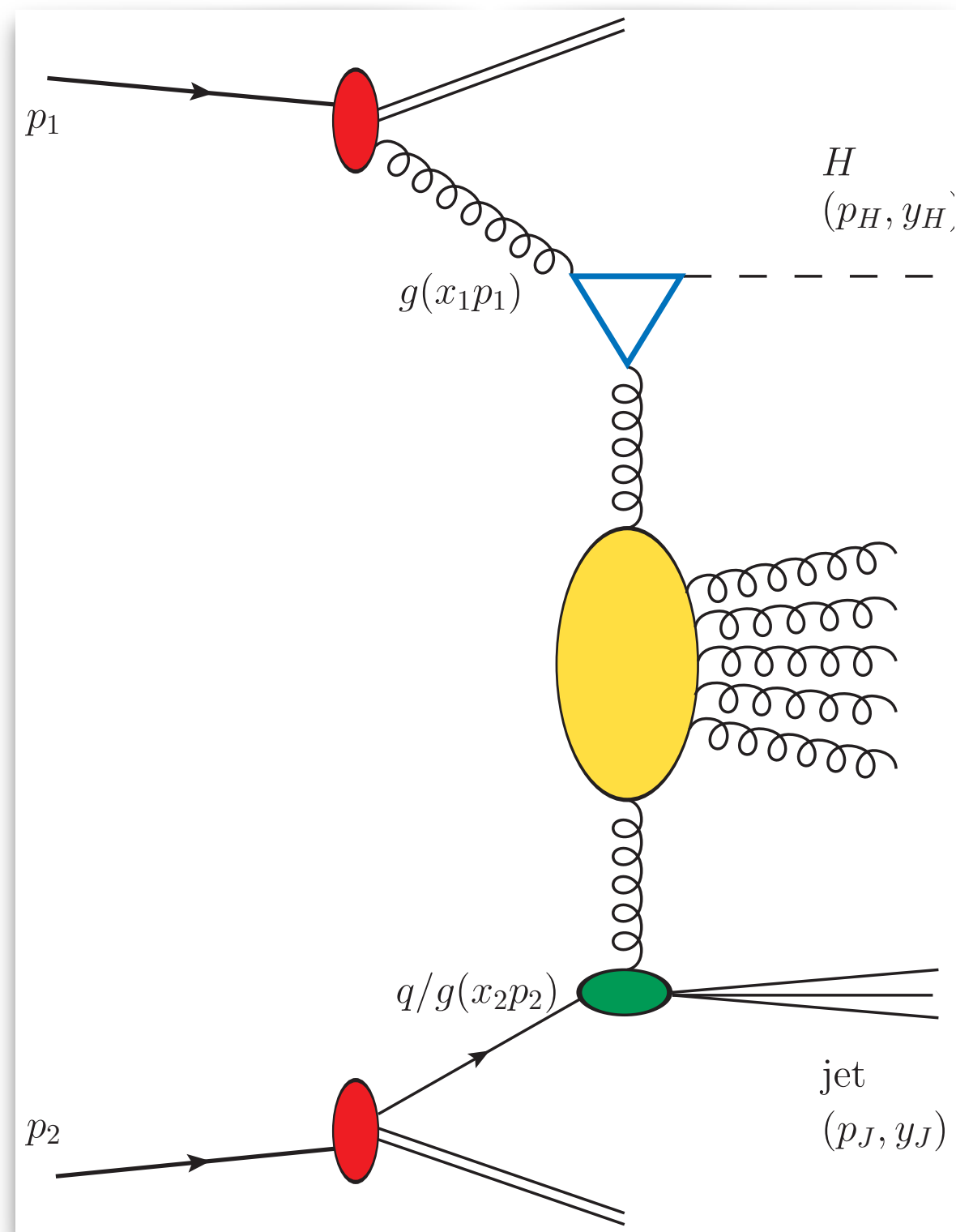
From Mueller-Navelet to Higgs and heavy flavor



Pheno path: hunt for channels leading to a NLL **stabilization pattern** at **natural scales** (¡!)

HIGGS BOSON

Stabilizers $\Leftrightarrow$ large Higgs transverse masses

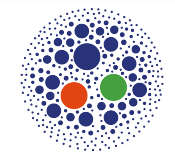


(Higgs + jet, NLL/NLO*) [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

(NLO Higgs coeff. function) [F. G. C., M. Fucilla et al., JHEP 08 (2022) 092]

[Today's talk by Michael]

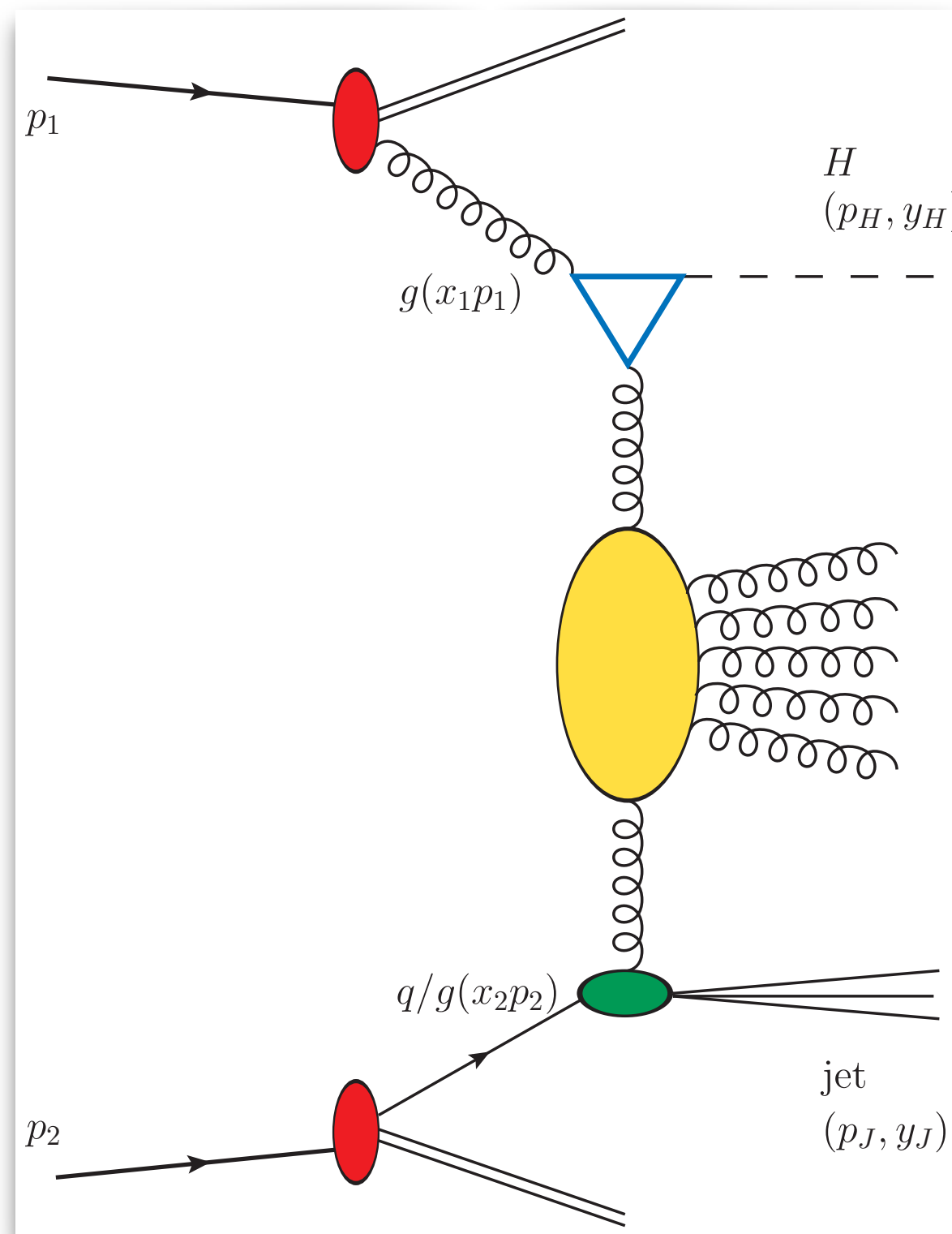
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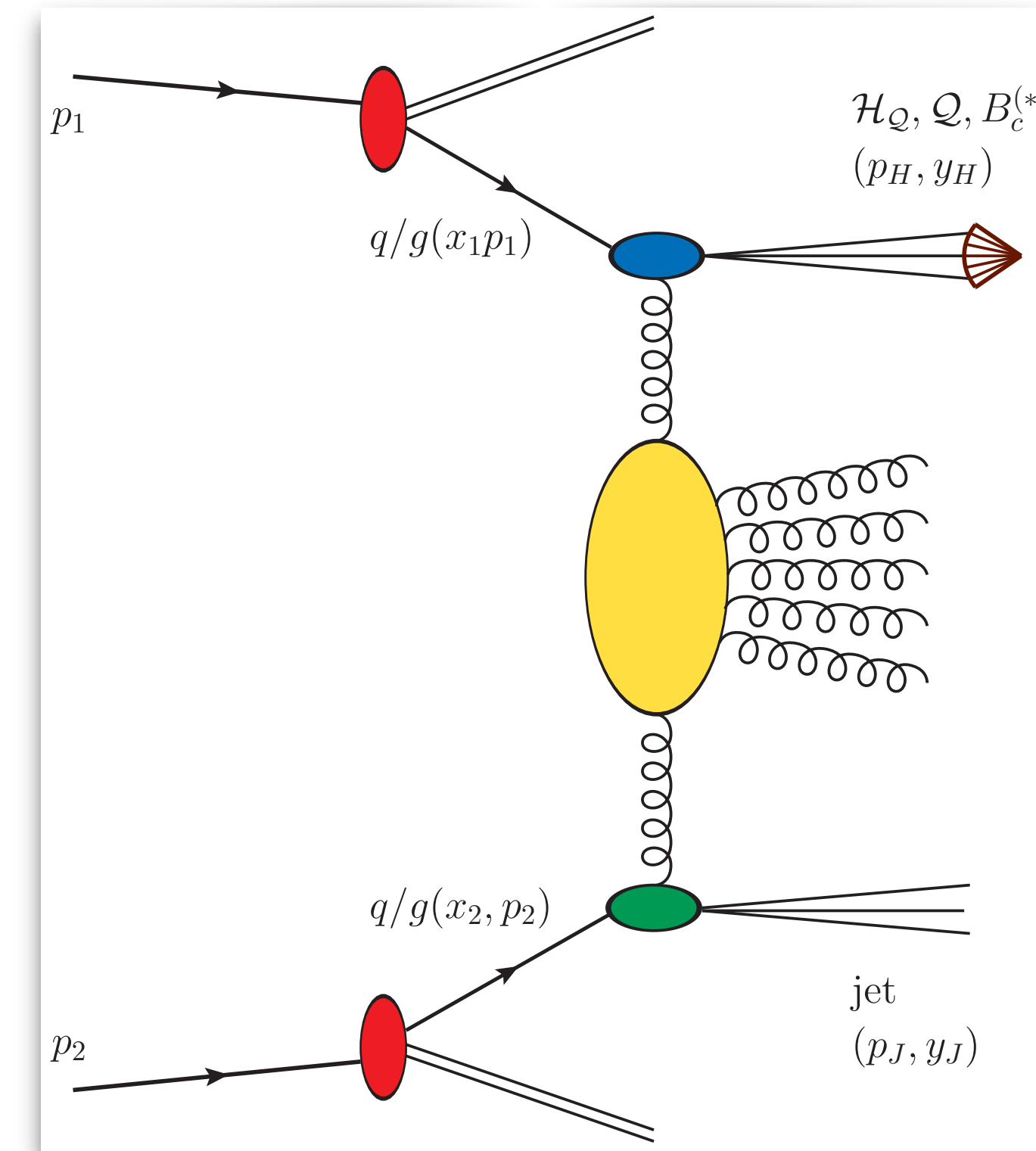
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$\otimes$ [Today's talk by Michael]

HEAVY FLAVOR AT LARGE P_T

Stabilizers $\Leftrightarrow$ gluon fragmentation channels

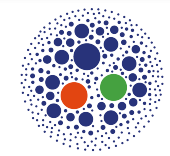


($\Lambda_c^\pm$ baryons, NLL/NLO) $\otimes$ [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

(J/ψ or Υ , NLL/NLO) $\otimes$ [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

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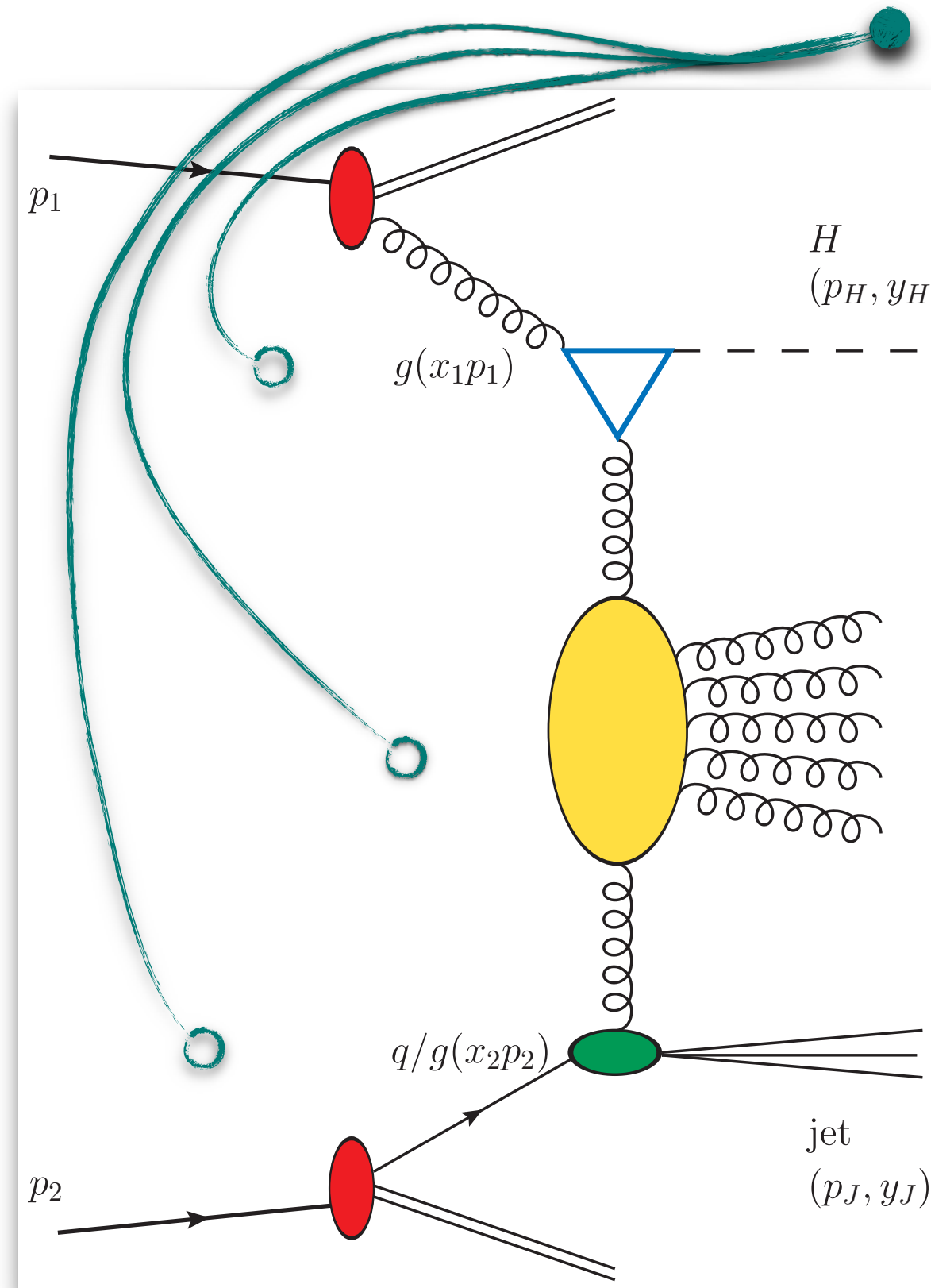
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HIGGS BOSON

Stabilizers $\Leftrightarrow$ large Higgs **transverse masses**



$$\mu_{F,R} \sim M_{H,\perp}$$

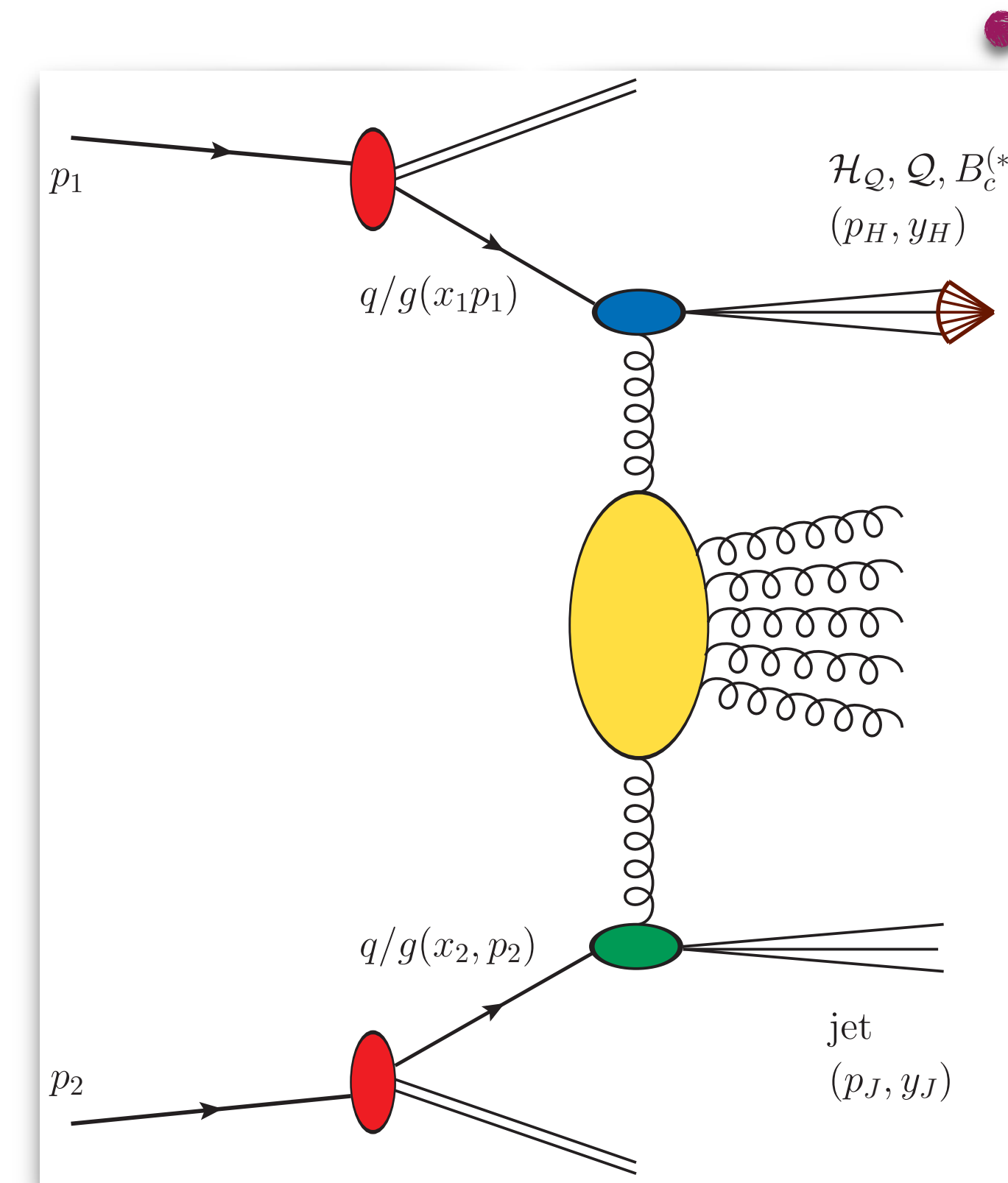
NLO*

NLL

NLO*

HEAVY FLAVOR AT LARGE P_T

Stabilizers $\Leftrightarrow$ **gluon fragmentation** channels



NLO(+)

NLL

NLO(+)

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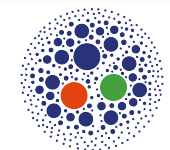
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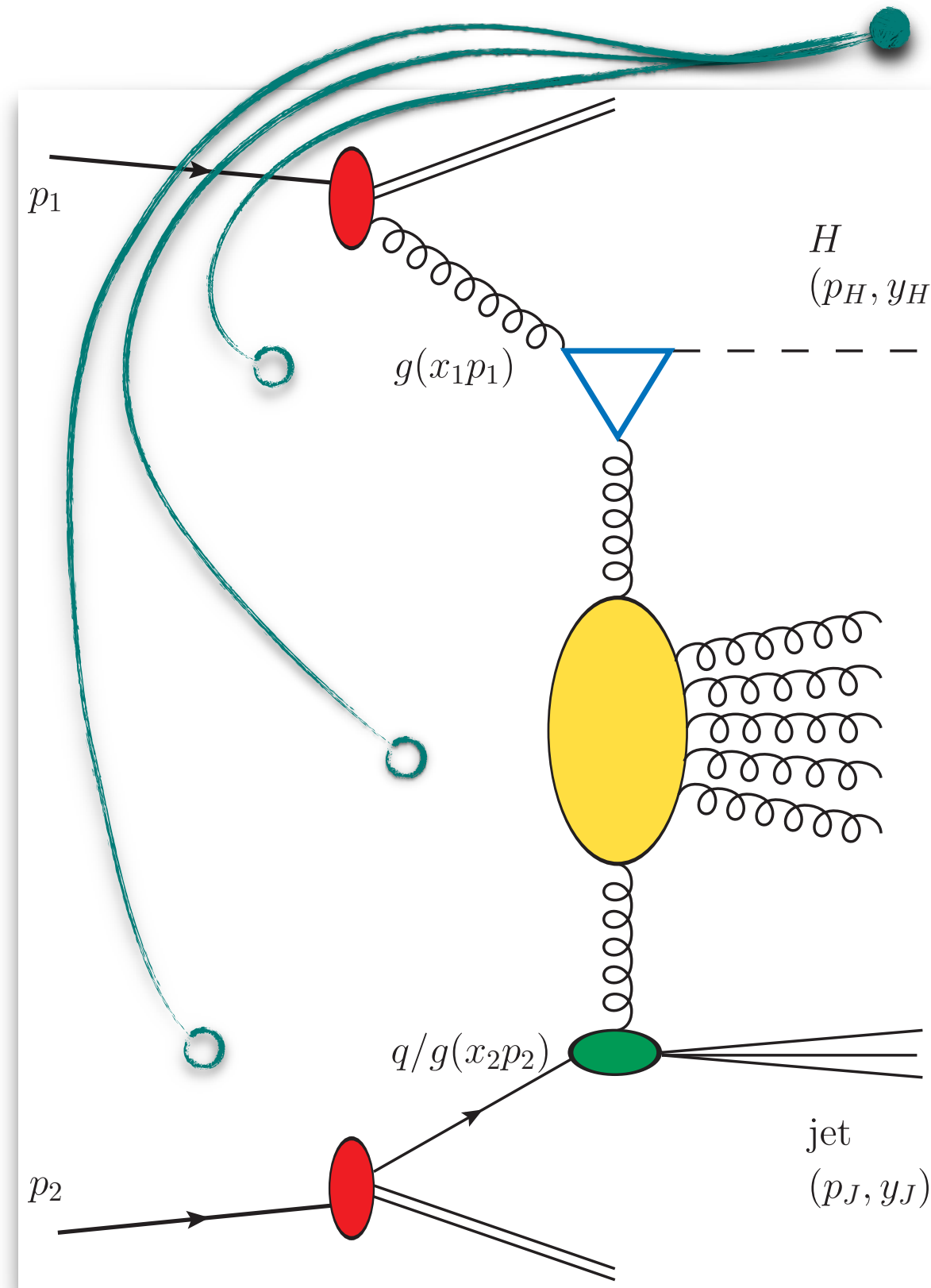
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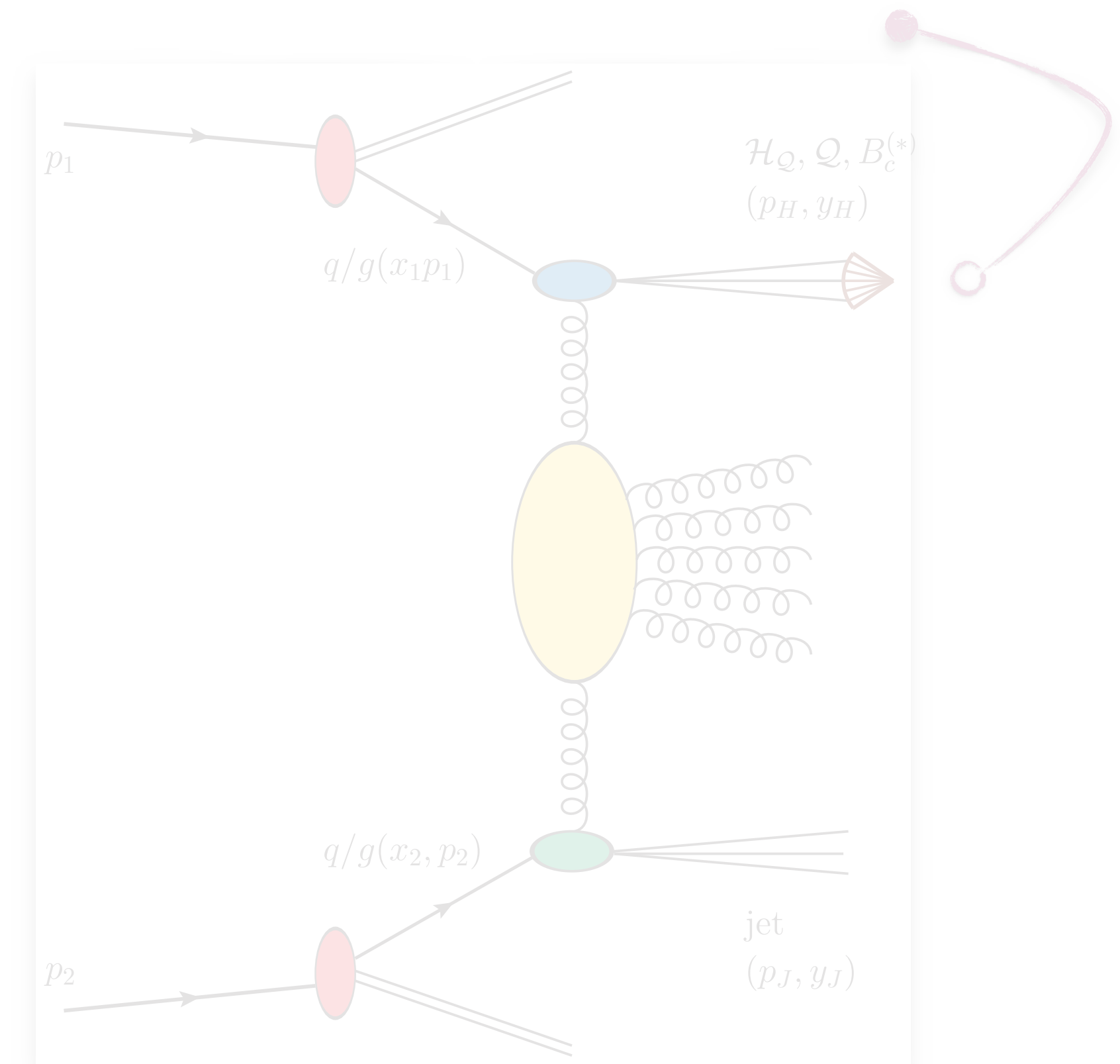
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NLO(+)

NLL

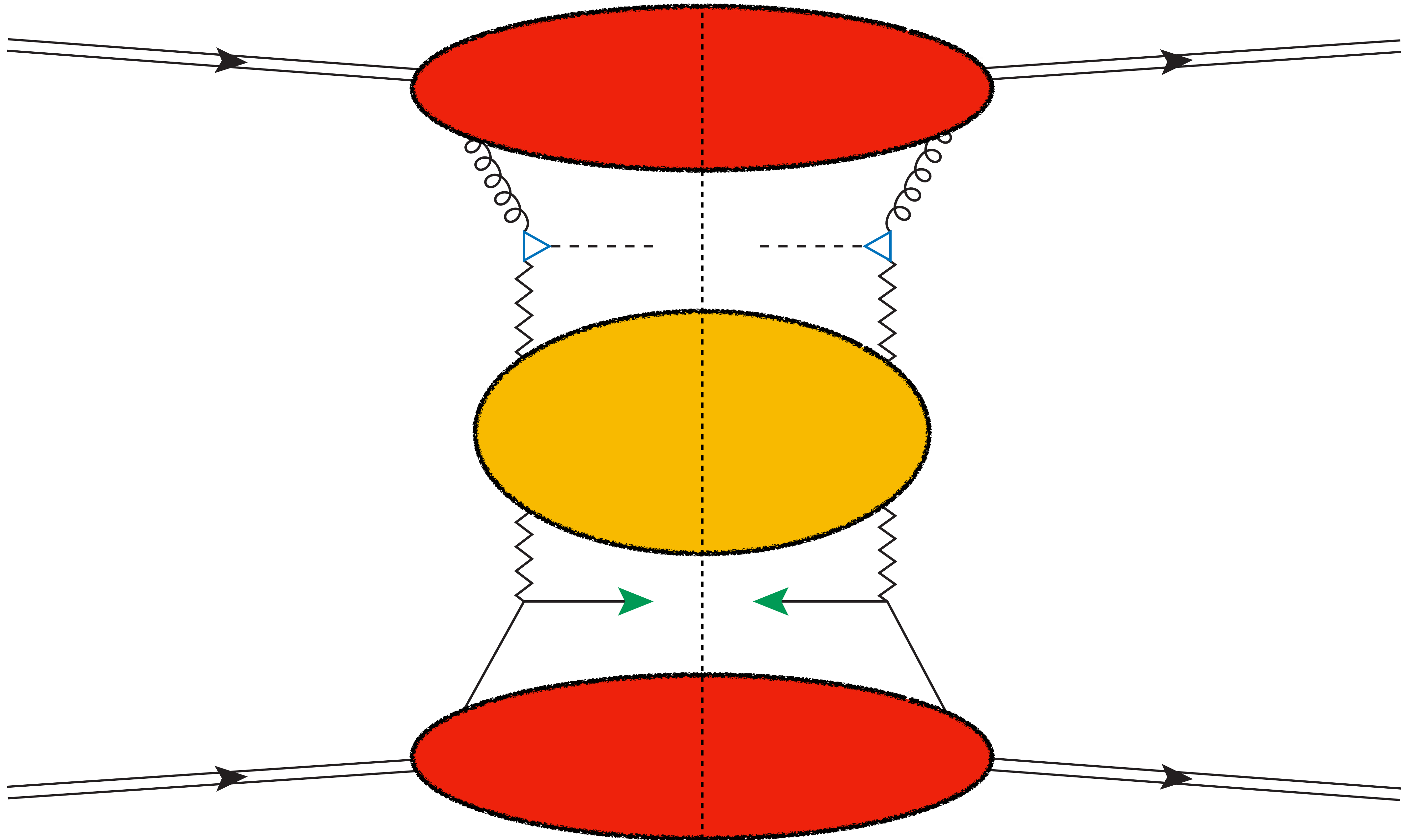
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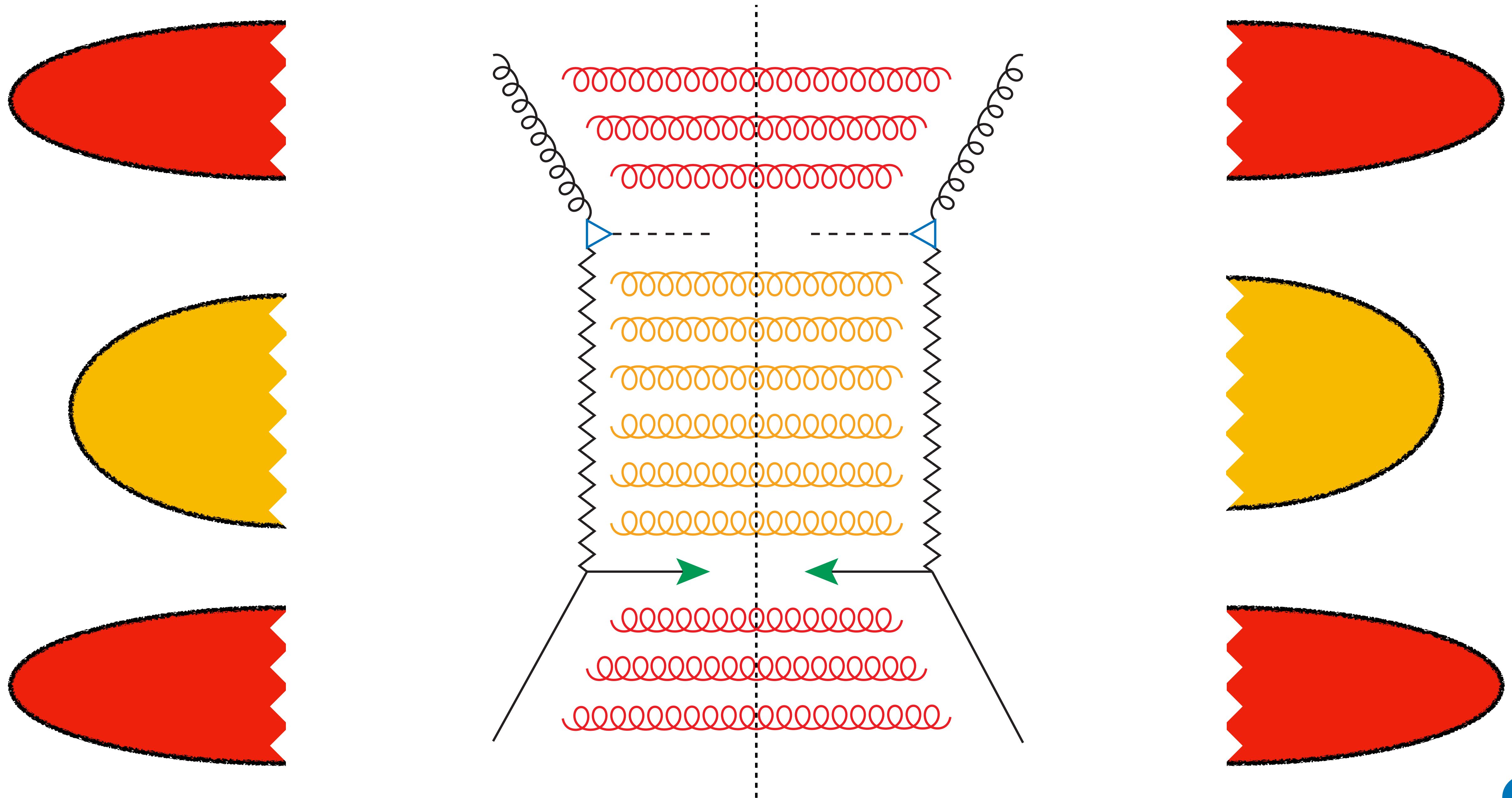
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Anatomy of Higgs + jet in hybrid factorization (HyF)

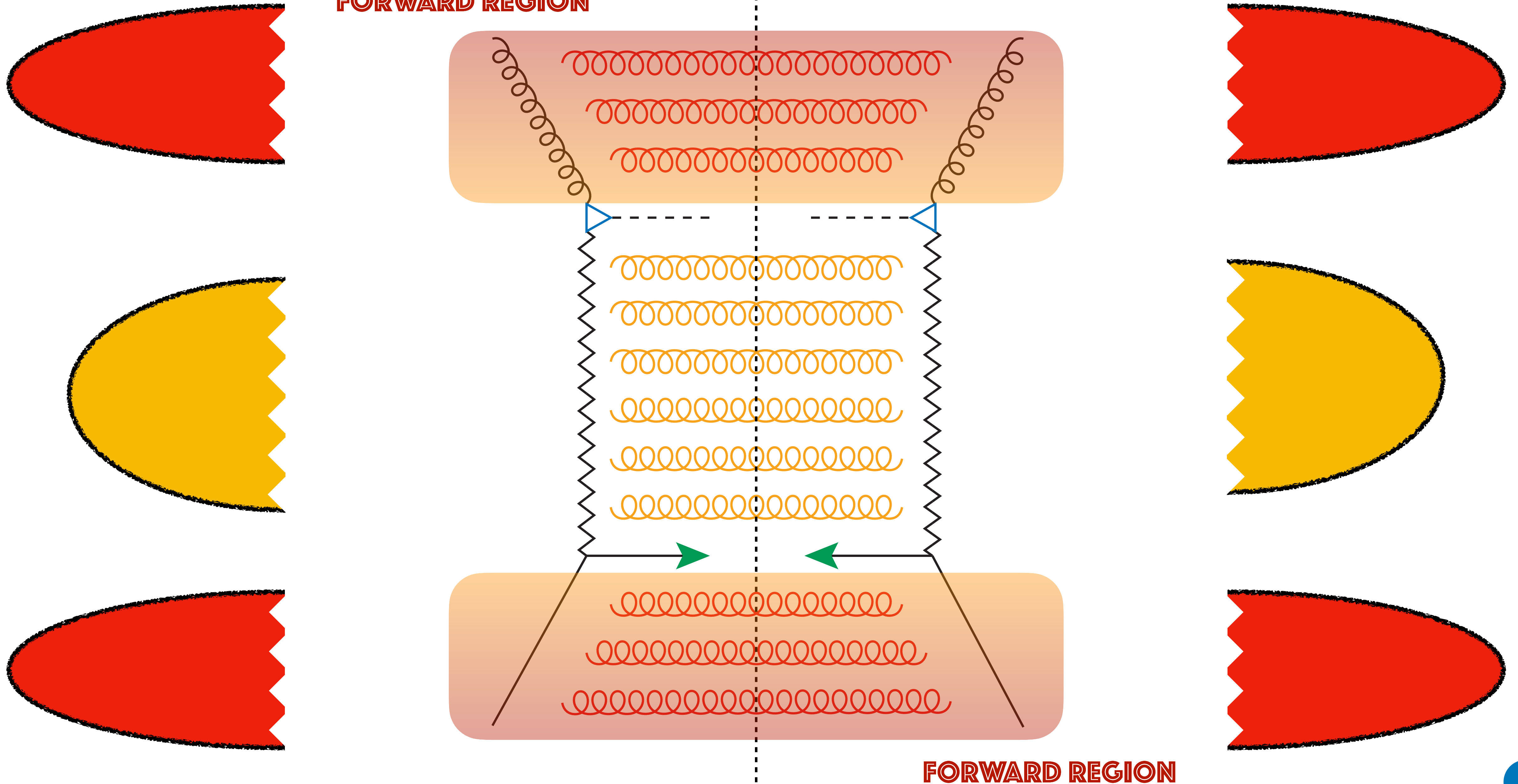


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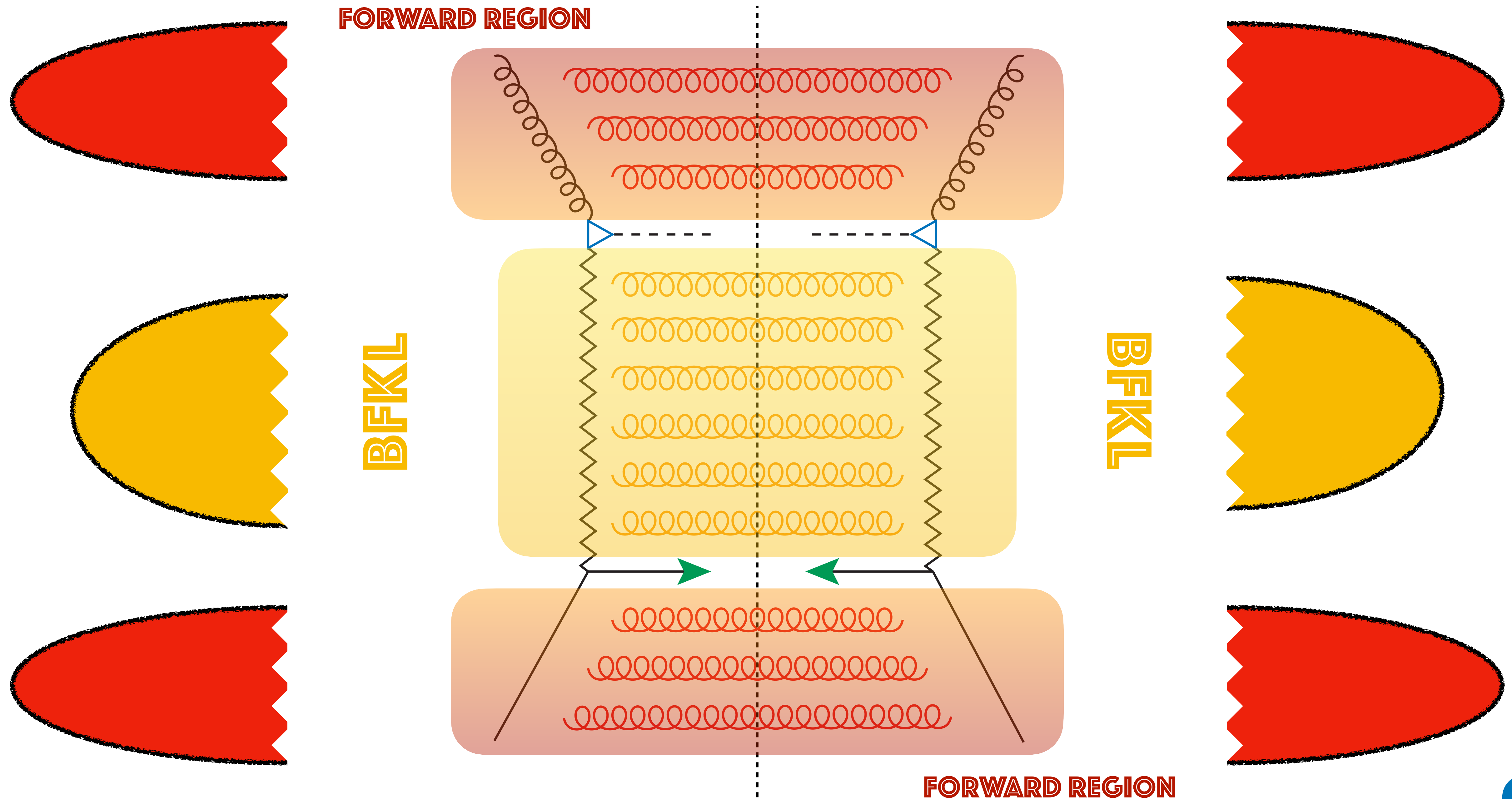


Anatomy of Higgs + jet in hybrid factorization (HyF)

FORWARD REGION



Anatomy of Higgs + jet in hybrid factorization (HyF)

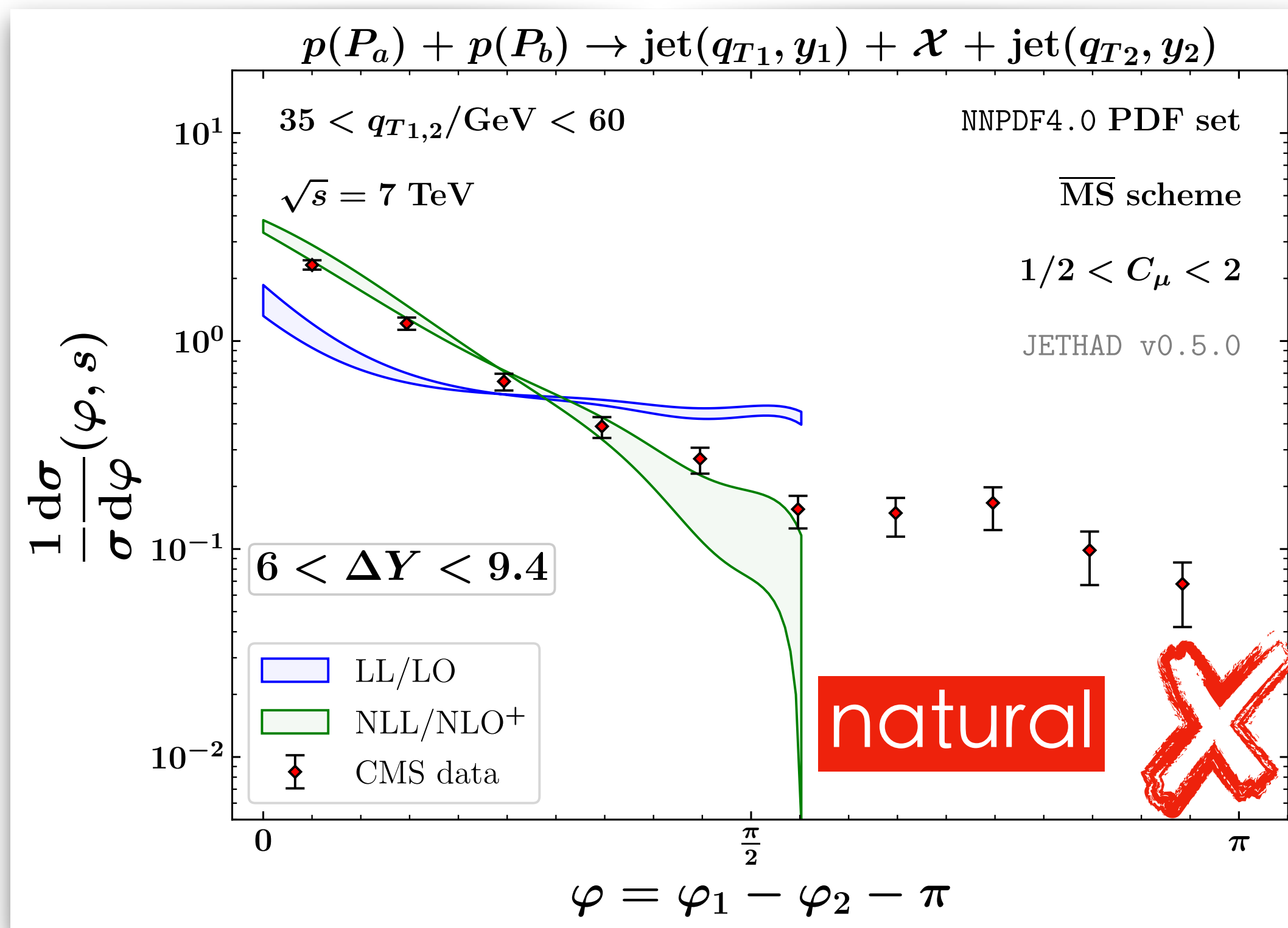


Azimuthal-angle multiplicity

$$\frac{1}{\sigma} \frac{d\sigma(\Delta Y, s)}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\}$$

MUELLER-NAVELET JETS

[\[B. Ducloué, L. Szymanowski, S. Wallon, Phys. Rev. Lett. 112 \(2014\) 082003\]](#)
 (figure below) [\[F. G. C., A. Papa, Phys. Rev. D 106 \(2022\) 11, 114004\]](#)



Azimuthal-angle multiplicity

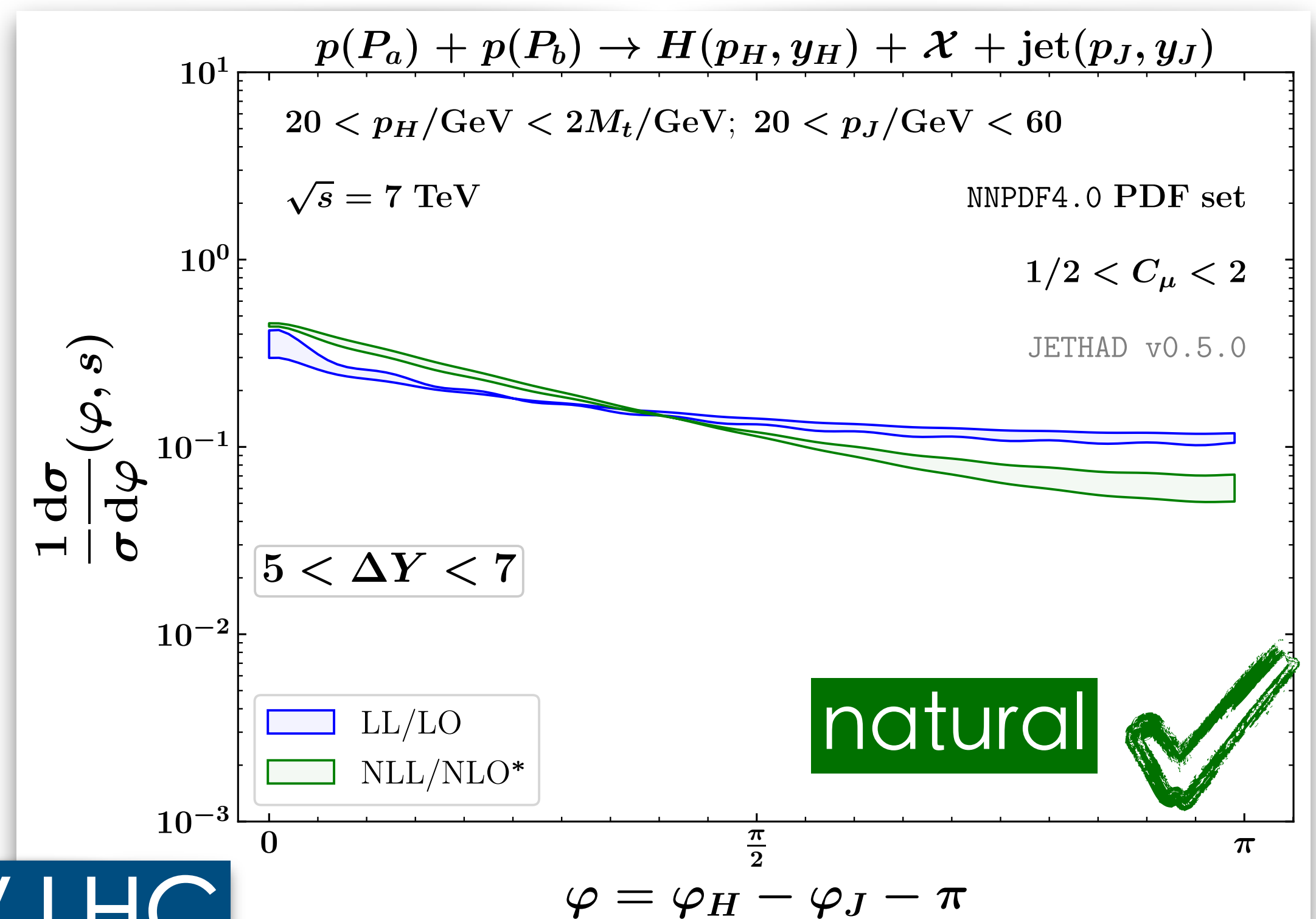
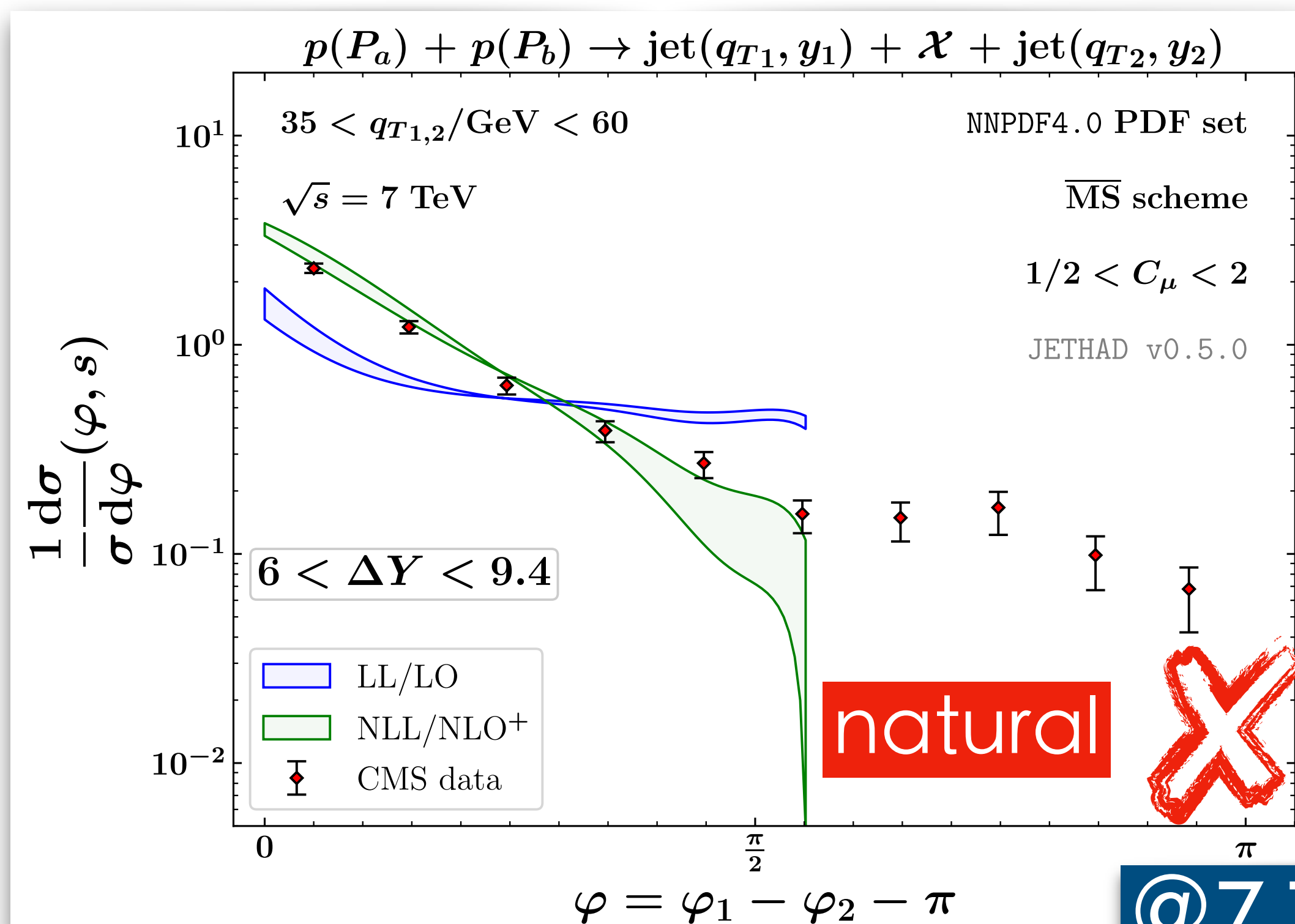
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(figure below) ⌋ [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

HIGGS + JET

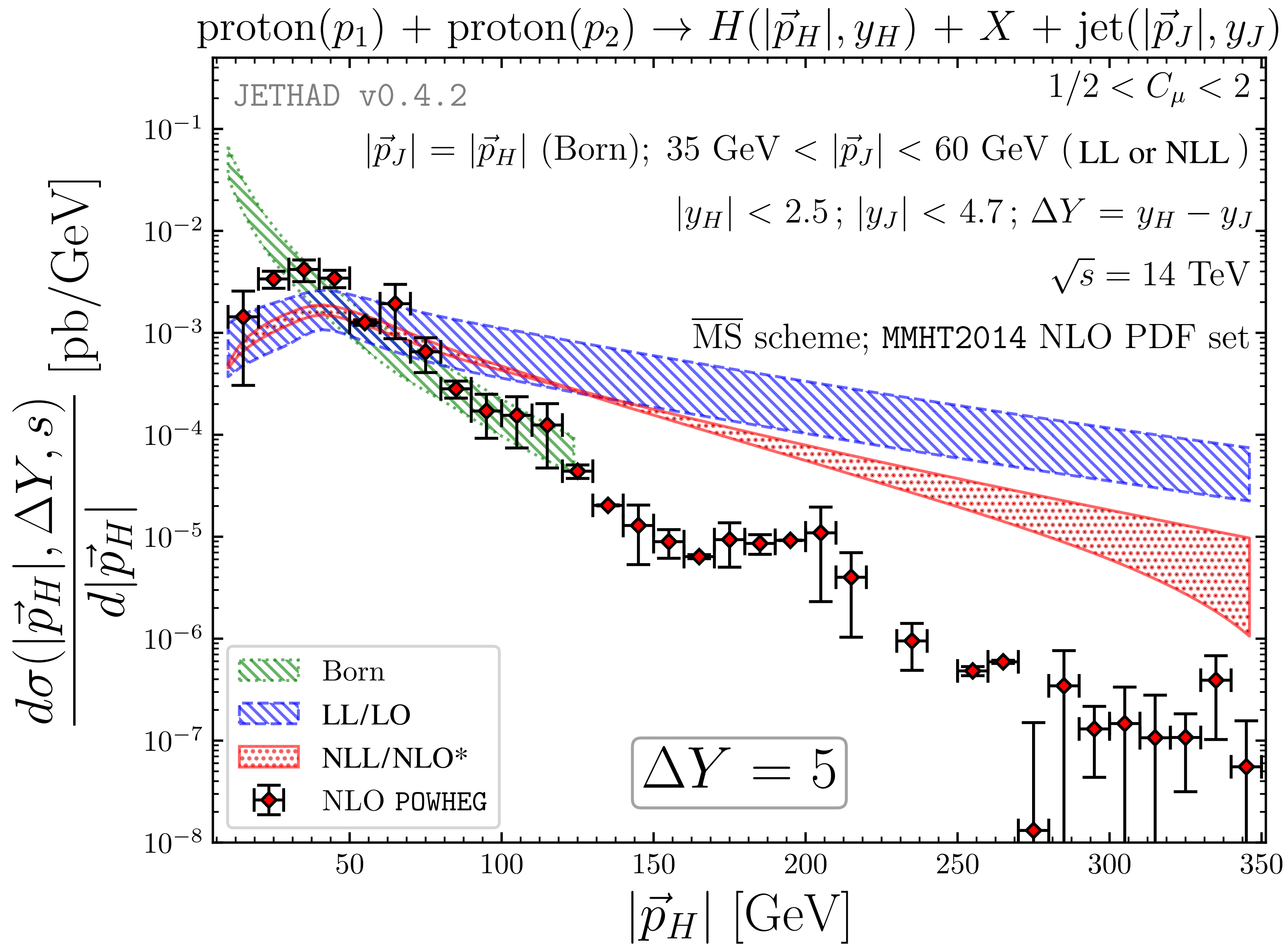
(figure below) ⌋ [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]
(NLO Higgs coefficient function) ⌋ [F. G. C. et al., JHEP 08 (2022) 092]



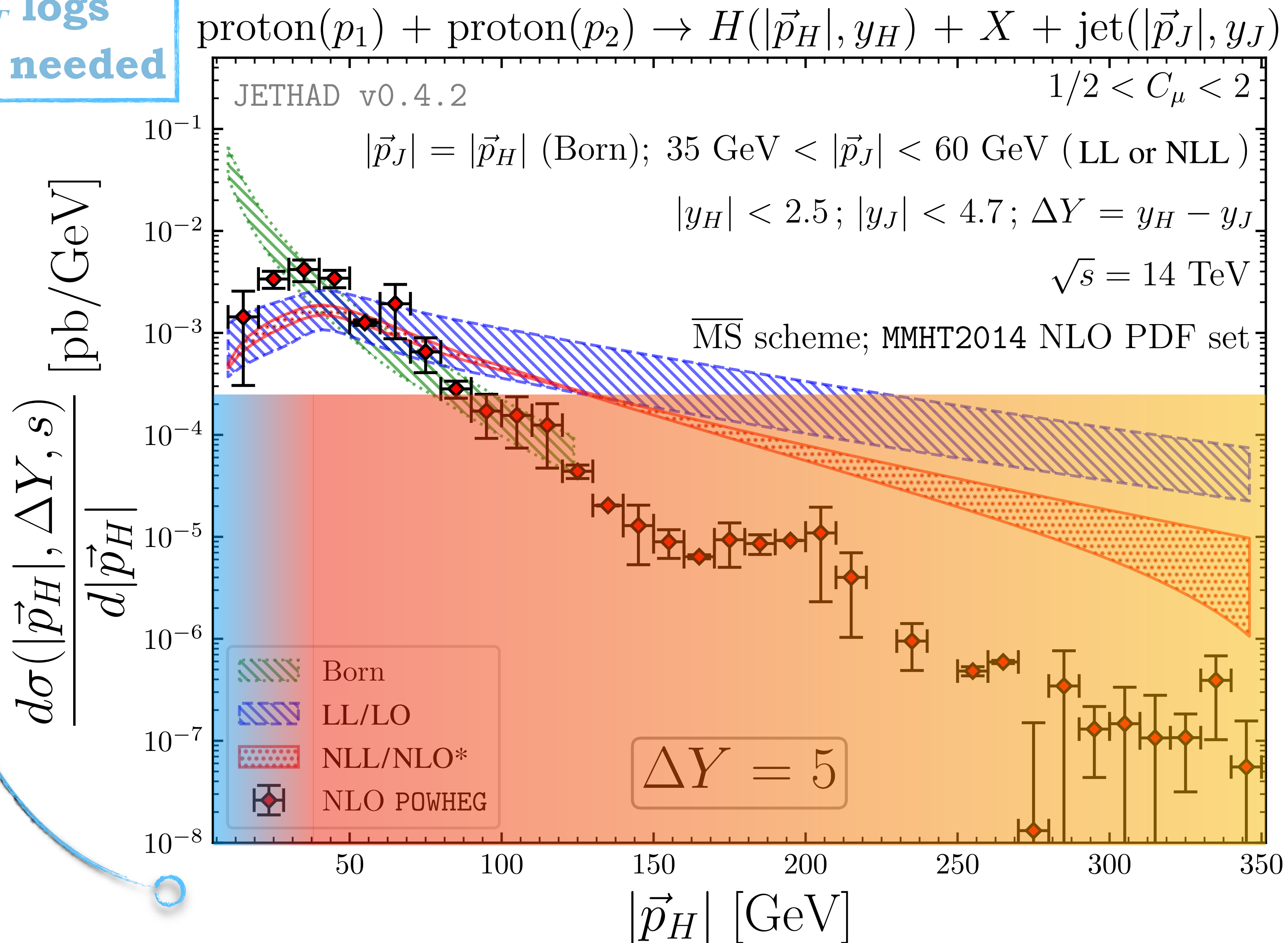
@7 TeV LHC

The background of the slide features several overlapping, semi-transparent diagrams of particle physics processes. These diagrams show various particles, represented by colored spheres (red, blue, green) and wavy lines (yellow), interacting in circular or semi-circular paths. Some diagrams include arrows indicating the direction of particle flow. The overall aesthetic is scientific and technical, with a light blue and green color palette.

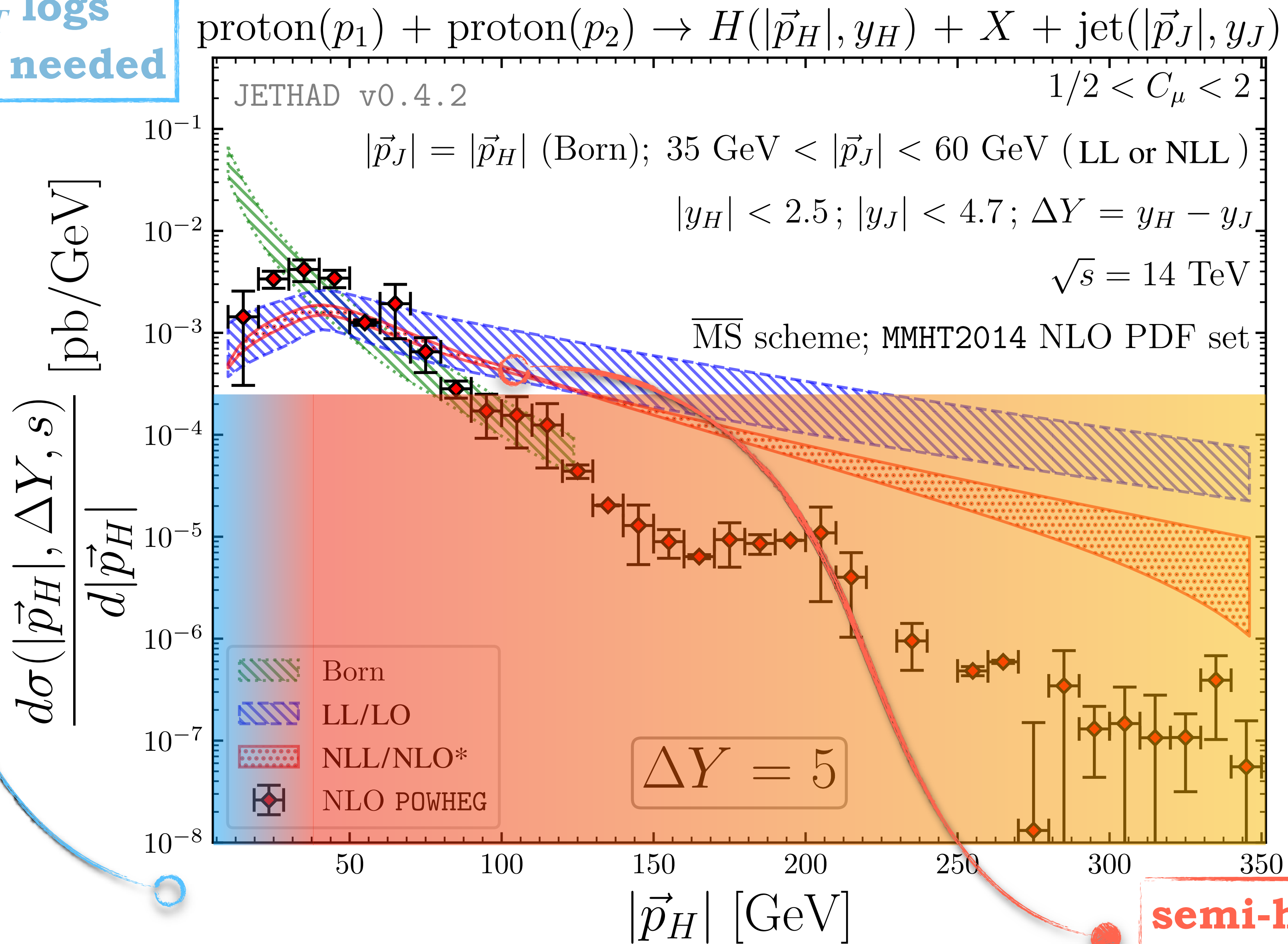
NLL accurate predictions
matched to NLO
via the JETHAD Method



large p_T logs
 p_T -resum. needed



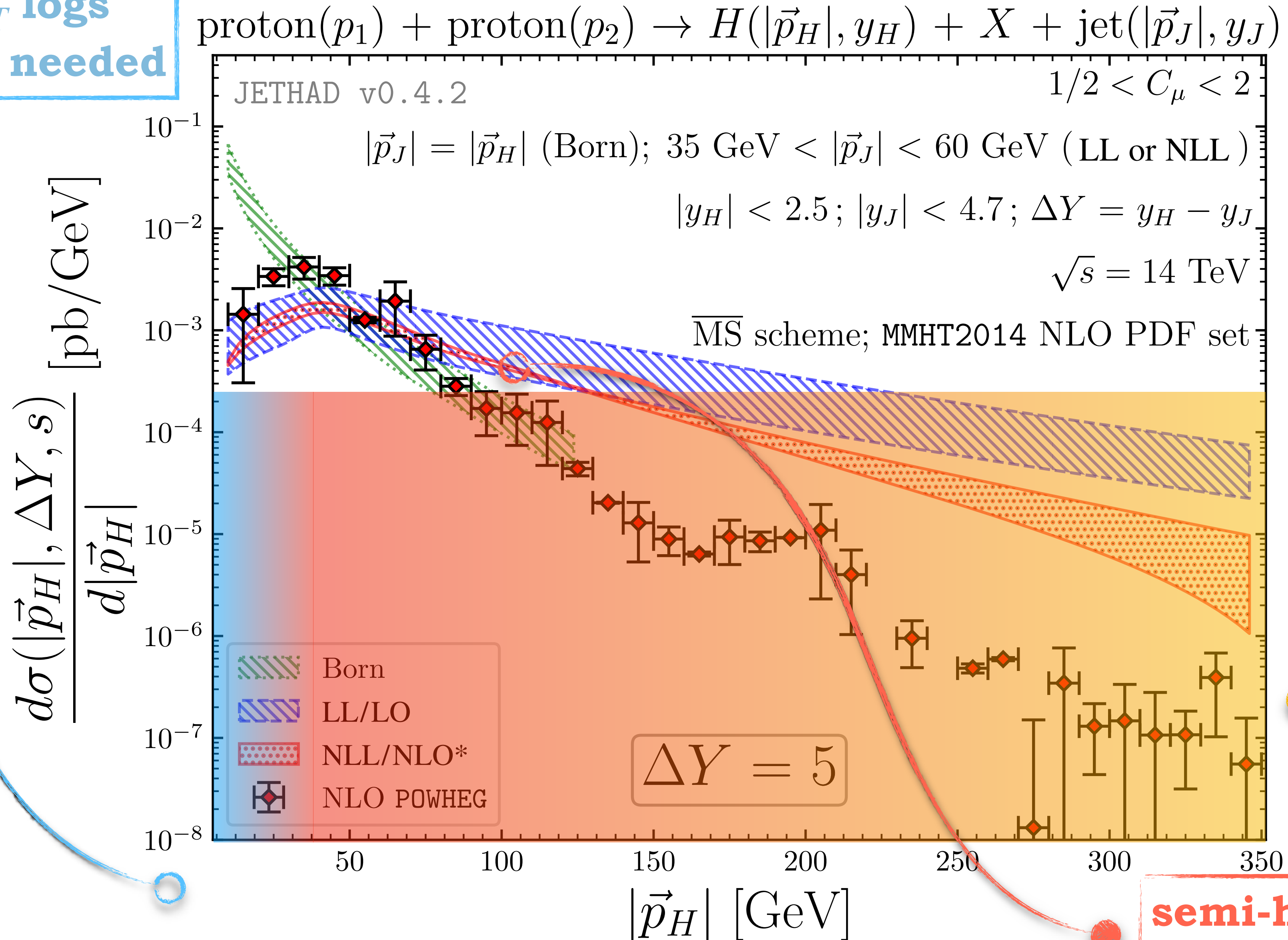
large p_T logs
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semi-hard regime
 BFKL expected

DGLAP-type + large- x threshold logs $\rightarrow$ BFKL decoupling

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 p_T -resum. needed

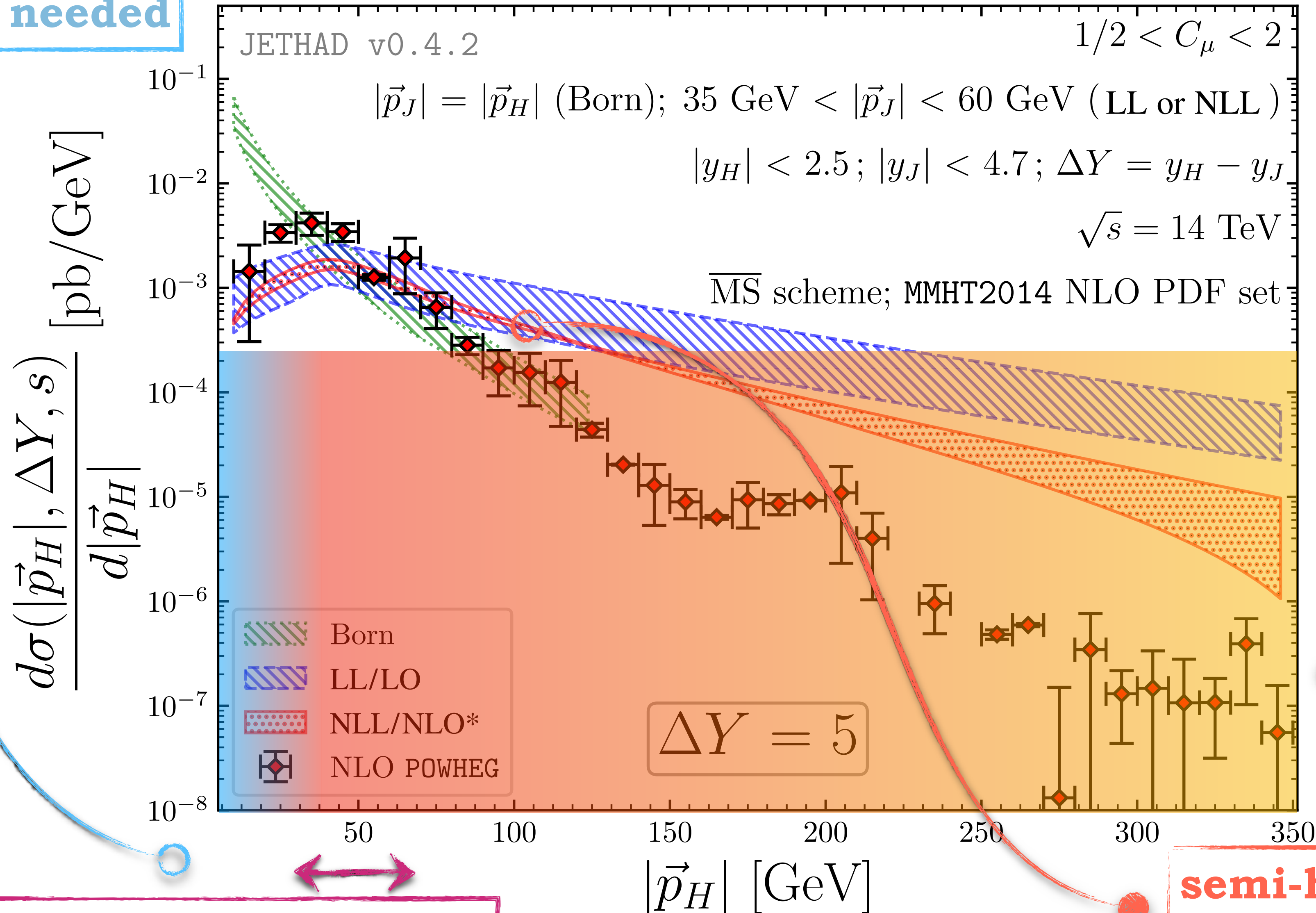


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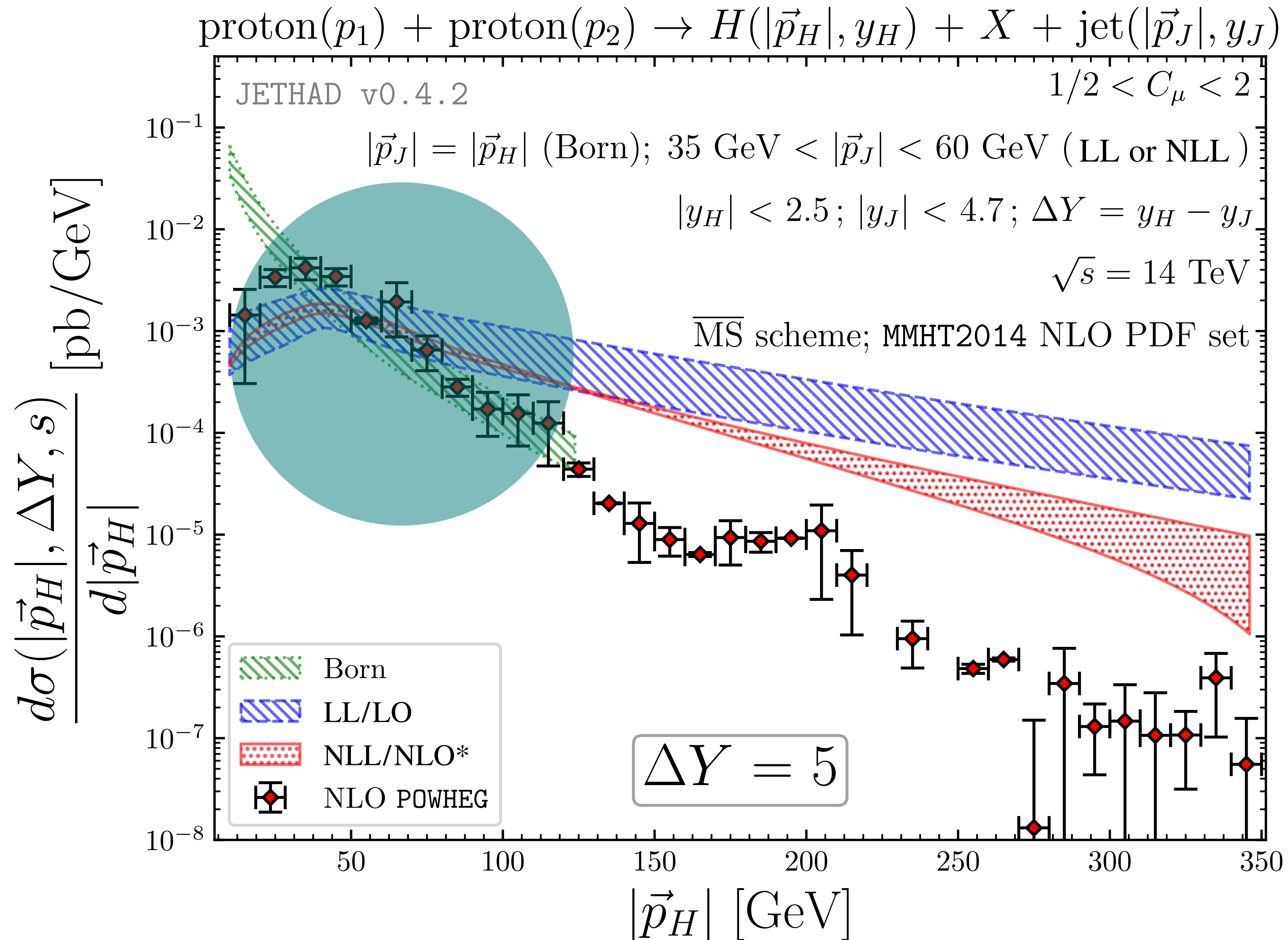
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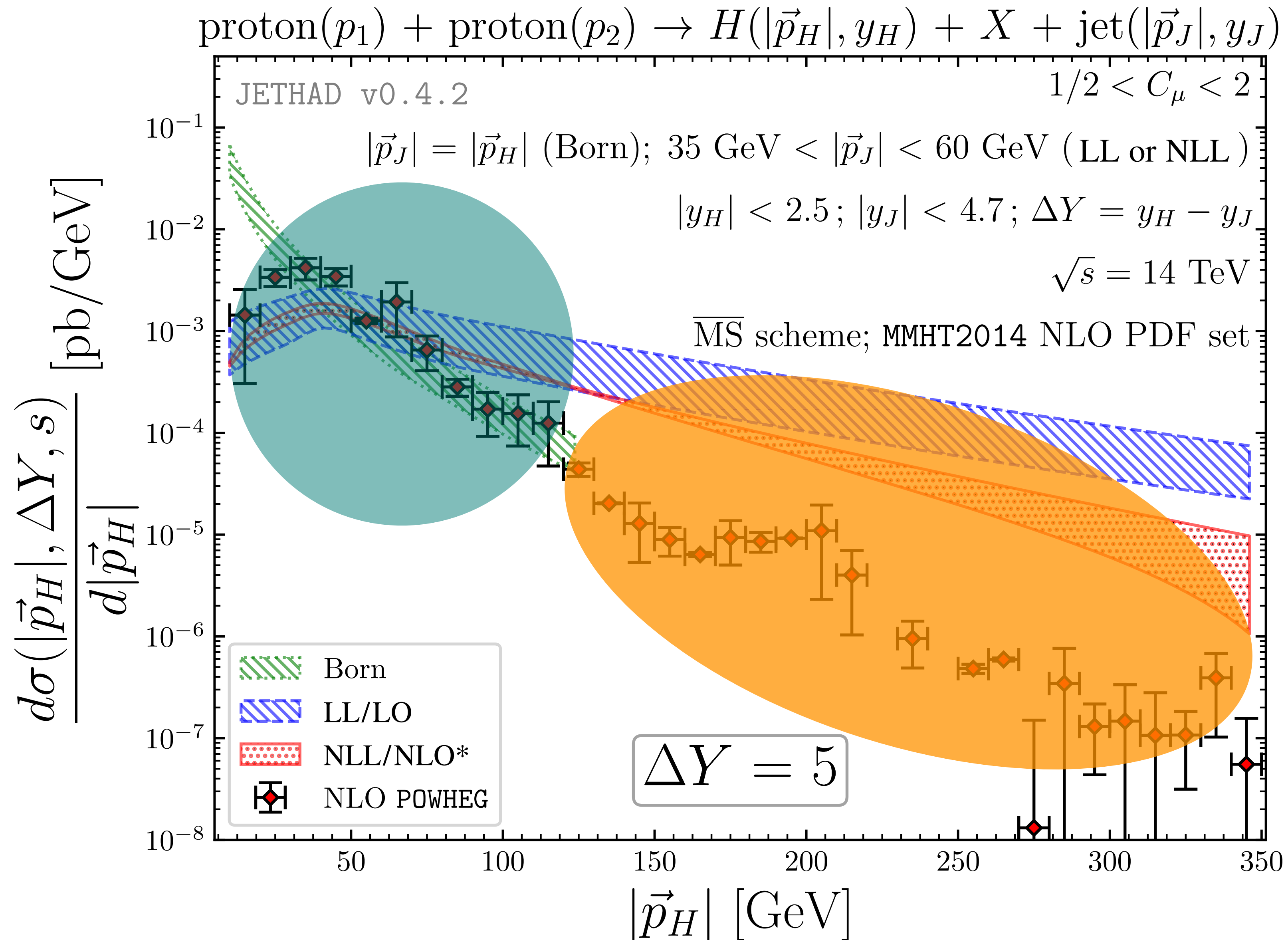
$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$



almost back-to-back emissions
large imbalance double logs

semi-hard regime
BFKL expected

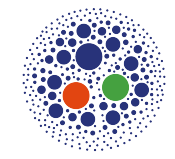




! **Precision corrections** *expected*, but **hybrid factorization** *predicts* **large deviations** from **f.o.** !

Matching NLL to NLO with JETHAD

! **Precision corrections** *expected* $\Leftrightarrow$ *need* for an accurate NLL-to-NLO **Matching procedure** !

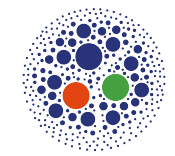


JETHAD Method $\rightarrow$ NLL/NLO Additive Matching (analytic: BFKL kernel + coefficient functions)

$$d\sigma^{\text{NLL/NLO}^*}(\Delta Y, \varphi, s) = \underbrace{d\sigma^{\text{NLO}}(\Delta Y, \varphi, s)}_{\text{NLO POWHEG w/o PS}} + d\sigma^{\text{NLL}^*}(\Delta Y, \varphi, s) - \Delta d\sigma^{\text{NLL/NLO}^*}(\Delta Y, \varphi, s)$$

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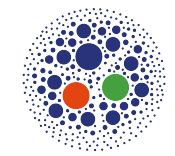


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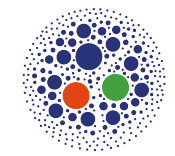
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NLL* JETHAD w/o NLO* double counting

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HELL + ggHiggs

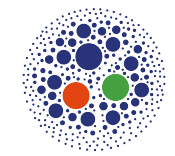
N³LL_{lx}/LL_{sx}/N³LO

Inclusive Higgs

 [M. Bonvini, S. Marzani (2018)]

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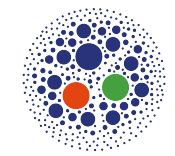
Higgs + jet(s)

 [J. R. Andersen et al. (2022)]

  [Today's talks by Jeppe & Andreas]

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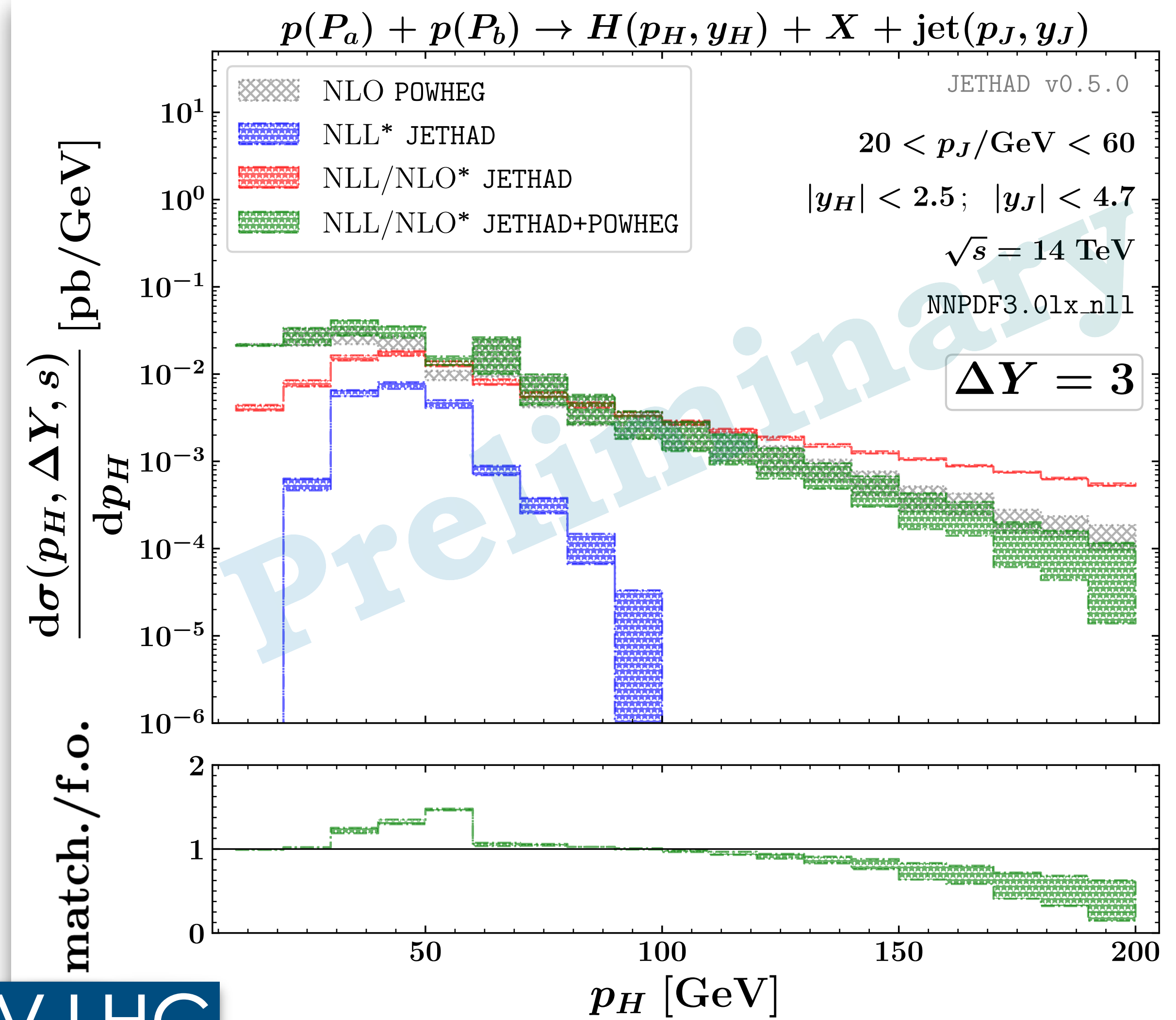
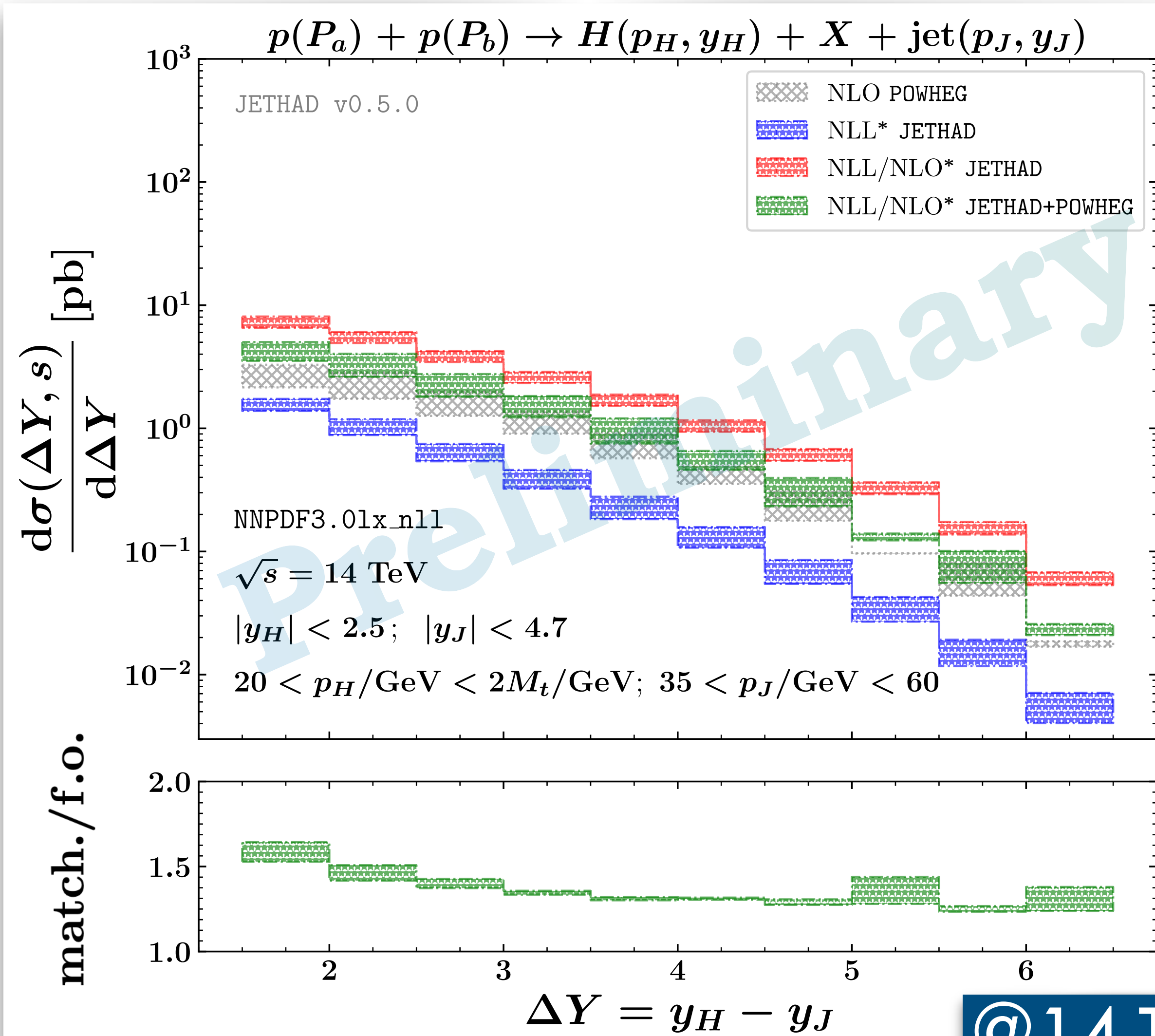
RadISH + MCFM-8.3

NNLL_{TM}/NLO

Higgs + jet

 [P.F. Monni et al. (2020)]

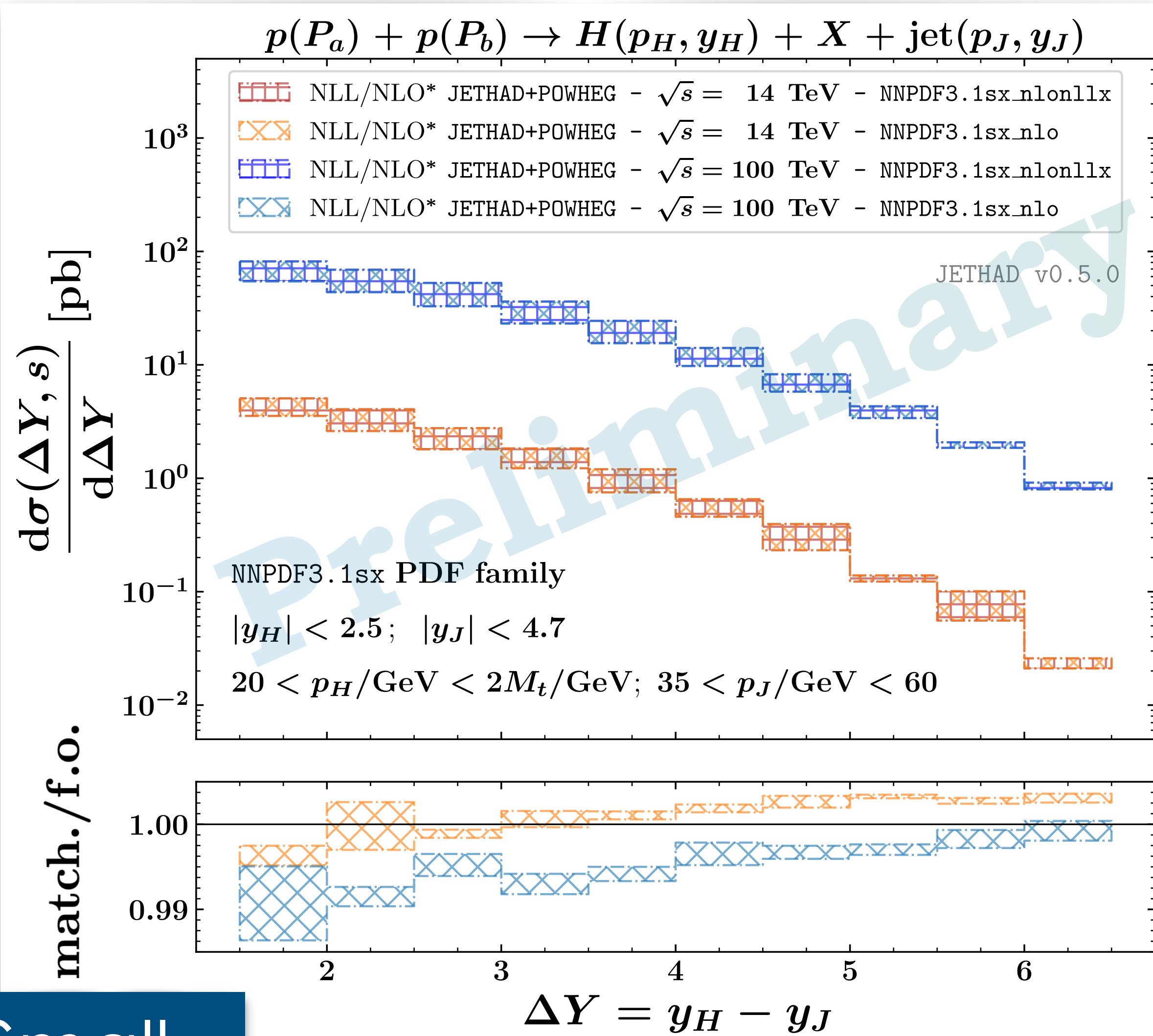
The Higgs + jet spectrum from POWHEG + JETHAD



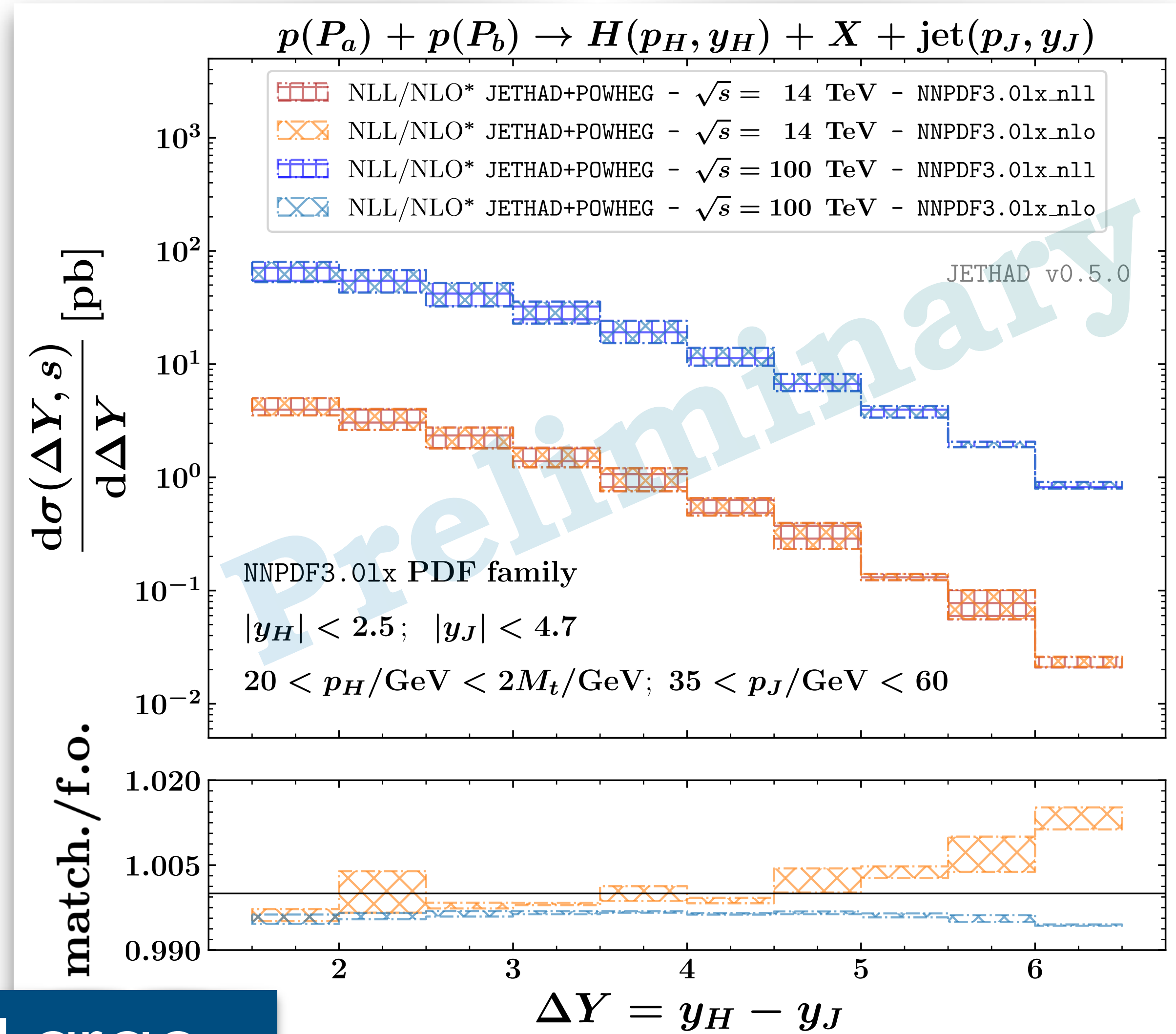
@14 TeV LHC

NLL matched to NLO fixed-order JETHAD + POWHEG (in progress)

Small- x and large- x enhancement from PDFs

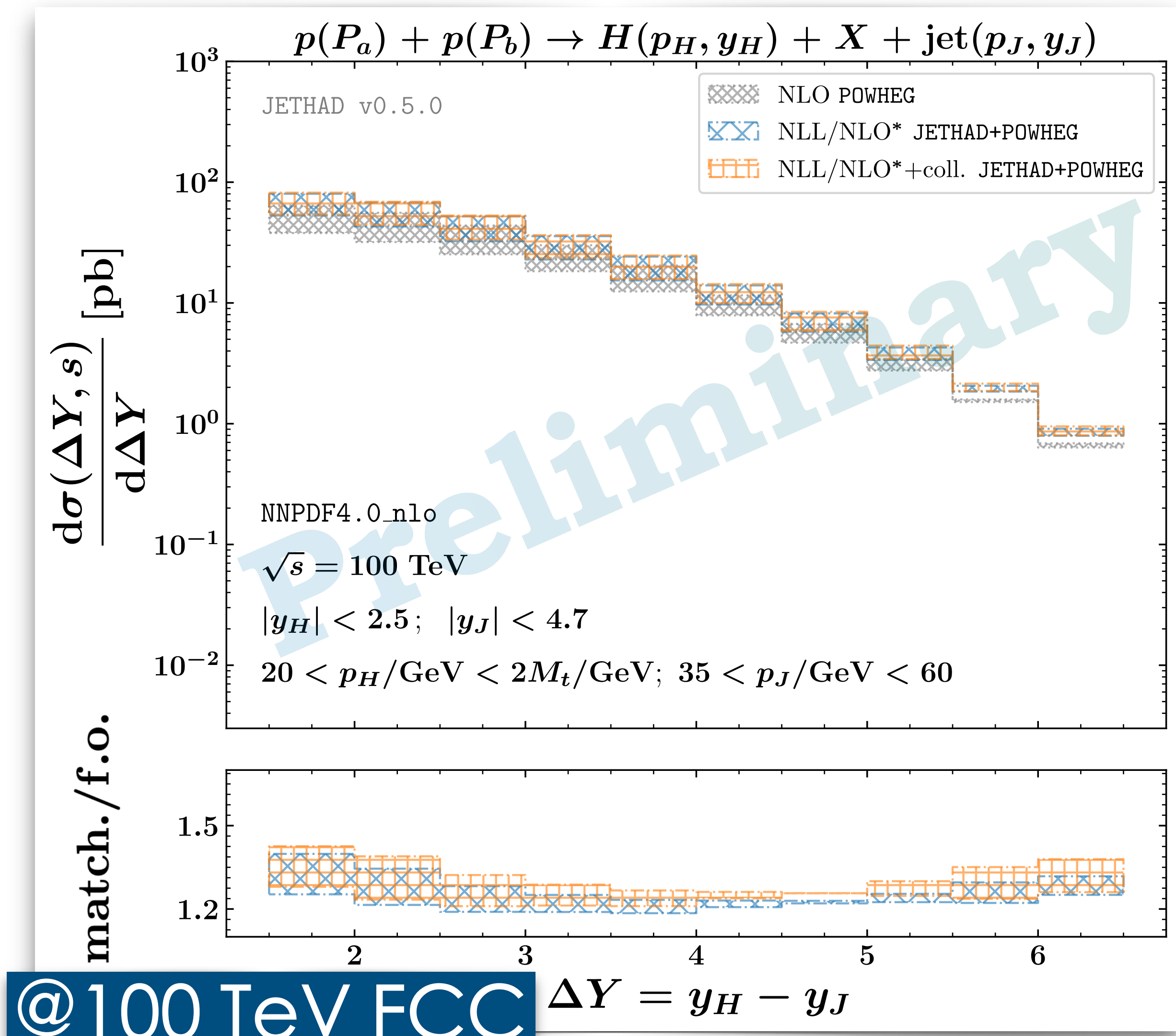
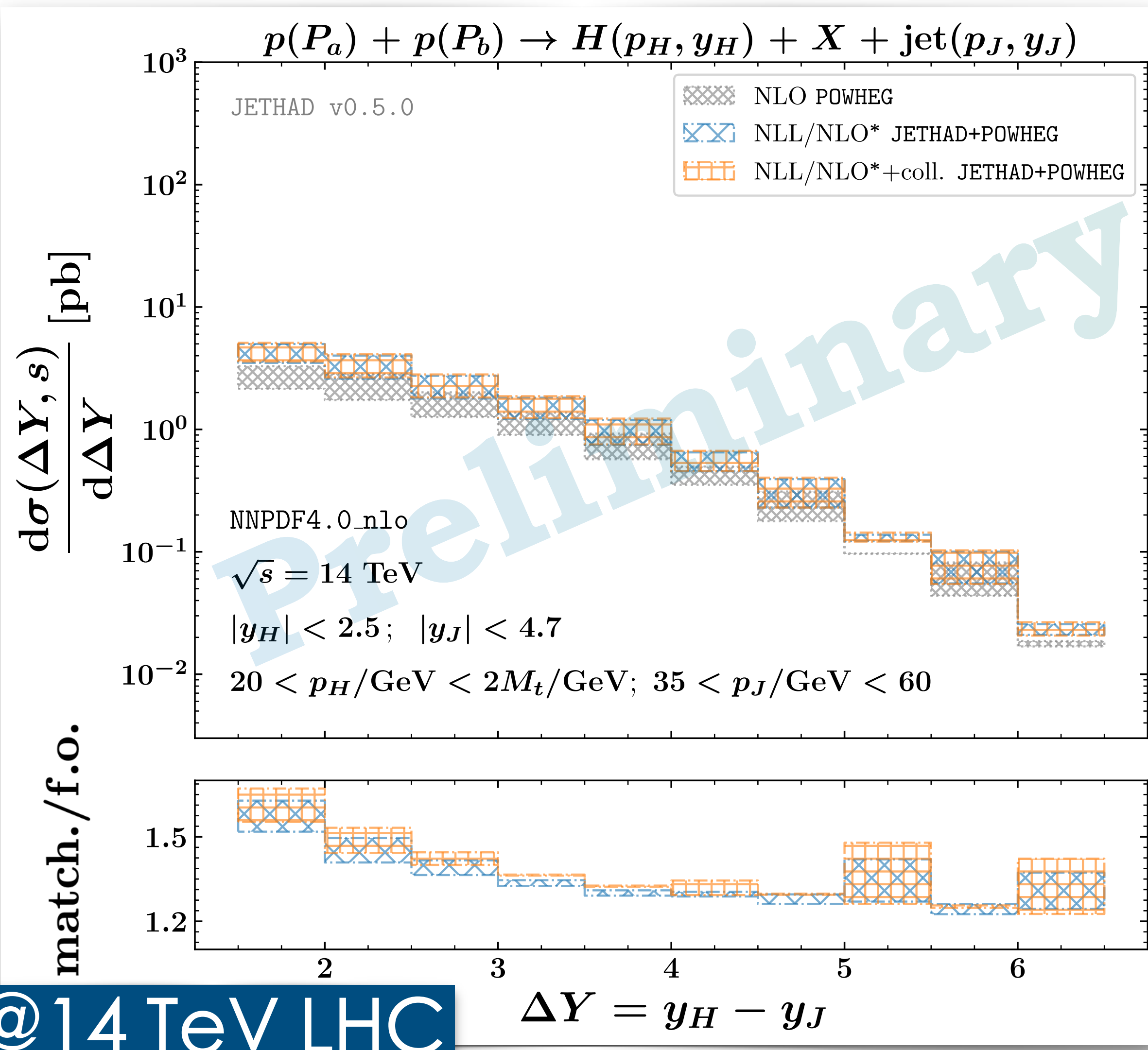


Small- x



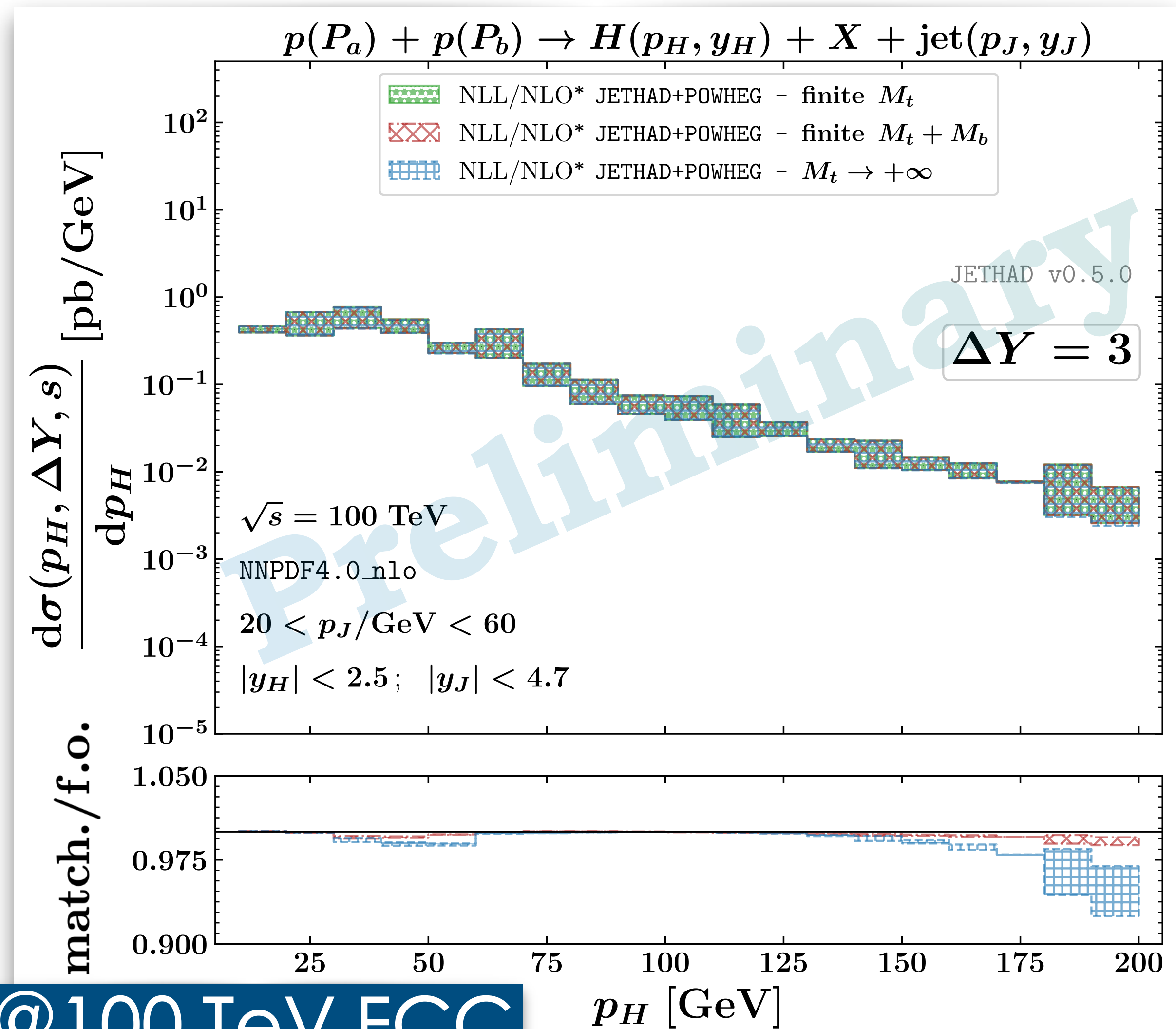
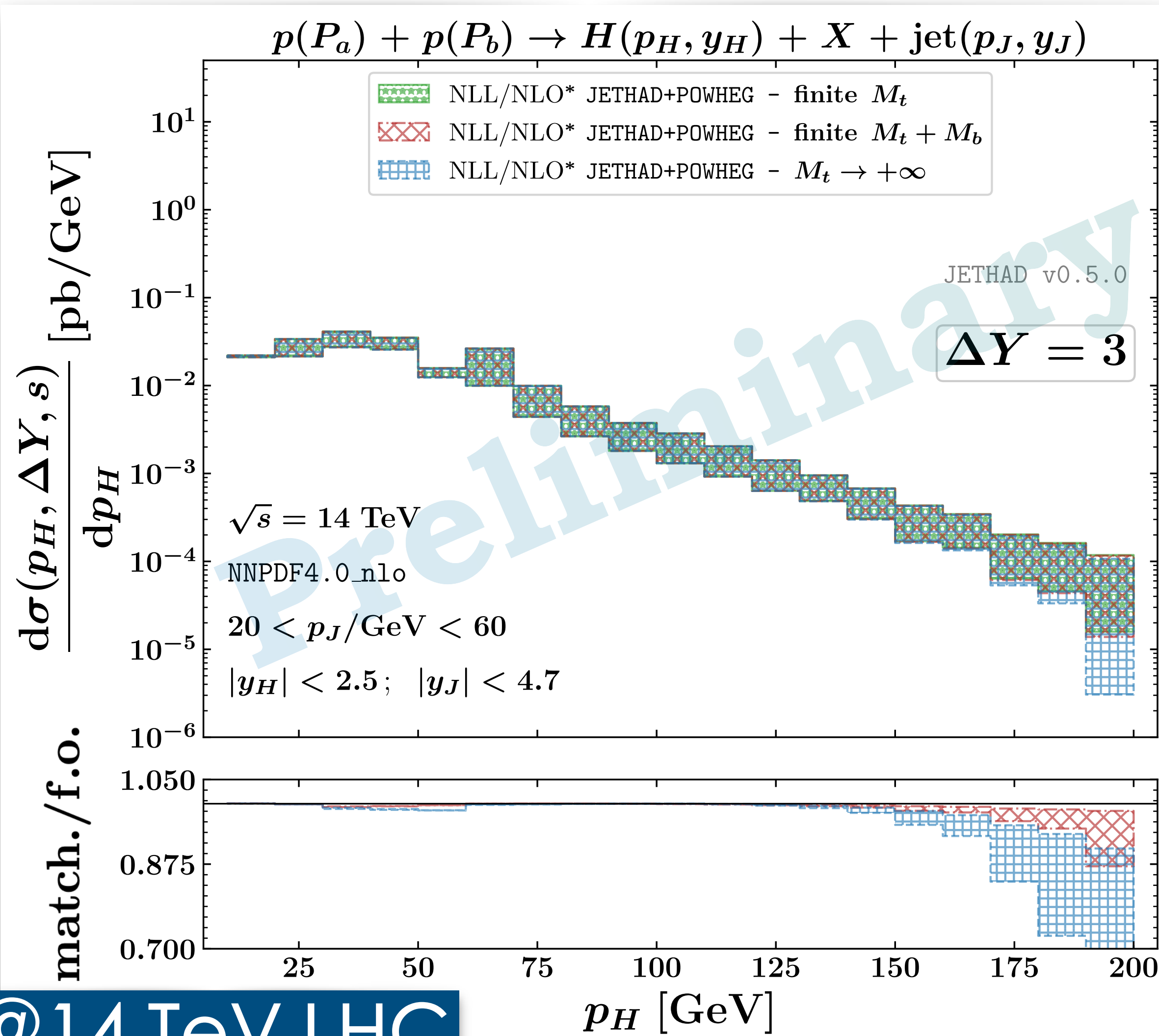
Large- x

Effect of collinear improvement on NLL BFKL kernel



(Collinear improvement) [G.P. Salam, JHEP 07 (1998) 019]; [M. Ciafaloni et al., Phys.Lett.B 587 (2004) 87-94]; [A. Sabio Vera, Nucl.Phys.B 722 (2005) 65-80]

Finite top- and bottom-mass corrections



Effect of finite heavy-quark masses on NLO coeff. functions to be gauged

(NLO Higgs) [\[F. G. C., M. Fucilla, D. Yu. Ivanov, M. M. A. Mohammed, A. Papa, JHEP 08 \(2022\) 092\]](#)

Paving the way toward precision

- ☑ Semi-inclusive **Higgs + jet** as novel probe for High-Energy QCD
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- ☐ Precision studies $\Leftrightarrow$ NLL/NLO Matching via the **JETHAD** Method
- ☐ Full NLL/NLO analysis: NLO Higgs [**Michael's talk**], jet algorithm
- ☐ Systematic uncertainties: top & bottom masses, PDF impact, Matching
- ☐ **Transversal formalism** as underlying staging for several **resummations**





Extras

Mueller-Navelet jets @LHC & resummation instabilities

-  Strong manifestation of **higher-order instabilities** via scale variation (⚠️)
-  ⚠️ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
-  **BLM** scales, theory vs experiment: CMS @7TeV with **symmetric** p_T -ranges, only

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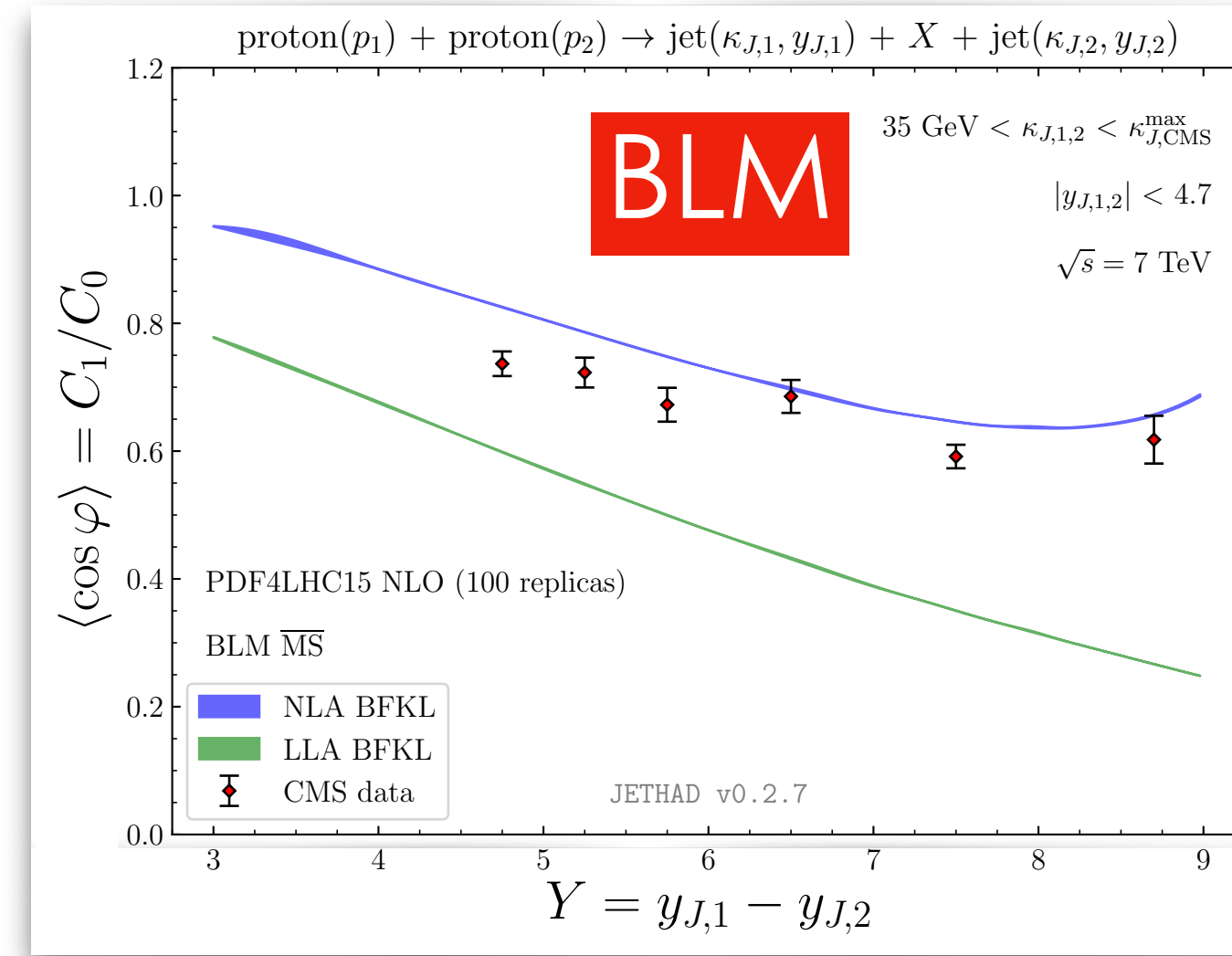
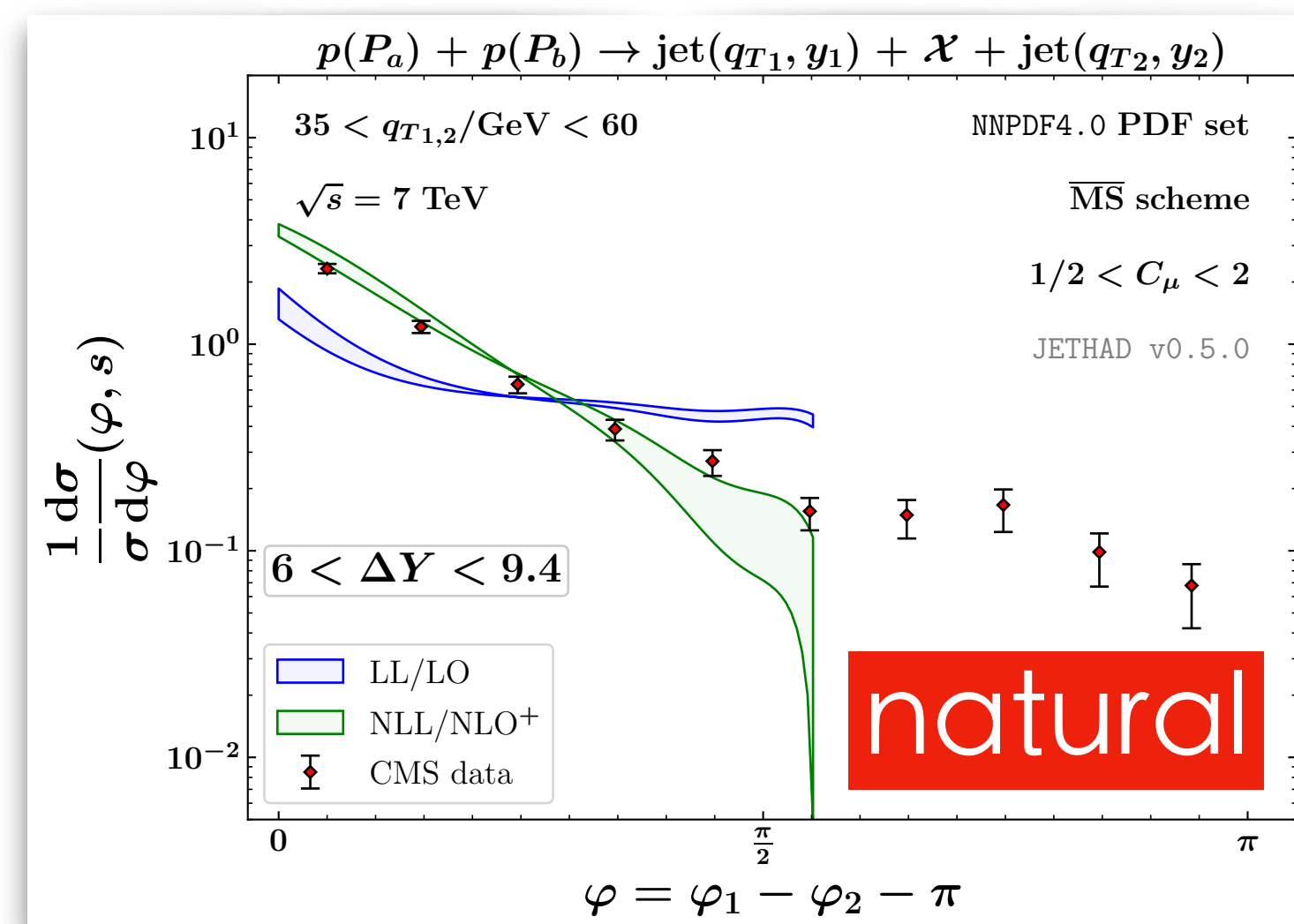
[CMS Collaboration, JHEP 08 (2016) 139]

[B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]

[F. Caporale et al., Eur. Phys. J. C 74 (2014) 10, 3084]

(left figure) [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

(right figure) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]



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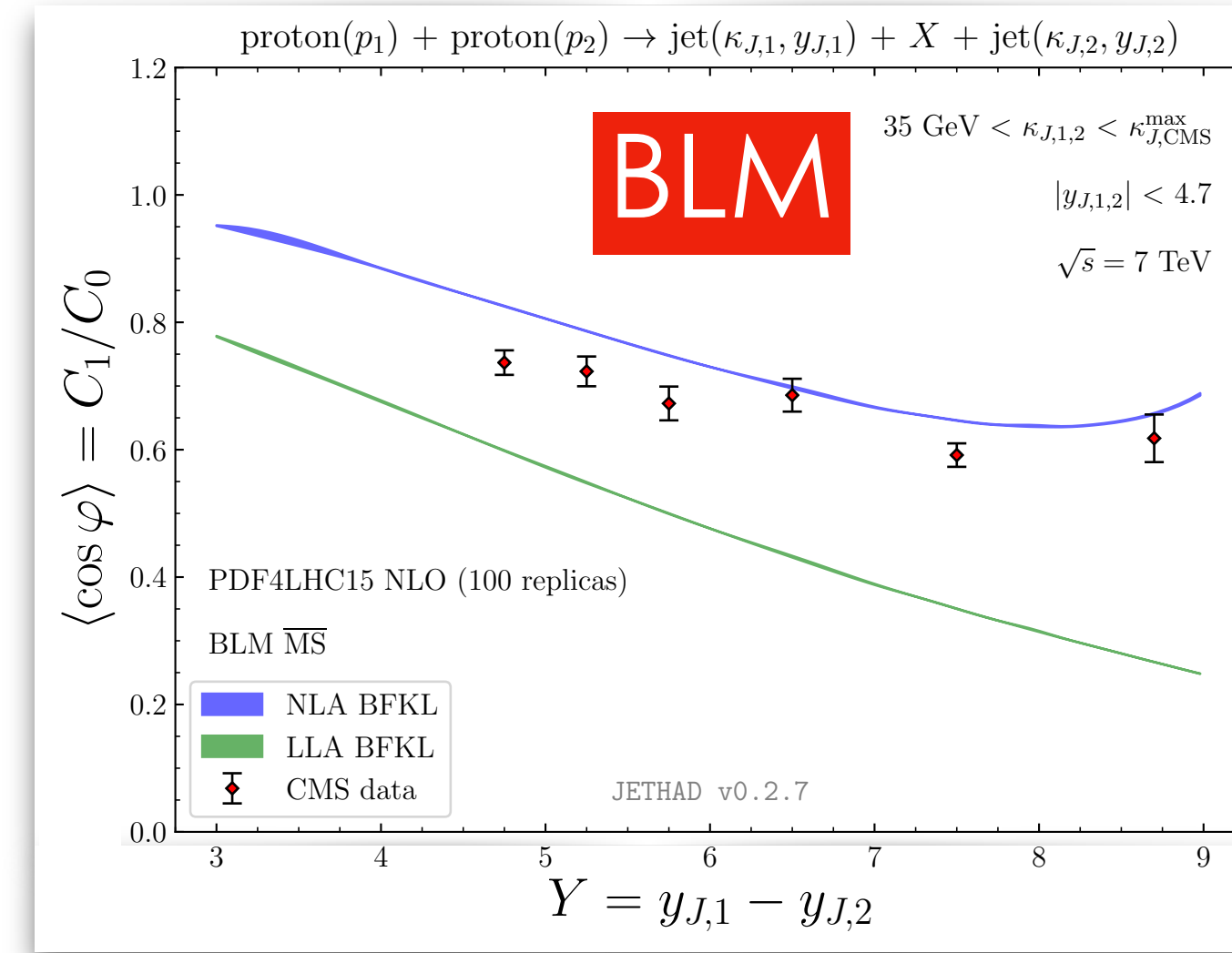
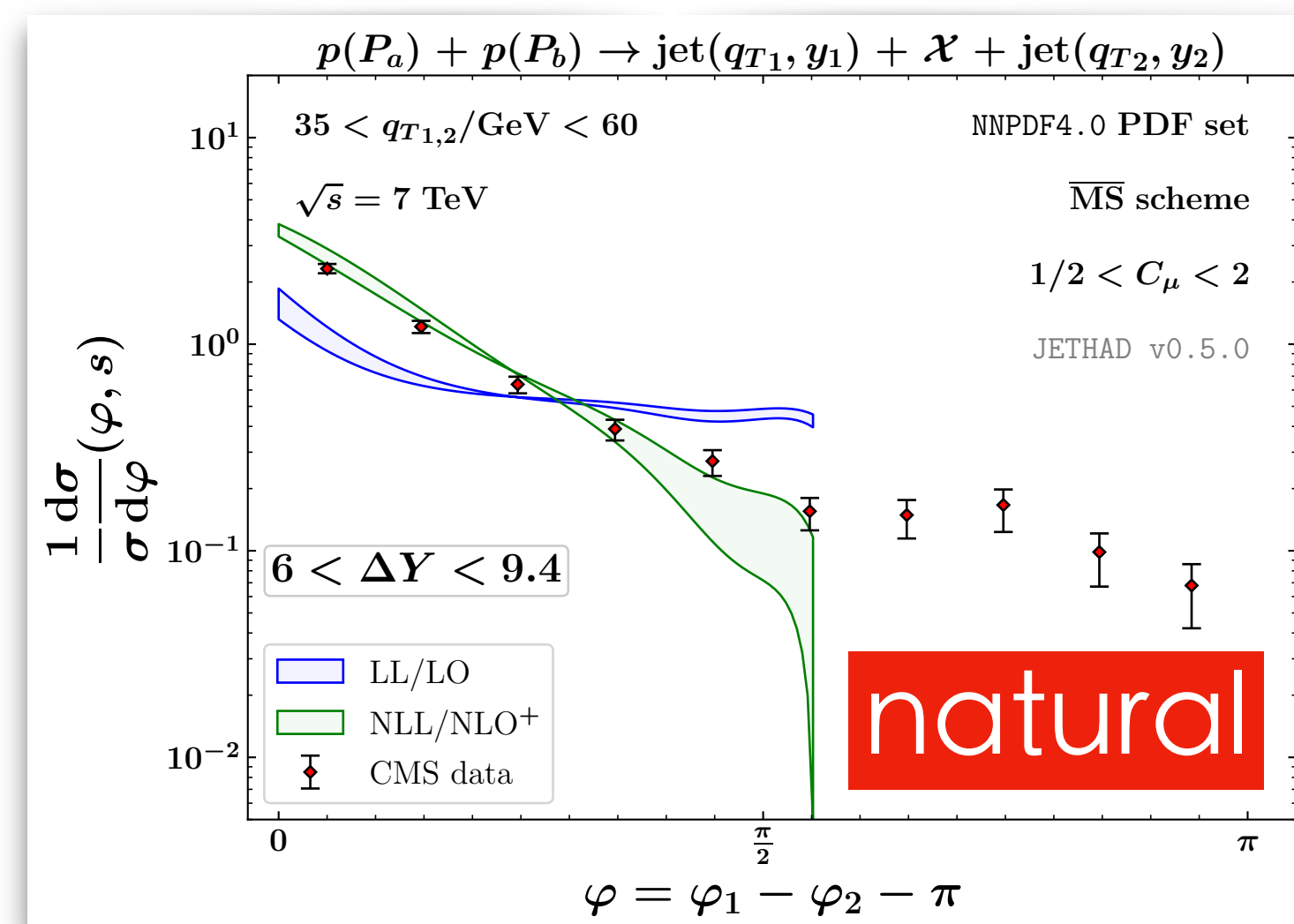
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$\mu_R^{\text{BLM}} \gg \mu_R^{\text{nat.}} \Rightarrow d\sigma^{\text{BLM}}/d\sigma^{\text{nat.}} \sim 10^{-(1\div 2)} \Rightarrow$ precision studies hampered



Unsuccessful scale optimization → processes featuring **natural stability** (¿?)

Higgs + jet highlights from the FCC Week 2022

The high-energy QCD dynamics from Higgs+jet correlations at FCC

Francesco G. Celiberto^{1,2,3} and Alessandro Papa^{4,5}

FCC Week 2022, Sorbonne Université, France

Hors d'œuvre

Higgs sector → SM benchmarks, BSM portals
Gluon fusion → key ingredient for precision QCD
Fixed-order ← improved by resummations
FCC energies ↔ high-energy (HE) resummation
Higgs+jet → golden channel to hunt for HE signals

NLL/NLO differential cross section

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2 f_r(x_1, \mu_F) f_s(x_2, \mu_F) \frac{d\hat{\sigma}_{rs}(x_1 x_2 s, \mu_F)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$
$$\frac{d\hat{\sigma}_{rs}(x_1 x_2 s, \mu)}{dy_H dy_J d^2\vec{p}_H d^2\vec{p}_J} = \frac{1}{(2\pi)^2} \times \int \frac{d^2\vec{q}_1}{q_1^2} V_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \times \int_{s-i\infty}^{s+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0} \right)^\omega g_\omega(\vec{q}_1, \vec{q}_2) \times \int \frac{d^2\vec{q}_2}{q_2^2} V_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

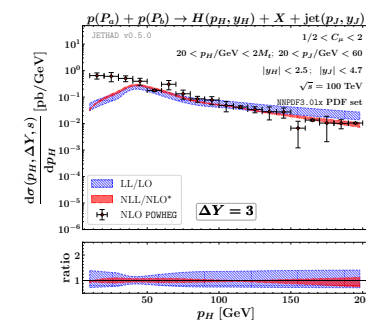
PDFs with threshold

NLO Higgs vertex

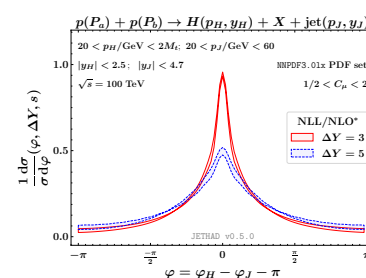
NLL BFKL kernel

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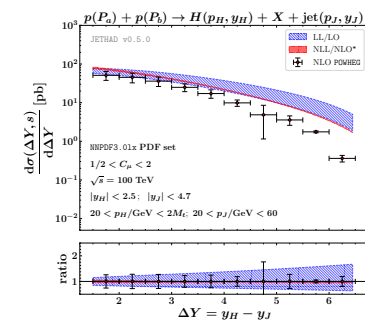
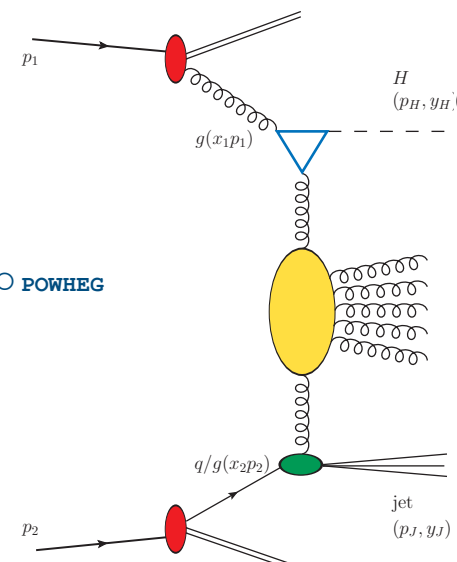
Hybrid high-energy and collinear factorization at work



Higgs p_T distribution: NLL/NLO JETHAD vs NLO POWHEG



Azimuthal distribution at NLL/NLO



Rapidity distribution: NLL/NLO JETHAD vs NLO POWHEG

HE resummation from JETHAD

Large-x NNPDF3.01x PDFs with threshold

Comparison with fixed-order from POWHEG

Distributions stable under NLL corrections

A path towards precision

- ✓ NLL bands nested inside LL ones → solid stability
- ✓ HE signal clearly disengaged from NLO background
- ✓ Way toward precision studies of HE QCD (μ !)
- Multilateral formalism → encode other resummations
- A window on proton structure at small-x (μ ?)

Further information

- ¹ ECT*, I-38123 Villazzano, Trento, Italy
- ² Fondazione Bruno Kessler (FBK), I-38123 Povo, Trento, Italy
- ³ INFN-TIFPA, I-38123 Povo, Trento, Italy
- ⁴ Università della Calabria, I-87036 Rende, Cosenza, Italy
- ⁵ INFN-Cosenza, I-87036 Rende, Cosenza, Italy

Contact: fceliberto@ectstar.eu

Take a picture to the QR code to download the paper on Higgs+jet resummed distributions at 14TeV LHC: [FGC et al., EPJ C 81 (2021) 4, 293]



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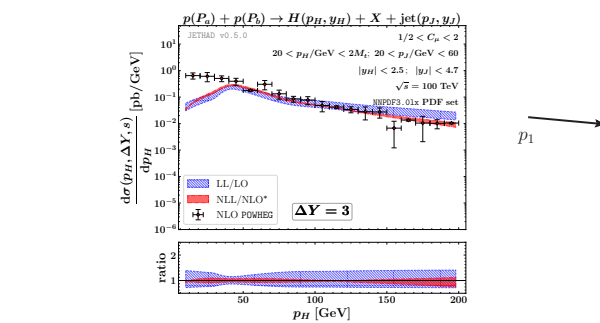
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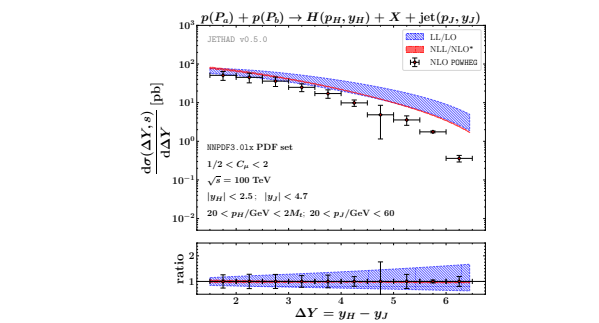
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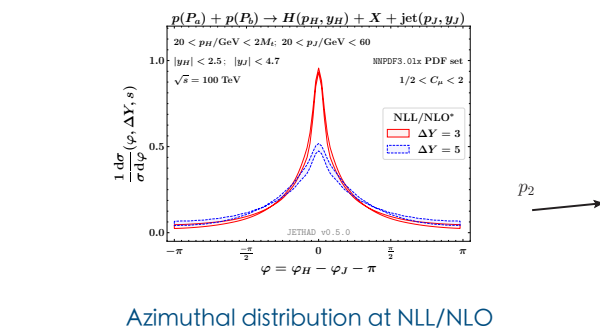
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Azimuthal distribution at NLL/NLO

A path towards precision

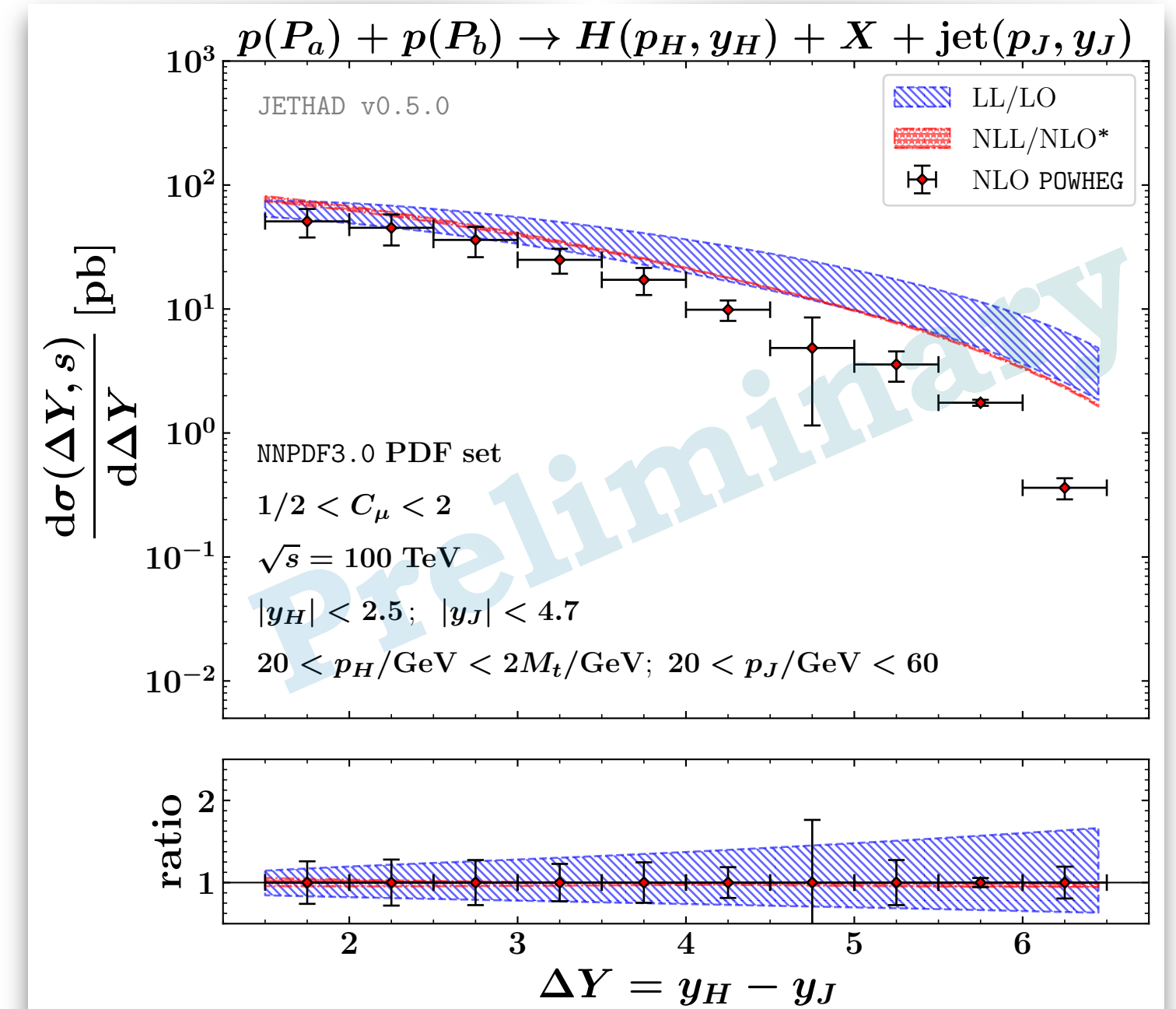
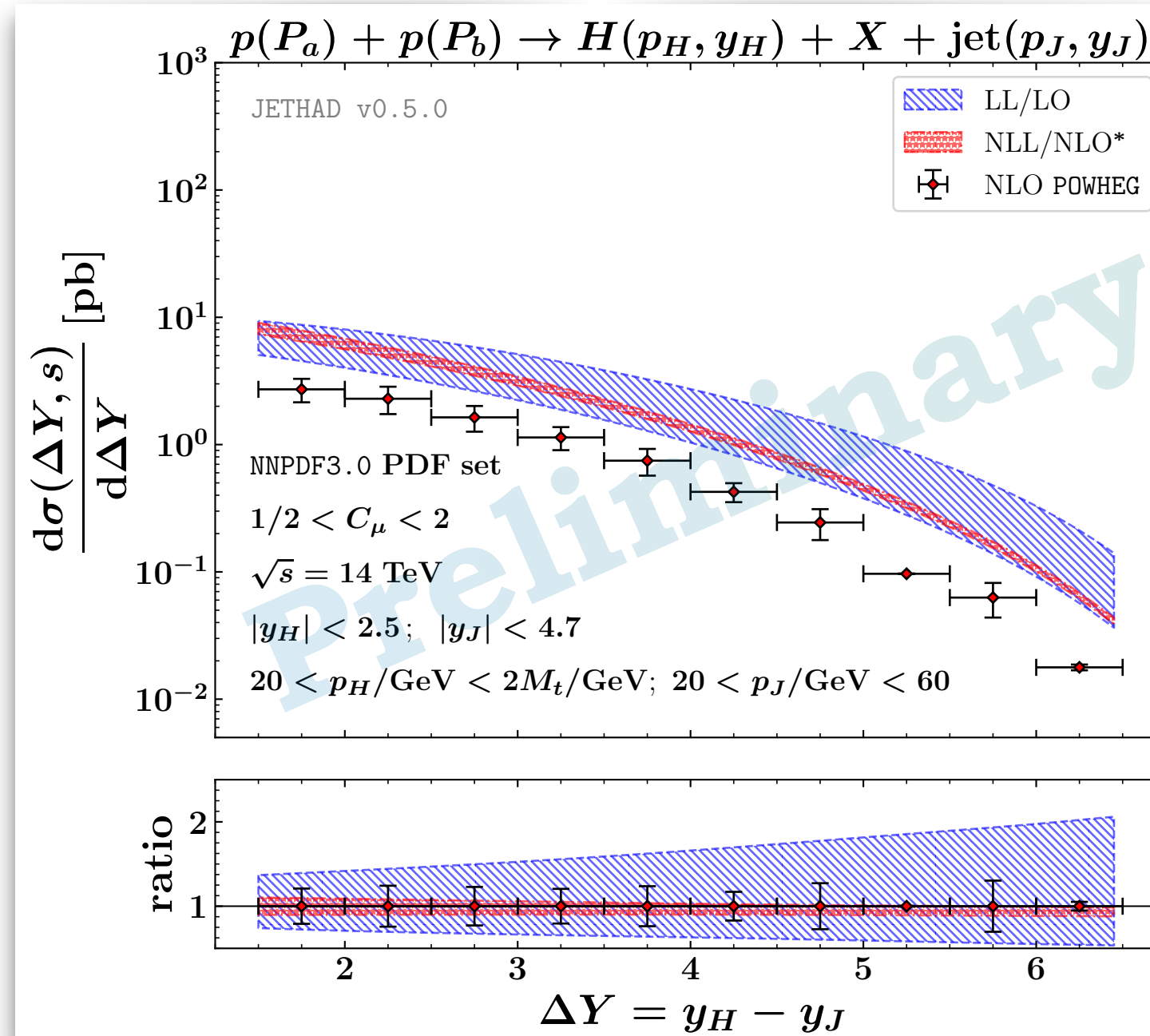
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Rapidity distribution: NLL/NLO* JETHAD vs NLO POWHEG

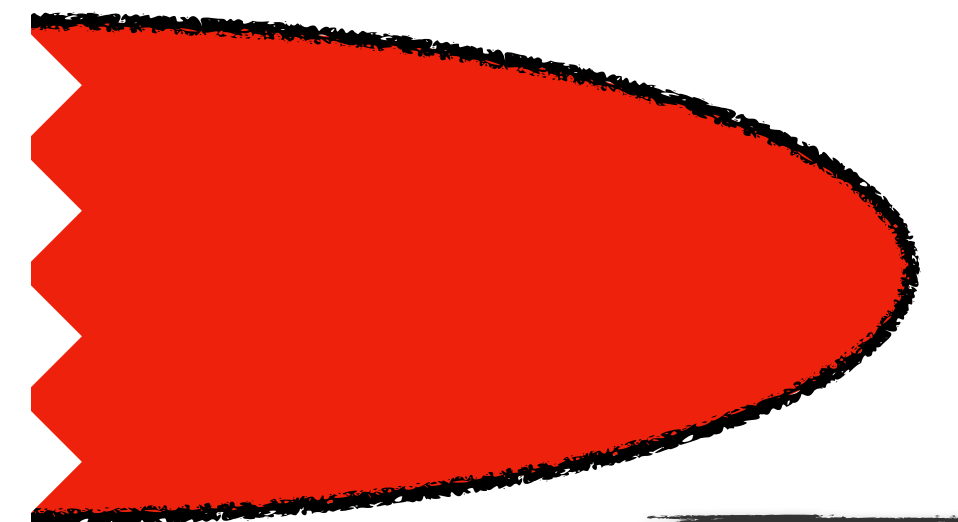
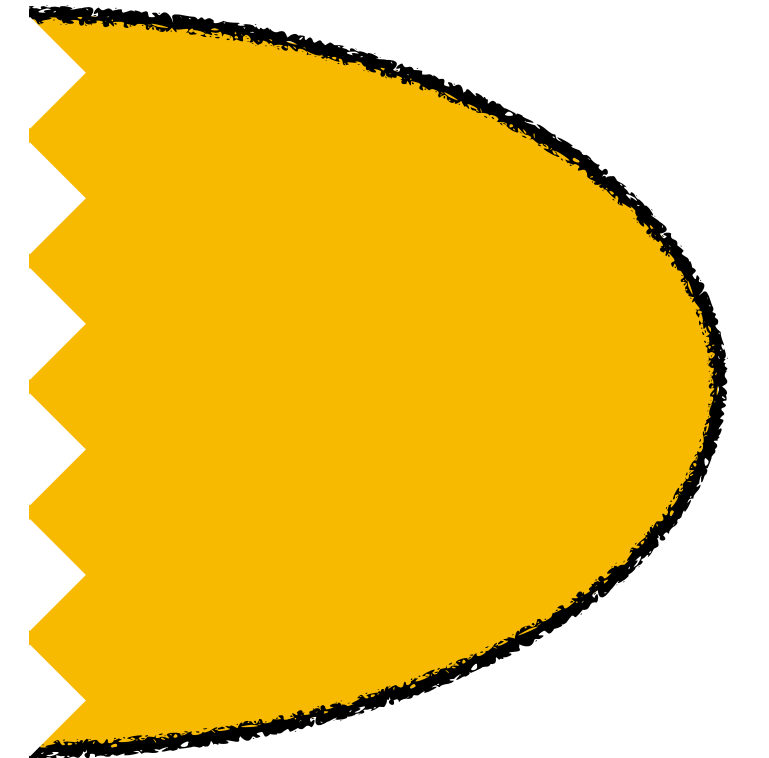
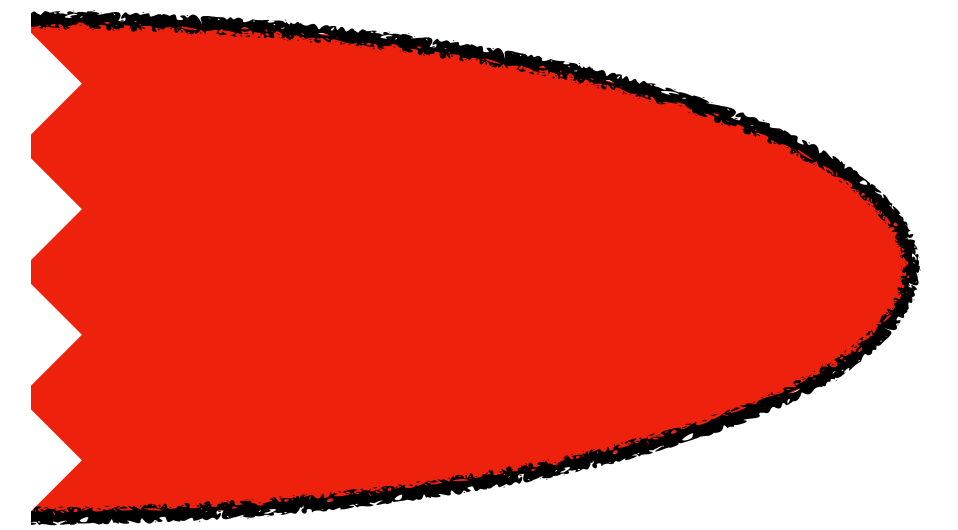
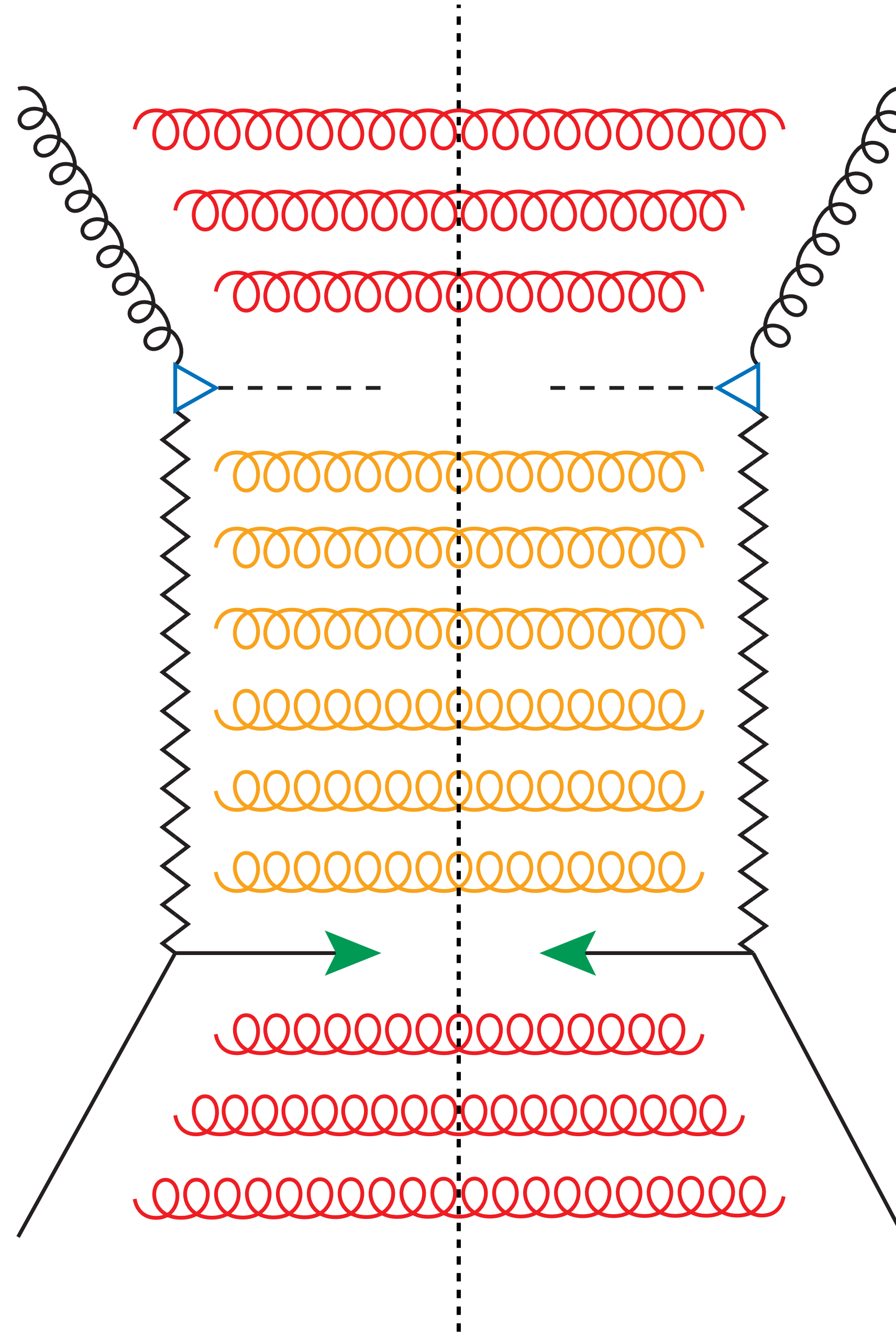
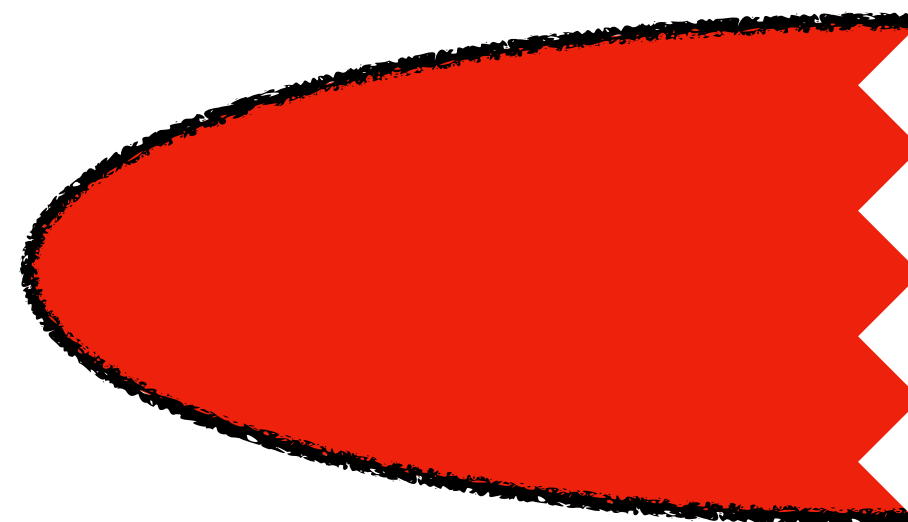
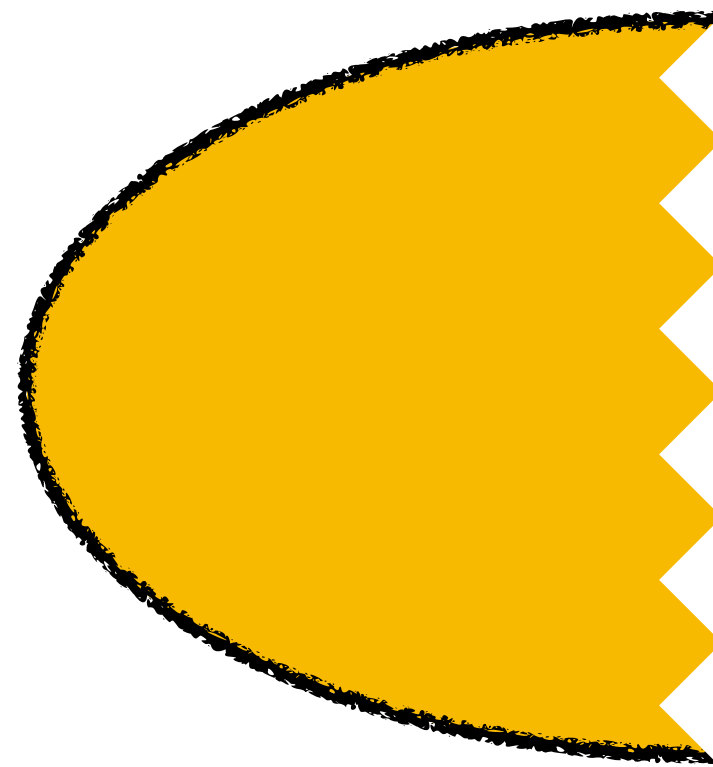
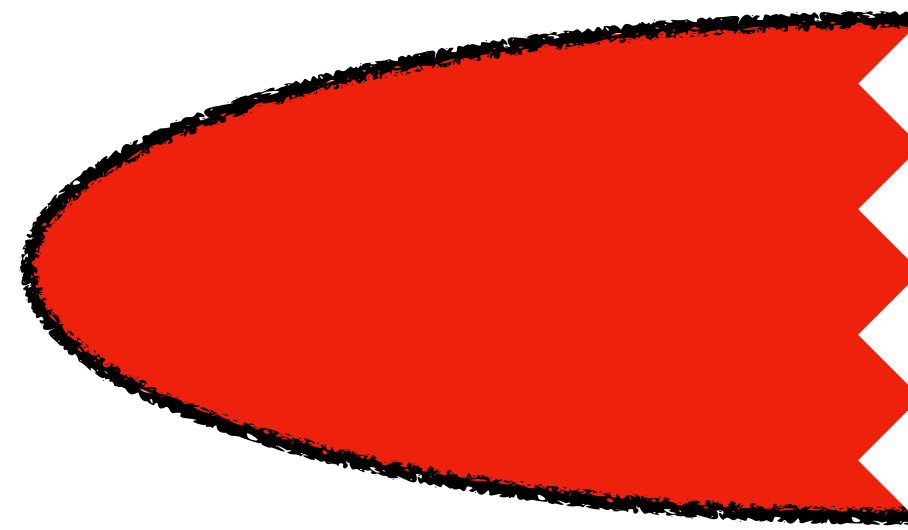


14 TeV LHC

100 TeV FCC

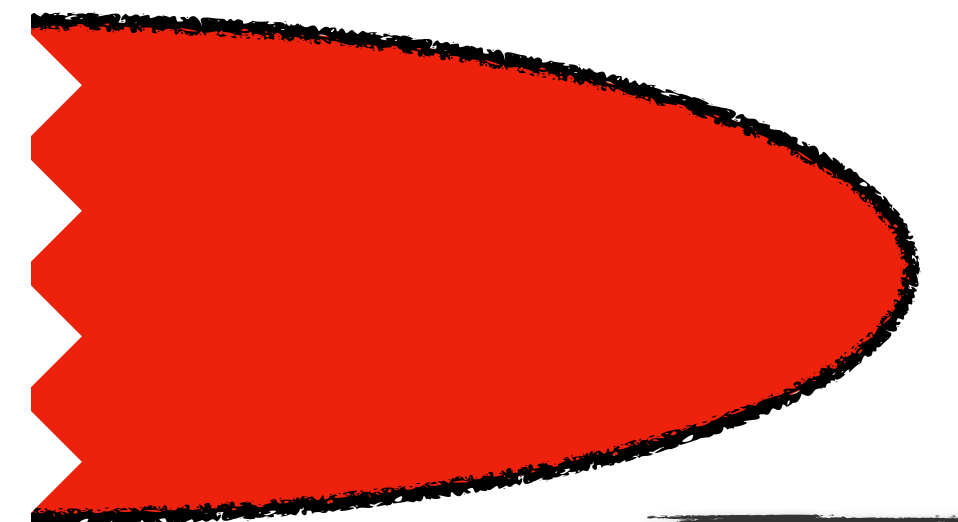
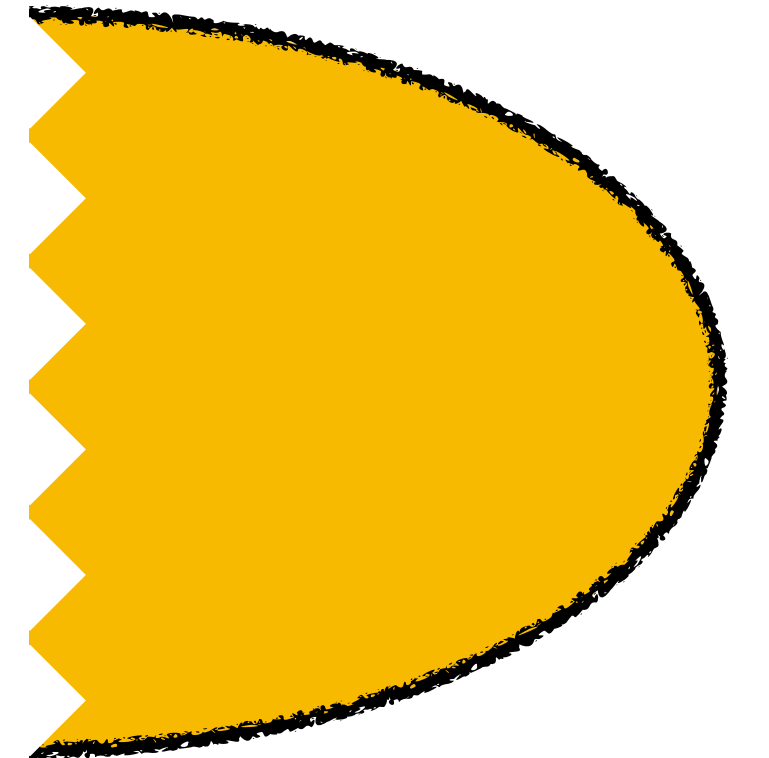
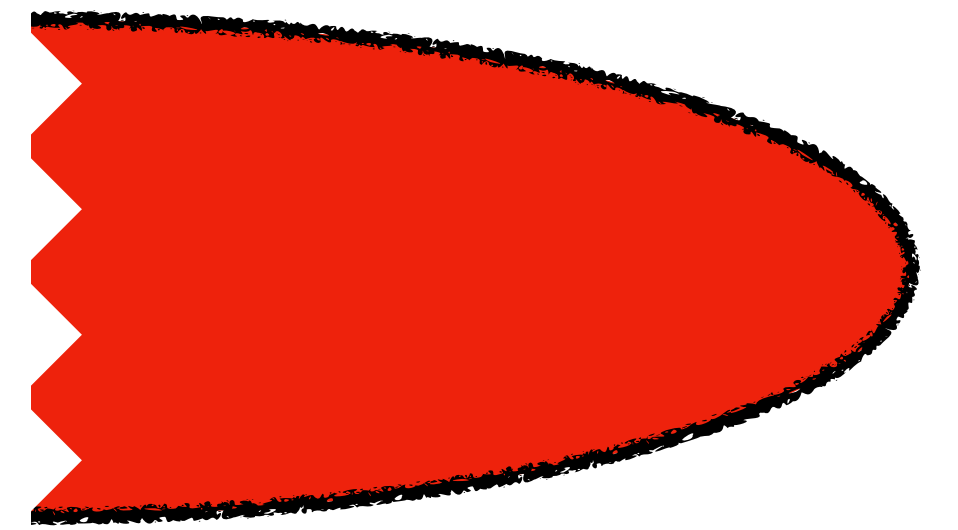
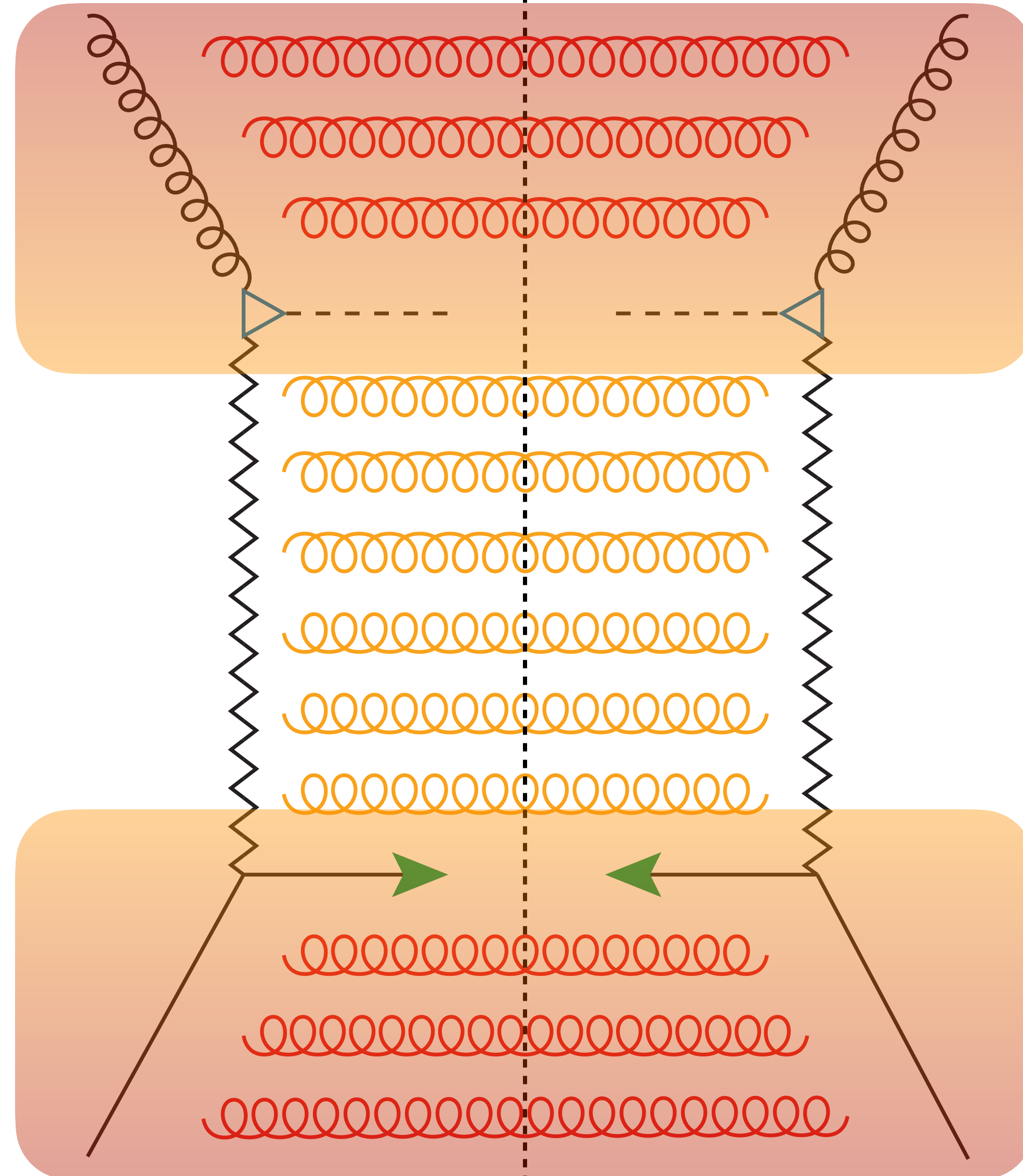
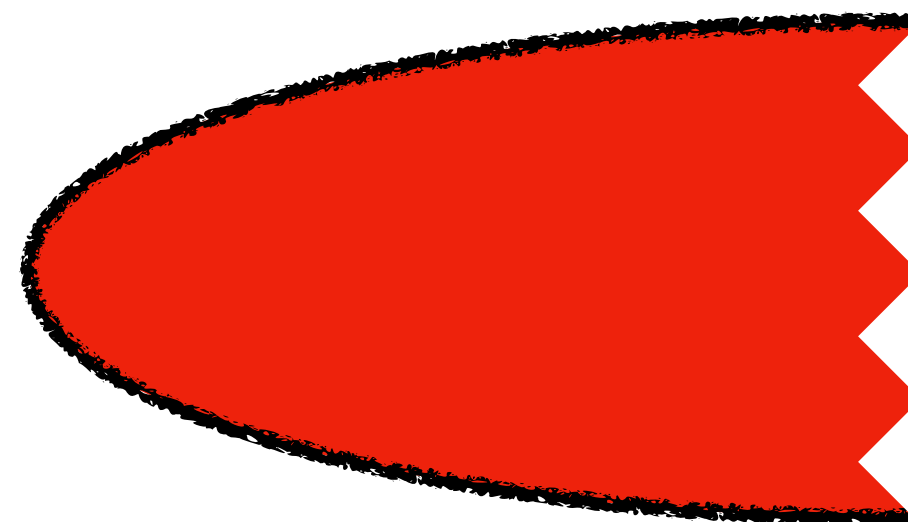
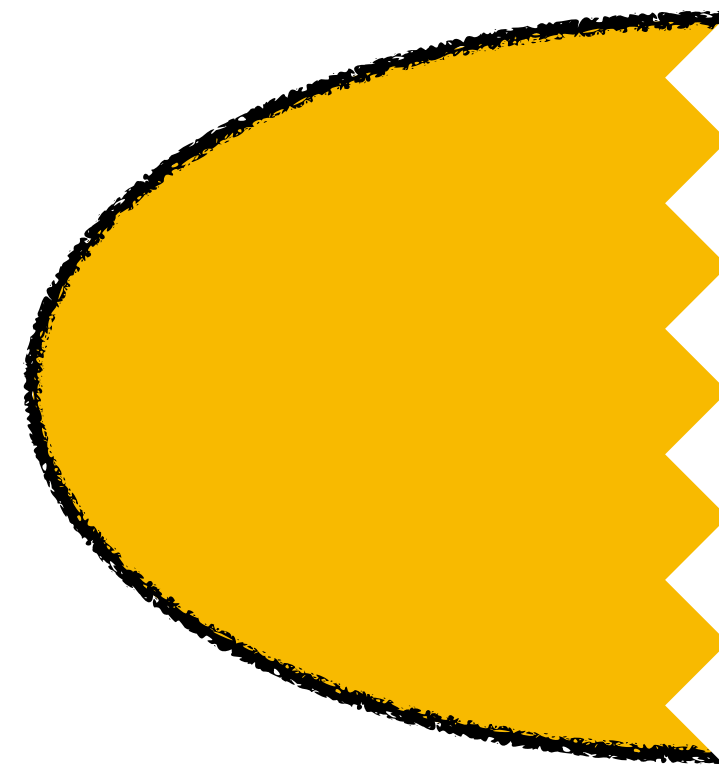
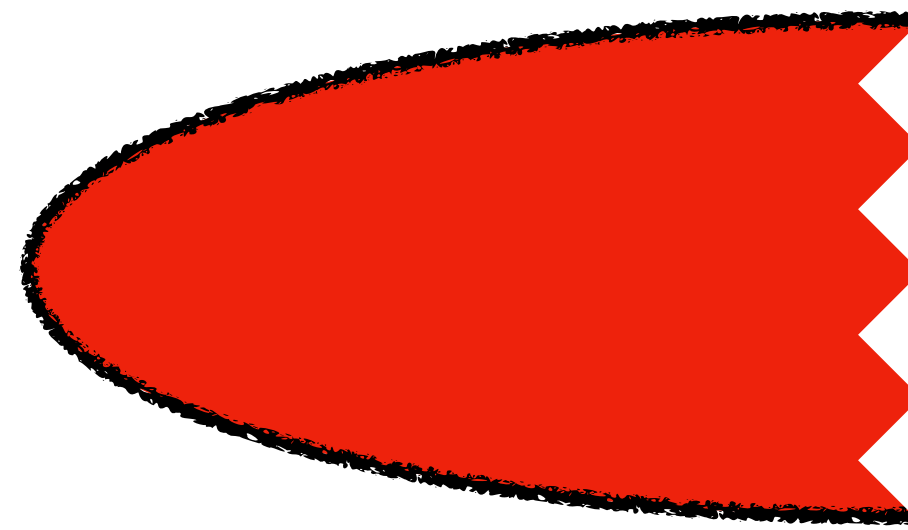
Backup

Anatomy of Higgs + jet in hybrid factorization (HyF)



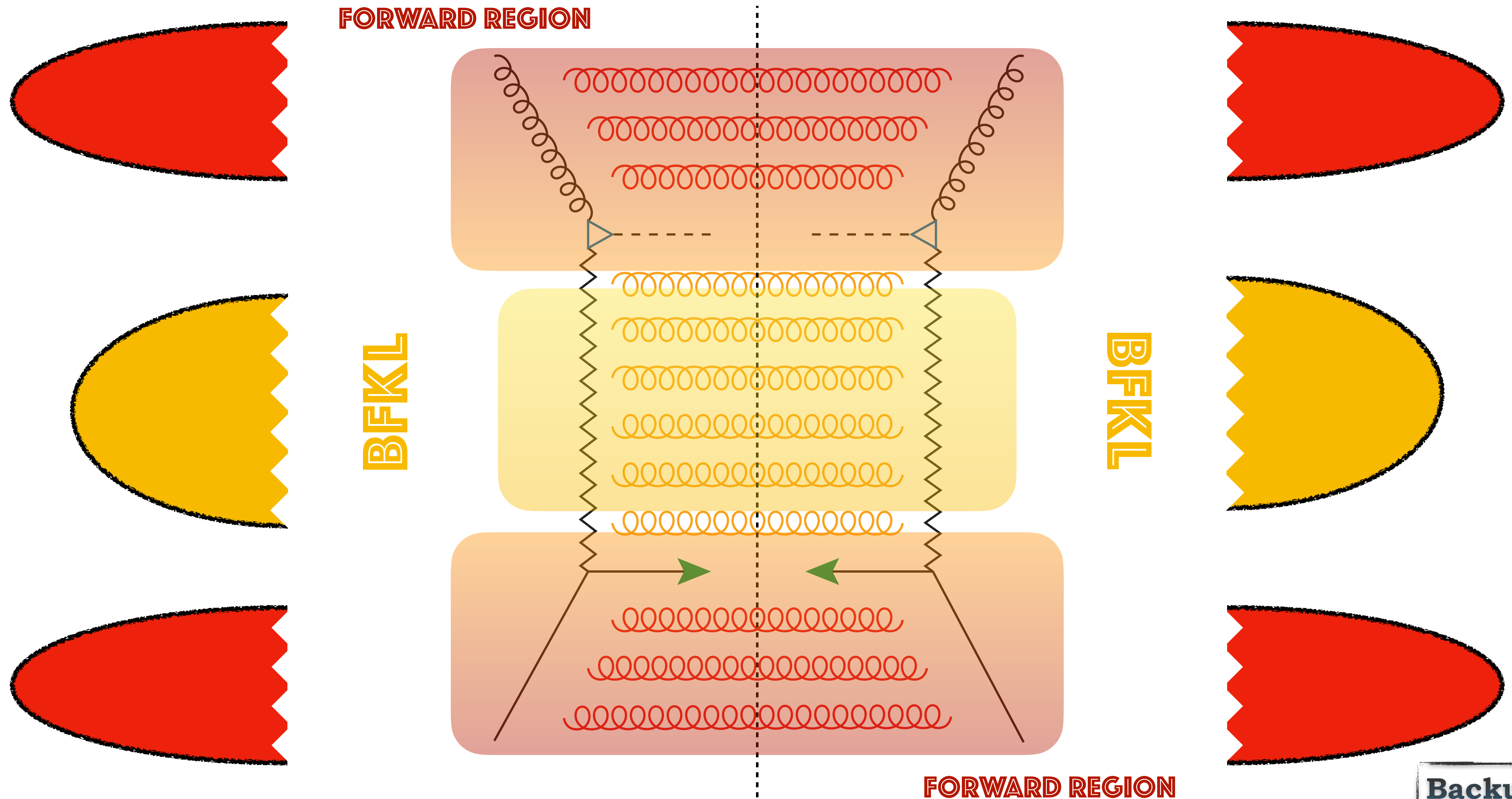
Anatomy of Higgs + jet in hybrid factorization (HyF)

FORWARD REGION



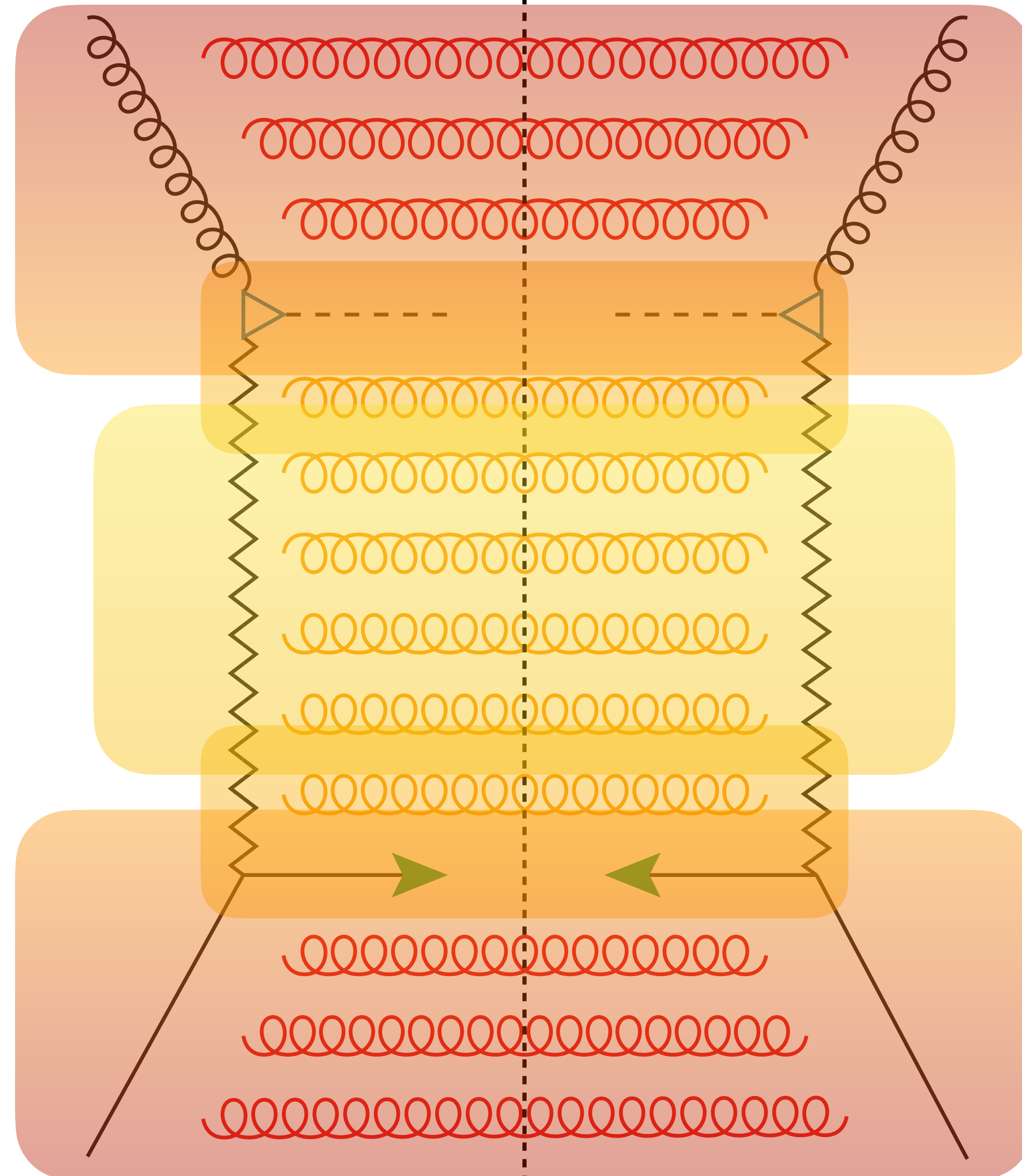
FORWARD REGION

Anatomy of Higgs + jet in hybrid factorization (HyF)



Anatomy of Higgs + jet in hybrid factorization (HyF)

FORWARD REGION



NEXT-TO-FORWARD REGION

BFKL

BFKL

NEXT-TO-FORWARD REGION

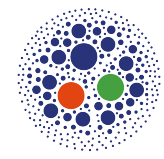
FORWARD REGION

Backup

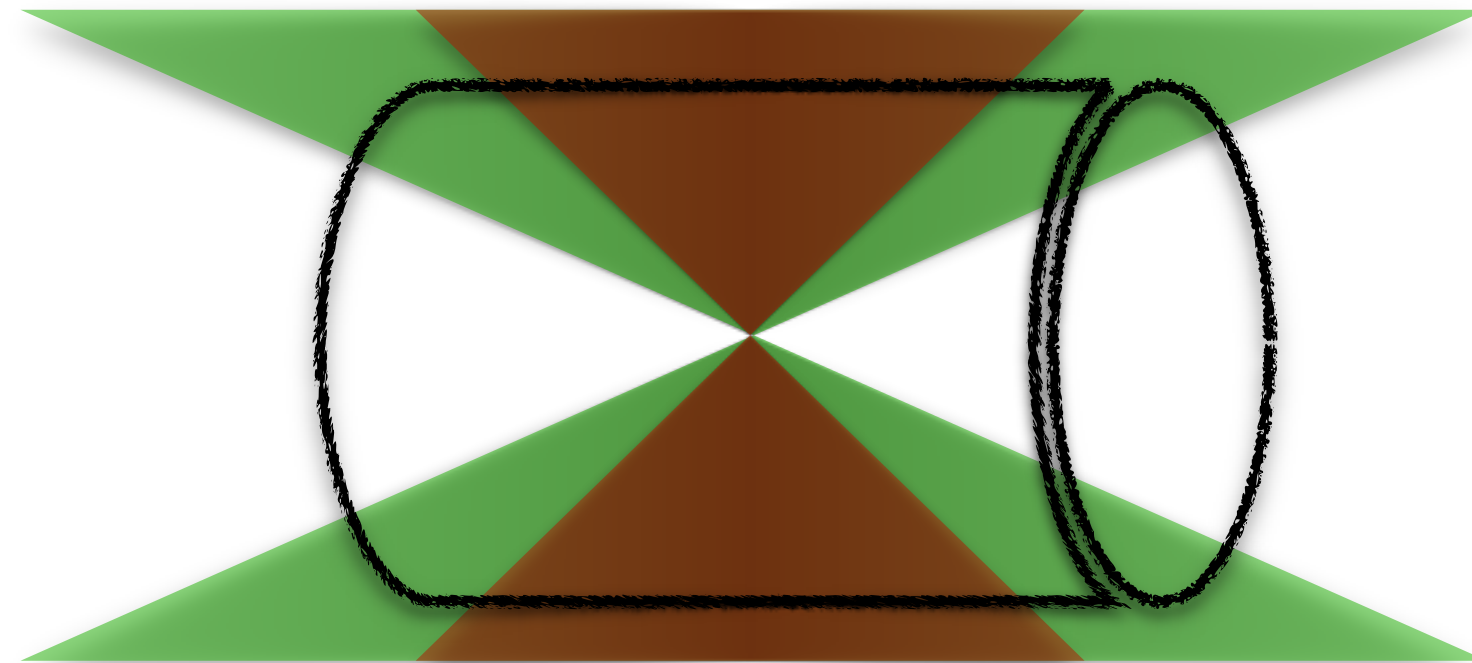
The background of the slide is a light blue gradient with several faint, overlapping Feynman diagrams. These diagrams depict particle interactions using various symbols: red and blue spheres for fermions, green spheres for scalars, and yellow wavy lines for gauge bosons. Arrows indicate the direction of particle flow. The diagrams are arranged in a grid-like pattern, creating a scientific and technical atmosphere.

Ultraforward charm + Higgs production at 14 TeV FPF+ATLAS

High-energy QCD at new-gen Forward Facilities



Forward + backward CMS detections: Mueller-Navelet, hadron-jet, di-hadron



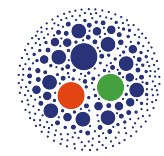
$$|y_{\text{jet}}| < 4.7$$

barrel + endcap

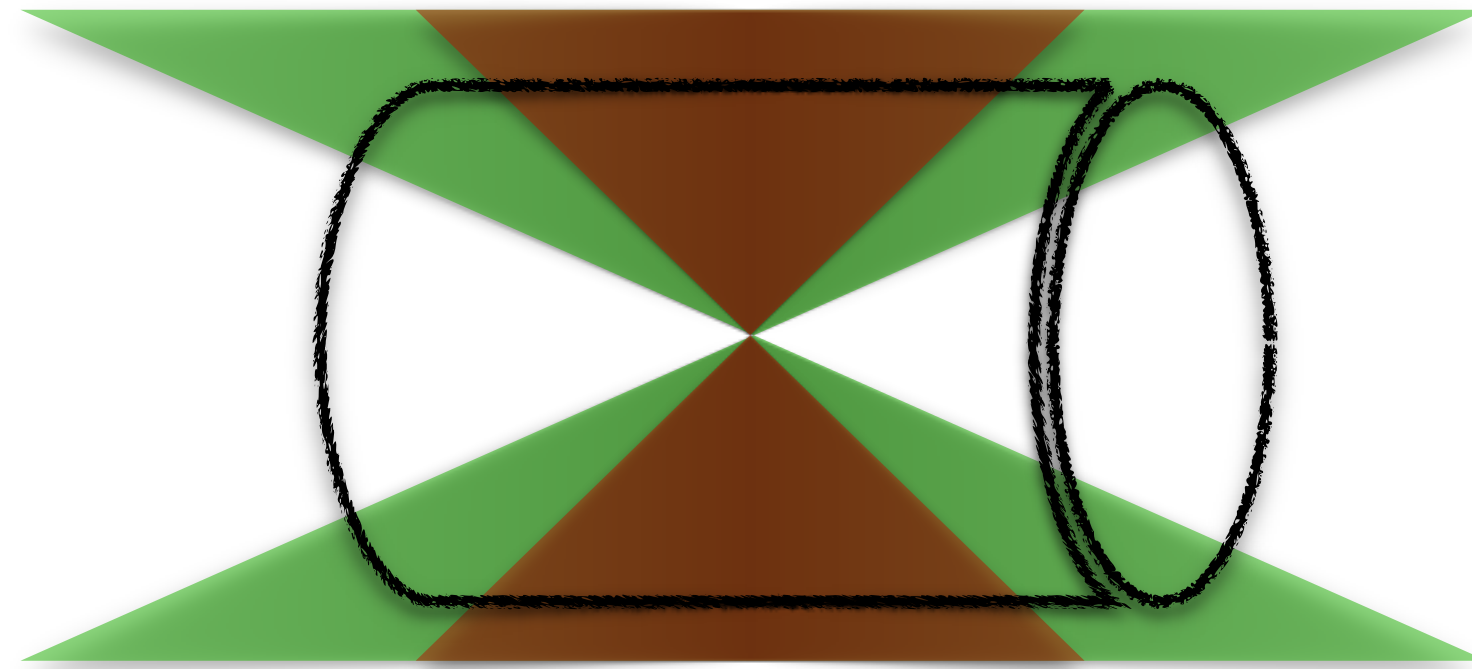
$$|y_{\text{hadron}}| < 2.4$$

barrel

High-energy QCD at new-gen Forward Facilities



Forward + backward CMS detections: Mueller-Navelet, hadron-jet, di-hadron

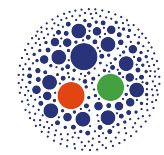


$$|y_{\text{jet}}| < 4.7$$

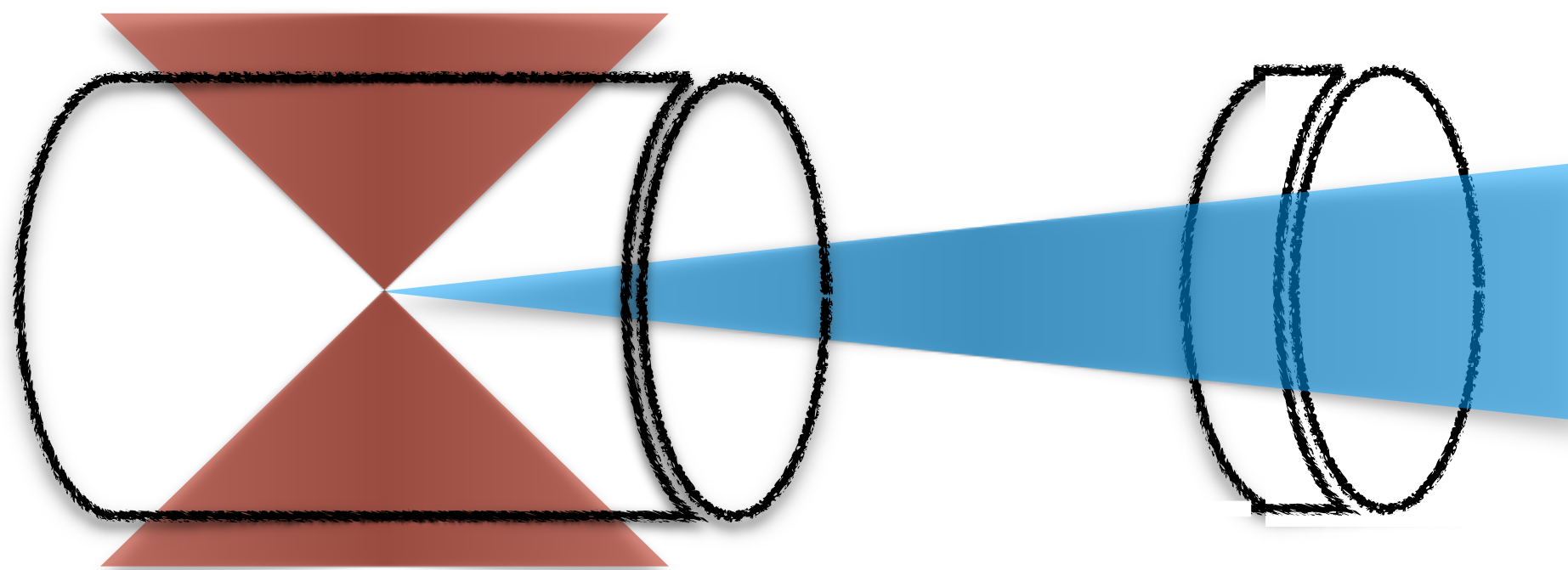
barrel + endcap

$$|y_{\text{hadron}}| < 2.4$$

barrel



Ultra-forward FPF + central ATLAS detections: single-charmed hadrons + Higgs



$$5 < |y_{D^*, \Lambda_c}| < 7$$

FPF

$$|y_{\text{Higgs}}| < 2.5$$

ATLAS barrel

(charm + Higgs) [F. G. C. et al., Phys. Rev. D 105 (2022) 11, 114056]

(light mesons + heavy flavor) [F. G. C., Phys. Rev. D 105 (2022) 11, 114008]

Backup

High-energy QCD in ultraforward directions

Backup



The Forward Physics Facility: Sites, experiments, and physics potential

Luis A. Anchordoqui¹, Akitaka Ariga^{2,3}, Tomoko Ariga⁴, Weidong Bai⁵, Kinso Balazs⁶, Brian Batell⁷, Jamie Boyd⁶, Joseph Bramante⁸, Mario Campanelli⁹, Adrian Carmona¹⁰, **Francesco G. Celiberto**^{11,12,13}, Grigorios Chachamis¹⁴, Matthew Citron¹⁵, Giovanni De Lellis^{16,17}, Albert De Roeck⁶, Hans Dembinski¹⁸, Peter B. Denton¹⁹, Antonia Di Crescenzo^{16,17,6}, Milind V. Diwan²⁰, Liam Dougherty²¹, Herbi K. Dreiner²², Yong Du²³, Rikard Enberg²⁴, Yasaman Farzan²⁵, Jonathan L. Feng^{26,*}, Max Fieg²⁶, Patrick Foldenauer²⁷, Saeid Foroughi-Abari²⁸, Alexander Friedland²⁹, Michael Fucilla^{30,31}, Jonathan Gall³², Maria Vittoria Garzelli^{33,*}, Francesco Giuliani³⁴, Victor P. Goncalves³⁵, Marco Guzzi³⁶, Francis Halzen³⁷, Juan Carlos Helo^{38,39}, Christopher S. Hill⁴⁰, Ahmed Ismail⁴¹, Ameen Ismail⁴², Richard Jacobsson⁶, Sudip Jana⁴³, Yu Seon Jeong⁴⁴, Krzysztof Jodłowski⁴⁵, Kevin J. Kelly⁴⁶, Felix Kling^{29,47,**}, Fnu Karan Kumar²⁰, Zhen Liu⁴⁸, Rafał Maciuła⁴⁹, Roshan Mammen Abraham⁴¹, Julien Manshanden³³, Josh McFayden⁵⁰, Mohammed M.A. Mohammed^{30,31}, Pavel M. Nadolsky⁵¹, Nobuchika Okada⁵², John Osborne⁶, Hidetoshi Otono⁴, Vishvas Pandey^{53,46}, Alessandro Papa^{30,31}, Digesh Raut⁵⁴, Mary Hall Reno⁵⁵, Filippo Resnati⁶, Adam Ritz²⁸, Juan Rojo⁵⁶, Ina Sarcevic⁵⁷, Christiane Scherb⁵⁸, Holger Schulz⁵⁹, Pedro Schwaller⁶⁰, Dipan Sengupta⁶¹, Torbjörn Sjöstrand⁶², Tyler B. Smith²⁶, Dennis Soldin⁵⁴, Anna Stasto⁶³, Antoni Szczurek⁴⁹, Zahra Tabrizi⁶⁴, Sebastian Trojanowski^{65,66}, Yu-Dai Tsai^{26,46}, Douglas Tuckler⁶⁷, Martin W. Winkler⁶⁸, Keping Xie⁷, Yue Zhang⁶⁷

¹ Department of Physics and Astronomy, Lehman College, City University of New York, Bronx, NY 10468, USA

² Albert Einstein Center for Fundamental Physics, Laboratory for High Energy Physics, University of Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland

³ Department of Physics, Chiba University, 1-33 Yayoi-cho Inage-ku, Chiba, 263-8522, Japan

⁴ Kyushu University, Nishi-ku, 819-0395 Fukuoka, Japan

⁵ Sun Yat-sen University, School of Physics, No. 135, Xingang Xi Road, Guangzhou, 510275, PR China

⁶ CERN, CH-1211 Geneva 23, Switzerland

⁷ PITT PACC, Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

⁸ Department of Physics, Queen's University, Kingston, ON K7L 2S8, Canada

⁹ Department of Physics & Astronomy, University College London, Gower Street, London, WC1E 6BT, United Kingdom

¹⁰ CAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada, E18071 Granada, Spain

¹¹ European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT*), I-38123 Villazzano, Trento, Italy

¹² Fondazione Bruno Kessler (FBK), I-38123 Povo, Trento, Italy

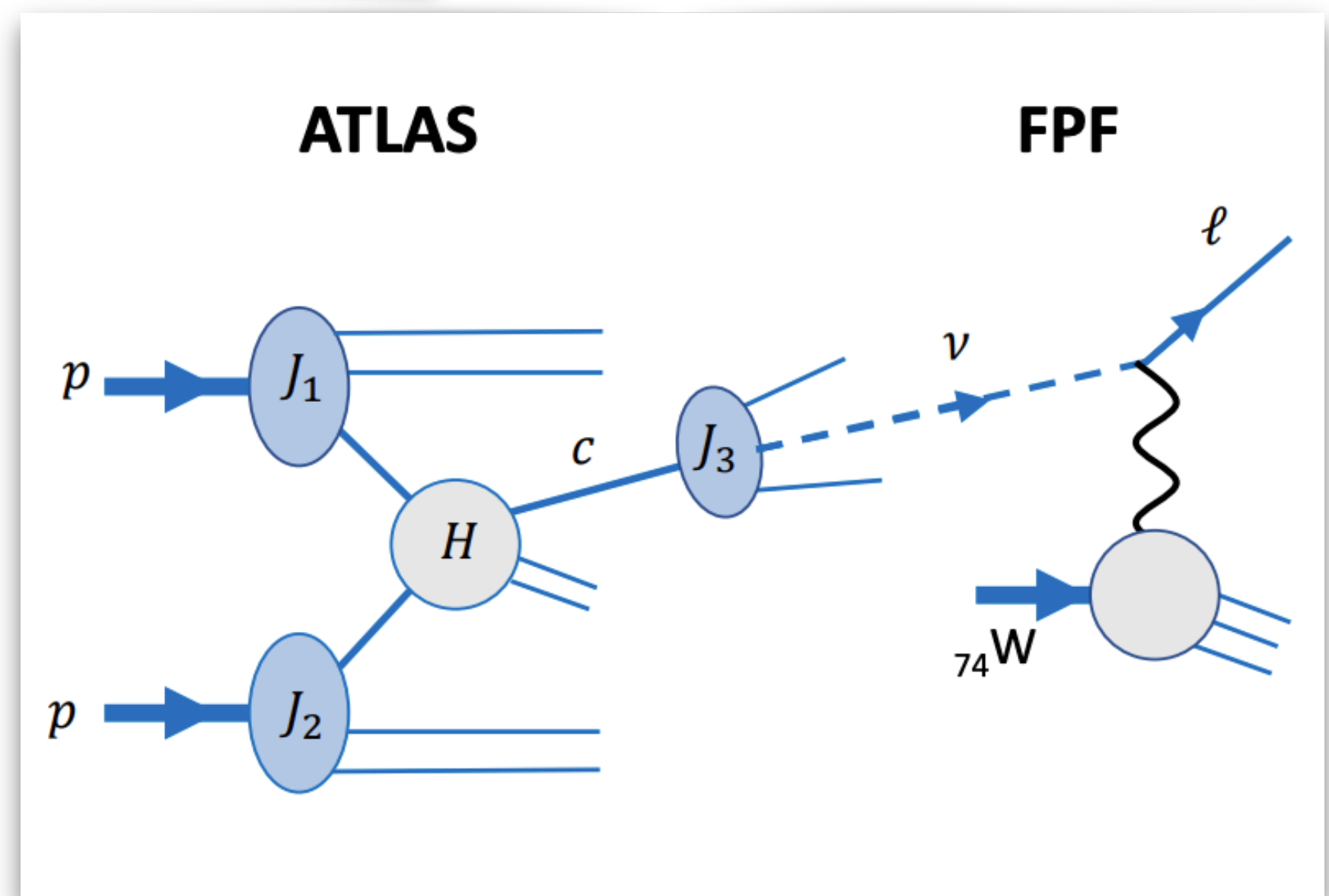
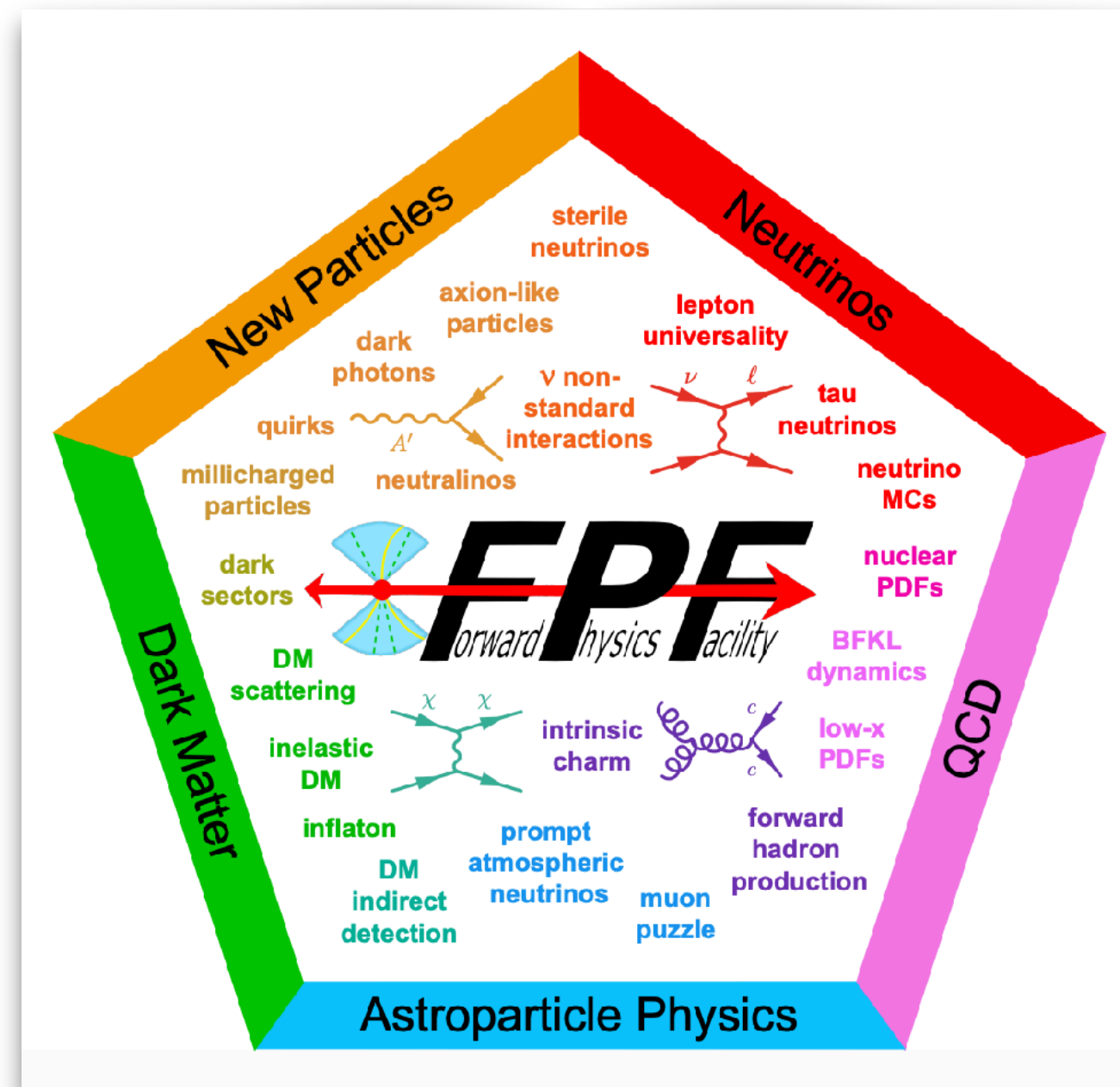
¹³ INFN-TIFPA Trento Institute of Fundamental Physics and Applications, I-38123 Povo, Trento, Italy

* Corresponding authors.

** Corresponding author at: Deutsches Elektronen-Synchrotron DESY, Notkestrasse 85, 22607 Hamburg, Germany.
E-mail addresses: jlf@uci.edu (J.L. Feng), maria.vittoria.garzelli@desy.de (M.V. Garzelli), felix.kling@desy.de (F. Kling).

<https://doi.org/10.1016/j.physrep.2022.04.004>

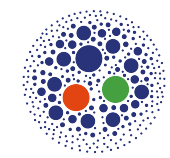
0370-1573/© 2022 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).



The background of the slide is a light blue gradient with several faint, overlapping Feynman diagrams. These diagrams represent particle interactions, featuring various colored spheres (red, blue, green) and wavy lines (yellow, blue) with arrows indicating the flow of particles and forces. The diagrams are arranged in a circular, repeating pattern, creating a sense of depth and scientific complexity.

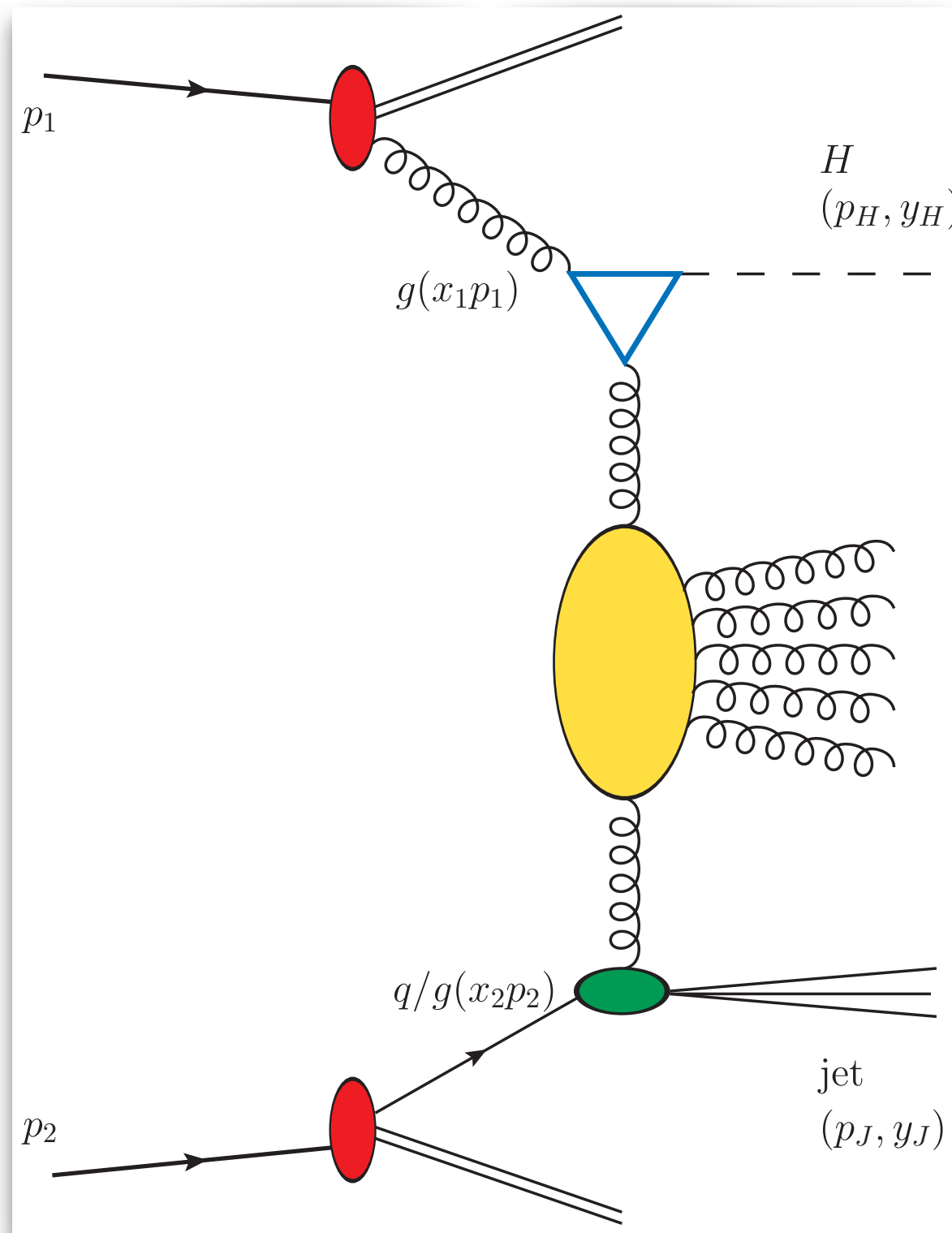
Higgs + jet distributions

Inclusive Higgs + jet at the LHC

-  Inclusive h.p. of a Higgs + jet system with high p_T and large rapidity separation, ΔY
-  Large energy scales expected to **stabilize** the high-energy resummed series

$$\frac{d\sigma}{dx_1 dx_2 d|\vec{p}_H| d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2 \cos(n\varphi) \mathcal{C}_n \right]$$

$$\varphi = \varphi_H - \varphi_J - \pi$$



Inclusive Higgs + jet at the LHC

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$$\varphi = \varphi_H - \varphi_J - \pi$$

NLO*

NLL

NLO*

Higgs vertex
(off-shell coefficient function)

jet vertex
(off-shell coefficient function)

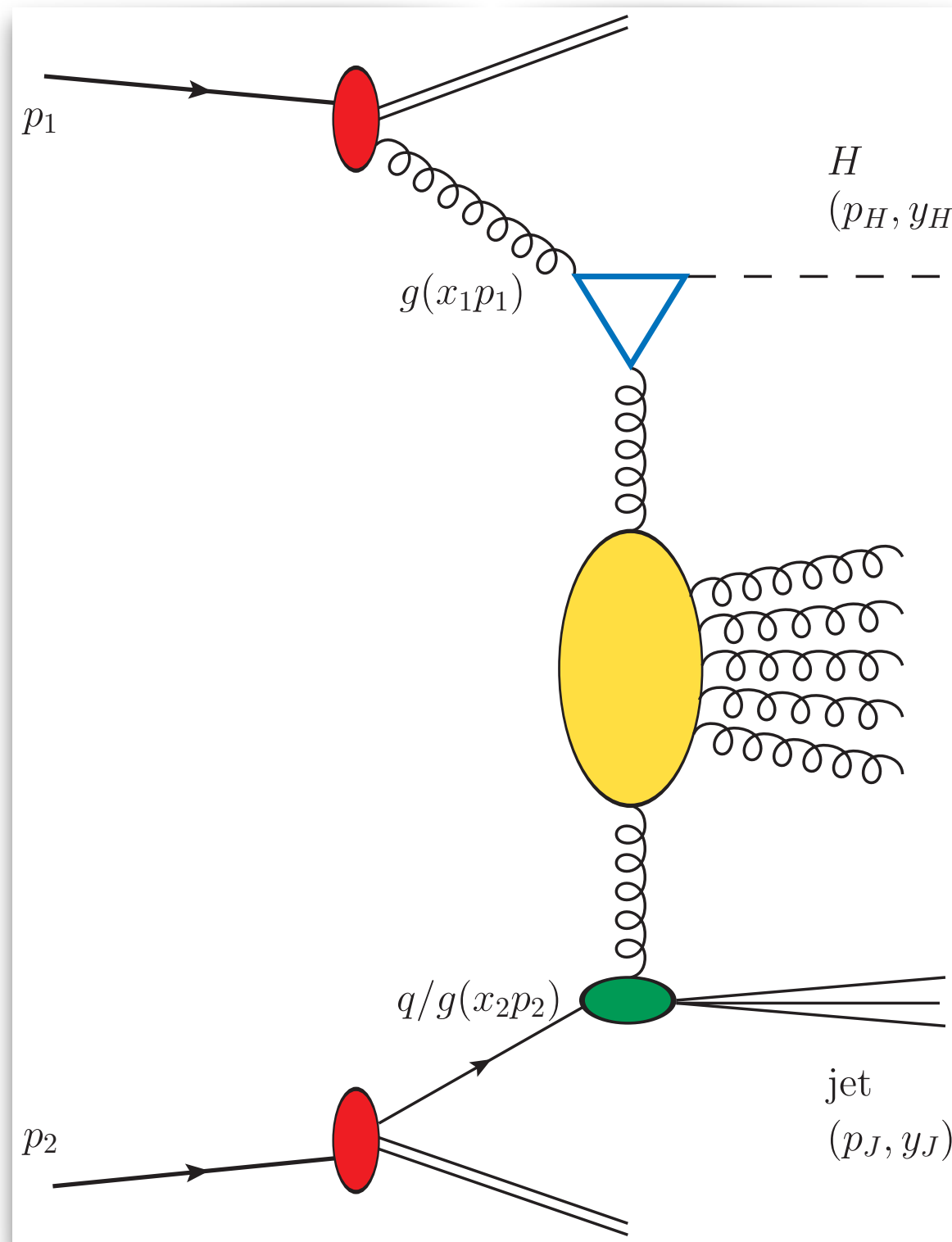
$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H dy_J d^2\vec{p}_H d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^2} \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H)$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0} \right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

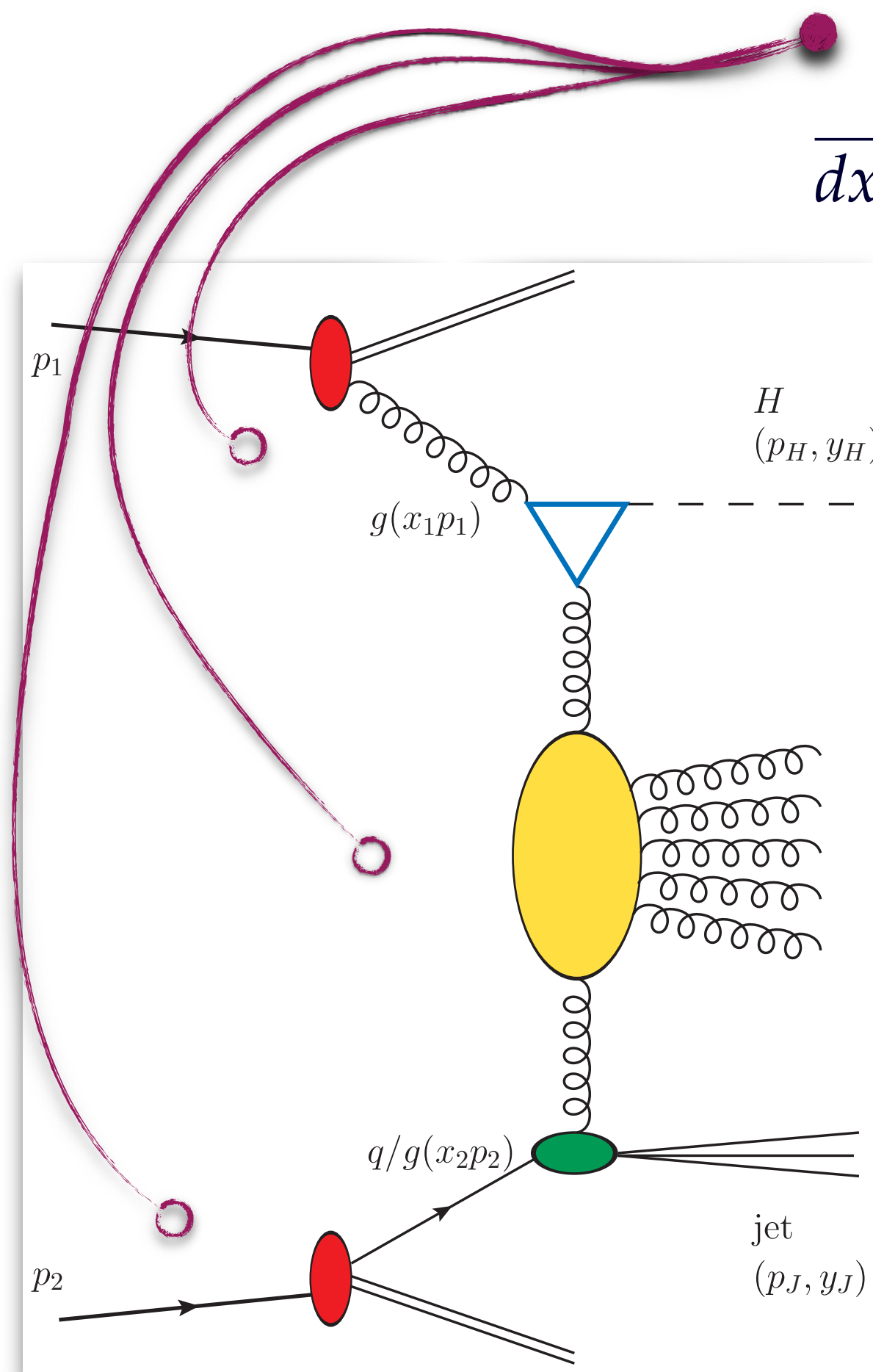
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^2} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

BFKL Green's function



Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high p_T and large rapidity separation, ΔY
- Large energy scales expected to stabilize the high-energy resummed series



$$\frac{d\sigma}{dx_1 dx_2 d|\vec{p}_H| d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2 \cos(n\varphi) \mathcal{C}_n \right]$$

Higgs vertex
(off-shell coefficient function)

jet vertex
(off-shell coefficient function)

$$\varphi = \varphi_H - \varphi_J - \pi$$

$$\mu_{F,R} \sim M_{H,\perp}$$

$$\mu_R \sim \sqrt{M_{H,\perp} p_J}$$

$$\mu_{F,R} \sim p_J$$

NLO*

NLL

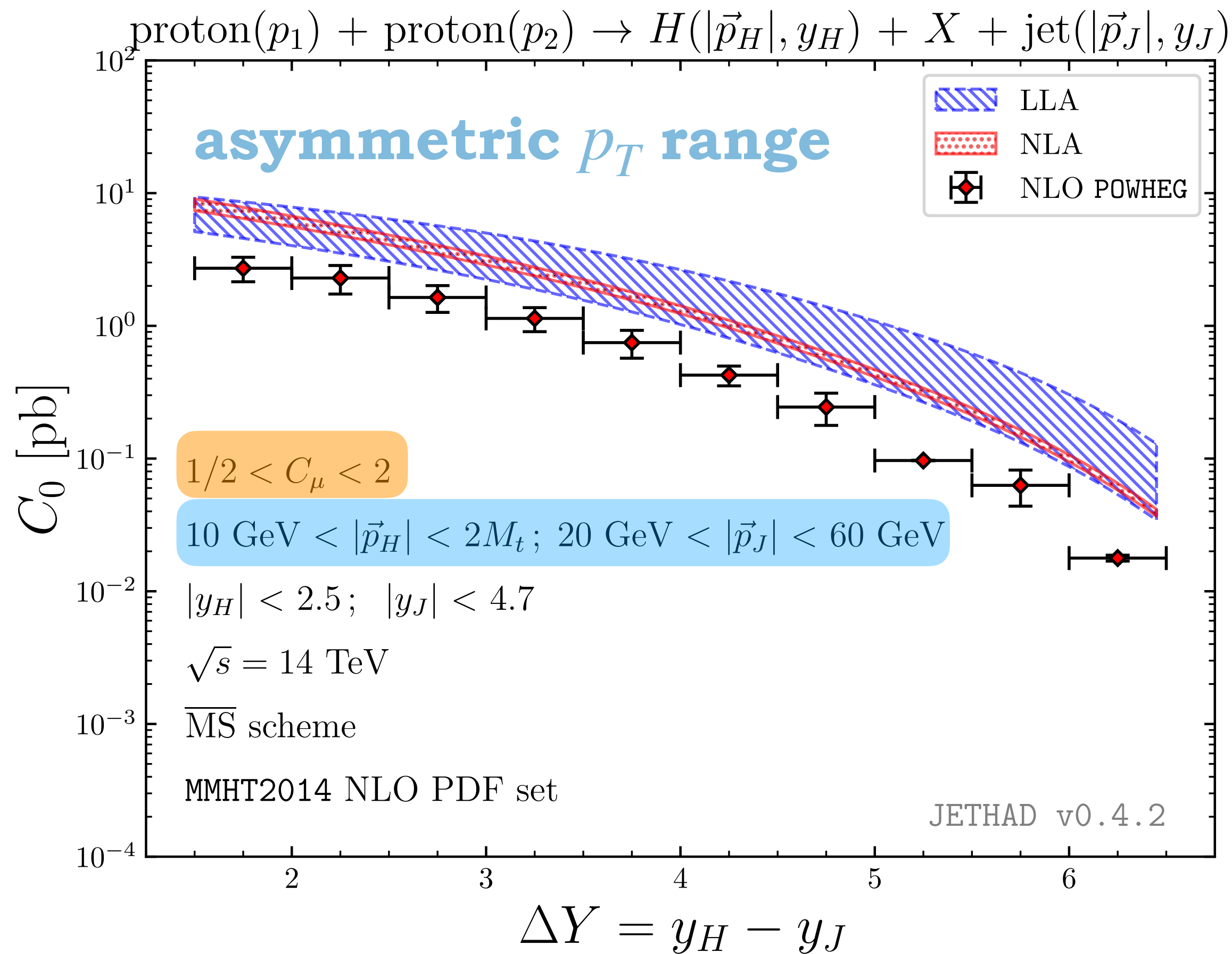
NLO*

$$\begin{aligned} \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H dy_J d^2\vec{p}_H d^2\vec{p}_J} &= \frac{1}{(2\pi)^2} \\ &\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^2} \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \\ &\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0} \right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2) \\ &\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^2} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \end{aligned}$$

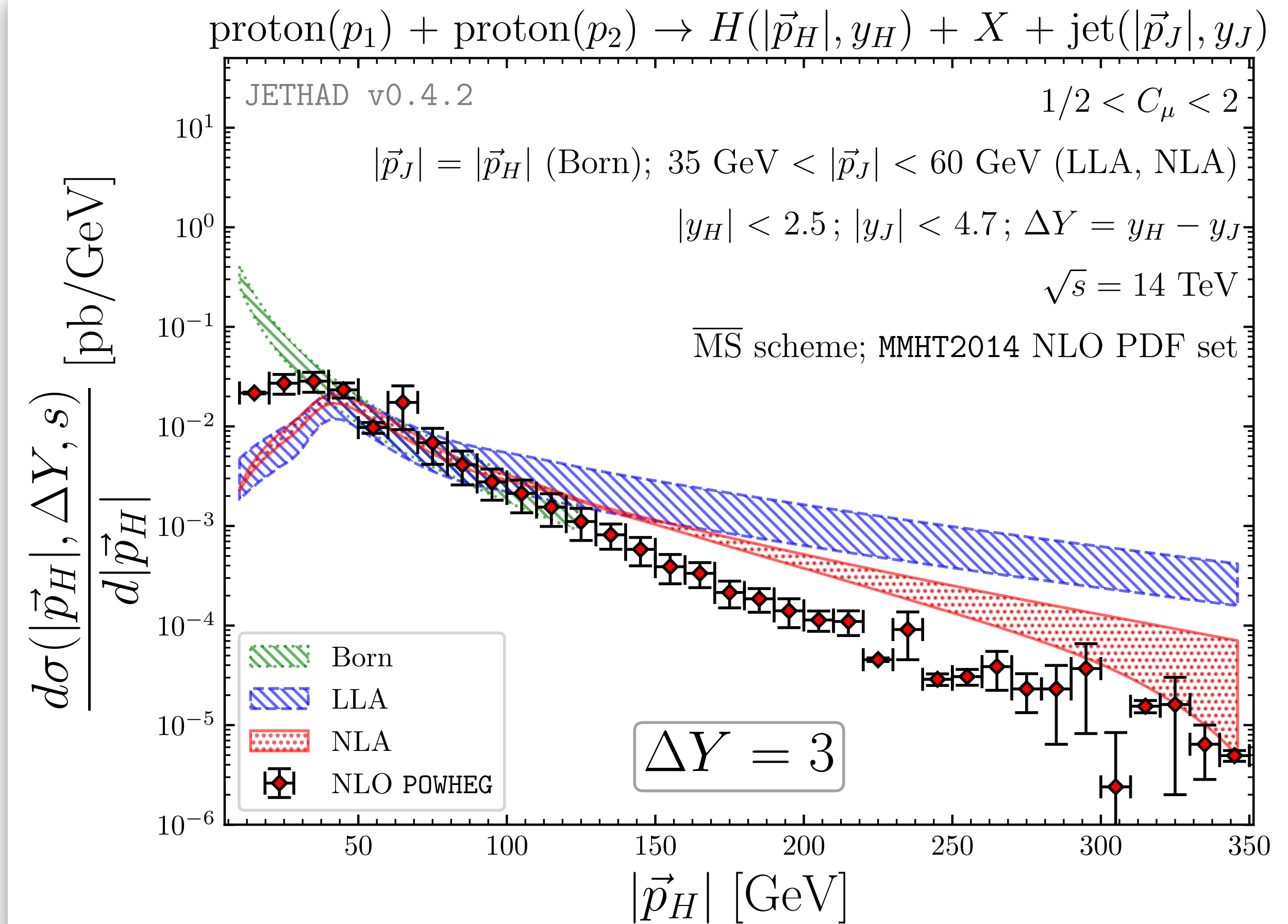
BFKL Green's function

The Higgs + jet spectrum in hybrid factorization

ΔY spectrum



p_H spectrum

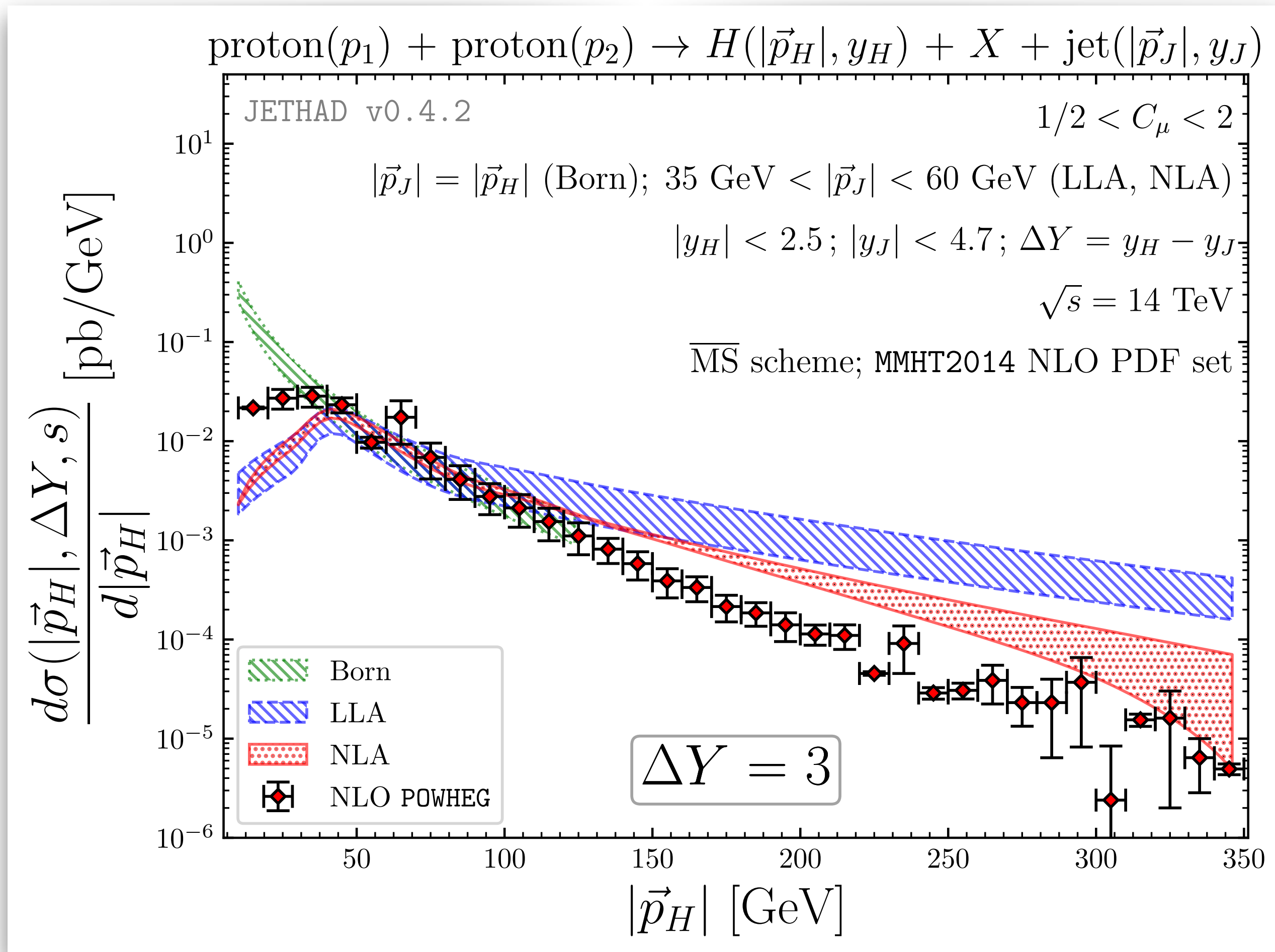


(in this slide) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 4, 293\]](#)

(JETHAD) [\[F. G. C., Eur. Phys. J. C 81 \(2021\) 8, 691\]](#)

Higgs transverse-momentum distribution

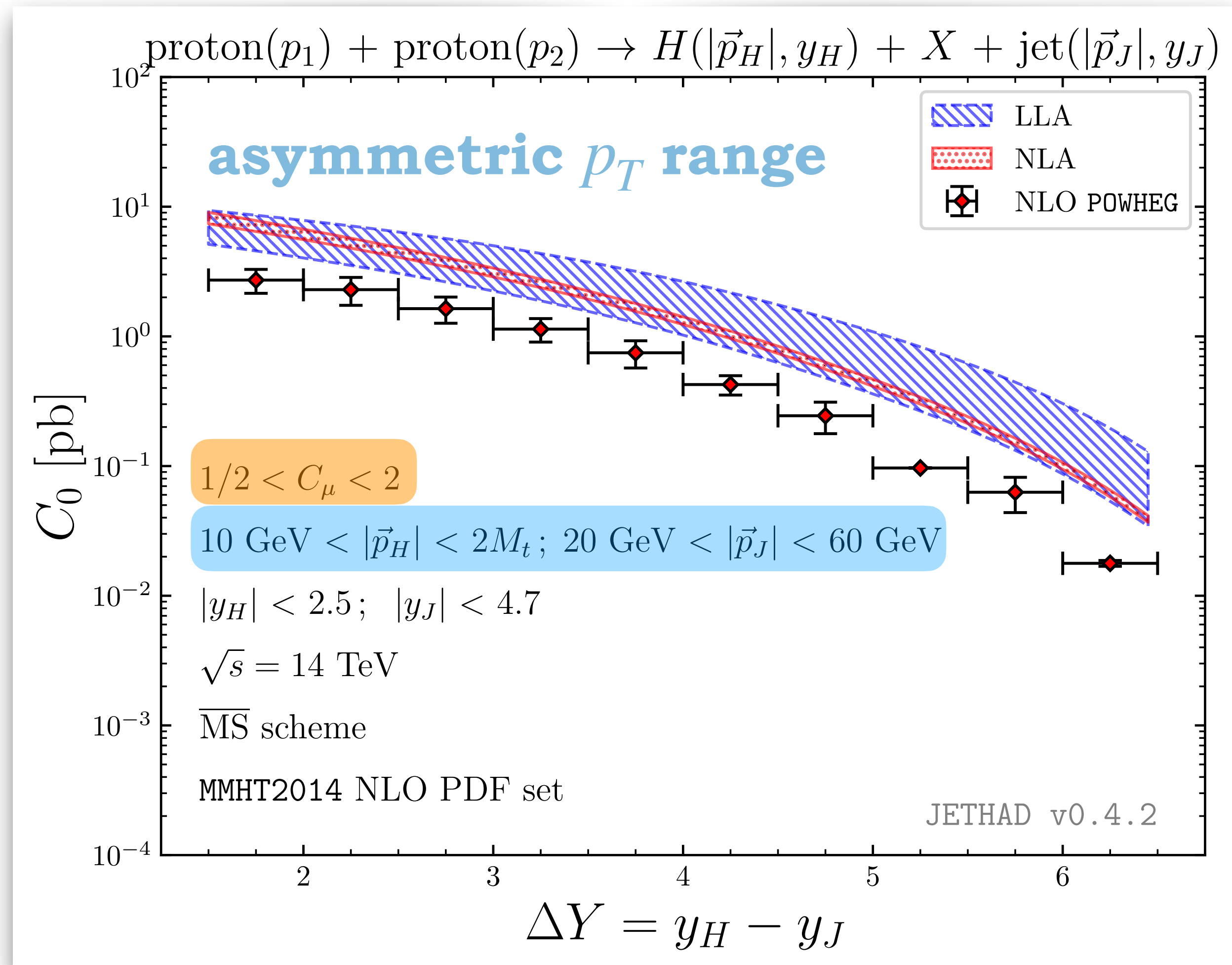
$$\frac{d\sigma(|\vec{p}_H|, \Delta Y, s)}{d|\vec{p}_H|d\Delta Y} = \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) \mathcal{C}_0$$



- ✓ HE resummation from JETHAD
- ✓ Comparison with fixed-order POWHEG
- ✓ Distributions stable under NLL corrections

ΔY -distribution

$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$



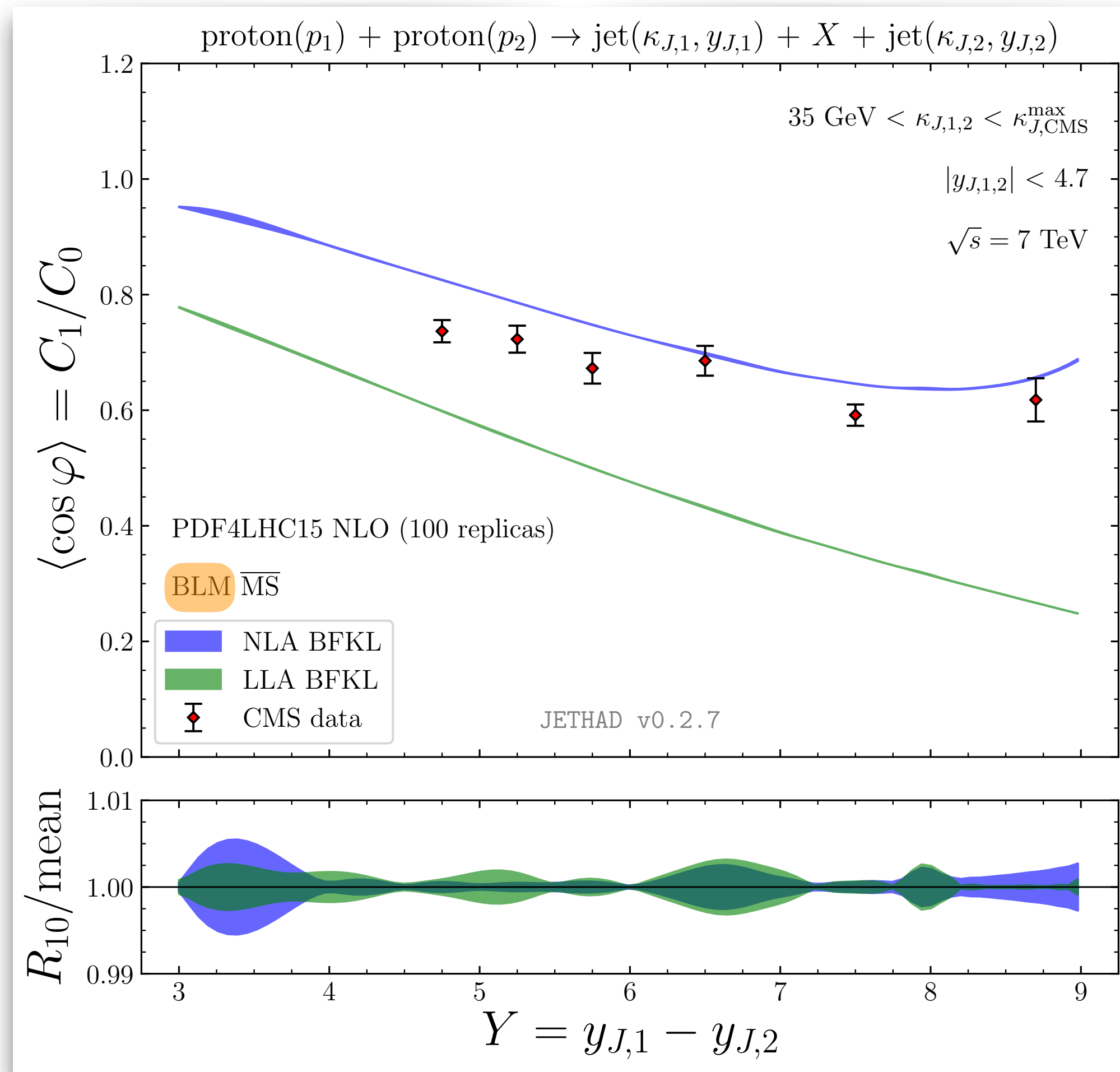
Angular correlations

$$R_{n0}(\Delta Y, s) = C_n/C_0 \equiv \langle \cos n\varphi \rangle$$

Mueller-Navelet jets

[\[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 \(2014\) 082003\]](#)

(figure below) [\[F. G. C., Eur. Phys. J. C 81 \(2021\) 8, 691\]](#)



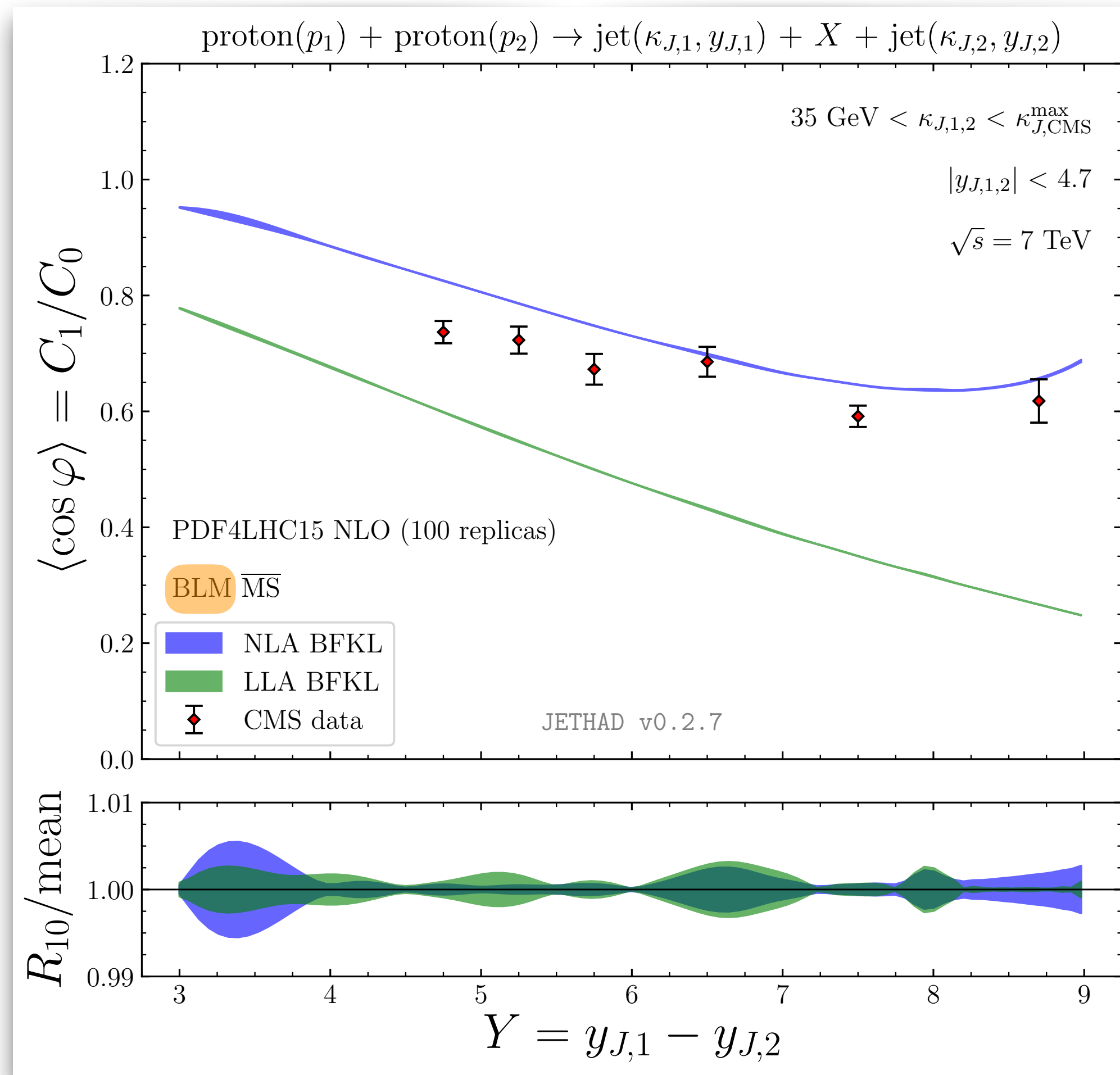
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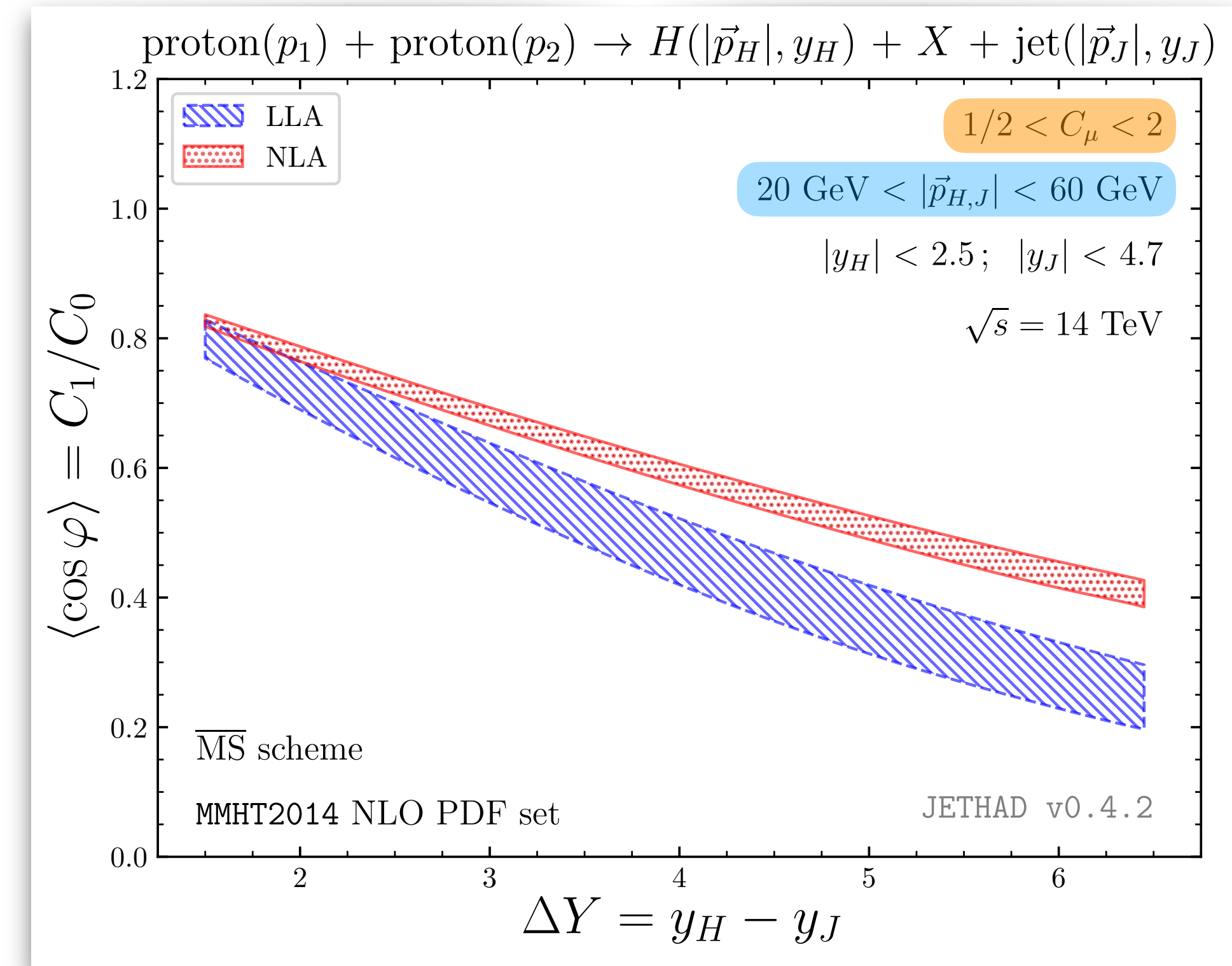
(figure below) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]



Higgs + jet

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

(NLO Higgs impact factor) [F. G. C. et al., under review (2022)]

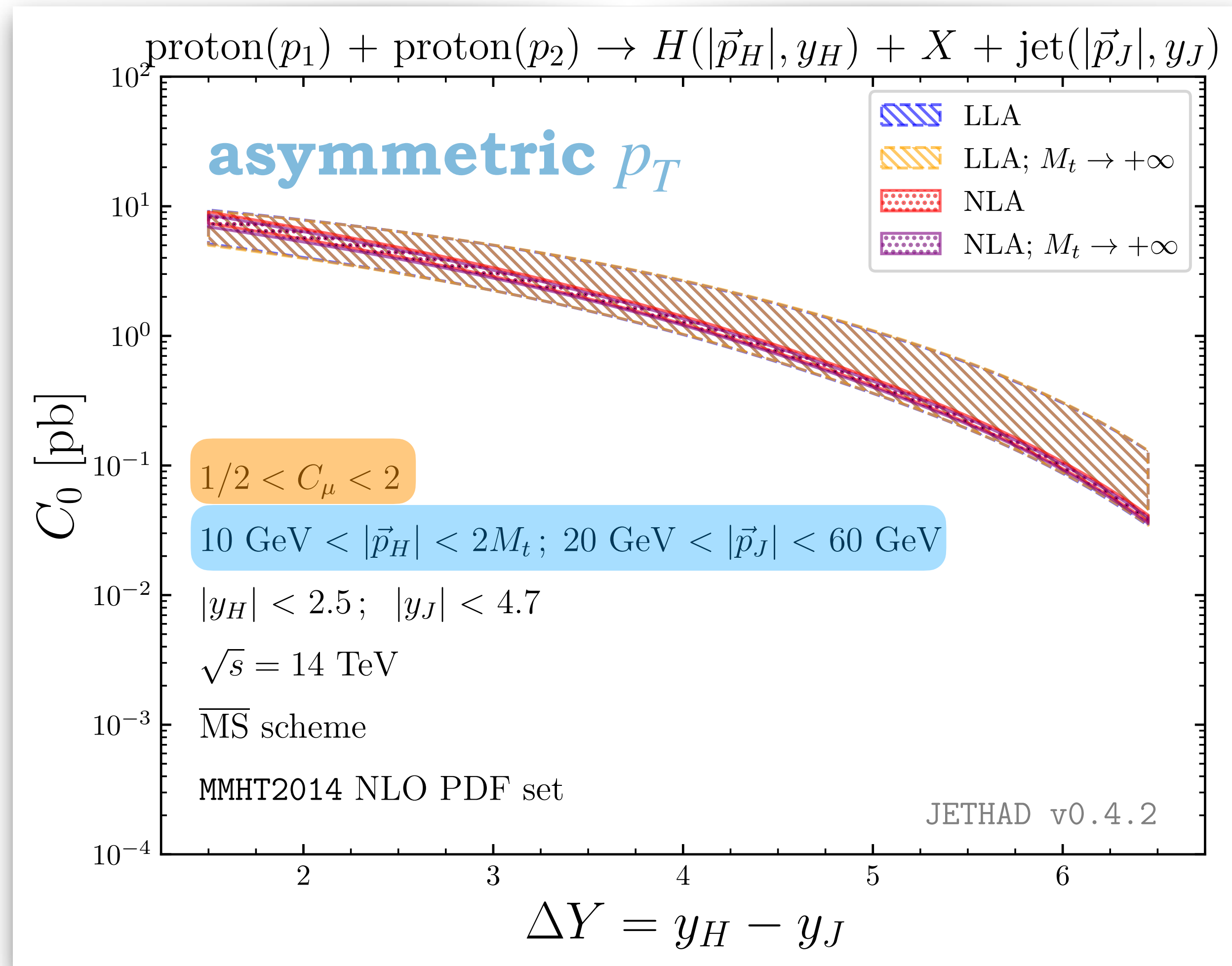


natural scales

symmetric p_T range

ΔY -distribution in the infinite top-mass limit

$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$



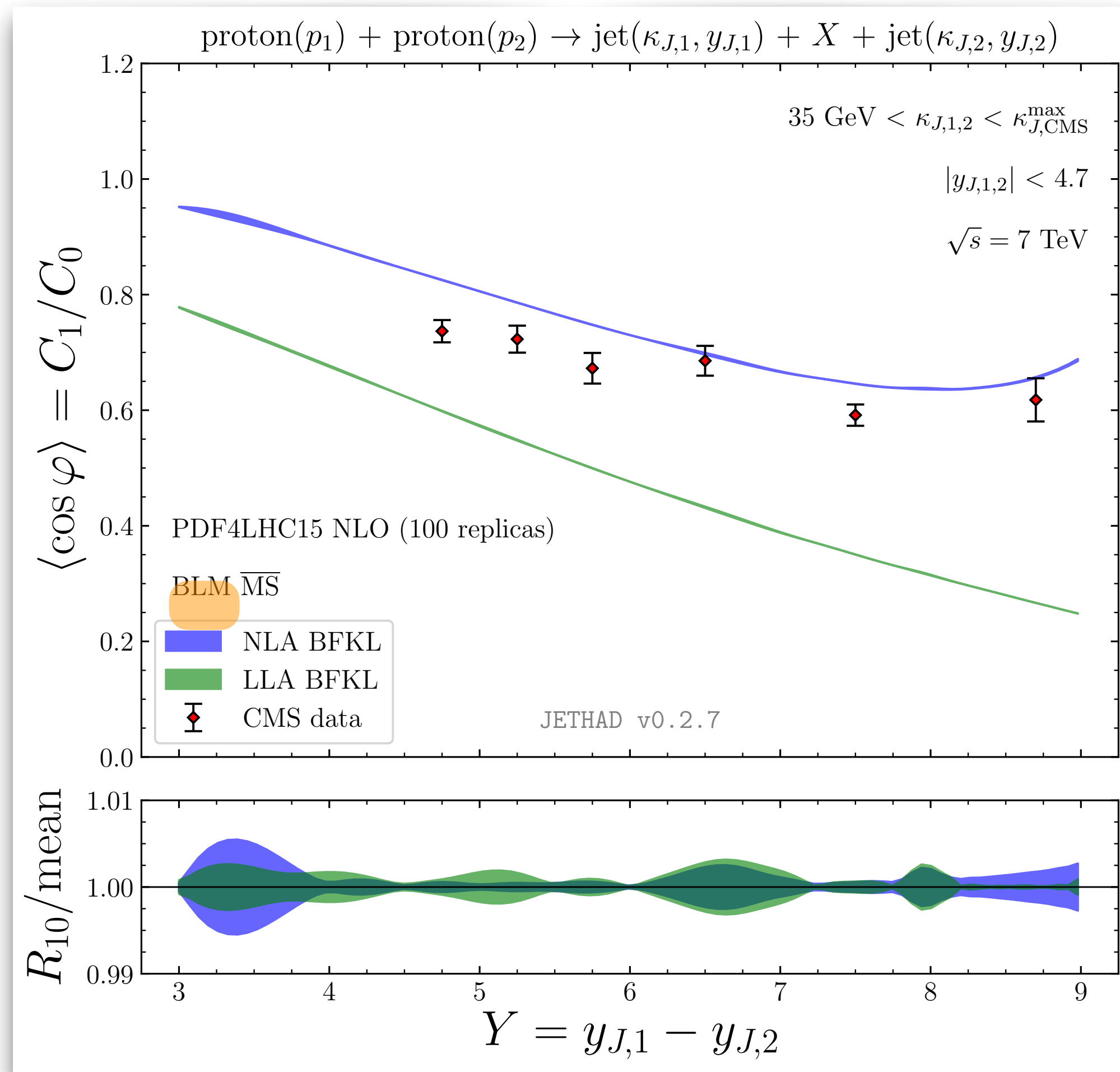
Angular correlations in the infinite top-mass limit

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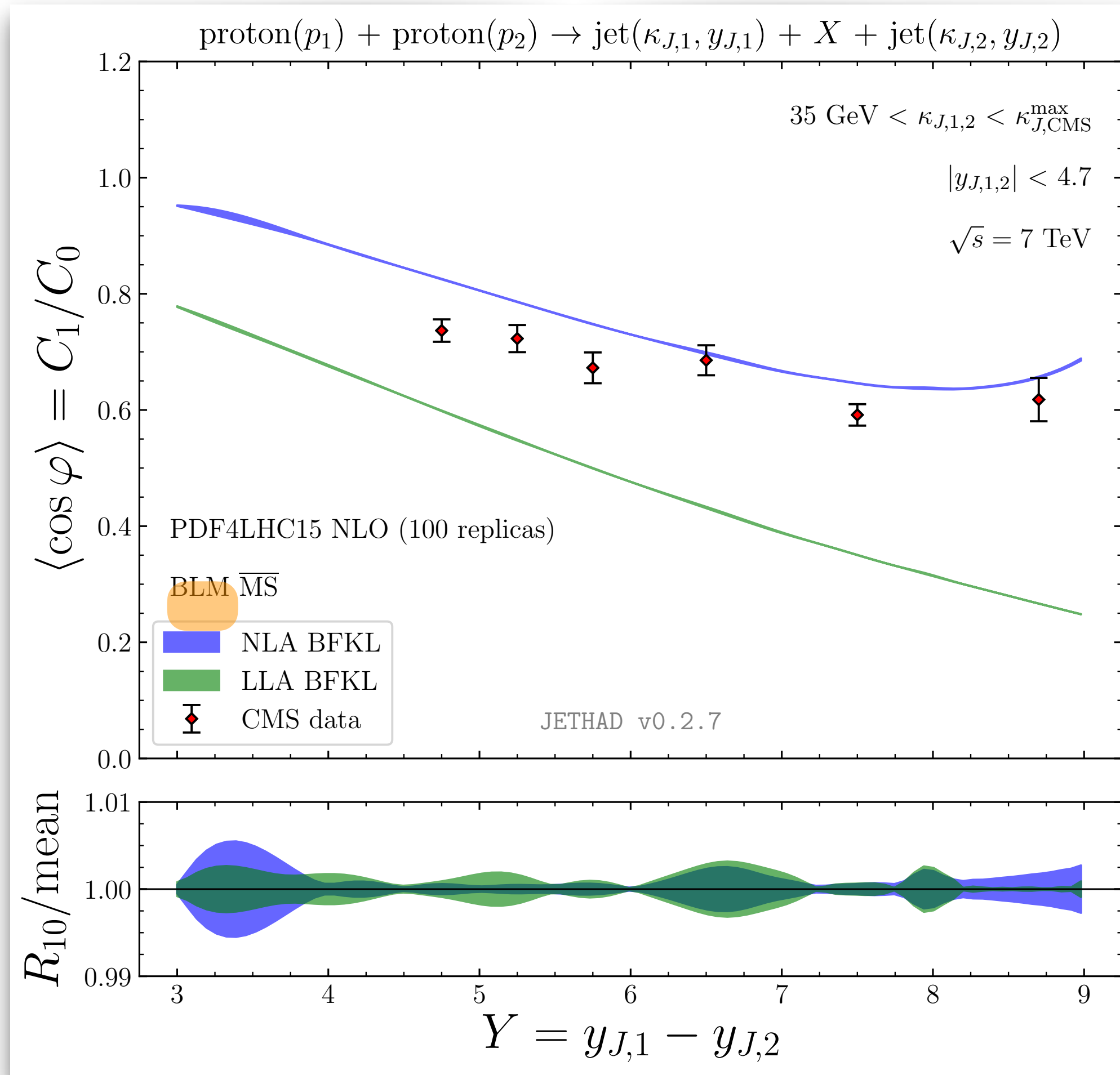
Angular correlations in the infinite top-mass limit

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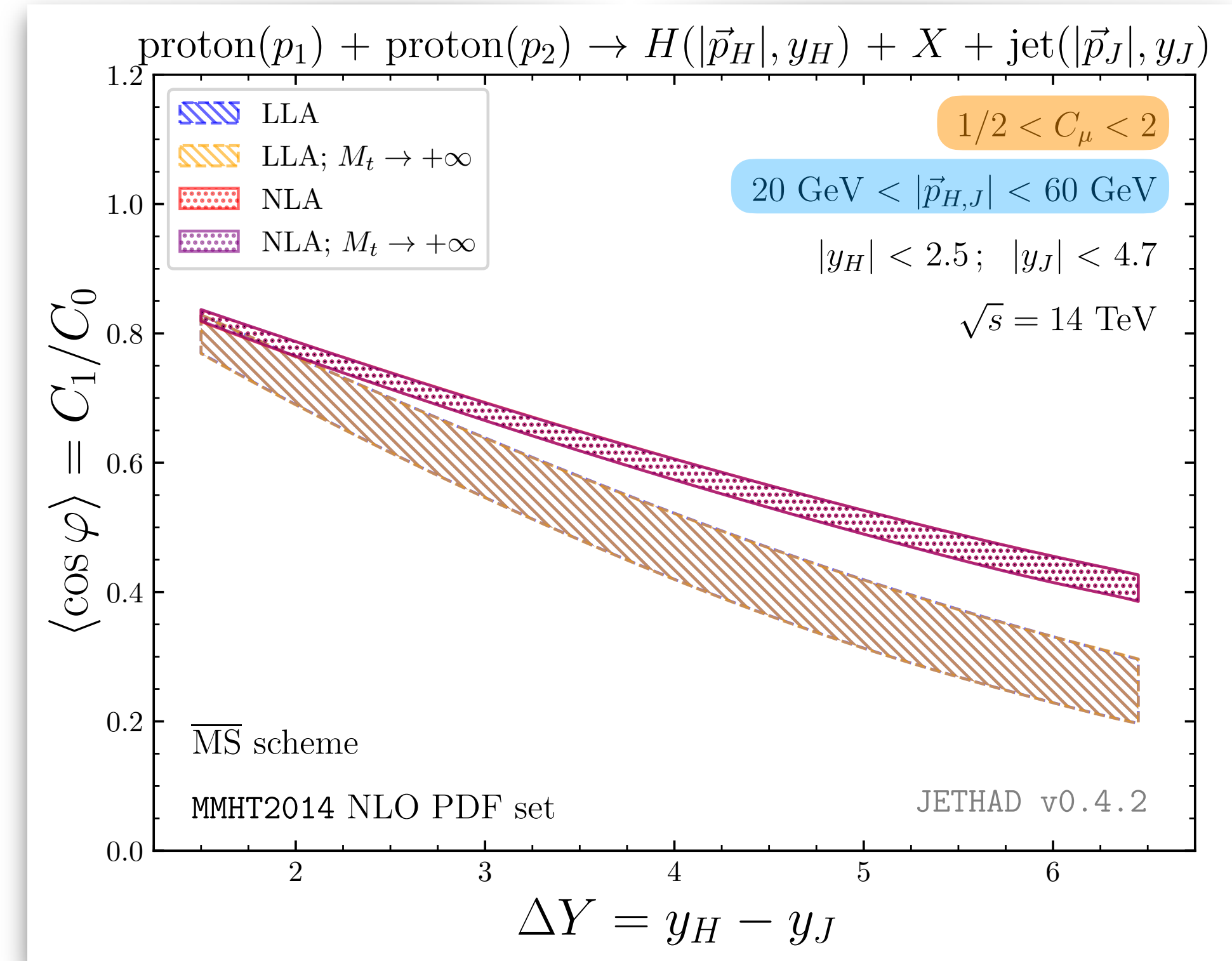
(figure below) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]



Higgs + jet

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

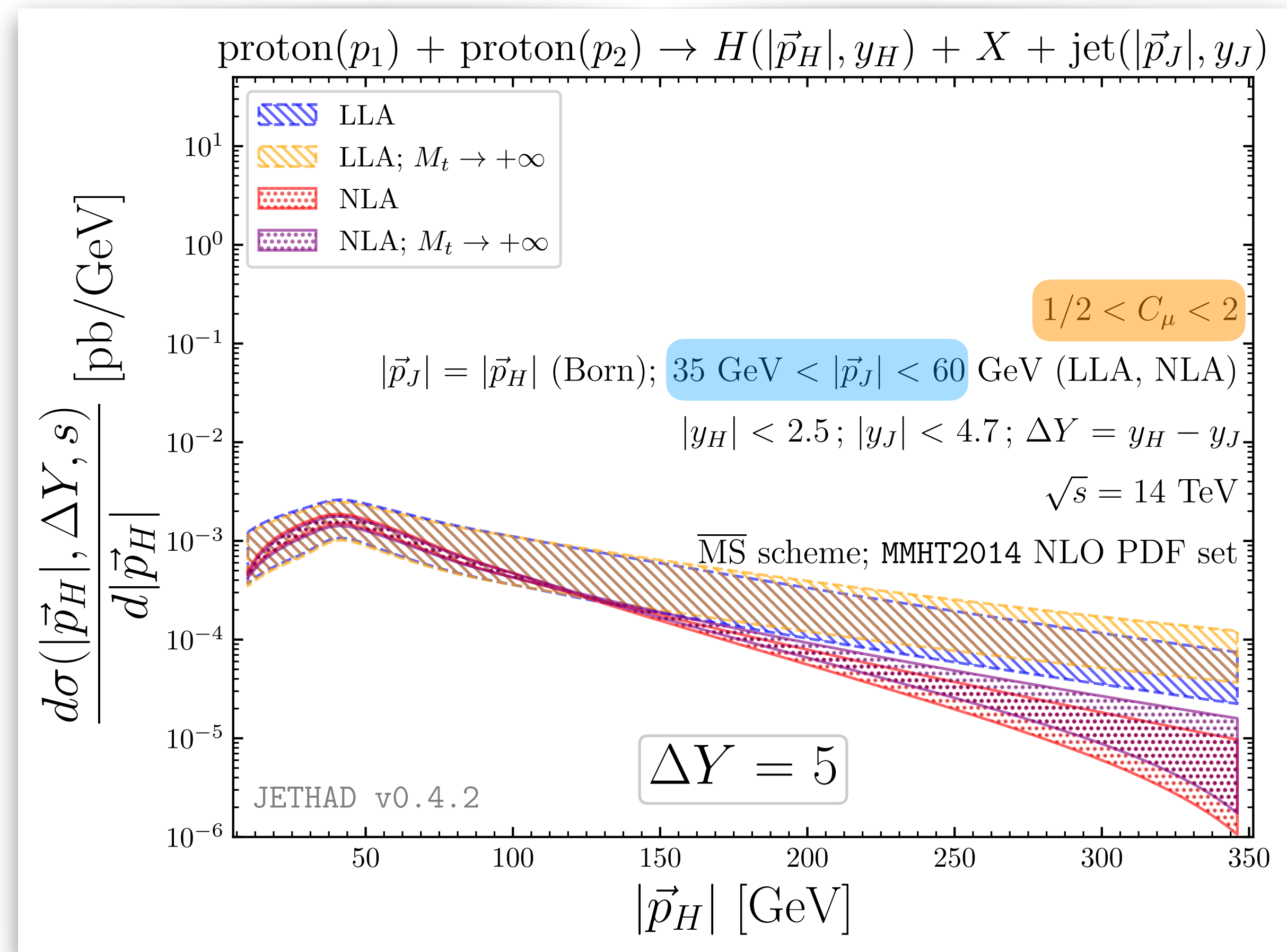
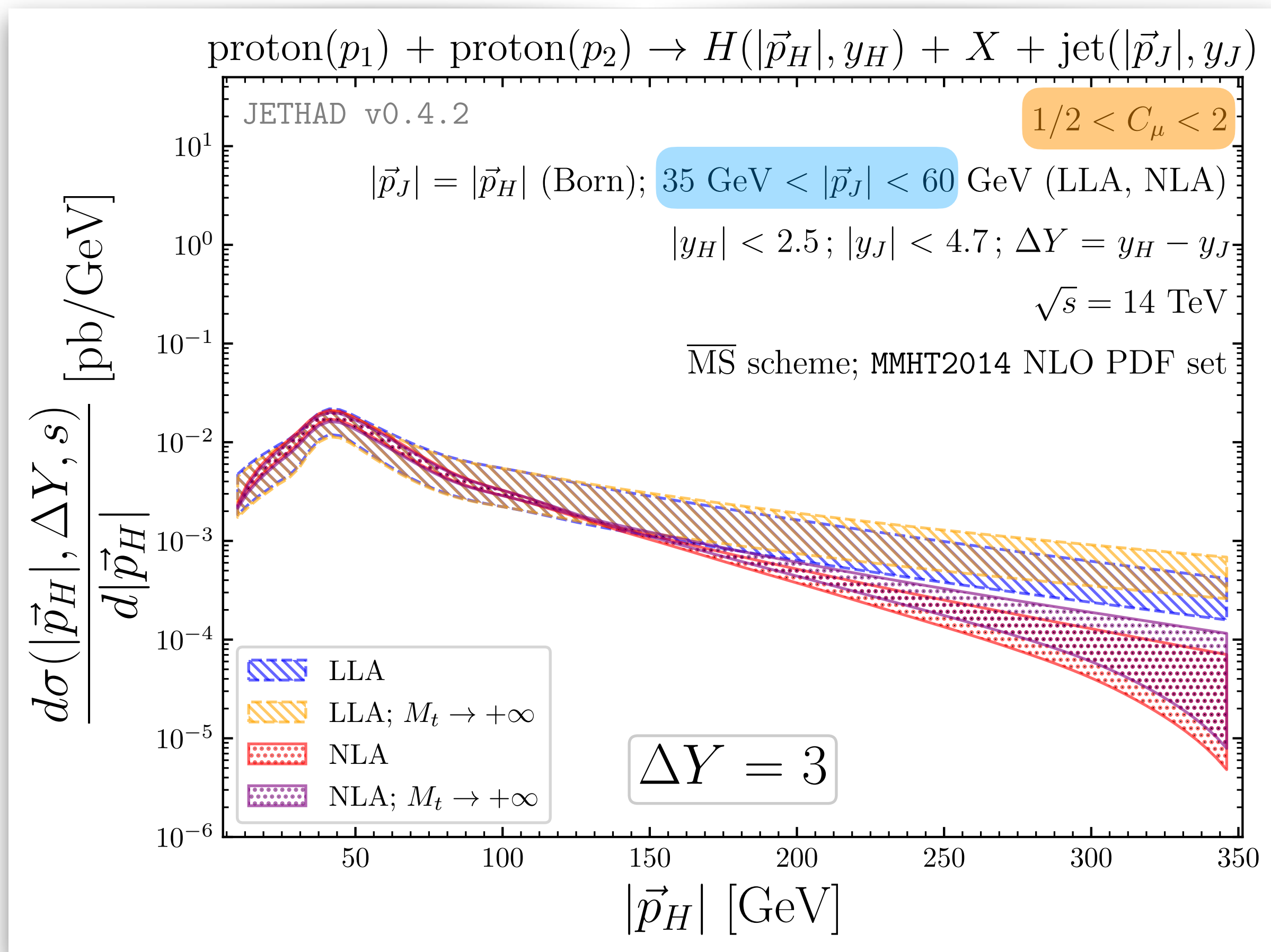
(NLO Higgs impact factor) [F. G. C. et al., under review (2022)]



natural scales

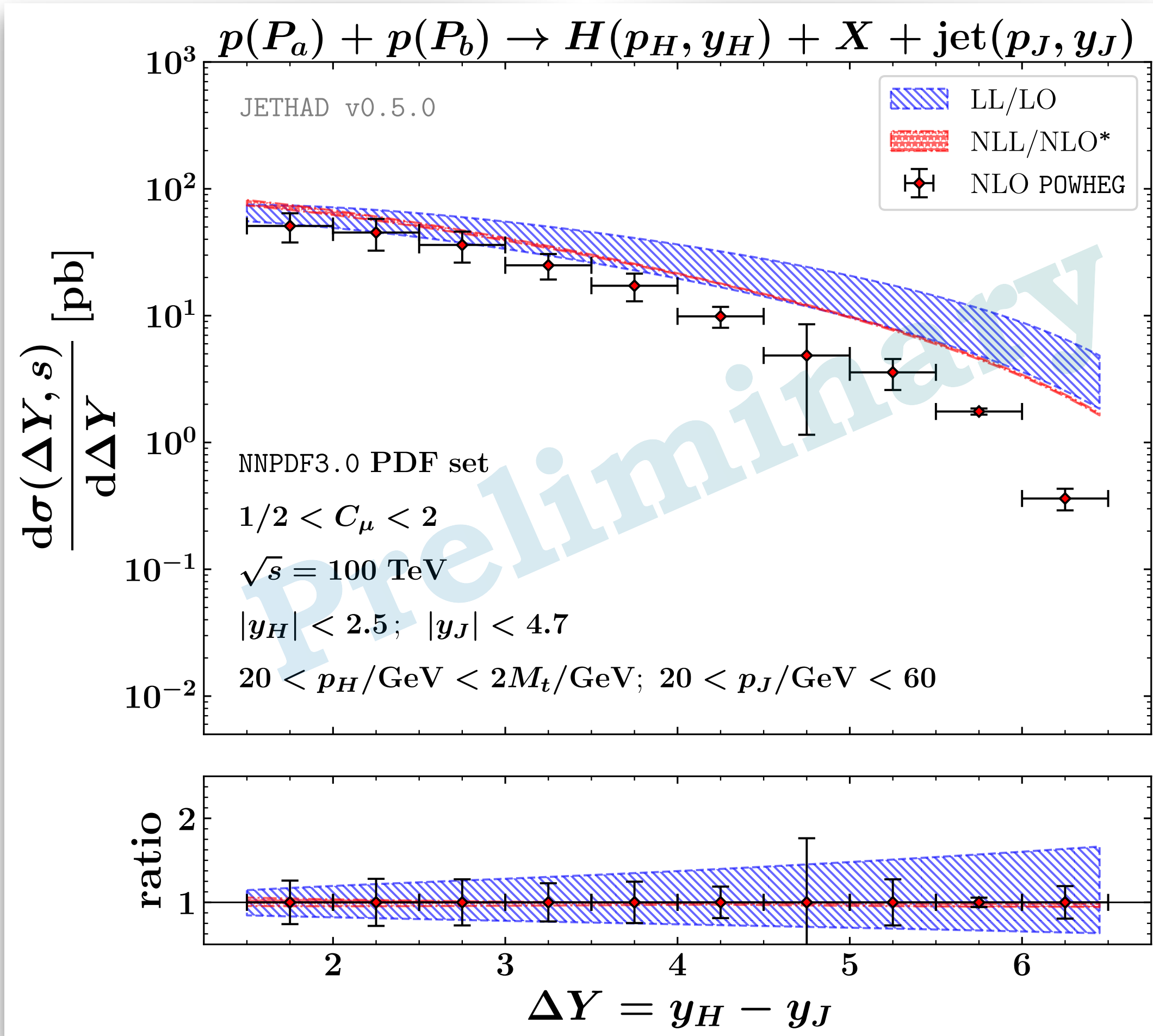
symmetric p_T range

Higgs transverse-momentum distribution for ($M_t \rightarrow +\infty$)



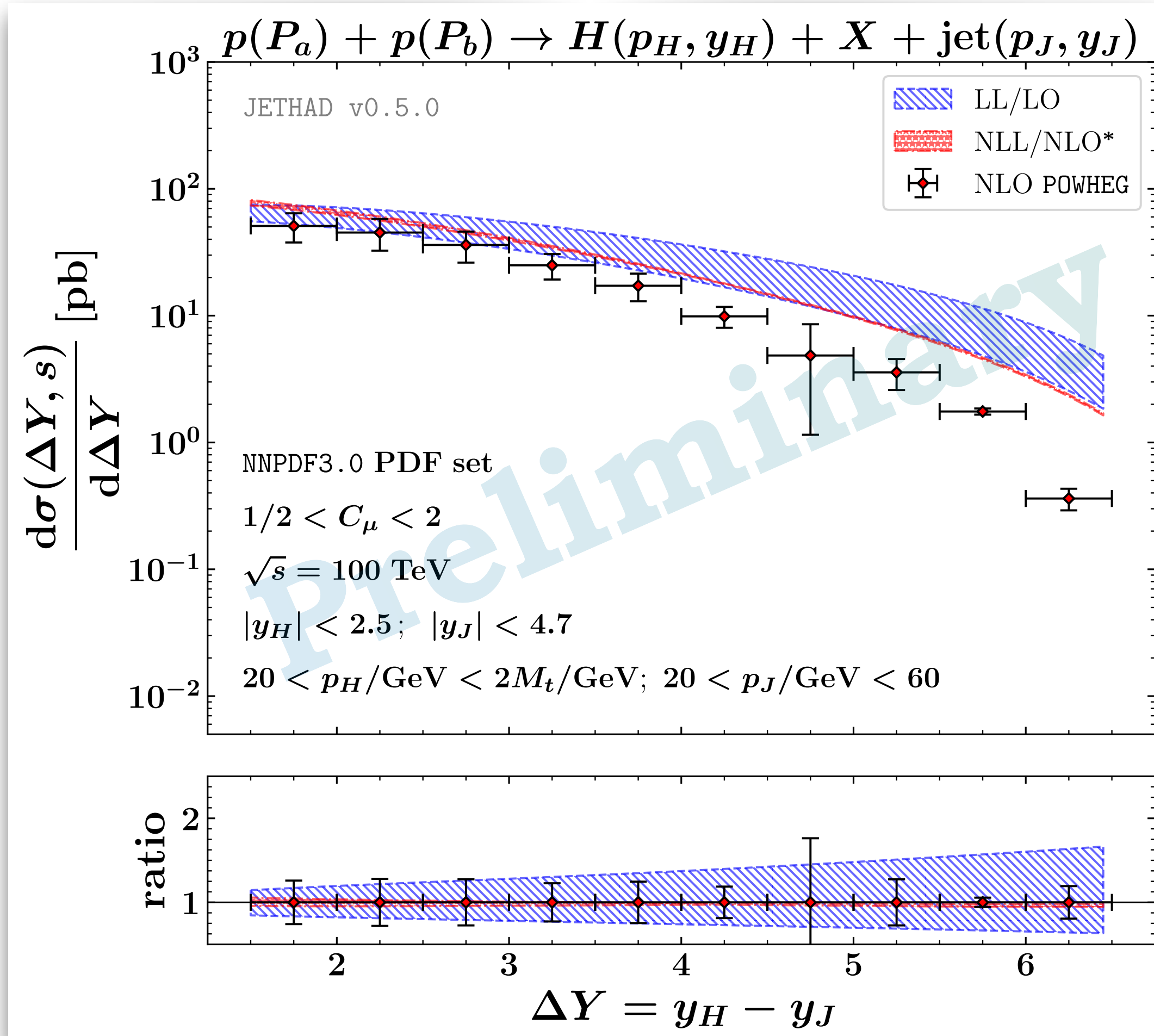
Higgs + jet at @FCC: small- x enhancement from PDFs

High-energy resummation + NNPDF3.0 

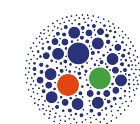
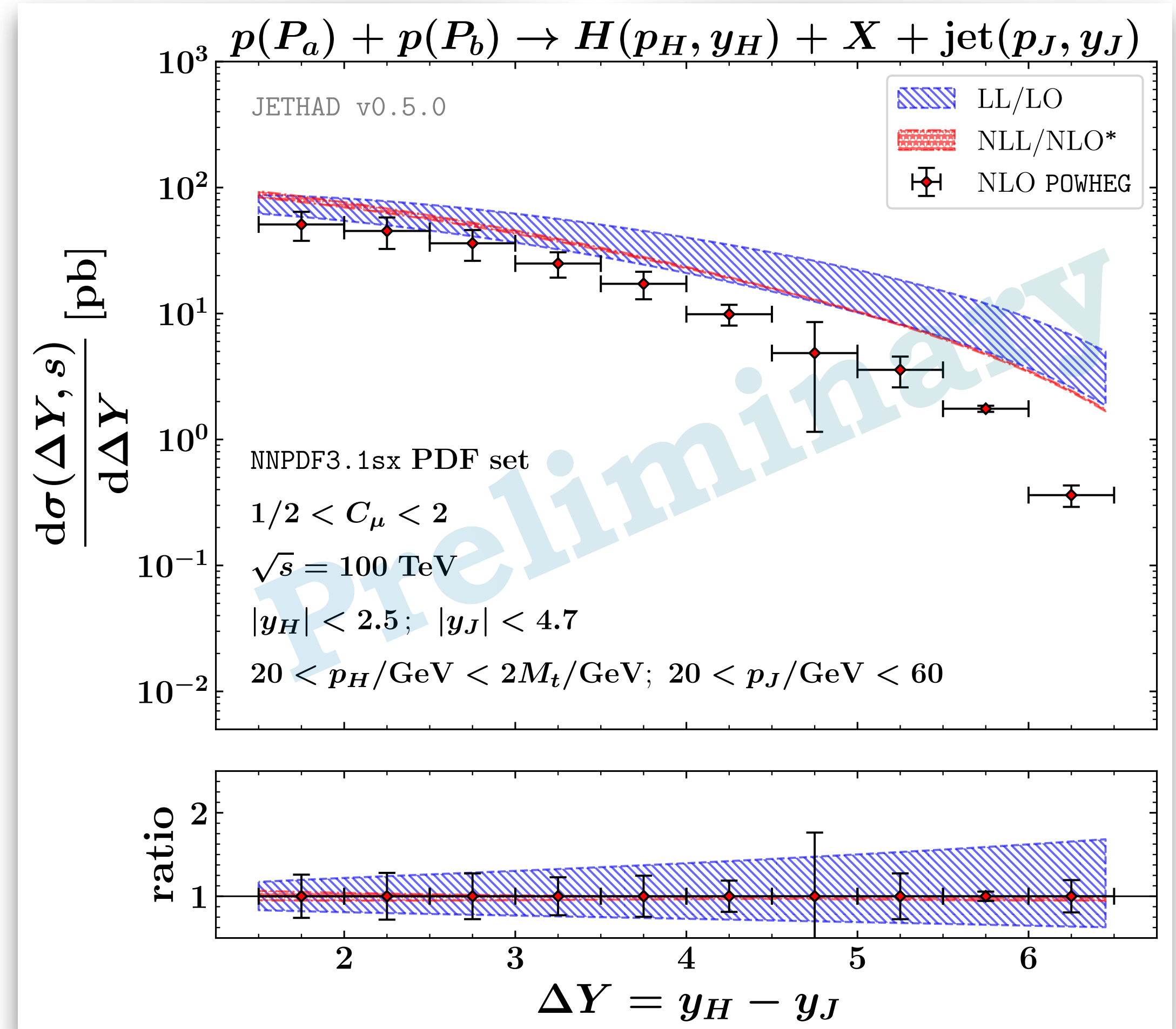


Higgs + jet at @FCC: small- x enhancement from PDFs

High-energy resummation + **NNPDF3.0** 



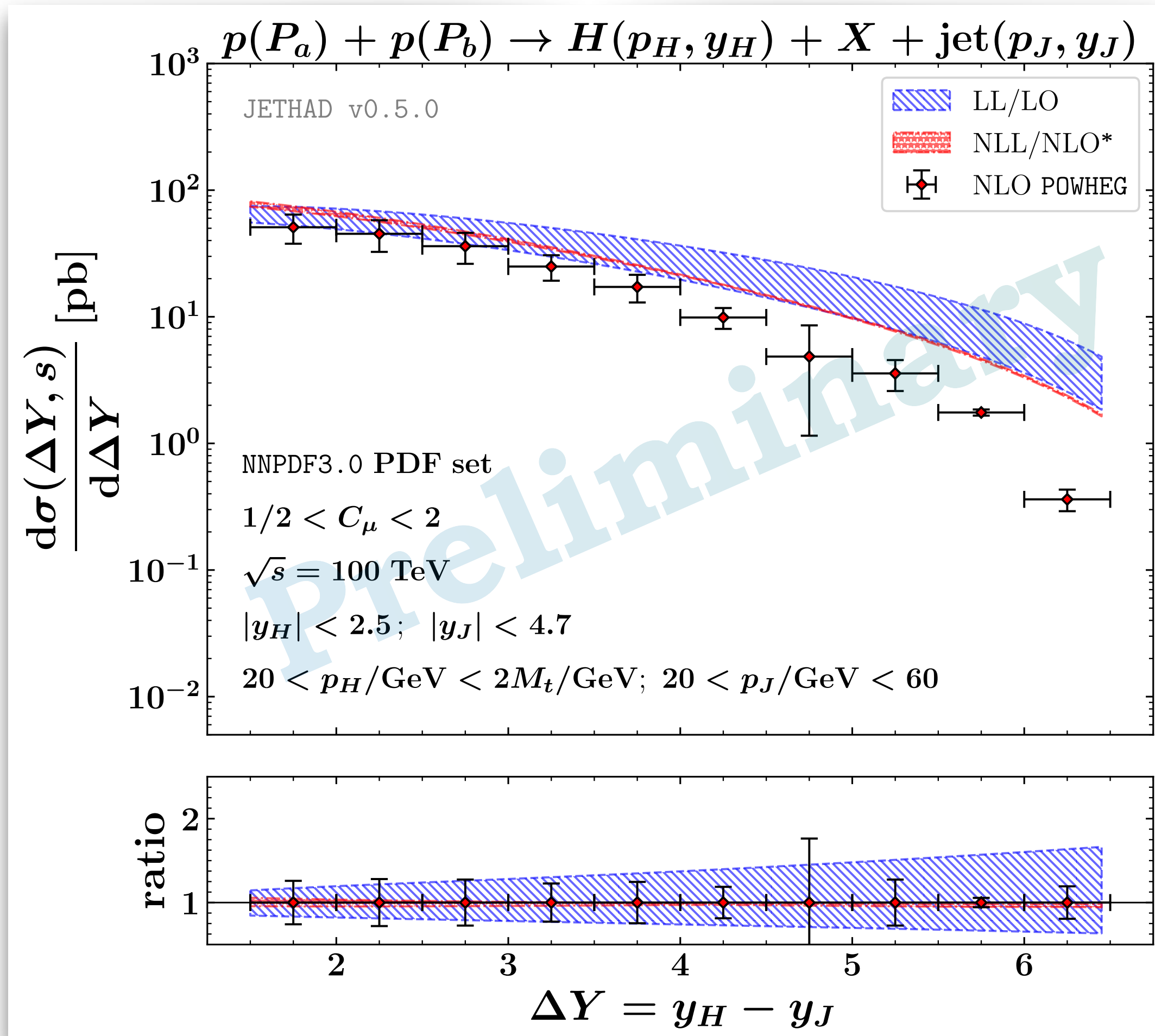
High-energy resummation + **NNPDF3.1sx** 



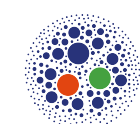
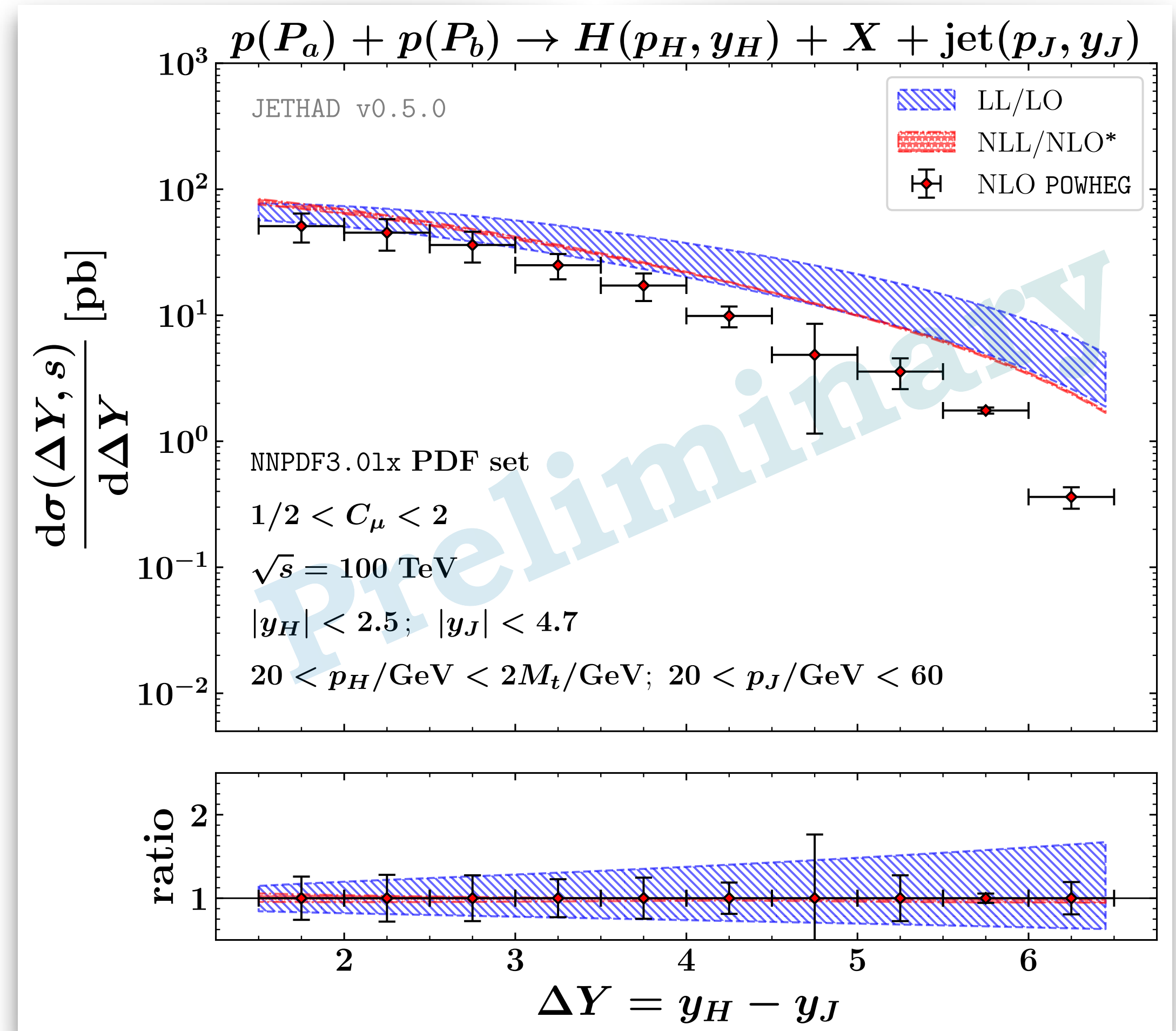
Small-x resummation on PDFs $\Rightarrow + (13.5 \div 2.10) \%$ @NLL/NLO*

Higgs + jet at @FCC: large- x enhancement from PDFs

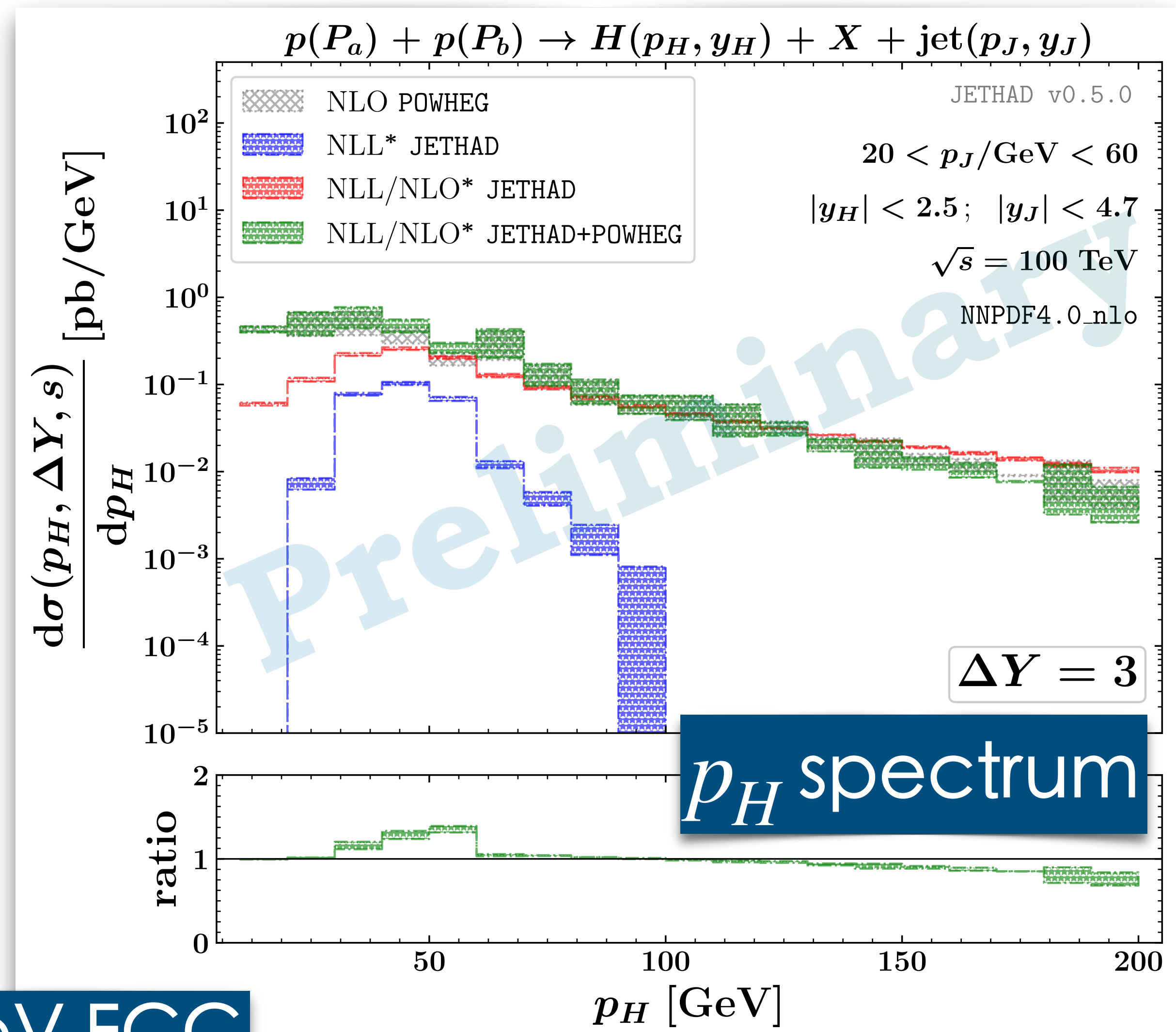
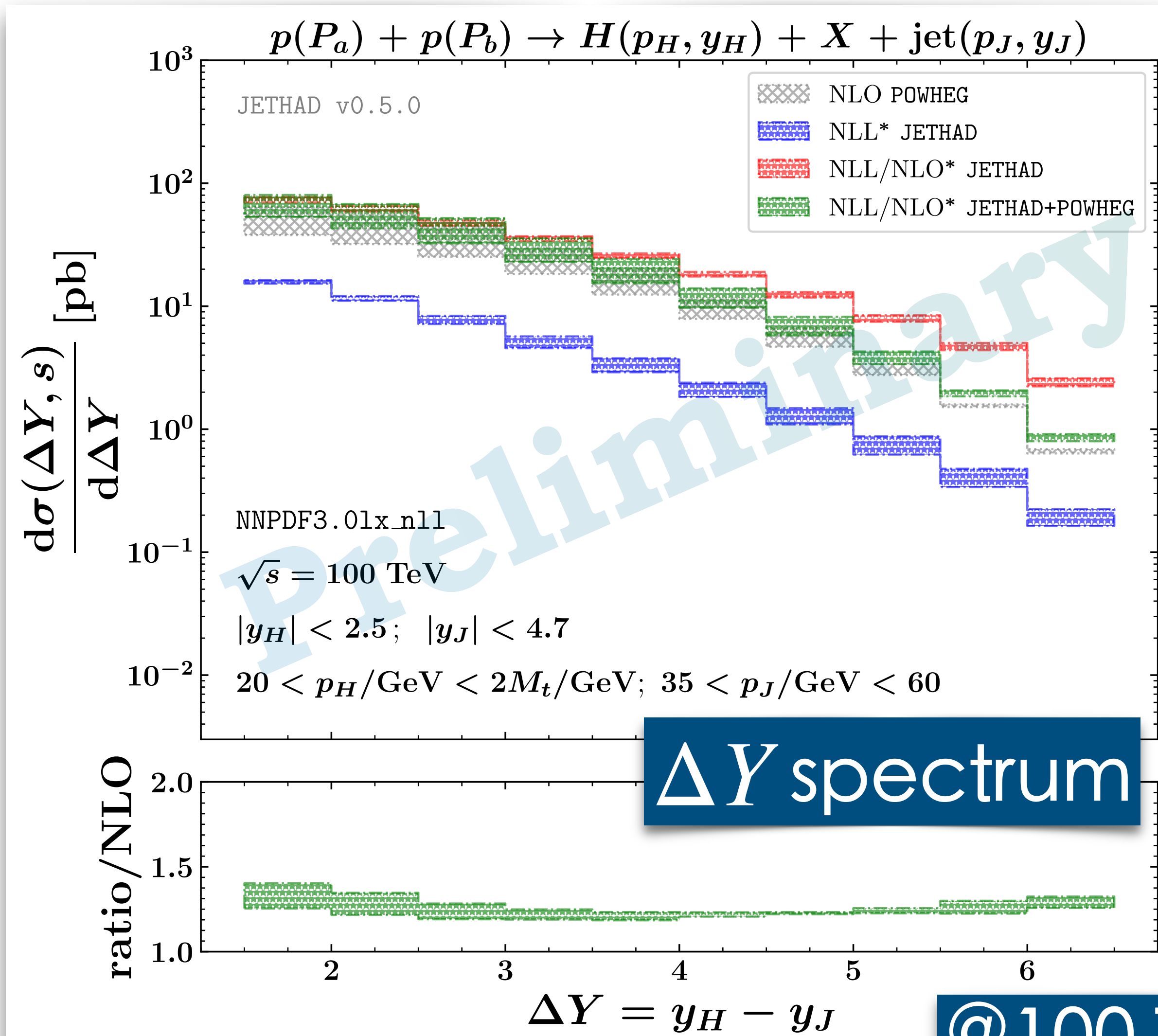
High-energy resummation + **NNPDF3.0** 



High-energy resummation + **NNPDF3.01x** 



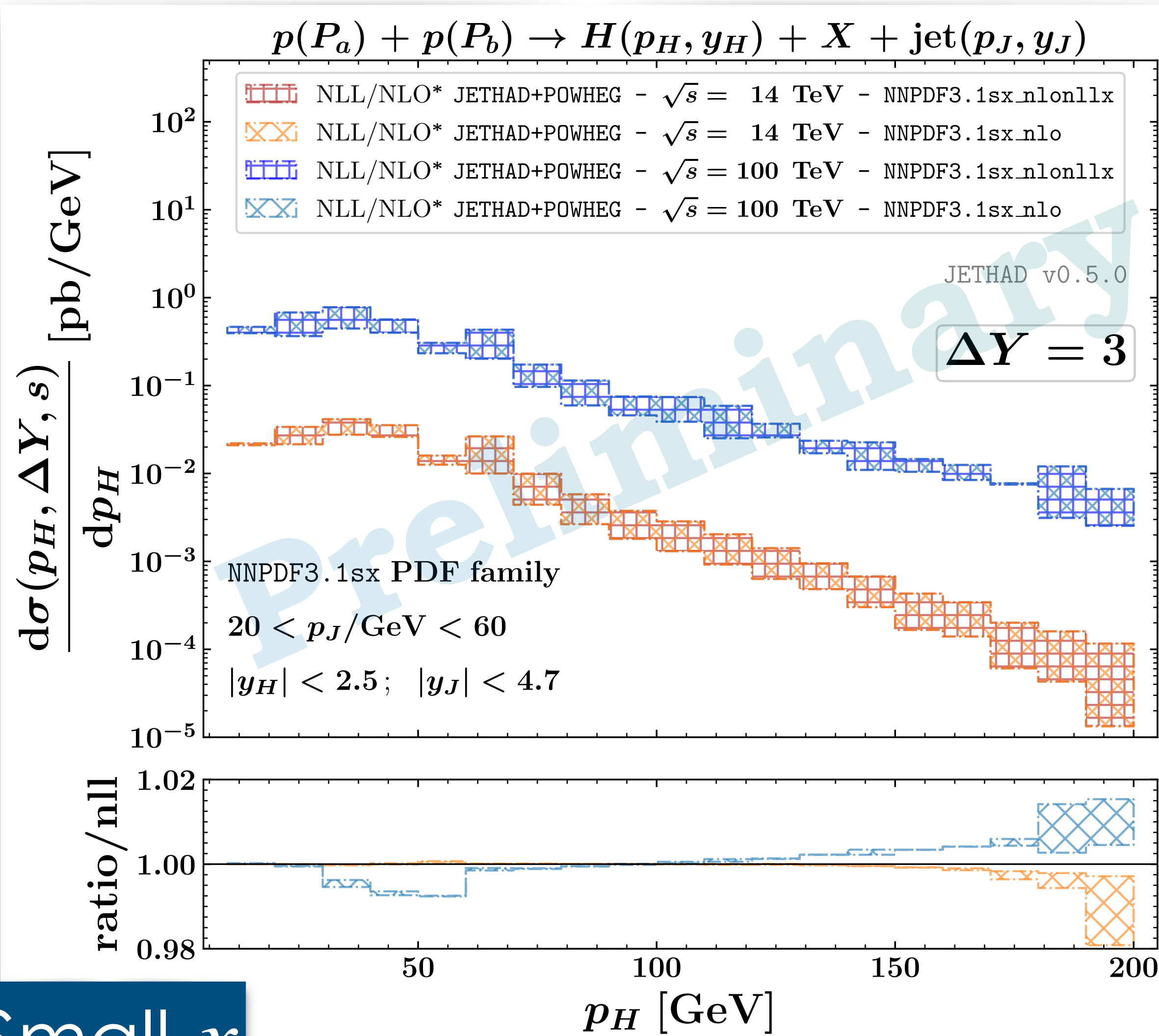
The Higgs + jet spectrum from POWHEG + JETHAD



@100 TeV FCC

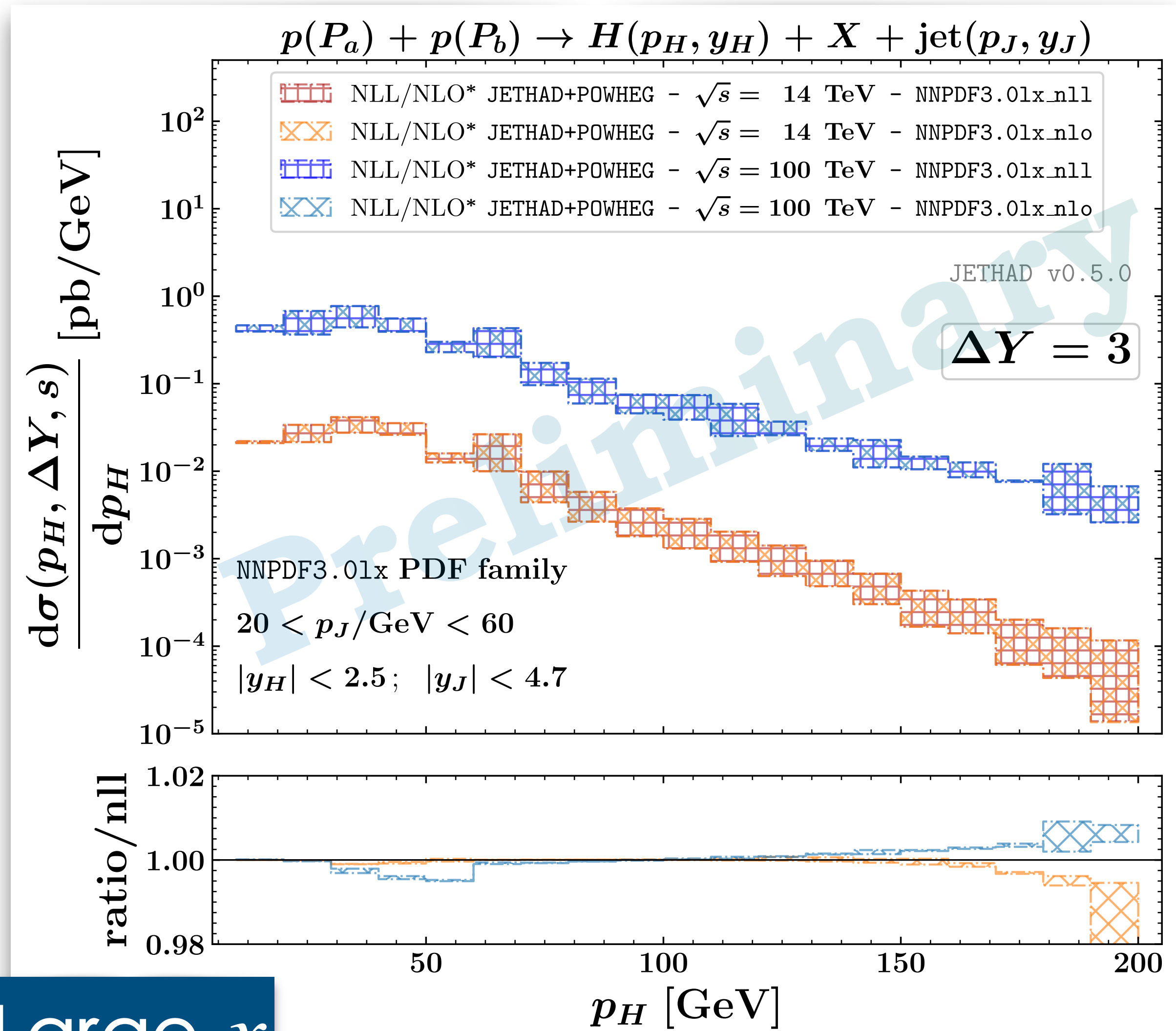
NLL matched to NLO fixed-order JETHAD + POWHEG (in progress)

Small- x and large- x enhancement from PDFs



Small- x

Backup



Large- x

Impact of large- x threshold logs on NLO impact factors to be gauged

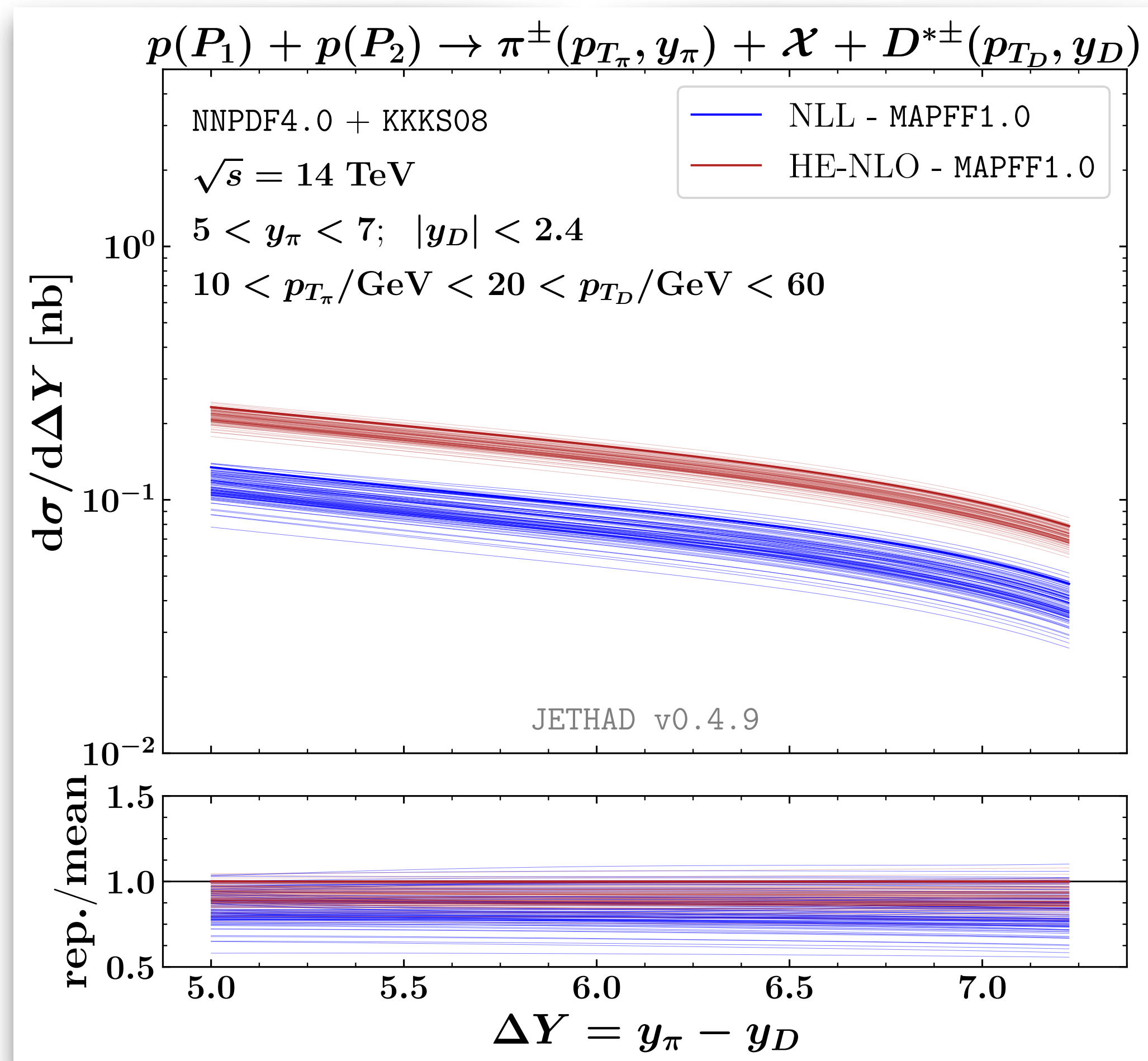
Heavy-light hadrons

The background of the slide features a repeating pattern of particle physics diagrams. These diagrams depict various types of hadrons, including mesons (quark-antiquark pairs) and baryons (three-quark systems), connected by wavy lines representing gluons. The particles are shown in different colors (red, blue, green) and some have arrows indicating spin or momentum. The overall aesthetic is scientific and colorful, with a light blue and green gradient background.

Rapidity distributions @FPF+ATLAS

Inclusive $\pi^\pm$ (FPF) + $D^{*\pm}$ (ATLAS) production

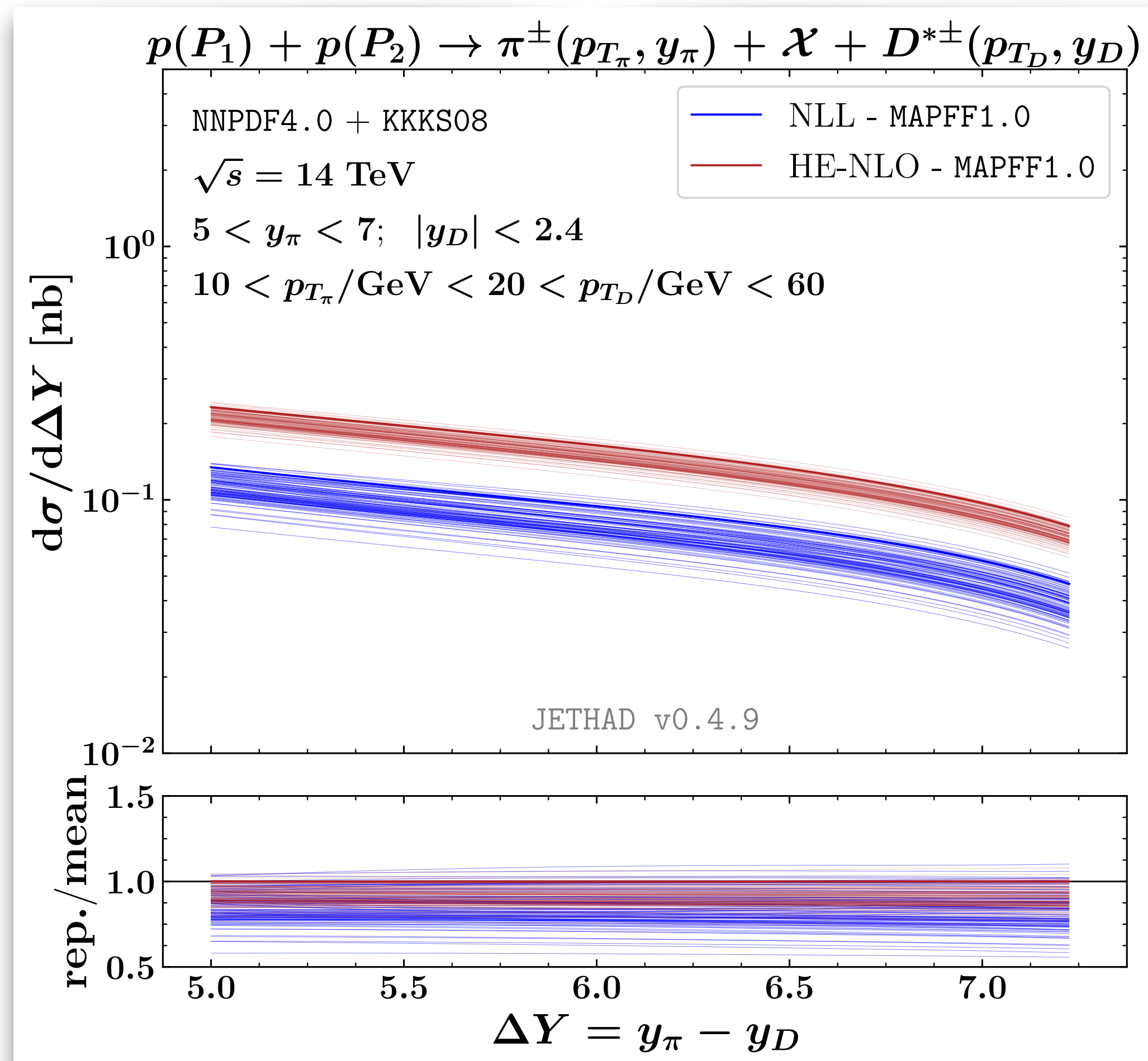
[FPF Snowmass Whitepaper]



Rapidity distributions @FPF+ATLAS

Inclusive $\pi^\pm$ (FPF) + $D^{*\pm}$ (ATLAS) production

[FPF Snowmass Whitepaper]



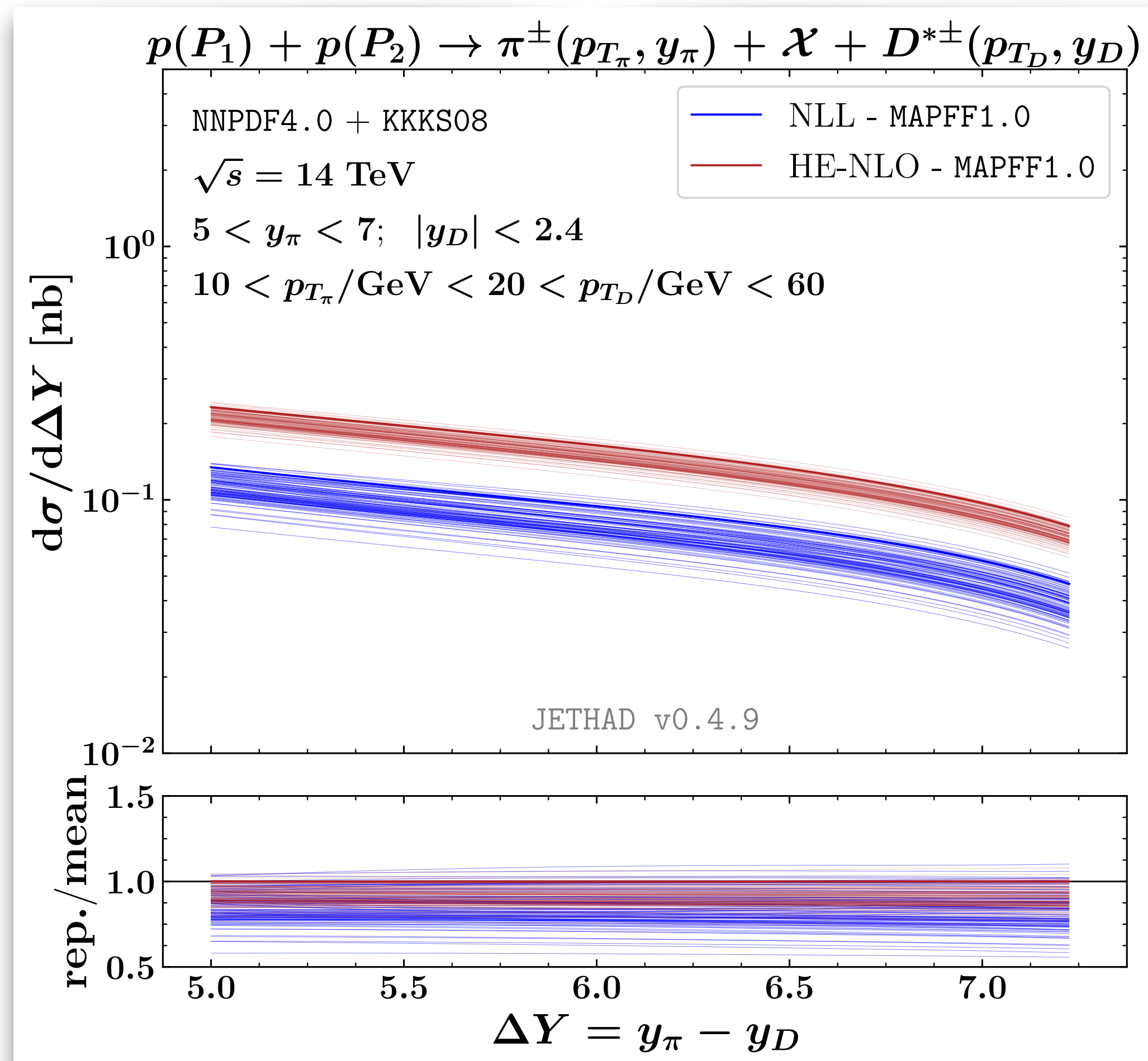
* Impact of collinear FFs on ΔY -distribution

* Replica method at work

Rapidity distributions @FPF+ATLAS

Inclusive $\pi^\pm$ (FPF) + $D^{*\pm}$ (ATLAS) production

[FPF Snowmass Whitepaper]



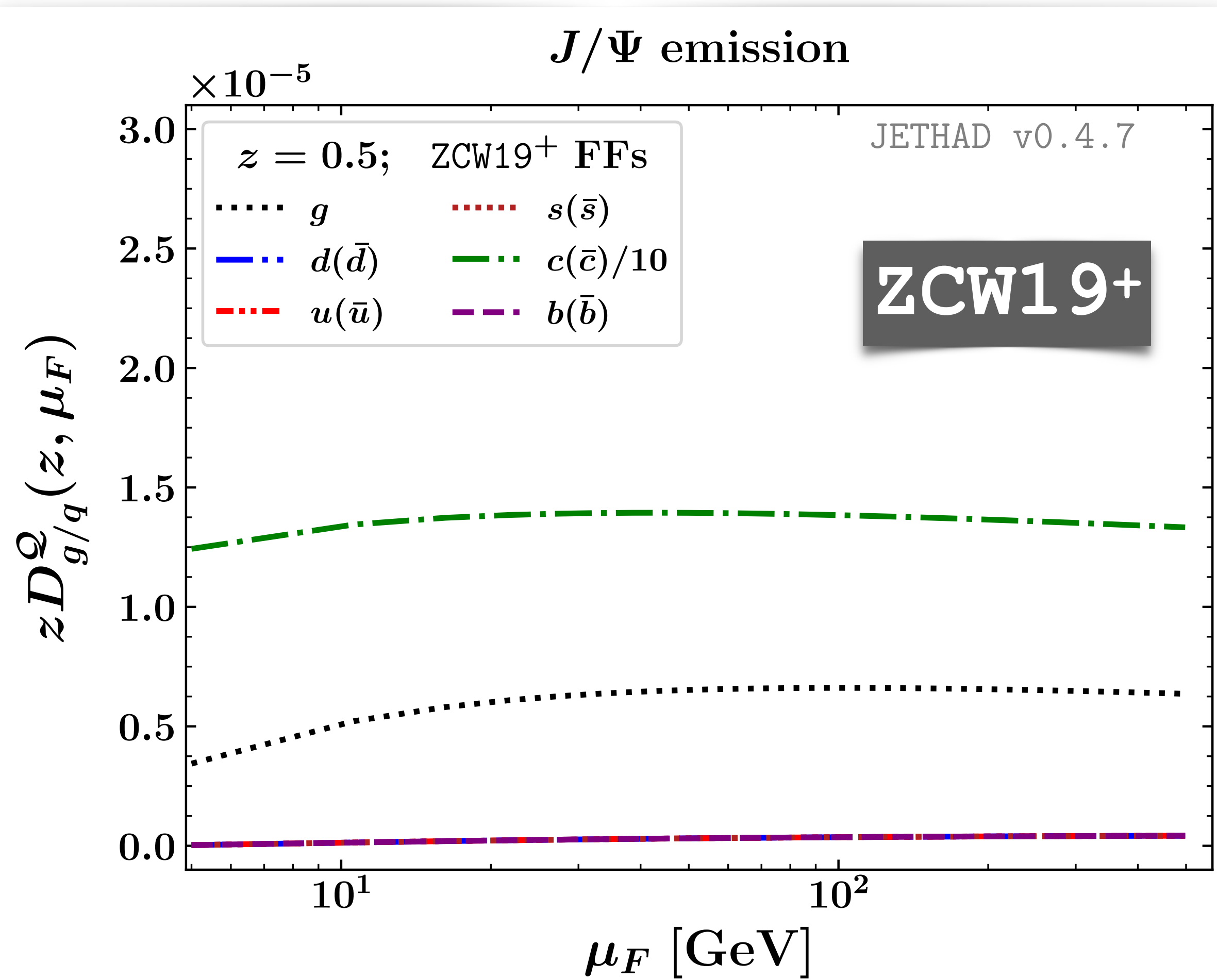
- * Impact of collinear FFs on ΔY -distribution
- * Replica method at work
- * Larger spread of replicas at NLL
- * Probe FFs in complementary ranges
 - Weight of FF replicas in the same set
 - Different sets via functional correlation?
- * Complementary studies on FFs

Vector quarkonia

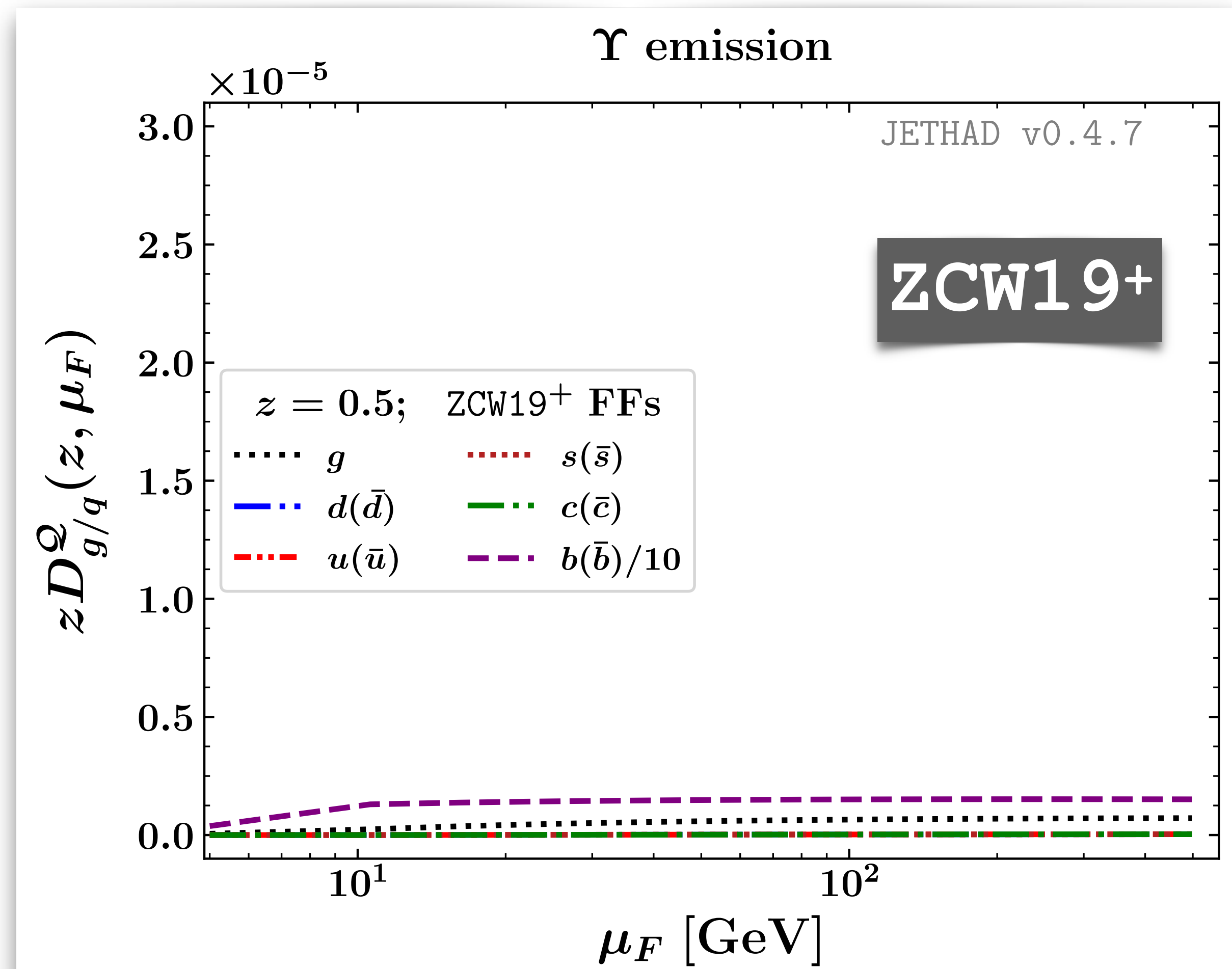


Vector quarkonium + jet at the LHC

J/ψ collinear FFs

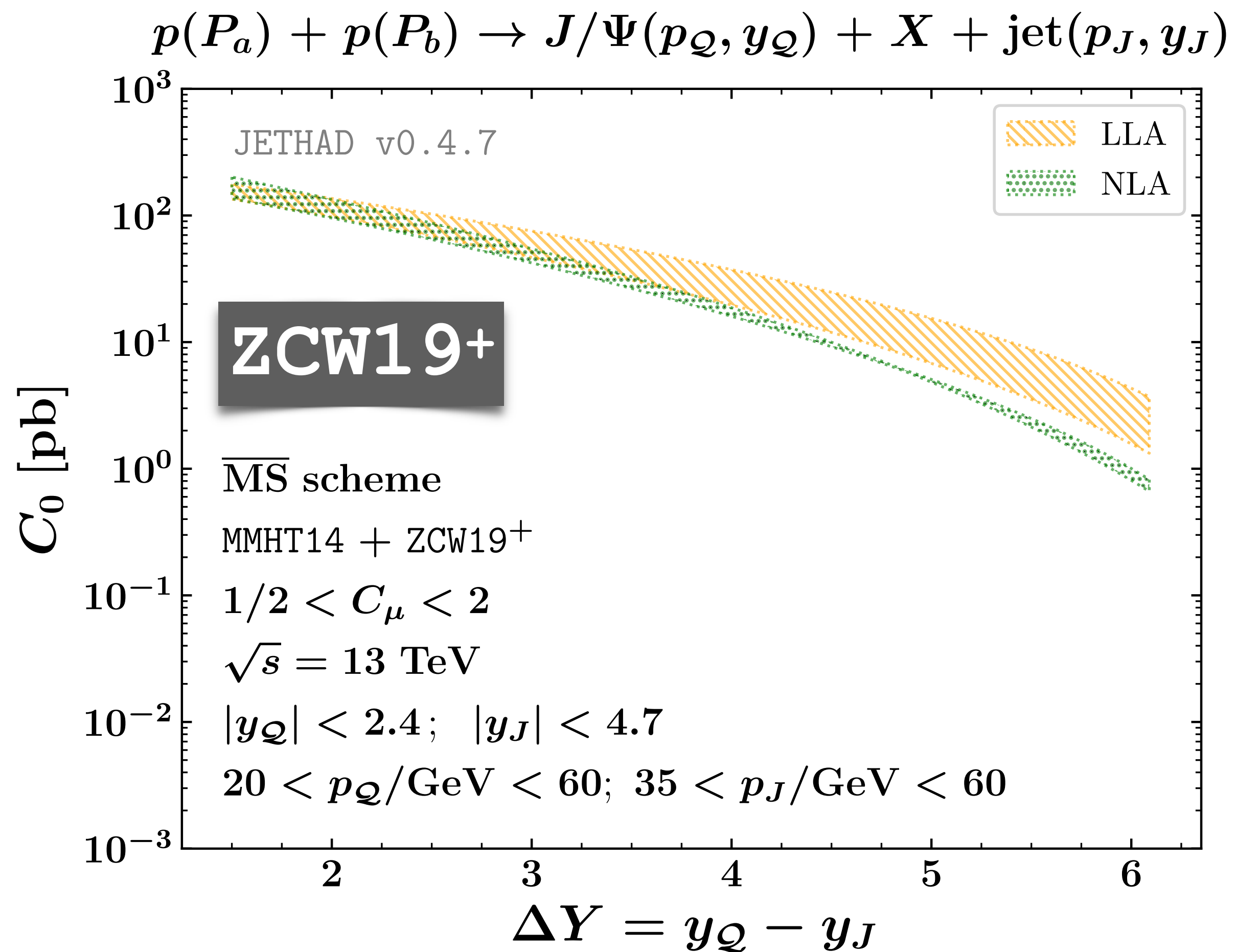


Υ collinear FFs

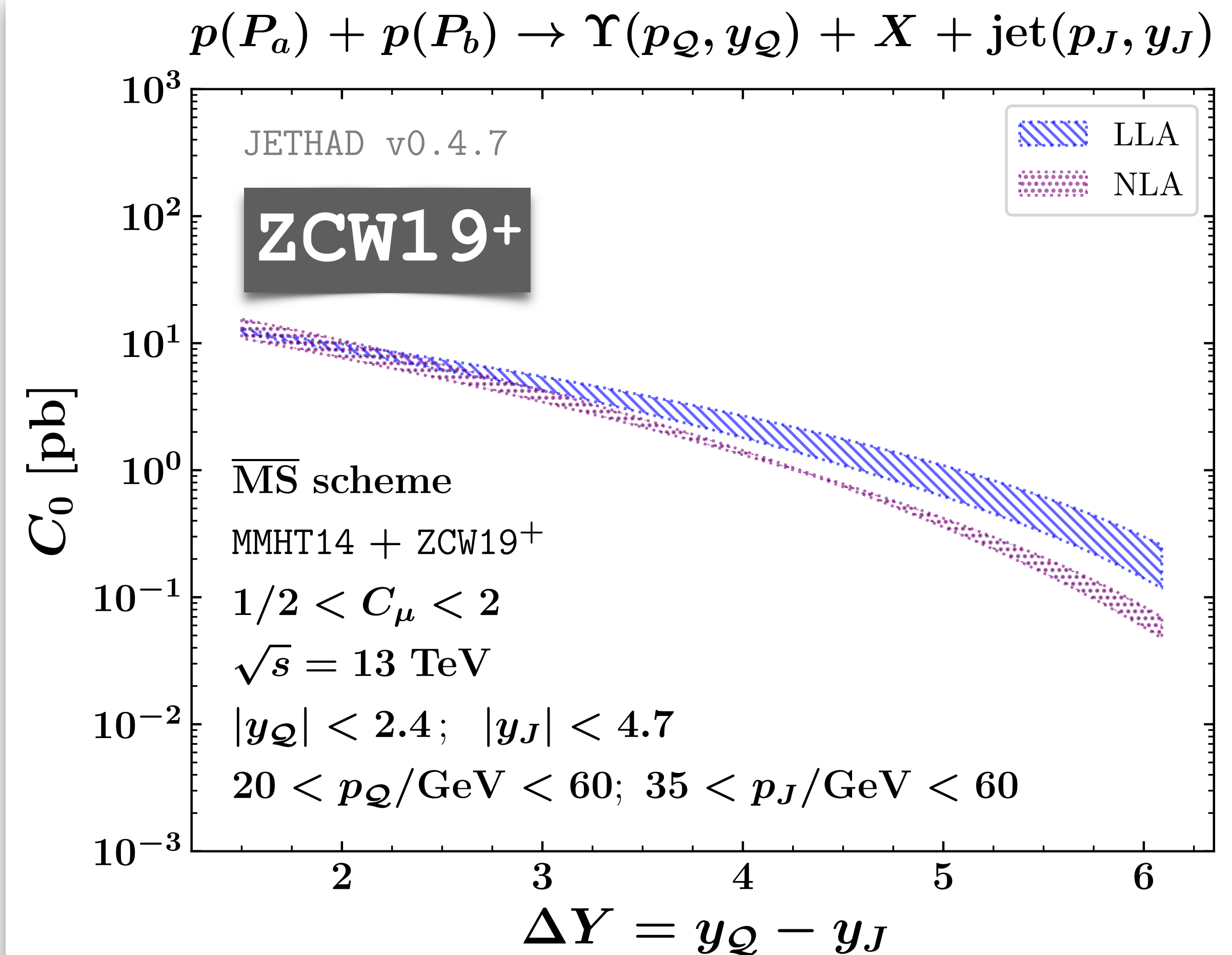


Vector quarkonium + jet at the LHC

$J/\psi + \text{jet}$

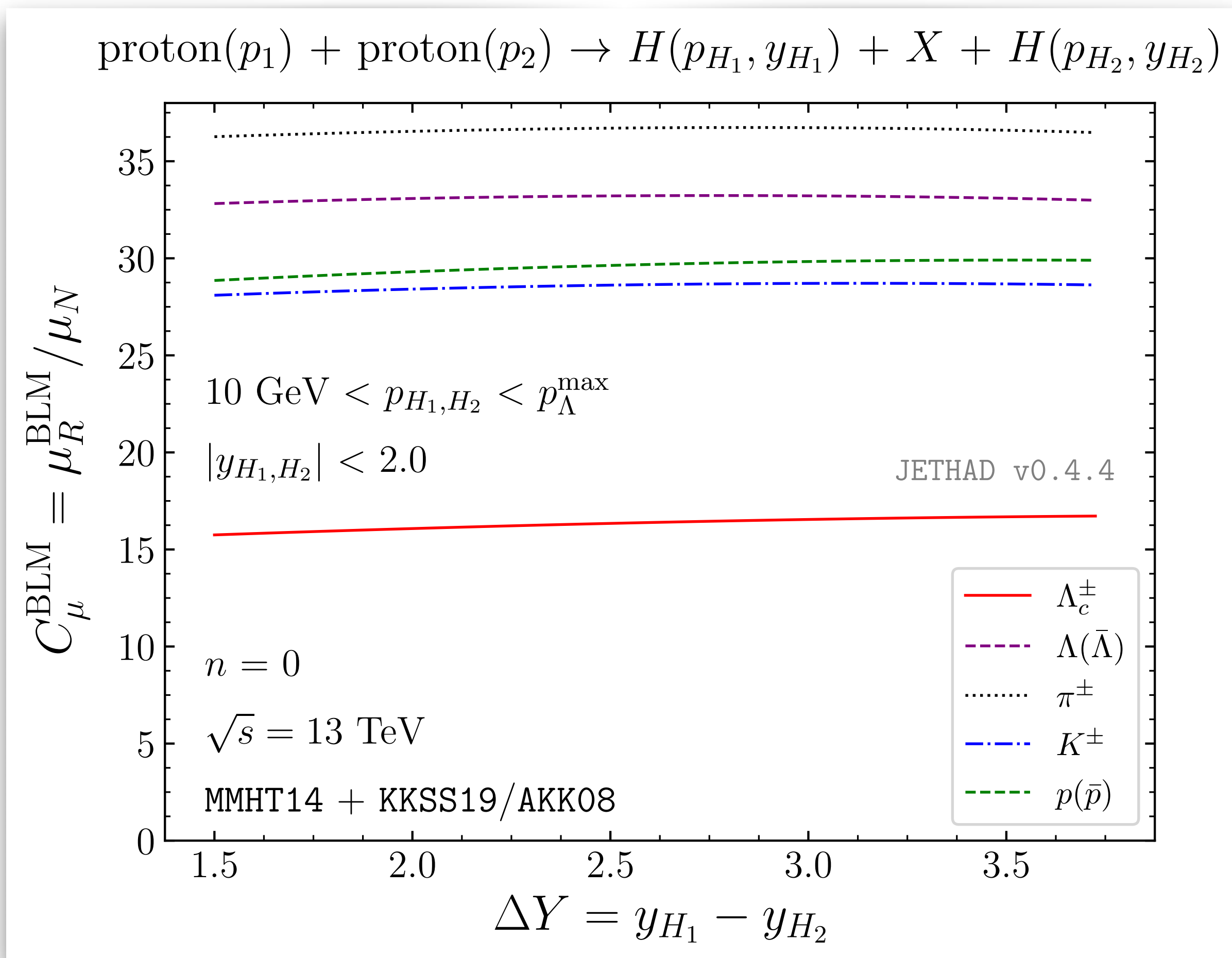


$\Upsilon + \text{jet}$

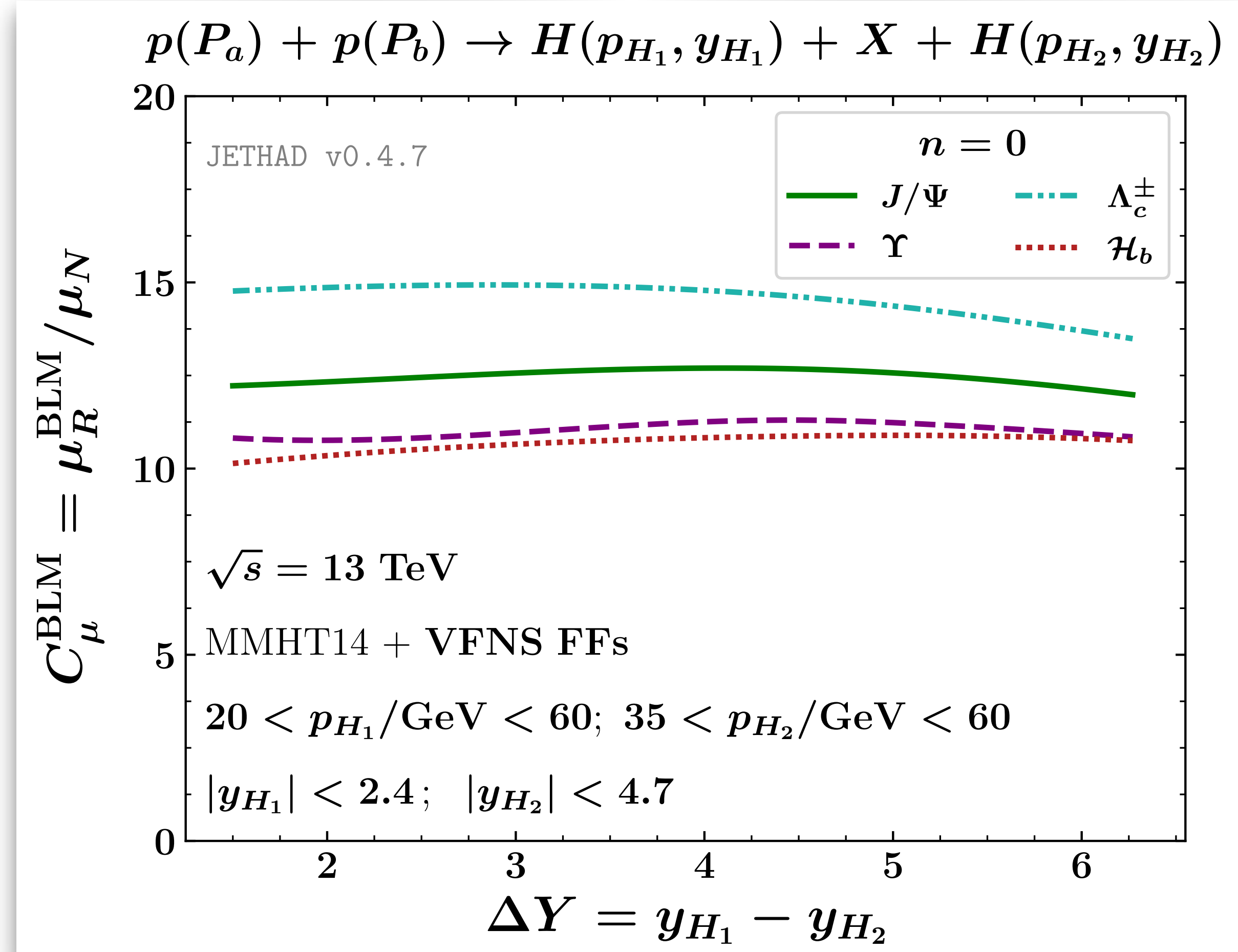


Heavy flavor at the LHC: BLM scales

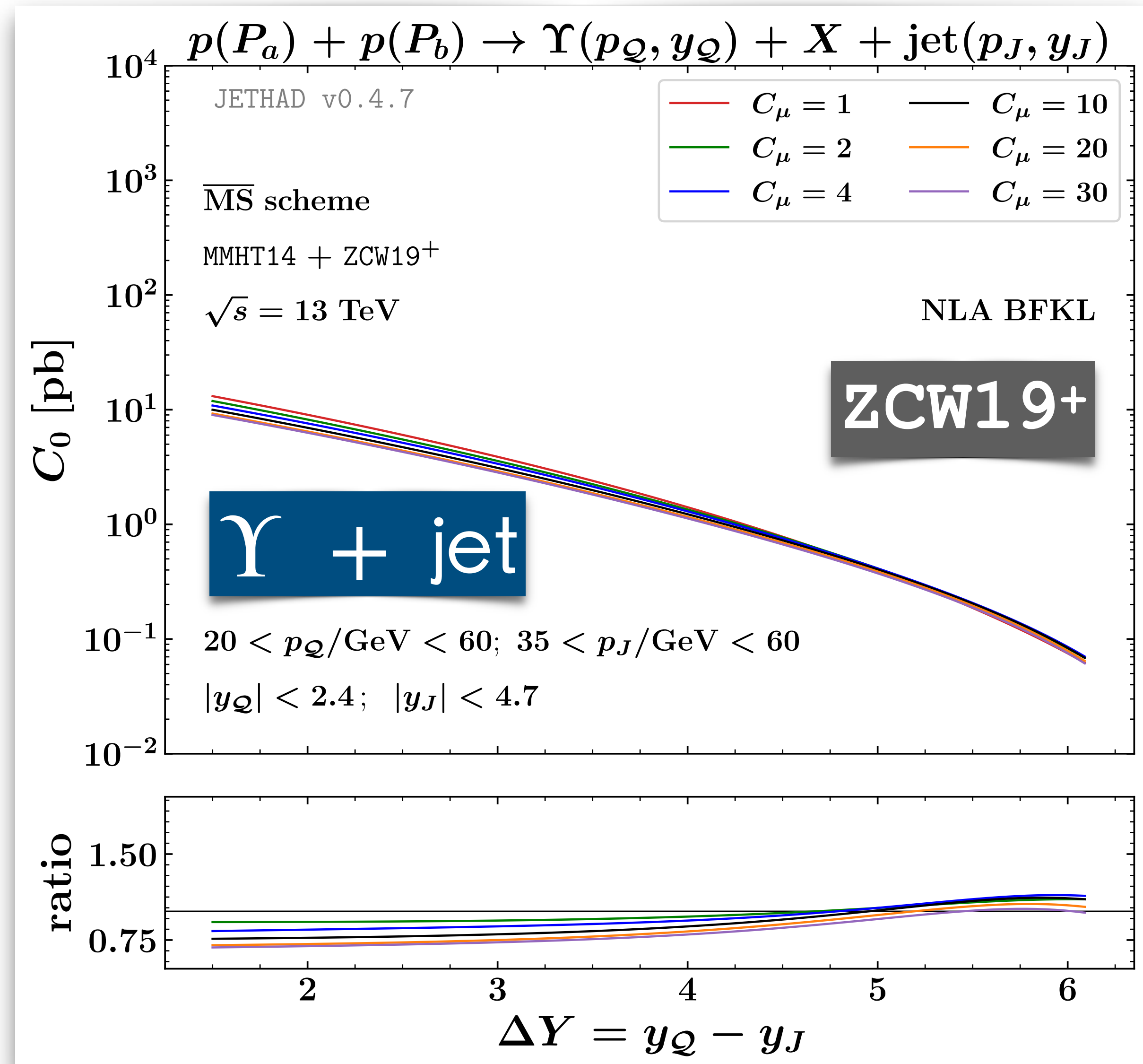
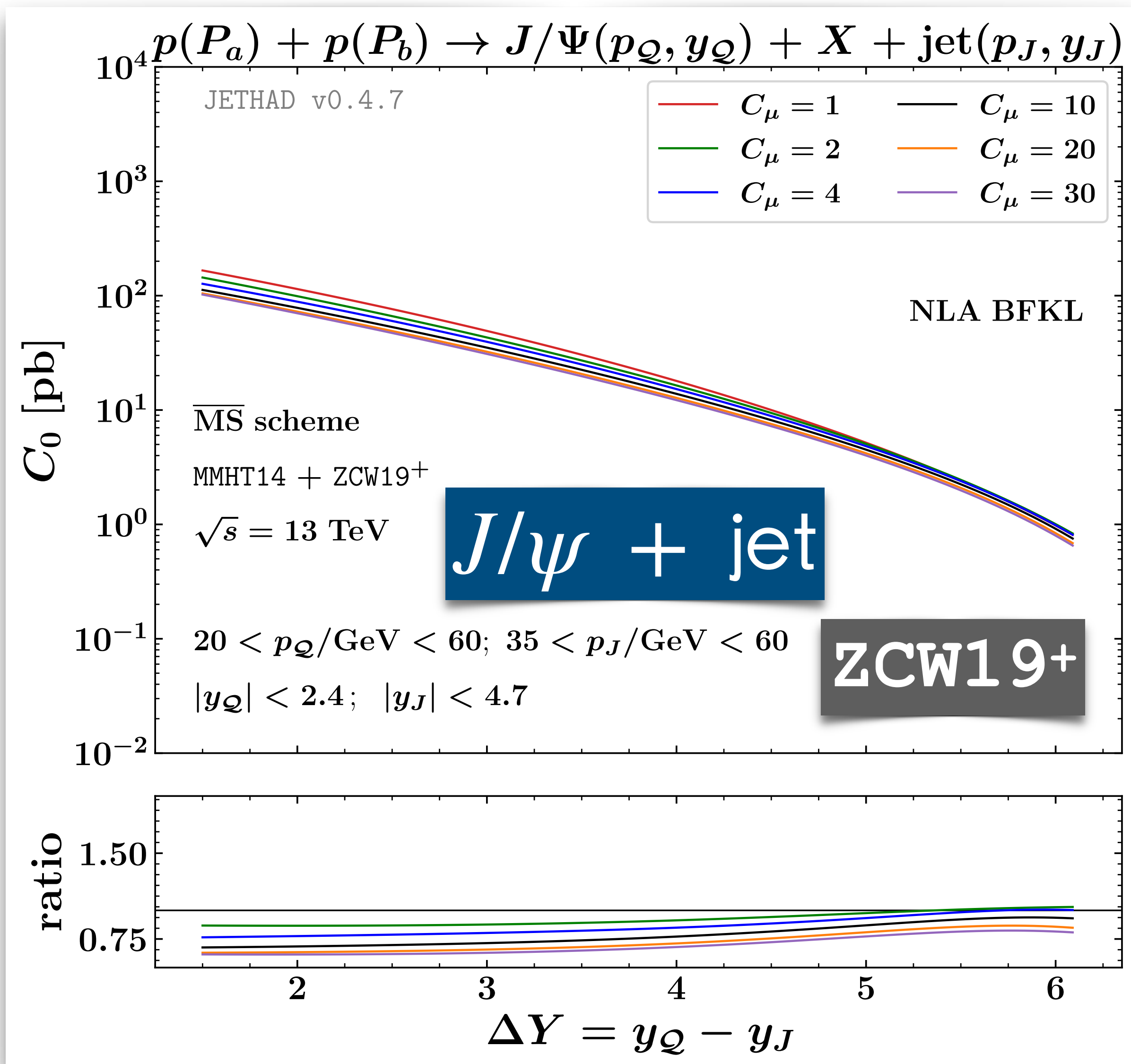
Heavy-light hadrons



Vector quarkonia



Vector quarkonium + jet at the LHC



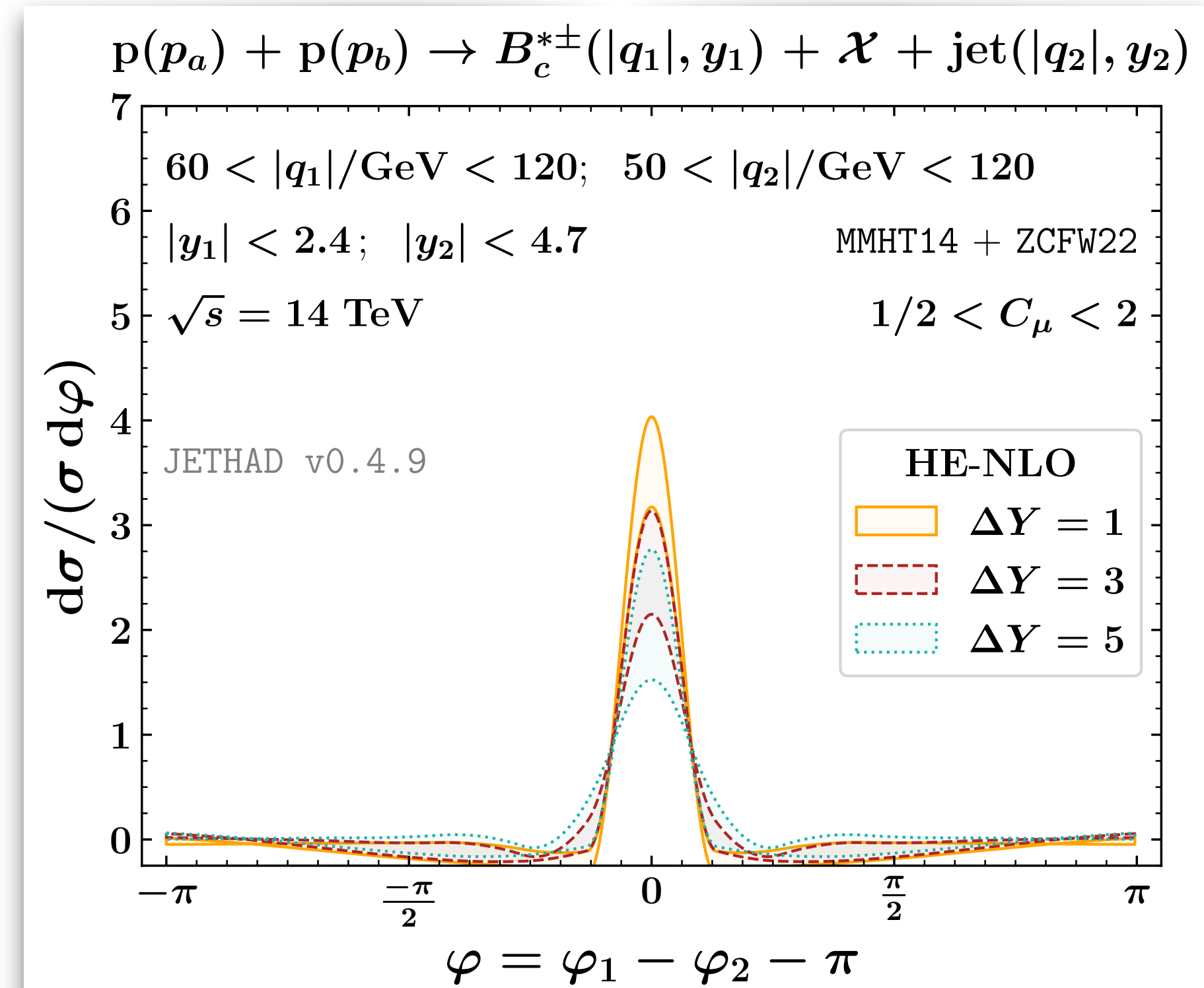
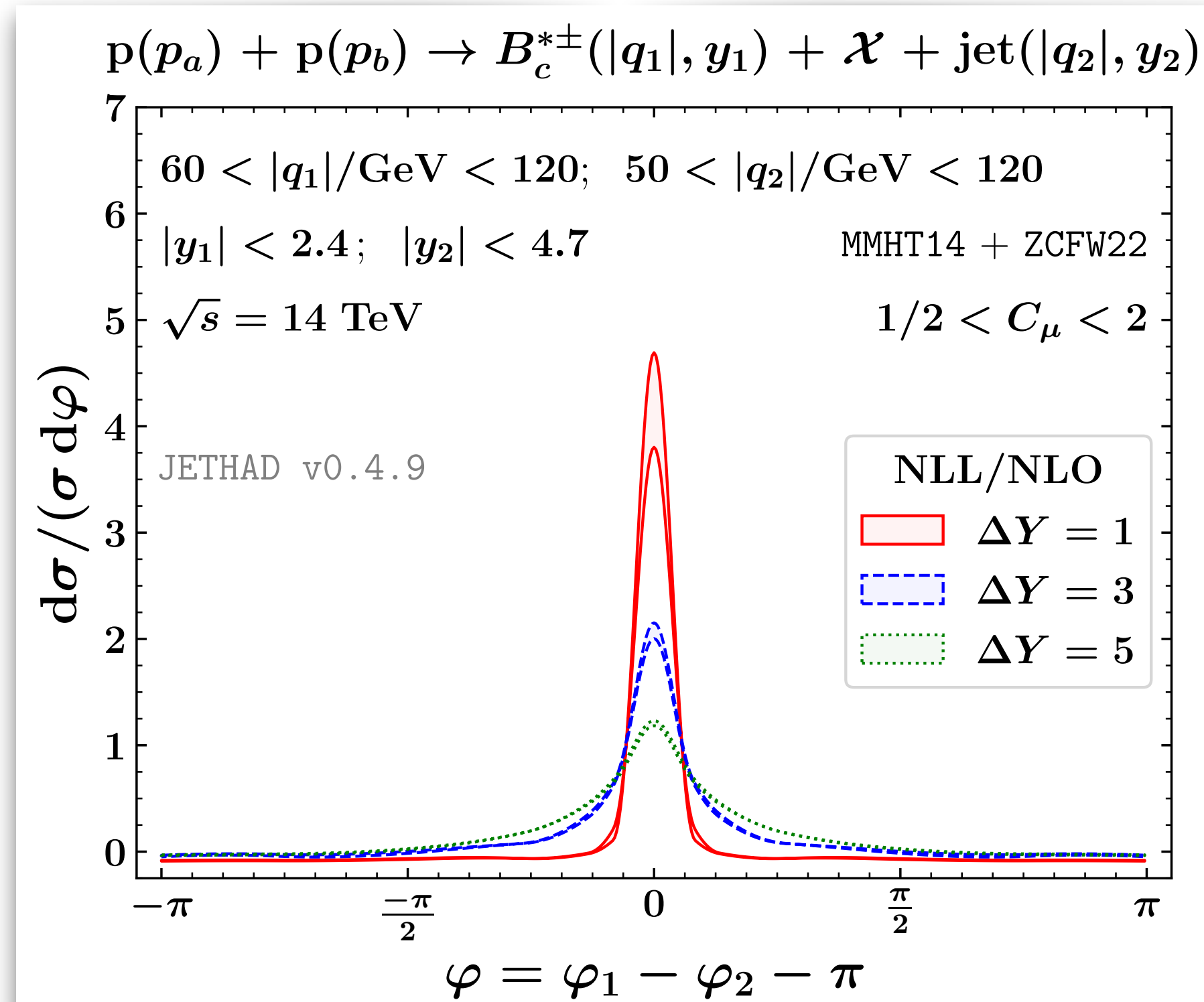
Charmed B -mesons

Charmed B -mesons from single-parton fragmentation

(2) ! Let us consider $B_c(^1S_0)$ and $B_c(^3S_1)$ at large $p_T \rightarrow$ single-parton fragmentation from **NRQCD** !

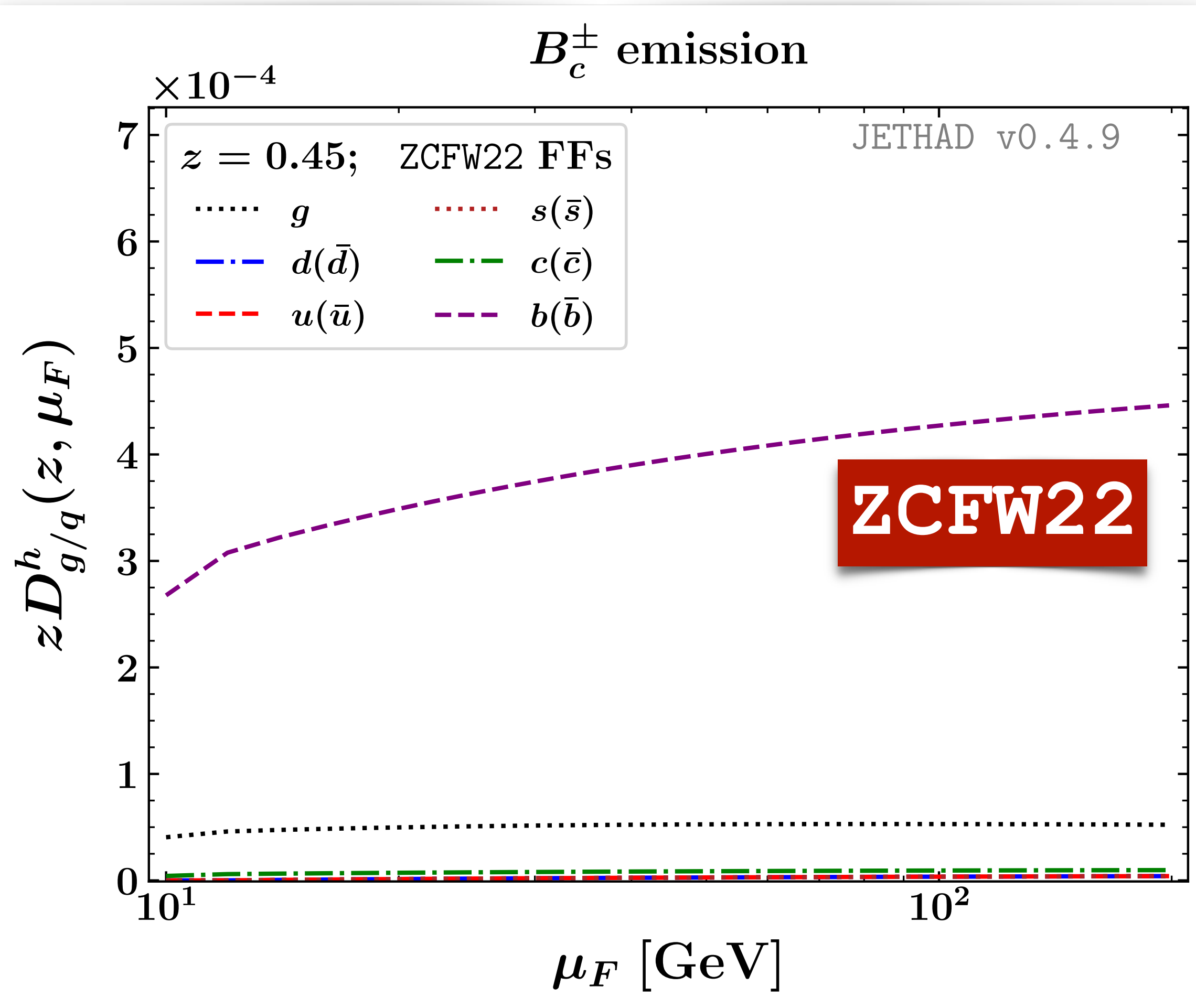
(NLO heavy quark) [\[X. Zheng et al., Phys. Rev. D 100 \(2019\) 3, 034004\]](#)

(NLO gluon) [\[X. Zheng et al., JHEP 05 \(2022\) 036\]](#)

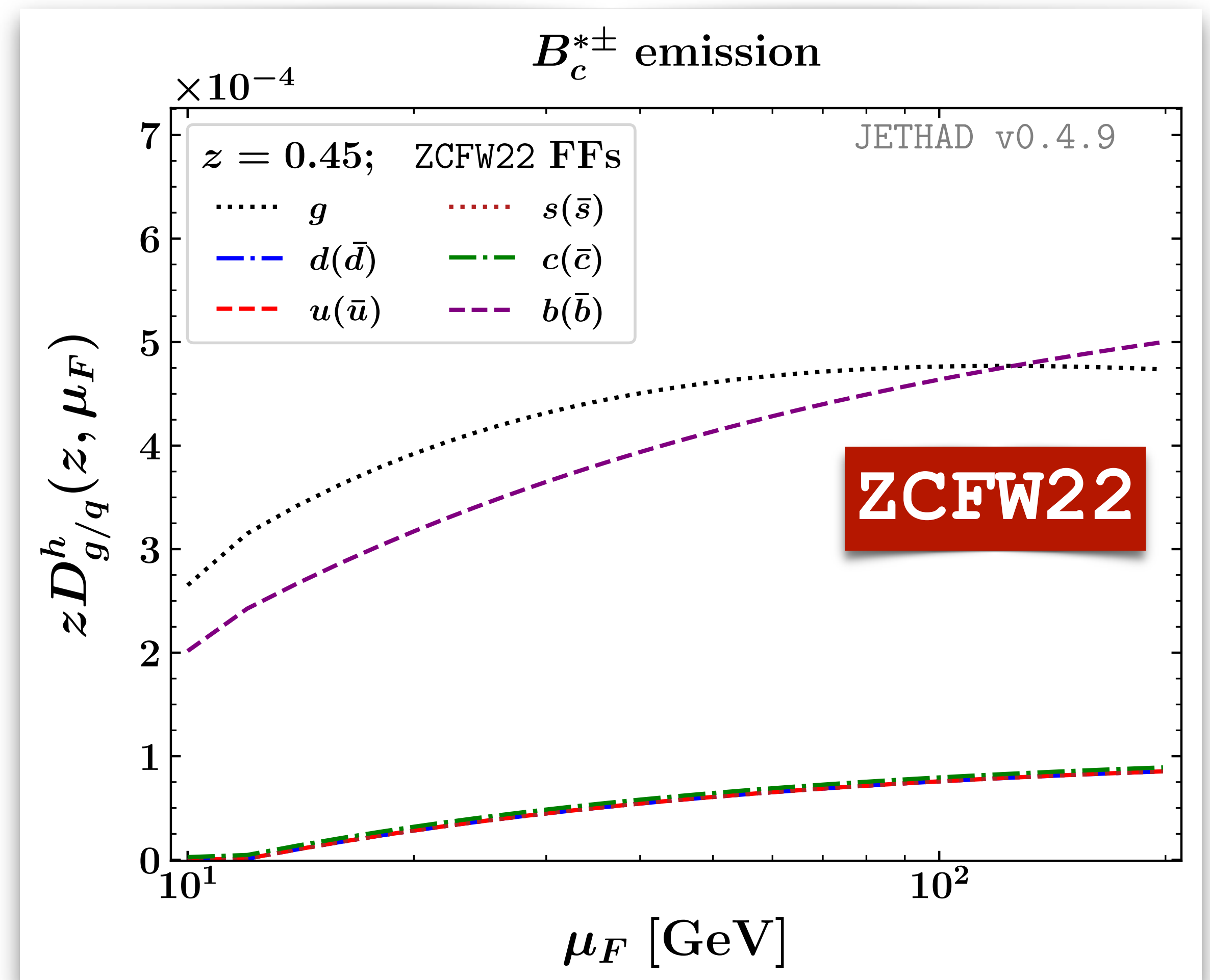


Charmed B -mesons + jet at the HL-LHC

$B_c^\pm(^1S_0)$ collinear FFs

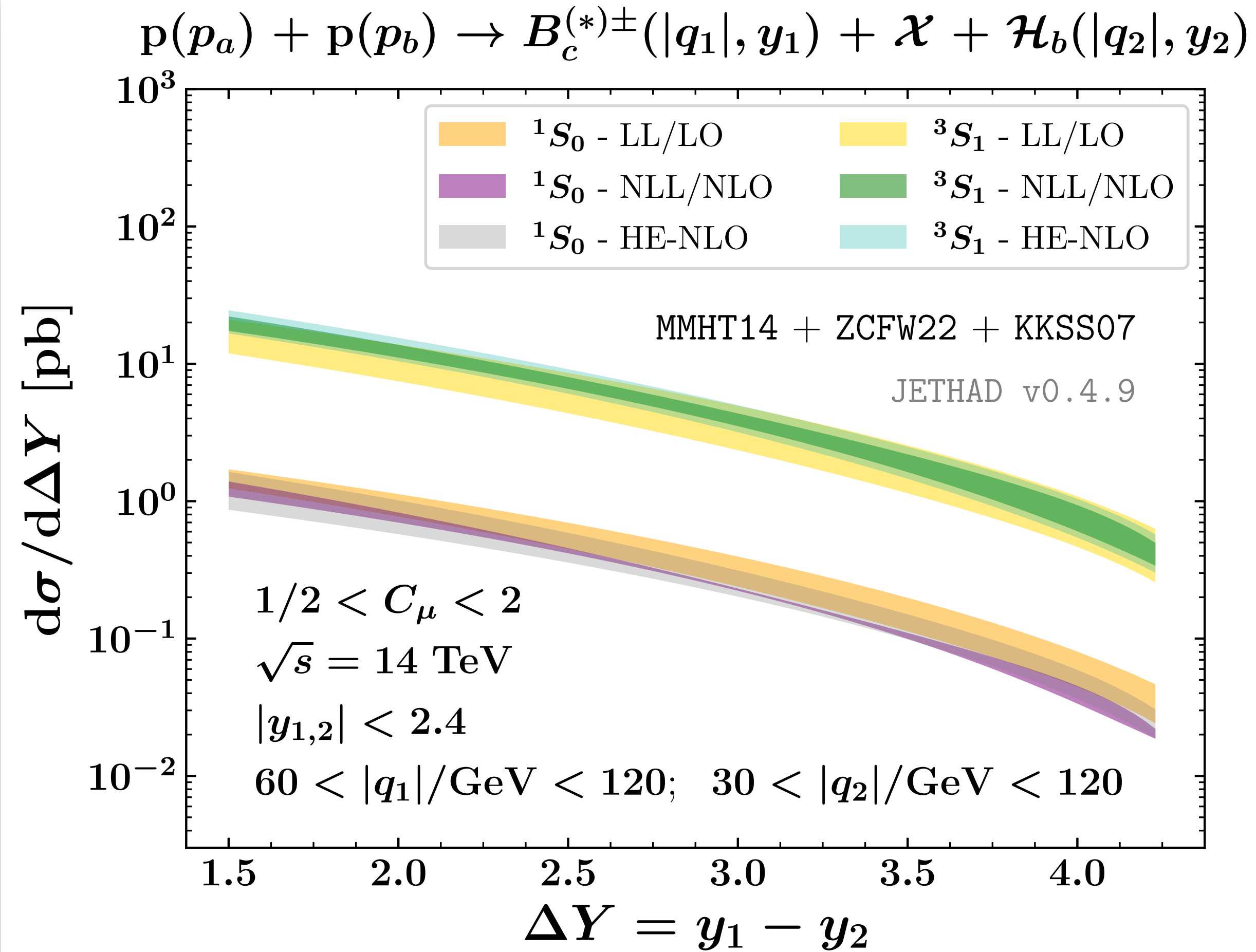


$B_c^\pm(^3S_1)$ collinear FFs

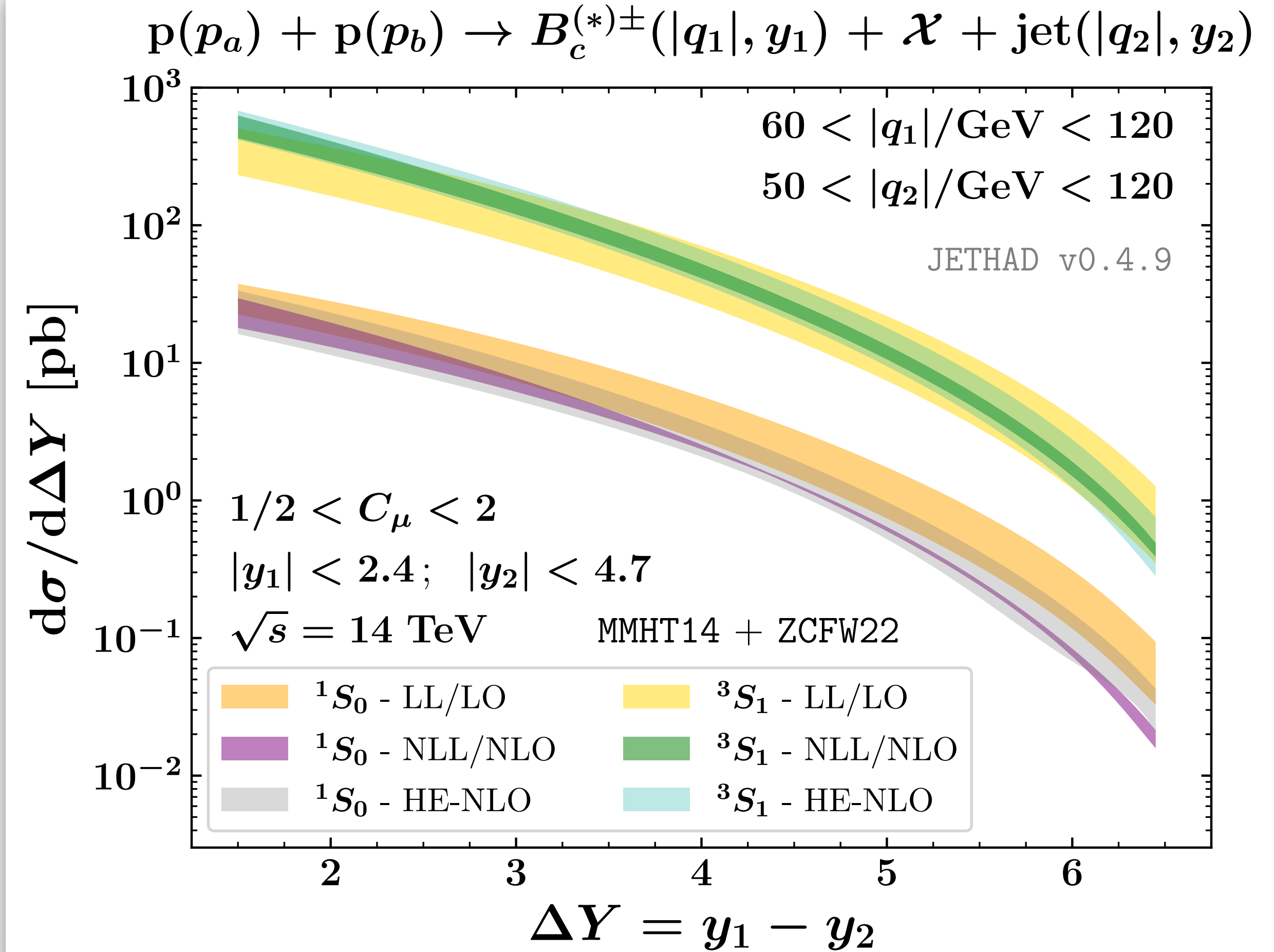


Charmed B -mesons + jet at the HL-LHC

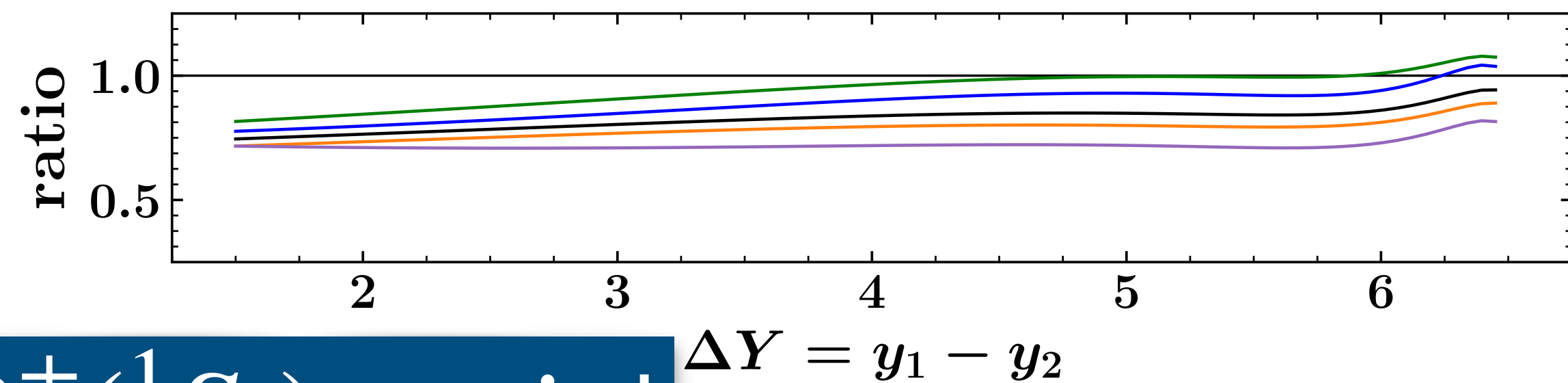
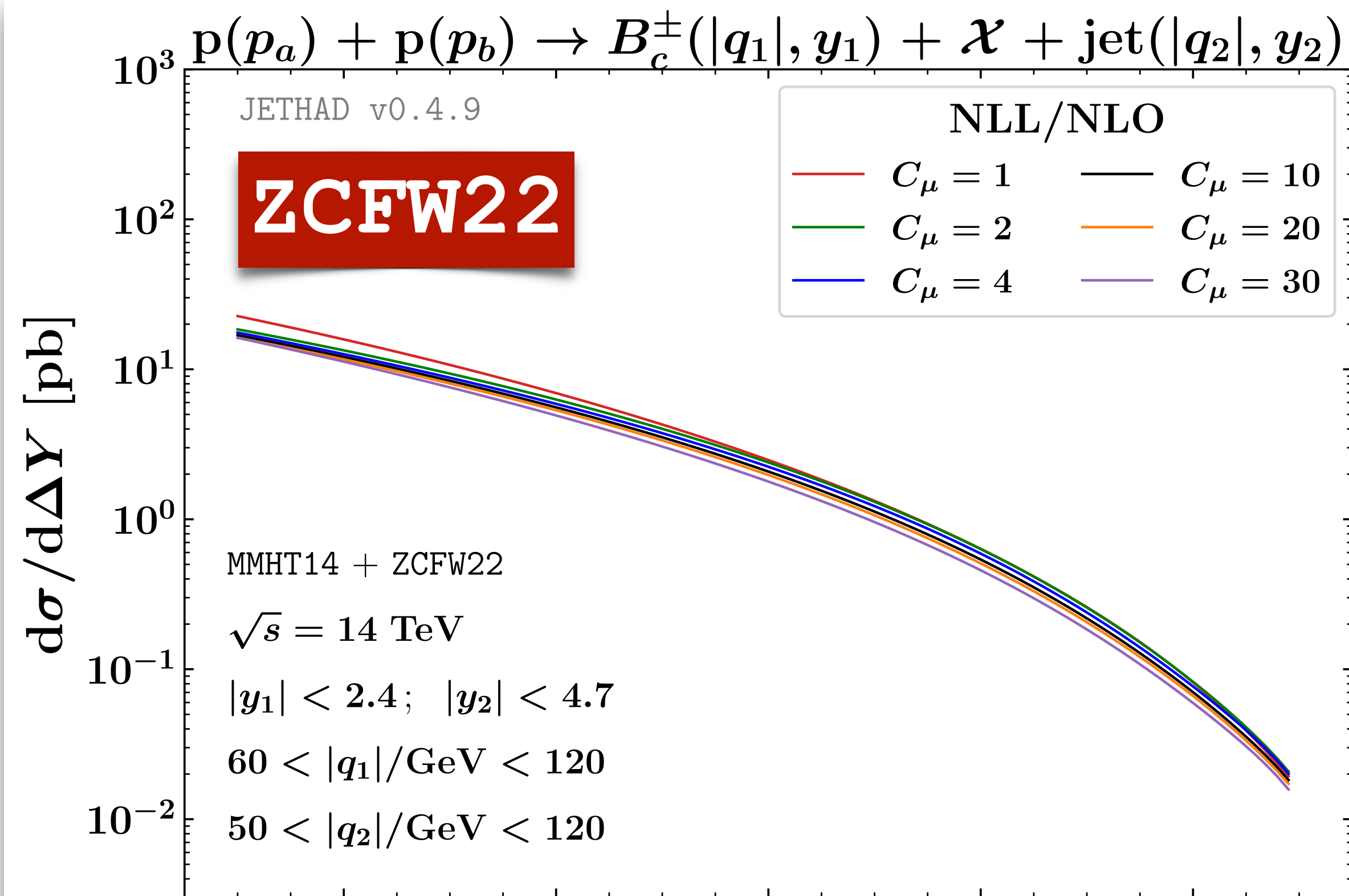
$B_c^\pm(^1S_0) + \text{b-hadron}$



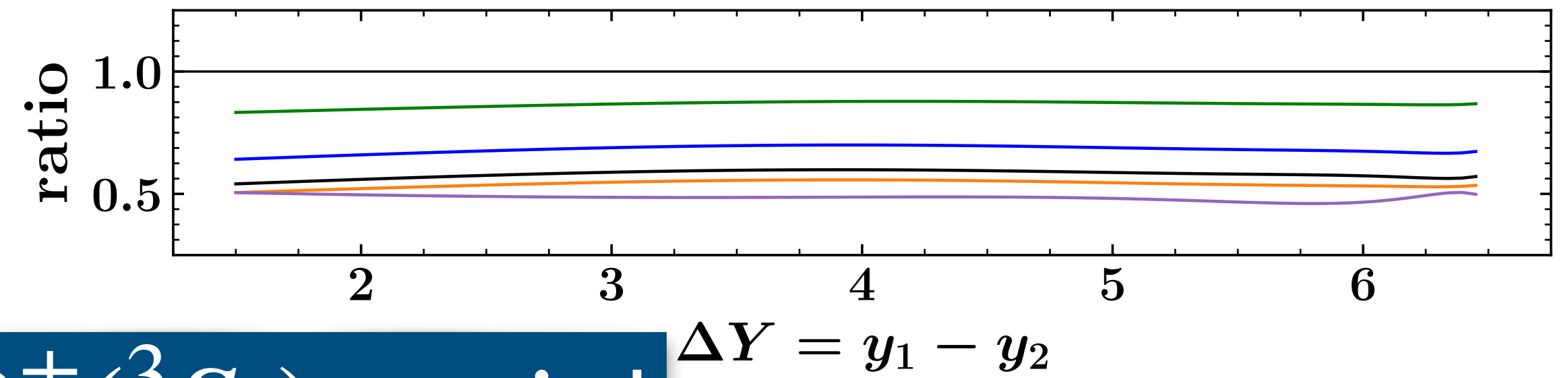
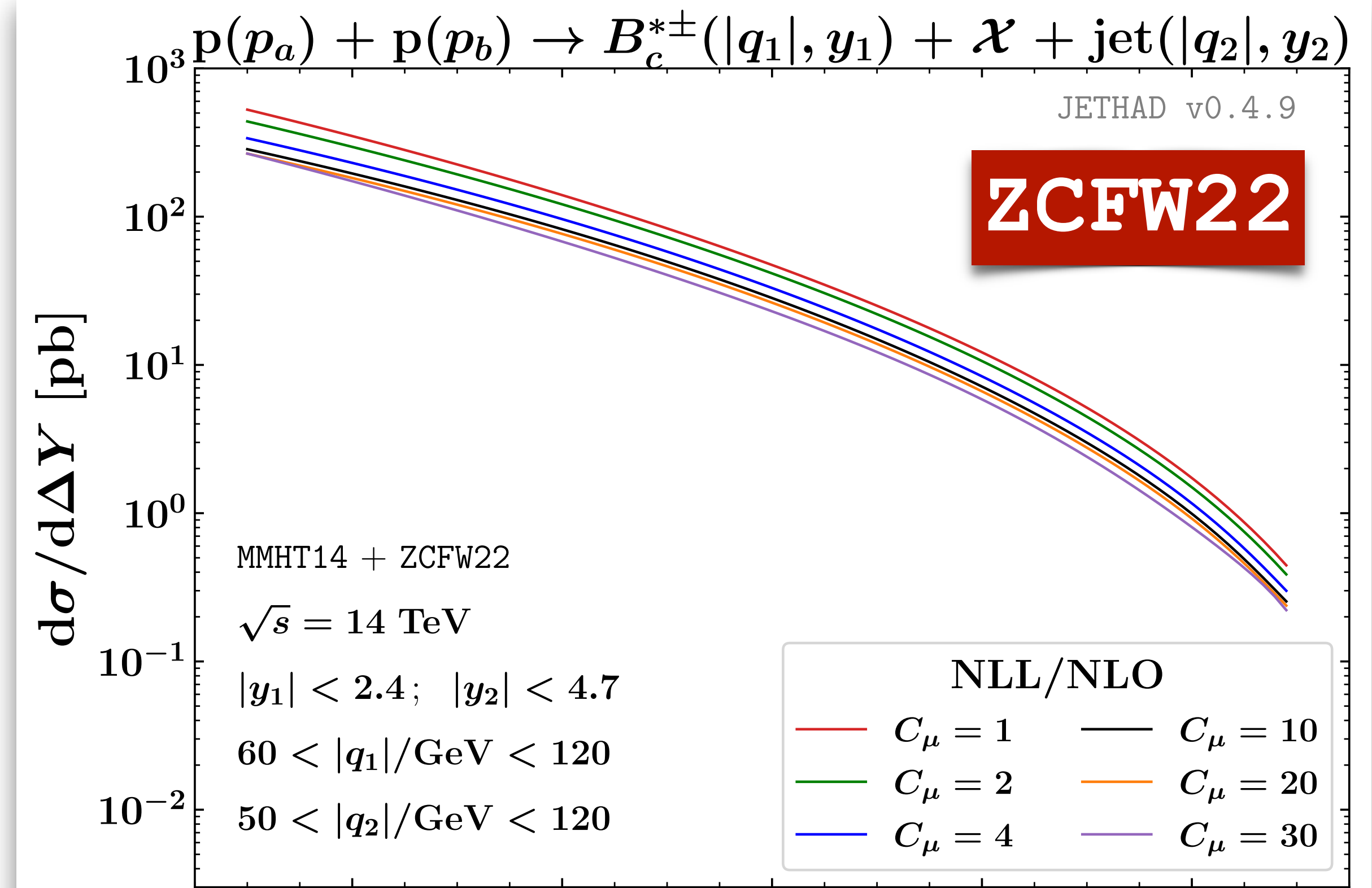
$B_c^\pm(^3S_1) + \text{jet}$



Charmed B -mesons + jet at the HL-LHC



$B_c^\pm(^1S_0) + \text{jet}$



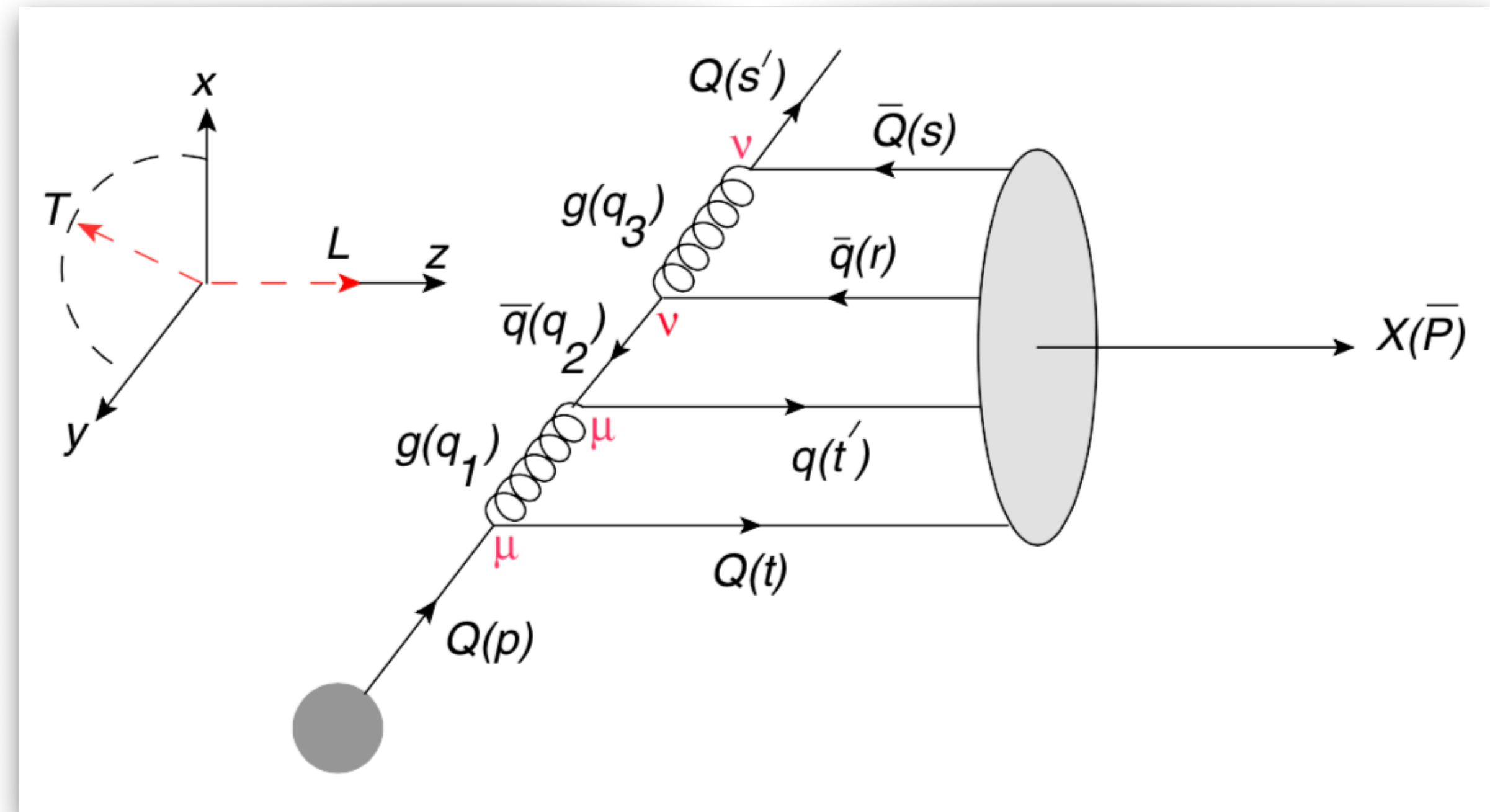
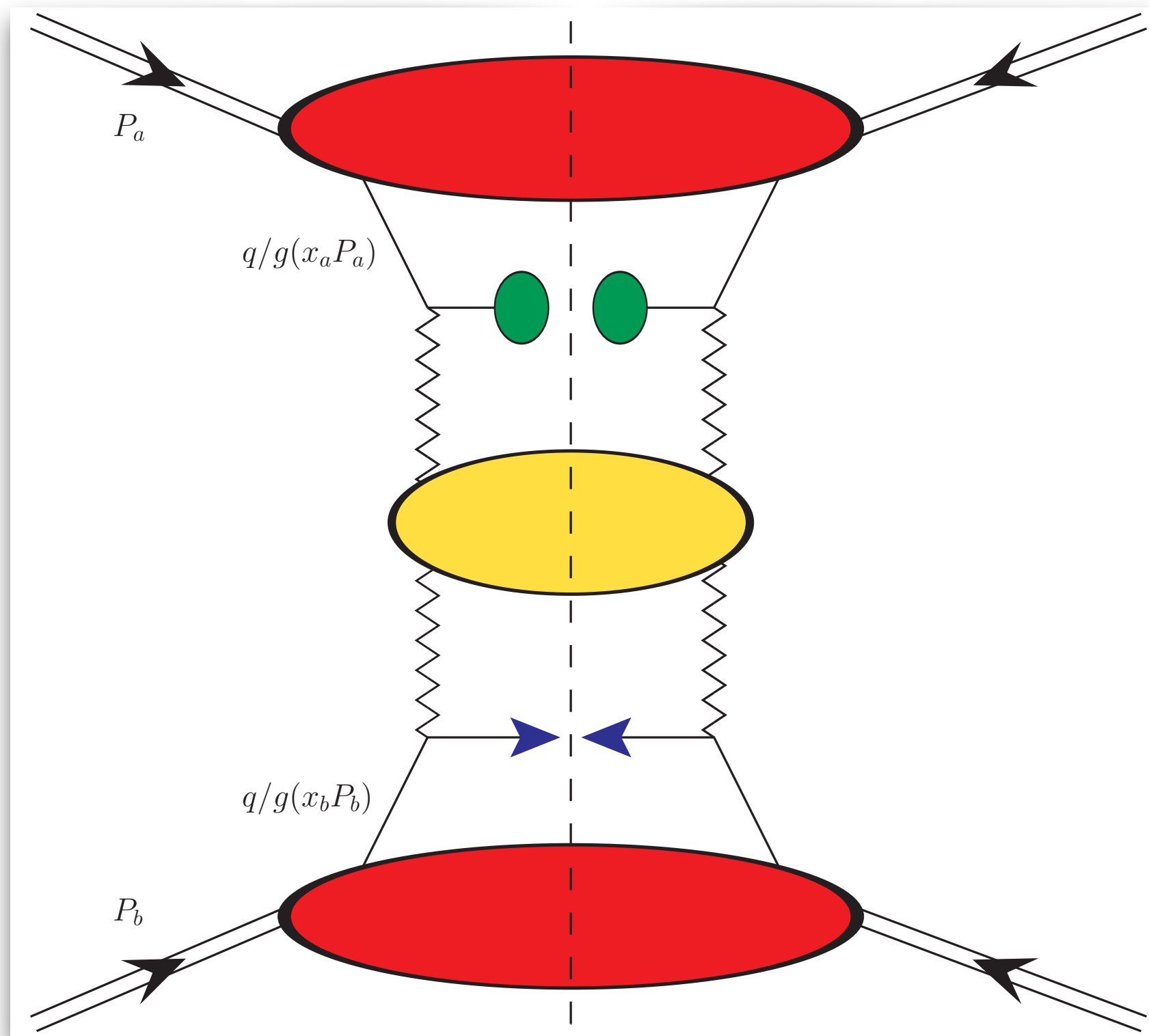
$B_c^\pm(^3S_1) + \text{jet}$

The background is a light blue gradient with several overlapping circular patterns. Inside these circles are stylized particle tracks, represented by yellow and orange wavy lines. Various colored spheres (red, blue, green) and arrows are placed along these tracks, suggesting a complex particle interaction or decay process. The overall aesthetic is scientific and futuristic.

A high-energy QCD portal to exotic matter

Heavy-light tetraquark from single-parton fragmentation

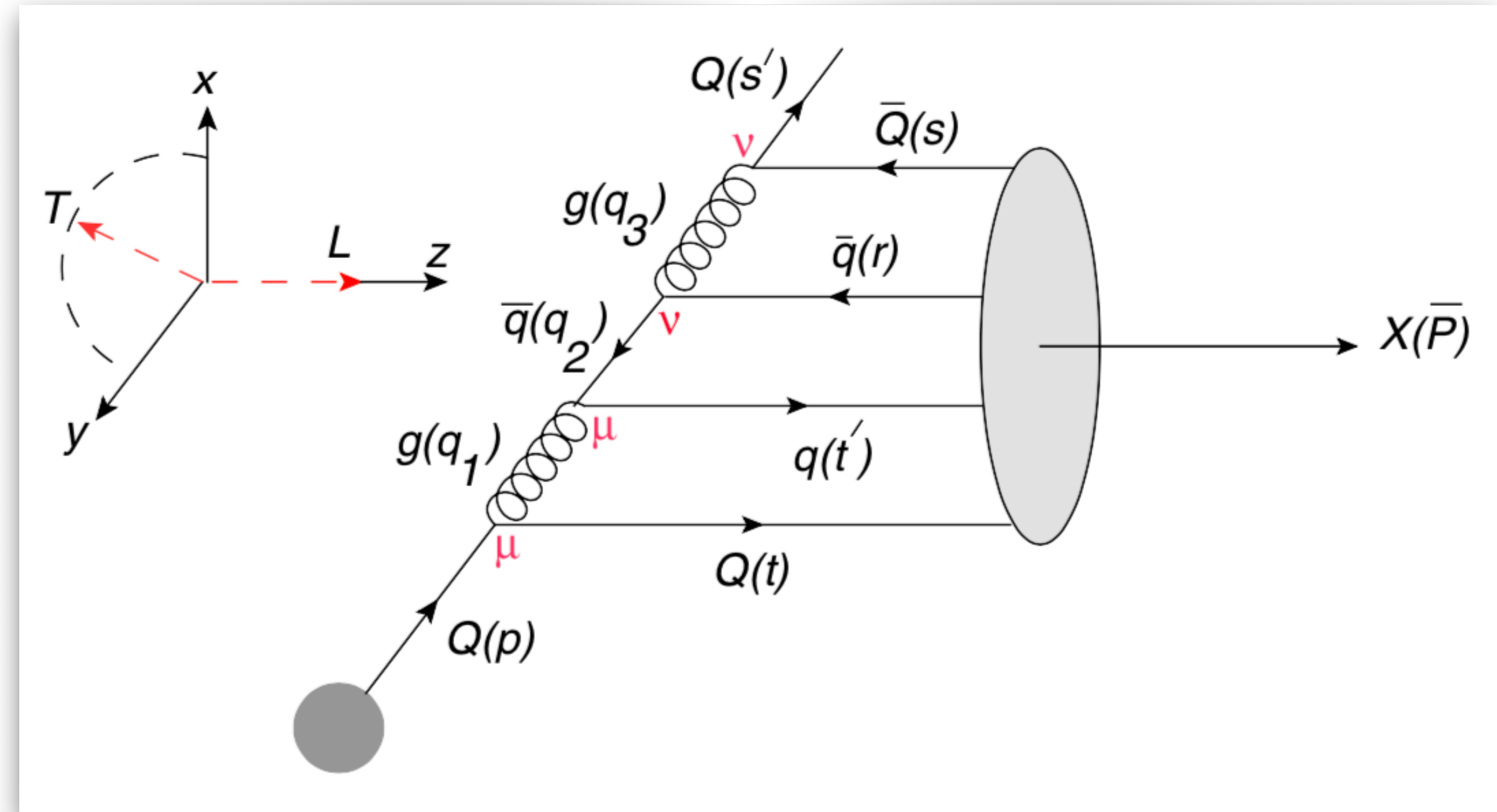
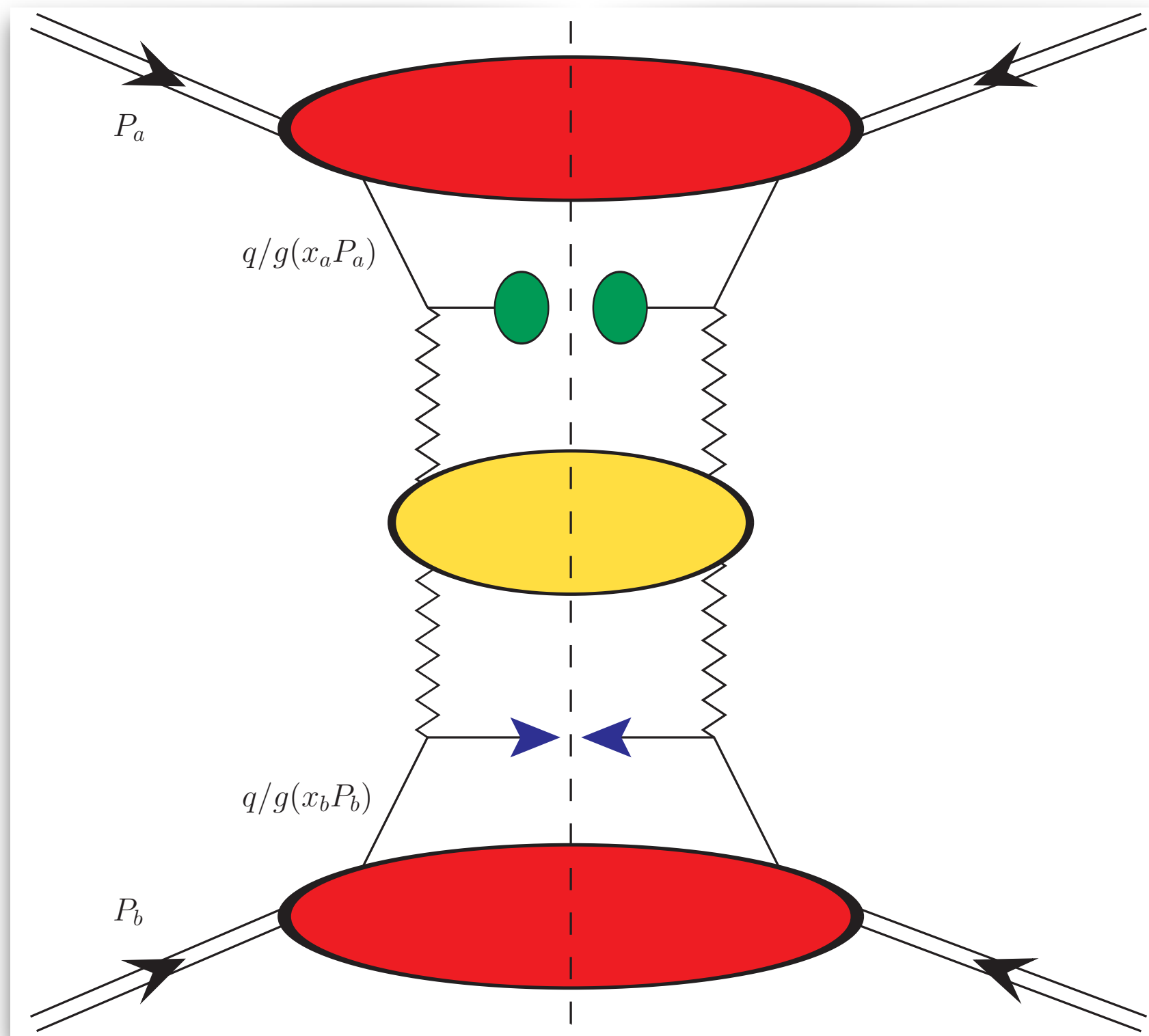
! Let us consider heavy-light $X_{Qq\bar{Q}\bar{q}}$ tetraquarks at large $p_T \rightarrow$ single-parton fragmentation !



[F. G. C., A. Papa, in preparation]

Heavy-light tetraquark from single-parton fragmentation

! Let us consider heavy-light $X_{Qq\bar{Q}\bar{q}}$ tetraquarks at large $p_T \rightarrow$ single-parton fragmentation !



[F. G. C., A. Papa, in preparation]

S-wave

$$D_Q^X(z, \mu_0) = N \frac{z \times \Sigma_{\text{spin}} \Gamma \bar{\Gamma}}{(m_X^2 - 2m_Q^2 + 2p \cdot s')^2}$$

$$= N \frac{z \times \Sigma_{\text{spin}} \Gamma \bar{\Gamma}}{[m_X^2 - (m_Q^2 + \langle p_T^2 \rangle)(1 + z - \frac{1}{1-z})]^2}$$

TQHL1.0 FFs: $(Q \rightarrow X_{Qq\bar{Q}\bar{q}}) \otimes$ **APFEL++**
 $[\mu_0 = m_X + m_Q]$

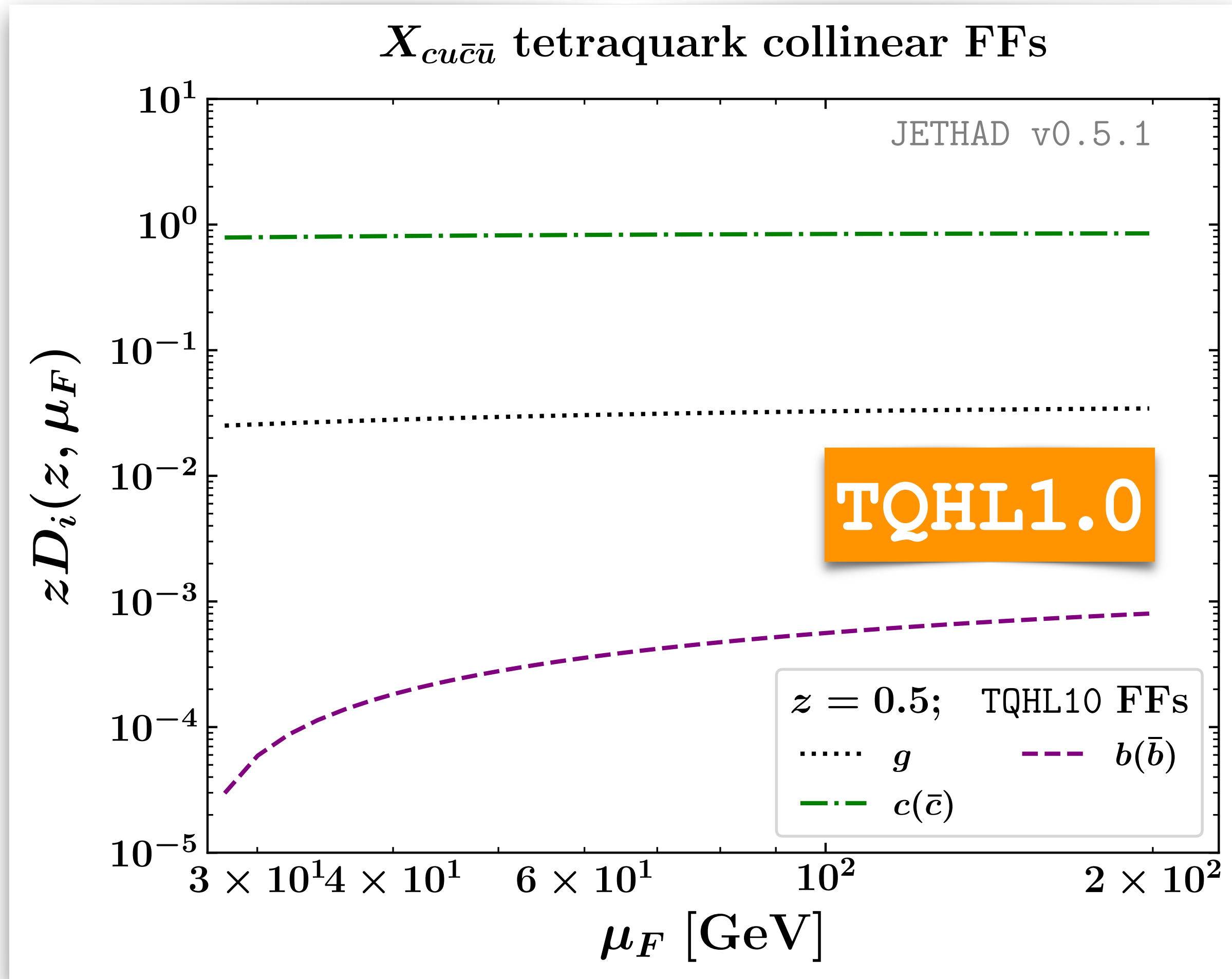
(LO) [S. M. Moosavi Nejad, Phys. Rev. D 05 (2022) 3, 034001]

(framework) [M. Suzuki, Phys. Rev. D 33 (1986) 676]

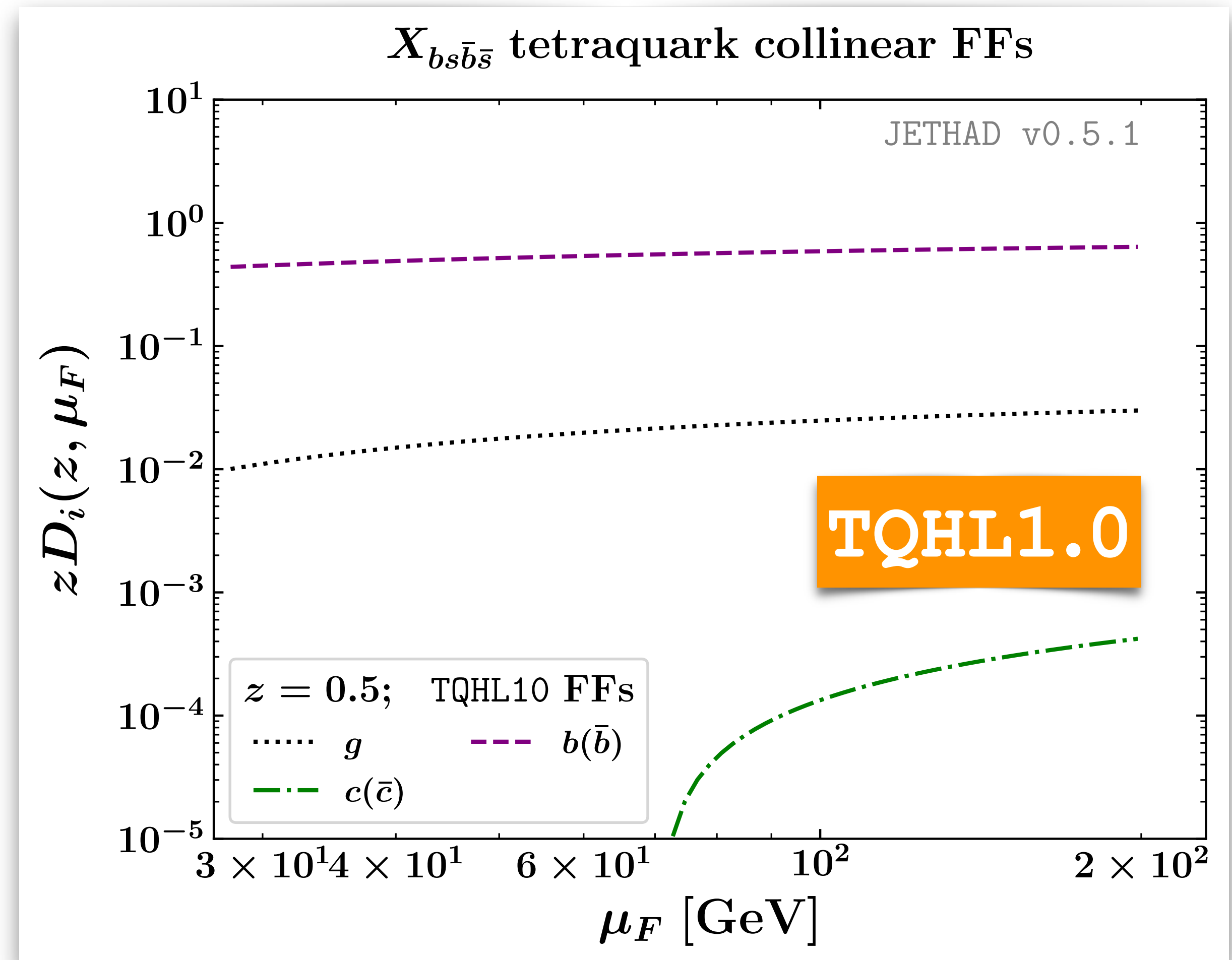
Backup

Heavy-light tetraquarks at the HL-LHC

$X_{cu\bar{c}\bar{u}}$ collinear FFs

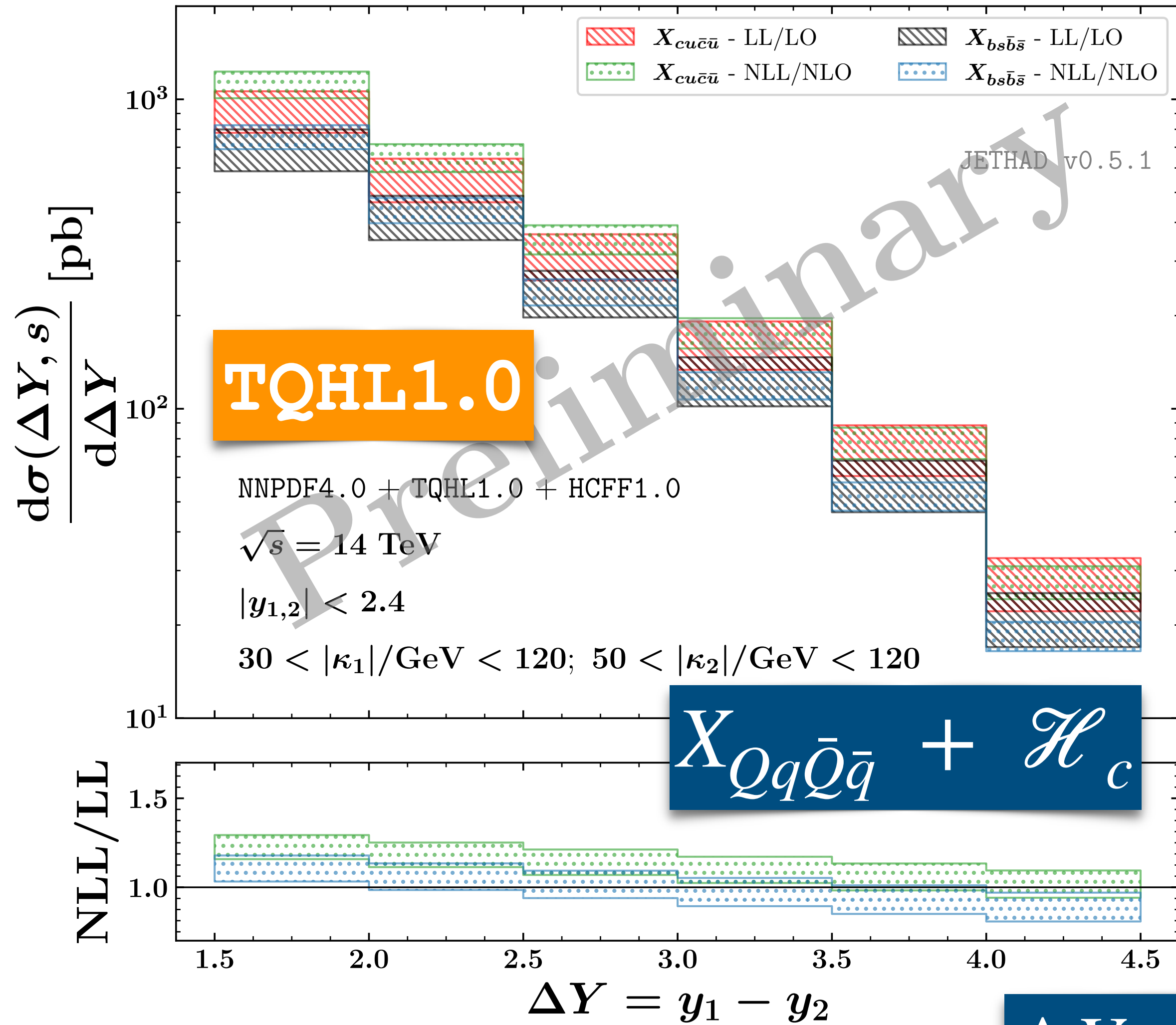


$X_{bs\bar{b}\bar{s}}$ collinear FFs



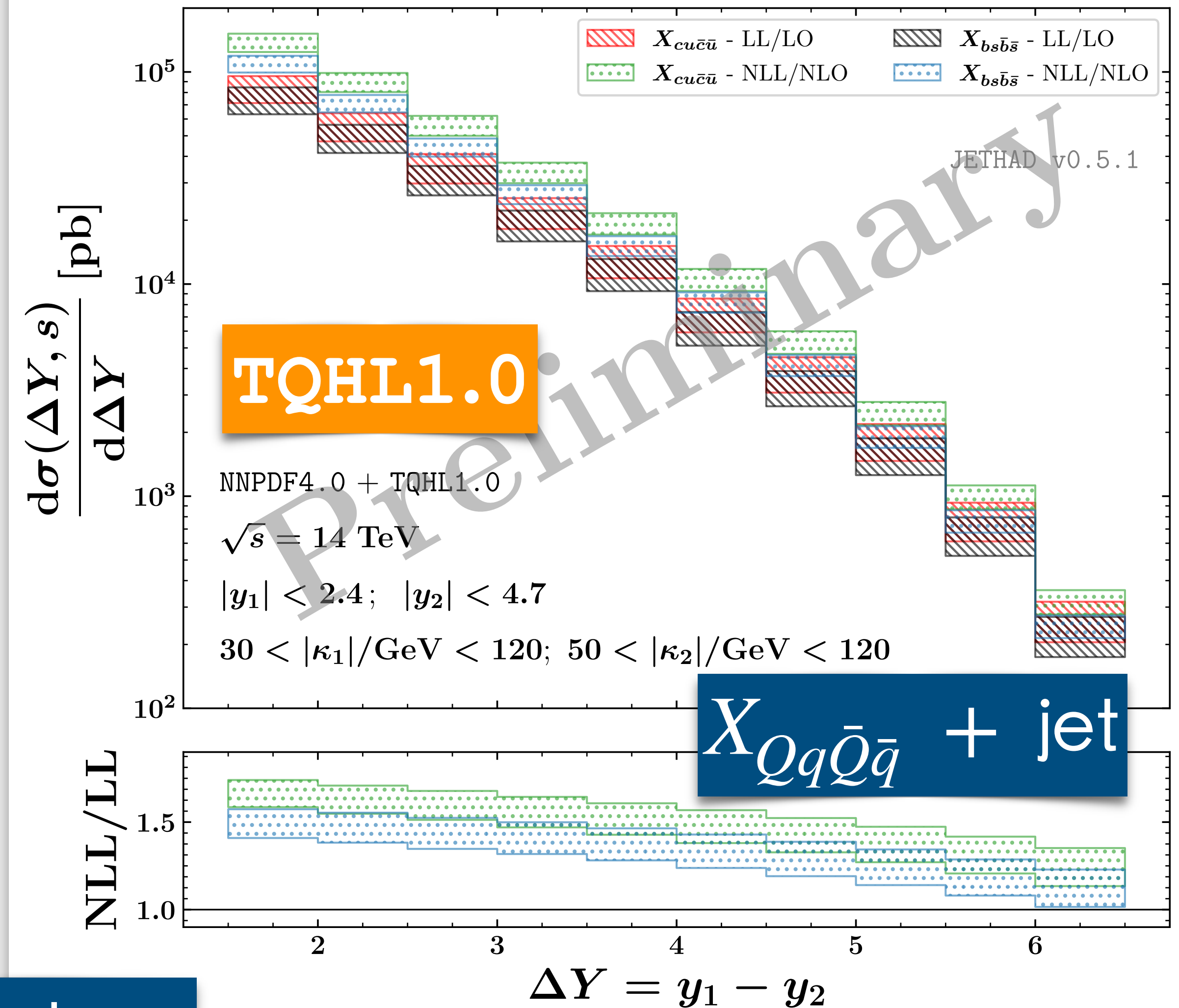
Heavy-light tetraquarks at the HL-LHC

$$p(p_a) + p(p_b) \rightarrow X_{Qq\bar{Q}\bar{q}}(|\kappa_1|, y_1) + \mathcal{X} + \mathcal{H}_c(|\kappa_2|, y_2)$$

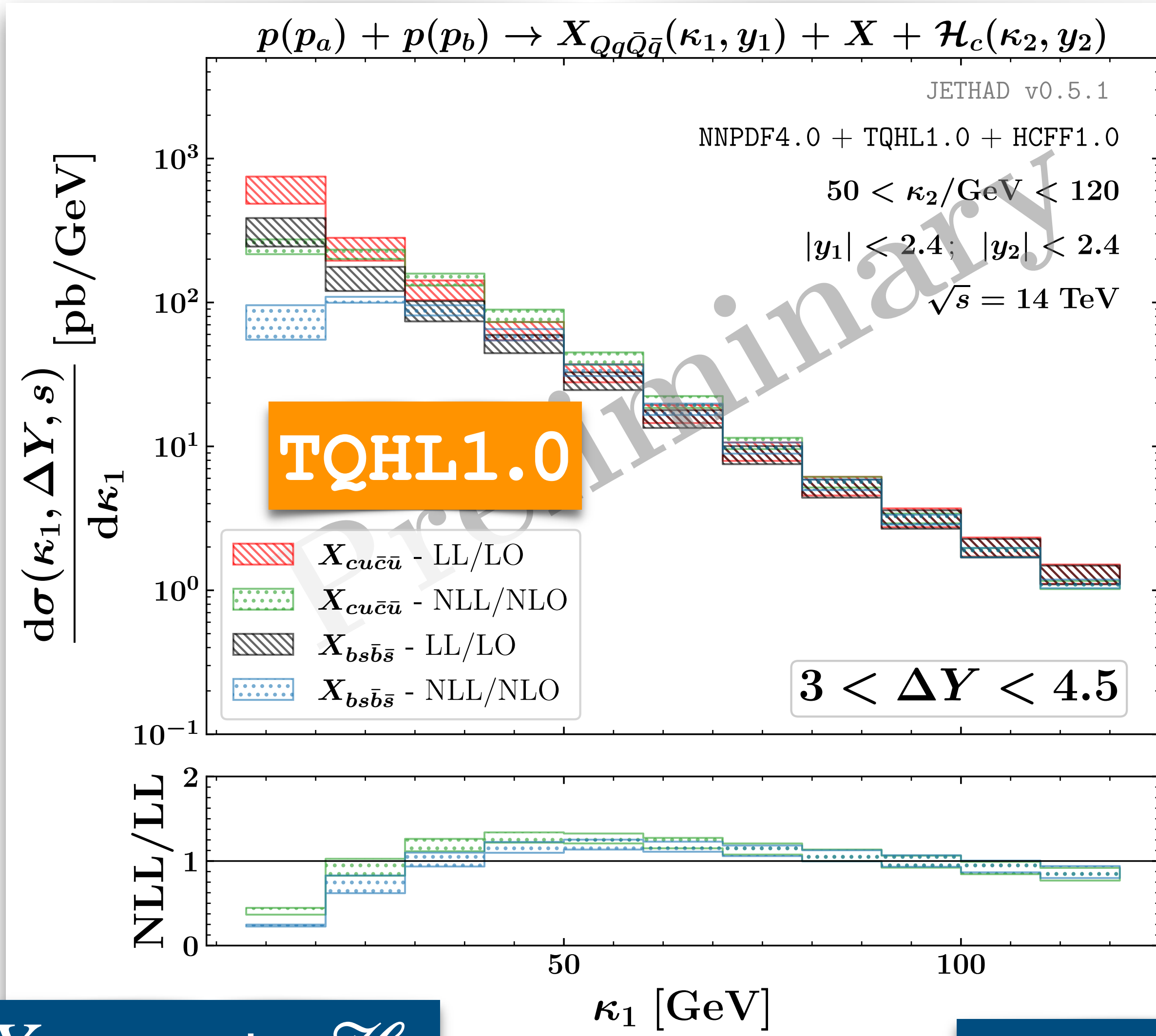


ΔY spectrum

$$p(p_a) + p(p_b) \rightarrow X_{Qq\bar{Q}\bar{q}}(|\kappa_1|, y_1) + \mathcal{X} + \text{jet}(|\kappa_2|, y_2)$$



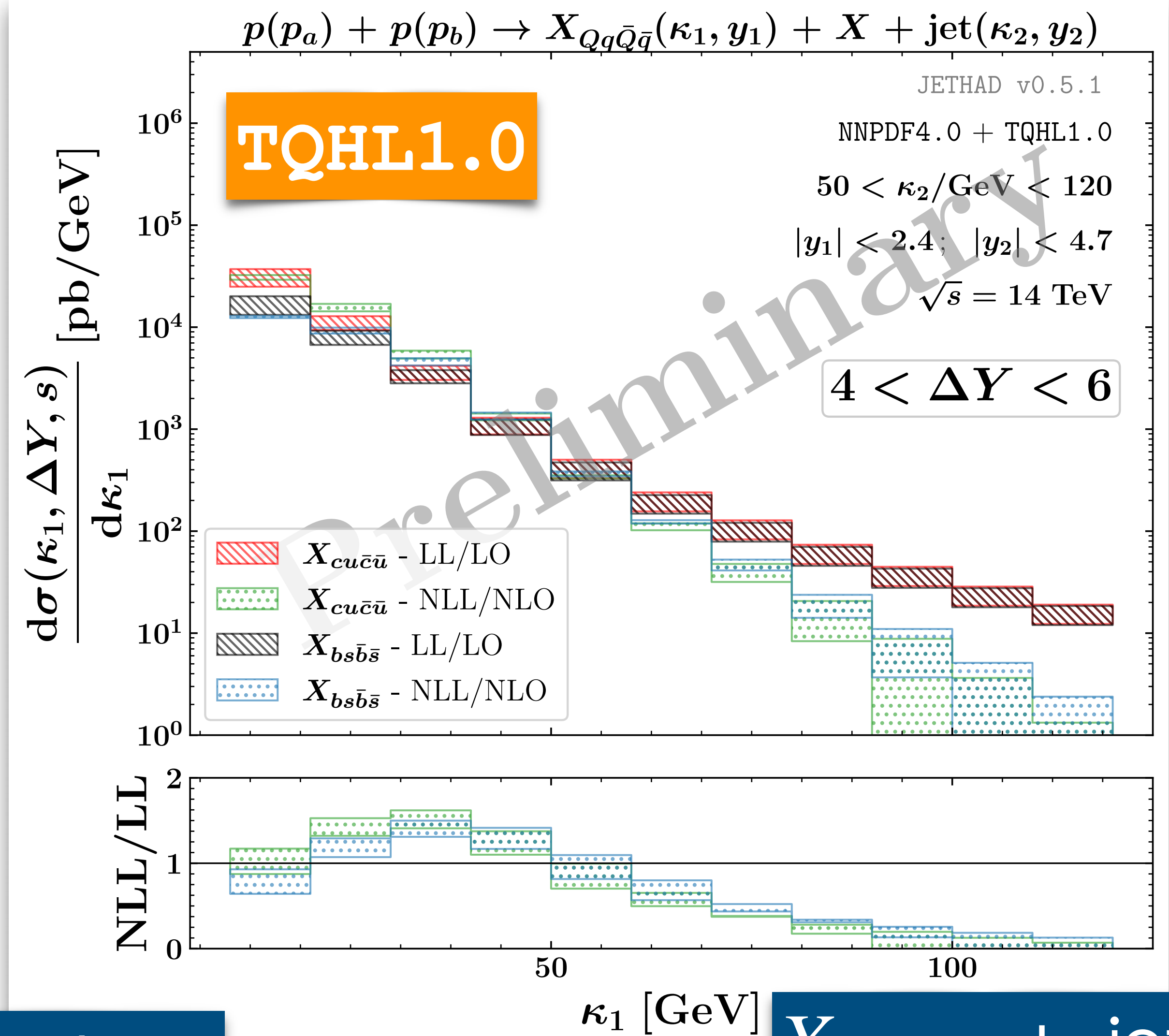
Heavy-light tetraquarks at the HL-LHC



$X_{Qq\bar{Q}\bar{q}} + \mathcal{H}_c$

$|\kappa_1|$ spectrum

[F. G. C., A. Papa, in preparation]



$X_{Qq\bar{Q}\bar{q}} + \text{jet}$

Backup

The background is a light blue gradient with several overlapping circular diagrams. Each diagram contains a yellow wavy line representing a particle path, with various colored spheres (red, blue, green) and arrows indicating interactions or emissions. The text 'Heavy-flavored emissions' is centered in a bold blue font.

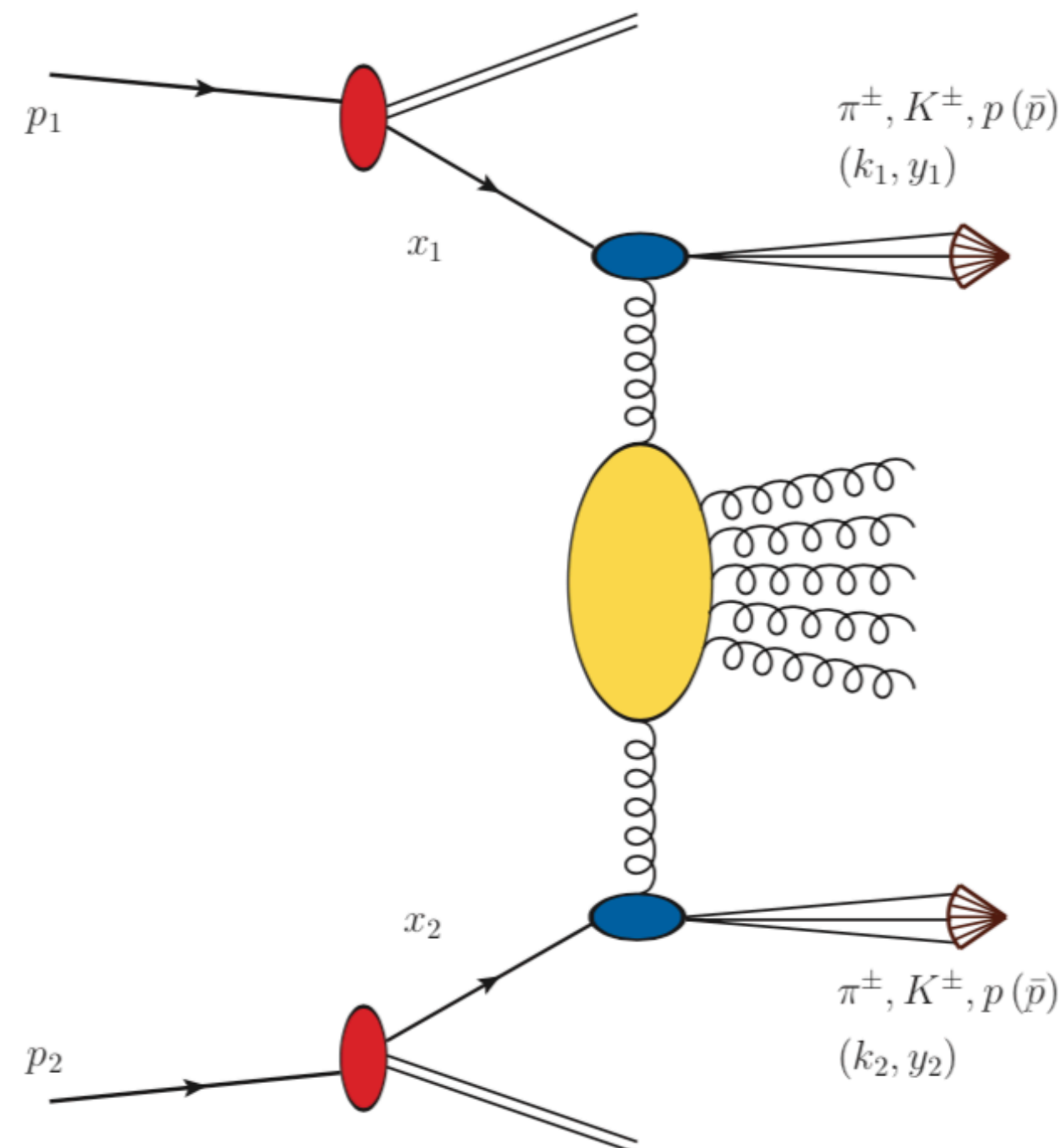
Heavy-flavored emissions

From Higgs + jet to bound states

Di-hadron and hadron-jet correlations

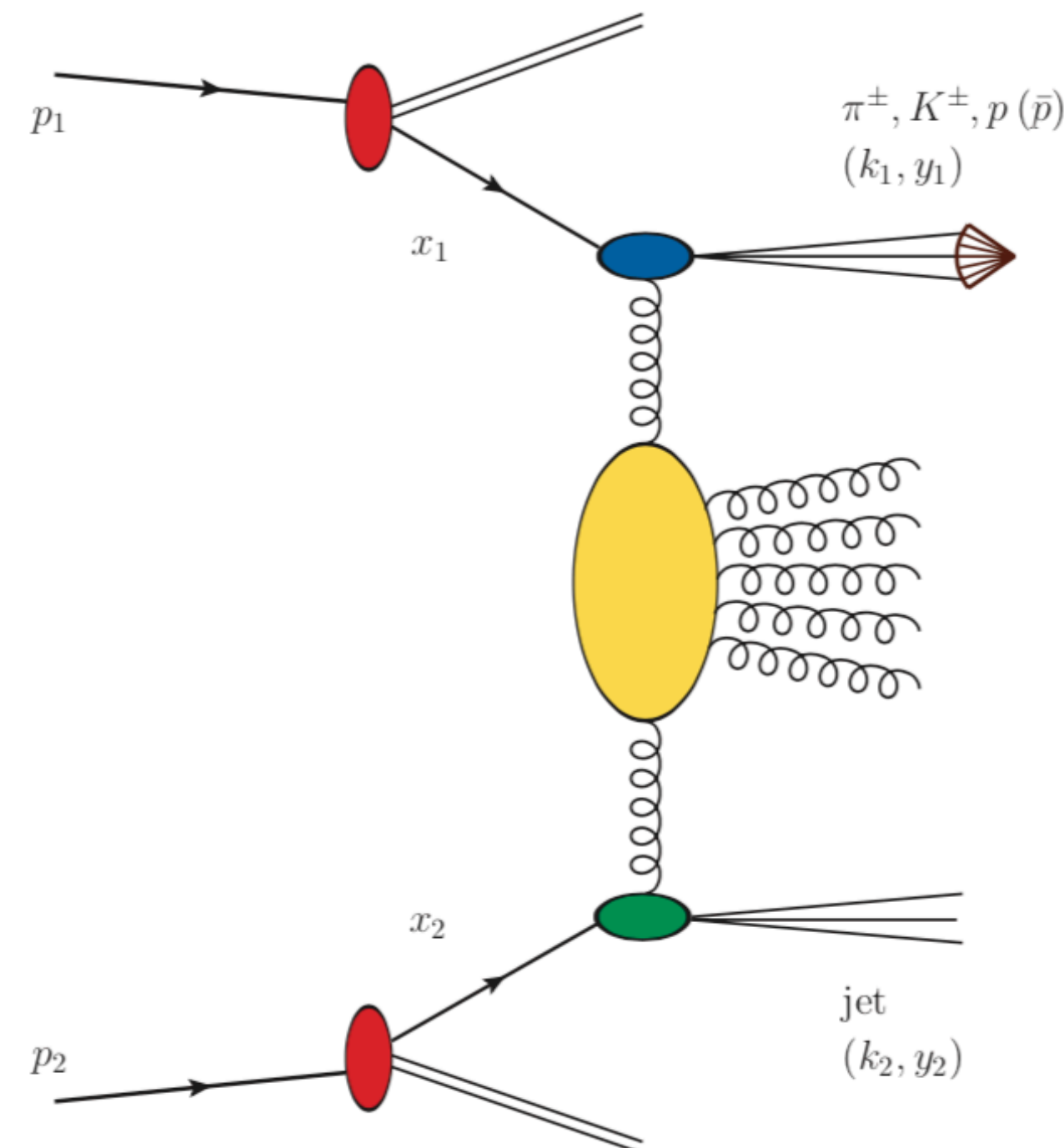
Inclusive di-hadron production

[D.Yu. Ivanov, A. Papa (2012)] (NLO forward-hadron impact factor)
[F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]



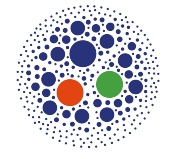
Inclusive hadron-jet production

[A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]
[F.G.C. (in preparation)]



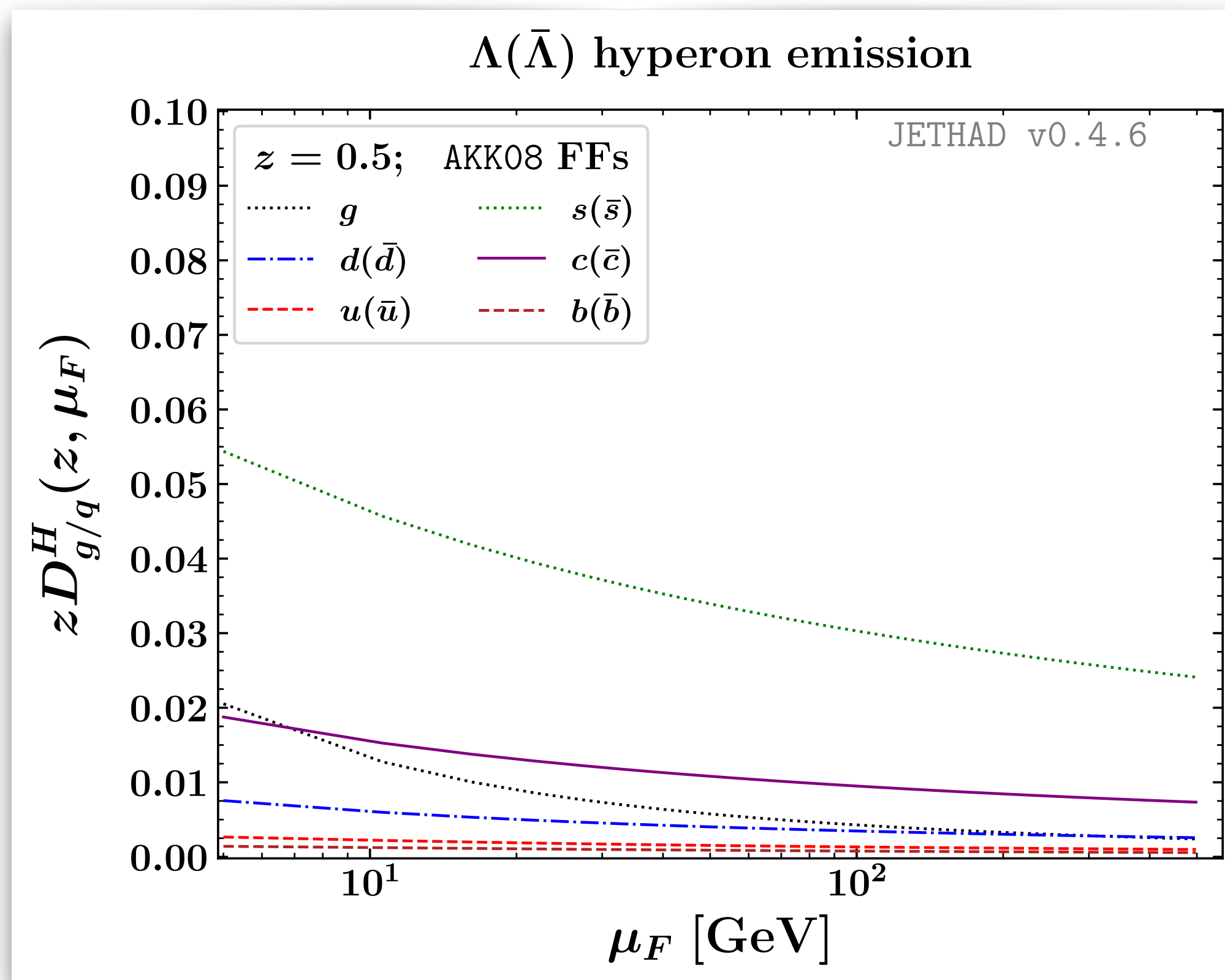
- ◇ NLO impact factors known $\Rightarrow$ full NLA BFKL analysis feasible
- ◇ PDFs + FFs at work (both), hadrons at smaller rapidities than jets (di-hadron)
- ◇ genuine *asymmetric* cuts in transverse momenta (hadron-jet)

Stabilizing effects of heavy-flavor fragmentation



AKK08 VFNS collinear FFs for Λ hyperon: $|uds\rangle$

[\[S. Albino et al., Nucl. Phys. B 803 \(2008\) 42-104\]](#)

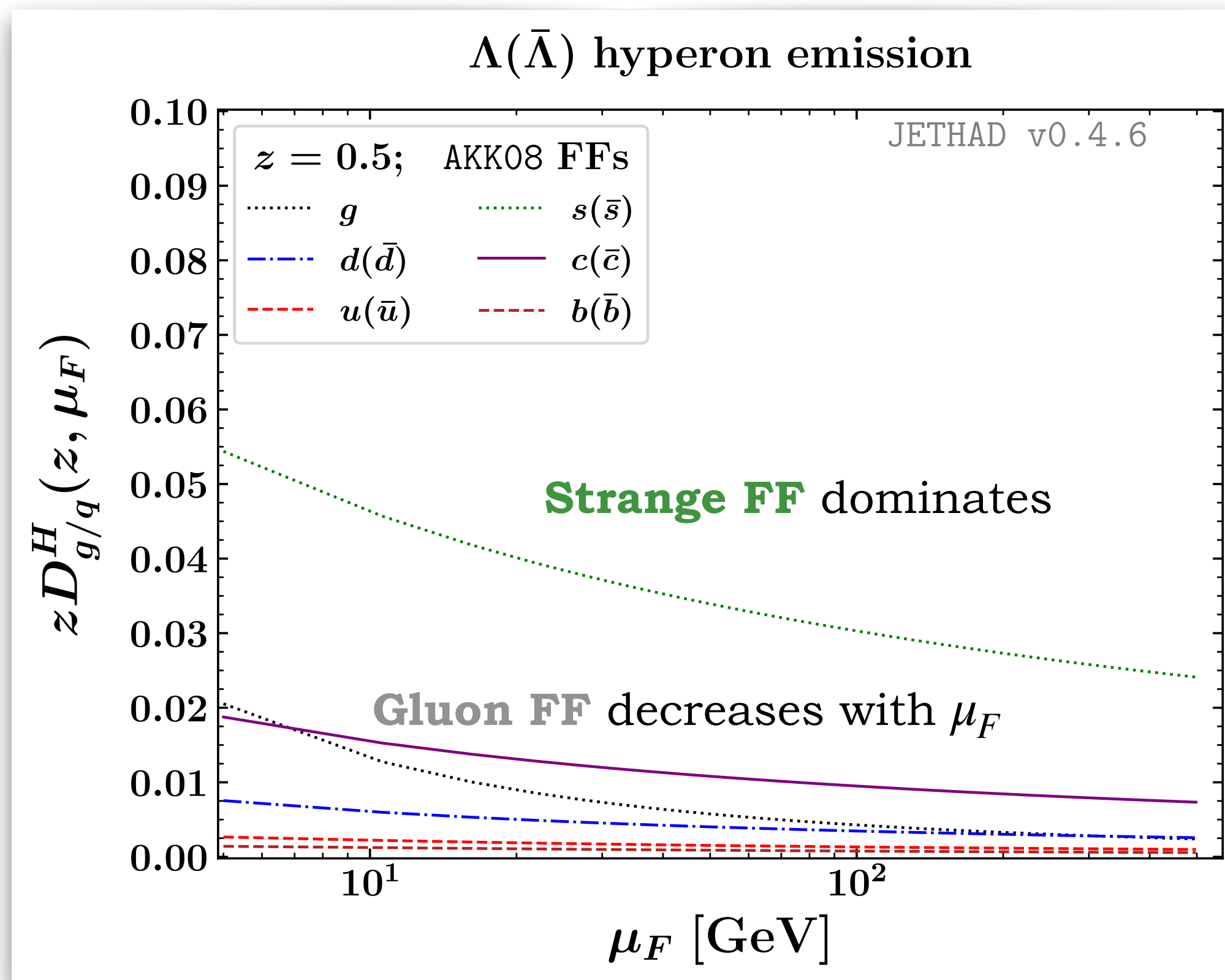


Stabilizing effects of heavy-flavor fragmentation



AKK08 VFNS collinear FFs for Λ hyperon: $|uds\rangle$

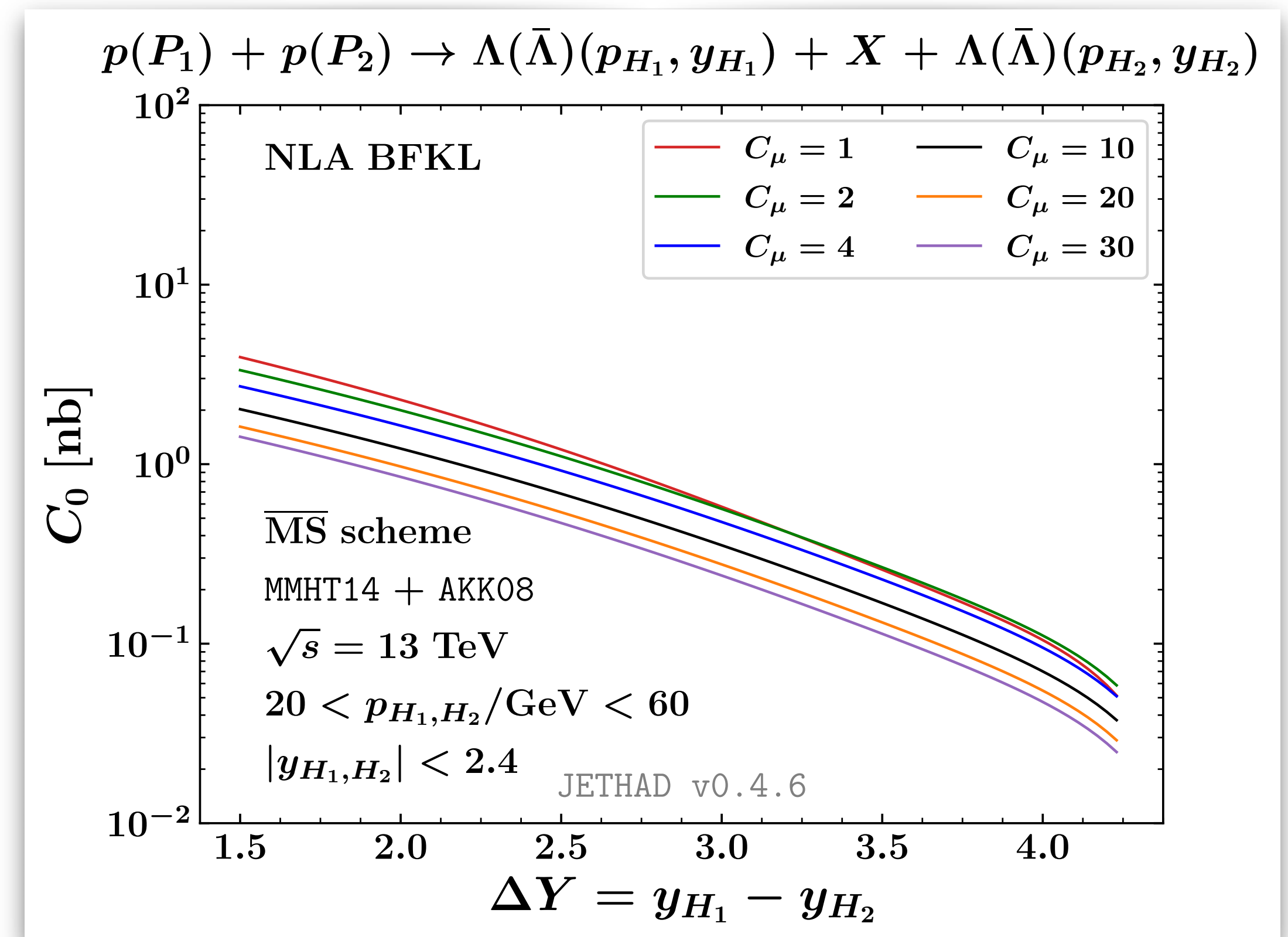
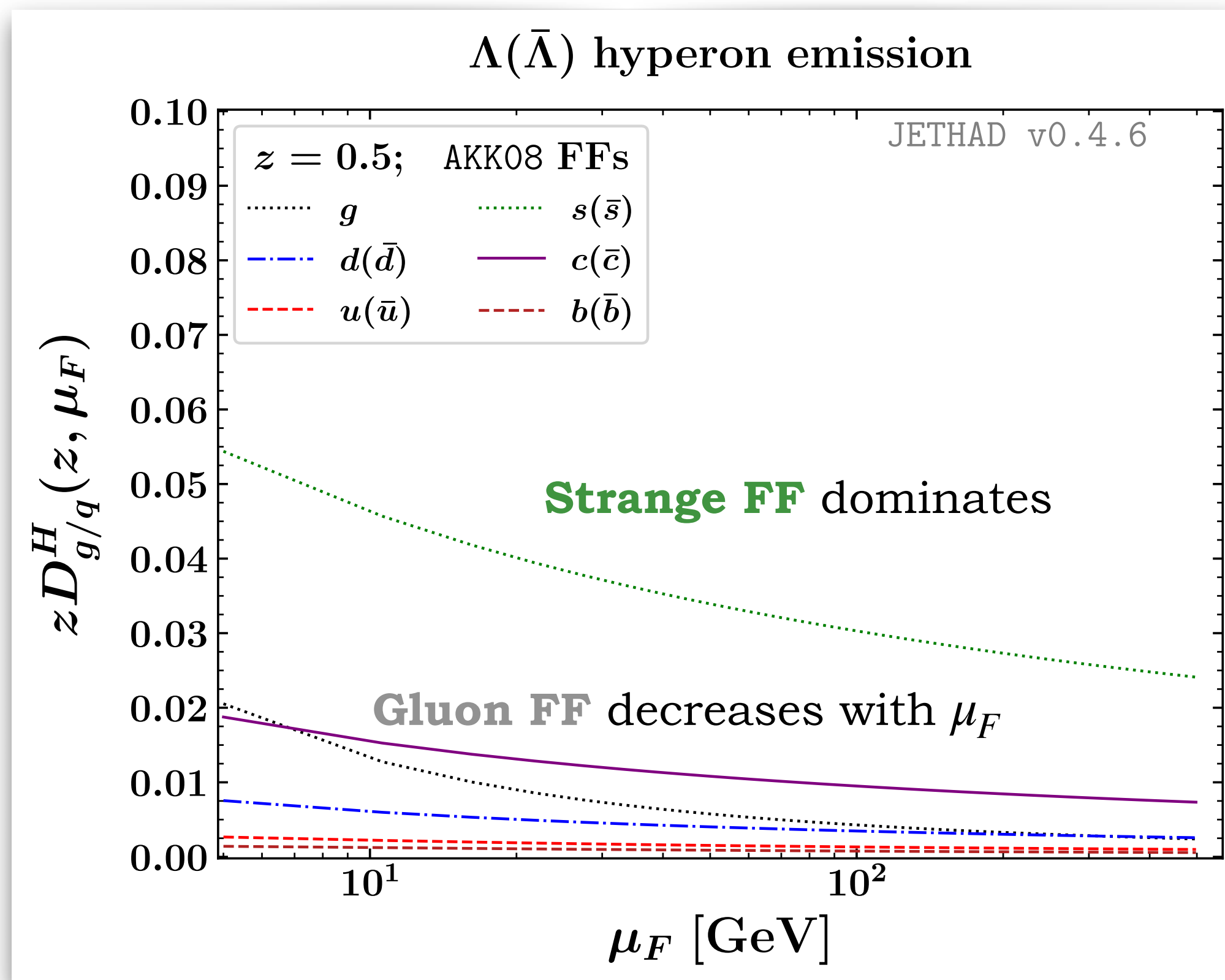
[\[S. Albino et al., Nucl. Phys. B 803 \(2008\) 42-104\]](#)



Stabilizing effects of heavy-flavor fragmentation

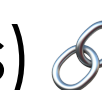
 **AKK08** VFNS collinear FFs for Λ hyperon: $|uds\rangle$

 [S. Albino et al., Nucl. Phys. B 803 (2008) 42-104]

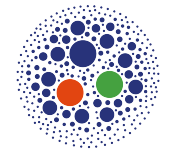


 Rapidity distribution **sensitive** to scale variations

(Λ hyperons)  [F. G. C. et al., Phys. Rev. D 102 (2020) 9, 094019]

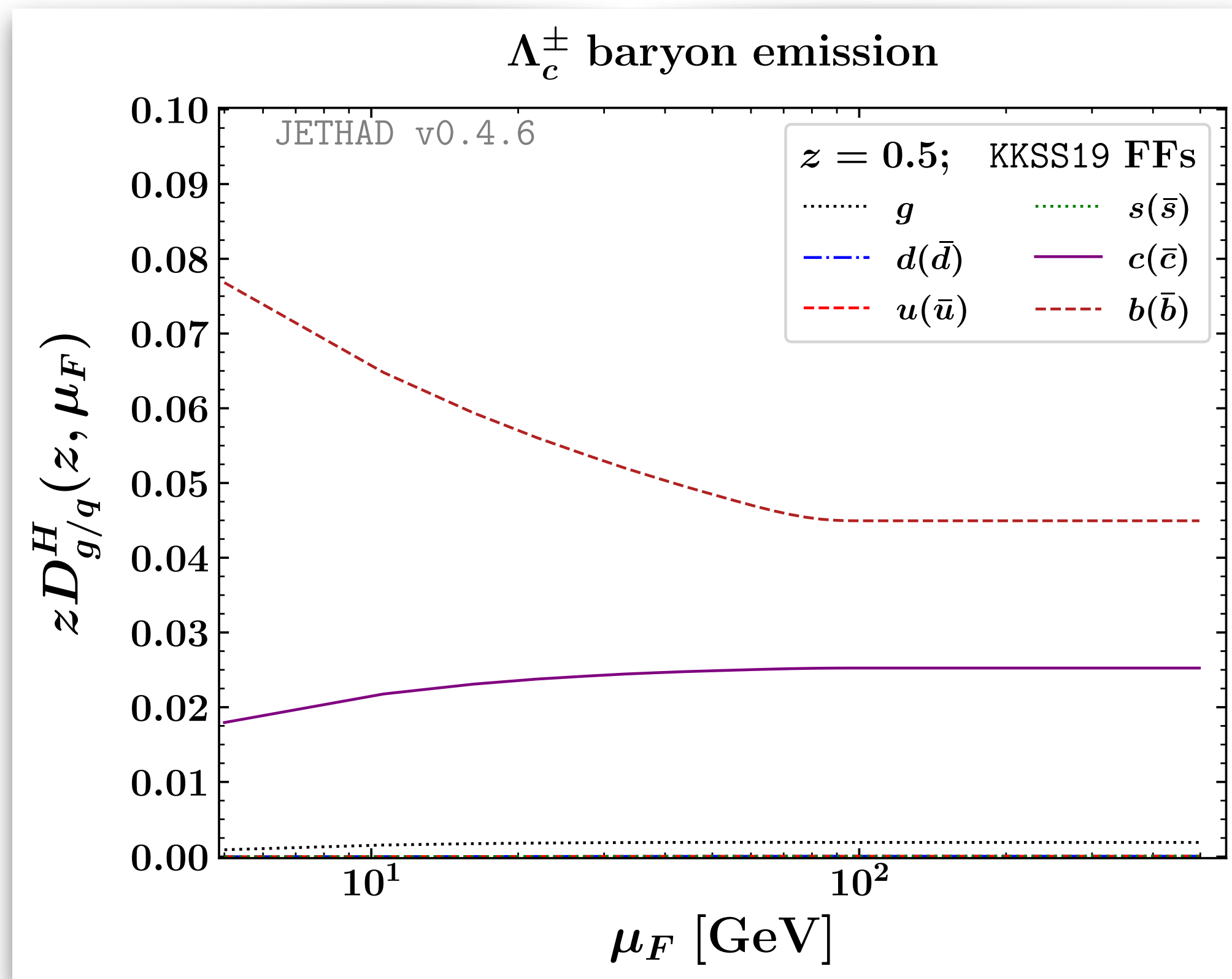
(cascade Ξ baryons)  [F. G. C., Eur. Phys. J. C (in press)]

Stabilizing effects of heavy-flavor fragmentation

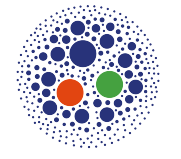


KKSS19 VFNS collinear FFs for Λ_c baryons: $|udc\rangle$

[\[B. A. Kniehl et al., Phys. Rev. D 101 \(2020\) 11, 114021\]](#)

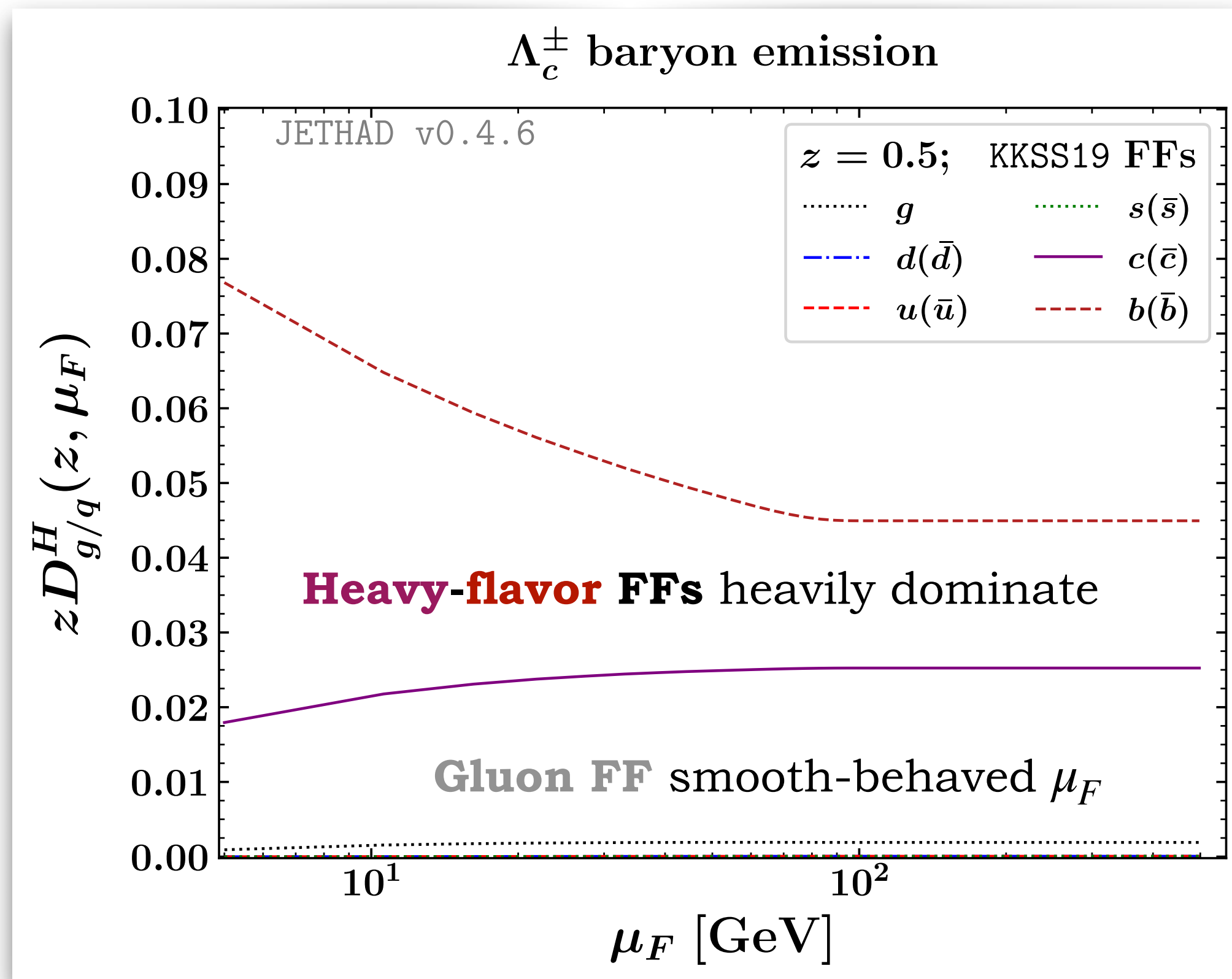


Stabilizing effects of heavy-flavor fragmentation

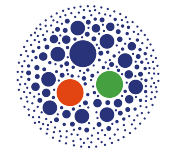


KKSS19 VFNS collinear FFs for Λ_c baryons: $|udc\rangle$

[\[B. A. Kniehl et al., Phys. Rev. D 101 \(2020\) 11, 114021\]](#)

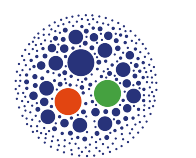
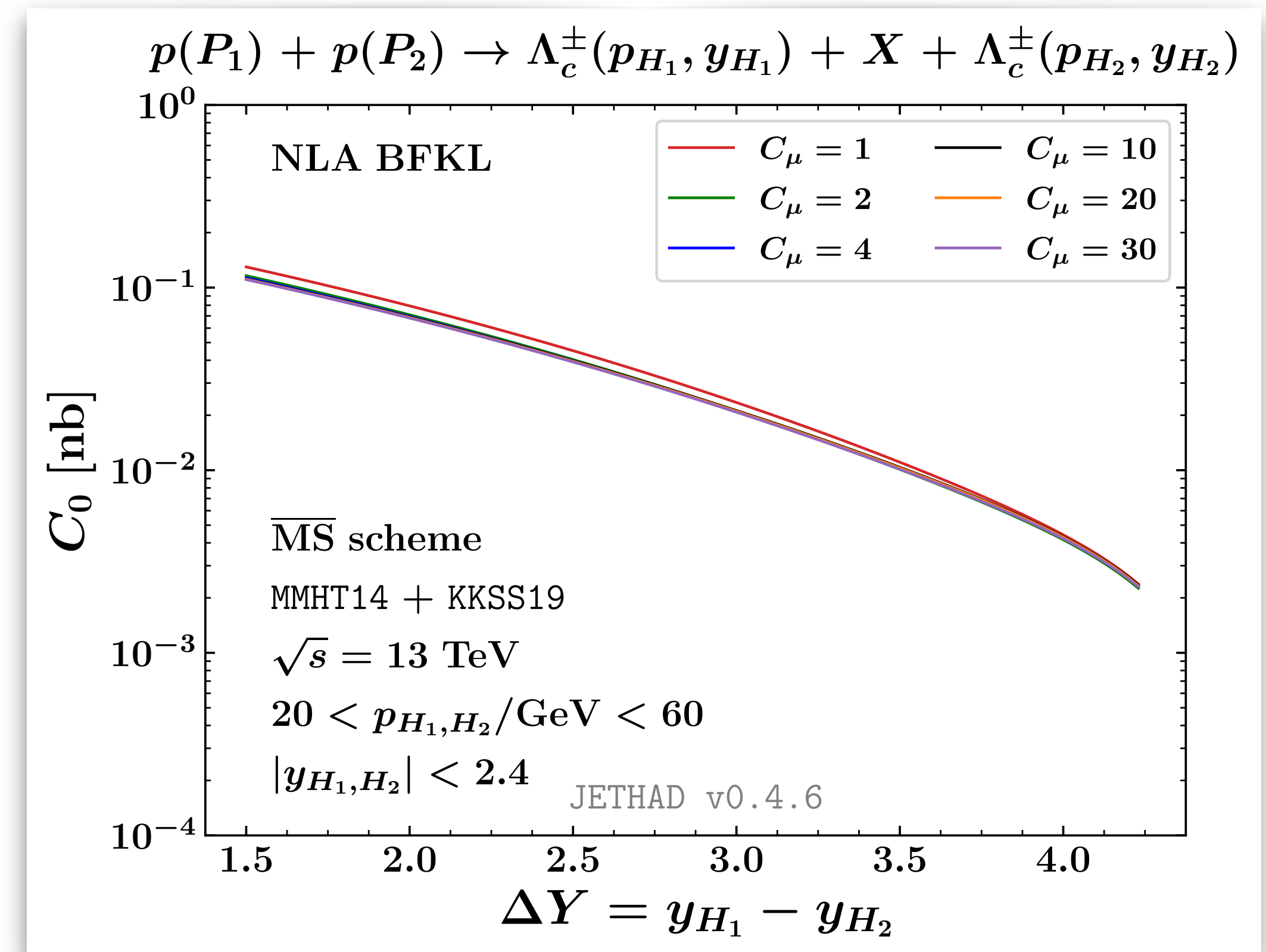
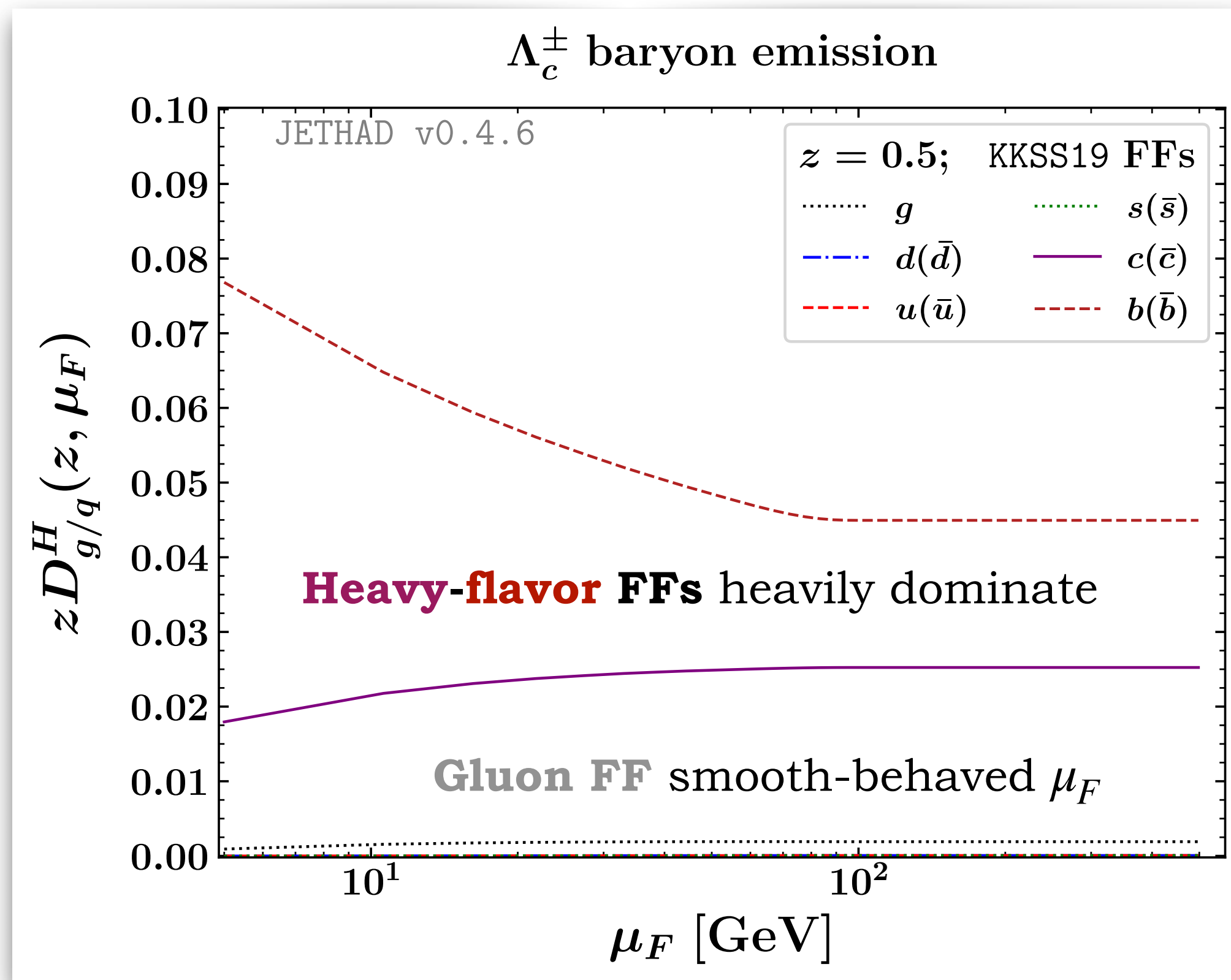


Stabilizing effects of heavy-flavor fragmentation



KKSS19 VFNS collinear FFs for Λ_c baryons: $|udc\rangle$

[\[B. A. Kniehl et al., Phys. Rev. D 101 \(2020\) 11, 114021\]](#)



Rapidity distribution **stable** under scale variations

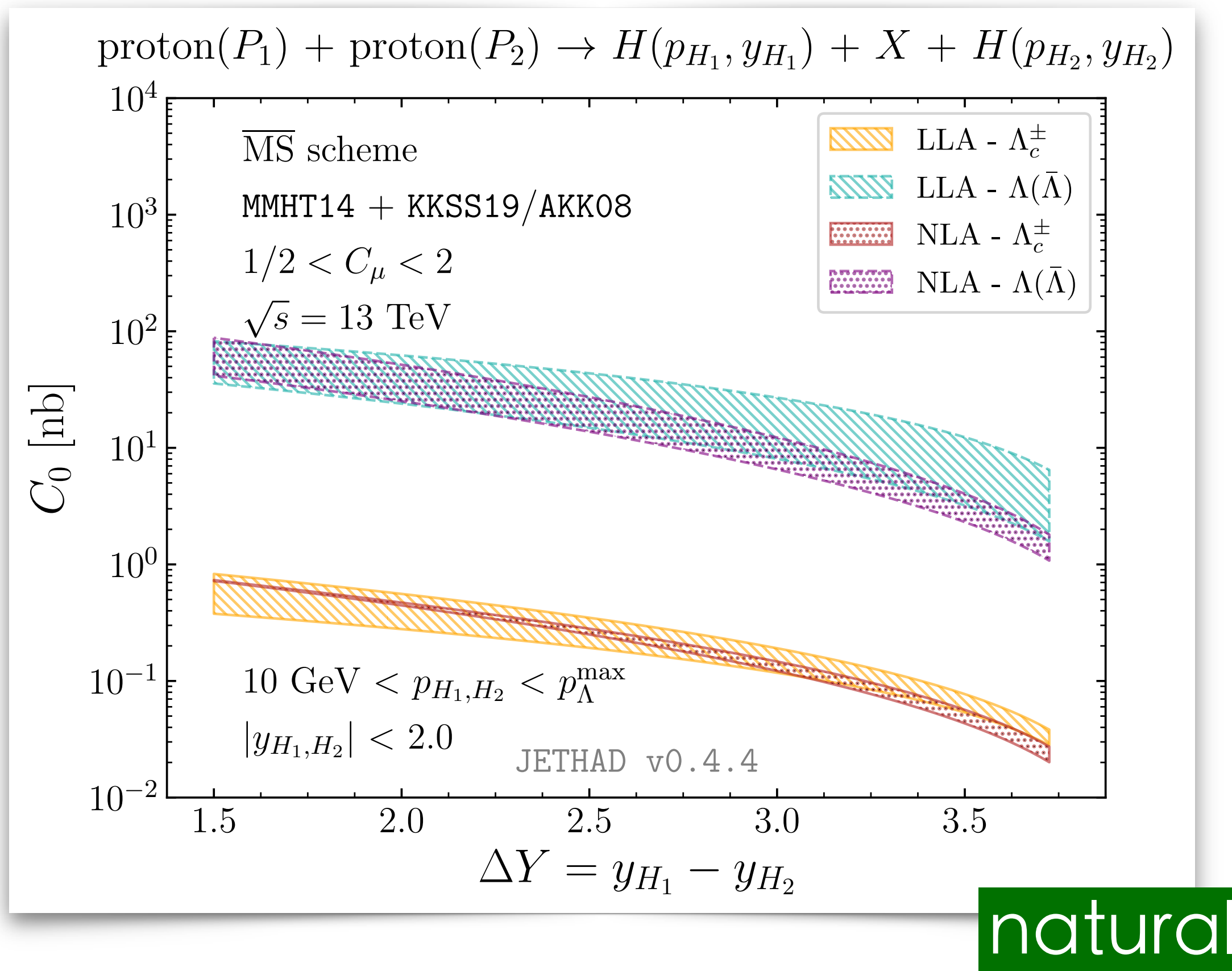
($B_c^{(*)}$ hadrons) [\[F. G. C., Phys. Lett. B 835 \(2022\) 137554\]](#)

(Λ_c baryons, in this slide) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 8, 780\]](#)

(H_b hadrons) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

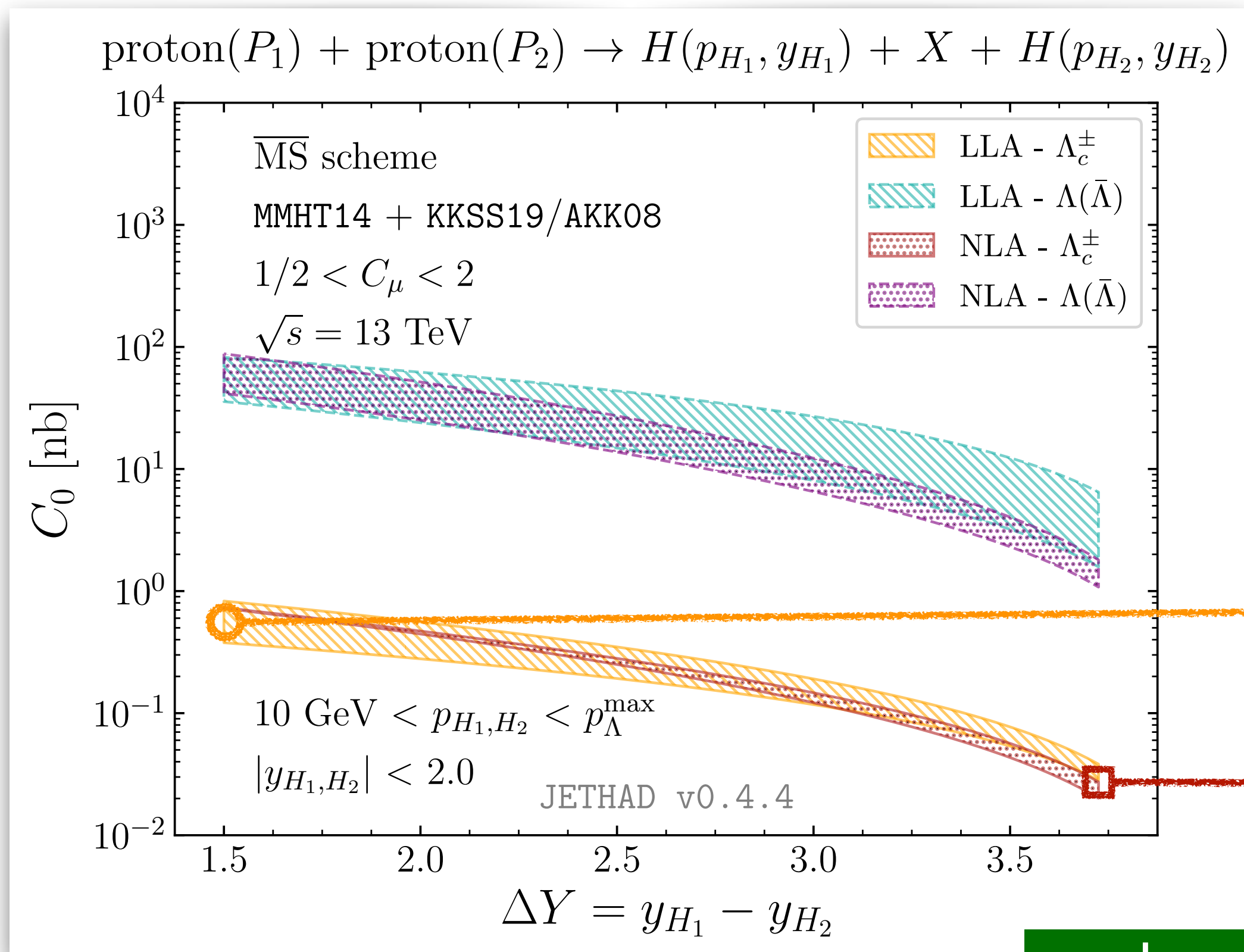
Stability under scale variations & NLL corrections

Hybrid factorization @work: Λ_c baryons $|udc\rangle$ versus Λ hyperons $|uds\rangle$



Stability under scale variations & NLL corrections

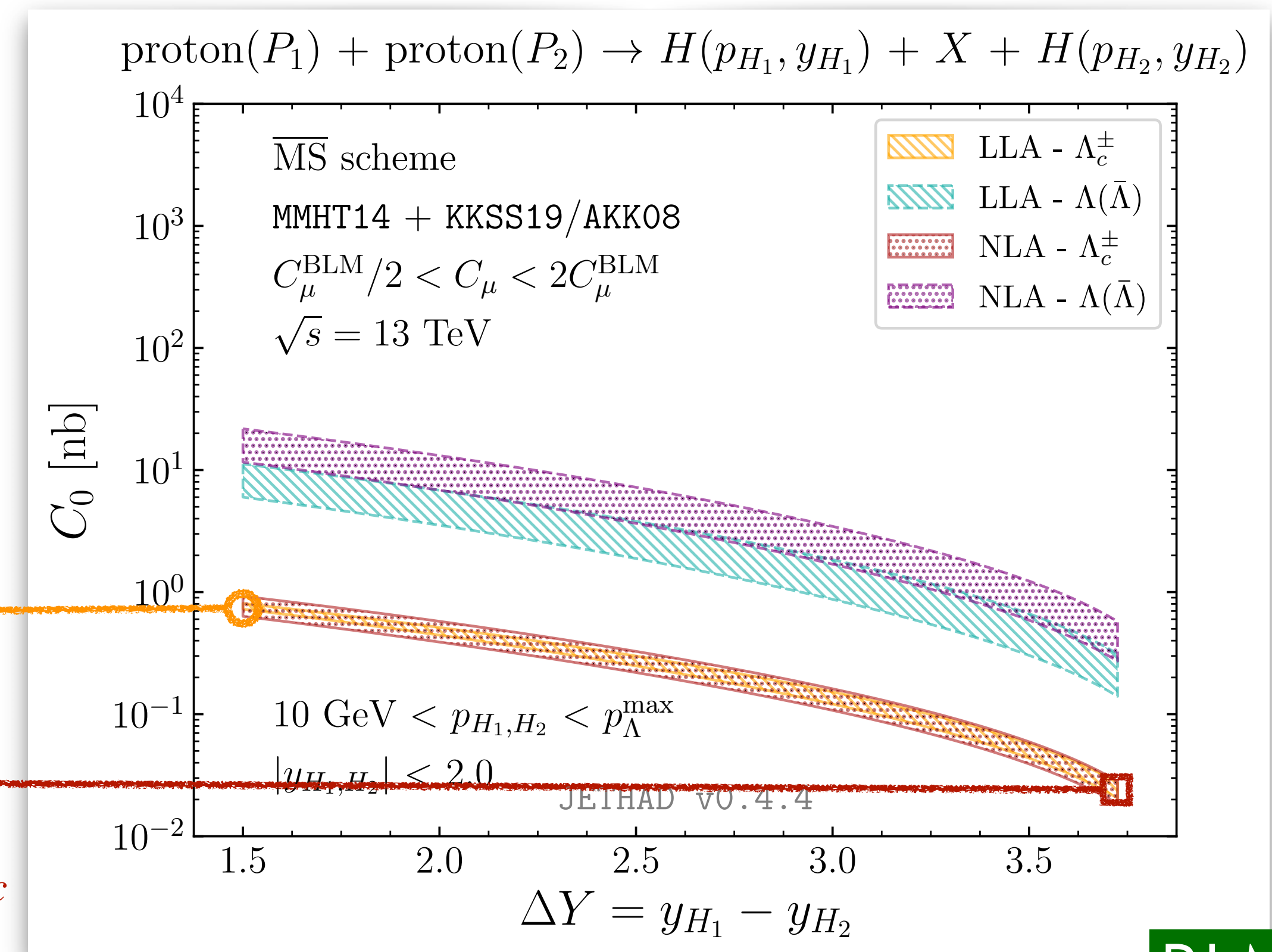
Hybrid factorization @work: Λ_c baryons $|udc\rangle$ versus Λ hyperons $|uds\rangle$



LL Λ_c

NLL Λ_c

natural



BLM

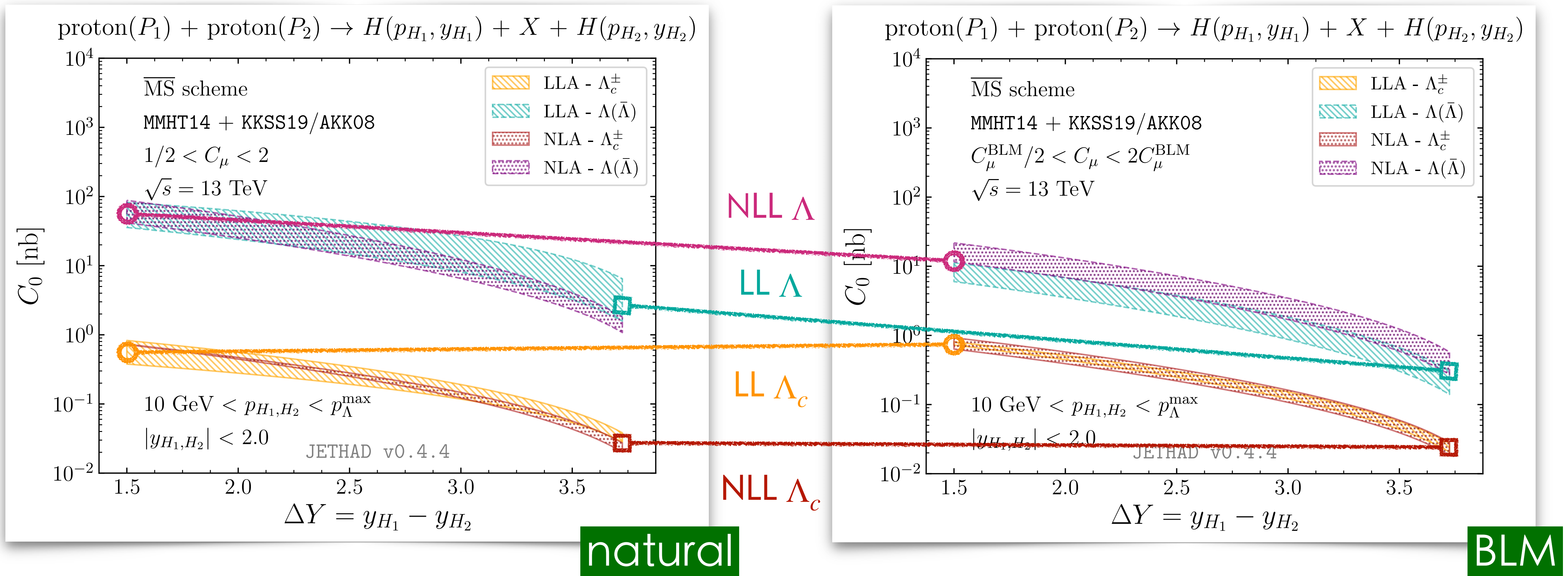
NLL corrections: rapidity distribution **stable** for Λ_c

(Λ_c baryons, in this slide) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 8, 780\]](#)

(H_b hadrons) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

Stability under scale variations & NLL corrections

Hybrid factorization @work: Λ_c baryons $|udc\rangle$ versus Λ hyperons $|uds\rangle$

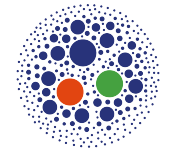


NLL corrections: rapidity distribution **stable** for Λ_c , loses $\sim 10^1$ magnitude for Λ

(Λ_c baryons, in this slide) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 8, 780\]](#)

(H_b hadrons) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

Stabilizing effects of heavy-flavor fragmentation

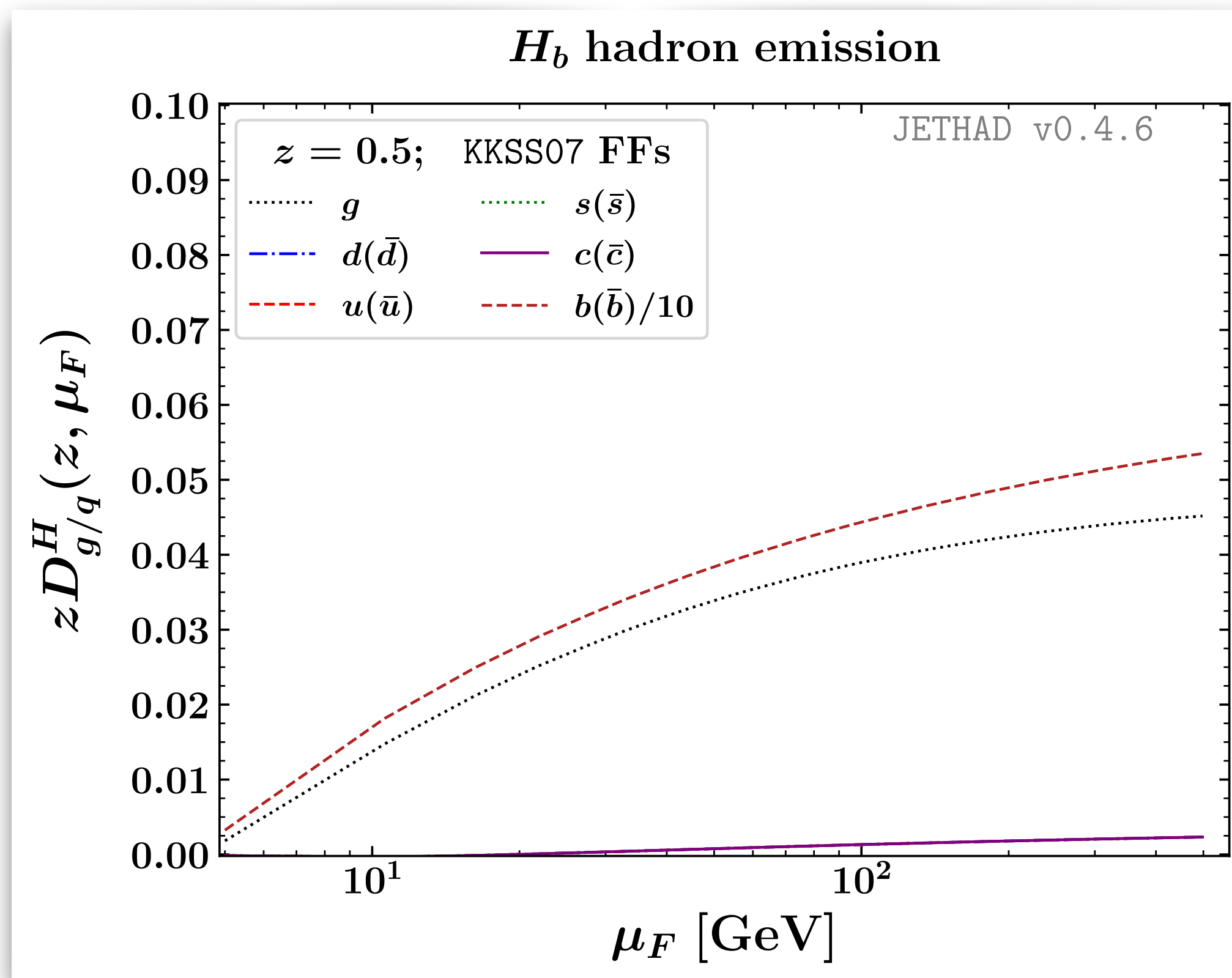


KKSS07 VFNS collinear FFs for:

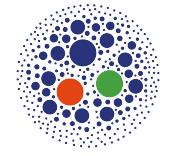
$$H_b = B^\pm, B^0, B_s^0, \Lambda_b$$

[\[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 \(2018\) 11, 114010\]](#)

[\[B. A. Kniehl et al., Phys. Rev. D 77 \(2008\) 11, 014011\]](#)



Stabilizing effects of heavy-flavor fragmentation

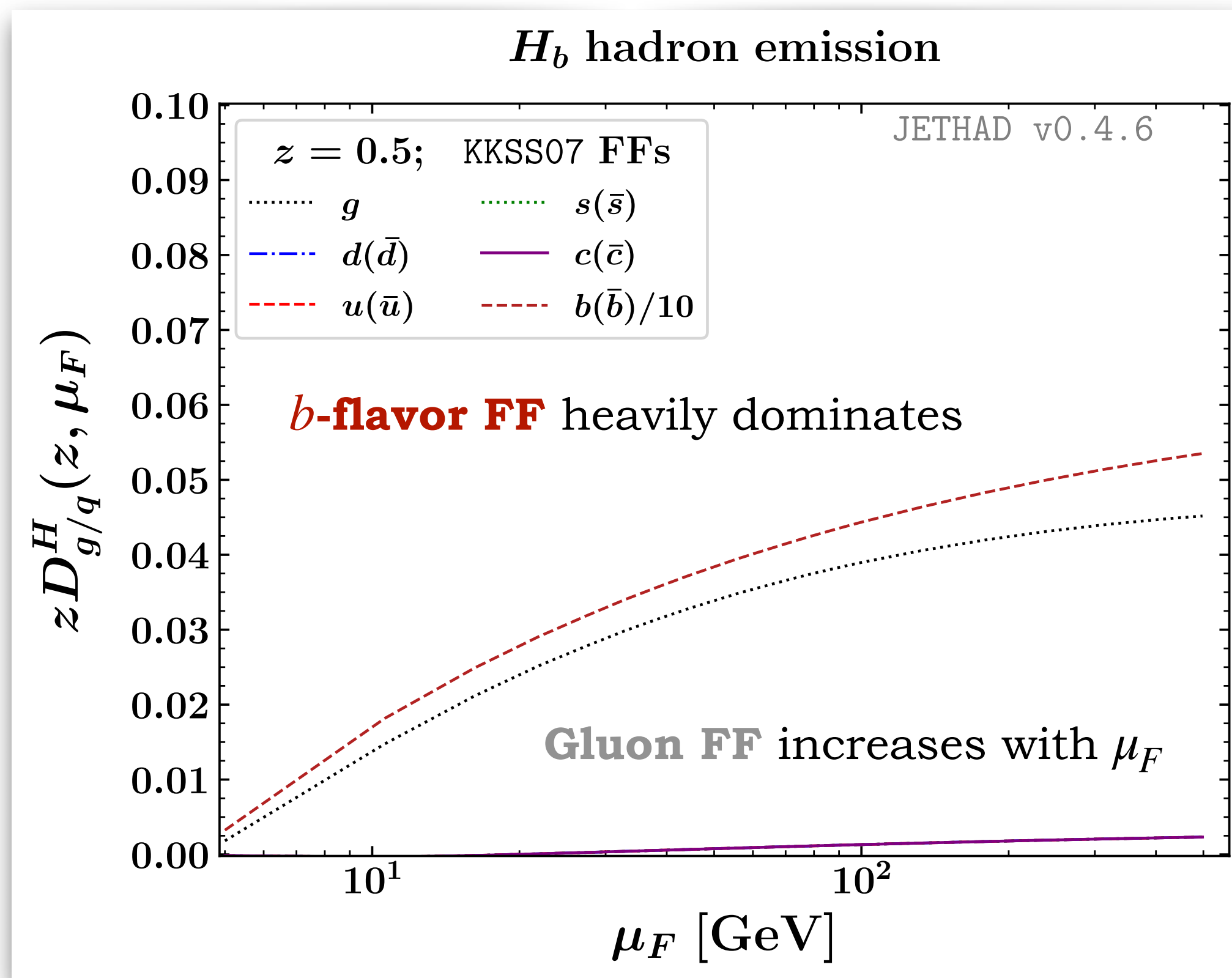


KKSS07 VFNS collinear FFs for:

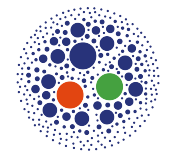
$$H_b = B^\pm, B^0, B_s^0, \Lambda_b$$

[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 (2018) 11, 114010]

[B. A. Kniehl et al., Phys. Rev. D 77 (2008) 11, 014011]



Stabilizing effects of heavy-flavor fragmentation

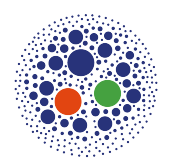
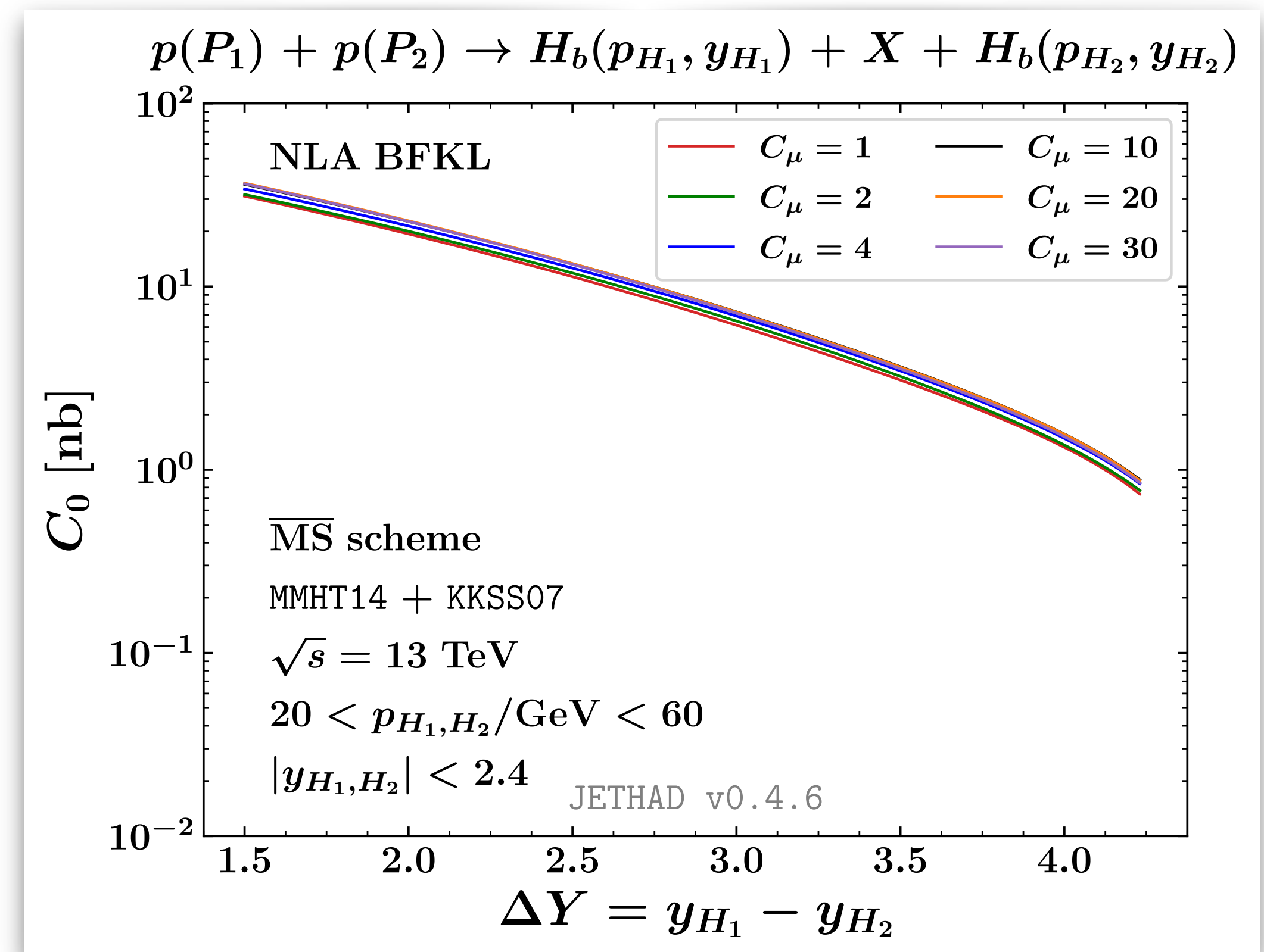
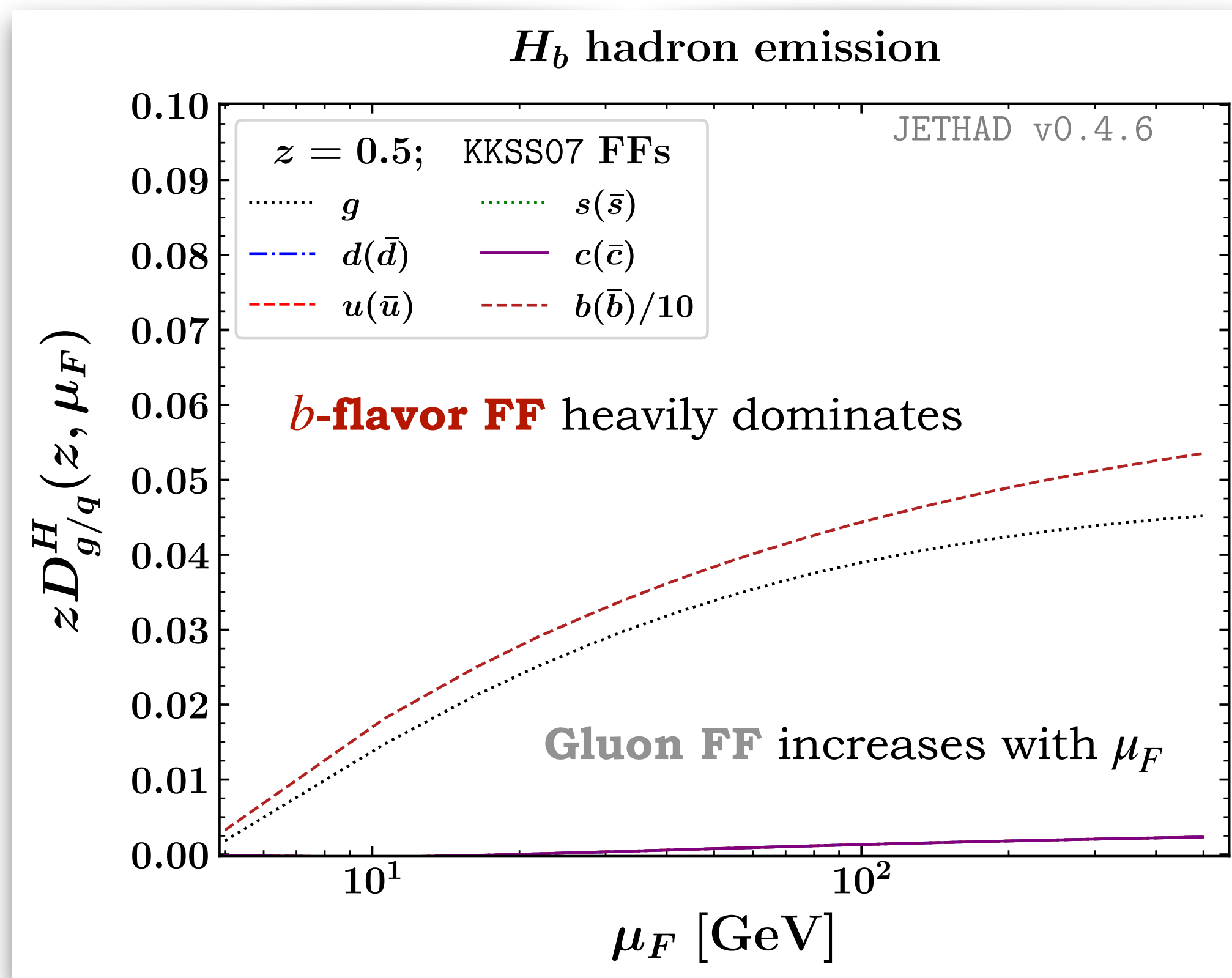


KKSS07 VFNS collinear FFs for:

$$H_b = B^\pm, B^0, B_s^0, \Lambda_b$$

[\[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 \(2018\) 11, 114010\]](#)

[\[B. A. Kniehl et al., Phys. Rev. D 77 \(2008\) 11, 014011\]](#)



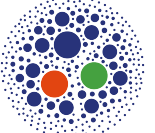
Rapidity distribution **very stable** under scale variations

(H_b hadrons, in this slide) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

(Λ_c baryons) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 8, 780\]](#)

Stabilizing effects of heavy-flavor fragmentation

 Stabilization mechanism encoded in the heavy-flavor **gluon FF**

 Forward-hadron LO impact factor $\Rightarrow$ **gluon FF** enhanced by **gluon PDF** in collinear convolution

$$c_{\Lambda}(n, \nu, |\vec{p}|, x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}|^2)^{i\nu-1/2} \int_x^1 \frac{dz}{z} \left(\frac{z}{x}\right)^{2i\nu-1} \left[\frac{C_A}{C_F} \boxed{f_g(z)} \boxed{D_g^{\Lambda}\left(\frac{x}{z}\right)} + \sum_{a=q,\bar{q}} f_a(z) D_a^{\Lambda}\left(\frac{x}{z}\right) \right]$$

Stabilizing effects of heavy-flavor fragmentation

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Forward-hadron NLO impact factor $\Rightarrow$ a **non-diagonal heavy-flavor** channel open...

$$\begin{aligned} c_1^{(1)}(n, \nu, |\vec{k}_1|, \alpha_1) &= 2\sqrt{\frac{C_F}{C_A}}(\vec{k}_1^2)^{i\nu-\frac{1}{2}} \frac{1}{2\pi} \int_{\alpha_1}^1 \frac{dx}{x} \int_{\frac{\alpha_1}{x}}^1 \frac{d\zeta}{\zeta} \left(\frac{x\zeta}{\alpha_1}\right)^{2i\nu-1} \\ &\times \left[\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_1}{x\zeta}\right) C_{gg}(x, \zeta) + \sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_1}{x\zeta}\right) C_{qq}(x, \zeta) \right. \\ &+ \left. D_g^h\left(\frac{\alpha_1}{x\zeta}\right) \sum_{a=q,\bar{q}} f_a(x) C_{qg}(x, \zeta) + \frac{C_A}{C_F} f_g(x) \sum_{a=q,\bar{q}} D_a^h\left(\frac{\alpha_1}{x\zeta}\right) C_{gq}(x, \zeta) \right] \end{aligned}$$

...but $|C_{gg}| \sim 50 \div 10^4 |C_{gq}|$

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...but $\boxed{|C_{gg}|} \sim 50 \div 10^4 \boxed{|C_{gq}|}$

 **Gluon FF** rises with energy $\Rightarrow$ this **compensates** PDF and BFKL kernel decreasing behavior

The background is a complex, abstract illustration featuring several overlapping circular diagrams. Each diagram contains a network of yellow wavy lines, likely representing gluons, and various colored spheres (red, blue, green) with arrows, possibly representing quarks and their interactions. The overall color palette is soft, with pastel blues, greens, and yellows. A bright, multi-colored starburst or light source is visible on the left side, with rays emanating from it. The word "Quarkonia" is centered in a bold, blue, sans-serif font.

Quarkonia

Is the natural stability robust?

(1) **KKSS07** and **KKSS19** VFNS collinear FFs share the same extraction technology

⌚ *Might natural stability be related to the given FF determination(s) ?*

Is the natural stability robust?

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⚠ *Might natural stability be related to the given FF determination(s) ?*

(2) **KKSS07** and **KKSS19** VFNS collinear FFs assume no initial-scale gluon, but evolution-driven

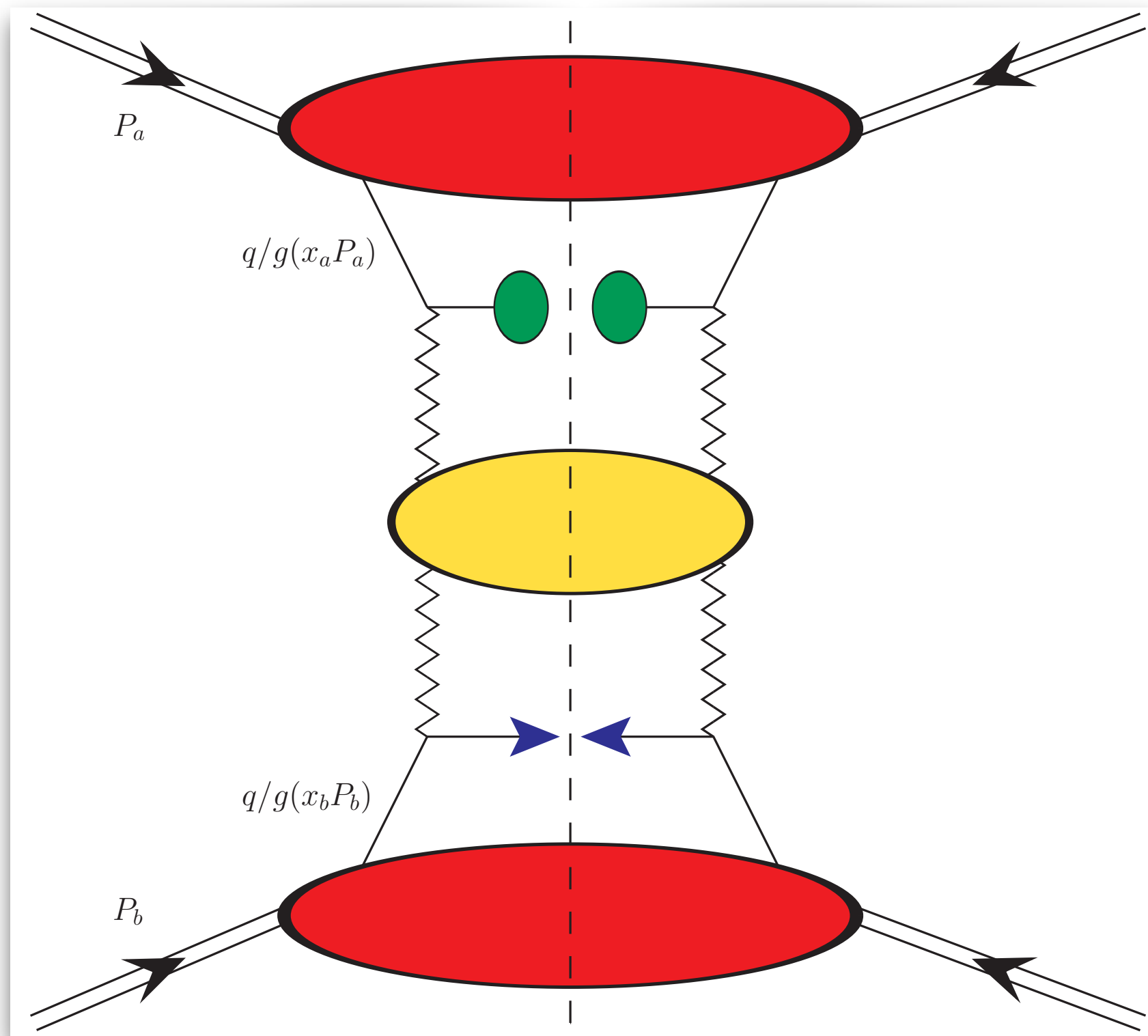
⚠ *Might natural stability be artificially generated by this Ansatz ?*

Vector quarkonium from single-parton fragmentation

- (1) *! Let us consider J/ψ and Υ at large $p_T \rightarrow$ single-parton **fragmentation** from **NRQCD** !*

Vector quarkonium from single-parton fragmentation

(1) **!** Let us consider J/ψ and Υ at large $p_T \rightarrow$ single-parton fragmentation from **NRQCD** !



2.1 High-energy resummed cross section

The process under investigation is

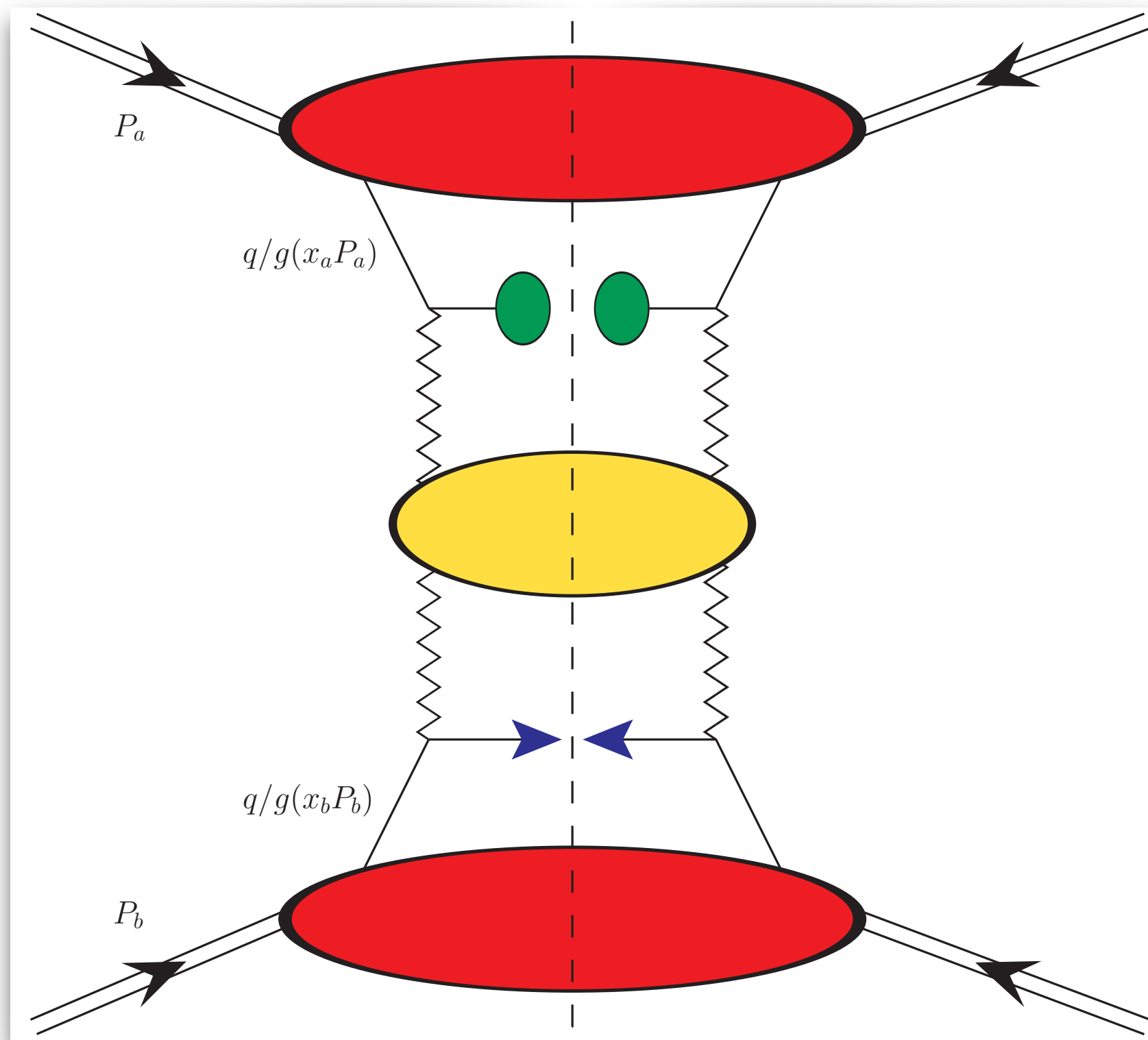
$$p(P_a) + p(P_b) \rightarrow \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J), \quad (1)$$

where $p(P_{a,b})$ stands for an initial proton with momentum $P_{a,b}$, $\mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}})$ is a J/ψ or a Υ emitted with momentum $p_{\mathcal{Q}}$ and rapidity $y_{\mathcal{Q}}$, the light jet is tagged with momentum p_J and rapidity y_J , and X denotes all the undetected products of the reaction. High observed transverse momenta, $|\vec{p}_{\mathcal{Q},J}|$, together with a large rapidity separation, $\Delta Y = y_{\mathcal{Q}} - y_J$, are required conditions to get a diffractive semi-hard configuration in the final state. Furthermore the transverse-momentum ranges need to be enough large to ensure the validity of description of the quarkonium production mechanism in terms of single-parton VFNS collinear fragmentation.

 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

Vector quarkonium from single-parton fragmentation

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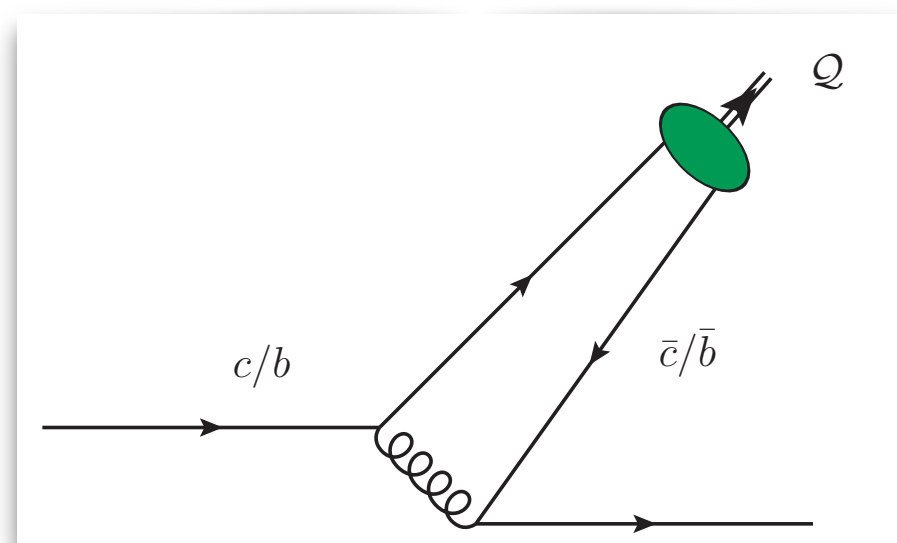
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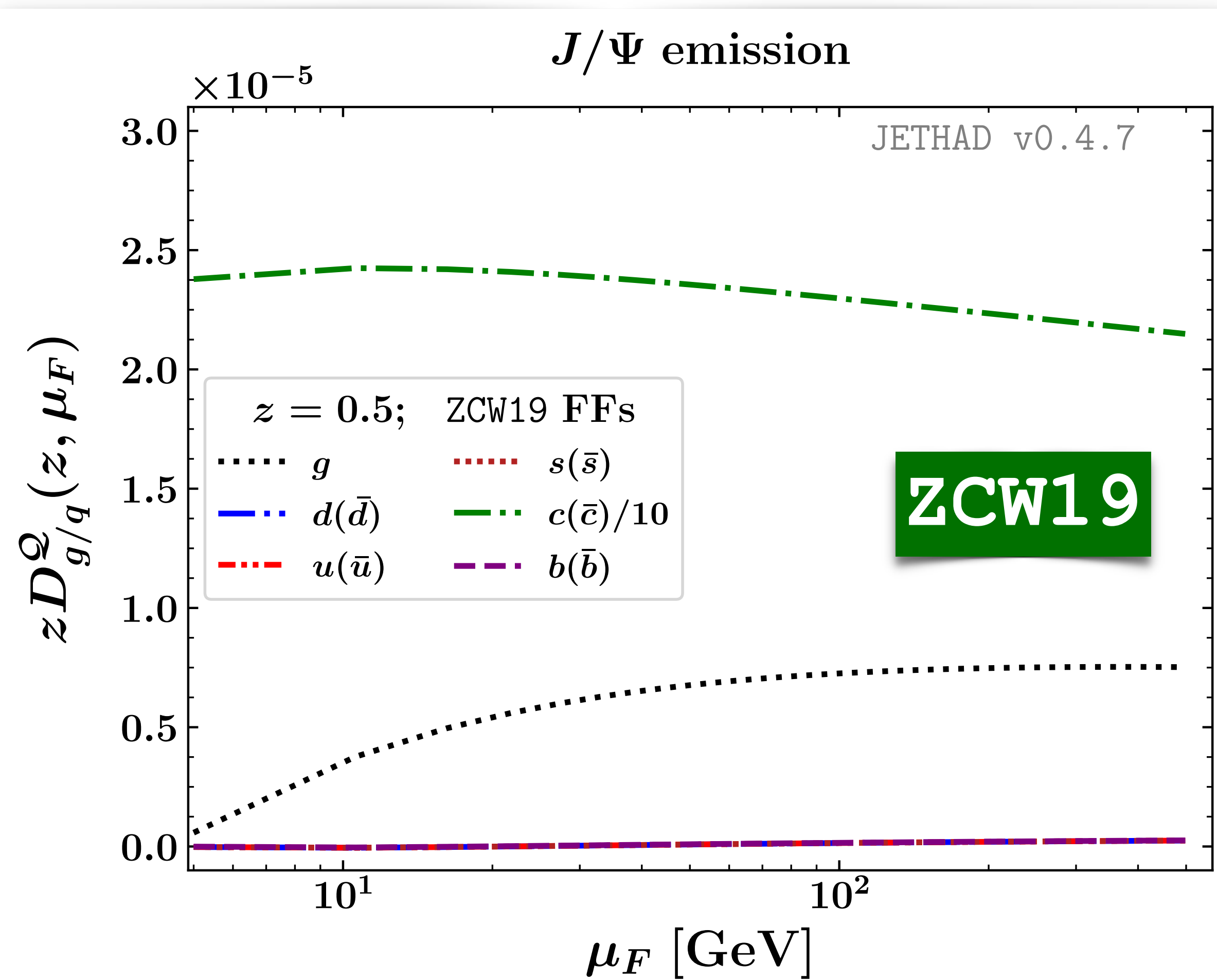


(LO) [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]

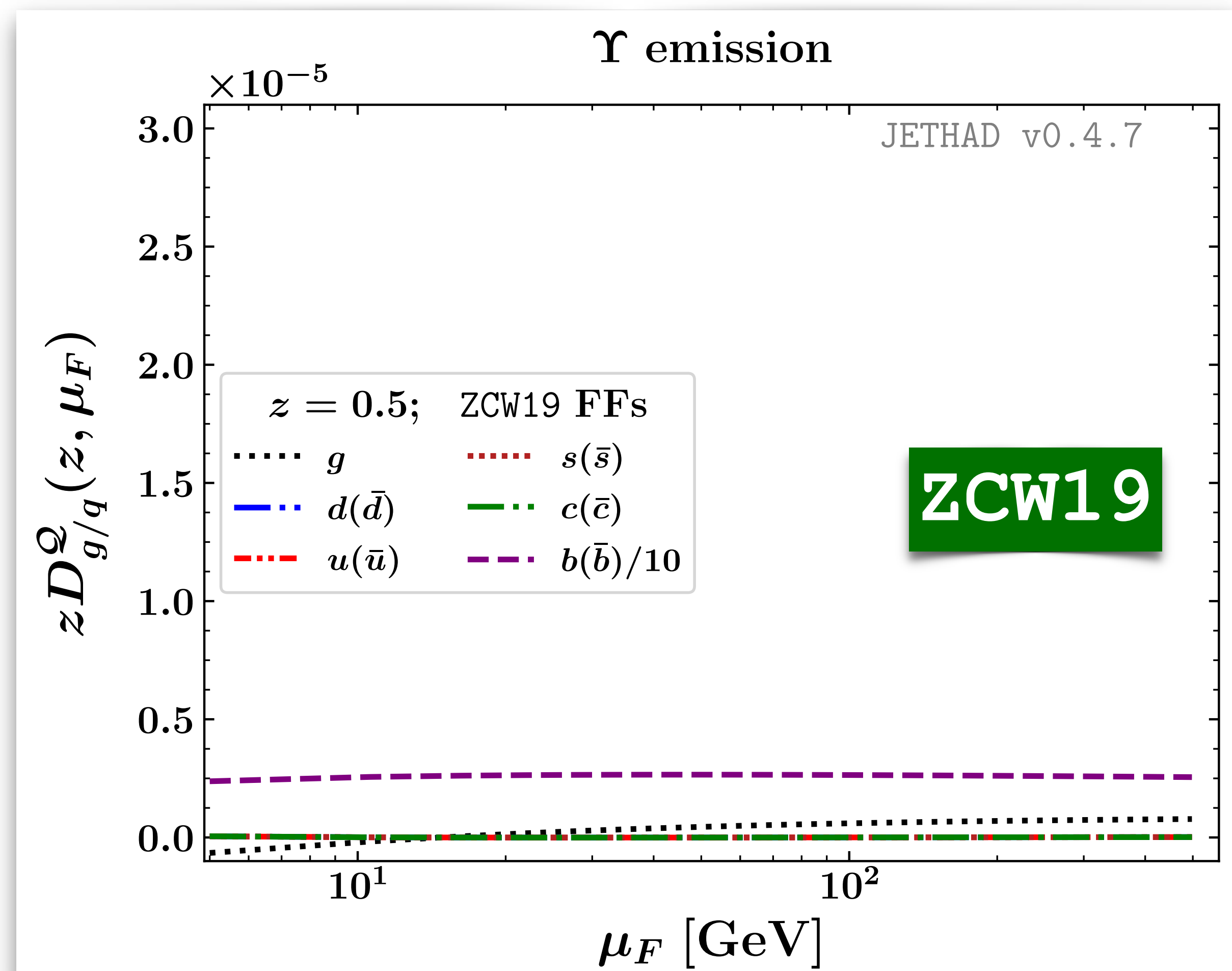
(NLO) [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

Vector quarkonium + jet at the LHC

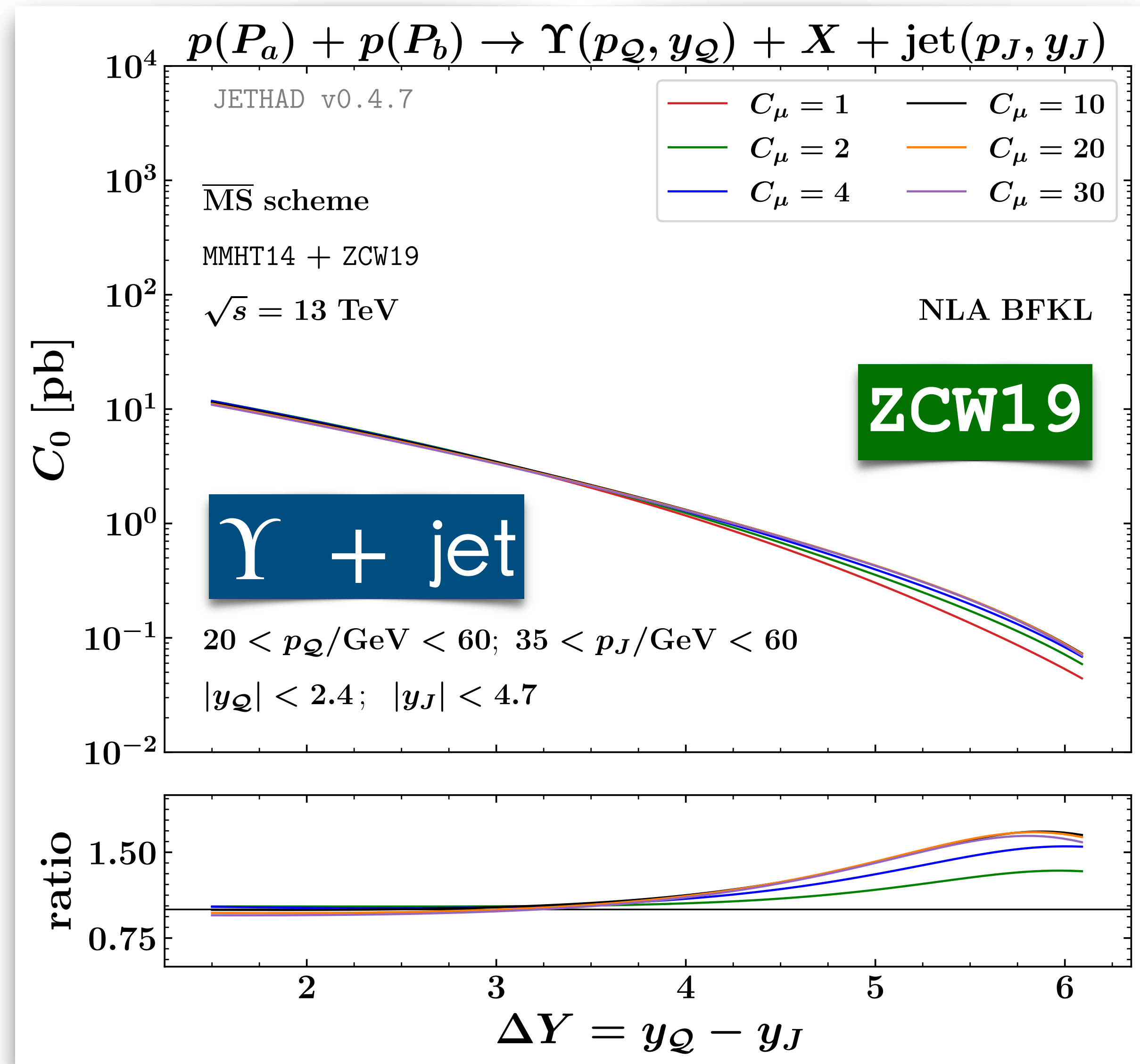
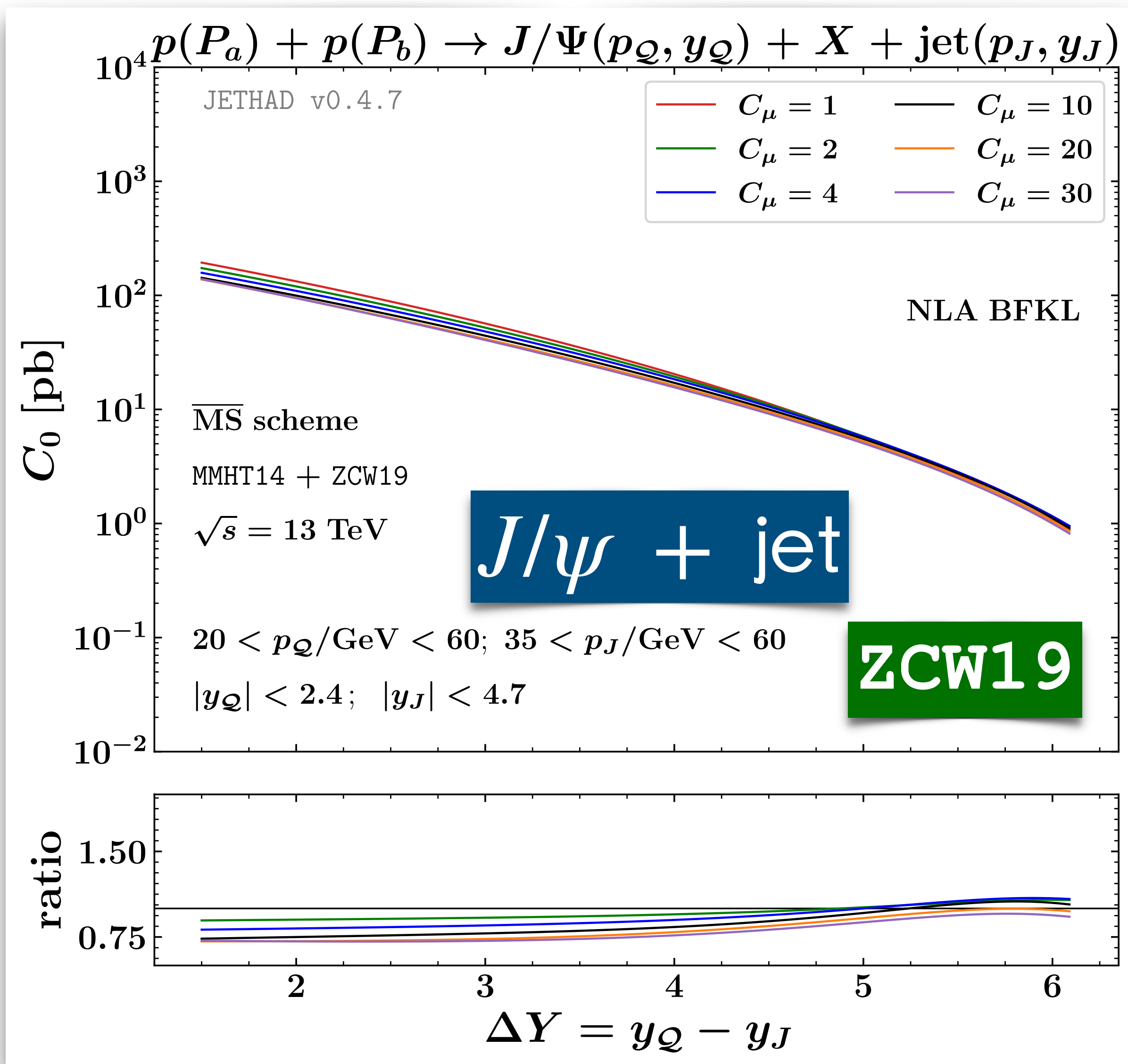
J/ψ collinear FFs



Υ collinear FFs

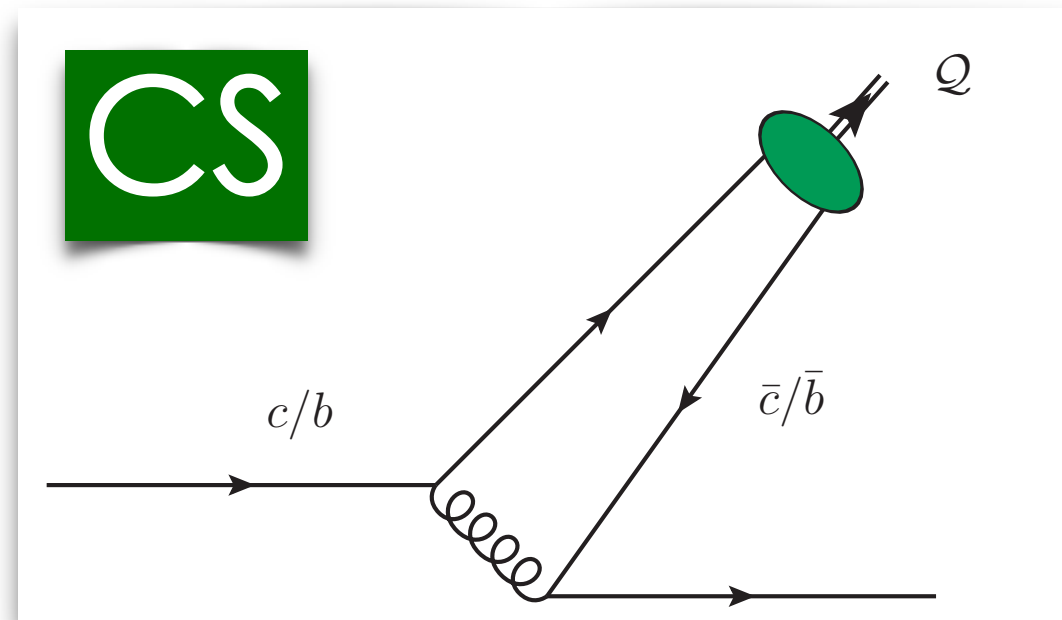


Vector quarkonium + jet at the LHC



Vector quarkonium from single-parton fragmentation

(2)  Let us consider J/ψ and Υ at large $p_T \rightarrow$ initial-scale **heavy-quark** + **gluon** from **NRQCD** !



$$D_Q^Q(z, \mu_F \equiv \mu_0) = D_Q^{Q, \text{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_Q(0)|^2 \Gamma^{Q, \text{NLO}}(z)$$

$$(Q \rightarrow Q Q) \text{ at } \mu_0 = 3m_Q$$

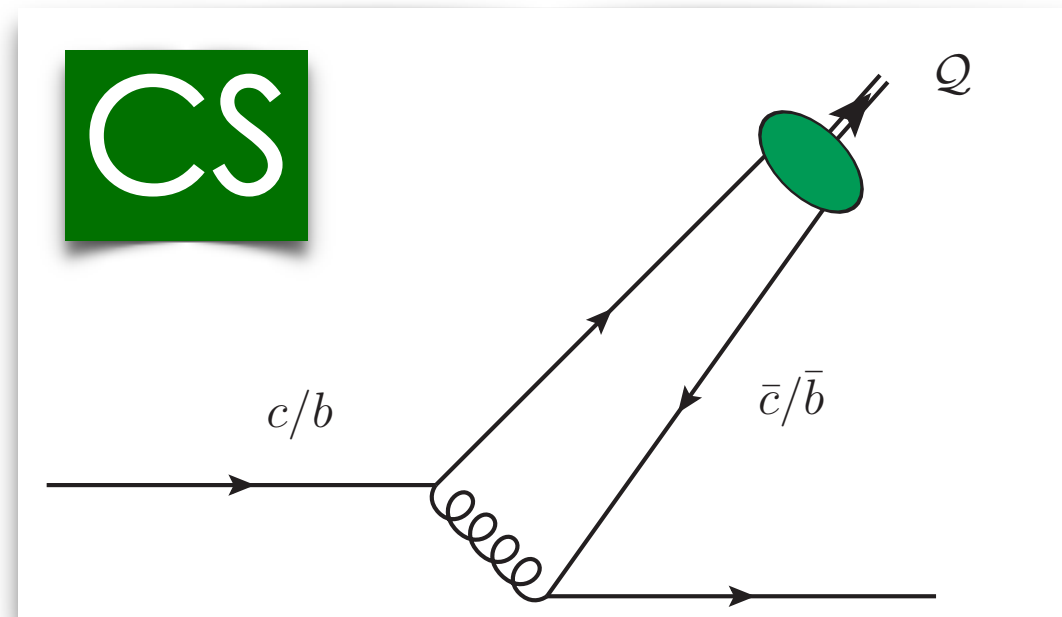
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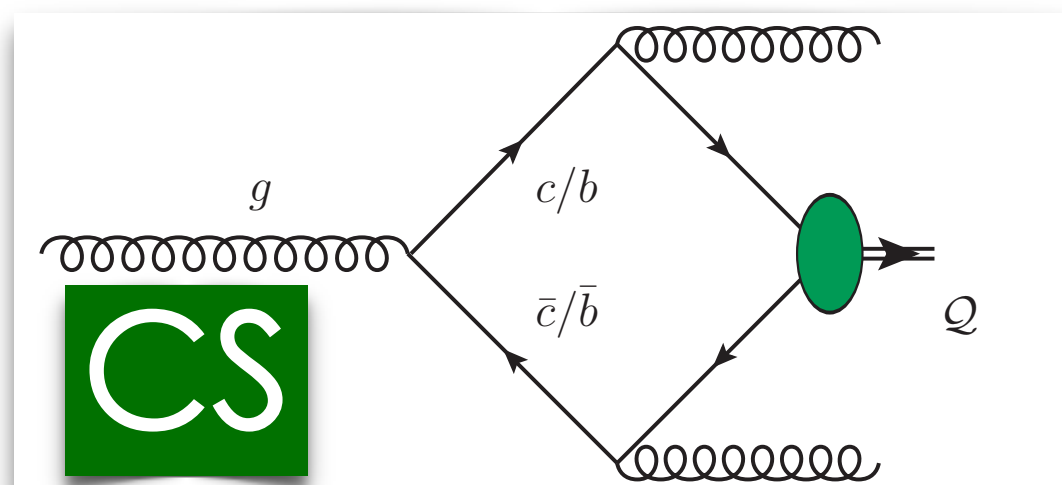
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+



$$D_g^Q(z, 2m_Q) = \frac{5}{36(2\pi)^2} \alpha_s^3(2m_Q) \frac{|\mathcal{R}_Q(0)|^2}{m_Q^3} \int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2} \\ \sum_{i=1}^2 z^i \left[f_i^{(g)}(\xi, \tau) + g_i^{(g)}(\xi, \tau) \frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln \left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}} \right) \right],$$

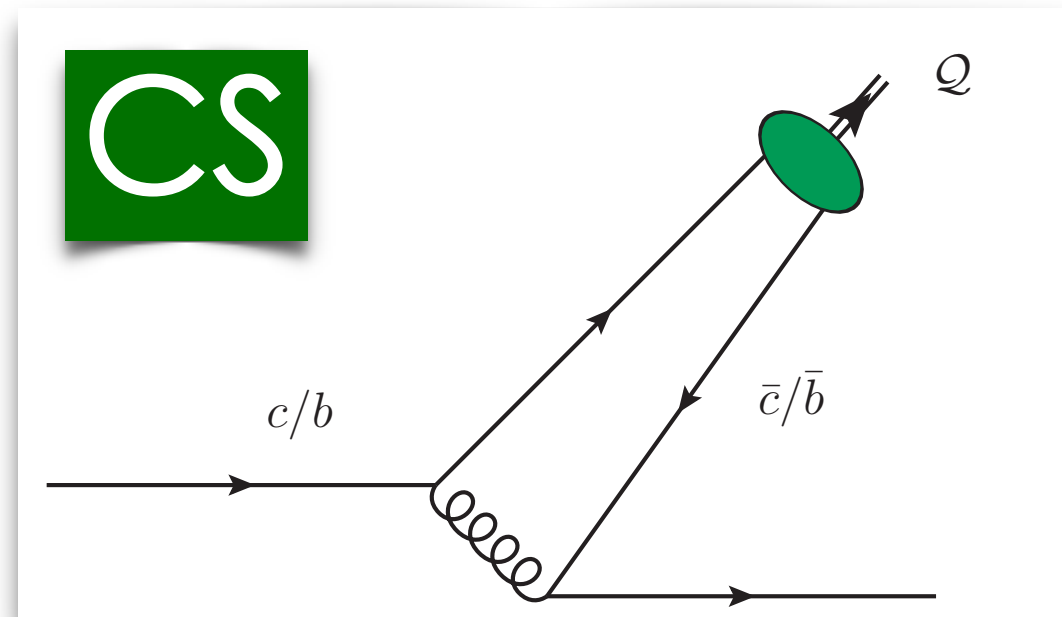
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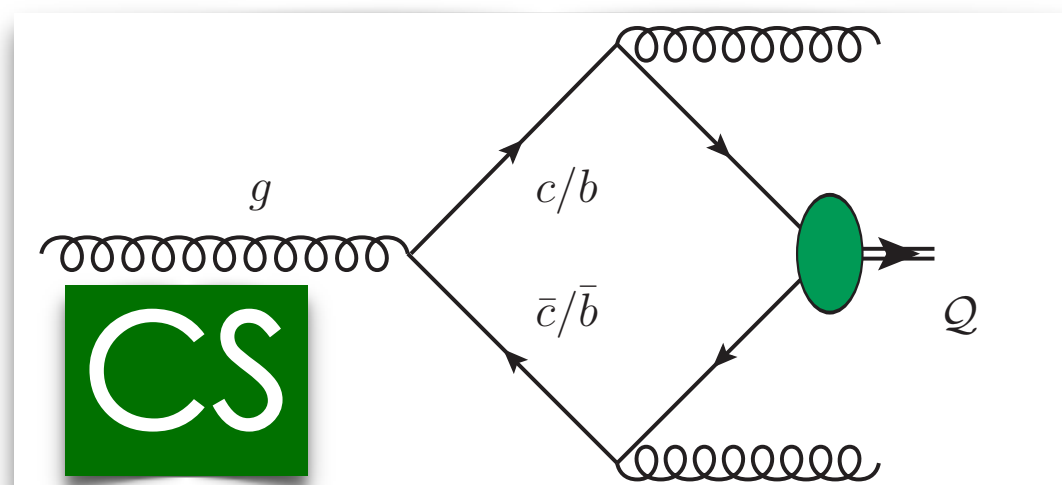
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⊗

[F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

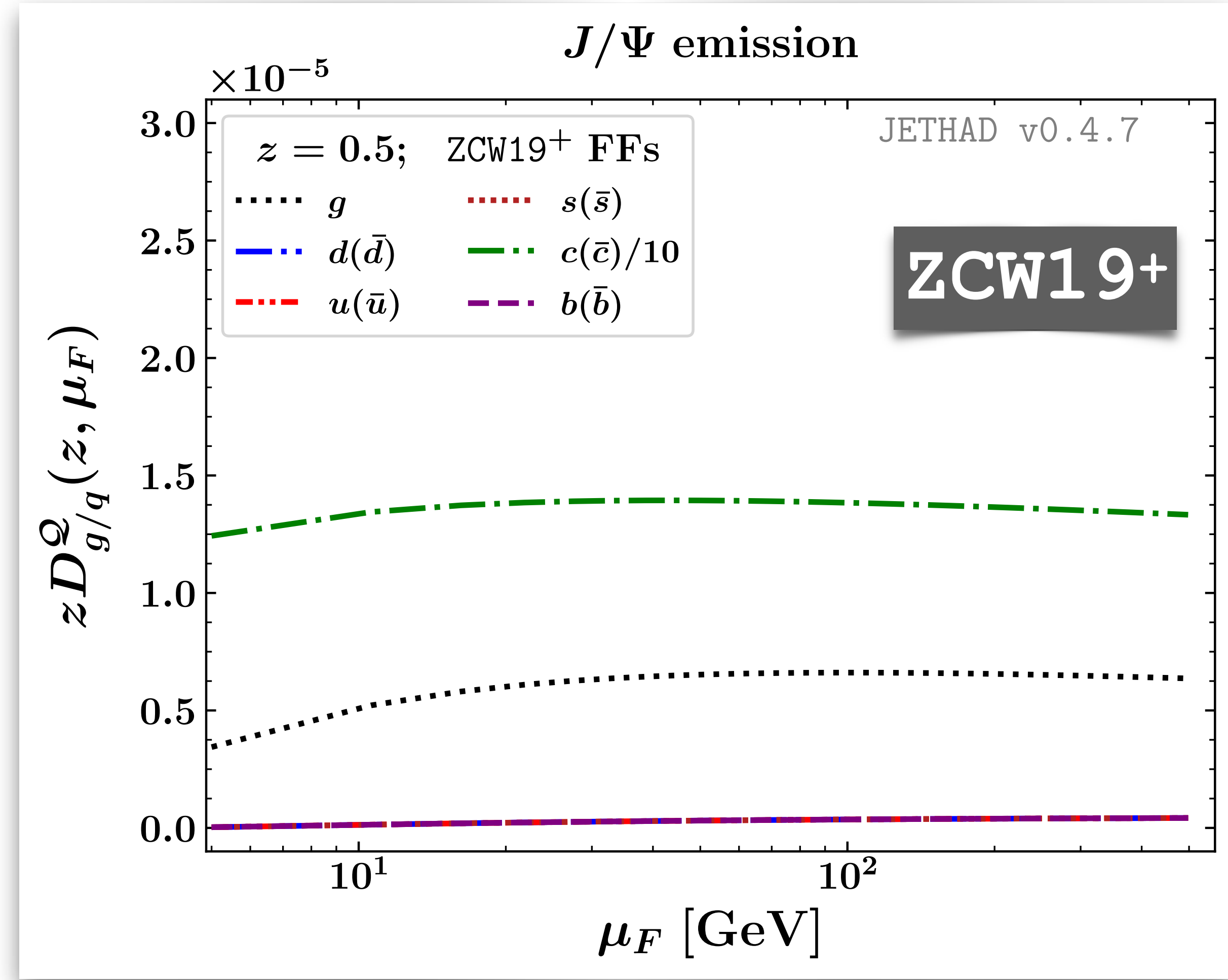
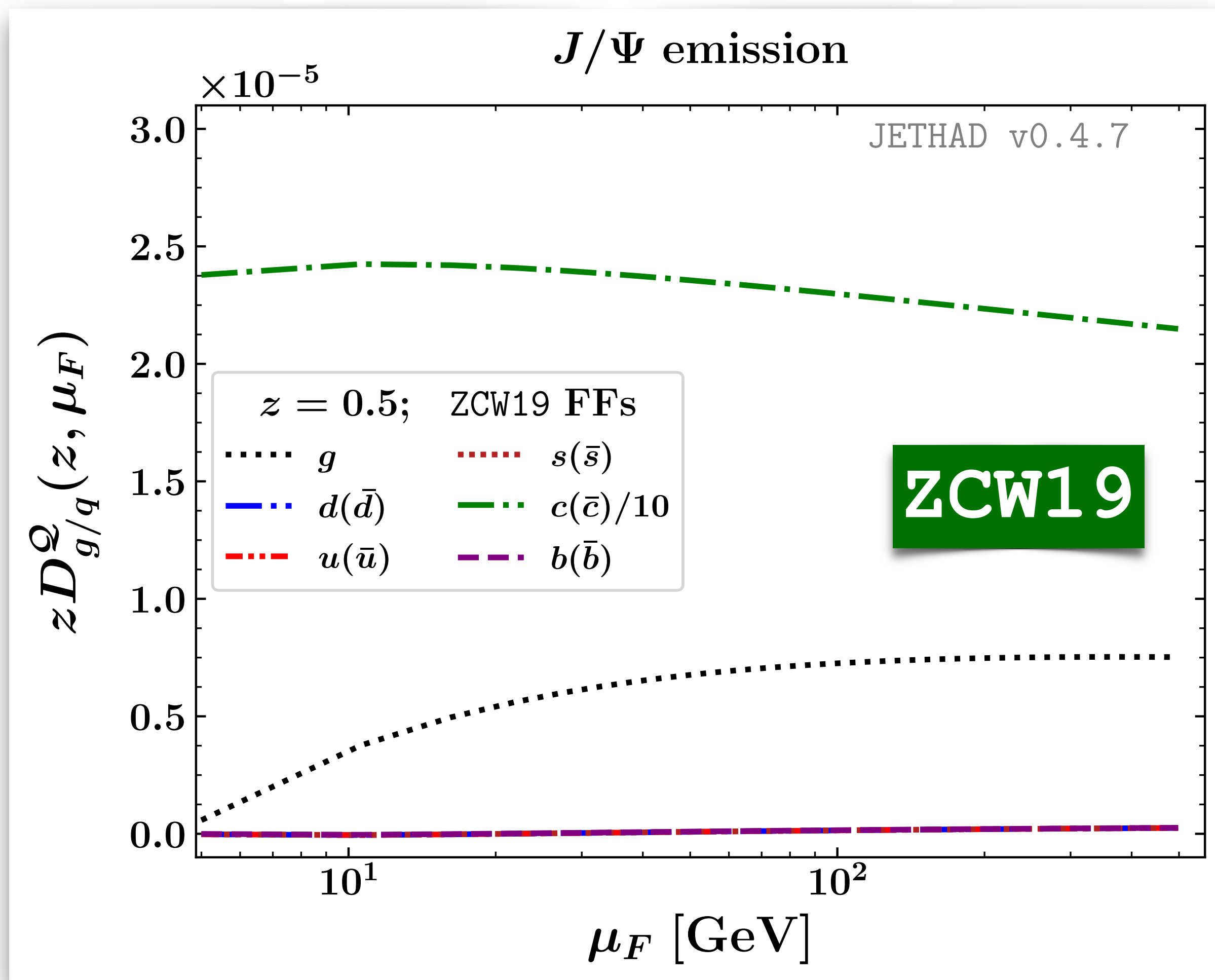
ZCW19+
onium FFs

=

APFEL++

Vector quarkonium + jet at the LHC

J/ψ collinear FFs



Basics of BFKL

The background of the slide is a vibrant, abstract composition. It features several overlapping, semi-transparent circular shapes in shades of blue, green, and purple. Within and around these circles are various particle-like symbols: red spheres with upward-pointing arrows, blue spheres with rightward-pointing arrows, and green spheres with downward-pointing arrows. These symbols are connected by thin, yellow, wavy lines that resemble particle tracks or field lines. The overall effect is a sense of dynamic energy and scientific complexity, fitting for a presentation on high-energy physics.

The high-energy resummation

- **BFKL resummation:** [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

based on $\longrightarrow$ **gluon Reggeization**

leading logarithmic approximation (LL):

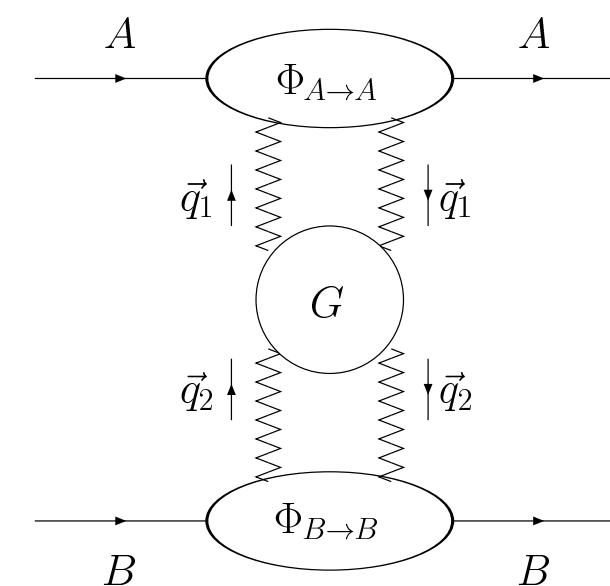
$$\alpha_s^n (\ln s)^n$$

$$\mathcal{A} = \underbrace{\text{Diagram 1}}_{\sim s} + \left(\underbrace{\text{Diagram 2}}_{\sim s (\alpha_s \ln s)} + \underbrace{\text{Diagram 3}}_{\sim s (\alpha_s \ln s)^2} + \dots \right) + \left(\dots \right) + \dots$$

next-to-leading logarithmic approximation (NLL):

$$\alpha_s^{n+1} (\ln s)^n$$

Total cross section for $A + B \rightarrow X$: $\sigma_{AB}(s) = \frac{\text{Im}_s \{ \mathcal{A}_{AB}^{AB} \}}{s} \Leftarrow$ **optical theorem**



► $\text{Im}_s \{ \mathcal{A}_{AB}^{AB} \}$ factorization:

convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLL just for few processes**

The high-energy resummation

$$\text{Im}_s (\mathcal{A}) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^2} \Phi_A(\vec{q}_1, \mathbf{s}_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^2} \Phi_B(-\vec{q}_2, \mathbf{s}_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{\mathbf{s}_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

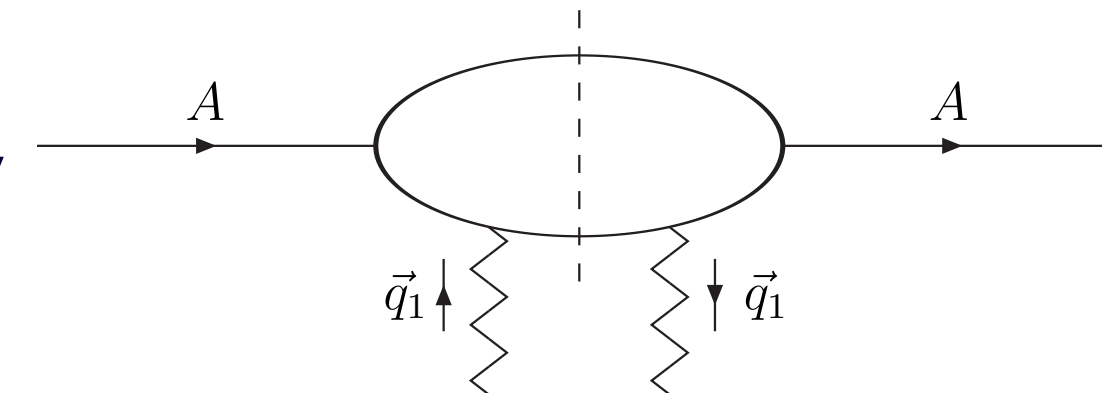
- **Green's function** is **process-independent** and takes care of the **energy dependence**

→ determined through the **BFKL equation**

[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy

→ known in the NLA just for few processes



- Successful tests of NLA BFKL in the **Mueller–Navelet** channel with the advent of the LHC; nevertheless, *new BFKL-sensitive observables* as well as *more exclusive final-state reactions* are needed (**di-hadron**, **hadron-jet**, **heavy-quark pair**, **multi-jet**, production processes,...)

(**MN jets**) [B. Ducloué, L. Szymanowski, S. Wallon (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]

(**di-hadron**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

(**four-jet**) [F. Caporale, F.G.C., G. Chachamis, A. Sabio Vera (2016)]

(**multi-jet**) [F. Caporale, F.G.C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017, 2017)]

(**heavy-quark pair**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M. Fucilla, A. Papa (2018)]

(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

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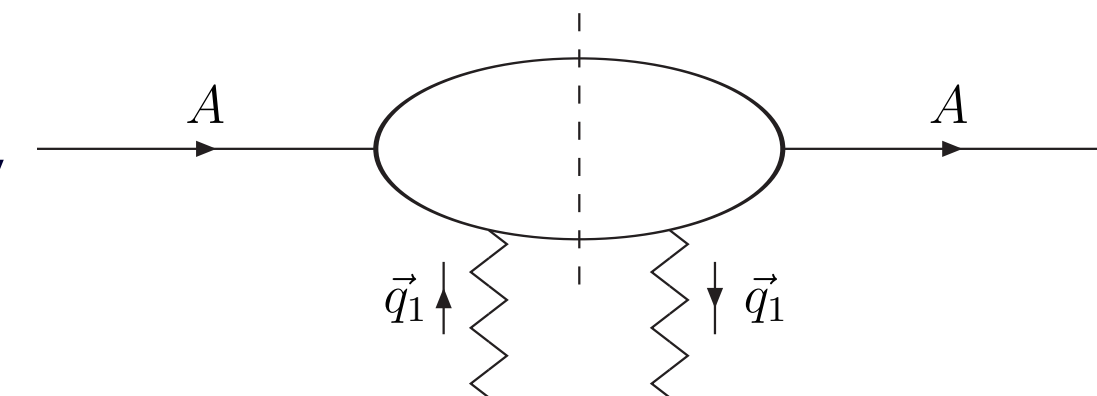
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(κ_T space) [M. Hentschinski, K. Kutak, A. Van Hameren (2021)]

(κ_T & Mellin) [F.G.C., M. Fucilla, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2022)]

NEW!
NLO HIGGS

Backup

Gluon Reggeization in perturbative QCD

◇ Gluon quantum numbers in the t -channel: 8^- representation

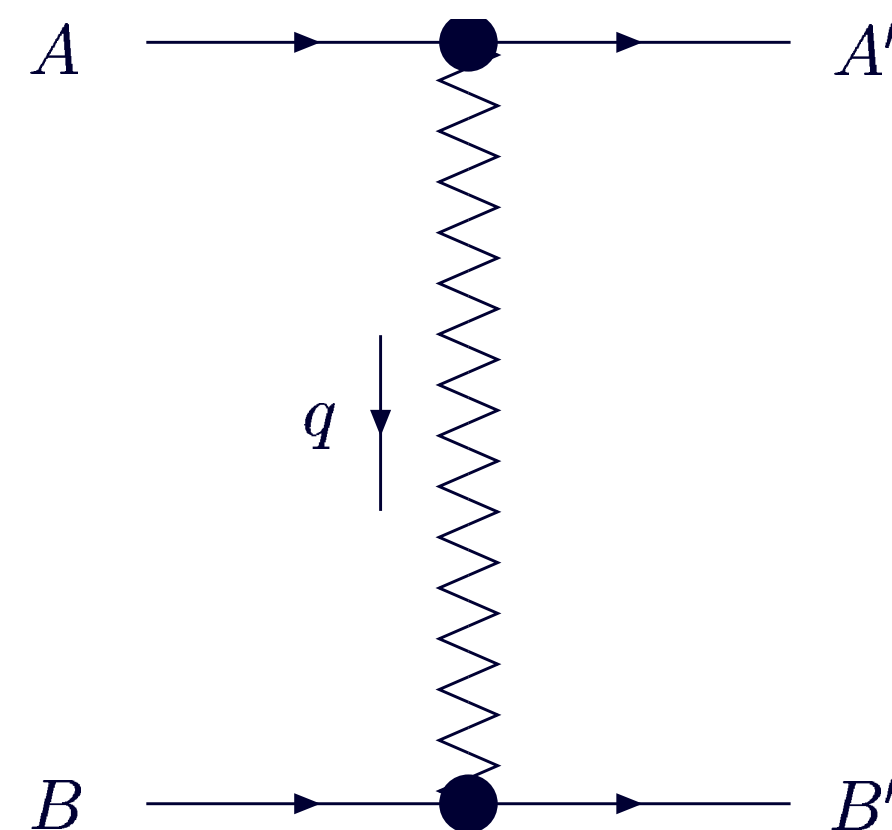
◇ Regge limit: $s \simeq -u \rightarrow \infty$, t not growing with s

→ amplitudes governed by **gluon Reggeization** $\rightarrow D_{\mu\nu} = -i \frac{g_{\mu\nu}}{q^2} \left(\frac{s}{s_0} \right)^{\alpha_g(q^2)-1}$

$\xrightarrow{\text{feature}}$ all-order resummation: **LLA** $[\alpha_s^n (\ln s)^n]$ + **NLA** $[\alpha_s^{n+1} (\ln s)^n]$

$\xrightarrow{\text{consequence}}$ factorization of elastic and real part of inelastic amplitudes

$\xrightarrow{\text{example}}$ Elastic scattering process: $A + B \rightarrow A' + B'$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t), \quad j(0) = 1$$

$\omega(t) \rightarrow$ Reggeized gluon trajectory

$$\Gamma_{A'A}^c = g \langle A' | T^c | A \rangle \Gamma_{A'A} \rightarrow \text{PPR vertex}$$

$T^c \rightarrow$ fundamental (q) or adjoint (g)

- QCD is the unique SM theory where all elementary particles reggeize
- Possible extensions: N=4 SYM, AdS/CFT,...

The high-energy resummation

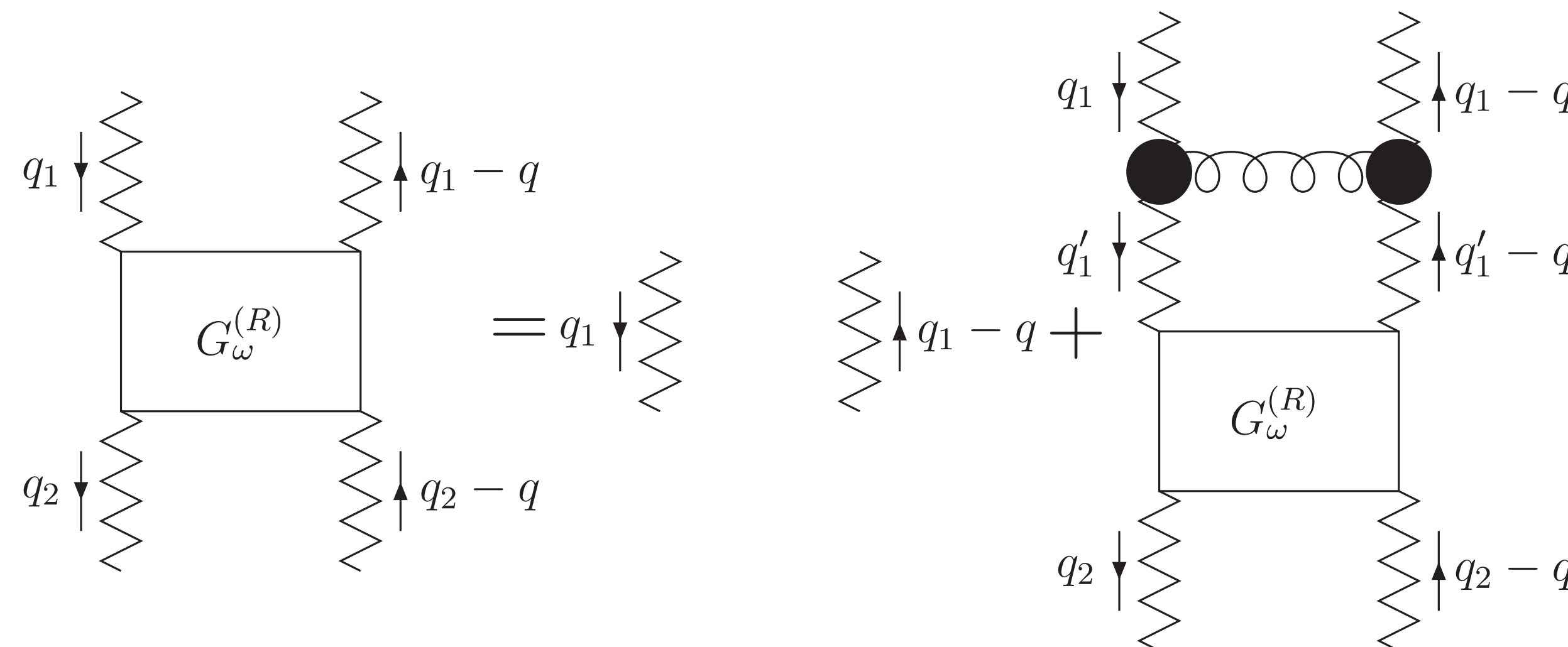
$$\text{Im}_s \{ \mathcal{A} \} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^2} \Phi_A(\vec{q}_1, \mathbf{s}_0) \int \frac{d^{D-2} q_2}{\vec{q}_2^2} \Phi_B(-\vec{q}_2, \mathbf{s}_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{\mathbf{s}_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

- **Green's function** is **process-independent** and takes care of the **energy dependence**

→ determined through the **BFKL equation**

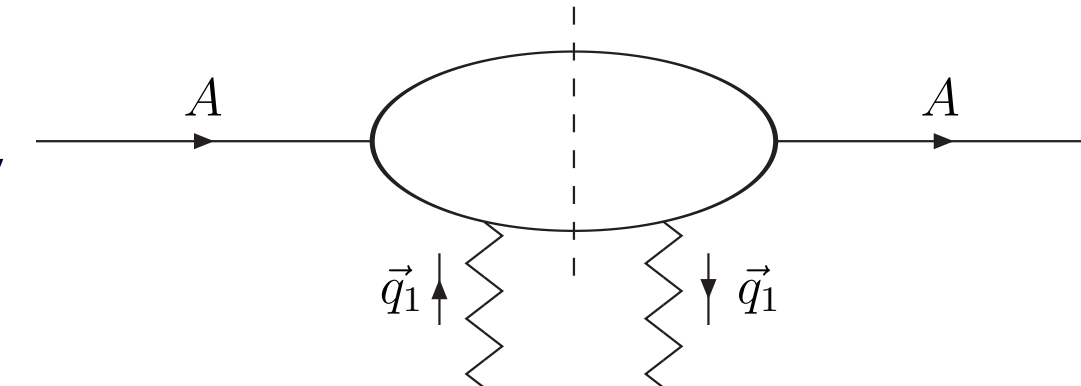
[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q K(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_1) .$$



The high-energy resummation

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy
→ known in the NLA just for few processes



- ◇ **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni, G. Rodrigo (2000)]

- ◇ $\gamma^* \longrightarrow V$, with $V = \rho^0, \omega, \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

- ◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

- ◇ forward identified hadron production

[D.Yu. Ivanov, A. Papa (2012)]

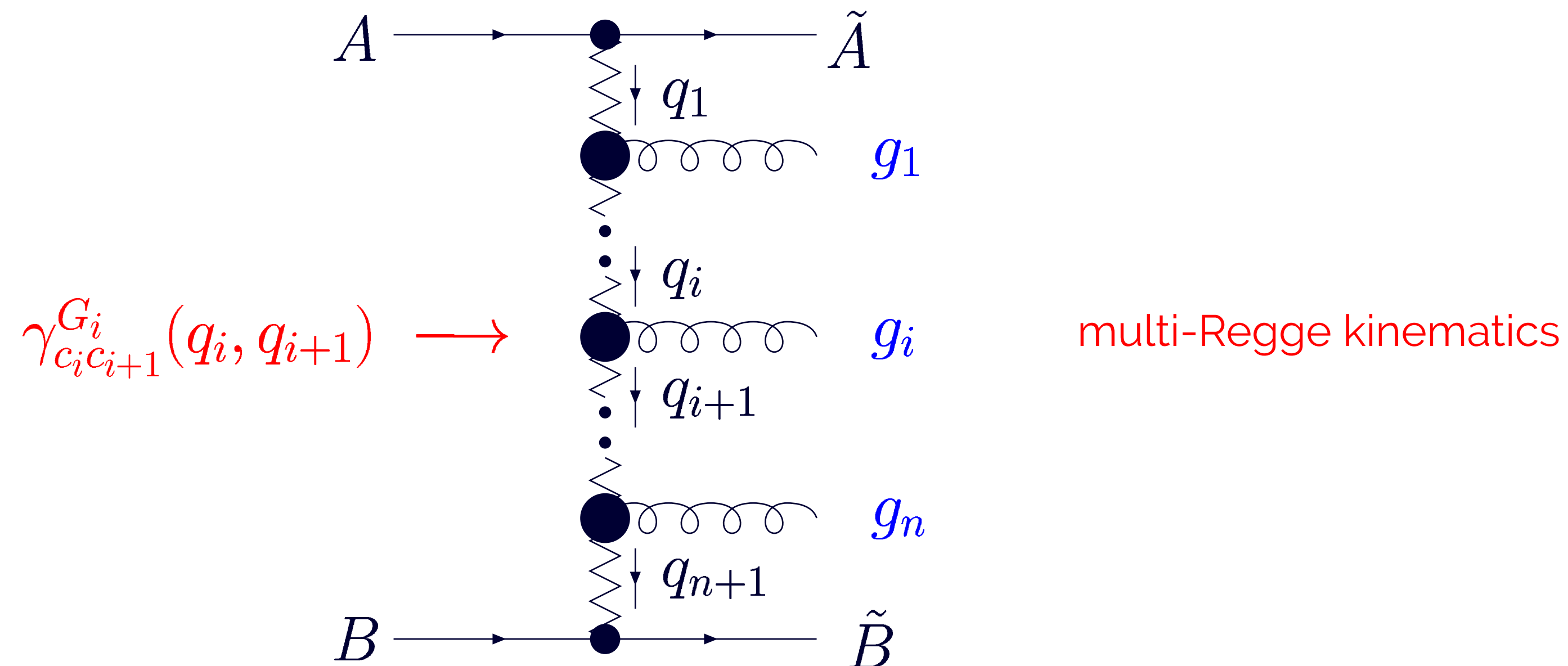
- ◇ $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

The high-energy resummation

BFKL in the LLA (I)

Inelastic scattering process $A + B \rightarrow \tilde{A} + \tilde{B} + n$ in the LLA



$$\text{Re} \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s \Gamma_{\tilde{A}A}^{c_1} \left(\prod_{i=1}^n \gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1}) \left(\frac{s_i}{s_R} \right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_R} \right)^{\omega(t_{n+1})} \Gamma_{\tilde{B}B}^{c_{n+1}}$$

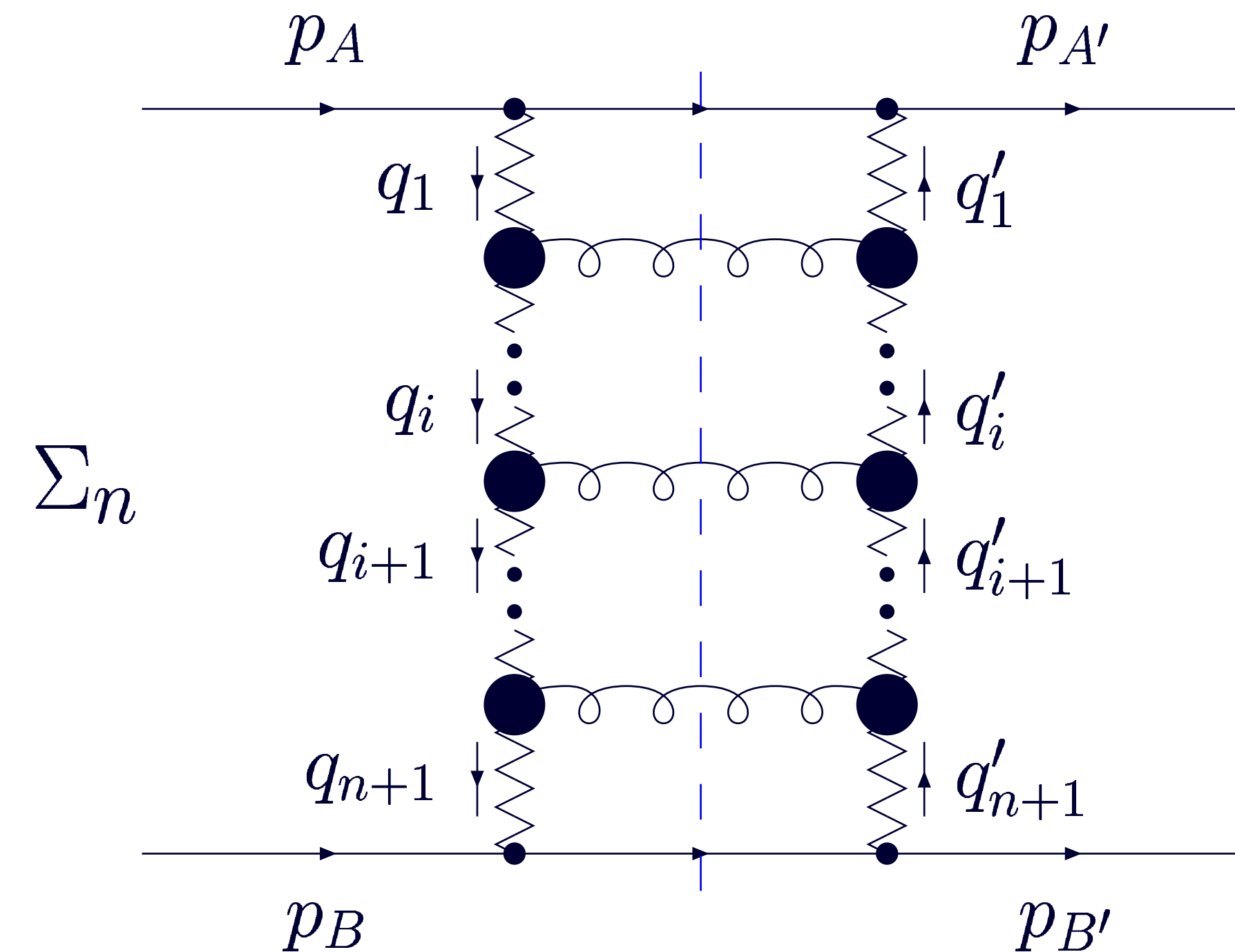
$\gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1}) \rightarrow$ RRG vertex

$s_R \rightarrow$ energy scale, irrelevant in the LLA

The high-energy resummation

BFKL in the LLA (II)

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s -channel unitarity



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})^{A'B'}_{AB}, \quad \mathcal{R} = 1 \text{ (singlet)}, 8^- \text{ (octet)}, \dots$$

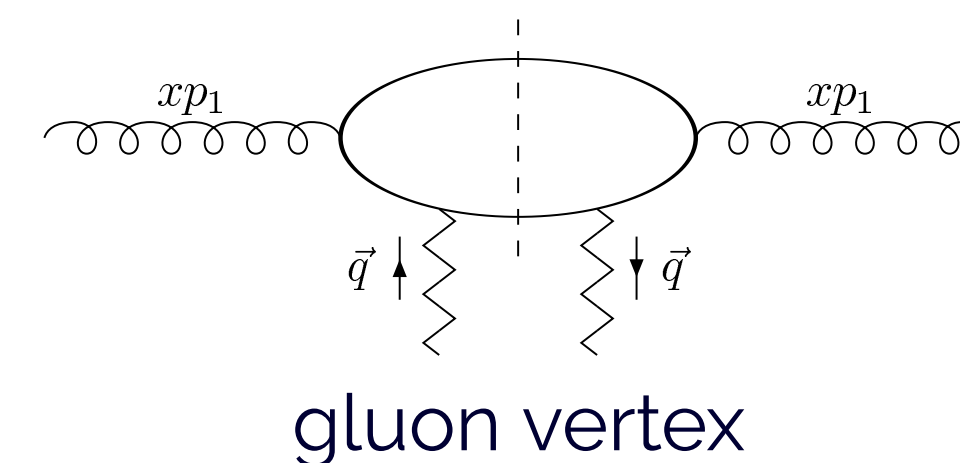
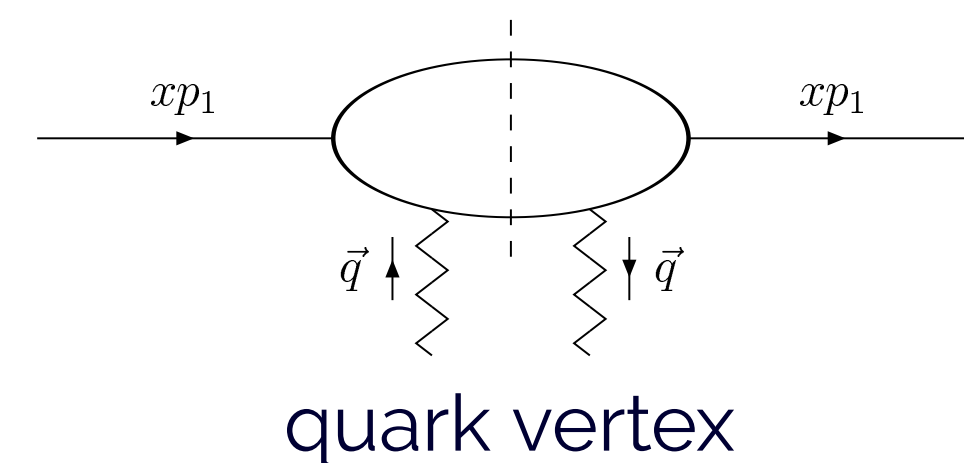
The 8^- color representation is important for the **bootstrap**, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

Forward-jet impact factor

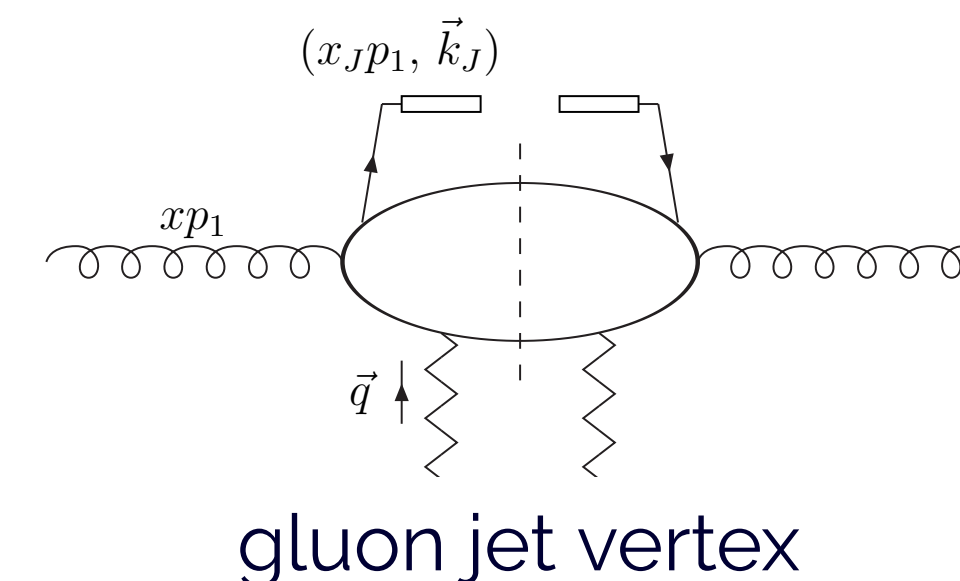
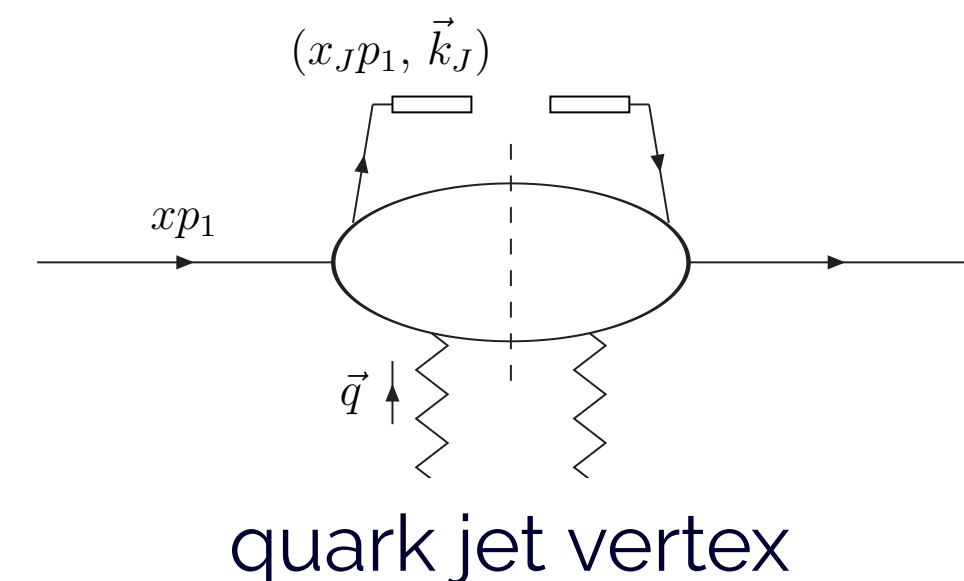
- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]

[M. Ciafaloni and G. Rodrigo (2000)]

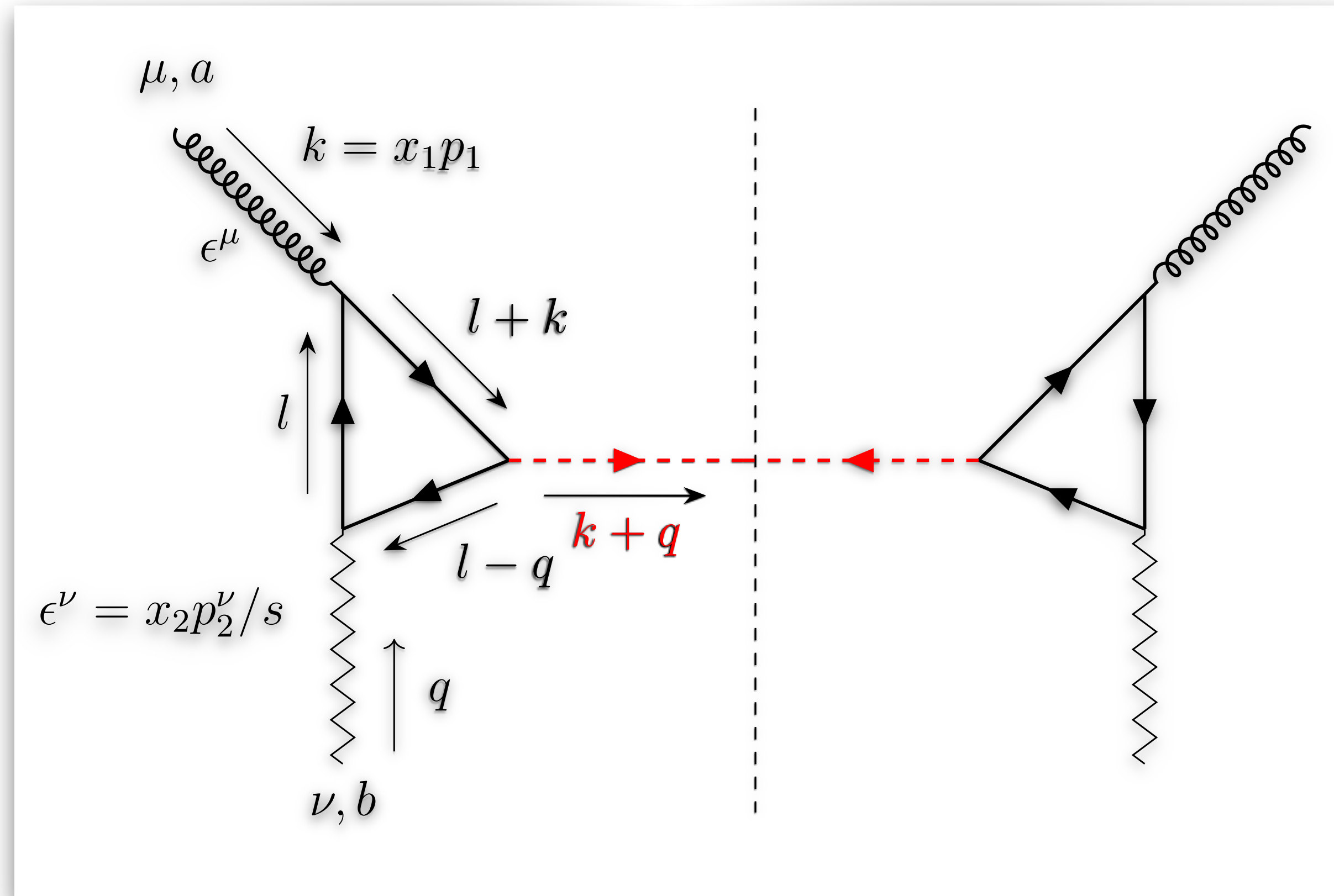


- “open” one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes [\text{quark vertex}] + f_g \otimes [\text{gluon vertex}]$

Forward-Higgs LO impact factor



$$\frac{d\Phi_J^{(0)}(\nu, n)}{dx_J d^2\vec{p}_J} = 2\alpha_s \sqrt{\frac{C_F}{C_A}} (\vec{p}_J^2)^{i\nu-3/2} \left(\frac{C_A}{C_F} f_g(x_J) + \sum_{a=q\bar{q}} f_a(x_J) \right) e^{in\phi_J}$$

Forward-Higgs NLO-RG impact factor

$$\begin{aligned}\tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H) = c_H(n, \nu, |\vec{p}_H|, x_H) & \left\{ \frac{\beta_0}{4N_c} \left(2 \ln \frac{\mu_{R_1}}{|\vec{p}_H|} + \frac{5}{3} \right) + \frac{\chi(n, \nu)}{2} \ln \left(\frac{s_0}{M_{H,\perp}^2} \right) \right. \\ & + \frac{\beta_0}{4N_c} \left(2 \ln \frac{\mu_{R_1}}{M_{H,\perp}} \right) \\ & \left. - \frac{1}{2N_c f_g(x_H, \mu_{F_1})} \ln \frac{\mu_{F_1}^2}{M_{H,\perp}^2} \int_{x_H}^1 \frac{dz}{z} \left[P_{gg}(z) f_g \left(\frac{x_H}{z}, \mu_{F_1} \right) + \sum_{a=q, \bar{q}} P_{ga}(z) f_a \left(\frac{x_H}{z}, \mu_{F_1} \right) \right] \right\}\end{aligned}$$

Forward-jet NLO-RG impact factor

$$\begin{aligned}
 \tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J) = & c_J(n, \nu, |\vec{p}_J|, x_J) \left\{ \frac{\beta_0}{4N_c} \left(2 \ln \frac{\mu_{R_2}}{|\vec{p}_J|} + \frac{5}{3} \right) + \frac{\chi(n, \nu)}{2} \ln \left(\frac{s_0}{|\vec{p}_J|^2} \right) \right. \\
 & - \frac{1}{2N_c \left(\frac{C_A}{C_F} f_g(x_J, \mu_{F_2}) + \sum_{a=q, \bar{q}} f_a(x_J, \mu_{F_2}) \right)} \ln \frac{\mu_{F_2}^2}{|\vec{p}_J|^2} \\
 & \times \left(\frac{C_A}{C_F} \int_{x_J}^1 \frac{dz}{z} \left[P_{gg}(z) f_g \left(\frac{x_J}{z}, \mu_{F_2} \right) + \sum_{a=q, \bar{q}} P_{ga}(z) f_a \left(\frac{x_J}{z}, \mu_{F_2} \right) \right] \right. \\
 & \left. \left. + \sum_{a=q, \bar{q}} \int_{x_J}^1 \frac{dz}{z} \left[P_{ag}(z) f_g \left(\frac{x_J}{z}, \mu_{F_2} \right) + P_{aa}(z) f_a \left(\frac{x_J}{z}, \mu_{F_2} \right) \right] \right) \right\} .
 \end{aligned}$$

Inclusive Higgs + jet: NLL/NLO* azimuthal coefficients

$$\begin{aligned}
 \mathcal{C}_n &= \frac{e^{\Delta Y}}{s} \frac{M_{H,\perp}}{|\vec{p}_H|} \\
 &\times \int_{-\infty}^{+\infty} d\nu \left(\frac{x_J x_H s}{s_0} \right)^{\bar{\alpha}_s(\mu_{R_c})} \left\{ \chi(n, \nu) + \bar{\alpha}_s(\mu_{R_c}) \left[\bar{\chi}(n, \nu) + \frac{\beta_0}{8N_c} \chi(n, \nu) \left[-\chi(n, \nu) + \frac{10}{3} + 4 \ln \left(\frac{\mu_{R_c}}{\sqrt{\vec{p}_H \vec{p}_J}} \right) \right] \right] \right\} \\
 &\times \left\{ \alpha_s^2(\mu_{R_1}) c_H(n, \nu, |\vec{p}_H|, x_H) \right\} \left\{ \alpha_s(\mu_{R_2}) [c_J(n, \nu, |\vec{p}_J|, x_J)]^* \right\} \\
 &\times \left\{ 1 + \bar{\alpha}_s(\mu_{R_1}) \frac{\tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H)}{c_H(n, \nu, |\vec{p}_H|, x_H)} + \bar{\alpha}_s(\mu_{R_2}) \left[\frac{\tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J)}{c_J(n, \nu, |\vec{p}_J|, x_J)} \right]^* \right\} .
 \end{aligned}$$