Elimination of QCD Renormalization Scale and Scheme Ambiguities

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In collaboration with S.J.Brodsky, X.G.Wu, S.Q.Wang, F. Sannino, P.G.Ratcliffe.

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Crieff, Scotland.

Outline

- The Renormalization Scale Setting in QED/QCD
- State of the art about renormalization in QCD
- The iCF and the PMC_∞ scale setting procedure
- PMC_∞ and the Event Shape Variables
- Comparison with CSS
- QED and IR Conformal limit for Thrust
- *$\alpha s(Q)$: New method to determine the coupling*
- HQ production
- Work in progress / future perspectives

Why The Scale Setting in QCD is a key issue?...truncated series (NLO,NNLO,N3LO...)

Stückelberg and Peterman

Renormalization: subtraction of infinities at μ0.
RGE- R.Group equations

$$Z_{\alpha_s}^{-1} = (\sqrt{Z_3} Z_2 / Z_1)^2,$$

$$Z_\alpha\left(Q^2\right) = 1 - \frac{\beta_0 \alpha_s\left(Q^2\right)}{4\pi\varepsilon},$$

$$\frac{1}{4\pi} \frac{d\alpha_s(Q^2)}{d\log Q^2} = \beta(\alpha_s),$$

$$\beta\left(\alpha_s\right) = -\left(\frac{\alpha_s}{4\pi}\right)^2 \sum_{n=0} \left(\frac{\alpha_s}{4\pi}\right)^n \beta_n.$$

Not only confinement mechanism...

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_0^2)}{1 + \beta_0 \frac{\alpha_s(\mu_0^2)}{4\pi} \ln(Q^2/\mu_0^2)}$$

- *To determine αs(Q^2) to the highest precision;*
- *To make precision tests of the QCD;*
- *To eliminate the renormalization scale ambiguity and the scheme dependence in the observables;*
- *To assess and reach the maximum sensitivity to NP.*

The Renormalization Scale Problem in QED

QED is not only perturbative :

- *No ambiguity in the renormalization scale in QED;*
- *The renormalization scale in QED is physical and set by the exchanged photon virtuality;*
- *An infinite series of Vacuum Polarization diagrams is resummed;*
- *The QED coupling is defined from physical observables (Gell Mann-Low scheme);*
- *No scheme dependence is left;*
- *Analyticity (space-like/time-like);*
- *Exact number of active leptons is set;*
- *Recover of a conformal-like series;*

Effective coupling

$$\boldsymbol{\alpha(Q) = \frac{\alpha_0}{1 - \Pi(Q)}}$$

The VPF contains all β-terms

QED: a Theoretical Constraint for QCD

QCD $\longrightarrow$ ***Abelian Gauge Theory***

In the limit : $Nc \longrightarrow 0$,

at fixed $\alpha = C_F\,\alpha_s\,,\; n_l = T\,n_F\,/\,C_F$

The scale setting procedure used in QCD must be consistent with the QED

Huet, S.J.Brodsky

"In the perspective of a theory unifying all the interactions, electromagnetic, weak and strong nuclear, such as a so-called grand unified theory or GUT, we are constrained to apply the same scale-setting procedure in all sectors of the theory."

S.J. Brodsky, L.D.G.

The road to scale setting is paved with some misbeliefs

According to the Conventional practice (CSS) :

- ***Scale is set to the «proper» scale of the process and varied in the range of 2;***
- ***Scales are judged by results «a posteriori»;***
- ***The renormalization scale is unique for each process;***
- ***The renormalization scale is a simple unphysical parameter;***
- ***The renormalization and factorization scales are equal;***

These assumptions are wrong for QED and thus too for QCD!
The CSS appears to be more a "lucky guess"!

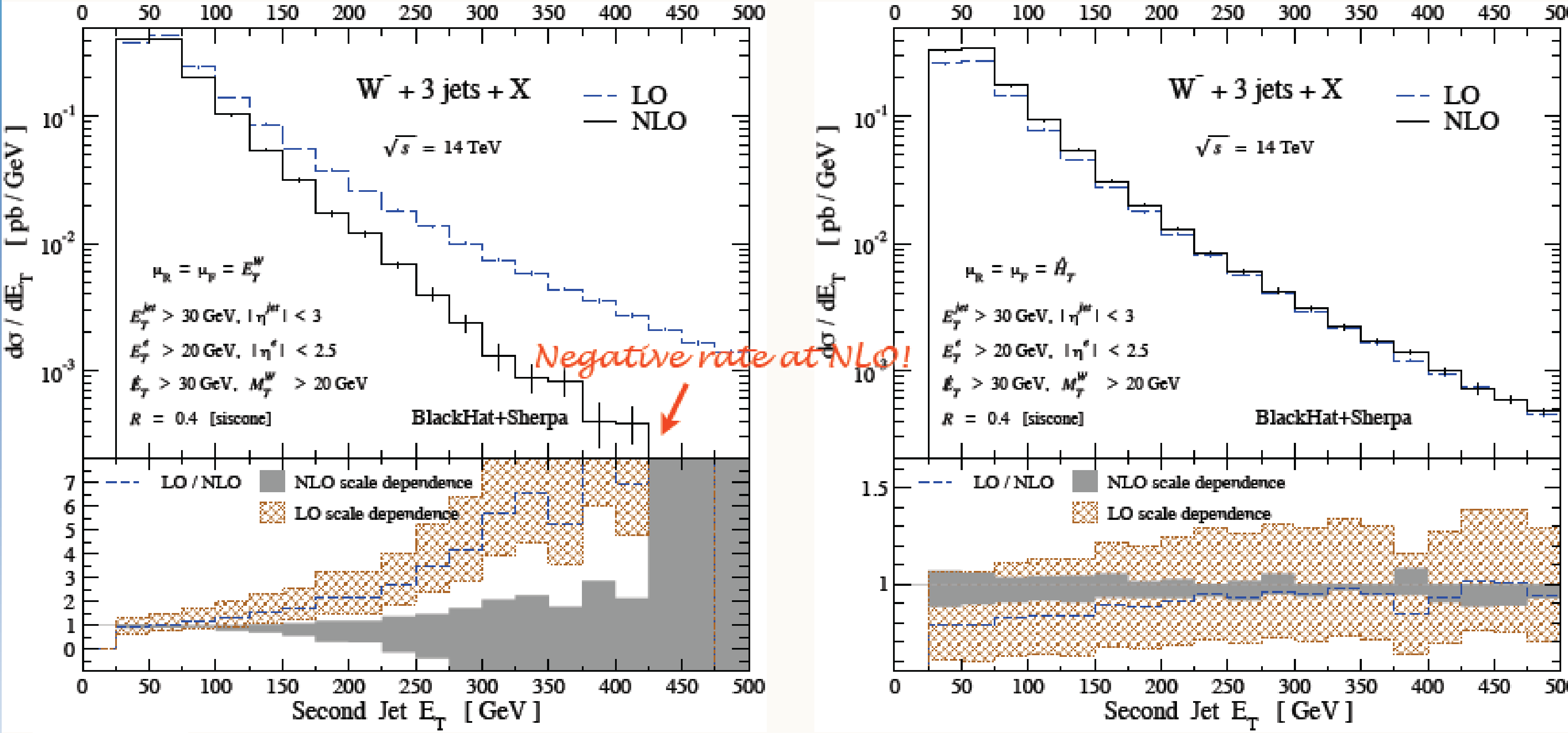
- In general, no one knows the proper renormalization scale value, *Q*;
- pQCD convergence is affected by Large logs and by Renormalons;
- CSS does not agree with QED;
- No distinction is made among different sources of errors and their relative contributions, MHO contributions must be separated from the scale ambiguities;
- Higher Th. precision is required by exp data;

In QCD…

NLO QCD predictions for W+ 3Jet distributions at LHC

Black Hat

$$\mu_R = \mu_F = E_T^W \qquad\qquad \mu_R = \mu_F = \hat{H}_T$$



F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower, and D. Maître

The Reliable Scale-Setting method

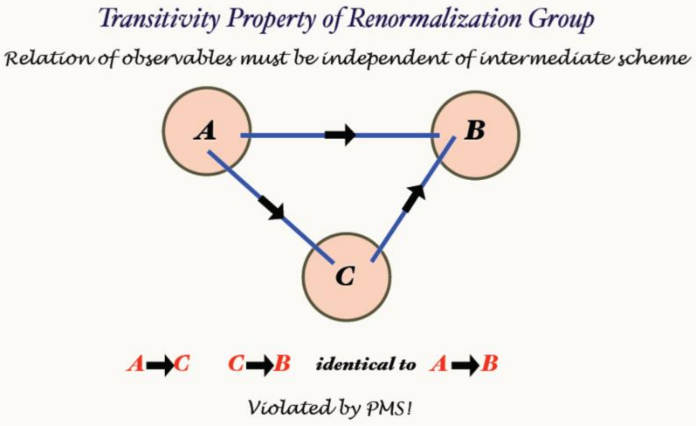
- RG properties: ***uniqueness, reflexivity, symmetry,* and *transitivity;***

Other requirements are

- Constraints given by other theories,

(dual theories LFHQCD ,QED, Conformal theories, non-perturbative results)

- **scheme independence**;
- ..Physical requirements;
- ...phenomenological results;



PMS,FAC:Optimization Procedures

- PMS

Scale and scheme parameters are treated as independent variables;

$$
\begin{aligned}
\frac{\partial \rho_n}{\partial \tau} &= \left(\frac{\partial}{\partial \tau} + \beta\left(\alpha_s\right)\frac{\partial}{\partial \alpha_s}\right)\rho_n \equiv 0 \\
\frac{\partial \rho_n}{\partial \beta_j} &= \left(\frac{\partial}{\partial \beta_j} - \beta\left(\alpha_{\mathrm{s}}\right)\int_0^{\alpha_{\mathrm{s}}} \mathrm{d}\alpha' \frac{\alpha'^{j+2}}{\left[\beta\left(\alpha'\right)\right]^2}\frac{\partial}{\partial \alpha_{\mathrm{s}}}\right)\rho_n \equiv 0
\end{aligned}
$$

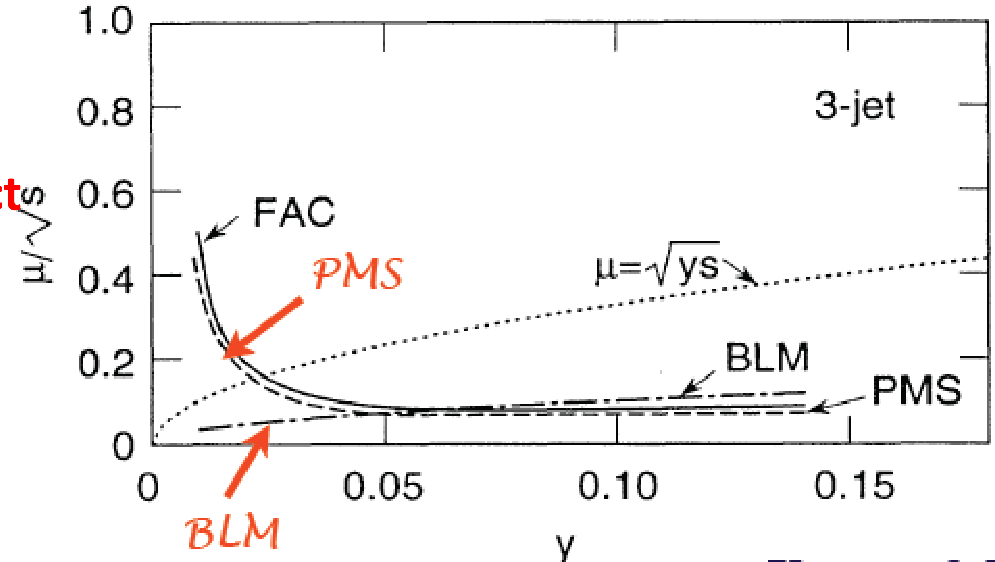
- FAC : physical quantities define «effective charges»:

$$
\alpha_R \equiv \left(\frac{R}{\mathcal{C}_0}\right)^{1/p}.
$$

- ***Optimization procedures such as PMS (xRGE) and FAC (ECH) lead to incorrect and unphysical results violating important RG properties;***

e+ e- to 3-jets

PMS and FAC lead to incorrect and unphysical results



Jet Invariant mass squared:

$$\mathcal{M}^2 = ys$$

PMS violates Transitivity property

Kramer & Lampe

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

The Principle of Maximum Conformality is the principle underlying the BLM

S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D 28, **Nov 23° 1982**

Observable in the initial parametrization

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0} a(Q) + [r_{2,0} + \beta_0 r_{2,1}] a(Q)^2 \\ & + [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}] a(Q)^3 \\ & + [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ & + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}] a(Q)^4 + \mathcal{O}(a^5) \qquad (6)\end{aligned}$$

Stanley J. Brodsky, L.D.G.:
Phys. Rev. D 86, 085026 (2011)

Mojaza, Matin and Brodsky, Stanley J. and Wu, Xing-Gang
Phys.Rev.Lett. 110 (2013) 192001

$r_{n,0}$ conformal coefficients

The β-terms are reabsorbed by RGE

$$\boxed{\rho(Q^2) = r_{0,0} + r_{1,0} a(Q_1) + r_{2,0} a(Q_2)^2 + r_{3,0} a(Q_3)^3 + r_{4,0} a(Q_4)^4 + \mathcal{O}(a^5) \ ,}$$

Conformal-like expansion

$$\ln \frac{Q_k^2}{Q^2} = \frac{R_{k,1} + \Delta_k^{(1)}(a) R_{k,2} + \Delta_k^{(2)}(a) R_{k,3}}{1 + \Delta_k^{(1)}(a) R_{k,1} + \left(\Delta_k^{(1)}(a)\right)^2 (R_{k,2} - R_{k,1}^2) + \Delta_k^{(2)}(a) R_{k,1}^2} \ .$$

PMC scales: reabsorb the RS dependence!

Features of the PMC/PMC$_\infty$

- All terms associated with the beta-function are included into the running coupling;
- PMC agrees with the QED in the Abelian limit;
- PMC is consistent with the IR Conformal limit;
- No scale ambiguities;
- Results are scheme independent ;
- The PMC scale sets the correct number of active flavors;
- Transitivity Property and all RG properties are all preserved;
- No renormalon n! growth in pQCD associated with the beta function;
- Resulting series is identical to conformal series! (CSR - Crewther Relation ;)
- PMC: One procedure from first principles for the whole SM and also for a GUT.

PMC∞ - Results for Event Shape Variables distributions at NNLO

Work in collaboration with S. J. Brodsky, S.Q.Wang , X.G. Wu and F. Sannino

- arXiv: 2104.12132 [hep]
- *Phys.Rev.D* 102 (2020) 1, 014015

Thrust and C-Par distribution at NNLO: process: e+e- →3jets

$$T = \frac{\max_{\vec{n}} \sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|},$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p_i}||\vec{p_j}| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p_i}|\right)^2},$$

Sheng-Quan Wang, S.J. Brodsky, Xing-Gang Wu , Jian-Ming Shen, L.D.G., *Phys.Rev.D* 100 (2019) 9, 094010

Sheng-Quan Wang, S.J. Brodsky, Xing-Gang Wu , L.D.G., Jian-Ming Shen, *Phys.Rev.D* 102 (2020) 1, 014005

Sheng-Quan Wang, S.J. Brodsky, Xing-Gang Wu , L.D.G., Phys. Rev. D 99, no.11, 114020 (2019)

Distributions from::

A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, Phys. Rev. Lett. **99**, 132002 (2007).
A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, JHEP **0712**, 094 (2007).
S. Weinzierl, JHEP **0906**, 041 (2009).
S. Weinzierl, Phys. Rev. Lett. **101**, 162001 (2008).

Strong Coupling from RunDec program :

K. G. Chetyrkin, J. H. Kuhn and M. Steinhauser, Comput. Phys. Commun. **133**, 43 (2000).

PMC$_\infty$ preserves the iCF: *the intrinsic Conformality*

Observable: Single variable distribution at NNLO calculated at the initial scale μ_0

$$\frac{1}{\sigma_{tot}}\frac{Od\sigma(\mu_0)}{dO}=\frac{\alpha_s(\mu_0)}{2\pi}\frac{Od\overline{A}_O(\mu_0)}{dO}+\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2\frac{Od\overline{B}_O(\mu_0)}{dO}+\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3\frac{Od\overline{C}_O(\mu_0)}{dO}+\mathcal{O}(\alpha_s^4), \qquad (3)$$

The iCF is an RG invariant parametrization with conformal coefficients and scales.

$$\begin{aligned}A_O(\mu_0)&=A_{Conf},\\ B_O(\mu_0)&=B_{Conf}+\frac{1}{2}\beta_0\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)A_{Conf},\\ C_O(\mu_0)&=C_{Conf}+\beta_0\ln\left(\frac{\mu_0^2}{\mu_{II}^2}\right)B_{Conf}+\\ &+\frac{1}{4}\left[\beta_1+\beta_0^2\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\right]\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)A_{Conf}\end{aligned} \qquad (4)$$

- **No redefinition of the conformal terms at higher orders;**
- **No initial scale dependence left under a global change of scale;**
- **The scale dependence is explicit.**
- **The iCF is the most general RG invariant parametrization;**
- **Other parametrizations can be reduced to the iCF;**

$$\begin{aligned}\sigma_I&=\left\{\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)+\frac{1}{2}\beta_0\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2\right.\\ &\left.+\frac{1}{4}\left[\beta_1+\beta_0^2\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\right]\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3\right\}A_{Conf}\\ \sigma_{II}&=\left\{\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2+\beta_0\ln\left(\frac{\mu_0^2}{\mu_{II}^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3\right\}B_{Conf}\\ \sigma_{III}&=\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3C_{Conf}\end{aligned} \qquad (6)$$

The conformal subsets are the fundamental blocks of the iCF

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)\sigma_N=0.$$

- **Each subset is scale invariant.**
- **Any combination of conformal subsets is an invariant.**
- **I can define a scale for each subset preserving the scale invariance.**
- **If Aconf=0 the whole subset becomes null.**

Ordered scale invariance: **scale invariance is preserved perturbatively independently from process, kinematics and order.**

New «*How to*» method

$$B_O(N_f) = C_F\left[C_A B_O^{N_c} + C_F B_O^{C_F} + T_F N_f B_O^{N_f}\right] \quad (12)$$

either for numerical or analytic calculations.

$$B_{Conf} = B_O\left(N_f \equiv \frac{33}{2}\right),$$

$$B_{\beta_0} \equiv \log\frac{\mu_0^2}{\mu_I^2} = 2\frac{B_O - B_{Conf}}{\beta_0 A_{Conf}}$$

Find the roots of the β terms and vary the number of flavors.

$$C_O(N_f) = \frac{C_F}{4}\left\{N_c^2 C_O^{N_c^2} + C_O^{N_c^0} + \frac{1}{N_c^2} C_O^{\frac{1}{N_c^2}} + N_f N_c \cdot C_O^{N_f N_c} + \frac{N_f}{N_c} C_O^{N_f/N_c} + N_f^2 C_O^{N_f^2}\right\}.$$

$$C_{Conf} = C_O\left(N_f \equiv \frac{33}{2}\right) - \frac{1}{4}\overline{\beta}_1 B_{\beta_0} A_{Conf}$$

β0 killing value

$$\overline{\beta}_1 \equiv \beta_1\boxed{(N_f = 33/2)} = -107.$$

$$C_{\beta_0} \equiv \log\left(\frac{\mu_0^2}{\mu_{II}^2}\right) = \frac{1}{\beta_0 B_{Conf}}\left(C_O - C_{Conf} - \frac{1}{4}\beta_0^2 B_{\beta_0}^2 A_{Conf} - \frac{1}{4}\beta_1 B_{\beta_0} A_{Conf}\right),$$

PMC∞ scales

$$\frac{1}{\sigma_{tot}}\frac{O d\sigma(\mu_I,\mu_{II},\mu_0)}{dO} = \left\{\overline{\sigma}_I + \overline{\sigma}_{II} + \overline{\sigma}_{III} + \mathcal{O}(\alpha_s^4)\right\}, \quad (20)$$

$$\alpha_I \equiv \alpha_s(\mu_I),\ \alpha_{II} \equiv \alpha_s(\tilde{\mu}_{II})$$

Conformal subsets

$$\eta = 3.51$$

$$\begin{aligned}
\overline{\sigma}_I &= A_{Conf}\frac{\alpha_I}{2\pi} \\
\overline{\sigma}_{II} &= (B_{Conf} + \eta A_{tot}A_{Conf})\left(\frac{\alpha_{II}}{2\pi}\right)^2 - \eta A_{tot}A_{Conf}\left(\frac{\alpha_0}{2\pi}\right)^2 \\
&\quad - A_{tot}A_{Conf}\frac{\alpha_0}{2\pi}\frac{\alpha_I}{2\pi} \\
\overline{\sigma}_{III} &= (C_{Conf} - A_{tot}B_{Conf} - (B_{tot} - A_{tot}^2)A_{Conf})\left(\frac{\alpha_0}{2\pi}\right)^3 \quad (23)
\end{aligned}$$

$$\mu_I = \sqrt{s}\cdot e^{-\frac{1}{2}B_{\beta_0}}, \quad (1-T)<0.33$$

$$\tilde{\mu}_{II} = \begin{cases} \sqrt{s}\cdot e^{-\frac{1}{2}C_{\beta_0}\cdot\frac{B_{Conf}}{B_{Conf}+\eta\cdot A_{tot}A_{Conf}}}, & (1-T)<0.33 \\ \sqrt{s}\cdot e^{-\frac{1}{2}\left(\frac{C_1}{11B_1-\frac{2}{3}B_0}\right)}, & (1-T)>0.33 \end{cases}$$

$$\mu_{III} = \sqrt{s}$$

Red

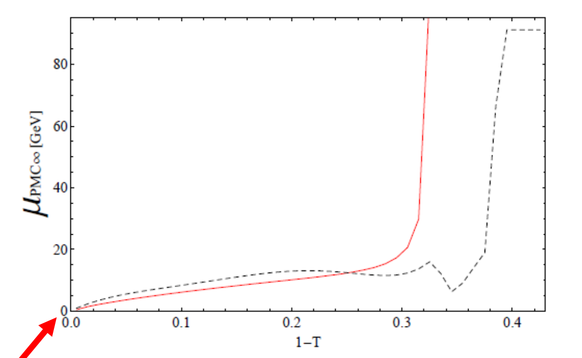
Dashed Black

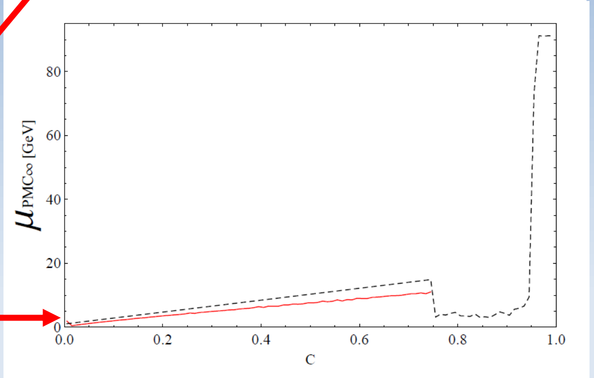
The PMC∞ scales are functions of the Event Shape Variable and of the physical scale of the process: $\sqrt{s}$

The PMC∞ determines the flow of the running coupling all over the spectrum of the observable reflecting the virtuality of quarks and gluons subprocesses

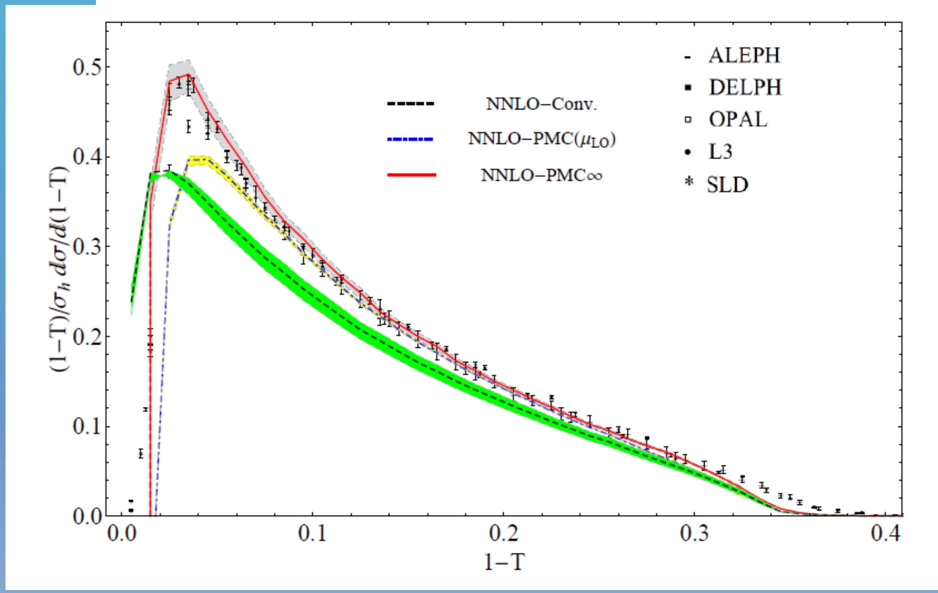
Kinematical constraints cancel the whole LO conformal subset: Aconf=0

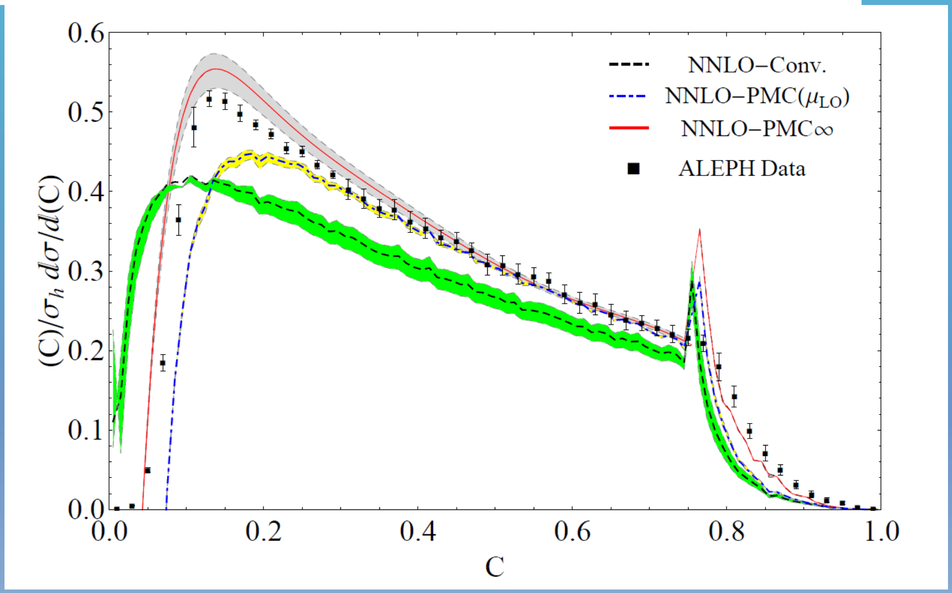
Correct physical behavior in the nonperturbative region (unlike other methods e.g. PMS and FAC)





Comparison with Conv. Scale Sett.





$\bar{\delta}[\%]$	Conv.	PMC(μ_{LO})	PMC$_\infty$
$0.10 < (1-T) < 0.33$	6.03	1.41	1.31
$0.21 < (1-T) < 0.33$	6.97	2.19	0.98
$0.33 < (1-T) < 0.42$	8.46		2.61
$0.00 < (1-T) < 0.33$	5.34	1.33	1.77
$0.00 < (1-T) < 0.42$	6.00	-	1.95

$\bar{\delta}[\%]$	Conv.	PMC(μ_{LO})	PMC$_\infty$
$0.00 < (C) < 0.75$	4.77	0.85	2.43
$0.75 < (C) < 1.00$	11.51	3.68	2.42
$0.00 < (C) < 1.00$	6.47	1.55	2.43

PMC∞ improves the precision of the pQCD predictions and the fit with data.

Errors: 85% depends on not-yet calculated orders.

The last unknown scale fixed to the last known leads to stable results.

We can use the standard criteria to evaluate the accuracy and the conformality at NNLO

The error due to the PMC∞ is 1.5% of the whole error $\approx 0.029 - 0.036\ \%$

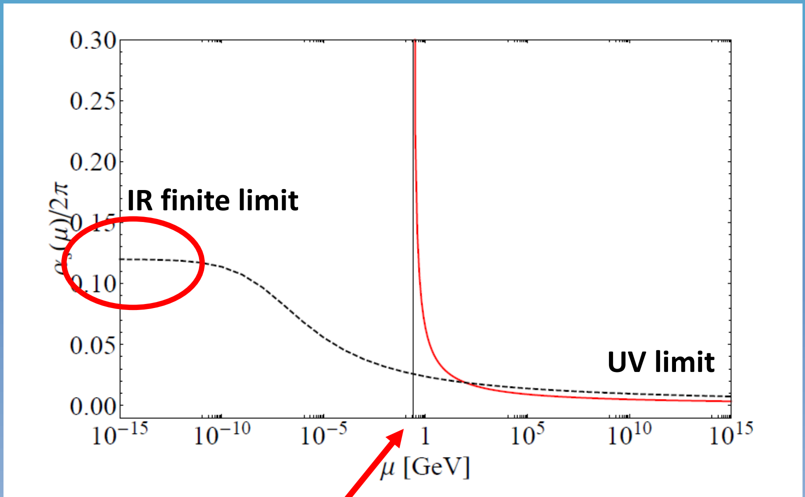
$$\delta = \left|\frac{\sigma(2M)) - \sigma(M/2)}{2\sigma(M)}\right|$$

M = $\sqrt{s}$ = Z0 mass

Thrust in the QCD conformal window

- Banks-Zaks: UV+IR fixed points

L.D.G. , F. Sannino, S.Q. Wang, X.G. Wu,
Phys.Lett.B 823 (2021) 136728



$$\Lambda = \mu_0 \left(1 + \frac{|x^*|}{x_0}\right)^{\frac{1}{2B|x^*|}} e^{-\frac{1}{2Bx_0}}$$

$$\mu^2 \frac{d}{d\mu^2}\left(\frac{\alpha_s}{2\pi}\right) = -\frac{1}{2}\beta_0\left(\frac{\alpha_s}{2\pi}\right)^2 - \frac{1}{4}\beta_1\left(\frac{\alpha_s}{2\pi}\right)^3 + O\left(\alpha_s^4\right)$$

2-loop solution:
Lambert function

$$\frac{dx}{dt} = -Bx^2(1+Cx)$$

$$We^W = z$$

with:

$$W = \left(-\frac{1}{Cx} - 1\right)$$

$$z = e^{-\frac{1}{Cx_0}-1}\left(-\frac{1}{Cx_0} - 1\right)\left(\frac{\mu^2}{\mu_0^2}\right)^{-\frac{B}{C}}.$$

The general solution for the coupling is:

$$x = -\frac{1}{C}\frac{1}{1+W}.$$

Raising the number of flavors Nf , we can compare the two methods CSS and PMC∞ all over the entire energy range from 0 up to ∞.

QCD Conformal Window:

$$\frac{34N_c^3}{13N_c^2-3} < N_f < \bar{N}_f$$

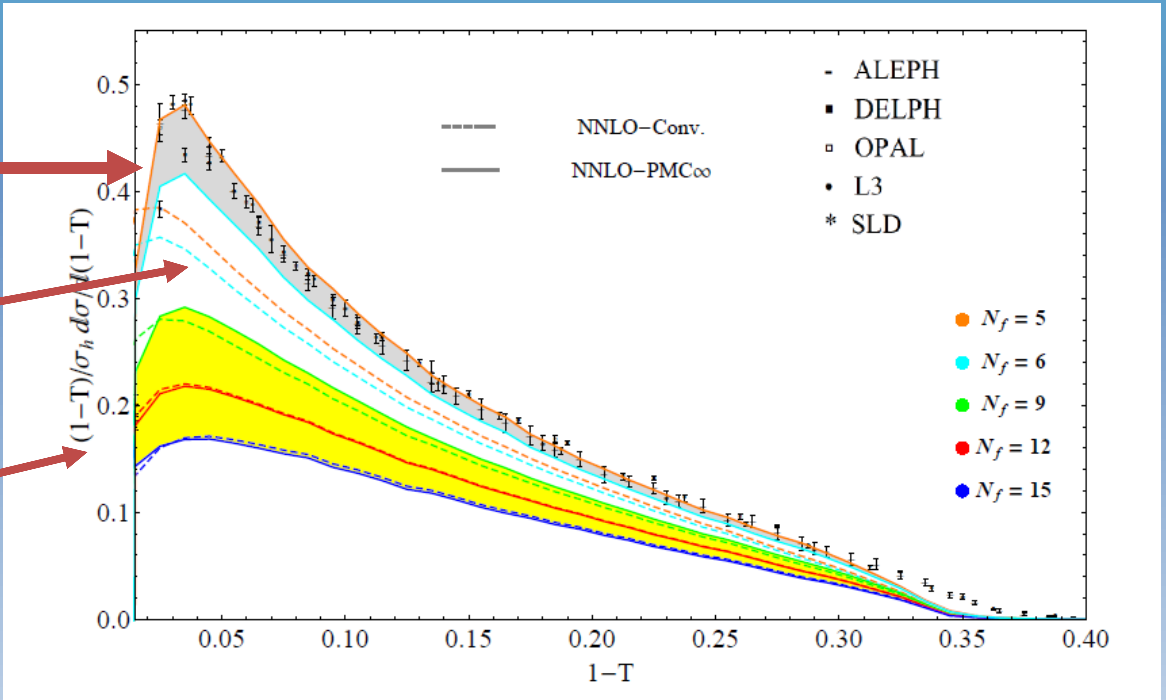
$$\bar{N}_f = x^{*-1}(x_0) \simeq 15.219 \pm 0.012,$$

Thrust in the Conformal Window: Conv.S.S. and PMC$_\infty$

PMC$_\infty$: the number of active flavors is in agreement with the SM: Gray : 5<nf<6

CSS deviates from the conformal distribution

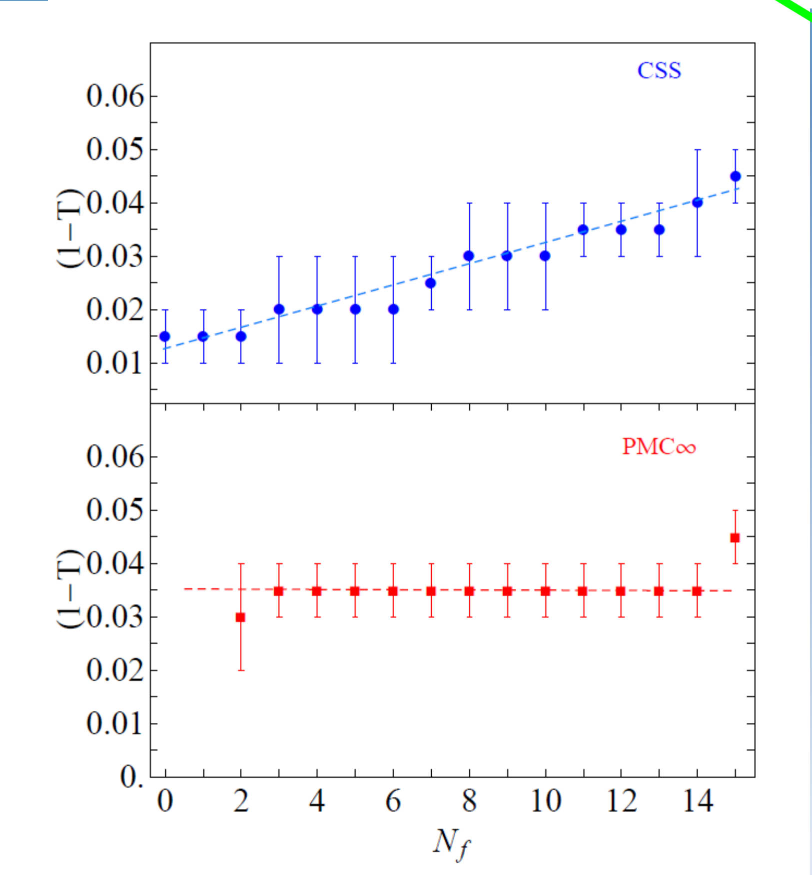
Yellow: Conformal thrust : 9<nf<15

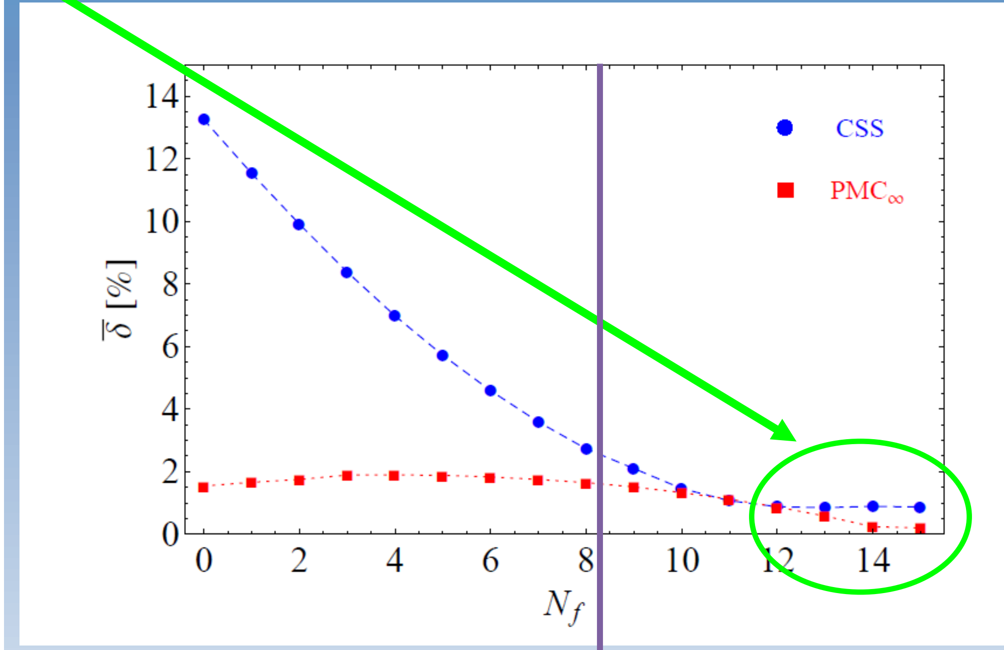


The PMC$_\infty$ is the natural extension of the conformal thrust out of the QCD conformal window.

New Features of the PMC$_\infty$

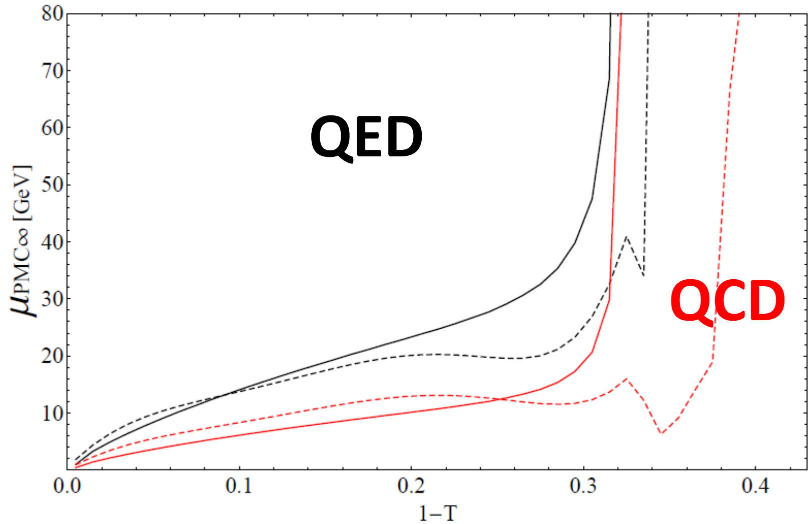
- Shape and the peak position are invariant
- Th. errors calculated with standard criteria show the correct limit in the conformal window





Phase transition

QED Thrust 3-Jet at NNLO, Limit Nc $\longrightarrow$ 0 is consistent with PMC$_\infty$



Color factor rescaling for QED:
NA=1, CF=1, TR=1,CA=0,Nc=0,NF=Nl

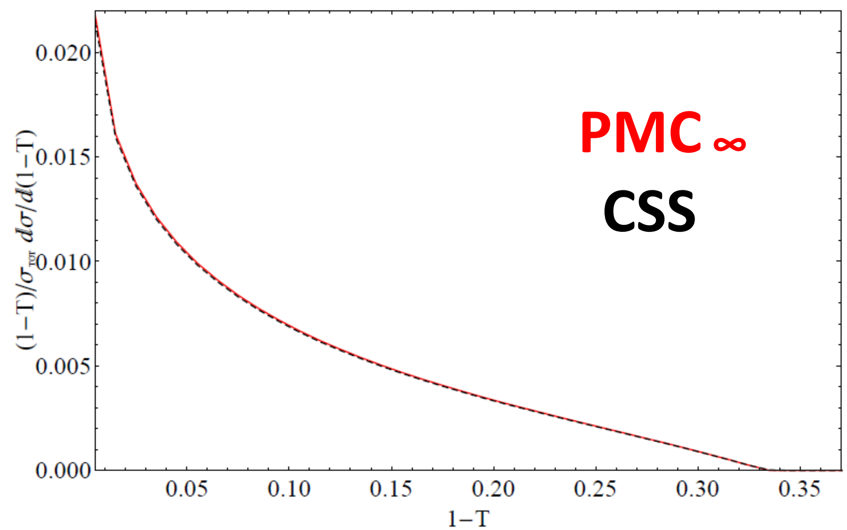
$$\beta_n / C_F^{n+1} \text{ and } \alpha_s \cdot C_F$$

$$\beta_0 = -\frac{4}{3} N_l \text{ and } \beta_1 = -4 N_l$$

QED/QCD PMC$_\infty$ scales differ by the scheme $\overline{\text{MS}}$ factor reabsorption 5/3

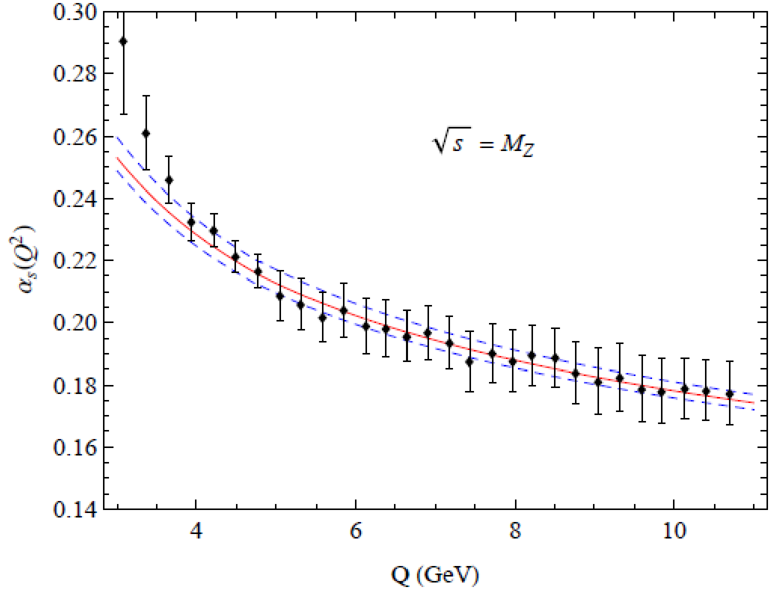
$$\alpha(Q^2) = \frac{\alpha}{\left(1 - \Re e \Pi^{\overline{\text{MS}}}(Q^2)\right)},$$

Analytic with leptons+quarks+W



The QED thrust

Novel method for the precise determination of $\alpha_s(Q)$



Sheng-Quan Wang, S.J. Brodsky,
Xing-Gang Wu , Jian-Ming Shen, L.D.G.,

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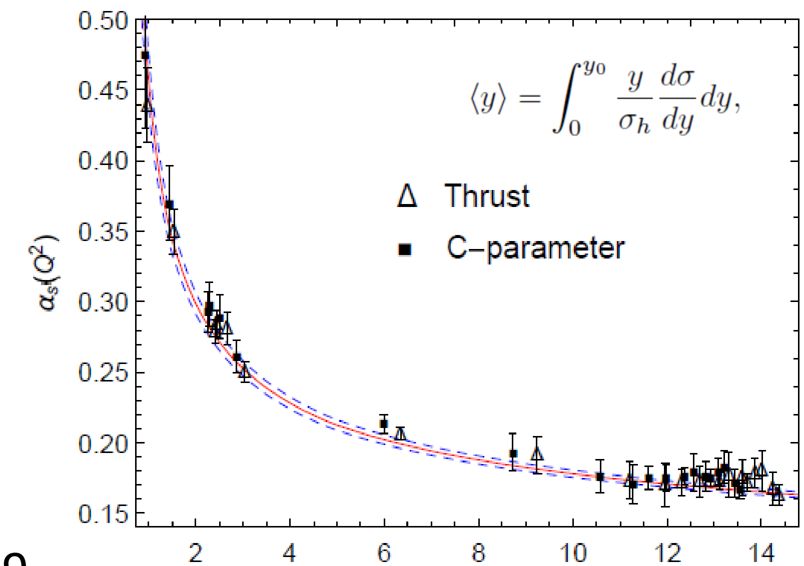
Asymptotic behavior of $\alpha_s(Q)$ determined from only one experiment

Mean values



Q (GeV)

T and C-par for e+e- → 3-Jet at a single $\sqrt{s}$

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0011(\text{exp.}) \pm 0.0005(\text{the.}) = 0.1185 \pm 0.0012,$$

1-T

$$\alpha_s(M_Z^2) = 0.1193^{+0.0009}_{-0.0010}(\text{exp.})^{+0.0019}_{-0.0016}(\text{the.}) = 0.1193^{+0.0021}_{-0.0019},$$

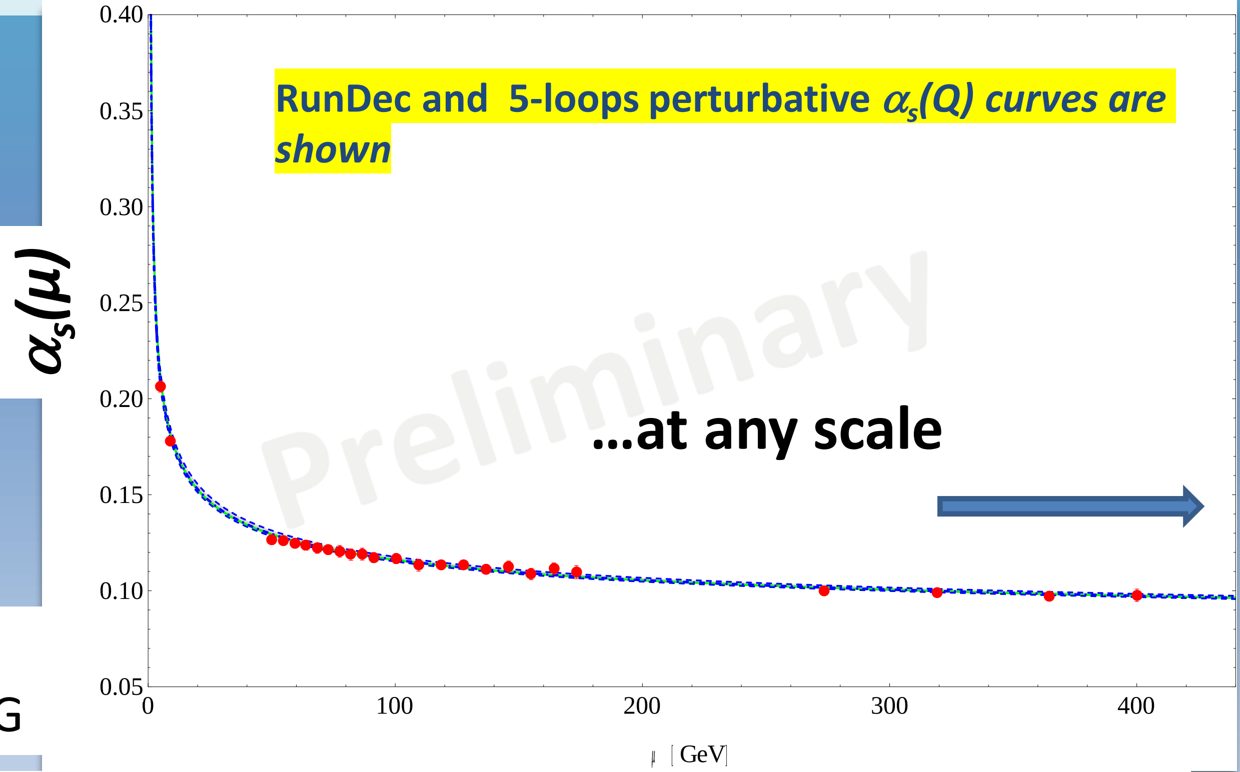
C-par

World average $\alpha_s(Mz)=0.1179$ ±0.0009 PDG

Preliminary results:

…Improved with the iCF to determine the Entire coupling $\alpha_s(Q)$.

One experiment Thrust distribution NNLO
e+e- → 3-Jet
at a single $\sqrt{s}$:
40 Bin -> coupling at any 40 different scales.
Errors by standard criteria



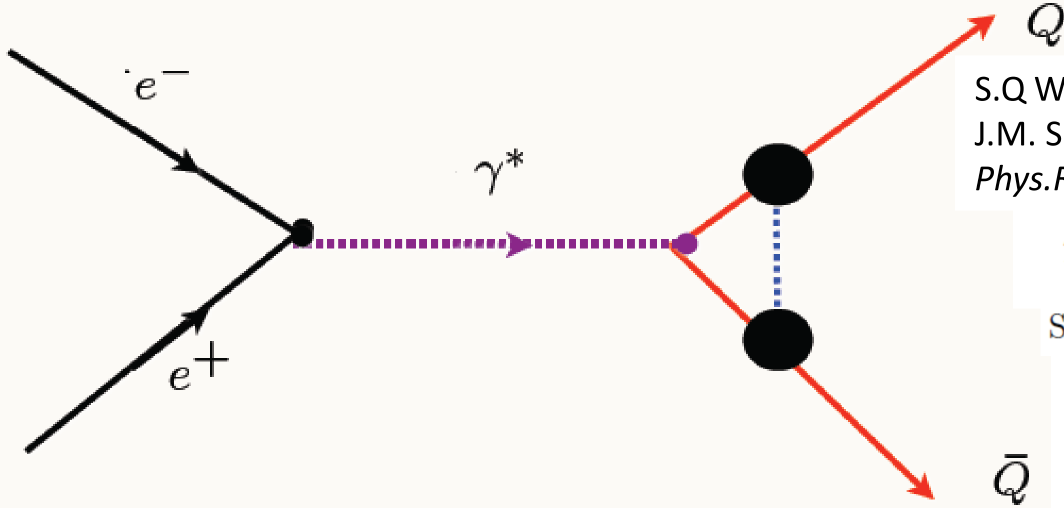
World average:
$\alpha_s(Mz)=0.1179 \pm 0.0009$ PDG

Significant improved precision: χ2 FIT or W.A. **(ALEPH data)**

$\alpha_s(M_Z)$ =0.1177+0.0003/-0.0002 (Exp)+0.0016/-0.0009 (Th.)

vs. the C.S.S. value: αs(MZ) = 0.1274 ± 0.0021± 0.0042

PMC: Heavy quark production at threshold



S.Q Wang, S.J. Brodsky, X.G. Wu , L.D.G., J.M. Shen,
Phys.Rev.D 102 (2020) 1, 014005

$$T = \frac{\eta}{1-e^{-\eta}}, \qquad \eta = \frac{2\pi\alpha}{v},$$

Sommerfeld rescattering formula

Multiple photon rescattering

QCD potential: $V_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r} + k\,r$: Coulomb-like potential $(\pi/v)^n$.
+ linear confining term (+Non-Coulomb contributions (pQCD and rel.)) → 2 scales at threshold →

$\overline{\mathrm{MS}}$ scheme

$$Q_h = e^{-11/24}\, m_Q$$

$$Q_v = 2\, e^{-5/6}\, v\, m_Q$$

τ-mass $e^+e^- \to \tau^+\tau^-$

c/b quark mass $e^+e^- \to c\bar{c}/b\bar{b}$

Heavy quark production at threshold PMC results

PMC Results are finite and consistent with QED

$$\sigma = \sigma^{(0)} \Big[1 + \delta_h^{(1)}\, a_s(\mu_r) + \left(\delta_{h,in}^{(2)}(\mu_r) + \delta_{h,n_f}^{(2)}(\mu_r)\, n_f\right) a_s^2(\mu_r)$$
$$\boxed{+ \left(\frac{\pi}{v}\right) \delta_v^{(1)}\, a_s(\mu_r) + \left(\frac{\pi}{v}\right) \left(\delta_{v,in}^{(2)}(\mu_r) + \delta_{v,n_f}^{(2)}(\mu_r)\, n_f\right) a_s^2(\mu_r) + \left(\frac{\pi}{v}\right)^2 \delta_{v^2}^{(2)}\, a_s^2(\mu_r) + \mathcal{O}(a_s^3)\Big]}.$$

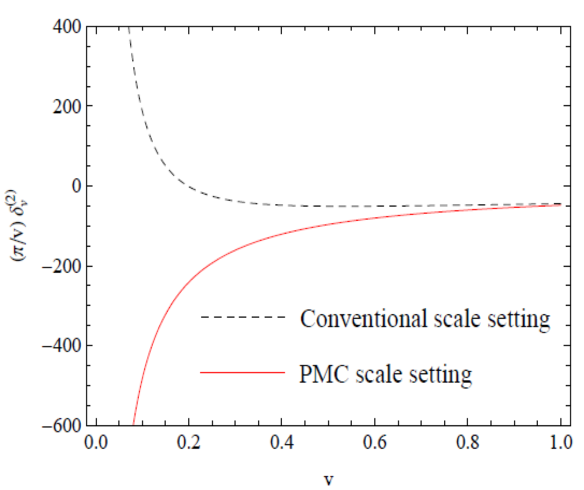
b quark pair production,

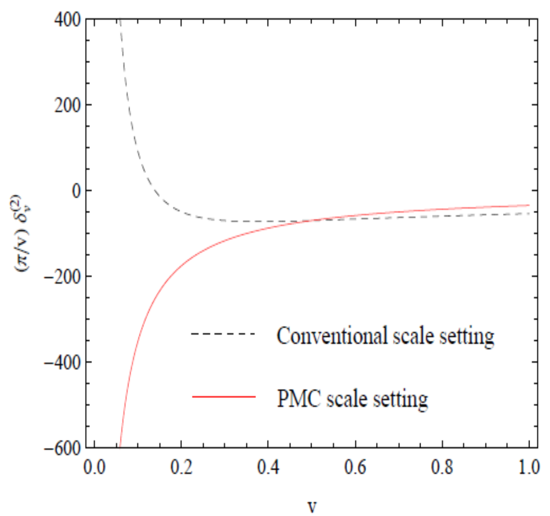
Coulomb corrections play an important role at threshold. 2 separate PMC scales for Coulomb and non-Coulomb terms. QCD and QED results in agreement under PMC.

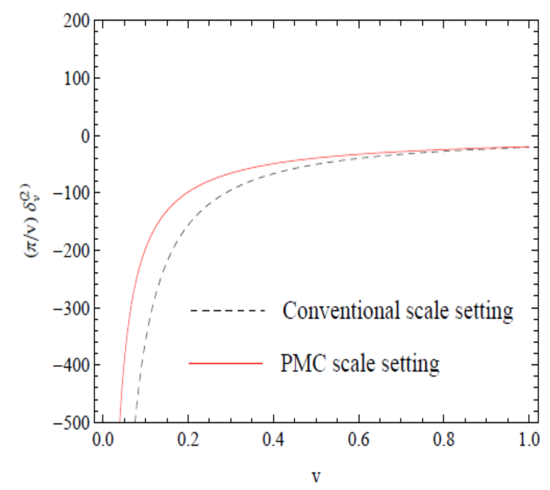
$\overline{\rm MS}$ scheme.

$$V(Q^2) = -\frac{4\,\pi^2\, C_F\, a_s^V(Q)}{Q^2},$$

QED







Summary

- The PMC_∞ is based on the PMC and it preserves the iCF;
- The iCF underlies an *ordered* scale invariance;
- Event shape variables results for T and C-par are in very good agreement with data in a wide range of values;
- Thrust in the IR Conformal Window and in the Nc=>0 limit shows consistency with the PMC_∞ ;
- The PMC_∞ eliminates the scale ambiguity and improves the precision of the QCD predictions at any order;
- Measurements of αs with PMC are in agreement with the world average and with the asymptotic behavior;
- Implementation of the PMC in the HQ production leads to finite and consistent results.
- ….Other PMC_∞ applications to :

$$\Gamma(H \to gg)\ R_{e^+e^-},\ R_\tau,\ \text{and}\ \Gamma(H \to b\bar{b})$$

also show improved predictions with respect to CSS.

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Thanks for your attention!