N^3LO Antenna Functions for Final-State Radiation

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Swiss National Science Foundation

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with Xuan Chen, Petr Jakubčík, Giovanni Stagnitto

Overview

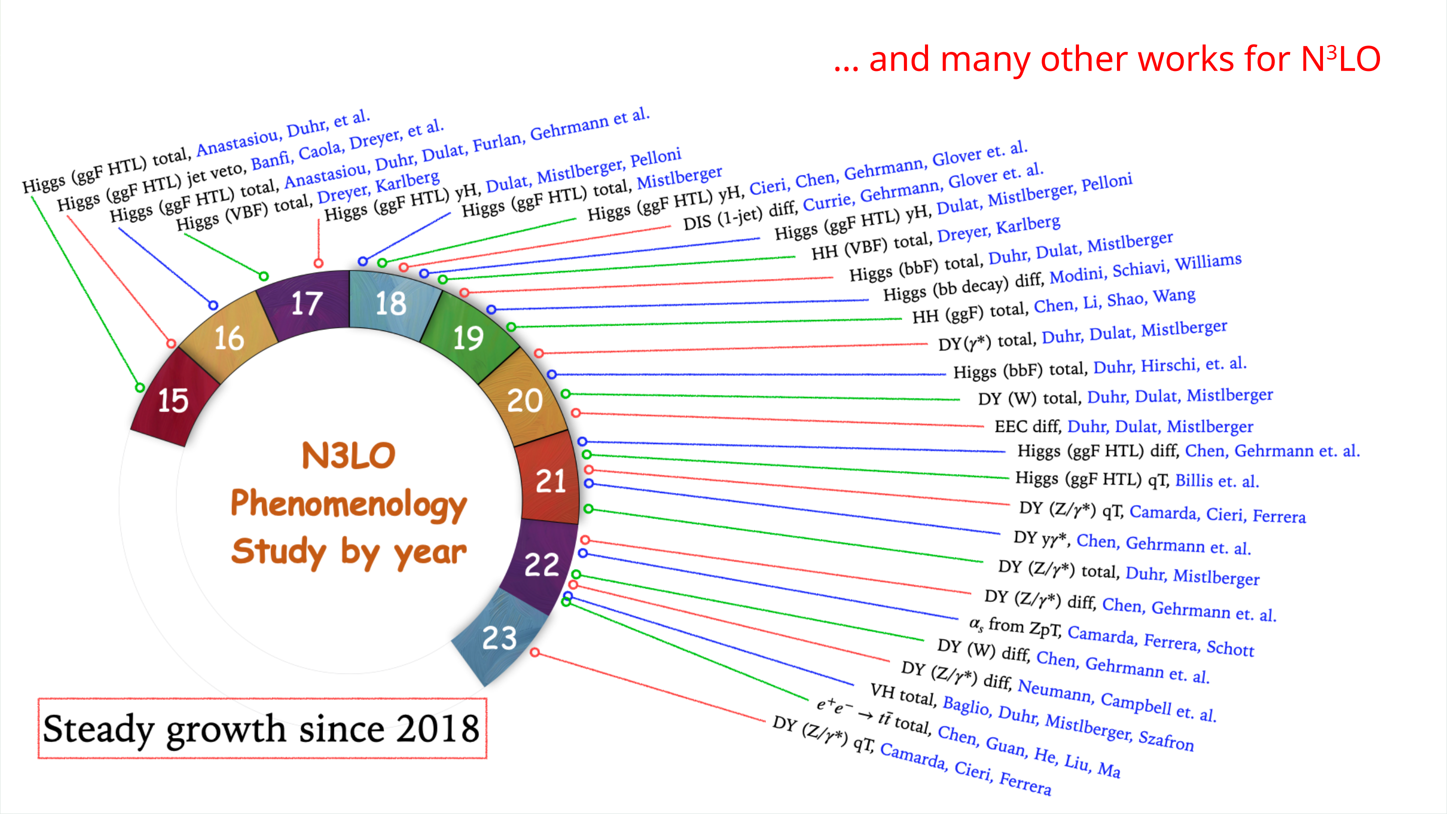
- Introduction and motivation
- Antenna subtraction up to NNLO [Gehrmann-De Ridder,Gehrmann,Glover ‘05] [Currie,Glover,Wells ‘13]
- N^3LO antenna functions
- Antenna subtraction at N^3LO: preliminary thoughts
- Conclusions

N³LO phenomenology

- Necessary to reach **percent-level** accuracy and keep up with experiments;

- **Inclusive** and **differential** for simple processes:
  - H,HH,VH
  - DIS
  - DY (γ*,Z,W)
  - $e^+e^- \to jj, t\bar{t}$

- Differential techniques:
  [Catani,Grazzini '07]
  - Slicing (qT);
  - Projection to Born;
  [Cacciari,Dreyer,Karlberg,Salam,Zanderighi '15]



Picture by Xuan Chen

Unresolved configurations at N^3LO

Probe **QCD** and its **factorization properties** beyond NNLO.

Exhaustive analysis of **unresolved configurations at N^3LO**:

2-loop

- **Single soft**

[Duhr,Gehrmann '13] [Li,Zhu '13]
[Dixon,Herrmann,Yan,Zhu '20]

- **Collinear**

[Badger,Glover '04]
[Duhr,Gehrmann,Jaquier '15]

1-loop

- **Double soft**

[Catani,Cieri '22] [Zhu '09]
[Czakon,Eschment,
Schellenberger '23]

- **Triple collinear**

[Catani,Florian,Rodrigo '04]
[Czakon,Sapeta '22]

0-loop

- **Triple soft**

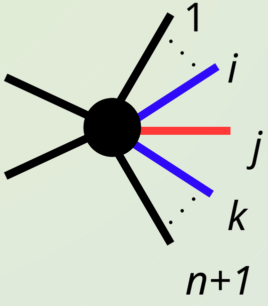
[Catani,Colferai,Torrini '20]
[Del Duca,Duhr,Haindl,Liu '23]
[Catani,Cieri,Colferai,Coradeschi '23]

- **Quadruple collinear**

[Del Duca,Duhr,Haindl,
Lazopoulos,Michel '20,'20]

Antenna subtraction at NLO

Real

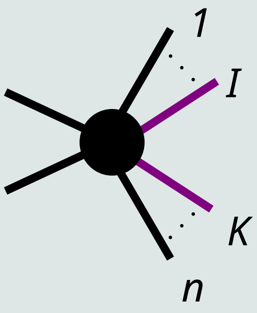


parton j soft or collinear to i or k

factorization properties of QCD

3-parton tree antenna

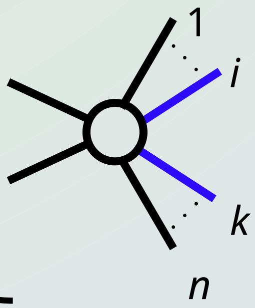
$$\underbrace{X_3^0(i,j,k)}_{\textbf{Antenna function}} \cdot$$



Analytical integration over PS of the unresolved radiation

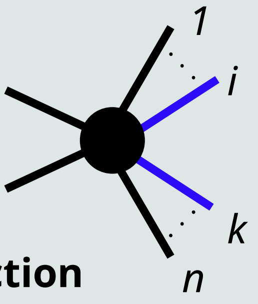
Virtual

Poles

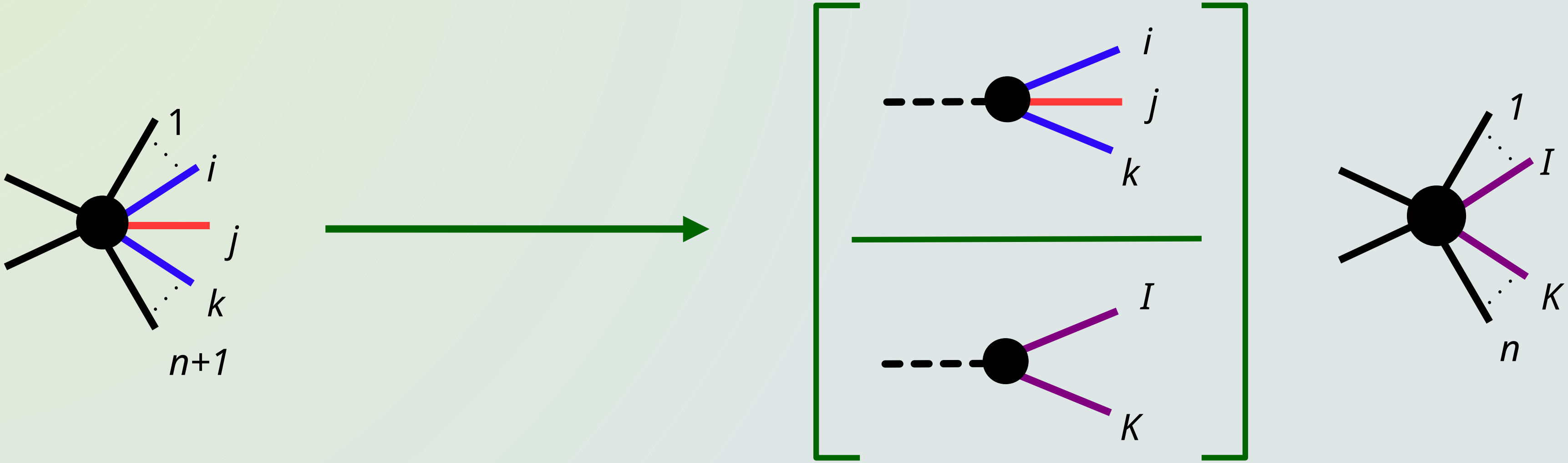


= Poles

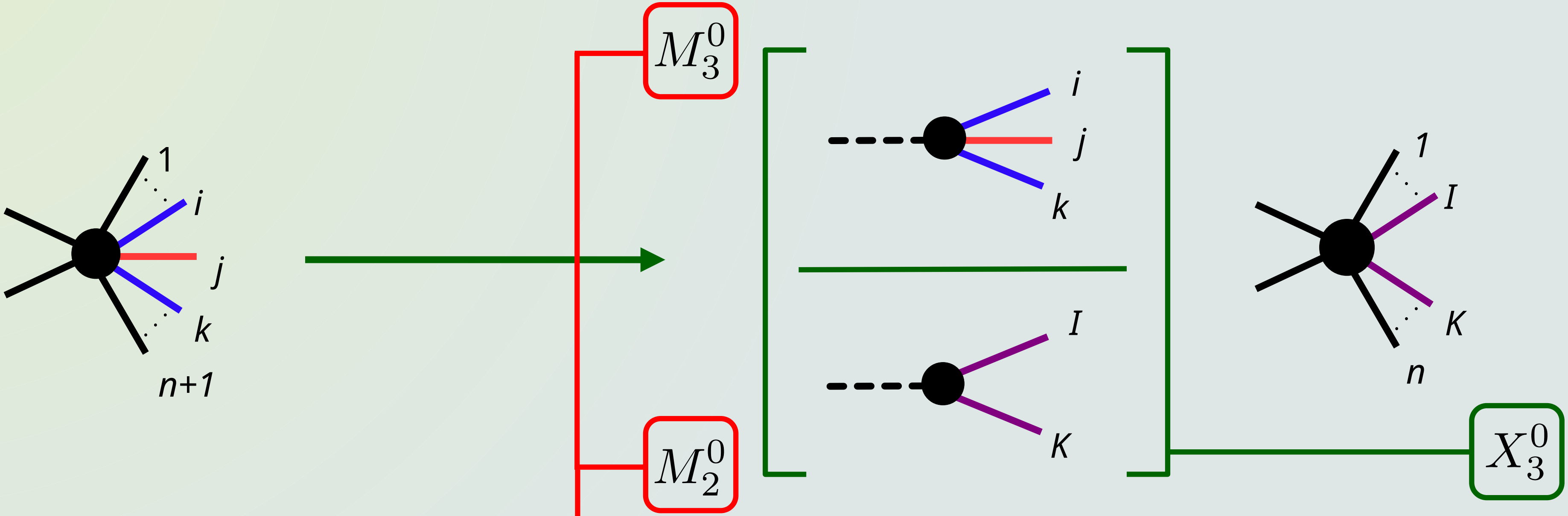
$$\underbrace{\mathcal{X}_3^0(s_{ik})}_{\textbf{Integrated antenna function}} \cdot$$



Antenna subtraction at NLO



Antenna subtraction at NLO

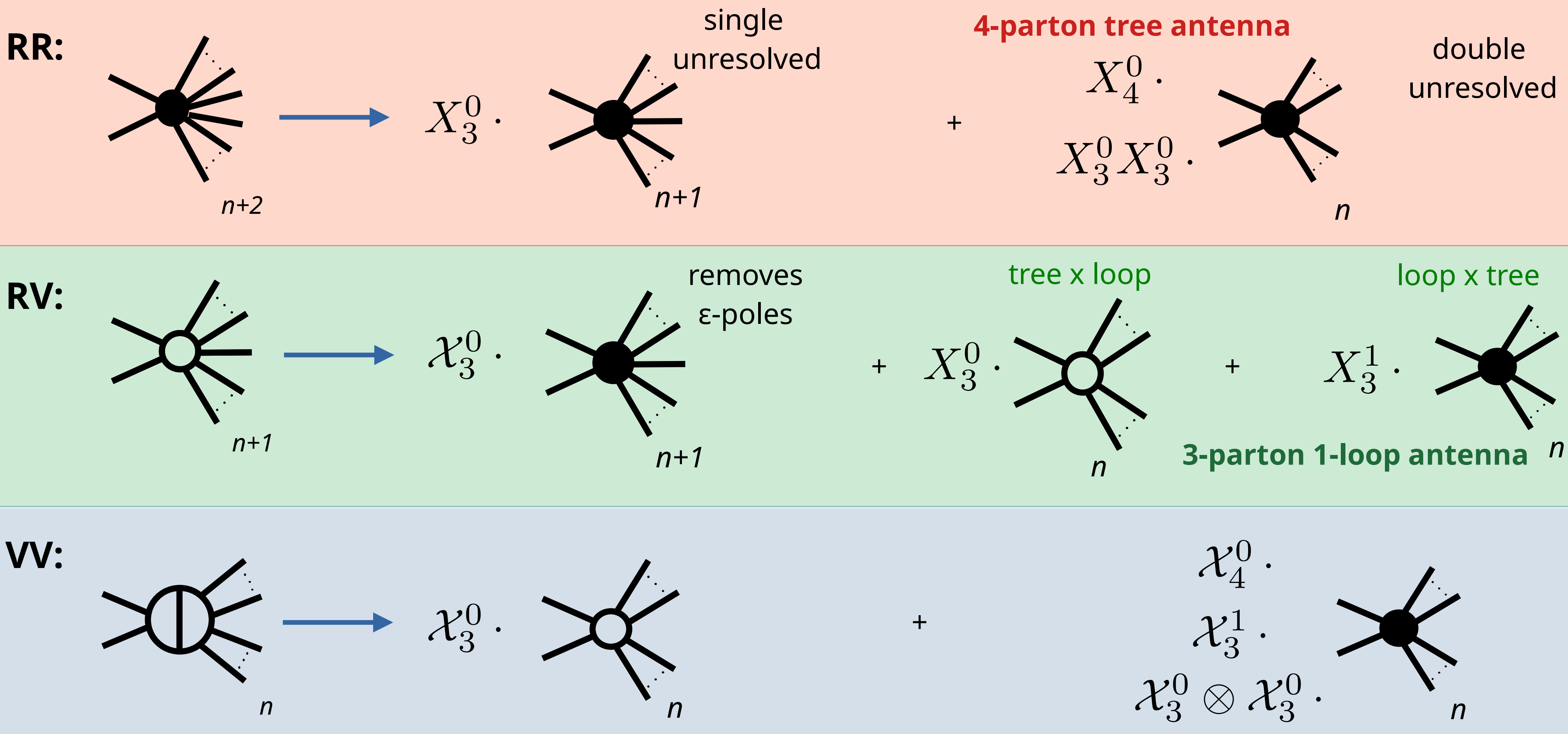


Matrix elements for suitable **colour singlet** decay.

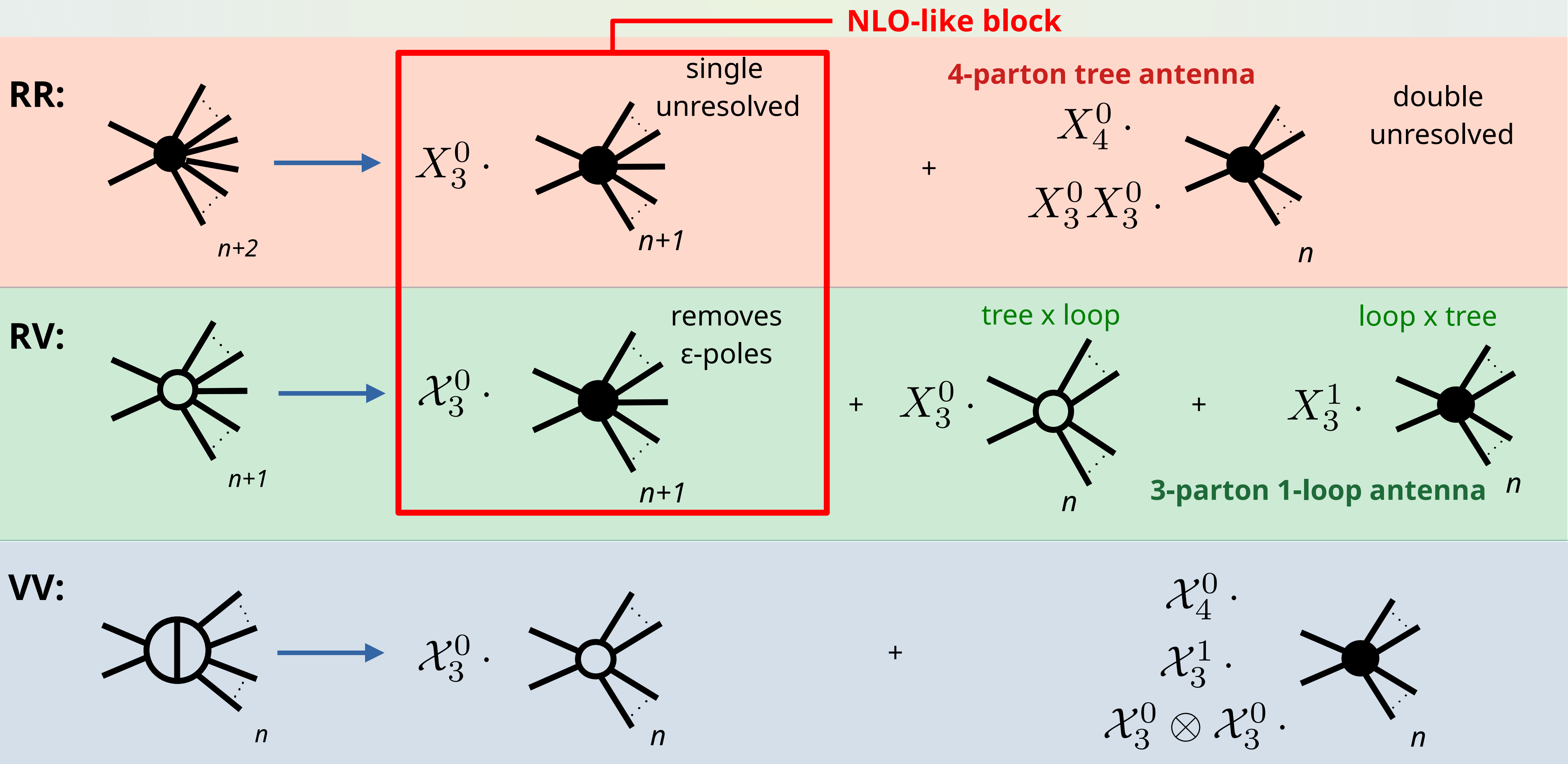
$$X_3^0 = \frac{M_3^0}{M_2^0}$$

$$\mathcal{X}_3^0 \propto \int \mathrm{d}\Phi_3 \, X_3^0$$

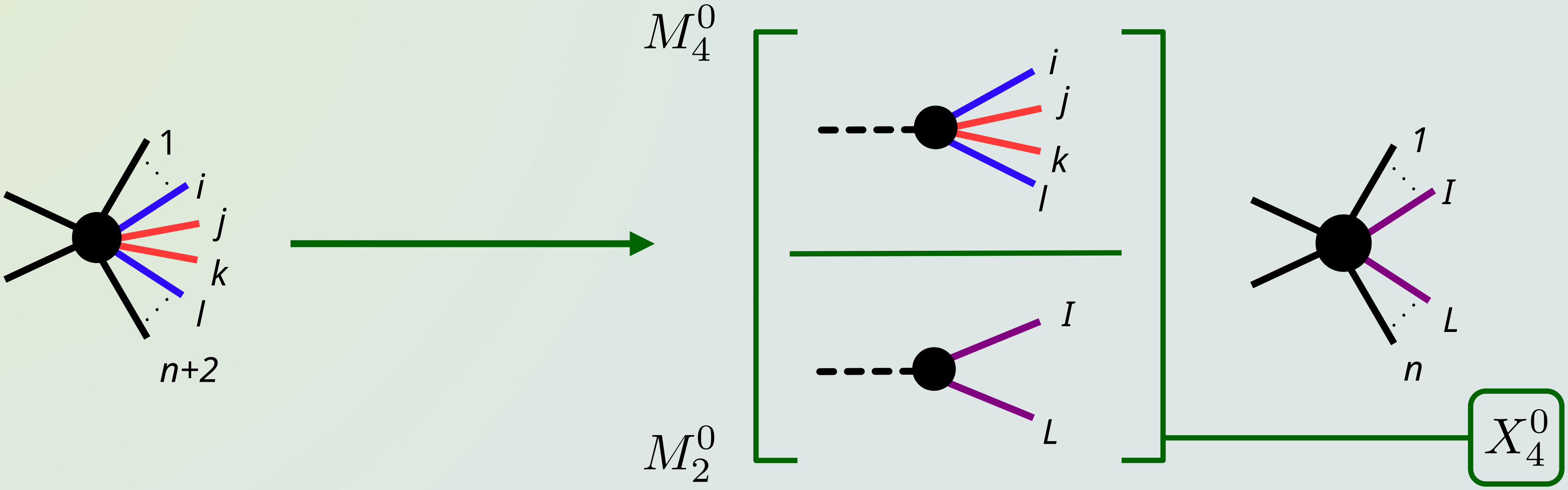
Antenna subtraction at NNLO



Antenna subtraction at NNLO



Antenna subtraction at NNLO

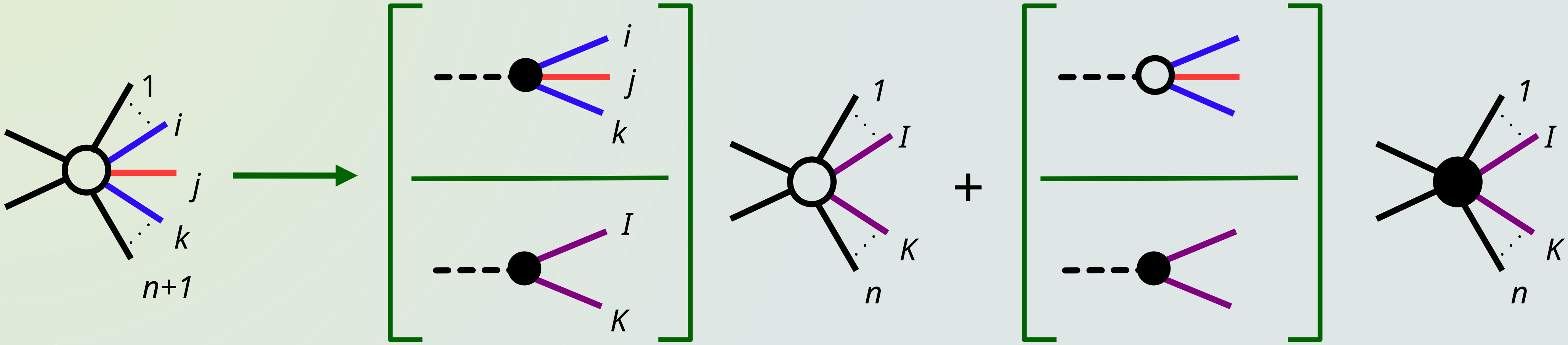


$$X_4^0 = \frac{M_4^0}{M_2^0}$$

$$\mathcal{X}_4^0 \propto \int \mathrm{d}\Phi_4 \, X_4^0$$

Extracted analogously to the three-parton case

Antenna subtraction at NNLO



Unresolved **tree-level** factor

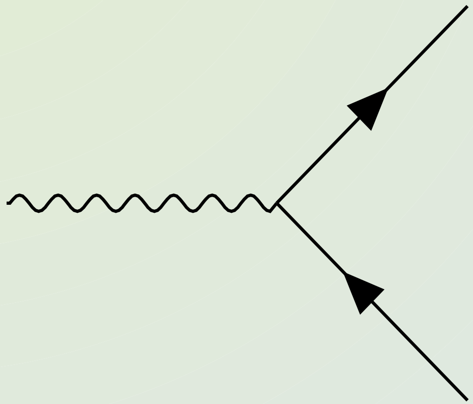
Unresolved **one-loop** factor

Three-parton one-loop antenna defined removing from the one-loop decay matrix element the unresolved tree-level configuration:

$$\boxed{X_3^1 = \frac{M_3^1}{M_2^0} - X_3^0 \frac{M_1^2}{M_2^0}, \qquad \mathcal{X}_3^1 \propto \int \mathrm{d}\Phi_3 \, X_3^1}$$

Colour singlet decays

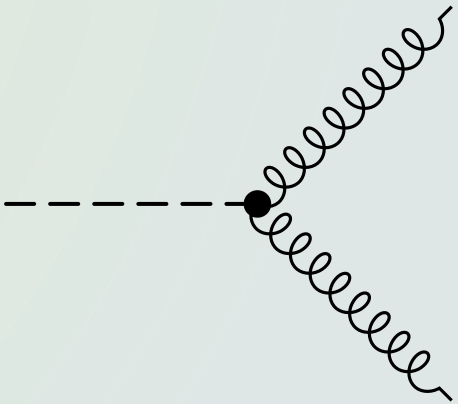
Photon decay: $\gamma^* \to q\bar{q}$



Quark-Antiquark

antenna functions

Higgs decay: $H \to gg$

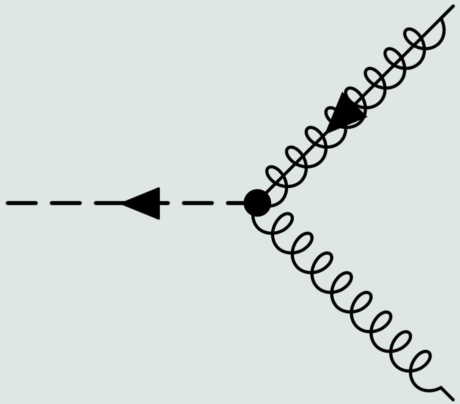


$$\mathcal{L} = -\frac{\lambda}{4} H F^{a,\mu\nu} F^a_{\mu\nu}$$

Gluon-Gluon

antenna functions

Neutralino decay: $\chi \to \tilde{g}g$



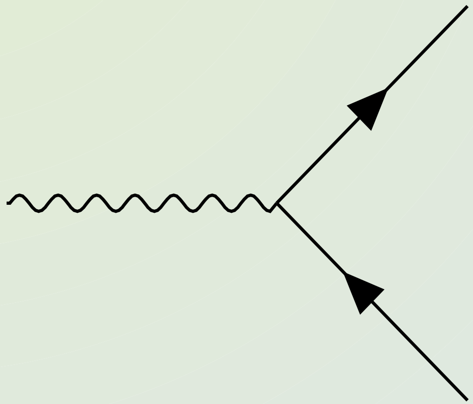
$$\mathcal{L} = i\eta\bar{\psi}^a_{\tilde{g}}\sigma^{\mu\nu}\psi_{\tilde{\chi}}F^a_{\mu\nu} + \text{h.c.}$$

Quark-Gluon

antenna functions

Colour singlet decays

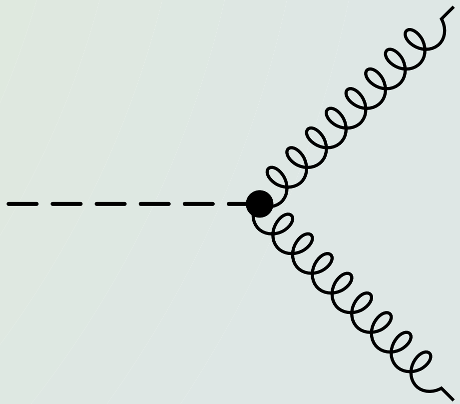
Photon decay: $\gamma^* \to q\bar{q}$



✓

Quark-Antiquark
antenna functions

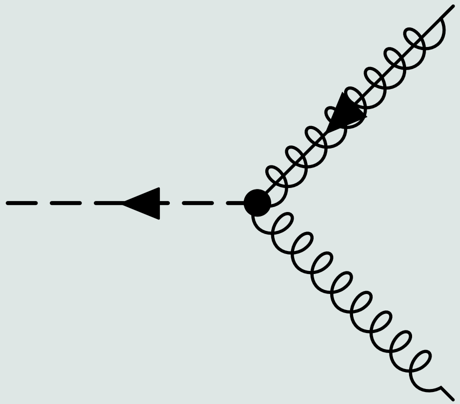
Higgs decay: $H \to gg$



✓

Gluon-Gluon
antenna functions

Neutralino decay: $\chi \to \tilde{g}g$



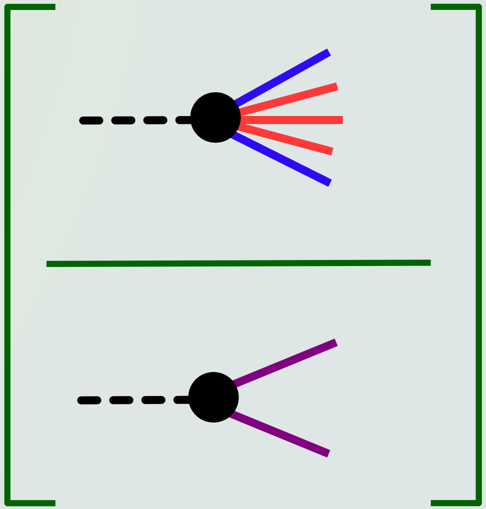
WORK IN PROGRESS

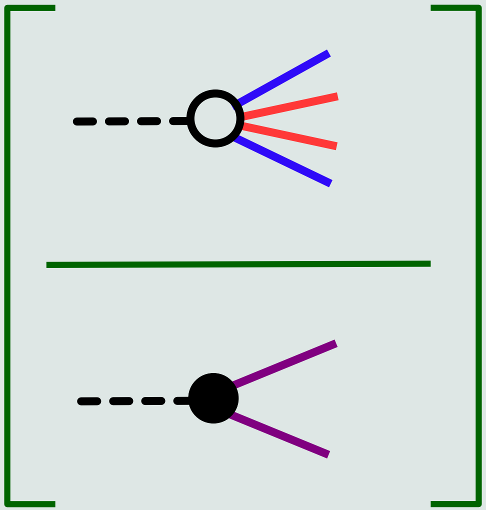
Quark-Gluon
antenna functions

Unintegrated Antenna Functions at N³LO

- **five-parton tree-level**
- **four-parton one-loop**

Extracted analogously to the NLO and NNLO cases.





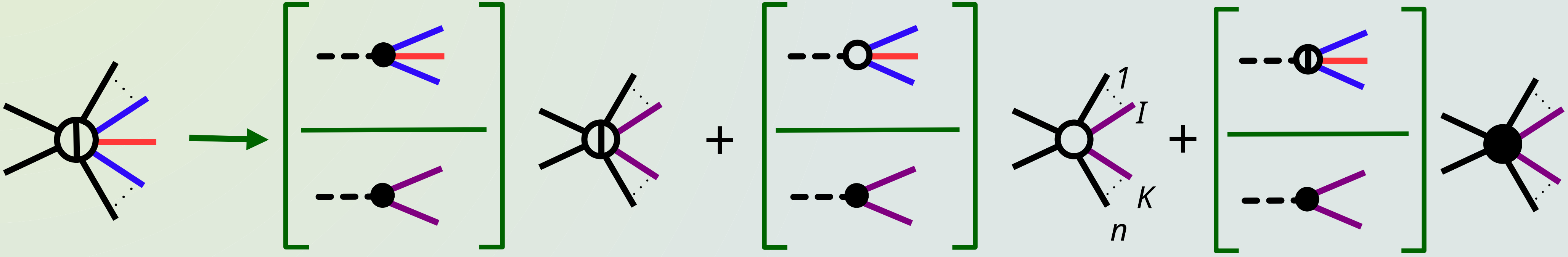
$$X_5^0 = \frac{M_5^0}{M_2^0}$$

$$\mathcal{X}_5^0 \propto \int \mathrm{d}\Phi_5 \, X_5^0$$

$$X_4^1 = \frac{M_4^1}{M_2^0} - X_4^0 \frac{M_2^1}{M_2^0}$$

$$\mathcal{X}_4^1 \propto \int \mathrm{d}\Phi_4 \, X_4^1$$

Unintegrated Antenna Functions at N³LO



Unresolved **tree-level** factor Unresolved **one-loop** factor Unresolved **two-loop** factor

Three-parton two-loop antenna is extracted removing from the two-loop decay process the one-loop and tree-level unresolved factors:

$$\boxed{X_3^2 = \frac{M_3^2}{M_2^0} - X_3^0 \frac{M_2^2}{M_2^0} - X_3^1 \frac{M_2^1}{M_2^0}, \qquad \mathcal{X}_3^2 = \int \mathrm{d}\Phi_3 X_3^2}$$

Integration strategy

- Integrated antennae derived from the integration of **renormalized matrix elements** for colour-singlet decay over the **fully inclusive phase space.** We compute:

$$\int \mathrm{d}\Phi_5 M_5^0, \quad \int \mathrm{d}\Phi_4 M_4^1, \quad \int \mathrm{d}\Phi_3 M_3^2, \quad \int \mathrm{d}\Phi_2 M_2^3$$

Two-parton three-loop, for completeness/validation:

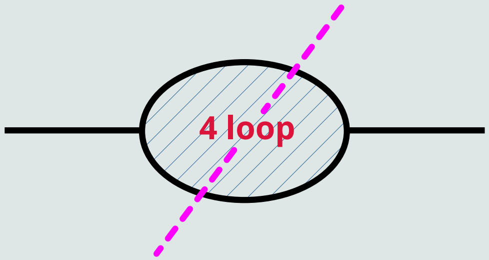
$$\int \mathrm{d}\Phi_5 M_5^0 + \int \mathrm{d}\Phi_4 M_4^1 + \int \mathrm{d}\Phi_3 M_3^2 + \int \mathrm{d}\Phi_2 M_2^3 = \quad \textbf{IR FINITE}$$

- **Reverse unitarity**:

$$2\pi i \delta^+(p^2) \to \frac{1}{p^2 + i0} - \frac{1}{p^2 - i0}$$

[Cutkosky '60]
[Anastasiou, Melnikov '02,'03]

- Phase space and (genuine) loop integrals addressed simultaneously;
- **Systematic treatment of all four layers** within a common framework;



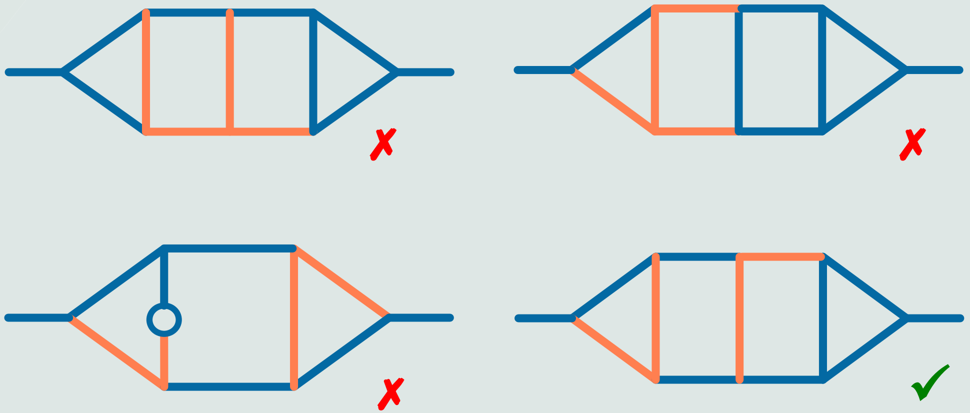
Diagrams generation

- Generation of 4-loop propagators;
- **Colouring** legs with **cuts**;
- Diagrams selection: Python
  - Number of **cuts**;
  - **Bisection** (two connected graphs, currents);
  - Number of **loops**;
  - Correct **cut species**;
  - **Momentum conservation**;
  - **No self-energy** on cut legs;
- Diagrams **tagging**: Python
  - Final state;
  - Loops type/configuration;

QGRAF [Nogueira ‘93]

	cuts	loops	loop config
VVV	2	3	$3\times0,\ 2\times1$
VRR	3	2	$2\times0,\ 1\times1$
VRR	4	1	1×0
RRR	5	0	-

Example with 4 cuts = cut propagator



diagrams:

$O(10^6)$

$O(10^3)$

Reduction to Master Integrals

- Diagrams evaluation: FORM [Vermaseren '00]
  - Insertion of **Feynman rules**;
  - **Colour** and **Dirac** algebra;
  - **Onshellness** conditions;
  - Reconstruction of **scalar integrals**;

O(10^5) integrals
max power in denominator: 11
max numerator rank: **process-dependent**

- Reduction to **Master Integrals**; Reduze2 [von Manteuffel,Studerus '12]

  0-scale reduction

	MIs
VVV	22
VRR	27
VRR	35
RRR	31

- **Master Integrals** insertion: MIs for any cut of a 4-loop propagator;

[Gituliar,Magerya,Pikelner '18]
[Magerya,Pikelner '19]

$\gamma^* \to q\bar{q}$

Presented in: 2211.08446

[Jakubčík,MM,Stagnitto '22]

Reduction:

- max numerator rank: **4**;
- full cost: O(week) on ~40 cores;

Checks:

- recovery of the **three-loop form factor**;

  [Baikov,Chetyrkin,Smirnov,Smirnov,Steinhauser '09]
  [Gehrmann,Glover,Huber,Ikizlerli,Studerus '10]

- full **cancellation of IR singularities**;

- recovery of the **total decay rate at N^{3}LO**;

  [Gorishny,Kataev,Larin '91] [Surguladze,Samuel '91]

$H \to gg$

Presented in: 2304.11180

[Chen,Jakubčík,MM,Stagnitto '23]

Reduction:

- max numerator rank: **5**;
- full cost: O(month) on ~40 cores;

Checks:

- recovery of the **three-loop form factor**;

  [Baikov,Chetyrkin,Smirnov,Smirnov,Steinhauser '09]
  [Gehrmann,Glover,Huber,Ikizlerli,Studerus '10]

- full **cancellation of IR singularities**;

- recovery of the **total decay rate at N^{3}LO**;

  [Baikov,Chetyrkin '06] [Moch,Vogt '07]

Results

- **Analytical results** up to trascendental **weight 6** for the inclusive integration, normalized to the respective LO and decomposed into:
  - layers: **V**, **R**, **VV**, **RV**, **RR**, **VVV**, **VVR**, **VRR**, **RRR**;
  - partonic channels;
  - colour factors;

- Inspection of IR poles structure and cancellation patterns:
  - understanding of **integrated unresolved structures at N3LO** is needed;
  - Comparison between $\gamma^* \to q\bar{q}$ and $H \to b\bar{b}$ (massless);
  - **abelian sector** of quark-antiquark dipole: **exponentiation of photon emission**;

$$
\begin{aligned}
\mathcal{T}^{(8)}_{q\bar{q}ggg}\Big|_{N^2} =\\
&+\frac{1}{\epsilon^6}\left(\frac{1}{2}\right)+\frac{1}{\epsilon^5}\left(\frac{331}{108}\right)+\frac{1}{\epsilon^4}\left(\frac{11843}{648}-\frac{31}{24}\pi^2\right)\\
&+\frac{1}{\epsilon^3}\left(\frac{259867}{2592}-\frac{10745}{1296}\pi^2-\frac{439}{18}\zeta_3\right)\\
&+\frac{1}{\epsilon^2}\left(\frac{6302057}{11664}-\frac{394223}{7776}\pi^2-\frac{6239}{36}\zeta_3+\frac{21853}{25920}\pi^4\right)\\
&+\frac{1}{\epsilon}\left(\frac{815913157}{279936}-\frac{26347837}{93312}\pi^2-\frac{181151}{162}\zeta_3\right.\\
&\qquad\left.+\frac{75767}{17280}\pi^4+\frac{13993}{216}\pi^2\zeta_3-\frac{10946}{45}\zeta_5\right)\\
&+\frac{736904809}{46656}-\frac{107045579}{69984}\pi^2-\frac{49920557}{7776}\zeta_3+\frac{7130357}{311040}\pi^4\\
&+\frac{67895}{144}\pi^2\zeta_3-\frac{103894}{45}\zeta_5-\frac{93257}{1306368}\pi^6+\frac{7861}{12}\zeta_3^2+\mathcal{O}(\epsilon)
\end{aligned}
$$

Abelian poles at **N^{k}LO** for **r** real emissions and **(k-r)** loops

$$\propto \binom{k}{r}(-1)^r\left(I^{(1)}_{q\bar{q}}\right)^k+\mathcal{O}(\epsilon^{-2k+2})$$

$$I^{(1)}_{q\bar{q}} = -\frac{2}{\epsilon^2}-\frac{3}{\epsilon}+\mathcal{O}(\epsilon^0) \quad (\mu^2=q^2)$$

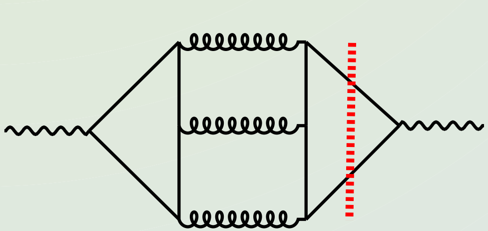
Singlet contribution in $\gamma^* \to q\bar{q}$

The **singlet contribution**, proportional to

$$\left(\sum_q e_q\right)^2 d^{abc}d^{abc} = \left(\sum_q e_q\right)^2 (N - 4/N)$$

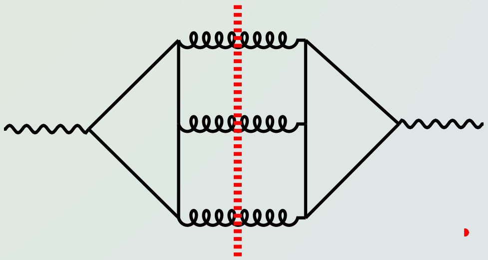
appears for the first time at three-loop order.

VVV



IR FINITE

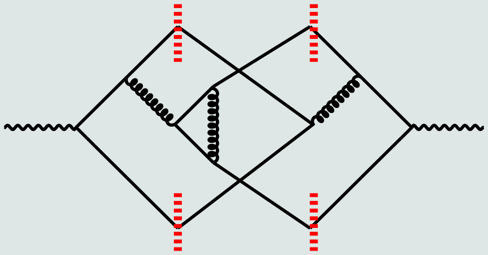
VVR



IR FINITE

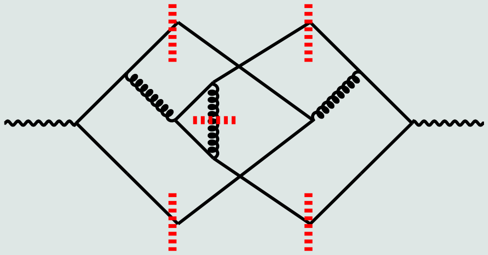
No counterterms

VRR



ε^{-1}

RRR



ε^{-1}

Related to antisymmetric contributions to:

- one-loop triple collinear splitting for $qQ\bar{Q}$
- tripole term in soft current for $gq\bar{q}$

[Catani,de Florian,Rodrigo '04]

[Del Duca,Duhr,Haindl,Liu '22]
[Catani,Cieri,Colferai,Coradeschi '22]

Antenna subtraction at N3LO

Consider a process with **h** hard partons at LO: **NLO** **NNLO** **N3LO**

	Loops	Emissions
M_n^ℓ	ℓ	$n-h$
X_n^ℓ	ℓ	$n-2$
$\mathcal{X}_n^\ell$	$\ell+n-2$	0

$$RRR: \; +X_3^0\tilde{M}_{h+2}^0 + \left(X_4^0 - X_3^0X_3^0\right)\tilde{M}_{h+1}^0 + \left(X_5^0 - X_4^0X_3^0 + X_3^0X_3^0X_3^0\right)\tilde{M}_h^0$$

$$VRR: \; +\mathcal{X}_3^0\tilde{M}_{h+2}^0 + \left(X_3^1 + \mathcal{X}_3^0X_3^0\right)\tilde{M}_{h+1}^0 + X_3^0\tilde{M}_{h+1}^1 + \left(X_4^0 + X_3^0X_3^0\right)\tilde{M}_h^1$$
$$+\left(\mathcal{X}_3^0X_3^0X_3^0 + \mathcal{X}_3^0X_4^0 + X_4^1 - X_3^1X_3^0\right)\tilde{M}_h^0$$

$$VVR: \; +\left(\mathcal{X}_4^0 + \mathcal{X}_3^1 + \mathcal{X}_3^0\mathcal{X}_3^0\right)\tilde{M}_{h+1}^0 + \mathcal{X}_3^0\tilde{M}_{h+1}^1 + X_3^2\tilde{M}_h^0 - X_3^1\tilde{M}_h^1$$
$$-X_3^0\tilde{M}_h^2 + \mathcal{X}_3^0X_3^0\tilde{M}_h^1 + \mathcal{X}_3^0X_3^1\tilde{M}_h^0 + \mathcal{X}_4^0X_3^0\tilde{M}_h^0$$
$$+\mathcal{X}_3^1X_3^0\tilde{M}_h^0 + \mathcal{X}_3^0X_3^0X_3^0\tilde{M}_h^0$$

$$VVV: \; +\left(\mathcal{X}_5^0 + \mathcal{X}_4^1 + \mathcal{X}_3^2 + \mathcal{X}_4^0\mathcal{X}_3^0 + \mathcal{X}_3^1\mathcal{X}_3^0 + \mathcal{X}_3^0\mathcal{X}_3^0\mathcal{X}_3^0\right)\tilde{M}_h^0$$
$$+\left(\mathcal{X}_4^0 + \mathcal{X}_3^1 + \mathcal{X}_3^0\mathcal{X}_3^0\right)\tilde{M}_h^1 + \mathcal{X}_3^0\tilde{M}_h^2$$

Removal of lower-order unresolved configurations:

- **NLO:** X_3^0
- **NNLO:** $X_4^0 - \sum X_3^0X_3^0\,, \quad X_3^1$
- **N3LO:** $X_5^0 - \sum X_4^0X_3^0 + \sum X_3^0X_3^0X_3^0\,, \quad X_4^1 - \sum X_3^1X_3^0$

Preliminary study for N3LO structures

Conclusions and outlook

- We computed integrated **quark-antiquark** and **gluon-gluon** antenna functions for **final-state radition at N^{3}LO**;

- Explicit identification of integrated real-emission **IR singularities at N^{3}LO**;

- First thoughts towards the extension of **antenna subtraction to N^{3}LO**;

- **Outlook:** computation of **quark-gluon** antenna functions;

- **Outlook:** detailed **catalogation and study** of antenna functions for final-state radiation at N^{3}LO;

Thank you for your attention!

Backup: Diagrams matching

- Diagrams matched onto **uncut families** first;

Reduze2 [von Manteuffel,Studerus '12]

F_1		F_2		F_3	
k_1	$k_2 - k_3$	k_1	$k_2 - k_3$	k_1	$k_4 - k_2$
k_2	$k_2 - k_4$	k_2	$k_2 - k_3$	k_2	$k_3 - k_4$
k_3	$k_3 - k_4$	k_3	$k_3 - k_4$	k_4	$q - k_1$
k_4	$q - k_1$	k_4	$q - k_1$	$k_1 - k_2$	$q - k_2$
$k_1 - k_2$	$q - k_2$	$k_1 - k_2$	$q - k_2$	$k_1 - k_3$	$q - k_3$
$k_1 - k_3$	$q - k_3$	$k_1 - k_3$	$q - k_4$	$k_1 - k_4$	$q + k_2 - k_3$
$k_1 - k_4$	$q - k_4$	$k_1 - k_4$	$q + k_3 - k_4$	$k_2 - k_3$	$q + k_4 - k_3 - k_1$

$$\dim(\mathrm{F}_n) = 14$$

- Diagrams matched onto **families with cut propagators:**
  - Pick a diagram;
  - Apply **mapping to uncut family** to the set of cut momenta;
  - If needed, add new cut family $\mathrm{F}_n\mathrm{c}_x$;

At N^{3}LO:
O(10)
cut families

Any residual **redundancy** is eliminated during the reduction process.

Goal: have a **sufficient** and **reasonably small** set of cut families.

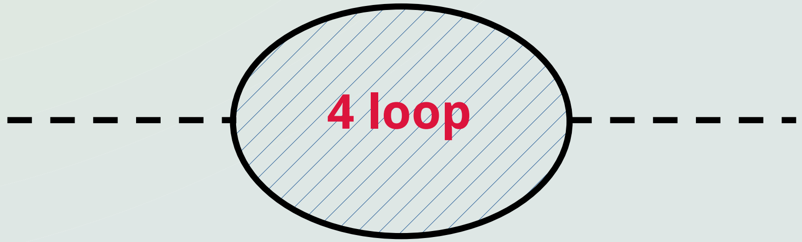
Backup: $\chi \to \tilde{g}g$

Neutralino decay for **quark-gluon antennae**: what is missing?

- N^{3}LO matrix elements;
- Three-loop renormalization coefficient for the effective coupling;
- Total decay rate at N^{3}LO: **crucial for validation**;

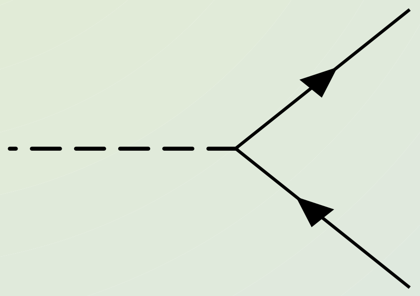
FORCER

[Ruijl,Ueda,Vermaseren ‘16,’17]

$Im[$  $]$

Backup: $H \to b\bar{b}$

Analogous calculation for the decay of a Higgs boson into a quark-antiquark pair:



- **Non-vanishing Yukawa coupling**
- **Massless quarks**

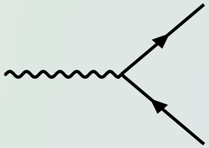
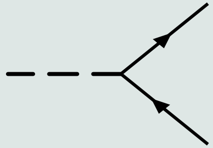
→ Vanishing singlet contribution

Reduction: same as the photon case;

Checks:

- recovery of the **three-loop form factor**; [Gehrmann,Kara '14]
- full **cancellation of IR singularities**;
- recovery of the **total decay rate at N^{3}LO**; [Baikov,Chetyrkin,Kühn '06] [Herzog,Ruijl,Ueda,Vermaseren,Vogt '17]

Backup: $H \to b\bar{b}$ vs $\gamma^* \to q\bar{q}$

Similarities are of course expected between  and  .

For each **partonic channel** (neglecting singlet) and **colour factor** up to N^{3}LO:

- the **two deepest non-vanishing IR poles** agree:
  - beyond this, finite parts and higher-ε terms of lower orders matter;
  - virtuals: predicatbility of IR poles; [Catani ‘98] [Becher,Neubert ‘09] [Almelid,Duhr,Gardi ‘16]
  - RR: structures analized in [Braun-White,Glover,Preuss ‘23]

- the terms with the **highest trascendental weight,** agree for every power of **ε**:
  - correspondence of highest weight terms in the **inclusive decay rates**;

In fact, up to **N^{4}LO**

[Herzog,Ruijl,Ueda,Vermaseren,Vogt ‘17]

Should be possible to infer this from the two-point function