Improved Antenna Subtraction at NNLO

Oscar Braun-White (he/him/his), IPPP Durham

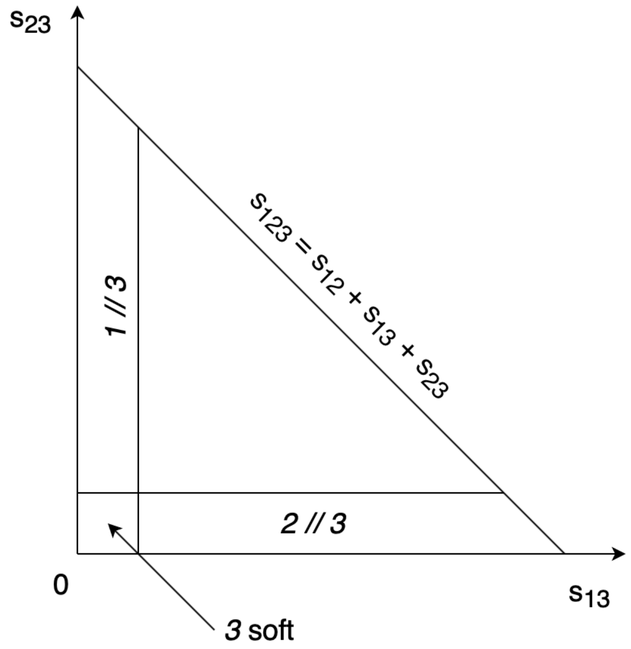
Based on work with Nigel Glover (IPPP Durham)
and Christian Preuss (ETH Zurich)
in arXiv:2302.12787 - accepted into JHEP, not yet published

RADCOR, Crieff, Scotland, 01/06/23

Introduction

Infrared Divergences

Subtraction/slicing at different orders in α_s



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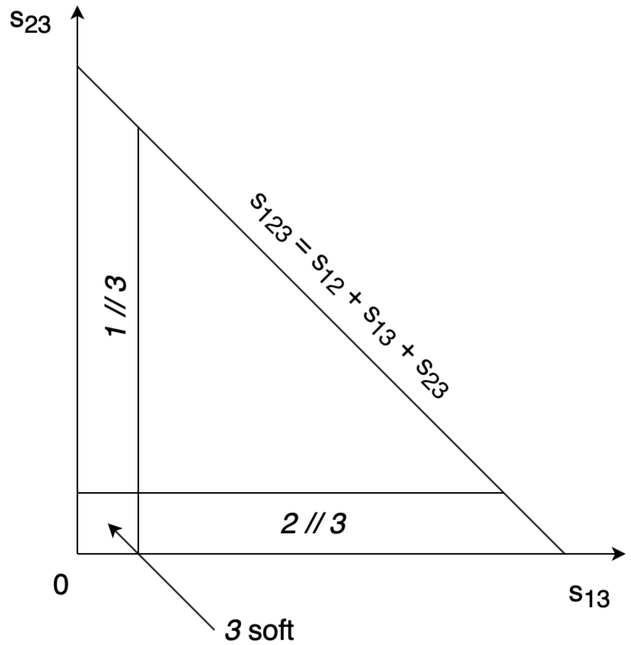

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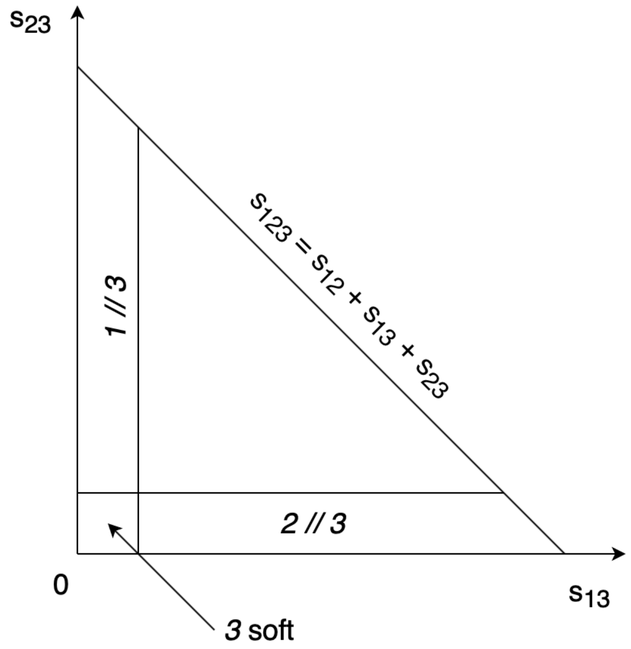

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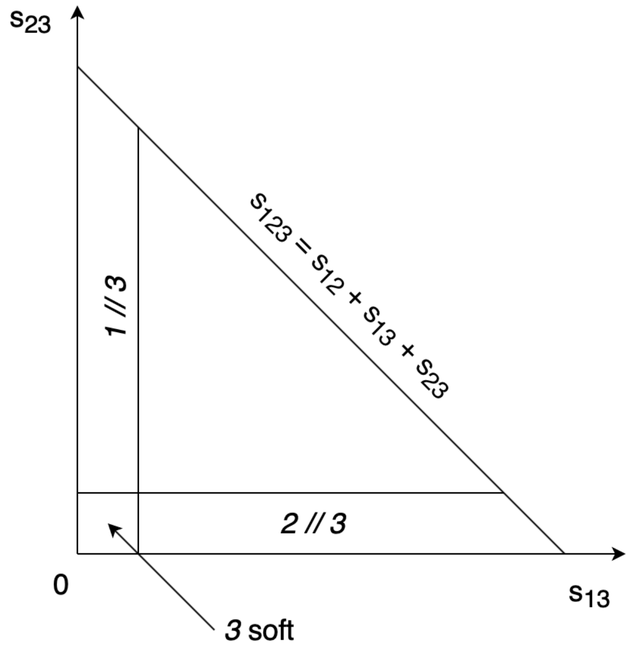

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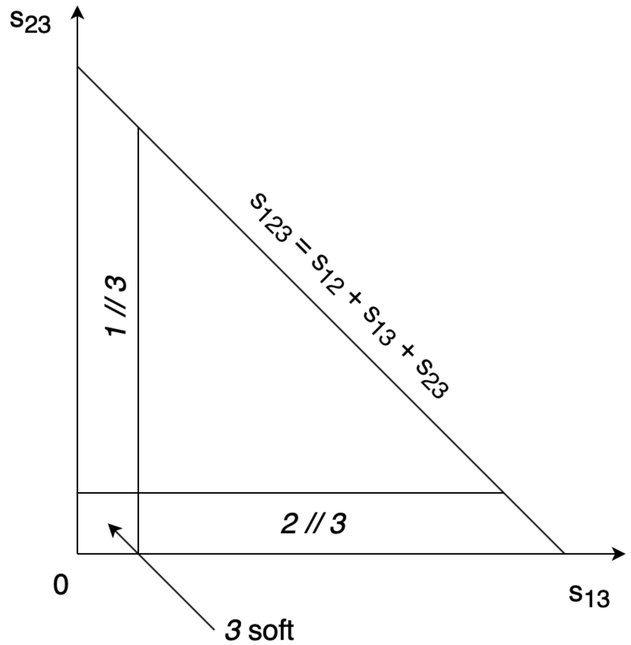
More Legs

More Loops



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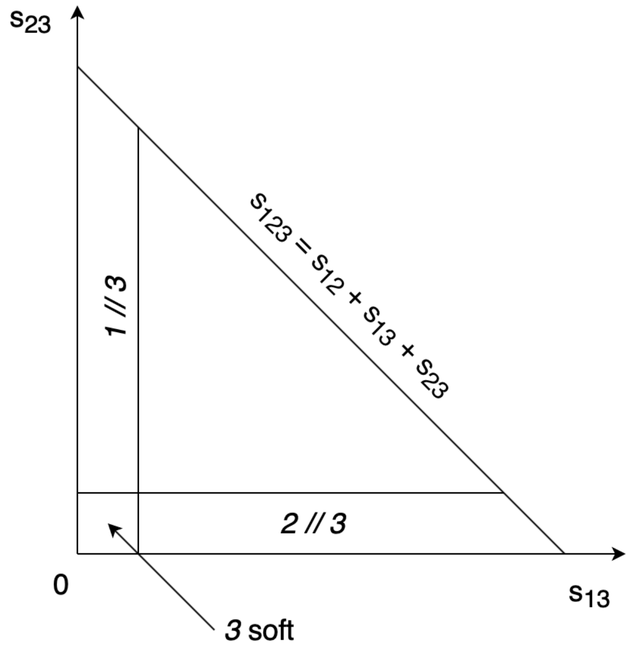

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- NNLOJET group uses antenna functions

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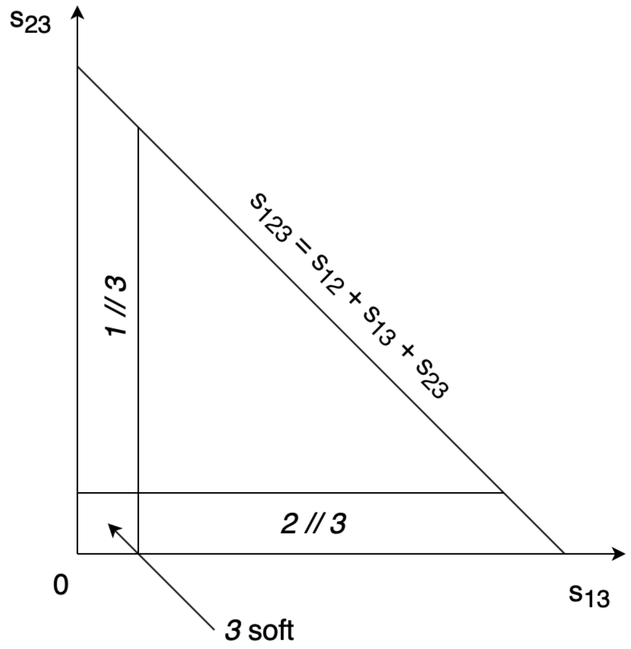

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Subtraction at NLO

m-jet cross section

$$d\hat{\sigma}^{NLO} = \left[\int_{d\Phi_m} d\hat{\sigma}_V^{NLO} + \int_{d\Phi_{m+1}} d\hat{\sigma}_S^{NLO}\right] + \int_{d\Phi_{m+1}} (d\hat{\sigma}_R^{NLO} - d\hat{\sigma}_S^{NLO})$$

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Combination is IR finite and numerically integrable.

No explicit ϵ poles in R or S

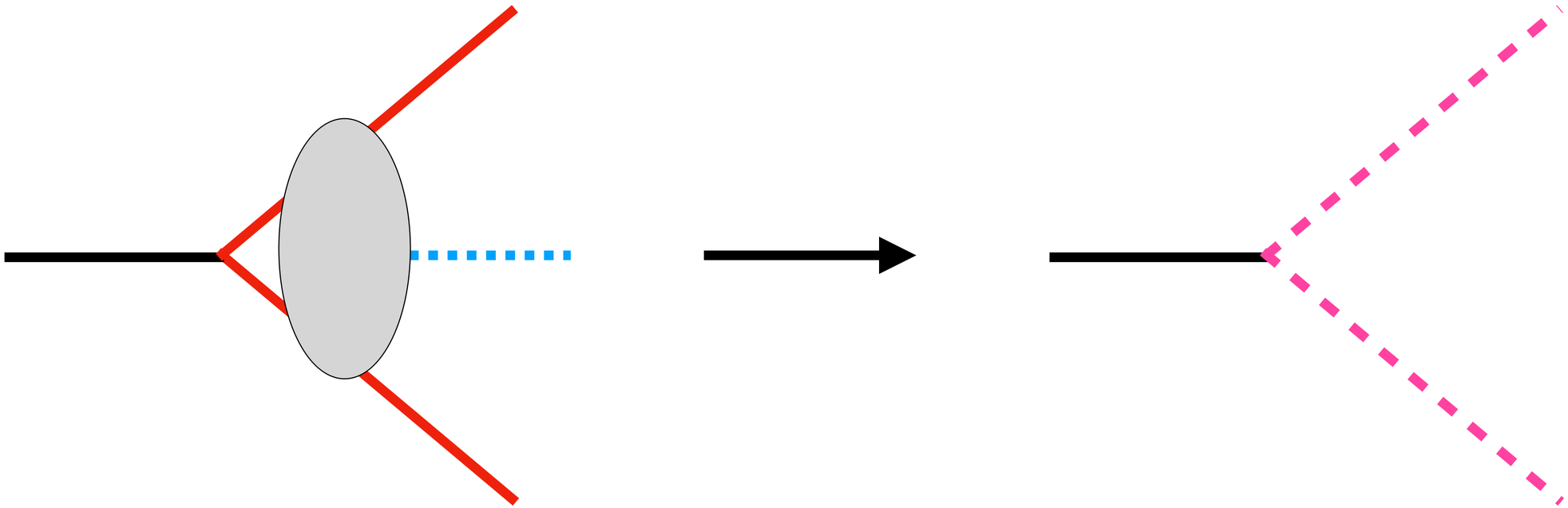
Antenna Subtraction

Started by Glover, Gehrmann and Gehrmann-De Ridder ~ 2005
NNLOJET

Subtraction Term at NLO - X_3^0

Two hard radiators, one unresolved particle

- Maps momentum of 3 → 2, where ..... is soft or collinear with either of
- Contains limits associated with one particle unresolved



Subtraction Term at NLO - X_3^0

Sum over one particle unresolved at a time

$$d\hat{\sigma}_S^{NLO} \approx \sum_{m+1} d\Phi_{m+1} \sum_j X_3^0(i^h, j, k^h) \, |\mathscr{M}_m|^2 J_m^{(m)}$$

Different X_3^0 expressions depending on particle types of $\{i, j, k\}$

Subtracts against colour-ordered sub-amplitudes

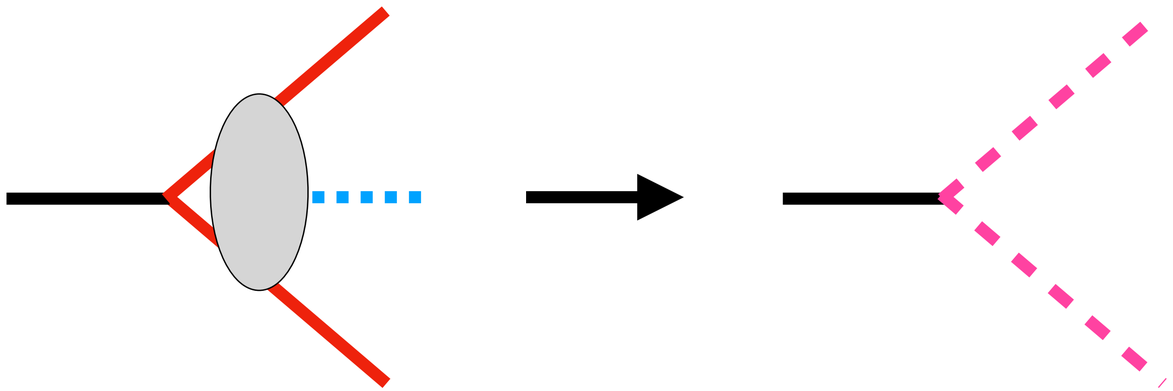
$$d\Phi_{m+1} = d\Phi_m \cdot d\Phi_{X_{ijk}} \qquad \mathscr{X}_3^0 \approx \int_{X_{ijk}} d\Phi_{X_{ijk}} X_3^0$$

Poles up to ϵ^{-2}

Antennae at NNLO

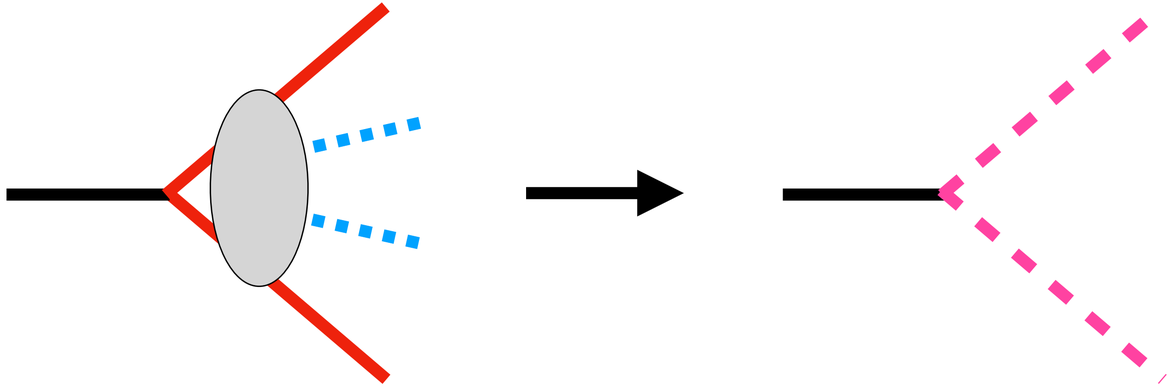
X_3^0 - Single unresolved limits of ..... .

No ϵ poles.



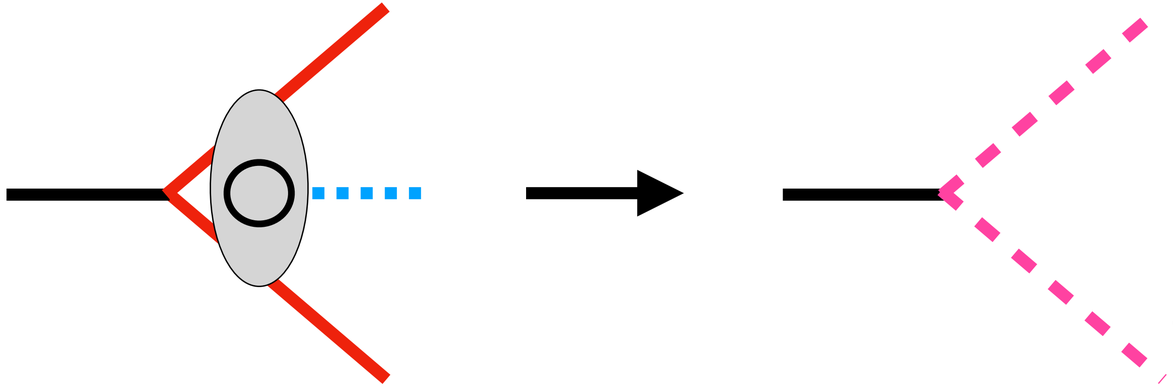
X_4^0 - Double and single unresolved limits of ..... .

No ϵ poles.



X_3^1 - Single unresolved limits of ..... .

Poles up to ϵ^{-2} from loop ○.



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$$+ \int_{d\Phi_{m+1}} (RV - S_{RV})$$
$$+ \int_{d\Phi_{m+2}} (RR - S_{RR}) + \int_{d\Phi_{m+2}} S_{RR}$$

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See Matteo's talk for a more detailed discussion…

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- X_4^0 sometimes have spurious infrared limits

Improved Antennae

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- Antenna functions obey physical symmetry relations (such as line reversal).

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Improved X_m^0 Antennae - $X_m^0(i_1^h, i_2, \ldots, i_{m-1}, i_m^h)$

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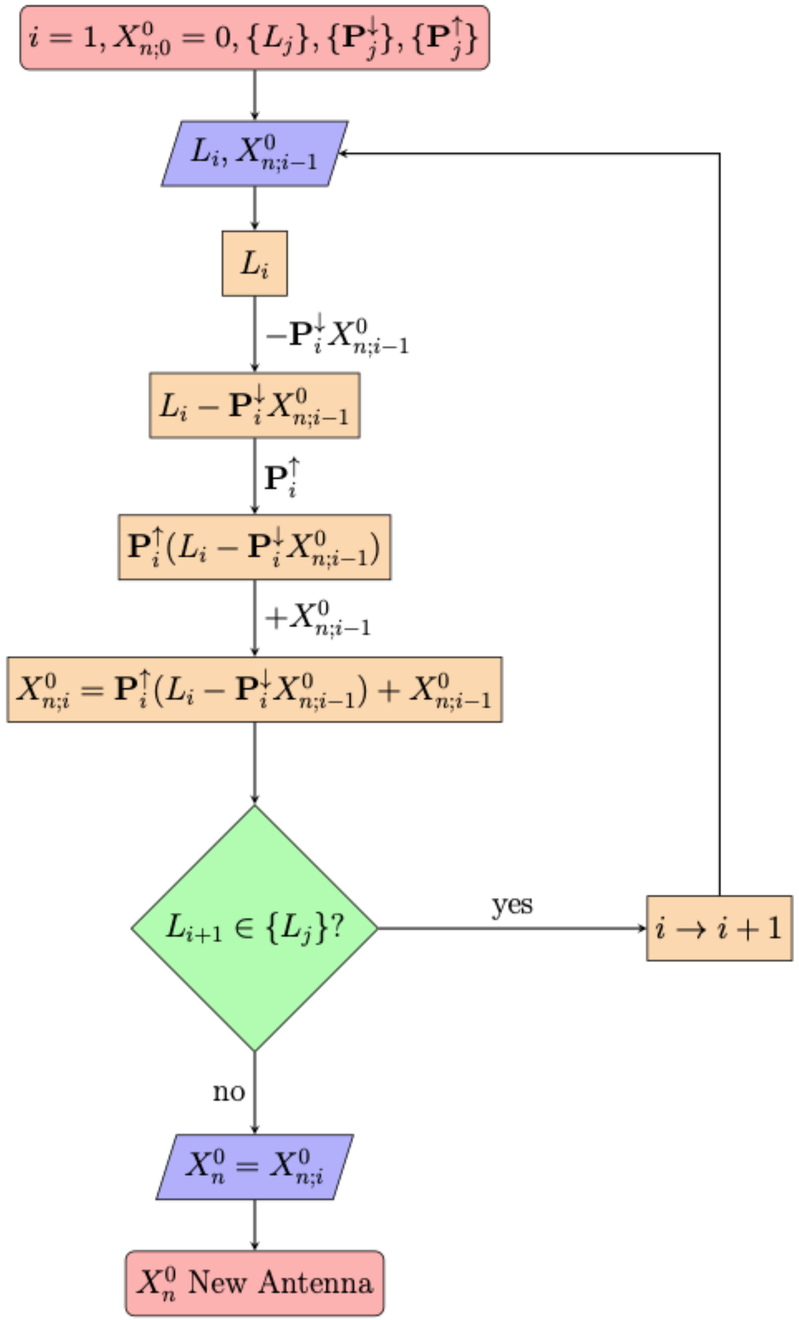
$$\begin{aligned} X_{m,1}^0 &= P_1^{\uparrow} L_1 \\ X_{m,2}^0 &= X_{m,1}^0 + P_2^{\uparrow}(L_2 - P_2^{\downarrow} X_{m,1}^0) \\ &\vdots \\ X_{m,N}^0 &= X_{m,N-1}^0 + P_N^{\uparrow}(L_N - P_N^{\downarrow} X_{m,N-1}^0) \end{aligned}$$

Flowchart


$i = 1, X^0_{n;0} = 0, \{L_j\}, \{\mathbf{P}^\downarrow_j\}, \{\mathbf{P}^\uparrow_j\}$
$L_i, X^0_{n;i-1}$
L_i
$-\mathbf{P}^\downarrow_i X^0_{n;i-1}$
$L_i - \mathbf{P}^\downarrow_i X^0_{n;i-1}$
$\mathbf{P}^\uparrow_i$
$\mathbf{P}^\uparrow_i(L_i - \mathbf{P}^\downarrow_i X^0_{n;i-1})$
$+X^0_{n;i-1}$
$X^0_{n;i} = \mathbf{P}^\uparrow_i(L_i - \mathbf{P}^\downarrow_i X^0_{n;i-1}) + X^0_{n;i-1}$
$L_{i+1} \in \{L_j\}?$
yes
$i \to i+1$
no
$X^0_n = X^0_{n;i}$
X^0_n New Antenna

Synthetic Antennae - $X_3^0(i^h, j, k^h)$

$$X_3^0(i^h, j, k^h) = \text{Ssoft}(i^h, j, k^h) + \text{Scol}(i^h, j; k^h) + \text{Scol}(k^h, j; i^h)$$

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Example - $A_3^0(i_q^h, j_g, k_{\bar{q}}^h)$

$$A_3^0(i_q^h, j_g, k_{\bar{q}}^h) = \frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{(1-\epsilon)s_{jk}}{s_{ijk}s_{ij}} + \frac{(1-\epsilon)s_{ij}}{s_{ijk}s_{jk}}$$

This differs from the old antenna derived directly from $|\mathscr{M}|^2$ for $\gamma^* \to qg\bar{q}$ at $\mathcal{O}(\epsilon)$

Example - $A_3^0(i_q^h, j_g, k_{\bar{q}}^h)$

$$A_3^0(i_q^h, j_g, k_{\bar{q}}^h) = \frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{(1-\epsilon)s_{jk}}{s_{ijk}s_{ij}} + \frac{(1-\epsilon)s_{ij}}{s_{ijk}s_{jk}}$$

This differs from the old antenna derived directly from $|\mathscr{M}|^2$ for $\gamma^* \to qg\bar{q}$ at $\mathcal{O}(\epsilon)$

Integrating over antenna phase space:

$$\mathscr{A}_3^0(s_{ijk}) = \left(s_{ijk}\right)^{-\epsilon}\left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{19}{4} - \frac{7\pi^2}{12} + \left(\frac{113}{8} - \frac{7\pi^2}{8} - \frac{25\zeta_3}{3}\right)\epsilon + \left(\frac{675}{16} - \frac{133\pi^2}{48} - \frac{71\pi^4}{1440} - \frac{25\zeta_3}{2}\right)\epsilon^2 + \mathcal{O}(\epsilon^3)\right]$$

Example - $F_3^0(i_g^h, j_g, k_g^h)$

$$F_3^0(i_g^h, j_g, k_g^h) = \frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{s_{ik}s_{jk}}{s_{ijk}^2 s_{ij}} + \frac{s_{ij}s_{ik}}{s_{ijk}^2 s_{jk}}$$

This differs from the old antenna derived directly from $|\mathscr{M}|^2$ for Higgs boson decay, for which any of the three gluons can be soft. Here only j can be soft.

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Integrating over antenna phase space:

$$\mathscr{F}_3^0(s_{ijk}) = \left(s_{ijk}\right)^{-\epsilon}\left[\frac{1}{\epsilon^2} + \frac{11}{6\epsilon} + \frac{65}{12} - \frac{7\pi^2}{12} + \left(\frac{129}{8} - \frac{77\pi^2}{72} - \frac{25\zeta_3}{3}\right)\epsilon + \left(\frac{771}{16} - \frac{455\pi^2}{144} - \frac{71\pi^4}{1440} - \frac{275\zeta_3}{18}\right)\epsilon^2 + \mathscr{O}(\epsilon^3)\right]$$

Example - $F_3^0(i_g^h, j_g, k_g^h)$

$$F_3^0(i_g^h, j_g, k_g^h) = \frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{s_{ik}s_{jk}}{s_{ijk}^2 s_{ij}} + \frac{s_{ij}s_{ik}}{s_{ijk}^2 s_{jk}}$$

Here $L_2 = \text{Collinear}\,(i^h, j) = \dfrac{P_{gg}(i^h, j)}{s_{ij}} = \dfrac{1}{s_{ij}}\left(\dfrac{2\left(1 - x_j\right)}{x_j} + x_j\left(1 - x_j\right)\right)$

where $P_{gg}(i^h, j) + P_{gg}(j^h, i) \equiv P_{gg}(x_j)$

L_3 similar

Synthetic Antennae - $X_4^0(i^h, j, k, l^h)$

Same algorithm, more limits

$$
\begin{aligned}
X_4^0(i^h, j, k, l^h) = {} & Dsoft(i^h, j, k, l^h) \\
& + Tcol(i^h, j, k; l^h) + Tcol(l^h, k, j; i^h) \\
& + Dcol(i^h, j; k, l^h) \\
& + Ssoft(i^h, j, k; l^h) + Ssoft(j, k, l^h; i^h) \\
& + Scol(i^h, j; k, l^h) + Scol(j, k; i^l, l^h) + Scol(l^h, k; j, i^h)
\end{aligned}
$$

Slightly different structure for sub-leading colour antennae

Synthetic Antennae - $X_4^0(i^h, j, k, l^h)$

Same algorithm, more limits

For $Tcol(i^h, j, k; l^h)$, need $P_{abc \to A}(i^h, j, k)$ for all $\{a, b, c\}$ QCD particle types.

Synthetic Antennae - $X_4^0(i^h, j, k, l^h)$

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For $Tcol(i^h, j, k; l^h)$, need $P_{abc\to A}(i^h, j, k)$ for all $\{a, b, c\}$ QCD particle types.

Most $P_{abc\to A}(i, j, k)$ have a hard particle by default, except $P_{g\bar{q}q\to g}(i, j, k)$ and $P_{ggg\to g}(i, j, k)$

Synthetic Antennae - $X_4^0(i^h, j, k, l^h)$

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Most $P_{abc\to A}(i, j, k)$ have a hard particle by default, except $P_{g\bar{q}q\to g}(i, j, k)$ and $P_{ggg\to g}(i, j, k)$

This work done in JHEP09(2022)059, arXiv: 2204.10755,
O. Braun-White and N. Glover,
Decomposition of triple collinear splitting functions

Example - $A_4^0(1_q^h, 2_g, 3_g, 4_{\bar{q}}^h)$

Built using double unresolved limits, single unresolved limits and A_3^0

$$
\begin{aligned}
\mathbf{S}_{23}^{\downarrow} A_4^0(1^h,2,3,4^h) &= S_{gg}(1^h,2,3,4^h) \\
\mathbf{TC}_{123}^{\downarrow} A_4^0(1^h,2,3,4^h) &= P_{qgg}(1^h,2,3) \\
\mathbf{TC}_{234}^{\downarrow} A_4^0(1^h,2,3,4^h) &= P_{qgg}(4^h,3,2) \\
\mathbf{DC}_{1234}^{\downarrow} A_4^0(1^h,2,3,4^h) &= P_{qg}(1^h,2)P_{qg}(4^h,3) \\
\mathbf{S}_{2}^{\downarrow} A_4^0(1^h,2,3,4^h) &= \frac{2s_{13}}{s_{12}s_{23}} A_3^0(1,3,4) \\
\mathbf{S}_{3}^{\downarrow} A_4^0(1^h,2,3,4^h) &= \frac{2s_{24}}{s_{23}s_{34}} A_3^0(1,2,4) \\
\mathbf{C}_{12}^{\downarrow} A_4^0(1^h,2,3,4^h) &= P_{qg}(1^h,2)A_3^0([1+2],3,4) \\
\mathbf{C}_{23}^{\downarrow} A_4^0(1^h,2,3,4^h) &= P_{gg}(2,3)A_3^0(1,[2+3],4) \\
\mathbf{C}_{34}^{\downarrow} A_4^0(1^h,2,3,4^h) &= P_{qg}(4^h,3)A_3^0(1,2,[3+4]) \\
\mathbf{SC}_{2;34}^{\downarrow} A_4^0(1^h,2,3,4^h) &= \frac{2s_{134}}{s_{12}s_{234}} P_{qg}(4^h,3) \\
\mathbf{SC}_{3;12}^{\downarrow} A_4^0(1^h,2,3,4^h) &= \frac{2s_{124}}{s_{123}s_{34}} P_{qg}(1^h,2)
\end{aligned}
$$

Example - $A_4^0(1_q^h, 2_g, 3_g, 4_{\bar{q}}^h)$

Built using double unresolved limits, single unresolved limits and A_3^0

$$\mathbf{S}_{23}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = S_{gg}(1^h, 2, 3, 4^h)$$

$$\mathbf{TC}_{123}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = P_{qgg}(1^h, 2, 3)$$

$$\mathbf{TC}_{234}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = P_{qgg}(4^h, 3, 2)$$

$$\mathbf{DC}_{1234}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = P_{qg}(1^h, 2) P_{qg}(4^h, 3)$$

$$\mathbf{S}_{2}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = \frac{2 s_{13}}{s_{12} s_{23}} A_3^0(1, 3, 4)$$

$$\mathbf{S}_{3}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = \frac{2 s_{24}}{s_{23} s_{34}} A_3^0(1, 2, 4)$$

$$\mathbf{C}_{12}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = P_{qg}(1^h, 2) A_3^0([1+2], 3, 4)$$

$$\mathbf{C}_{23}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = P_{gg}(2, 3) A_3^0(1, [2+3], 4)$$

$$\mathbf{C}_{34}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = P_{qg}(4^h, 3) A_3^0(1, 2, [3+4])$$

$$\mathbf{SC}_{2;34}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = \frac{2 s_{134}}{s_{12} s_{234}} P_{qg}(4^h, 3)$$

$$\mathbf{SC}_{3;12}^{\downarrow} A_4^0(1^h, 2, 3, 4^h) = \frac{2 s_{124}}{s_{123} s_{34}} P_{qg}(1^h, 2)$$

Example - $A_4^0(i_q^h, j_g, k_g, l_{\bar{q}}^h)$

Expression too long to write here but integrated antenna matches $\mathscr{A}_4^{0,\text{OLD}}$

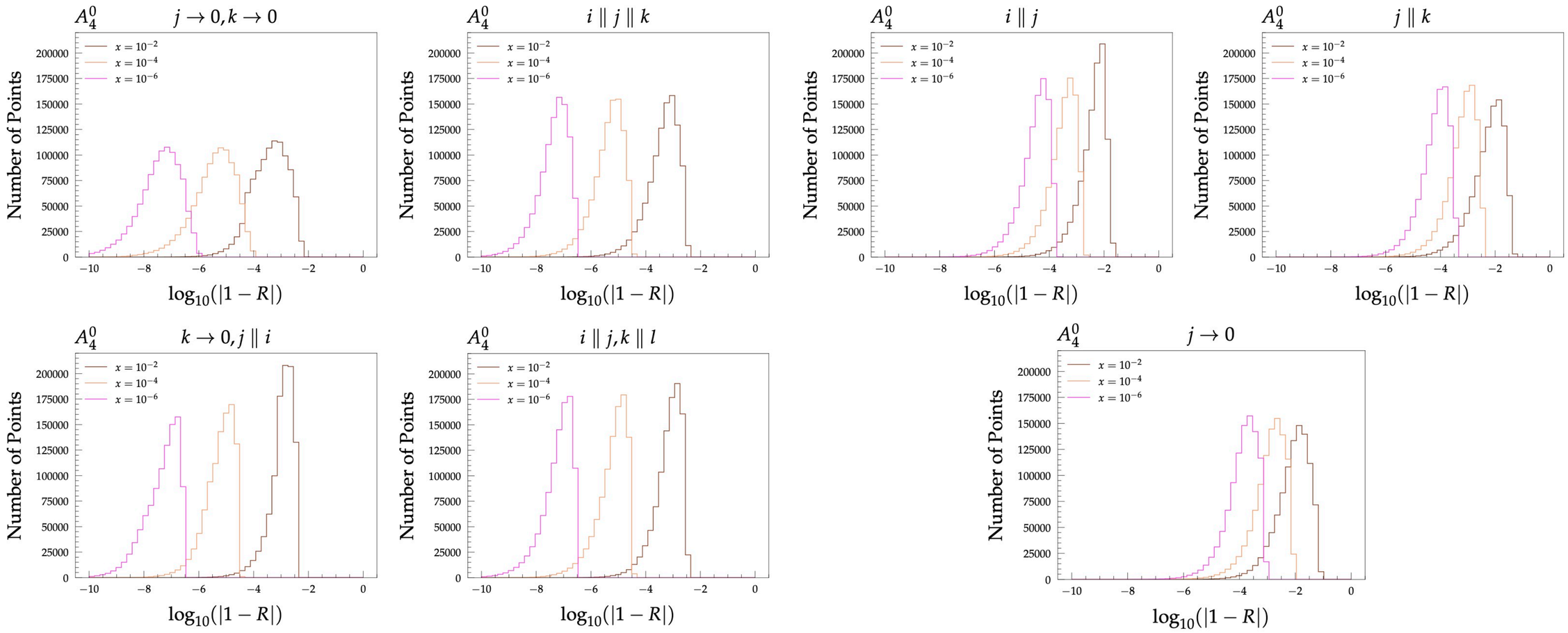
Example - $A_4^0(i_q^h, j_g, k_g, l_{\bar{q}}^h)$

Expression too long to write here but integrated antenna matches $\mathscr{A}_4^{0,\text{OLD}}$

$$\mathscr{A}_4^0(s_{ijkl}) = \left(s_{ijkl}\right)^{-\epsilon}\left[\frac{3}{4\epsilon^4} + \frac{65}{24\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{217}{18} - \frac{13\pi^2}{12}\right) + \frac{1}{\epsilon}\left(\frac{43223}{864} - \frac{589\pi^2}{144} - \frac{71\zeta_3}{4}\right) + \mathcal{O}(\epsilon^0)\right]$$

Old A_3^0 used as input here. Using new A_3^0 would change $\mathcal{O}(\epsilon^{-1})$ terms but algorithm is focus here.

Example - $A_4^0(i_q^h, j_g, k_g, l_{\bar{q}}^h)$



Numerical tests of A_4^0 against $A_4^{0,\text{OLD}}$ in all relevant singular limits. For three different values of scaling parameter x, the relative disagreement of the ratio $R = A_4^0/A_4^{0,\text{OLD}}$ is shown on the logarithmic axis.

Examples - $F_4^0(i_g^h, j_g, k_g, l_g^h)$ and $\tilde{F}_4^0(i_g^h, j_g, k_g, l_g^h)$

Old F_4^0 antenna had any pair soft, split into permutations of the new F_4^0 and $\tilde{F}_4^0$

Examples - $F_4^0(i_g^h, j_g, k_g, l_g^h)$ and $\tilde{F}_4^0(i_g^h, j_g, k_g, l_g^h)$

Old F_4^0 antenna had any pair soft, split into permutations of the new F_4^0 and $\tilde{F}_4^0$

$$F_4^{0,OLD} \sim F_4^0(i^h, j, k, l^h) + \text{ 3 cyclic permutations } + \tilde{F}_4^0(i^h, j, l, k^h) + \tilde{F}_4^0(l^h, i, k, j^h)$$

Examples - $F_4^0(i_g^h, j_g, k_g, l_g^h)$ and $\tilde{F}_4^0(i_g^h, j_g, k_g, l_g^h)$

Old F_4^0 antenna had any pair soft, split into permutations of the new F_4^0 and $\tilde{F}_4^0$

$$F_4^{0,OLD} \sim F_4^0(i^h, j, k, l^h) + \text{ 3 cyclic permutations } + \tilde{F}_4^0(i^h, j, l, k^h) + \tilde{F}_4^0(l^h, i, k, j^h)$$

Uses double unresolved limits, single unresolved limits and $F_3^0(i_g^h, j_g, k_g^h)$

Examples - $F_4^0(i_g^h, j_g, k_g, l_g^h)$ and $\tilde{F}_4^0(i_g^h, j_g, k_g, l_g^h)$

Old F_4^0 antenna had any pair soft, split into permutations of the new F_4^0 and $\tilde{F}_4^0$

$$F_4^{0,OLD} \sim F_4^0(i^h, j, k, l^h) + \text{ 3 cyclic permutations } + \tilde{F}_4^0(i^h, j, l, k^h) + \tilde{F}_4^0(l^h, i, k, j^h)$$

Uses double unresolved limits, single unresolved limits and $F_3^0(i_g^h, j_g, k_g^h)$

$$\mathscr{F}_4^0(s_{ijkl}) = \left(s_{ijkl}\right)^{-\epsilon}\left[\frac{3}{4\epsilon^4} + \frac{77}{24\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{559}{36} - \frac{13\pi^2}{12}\right) + \frac{1}{\epsilon}\left(\frac{59249}{864} - \frac{671\pi^2}{144} - \frac{69\zeta_3}{4}\right) + \mathcal{O}(\epsilon^0)\right]$$

$$\tilde{\mathscr{F}}_4^0(s_{ijkl}) = \left(s_{ijkl}\right)^{-\epsilon}\left[\frac{1}{\epsilon^4} + \frac{11}{3\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{313}{18} - \frac{3\pi^2}{2}\right) + \frac{1}{\epsilon}\left(\frac{34571}{432} - \frac{11\pi^2}{2} - \frac{86\zeta_3}{3}\right) + \mathcal{O}(\epsilon^0)\right]$$

Old f_3^0 sub-antenna used as input here. Using new F_3^0 would change $\mathcal{O}(\epsilon^{-2})$ terms but algorithm is focus here.

All X_4^0 complete for NNLO QCD

Quark-antiquark		
$qgg\bar{q}$	$X_4^0(i_q^h, j_g, k_g, l_{\bar{q}}^h)$	$A_4^0(i^h, j, k, l^h)$
$q\gamma\gamma\bar{q}$	$\widetilde{X}_4^0(i_q^h, j_\gamma, k_\gamma, l_{\bar{q}}^h)$	$\widetilde{A}_4^0(i^h, j, k, l^h)$
$q\bar{Q}Q\bar{q}$	$X_4^0(i_q^h, j_{\bar{Q}}, k_Q, l_{\bar{q}}^h)$	$B_4^0(i^h, j, k, l^h)$
$q\bar{q}q\bar{q}$	$X_4^0(i_q^h, j_{\bar{q}}, k_q, l_{\bar{q}}^h)$	$C_4^0(i^h, j, k, l^h)$
Quark-gluon		
$qggg$	$X_4^0(i_q^h, j_g, k_g, l_g^h)$	$D_4^0(i^h, j, k, l^h)$
	$\widetilde{X}_4^0(i_q^h, j_g, k_g, l_g^h)$	$\widetilde{D}_4^0(i^h, j, k, l^h)$
$q\bar{Q}Qg$	$X_4^0(i_q^h, j_{\bar{Q}}, k_Q, l_g^h)$	$E_4^0(i^h, j, k, l^h)$
$qg\bar{Q}Q$	$X_4^0(i_q^h, j_g, k_{\bar{Q}}, l_Q^h)$	$\overline{E}_4^0(i^h, j, k, l^h)$
$q\bar{Q}gQ$	$\widetilde{X}_4^0(i_q^h, j_{\bar{Q}}, k_g, l_Q^h)$	$\widetilde{E}_4^0(i^h, j, k, l^h)$
Gluon-gluon		
$gggg$	$X_4^0(i_g^h, j_g, k_g, l_g^h)$	$F_4^0(i^h, j, k, l^h)$
	$\widetilde{X}_4^0(i_g^h, j_g, k_g, l_g^h)$	$\widetilde{F}_4^0(i^h, j, k, l^h)$
$g\bar{Q}Qg$	$X_4^0(i_g^h, j_{\bar{Q}}, k_Q, l_g^h)$	$G_4^0(i^h, j, k, l^h)$
$gg\bar{Q}Q$	$X_4^0(i_g^h, j_g, k_{\bar{Q}}, l_Q^h)$	$\overline{G}_4^0(i^h, j, k, l^h)$
$g\bar{Q}gQ$	$\widetilde{X}_4^0(i_g^h, j_{\bar{Q}}, k_g, l_Q^h)$	$\widetilde{G}_4^0(i^h, j, k, l^h)$
$\bar{q}q\bar{Q}Q$	$X_4^0(i_{\bar{q}}^h, j_q, k_{\bar{Q}}, l_Q^h)$	$H_4^0(i^h, j, k, l^h)$

All X_4^0 complete for NNLO QCD

Quark-antiquark		
$qgg\bar{q}$	$X_4^0(i_q^h, j_g, k_g, l_{\bar{q}}^h)$	$A_4^0(i^h, j, k, l^h)$
$q\gamma\gamma\bar{q}$	$\widetilde{X}_4^0(i_q^h, j_\gamma, k_\gamma, l_{\bar{q}}^h)$	$\widetilde{A}_4^0(i^h, j, k, l^h)$
$q\bar{Q}Q\bar{q}$	$X_4^0(i_q^h, j_{\bar{Q}}, k_Q, l_{\bar{q}}^h)$	$B_4^0(i^h, j, k, l^h)$
$q\bar{q}q\bar{q}$	$X_4^0(i_q^h, j_{\bar{q}}, k_q, l_{\bar{q}}^h)$	$C_4^0(i^h, j, k, l^h)$
Quark-gluon		
$qggg$	$X_4^0(i_q^h, j_g, k_g, l_g^h)$	$D_4^0(i^h, j, k, l^h)$
	$\widetilde{X}_4^0(i_q^h, j_g, k_g, l_g^h)$	$\widetilde{D}_4^0(i^h, j, k, l^h)$
$q\bar{Q}Qg$	$X_4^0(i_q^h, j_{\bar{Q}}, k_Q, l_g^h)$	$E_4^0(i^h, j, k, l^h)$
$qg\bar{Q}Q$	$X_4^0(i_q^h, j_g, k_{\bar{Q}}, l_Q^h)$	$\overline{E}_4^0(i^h, j, k, l^h)$
$q\bar{Q}gQ$	$\widetilde{X}_4^0(i_q^h, j_{\bar{Q}}, k_g, l_Q^h)$	$\widetilde{E}_4^0(i^h, j, k, l^h)$
Gluon-gluon		
$gggg$	$X_4^0(i_g^h, j_g, k_g, l_g^h)$	$F_4^0(i^h, j, k, l^h)$
	$\widetilde{X}_4^0(i_g^h, j_g, k_g, l_g^h)$	$\widetilde{F}_4^0(i^h, j, k, l^h)$
$g\bar{Q}Qg$	$X_4^0(i_g^h, j_{\bar{Q}}, k_Q, l_g^h)$	$G_4^0(i^h, j, k, l^h)$
$gg\bar{Q}Q$	$X_4^0(i_g^h, j_g, k_{\bar{Q}}, l_Q^h)$	$\overline{G}_4^0(i^h, j, k, l^h)$
$g\bar{Q}gQ$	$\widetilde{X}_4^0(i_g^h, j_{\bar{Q}}, k_g, l_Q^h)$	$\widetilde{G}_4^0(i^h, j, k, l^h)$
$\bar{q}q\bar{Q}Q$	$X_4^0(i_{\bar{q}}^h, j_q, k_{\bar{Q}}, l_Q^h)$	$H_4^0(i^h, j, k, l^h)$

Simplified IR structure for these X_4^0 compared to previously

$\Longrightarrow$

Simplified subtraction terms

Conclusions and Outlook

Conclusions and Outlook

- General algorithm for creating more convenient idealised antennae
- Complete for all X_3^0 and X_4^0, meeting all design principles
- Should support construction of simplified NNLO subtraction terms, with no over-subtraction
- Extendable algorithm to X_3^1
  - Requires additional manipulation of explicit ϵ poles and hypergeometric functions
- Extendable algorithm to X_5^0 for future streamlined N3LO antenna subtraction scheme
  - Requires decomposition of quadruple collinear splitting functions into $P_{abcd\to A}(i^h, j, k, l)$ and similar for one-loop triple collinear splitting functions.
  - Create lists of required limits for X_5^0 out of these and new X_4^0 and new X_3^0.

Thank you very much! Questions?

Oscar Braun-White (he/him/his), IPPP Durham

Based on work with Nigel Glover (IPPP Durham)
and Christian Preuss (ETH Zurich) in arXiv:2302.12787

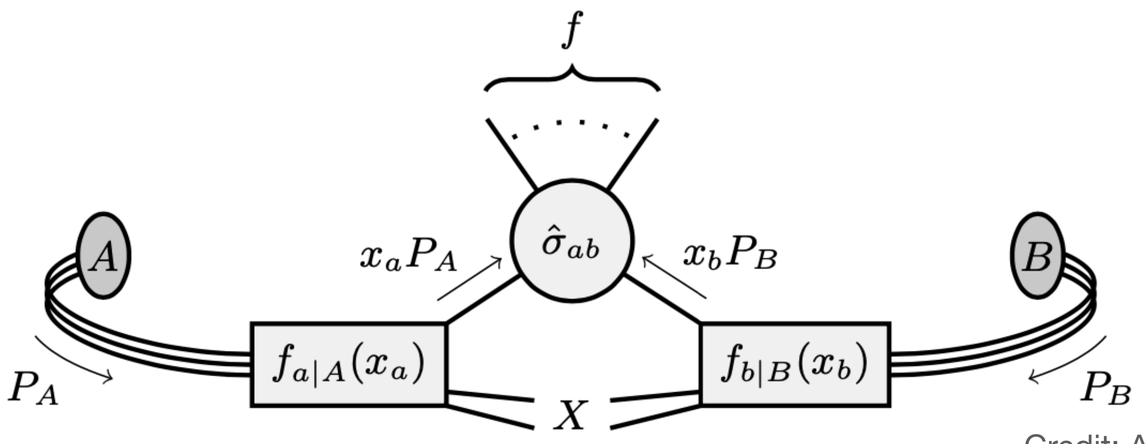
Backup - F_4^0

$$
\begin{aligned}
\mathbf{S}_{23}^{\downarrow} F_4^0(1^h,2,3,4^h) &= S_{gg}(1^h,2,3,4^h) \\
\mathbf{TC}_{123}^{\downarrow} F_4^0(1^h,2,3,4^h) &= P_{ggg}(1^h,2,3) \\
\mathbf{TC}_{234}^{\downarrow} F_4^0(1^h,2,3,4^h) &= P_{ggg}(4^h,3,2) \\
\mathbf{DC}_{1234}^{\downarrow} F_4^0(1^h,2,3,4^h) &= P_{gg}(1^h,2)P_{gg}(4^h,3) \\
\mathbf{S}_{2}^{\downarrow} F_4^0(1^h,2,3,4^h) &= \frac{2s_{13}}{s_{12}s_{23}} F_3^0(1,3,4) \\
\mathbf{S}_{3}^{\downarrow} F_4^0(1^h,2,3,4^h) &= \frac{2s_{24}}{s_{23}s_{34}} F_3^0(1,2,4) \\
\mathbf{C}_{12}^{\downarrow} F_4^0(1^h,2,3,4^h) &= P_{gg}(1^h,2)F_3^0([1+2],3,4) \\
\mathbf{C}_{23}^{\downarrow} F_4^0(1^h,2,3,4^h) &= P_{gg}(2,3)F_3^0(1,[2+3],4) \\
\mathbf{C}_{34}^{\downarrow} F_4^0(1^h,2,3,4^h) &= P_{gg}(4^h,3)F_3^0(1,2,[3+4]) \\
\mathbf{SC}_{2;34}^{\downarrow} F_4^0(1^h,2,3,4^h) &= \frac{2s_{134}}{s_{12}s_{234}} P_{gg}(4^h,3) \\
\mathbf{SC}_{3;12}^{\downarrow} F_4^0(1^h,2,3,4^h) &= \frac{2s_{124}}{s_{123}s_{34}} P_{gg}(1^h,2)
\end{aligned}
$$

Backup - $\tilde{F}_4^0$

$$\mathbf{S}_{23}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = S_{\gamma\gamma}(1^h,2,3,4^h)$$
$$\mathbf{TC}_{123}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{ggg}(2,1^h,3)$$
$$\mathbf{TC}_{234}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{ggg}(3,4^h,2)$$
$$\mathbf{DC}_{1234}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{gg}(1^h,2)P_{gg}(4^h,3)$$
$$\mathbf{DC}_{1324}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{gg}(1^h,3)P_{gg}(4^h,2)$$
$$\mathbf{S}_{2}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = \frac{2s_{14}}{s_{12}s_{24}}F_3^0(1,3,4)$$
$$\mathbf{S}_{3}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = \frac{2s_{14}}{s_{13}s_{34}}F_3^0(1,2,4)$$
$$\mathbf{C}_{12}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{gg}(1^h,2)F_3^0([1+2],3,4)$$
$$\mathbf{C}_{13}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{gg}(1^h,3)F_3^0([1+3],2,4)$$
$$\mathbf{C}_{24}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{gg}(4^h,2)F_3^0(1,3,[2+4])$$
$$\mathbf{C}_{34}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = P_{gg}(4^h,3)F_3^0(1,2,[3+4])$$
$$\mathbf{SC}_{2;34}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = \frac{2s_{134}}{s_{12}s_{234}}P_{gg}(4^h,3)$$
$$\mathbf{SC}_{3;24}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = \frac{2s_{124}}{s_{13}s_{234}}P_{gg}(4^h,2)$$
$$\mathbf{SC}_{3;12}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = \frac{2s_{124}}{s_{34}s_{123}}P_{gg}(1^h,2)$$
$$\mathbf{SC}_{2;13}^{\downarrow}\widetilde{F}_4^0(1^h,2,3,4^h) = \frac{2s_{134}}{s_{24}s_{123}}P_{gg}(1^h,3)$$

Cross Section σ_{AB}



Credit: Alexander Huss

$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \, f_{a|A}(x_a)\, f_{b|B}(x_b)\, \hat{\sigma}_{ab}(x_a, x_b)\, \left(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

parton distribution functions
(non-perturbative, universal)

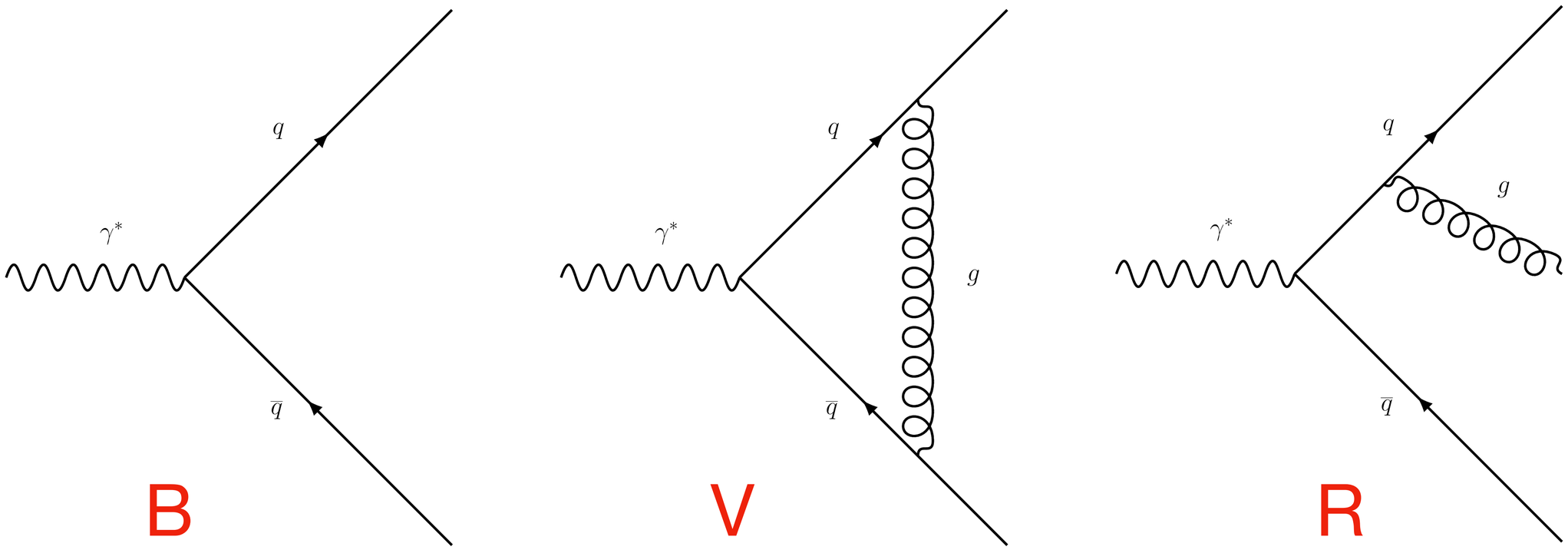
hard scattering
(perturbation theory)

non-perturbative effects
(no good understanding)
ultimately, limiting factor?

Partonic Cross Section $d\hat{\sigma}$

$$d\hat{\sigma} = \left(\frac{\alpha_s}{2\pi}\right)^m d\hat{\sigma}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^{m+1} d\hat{\sigma}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^{m+2} d\hat{\sigma}^{\text{NNLO}} + \left(\frac{\alpha_s}{2\pi}\right)^{m+3} d\hat{\sigma}^{\text{N3LO}} + \mathcal{O}(\alpha_s^{m+4})$$

- Theoretical predictions of QCD observables need to match experimental precision.



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