Next-to-soft resummation for color singlet processes at the LHC

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INTRODUCTION

- **What is Next-to-soft?**

In QCD improved parton model, the hadronic cross-section is computed via the convolution of parton distribution functions (PDFs), $f(x)$, and the partonic coefficient function, $\Delta_{ab}(z)$:

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2) \sum_{ab=q,\bar{q},g} \int dx_1 \int dx_2 \; f_a(x_1,\mu_F^2)\; f_b(x_2,\mu_F^2)\; \Delta_{ab}(q^2,\mu_R^2,\mu_F^2,z)$$

$q^2 \rightarrow$ Invariant mass of the final state

$\tau = \frac{q^2}{s} \rightarrow$ Hadronic scaling variable

$z = \frac{q^2}{\hat{s}} \rightarrow$ Partonic scaling variable

$\mu_R^2 \rightarrow$ Renormalisation scale

$\mu_F^2 \rightarrow$ Factorisation scale

$\sqrt{\hat{s}} \rightarrow$ Partonic centre of mass energy

$\sqrt{s} \rightarrow$ Hadronic centre of mass energy

INTRODUCTION

- **The Partonic coefficient function near the threshold : z->1 limit**

$$\mathbf{\Delta_{ab}} = \sum_{\mathbf{n=0}}^{\infty} \left(\frac{\alpha_{\mathbf{s}}}{4\pi}\right)^{\mathbf{n}} \sum_{\mathbf{m=0}}^{\mathbf{2n-1}} \left\{ \mathbf{c_{nm}} \left[\frac{\log^{\mathbf{m}}(\mathbf{1-z})}{(\mathbf{1-z})}\right]_+ + \mathbf{c_n}\, \delta(\mathbf{1-z}) + \mathbf{d_{nm}} \log^{\mathbf{m}}(\mathbf{1-z}) + \mathcal{O}(\mathbf{1-z}) \right\}$$

- Leading singular distributions, also known as leading power (LP)
- Corrections only from diagonal channels
- Contributions from soft real emissions + virtual corrections
- Well-understood
- Resummation known to N^3LL

Soft-Virtual (SV)

INTRODUCTION

- **The Partonic coefficient function near the threshold : z->1 limit**

$$\Delta_{ab} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n \sum_{m=0}^{2n-1} \left\{ c_{nm} \left[\frac{\log^m(1-z)}{(1-z)}\right]_+ + c_n\, \delta(1-z) + d_{nm}\, \log^m(1-z) + \mathcal{O}(1-z) \right\}$$

- Next-to-leading singular, also known as next-to-leading power (NLP)
- Collinear logarithms
- Corrections from diagonal and off-diagonal channels
- Not much understood
- Resummation known to LL

Next-to soft-virtual (NSV)

INTRODUCTION

- **Why Next-to-soft?**

➢ **For the Drell-Yan process at N3LO, Q= 200 GeV**

$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\mathcal{D}_5$	5.44%	$\ln^5(1-z)$	8.60%
	$\mathcal{D}_4$	2.62%	$\ln^4(1-z)$	9.82%
	$\mathcal{D}_3$	-2.73%	$\ln^3(1-z)$	-1.54%
	$\mathcal{D}_2$	-4.25%	$\ln^2(1-z)$	-8.98%
	$\mathcal{D}_1$	-1.94%	$\ln^1(1-z)$	-6.14%
	$\mathcal{D}_0$	-0.146%	$\ln^0(1-z)$	-1.28%
	$\delta(1-z)$	1.03%		
TOTAL		0.026%		0.47%

$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\ln^6 N$	-0.0025%	$\frac{\ln^6 N}{N}$	0%
	$\ln^5 N$	-0.001%	$\frac{\ln^5 N}{N}$	0.0004%
	$\ln^4 N$	0.0244%	$\frac{\ln^4 N}{N}$	0.006%
	$\ln^3 N$	0.171%	$\frac{\ln^3 N}{N}$	0.1%
	$\ln^2 N$	2.85%	$\frac{\ln^2 N}{N}$	0.56%
	$\ln N$	6.23%	$\frac{\ln N}{N}$	4.31%
	$\ln^0 N$	18.3%	$\frac{1}{N}$	3.30%
TOTAL		27.6%		8.28%

z-space **Mellin N- conjugate space**

$$\mathcal{D}_i(z) = \left(\frac{\log^i(1-z)}{1-z}\right)_+$$

INTRODUCTION

Higgs production in the gluon fusion | N3LO

JHEP 03 (2015) 091

Charalampos Anastasiou[a], Claude Duhr[b], Falko Dulat[a], Elisabetta Furlan[c], Thomas Gehrmann[d], Franz Herzog[e], Bernhard Mistlberger[a]

$$
\begin{aligned}
\eta_{gg}^{(3)}(z)\Big|_{(1-z)^0} &= -256\log^5(1-z) && (\rightarrow \quad 115.33\%)\\
&+ 959\log^4(1-z) && (\rightarrow \quad 101.07\%)\\
&+ 1254.029198\ldots\log^3(1-z) && (\rightarrow \quad -32.15\%)\\
&- 11089.328274\ldots\log^2(1-z) && (\rightarrow \quad -89.41\%)\\
&+ 15738.441212\ldots\log(1-z) && (\rightarrow \quad -55.50\%)\\
&- 5872.588877\ldots && (\rightarrow \quad -14.31\%)
\end{aligned}
$$

The total SV contribution in z-space -> -2.25 % of the Born

The total NSV contribution in z-space -> 25 % of the Born !

INTRODUCTION

The total SV contribution in Mellin N-space (conjugate space) -> 18 % of the Born
The total NSV contribution in N-space -> 11 % of the Born !

$$
\begin{aligned}
M\left[\eta_{gg}^{(3)}\right](N) \simeq\, & 36 \log^6 N && (\to 0.0013\%) \\
& + 170.679\ldots \log^5 N && (\to 0.0226\%) \\
& + 744.849\ldots \log^4 N && (\to 0.2570\%) \\
& + 1405.185\ldots \log^3 N && (\to 1.0707\%) \\
& + 2676.129\ldots \log^2 N && (\to 4.0200\%) \\
& + 1897.141\ldots \log N && (\to 5.1293\%) \\
& + 1783.692\ldots && (\to 8.0336\%) \\
& + 108 \frac{\log^5 N}{N} && (\to 0.0105\%) \\
& + 615.696\ldots \frac{\log^4 N}{N} && (\to 0.1418\%) \\
& + 2036.407\ldots \frac{\log^3 N}{N} && (\to 0.9718\%) \\
& + 3305.246\ldots \frac{\log^2 N}{N} && (\to 2.9487\%) \\
& + 3459.105\ldots \frac{\log N}{N} && (\to 5.2933\%) \\
& + 703.037\ldots \frac{1}{N} && (\to 1.7137\%).
\end{aligned}
$$

INTRODUCTION

Understanding the NSV sector is important because:

- Numerically sizeable owing to their large coefficients
- Provide a check of higher-order correction

- **Since the NSV sector gives rise to large logarithmic contributions in the threshold limit : spolis the perturbativity of the FO series**

- **Resolution : Find a way to resum these NSV logarithms beyond LL accuracy**

PREVIOUS WORKS ON NSV/NLP

- The earliest evidence that IR effects can be studied at NSV/NLP
  [Low, Burnett, Kroll]

- Early attempts :
  [Kraemer, Laenen, Spira (98)]
  [Akhoury, Sotiropoulos & Sterman (98)]

- Important Results & Predictions using Physical Kernel Approach & explicit computation:
  [Moch , Vogt et al. (09-20)]
  [Anastasiou, Duhr, Dulat et al.(14)]

- Universality of NSV/NLP effects and LL Resummation:
  [Laenen, Magnea, et al. (08-19)]
  [Grunberg & Ravindran (09)]
  [Ball, Bonvini, Forte, Marzani, Ridolfi (13)]
  [Del Duca et al. (17)]

- Subleading Factorisation and LL Resummation at NSV/NLP using SCET:
  [Larkoski, Nelli , Stewart et al. (14)]
  [Kolodrubetz, Moult, Neill ,Stewart et al. (17)]
  [Beneke et al. (19-20)]

And many other works…

OUR APPROACH

- The SV formalism was already known for color singlet processes : [Ravindran '05,'06]
- We extended the very formalsim to study the NSV effects in the diagonal channels for the color singlet processes

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Key points

- **Collinear factorisation**
- **Renormalisation group (RG) invariance**
- **Logarithmic structure present in the higher order results**

THE FORMALISM

Let's begin with the collinear factorisation

$$\frac{1}{z}\hat{\sigma}_{ab}(q^2, z, \varepsilon) = \sigma_0(\mu_R^2)\sum_{a'b'}\Gamma^{T}_{a'a}(z, \mu_F^2, \varepsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(q^2, \mu_R^2, \mu_F^2, z, \varepsilon)\right) \otimes \Gamma_{b'b}(z, \mu_F^2, \varepsilon)$$

THE FORMALISM

Let's begin with the collinear factorisation

Coliinear finite partonic coefficient function

bare partonic cross-section

$$\frac{1}{z}\hat{\sigma}_{ab}(q^2, z, \varepsilon) = \sigma_0(\mu_R^2) \sum_{a'b'} \Gamma^{T}_{a'a}(z, \mu_F^2, \varepsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(q^2, \mu_R^2, \mu_F^2, z, \varepsilon)\right) \otimes \Gamma_{b'b}(z, \mu_F^2, \varepsilon)$$

Altarelli-Parisi (AP) kernels

ϵ : Dimensional regularisation parameter

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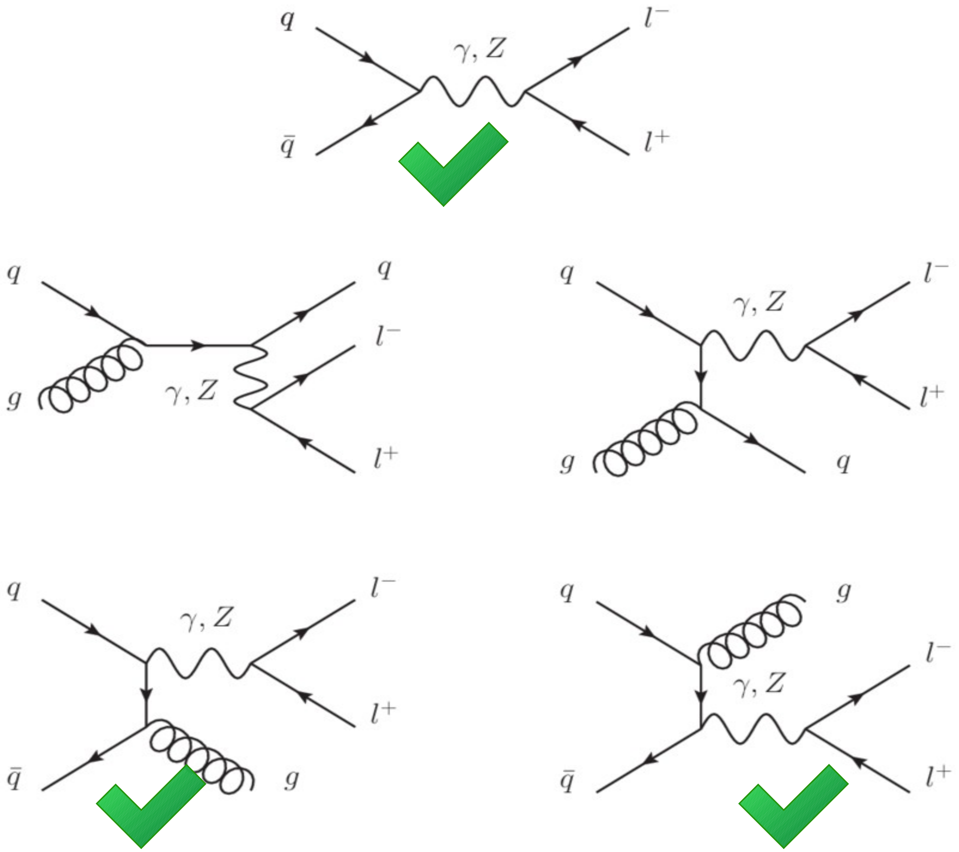
Altarelli-Parisi (AP) kernels

We only study the contribution from diagonal channel in the above factorisation formula keeping only SV and NSV terms

ϵ : Dimensional regularisation parameter

THE FORMALISM

For instance, we only consider the diagonal $q\bar{q}$ channel for the Drell-Yan process



THE FORMALISM

The factorisation in the diagonal $q\bar{q}$ channel for the Drell-Yan process:

$$\frac{\hat{\sigma}_{\mathbf{q\bar{q}}}}{\mathbf{z}\sigma_{\mathbf{0}}} = \mathbf{\Gamma}^{\mathbf{T}}_{\mathbf{qq}} \otimes \frac{\mathbf{\Delta_{q\bar{q}}}}{\mathbf{z}} \otimes \mathbf{\Gamma_{\bar{q}\bar{q}}} + \mathbf{\Gamma}^{\mathbf{T}}_{\mathbf{qq}} \otimes \cancel{\frac{\mathbf{\Delta_{qg}}}{\mathbf{z}}} \otimes \mathbf{\Gamma_{g\bar{q}}} + \cdots$$

Contribute to beyond NSV terms

$(1-z)^l \ln^k(1-z), l > 0, k \geq 0$

We can safely remove these terms

$$\boxed{\frac{\hat{\sigma}^{\mathbf{SV+NSV}}_{\mathbf{q\bar{q}}}}{\mathbf{z}\sigma_{\mathbf{0}}} = \mathbf{\Gamma}^{\mathbf{T}}_{\mathbf{qq}} \otimes \mathbf{\Delta}^{\mathbf{SV+NSV}}_{\mathbf{q\bar{q}}} \otimes \mathbf{\Gamma_{\bar{q}\bar{q}}}}$$

Remarkably simple form!!

THE FORMALISM : THE BUILDING BLOCKS

The perturbative structure of Finite mass-factorised SV+NSV partonic coefficient function for the diagonal channels:

$c\bar{c}$ -> $q\bar{q}$ for the DY
gg for g+g->H
$b\bar{b}$ for b+$\bar{b}$->H

$$\boxed{\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}}$$

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Altarelli-Parisi splitiing kernels

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Overall Renormalisation constant

Form factor (FF)

Soft-collinear function

Each of these building blocks obeys first order differential equations and additional evolution equations w.r.t various scales (Q, q, μF, μR)

THE FORMALISM : THE BUILDING BLOCKS

Form Factor (FF, captures pure virtual corrections)

$$\Delta_{c\bar{c}}(z, \epsilon, q^2 \mu_R^2, \mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2, \epsilon)|^2 S_c(q^2, z, \epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Sudakov diff eqn (K+G diff eqn)

The hard scale

$Q^2 = -q^2$

$$Q^2 \frac{d}{dQ^2} \log \hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^c\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

Poles

Finite

$$a_s(\mu_R^2) = \frac{g_s^2(\mu_R^2)}{16\pi^2}$$

RG invariance

$$\mu_R^2 \frac{d\hat{F}_c}{d\mu_R^2} = 0$$

Solution

$$\hat{F}_c(-q^2, \epsilon) = \exp\left(\int_0^{-q^2} \frac{d\lambda^2}{\lambda^2} \Gamma_{\hat{F},c}(\lambda^2, \epsilon)\right)$$

$\Gamma_{\hat{F},c} = (K_c + G_c)/2$ is the kernel

Function of

Cusp ano dim

$$\{A^c, B^c, f^c, \gamma^c, g^c\}$$

Collinear ano dim

Soft ano dim

UV ano dim

Process dependent

[Sen, Sterman,Magnea]

THE FORMALISM : THE BUILDING BLOCKS

$$\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Overall Renormalisation constant

Renormalisation scale

Renormalisation Group Eqn (RGE)

$$\mu_R^2 \frac{d}{d\mu_R^2} \log Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \;\; \gamma_{i-1}^c$$

UV anomalous dimension

THE FORMALISM : THE BUILDING BLOCKS

Altarelli-Parisi (AP) kernels

$$\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Altarelli-Parisi evolution eqn

Factorisation scale

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z,\mu_F^2,\epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}(z,a_s(\mu_F^2)) \otimes \Gamma_{a'b}(z,\mu_F^2,\epsilon)$$

We only need to consider diagonal AP splitting kernels and splitting function

Solution : $\Gamma_{cc}(\mu_F^2,z,\epsilon) = \mathcal{C} \exp\left(\frac{1}{2} \int_0^{\mu_F^2} \frac{d\lambda^2}{\lambda^2} P_{cc}(\lambda^2,z,\epsilon) \right)$

$$P_{cc}(z,\mu_F^2) = 2\left[\frac{A^c}{(1-z)_+} + B^c\,\delta(1-z) + C^c\,\log(1-z) + D^c + \mathcal{O}(1-z) \right]$$

beyond NSV

Collinear anomalous dimensions(contributes to pure NSV)

[Moch,Vogt,Vermaseren]

THE FORMALISM : THE BUILDING BLOCKS

- **Soft-collinear function, S_c:** captures the contributions with at least one real emission (R, RR, RV etc)
- Using the knowledge of the evolution equations and the correspoding solutions of other builiding blocks, we obtain a 1st order differential equation for the soft-collinear function S_c

K+G – type diff eqn

$$q^2 \frac{d}{dq^2} \ln \mathcal{S}_c = \left[\overline{K}^c \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) + \overline{G}^c \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) \right]$$

Poles

Finite

[Ajjath, P. Mukherjee, Ravindran]
PRD.105.094035
PRD.105.L091503

RG invariance

$$\mu_R^2 \frac{d \ln \mathcal{S}_c}{d\mu_R^2} = 0$$

- The solution to the above diff eqn is obtained using the RG invariance of S_c and studying the perturabative structure of state-of-the-art results for color singlet processes

SOFT-COLLINEAR FUNCTION

- **Inspired from explicit results, solution verified up to 3rd order, we propose the same structure to all orders**

$$\ln S^{\mathrm{SV+NSV}}(\hat{a}_s, q^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \, S_\epsilon^i \left(\frac{q^2(1-z)^2}{\mu^2 z}\right)^{i\frac{\epsilon}{2}} \left(\frac{i\epsilon}{1-z}\right) \left[\hat{\phi}^{\mathrm{SV},(i)}(\epsilon) + (1-z)\, \hat{\phi}^{\mathrm{NSV},(i)}(z,\epsilon)\right]$$

Pure SV part
Universal coefficients
$A^c, f^c, \overline{\mathcal{G}}^c$

$$\sum_{j=1}^{\infty} \sum_{k=0}^{i+j-1} \mathcal{G}_i^{(j,k)} \, \epsilon^j \, \log^k(1-z)$$

NSV part, z-dependency
Universal & Process dependent coeffcients
$C^c, D^c, \varphi_c(z)$

INTEGRAL REPRESENTATION

- Knowing the functional form of each buliding blocks, we derive an integral representation for Δ_c , which gives an understanding of the all-order structure

$$\mathbf{\Delta_c(q^2, \mu_R^2, \mu_F^2, z) = C_0^c(q^2, \mu_R^2, \mu_F^2)} \;\; \mathcal{C} \exp\left(\int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} \mathbf{P'_{cc}(a_s(\lambda^2), z)} + \mathcal{Q}^c(\mathbf{a_s(q^2(1-z)^2), z}) \right)$$

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- Proportional to $\delta(1-z)$
  Finite part after cancelling of poles
  between F.F & Soft-collinear function

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- Proportional to δ(1 − z)
Finite part after cancelling of poles
between F.F & Soft-collinear function

$$P'_{cc}\big(z, a_s(\mu_F^2)\big) \quad = \quad 2\Bigg[A^c(a_s(\mu_F^2))\mathcal{D}_0(z) + C^c(a_s(\mu_F^2))\log(1-z) + D^c(a_s(\mu_F^2))\Bigg]$$

- Finite part after
cancellation of poles between splitting
kernels and soft -collinear function

INTEGRAL REPRESENTATION

- Knowing the functional form of each buliding blocks, we derive an integral representation for Δ_c , which gives an understanding of the all-order structure

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Finite part after cancelling of poles between F.F & Soft-collinear function

$$P'_{cc}(z, a_s(\mu_F^2)) = 2\left[A^c(a_s(\mu_F^2))\mathcal{D}_0(z) + C^c(a_s(\mu_F^2))\log(1-z) + D^c(a_s(\mu_F^2))\right]$$

- Finite part after cancellation of poles between splitting kernels and soft -collinear function

$$\mathcal{Q}^c(a_s(q^2(1-z)^2), z) = \left(\frac{1}{1-z}\mathcal{G}^c_{SV}(a_s(q^2(1-z)^2))\right)_+ + \mathcal{G}^c_{NSV}(a_s(q^2(1-z)^2), z)$$

- The finite contribution comes completely from the soft-collinear function

RESUMMATION IN THE MELLIN N-SPACE

- We solve this integral representation in the Mellin N-space. Convolutions become normal products in the Mellin N-space, easy to handle.

- **Mellin transformation:** $\longrightarrow$

$$\boxed{\Delta_N^c = \int_0^1 dz\ z^{N-1} \Delta_c(z)}$$

- **The soft limit then converts to :**

Z→1 $\longrightarrow$ **N→∞**

RESUMMATION IN THE MELLIN N-SPACE

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- **The soft limit then converts to :**

Z→1 ⟶ N→∞

- **The large logarithms transform to:** →

SV NSV

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \to \frac{\ln^2 N}{2} - \frac{\ln N}{2N} + \frac{1}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\ln^k(1-z) \to \frac{\ln^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

RESUMMATION EXPONENTS

Solving the integral representation in Mellin N-space, we obtain :

$\omega = 2\beta_0 a_s(\mu_R^2) \log N$

$$\Delta_N^{\mathrm{SV+NSV}}(q^2,\mu_R^2,\mu_F^2) = \left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\tilde{g}_{0,i}(q^2,\mu_R^2,\mu_F^2)\right) \exp\left(\Psi^c_{\mathrm{sv,N}} + \Psi^c_{\mathrm{nsv,N}}\right)$$

N-independent

The SV part is well-known to third logarithmic accuracy for Color singlet processes

$$\Psi^c_{\mathrm{sv,N}} = g_1^c(\omega)\ln(N) + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g_{i+2}^c(\omega)$$

[Sterman]
[Catani, Trentedue]

The NSV part is the new result :

$$\Psi^c_{\mathrm{nsv,N}} = \frac{1}{N}\sum_{n=0}^{\infty} a_s^n \left[\bar{g}^c_{n+2}(\omega) + \sum_{k=0}^{n} h^c_{nk}(\omega)\ \log^k N\right]$$

ALL-ORDER PREDICTION: SV (ALREADY KNOWN)

$$\mathcal{L}^i = \ln^i(N)$$

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta^{(2)}_{c,N}$	$\Delta^{(3)}_{c,N}$	$\Delta^{(i)}_{c,N}$
SV-LL	$\tilde{g}^c_{0,0}, g^c_1$	$\mathcal{L}^4$	$\mathcal{L}^6$	$\mathcal{L}^{2i}$
SV-NLL	$\tilde{g}^c_{0,1}, g^c_2$		$\mathcal{L}^5, \mathcal{L}^4$	$\mathcal{L}^{2i-1}, \mathcal{L}^{2i-2}$
SV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3$			$\mathcal{L}^{2i-3}, \mathcal{L}^{2i-4}$
SV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$			$\mathcal{L}^{2i-2n-1}, \mathcal{L}^{2i-2n}$

ALL-ORDER PREDICTION: NSV (NEW)

$L_N^i = \frac{1}{N} \ln^i(N)$

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta_{c,N}^{(2)}$	$\Delta_{c,N}^{(3)}$	$\Delta_{c,N}^{(i)}$
NSV-LL	$\tilde{g}_{0,0}^c, g_1^c, \overline{g}_1^c, h_0^c$	L_N^3	L_N^5	L_N^{2i-1}
NSV-NLL	$\tilde{g}_{0,1}^c, g_2^c, \overline{g}_2^c, h_1^c$		L_N^4	L_N^{2i-2}
NSV-N^{2}LL	$\tilde{g}_{0,2}^c, g_3^c, \overline{g}_3^c, h_2^c$			L_N^{2i-3}
NSV-N^{n}LL	$\tilde{g}_{0,n}^c, g_{n+1}^c, \overline{g}_{n+1}^c, h_n^c$			$L_N^{2i-(n+1)}$

ALL-ORDER PREDICTION: NSV (NEW)

GIVEN		ONS	
Logarithmic Accuracy	Resummed Exponents		$\Delta^{(i)}_{c,N}$
NSV-LL	$\tilde{g}^c_{0,0}, g^c_1, \overline{g}^c_1, h^c_0$		L_N^{2i-1}
NSV-NLL	$\tilde{g}^c_{0,1}, g^c_2, \overline{g}^c_2, h^c_1$		L_N^{2i-2}
NSV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3, \overline{g}^c_3, h^c_2$		L_N^{2i-3}
NSV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$		$L_N^{2i-(n+1)}$

$$a_s \frac{1}{N} \log N$$

$$a_s^2 \frac{1}{N} \log^3 N$$

$$a_s^3 \frac{1}{N} \log^5 N$$

$$\vdots$$

$$a_s^i \frac{1}{N} \log^{2i-1} N$$

$$L_N^i = \frac{1}{N} \ln^i(N)$$

Contains only one-loop info

ALL-ORDER PREDICTION: NSV (NEW)

$L_N^i = \frac{1}{N}\ln^i(N)$

GIVEN		...ONS
Logarithmic Accuracy	Resummed	$\Delta^{(i)}_{c,N}$
NSV-LL		
NSV-NLL		
NSV-N^{2}LL		
NSV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$	$L_N^{2i-(n+1)}$

$$a_s \frac{1}{N} \log N$$

$$a_s^2 \frac{1}{N} \log^3 N$$

$$a_s^i \frac{1}{N} \log^{2i-1} N$$

- **The leading logarithm for SV+ NSV matches with the existing result :**

$$\boxed{\begin{aligned}\Delta_{LL}^{DY} &= g_0 \exp\left[\ln N g_1(\omega) + \frac{1}{N} h_0(\omega, N)\right] \\ &= \exp\left[8 C_F a_s \left(\ln^2 N + \frac{\ln N}{N}\right)\right]\end{aligned}}$$

[Beneke et al.]
[Laenen et al.]

Contains only one-loop info

ALL-ORDER PREDICTION: NSV (NEW)

GIVEN		ONS	
Logarithmic Accuracy	Resummed Exponents		$\Delta^{(i)}_{c,N}$
NSV-LL	$\tilde{g}^c_{0,0}, g^c_1, \overline{g}^c_1, h^c_0$		L_N^{2i-1}
NSV-NLL	$\tilde{g}^c_{0,1}, g^c_2, \overline{g}^c_2, h^c_1$		L_N^{2i-2}
NSV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3, \overline{g}^c_3, h^c_2$		L_N^{2i-3}
NSV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$		$L_N^{2i-(n+1)}$

$$a_s^2 \frac{1}{N} \log^2 N$$

$$a_s^3 \frac{1}{N} \log^4 N$$

$$a_s^4 \frac{1}{N} \log^6 N$$

$$\vdots$$

$$a_s^i \frac{1}{N} \log^{2i-2} N$$

$$L_N^i = \frac{1}{N} \ln^i(N)$$

Contains only two-loop info

ALL-ORDER PREDICTION: NSV (NEW)

GIVEN		ONS
Logarithmic Accuracy	Resummed Exponents	$\Delta^{(i)}_{c,N}$
NSV-LL	$\tilde{g}^c_{0,0}, g^c_1, \overline{g}^c_1, h^c_0$	L_N^{2i-1}
NSV-NLL	$\tilde{g}^c_{0,1}, g^c_2, \overline{g}^c_2, h^c_1$	L_N^{2i-2}
NSV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3, \overline{g}^c_3, h^c_2$	L_N^{2i-3}
NSV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$	$L_N^{2i-(n+1)}$

$$a_s^n \frac{1}{N} \log^n N$$

$$a_s^i \frac{1}{N} \log^{2i-n} N$$

$$L_N^i = \frac{1}{N} \ln^i(N)$$

Contains only n+1-loop info

ALL-ORDER PREDICTION: NSV (NEW)

$L_N^i = \frac{1}{N}\ln^i(N)$

GIVEN		ONS
Logarithmic Accuracy	Resummed Exponents	$\Delta^{(i)}_{c,N}$
NSV-LL	$\tilde{g}^c_{0,0}, g^c_1, \overline{g}^c_1, h^c_0$	L_N^{2i-1}
NSV-NLL	$\tilde{g}^c_{0,1}, g^c_2, \overline{g}^c_2, h^c_1$	L_N^{2i-2}
NSV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3, \overline{g}^c_3, h^c_2$	L_N^{2i-3}
NSV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$	$L_N^{2i-(n+1)}$

$$a_s^n \frac{1}{N} \log^n N$$

$$\vdots$$

$$a_s^i \frac{1}{N} \log^{2i-n} N$$

Contains only n+1-loop info

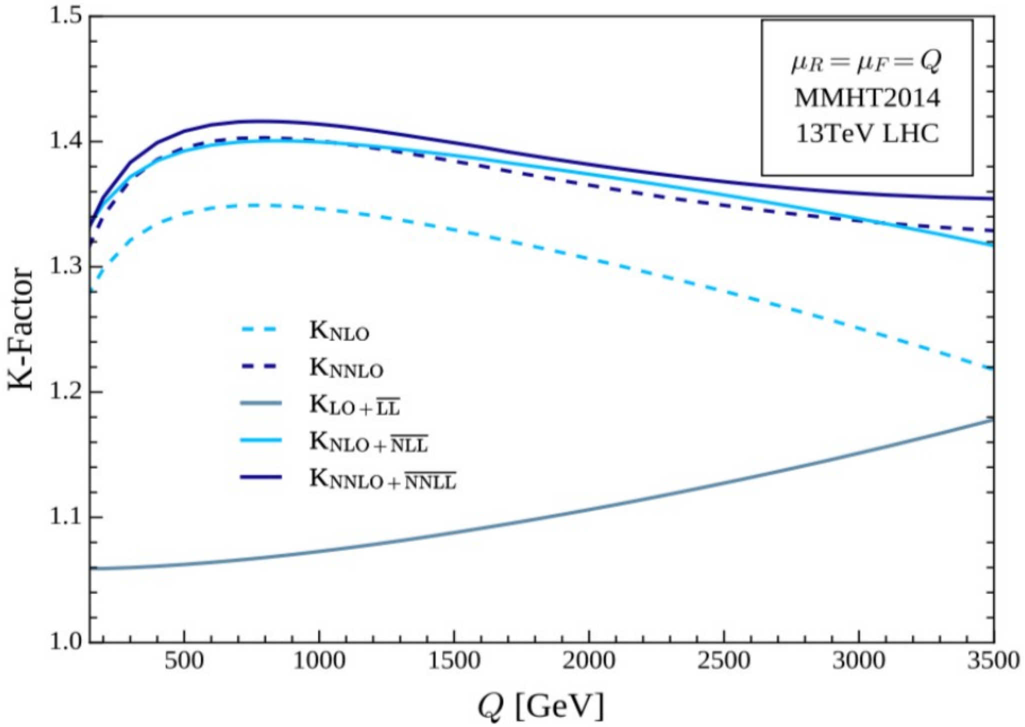
Verified our predictions up to 4th order in QCD

[Moch, Vogt et.al], [De Florian et al.], [Das et al.]

NSV Resummation Phenomenology | Drell-Yan Process
K- factor

$\mu_R = \mu_F = Q$(GeV)	LO + $\overline{\text{LL}}$	NLO	NLO + $\overline{\text{NLL}}$	NNLO	NNLO + $\overline{\text{NNLL}}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818

$$\mathrm{K}\left(Q\right) = \frac{\frac{d\sigma}{dQ}\left(\mu_R = \mu_F = Q\right)}{\frac{d\sigma^{\mathrm{LO}}}{dQ}\left(\mu_R = \mu_F = Q\right)}$$



- Increment from FO → resum at Q=2000 GeV

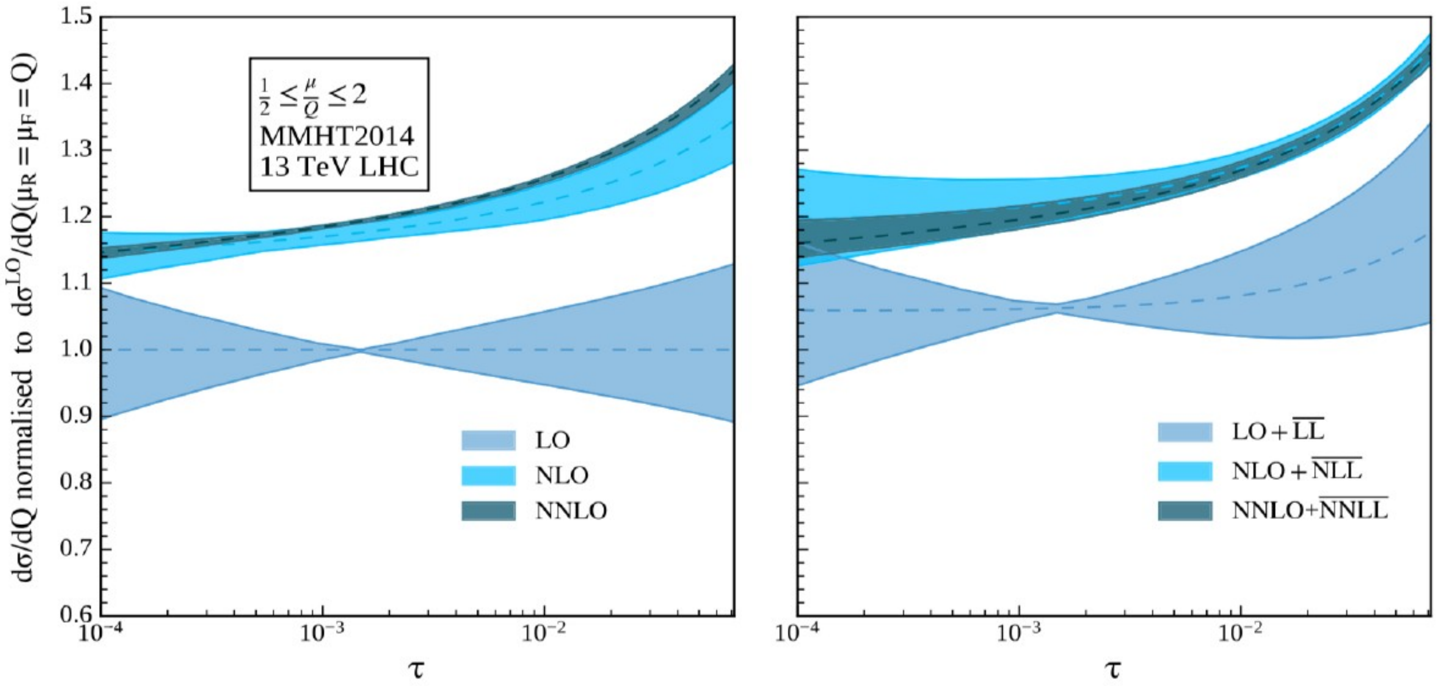
  10.6% : LO → LO + $\overline{\text{LL}}$
  5.2% : NLO → NLO + $\overline{\text{NLL}}$
  1.2% : NNLO → NNLO + $\overline{\text{NNLL}}$

- NLO + $\overline{\text{NLL}}$ from quark part mimics the NNLO

- Resummed predictions are closer compared FO
  Resummed corrections decrease as we go higher-order
  : Improves the reliability of perturbative predictions

NSV Resummation Phenomenology | Drell-Yan Process
7-point Scale variation

Impact of μ_R & μ_F scales in the predictions using canonical 7-point variation: $\frac{1}{2} \le \frac{\mu_F}{Q}, \frac{\mu_R}{Q} \le 2$



- FO from all channels & resummed predictions from only diagonal channels ($q\bar{q}$)

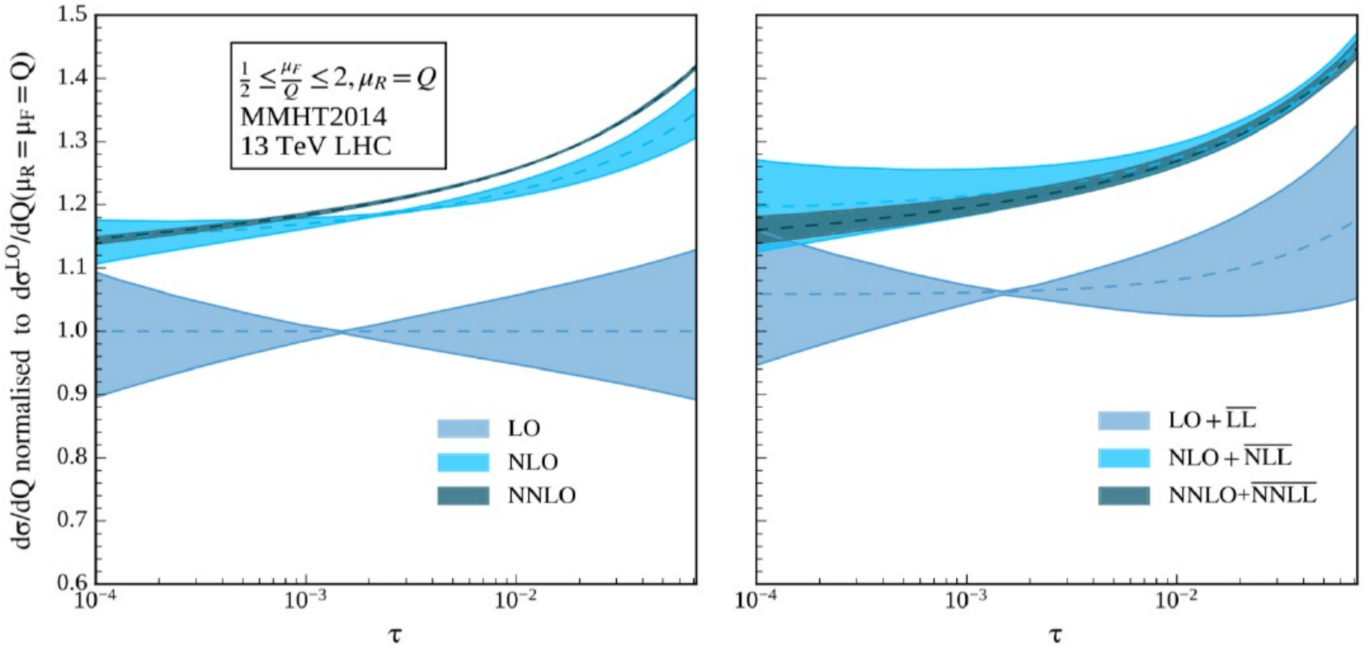
$$\tau = \frac{Q^2}{S}$$

- NLO to NLO + $\overline{\text{NLL}}$: band width decreases. Large scale uncertainity at NNLO + $\overline{\text{NNLL}}$
- NNLO + $\overline{\text{NNLL}}$ is within NLO + $\overline{\text{NLL}}$ unlike fixed order case at high energies

NSV Resummation Phenomenology | Drell-Yan Process
The Factorisation scale

In order to understand the cause of large uncertainty at NNLO + $\overline{\text{NNLL}}$, we study the scales separately.

μ_F - variation



- Bands in resummation plot look similar to 7-point scale variation : large band in 7-point scale is due to μ_{F} – uncertainities

- μ_F scale mixes up different channels:

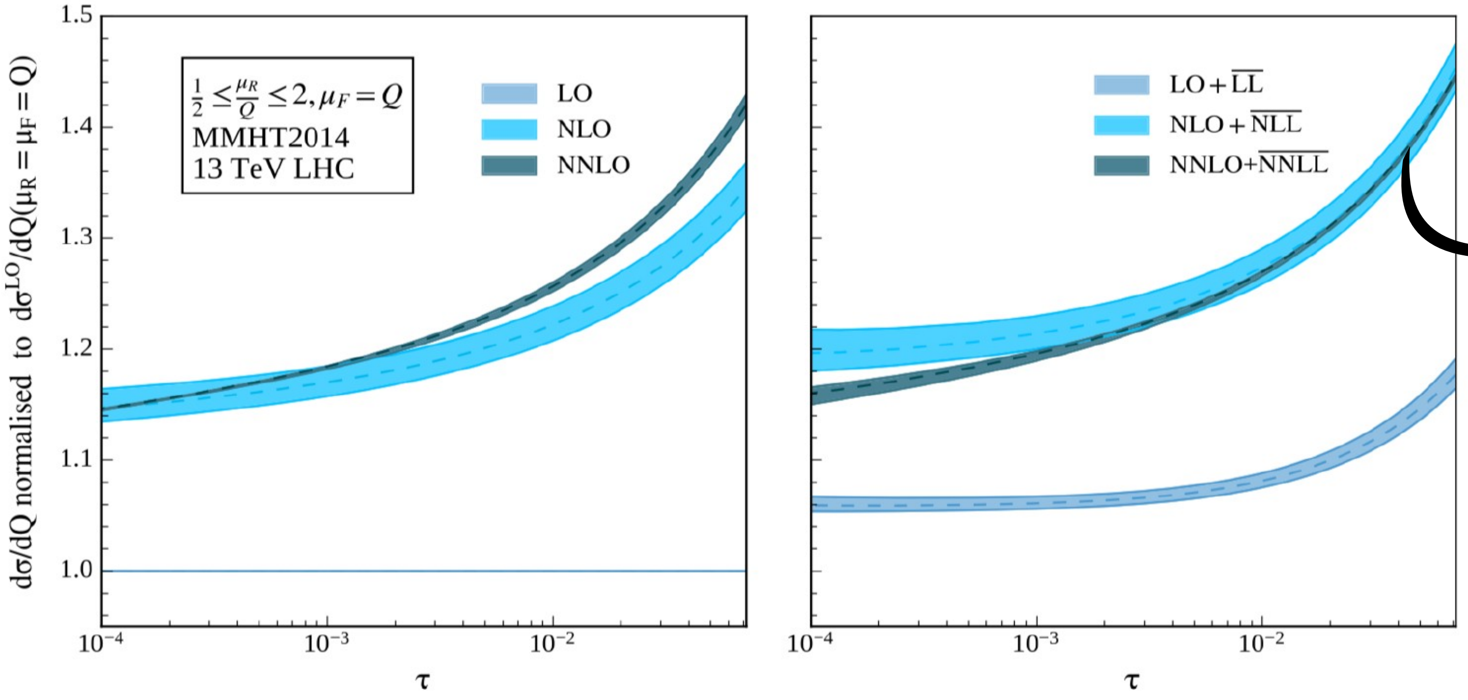
- Collinear logarithms arise from diagonal($q\bar{q}$) & off-diagonal (like qg) channels

- Large μ_F uncertainity could be due to lack of off-diagonal contribution!

NSV Resummation Phenomenology | Drell-Yan Process
The Renormalisation scale

To understand the role of collinear resummation, we see the μ_R - variation :

μ_R - variation



Substantial reduction in the μ_R – scale uncertainity at NNLO + $\overline{\text{NNLL}}$

NSV Resummation Phenomenology | Drell-Yan Process
The Renormalisation scale

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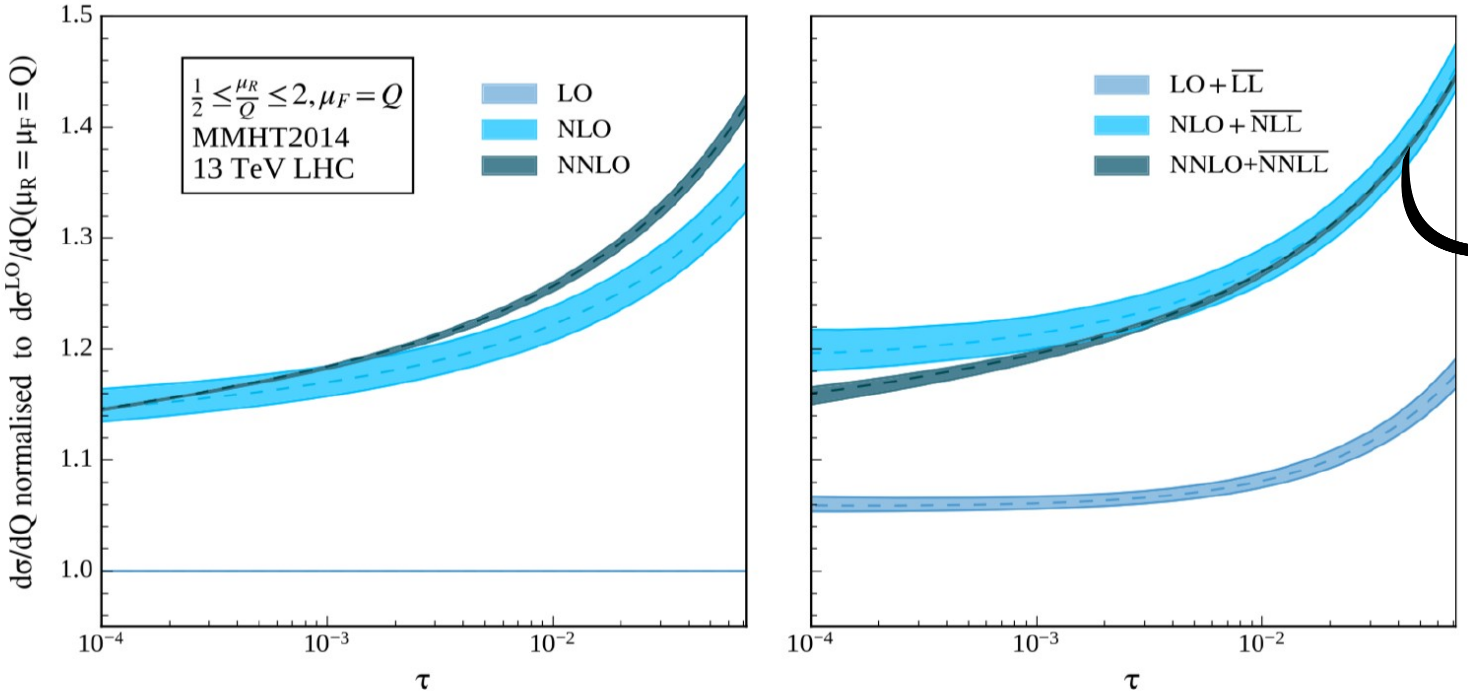


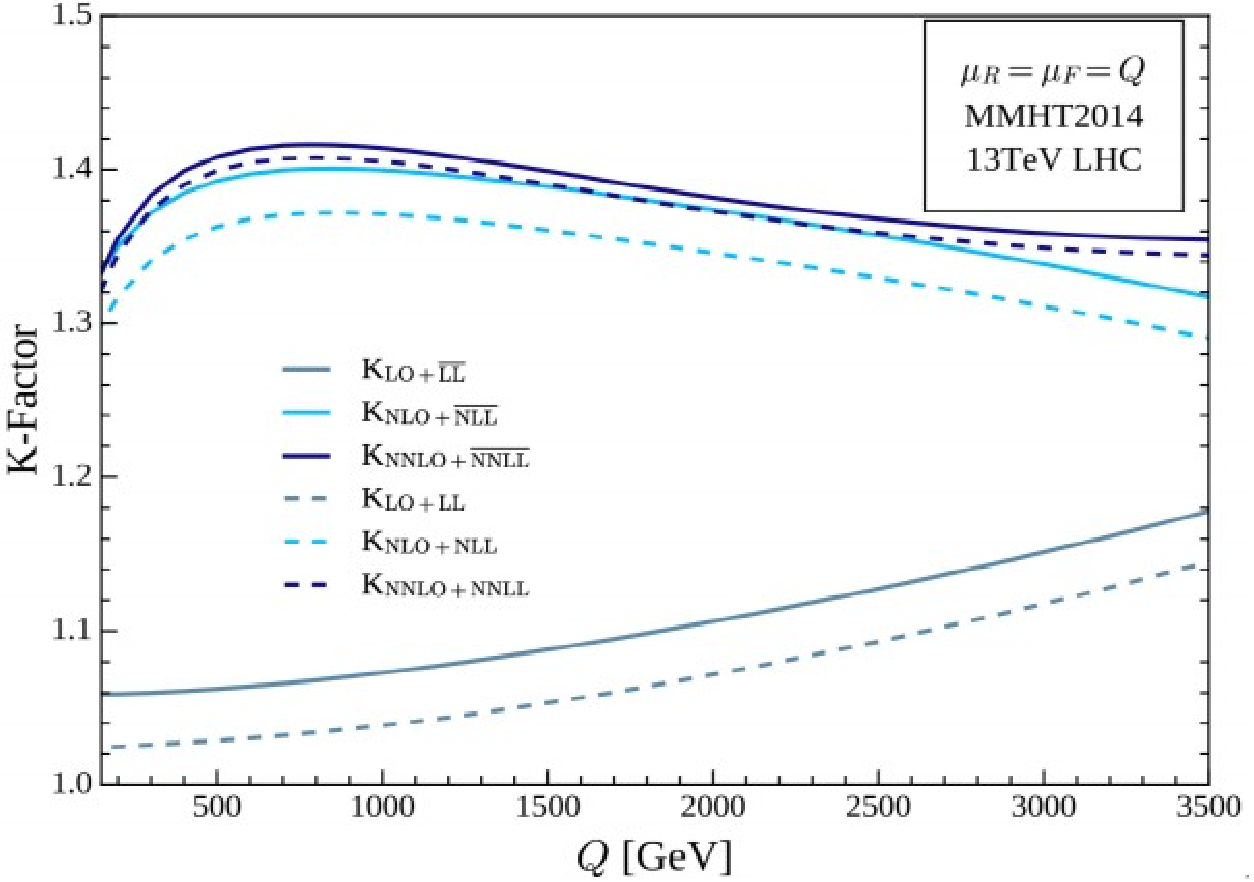
Substantial reduction in the μ_R – scale uncertainity at NNLO + $\overline{\text{NNLL}}$

The μ_R cancellation happens within each partonic channels.
Inclusion of resummed predictions improves the μ_R - scale uncertainity remarkably

NSV Resummation Phenomenology | Drell-Yan Process
SV -Resummation vs NSV -Resummation

Let's compare the SV resummation with SV+NSV resummation

Nomenclature :
N^nLL -> only SV Res
$\overline{\text{N}^n\text{LL}}$ -> SV+NSV Res



- Increment from SV resum → NSV resum at Q=2000 GeV

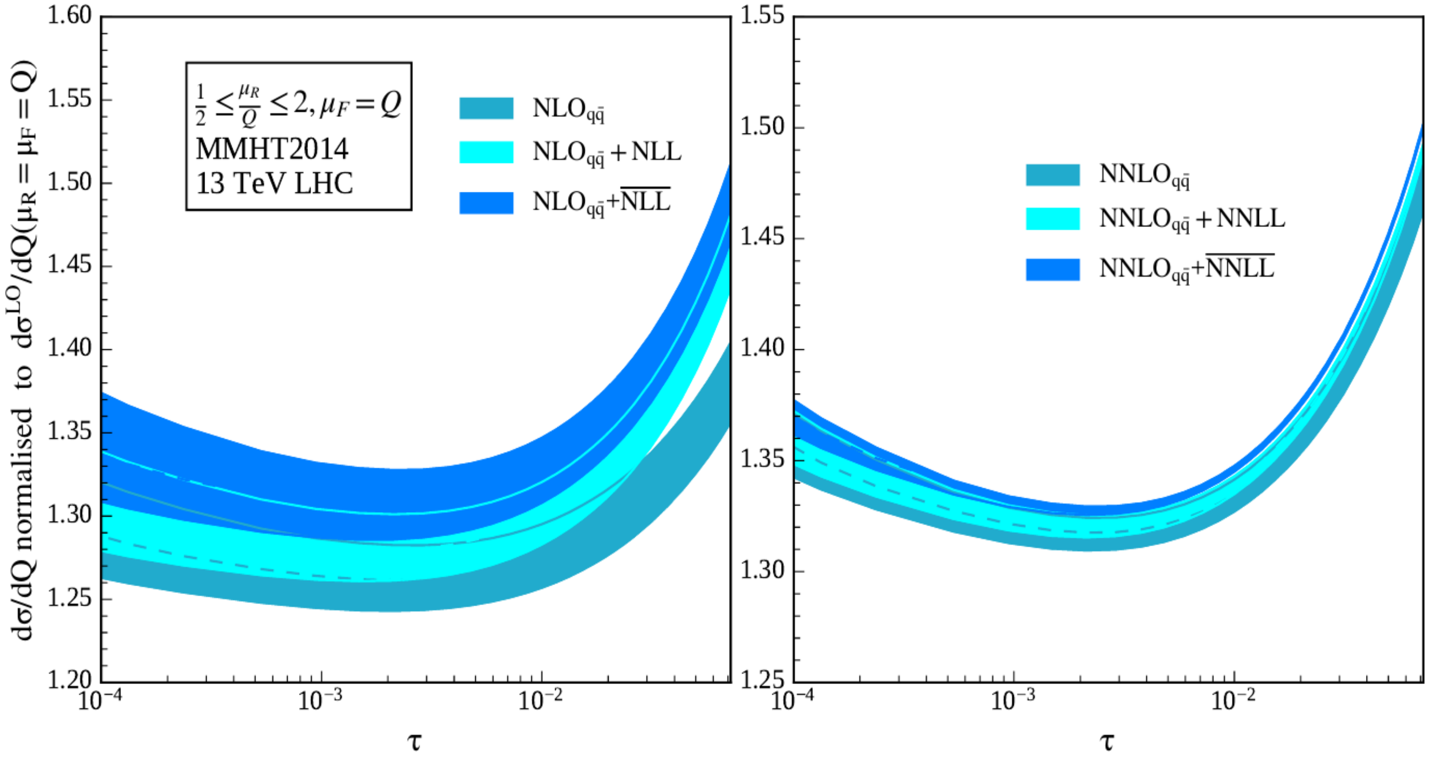
  2.1% for NLO+ NLL → NLO + $\overline{\text{NLL}}$
  0.64% for NNLO + NNLL → NNLO + $\overline{\text{NNLL}}$

- Perturbative convergence is improved after the inclusion of NSV resummation

NSV Resummation Phenomenology | Drell-Yan Process
SV -Resummation vs NSV -Resummation

Comparing SV resummation, SV+NSV resummation with NNLO results

μ_R scale variation within $q\bar{q}$- channel



- Behaviour of $NNLO_{q\bar{q}}+\overline{NNLL}$ is significantly improved from the corresponding SV $NNLO_{q\bar{q}}$ + NNLL

- Considerable improvement when adding the NSV resummation over the SV one leading to more reliable predictions

SUMMARY & OUTLOOK

- **Threshold resummation is essential in the kinematic limit $z \to 1$**

SUMMARY & OUTLOOK

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SUMMARY & OUTLOOK

- **Threshold resummation is essential in the kinematic limit $z \to 1$**

- **We extended the threshold resummation framework by including the subleading next-to-soft large logarithms in the diagonal channels to NNLL accuracy**

- **This brings in enhancement in the resummed predictions and improves the perturbative convergence**

SUMMARY & OUTLOOK

- Threshold resummation is essentail in the kinematic limit $z \to 1$

- We extended the threshold resummation framework by including the subleading next-to-soft large logarithms in the diagonal channels to NNLL accuracy

- This brings in enhancement in the resummed predictions and improves the perturbative convergence

- However, large scale uncertainities w.r.t the factorisation scale shows the need of including off-diagonal channles in the NSV resummation for large collinear logarithms, on-going work, stay tuned!

Thank you for your attention!

Additional slides

THE FORMALISM

Let's begin with the Factorisation

Unrenormalised Form Factor (FF)
(pure virtual corrections)

Soft-collinear function
(soft real emission contributions)

Partonic cross-section

$$\hat{\sigma}_{\mathbf{I}}(\mathbf{z},\epsilon) = \left(\mathbf{Z}_{\mathbf{I},\mathbf{UV}}\right)^{\mathbf{2}} \left|\hat{\mathbf{F}}_{\mathbf{I}}(\epsilon)\right|^{\mathbf{2}} \mathbf{S}_{\mathbf{I}}(\mathbf{z},\epsilon)\,, \quad \mathbf{I} = \mathbf{q}, \bar{\mathbf{q}}, \mathbf{g}$$

UV Renormalisation constant

Mass Factorisation

Coliinear finite Partonic coefficient function

$$\frac{\mathbf{1}}{\mathbf{z}}\hat{\sigma}_{\mathbf{ab}}(\mathbf{q^2},\mathbf{z},\varepsilon) = \sigma_{\mathbf{0}}(\mu_{\mathbf{R}}^{\mathbf{2}}) \sum_{\mathbf{a'b'}} \mathbf{\Gamma}^{\mathbf{T}}_{\mathbf{a'a}}(\mathbf{z},\mu_{\mathbf{F}}^{\mathbf{2}},\varepsilon) \otimes \left(\frac{\mathbf{1}}{\mathbf{z}}\mathbf{\Delta}_{\mathbf{a'b'}}(\mathbf{q^2},\mu_{\mathbf{R}}^{\mathbf{2}},\mu_{\mathbf{F}}^{\mathbf{2}},\mathbf{z},\varepsilon)\right) \otimes \mathbf{\Gamma}_{\mathbf{b'b}}(\mathbf{z},\mu_{\mathbf{F}}^{\mathbf{2}},\varepsilon)$$

Altarelli-Parisi (AP) kernels

OUR APPROACH

DIAGONAL AND OFF DIAGONAL

$$
\begin{aligned}
\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} &= \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{g\bar{q}} + \Gamma^T_{qq} \otimes \Delta_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \\
&+ \Gamma^T_{qg} \otimes \Delta_{gq} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qg} \otimes \Delta_{gg} \otimes \Gamma_{g\bar{q}} + \Gamma^T_{qg} \otimes \Delta_{g\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \\
&+ \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}q} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}g} \otimes \Gamma_{g\bar{q}} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.
\end{aligned}
$$

$$
\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.
$$

$$
\begin{aligned}
\frac{\hat{\sigma}_{qg}}{z\sigma_0} &= \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \Gamma^T_{qq} \otimes \Delta_{q\bar{q}} \otimes \Gamma_{\bar{q}g} \\
&+ \Gamma^T_{qg} \otimes \Delta_{gq} \otimes \Gamma_{qg} + \Gamma^T_{qg} \otimes \Delta_{gg} \otimes \Gamma_{gg} + \Gamma^T_{qg} \otimes \Delta_{g\bar{q}} \otimes \Gamma_{\bar{q}g} \\
&+ \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}q} \otimes \Gamma_{qg} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}g} \otimes \Gamma_{gg} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}\bar{q}} \otimes \Gamma_{\bar{q}g} \,.
\end{aligned}
$$

$$
\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{qg} \otimes \Gamma_{gg} \,.
$$

OUR APPROACH

Off-diagonal Channel: $$\frac{\hat{\sigma}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \cdots$$

In the threshold limit z -> 1 , keeping only $\log^k(1-z_i), \quad k = 0, \cdots \infty$ next to SV

$$\boxed{\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\mathrm{nsv}}_{qg} \otimes \Gamma_{gg} \,.}$$

dropping $(1-z_i)^k, \quad k = 1, \cdots \infty$

Getting complicated due to Mixing of channels

THE FORMALISM : THE BUILDING BLOCKS

$$a_s(\mu_R^2) = \frac{g_s^2(\mu_R^2)}{16\pi^2}$$

Form factor (pure virtual corrections)

$$Q^2 \frac{d}{dQ^2} \log \hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^c\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

Sudakov diff eqn (K+G eqn)

[Sen, Sterman,Magnea]

Overall Renormalisation constant

$$\mu_R^2 \frac{d}{d\mu_R^2} \log Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \ \ \gamma_{i-1}^c$$

Renormalisation Group Eqn (RGE)

UV anomalous dimension

Altarelli-Parisi Splitting kernels

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z, \mu_F^2, \epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}\left(z, a_s(\mu_F^2)\right) \otimes \Gamma_{a'b}(z, \mu_F^2, \epsilon)$$

Altarelli-Parisi evolution eqn

[Moch,Vogt,Vermaseren]

Altarelli-Parisi splitting function [Expanded in the limit z->1]

$$P_{cc}(z, \mu_F^2) = 2\left[\frac{A^c}{(1-z)_+} + B^c \ \delta(1-z) + C^c \ \log(1-z) + D^c + \mathcal{O}(1-z)\right]$$

$$\left\{A^c, B^c, C^c, D^c\right\}$$

Process independent IR anomalous dimensions

Phenomenology – Drell-Yan Process

GIVEN				PREDICTIONS		
$\Psi_c^{(1)}$	$\Psi_c^{(2)}$	$\Psi_c^{(3)}$	$\Psi_c^{(n)}$	$\Delta_c^{(2)}$	$\Delta_c^{(3)}$	$\Delta_c^{(i)}$
$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^3	$\mathcal{D}_5, \mathcal{D}_4$ L_z^5	$\mathcal{D}_{(2i-1)}, \mathcal{D}_{(2i-2)}$ $L_z^{(2i-1)}$
	$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^2, L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^4	$\mathcal{D}_{(2i-3)}, \mathcal{D}_{(2i-4)}$ $L_z^{(2i-2)}$
		$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^3, \cdots, L_z^0$				$\mathcal{D}_{(2i-5)}, \mathcal{D}_{(2i-6)}$ $L_z^{(2i-3)}$
			$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^n, \cdots, L_z^0$			$\mathcal{D}_{(2i-(2n-1))}, \mathcal{D}_{(2i-2n)}$ $L_z^{(2i-n)}$

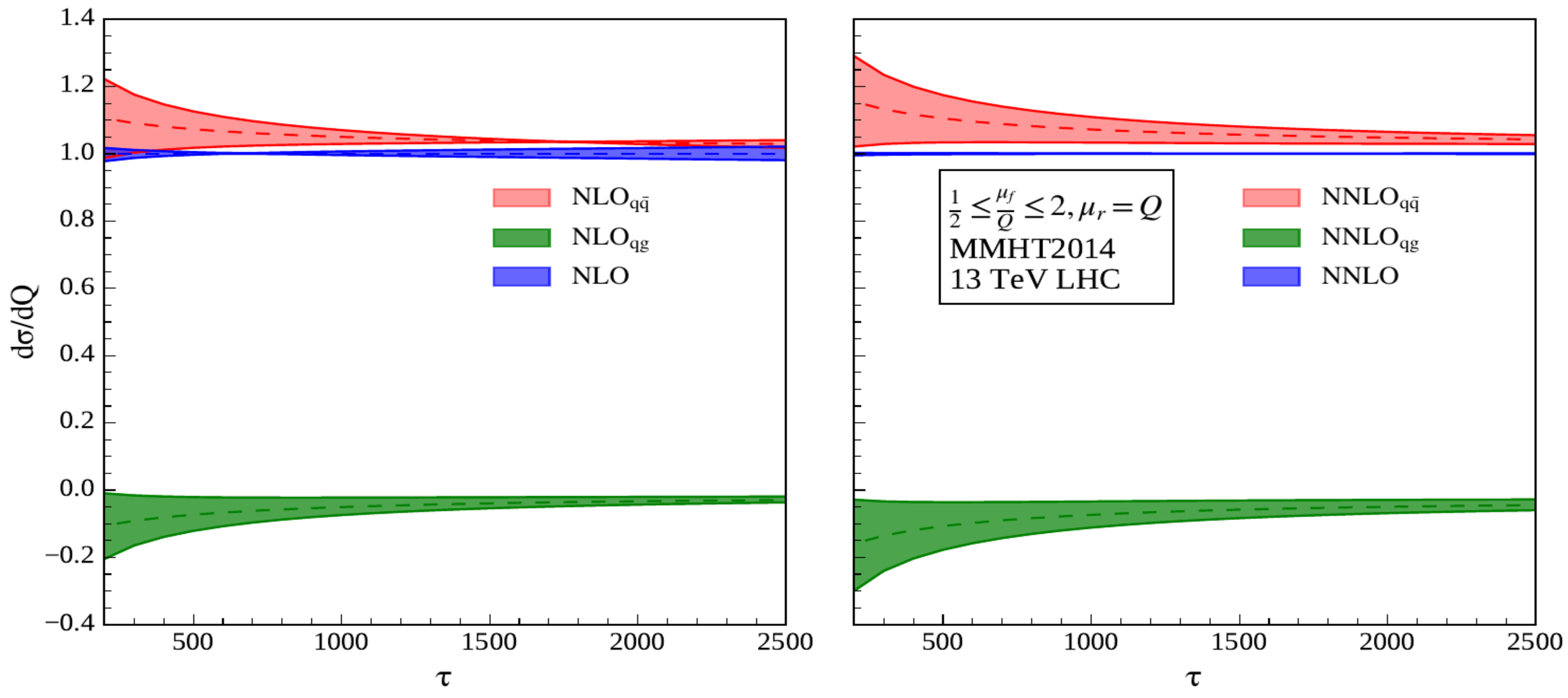
$$\mathcal{D}_j \equiv \left(\frac{\ln^j(1-z)}{1-z}\right)_+$$

$$L_z^i = \ln^i(1-z)$$

$$\delta = \delta(1-z)$$

The Predictions to all orders (z-space)

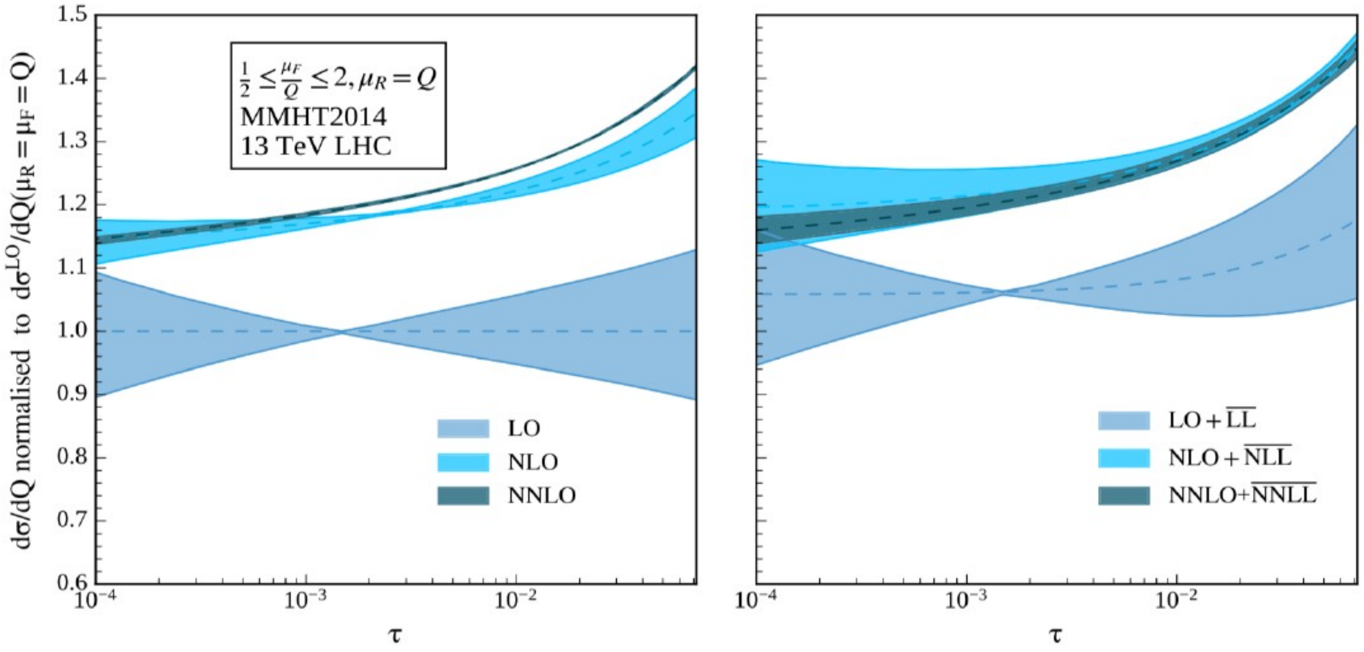
$q\bar{q}$ & qg contributions under μ_F variation keeping μ_R fixed



NSV Resummation Phenomenology | Drell-Yan Process
The Factorisation scale

In order to understand the cause of large uncertainty at NNLO + $\overline{\text{NNLL}}$, we study the scales separately.

μ_F - variation



- Bands in resummation plot look similar to 7-point scale variation : large band in 7-point scale is due to μ_F - uncertainties
- μ_F scale mixes up different channels:
- Collinear logarithms arises from $q\bar{q}$ & qg channels
- Large μ_F uncertainity could be due to lack of off-diagonal contribution!

At NLO : $q\bar{q} \rightarrow 22\%$ & $qg \rightarrow -5\,\%$

At NNLO : $q\bar{q} \rightarrow 4.9\%$ & $qg \rightarrow -2.5\,\%$

Bigger cancellation at NNLO.
Lack of qg resummed predictions
causes the larger uncertainty at NNLO + $\overline{\text{NNLL}}$