# Two-loop splitting in double parton distributions

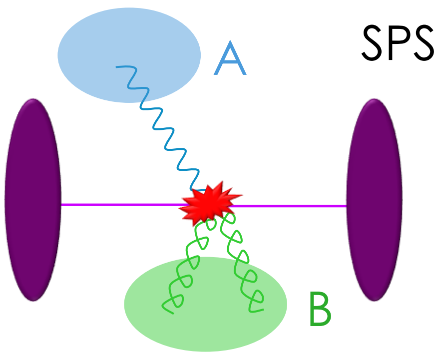
Jonathan Gaunt (U. of Manchester)

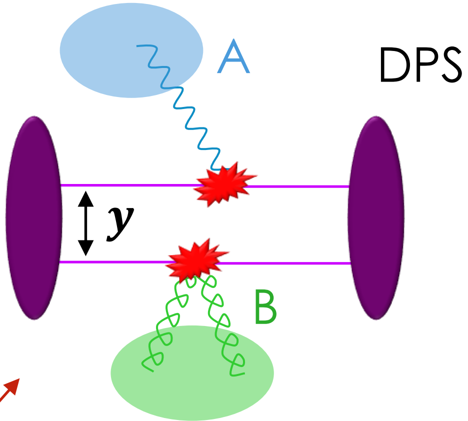
Based on SciPost Phys. 7 (2019) 2, 017, JHEP 08 (2021) 040 with Markus Diehl, Peter Plößl, Andreas Schäfer

RADCOR 2023
1st June 2023

# DOUBLE PARTON SCATTERING

Double parton scattering (DPS) is where we have **two separate hard scatters** in one collision




$$\sigma_S = f(x_1) \otimes \hat{\sigma}_{AB} \otimes f(x_1')$$

Single parton distributions (PDFs)

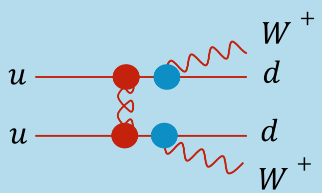
$$\sigma_D = \int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y}) \otimes \hat{\sigma}_A\ \hat{\sigma}_B \otimes F(x_1', x_2', \boldsymbol{y})$$

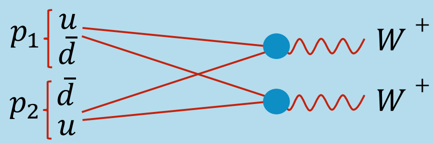
Double parton densities (DPDs)

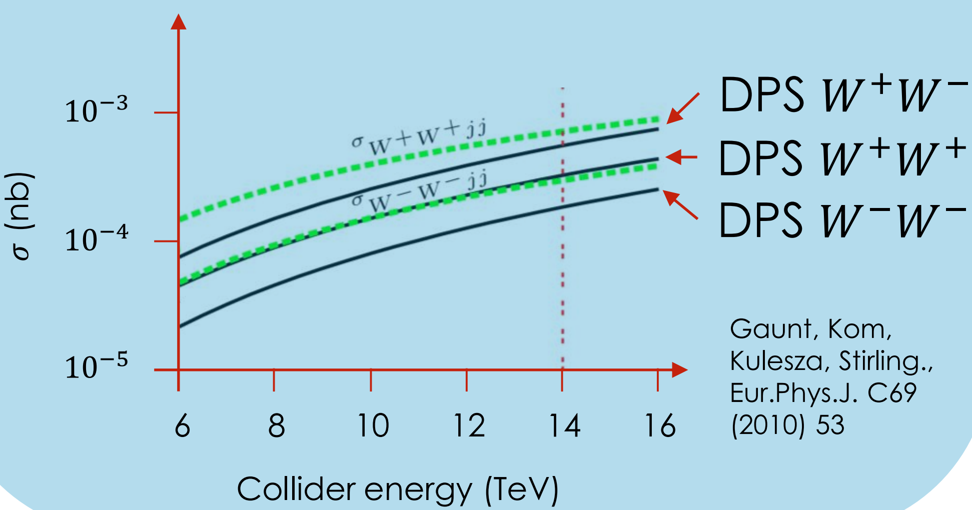
Paver, Treleani, Nuovo Cim. A70 (1982) 215
Mekhfi, Phys.Rev. D32 (1985) 2371
Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

# WHY STUDY DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

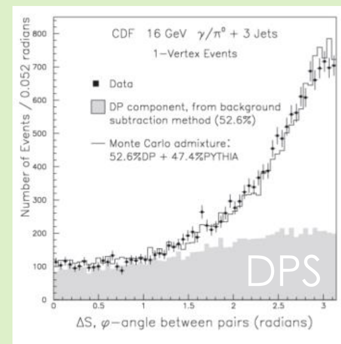
SPS: 


DPS: 

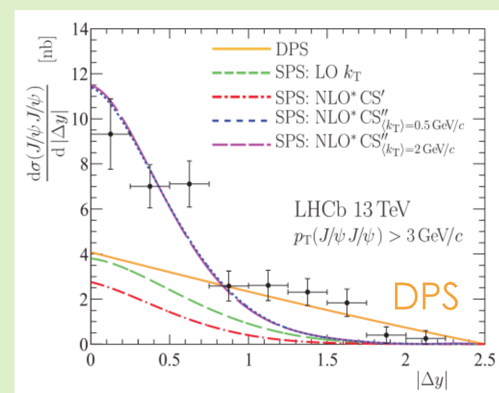

Gaunt, Kom, Kulesza, Stirling., Eur.Phys.J. C69 (2010) 53

…or in certain phase space regions




CDF, $\gamma + 3j$, Phys.Rev. D56 (1997) 3811-3832

LHCb, double $J/\psi$, JHEP 06, 047, (2017)



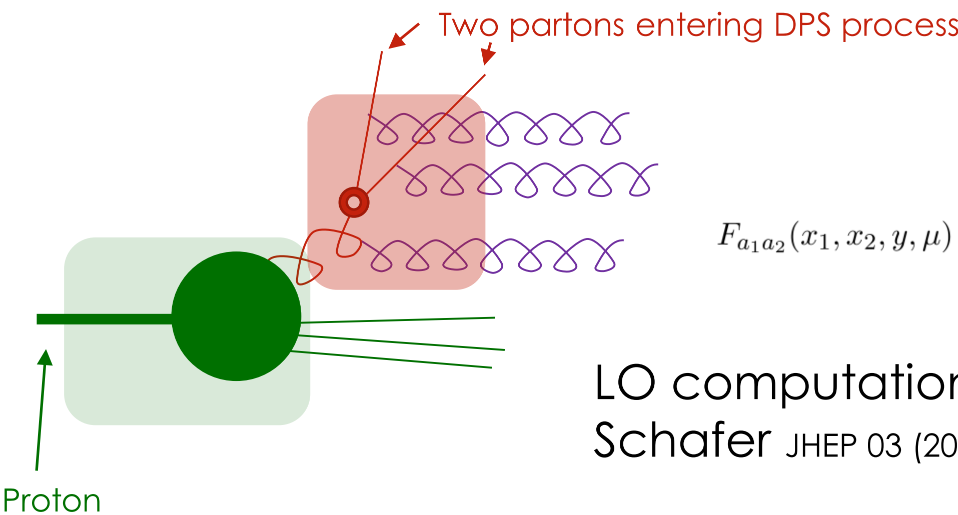
# DPDS AT SMALL Y

Theory predictions require DPDs: $F(x_1, x_2, \boldsymbol{y})$

For large $y \sim R_p$, need to use models, or compute on lattice.

See e.g. Bali et al., JHEP 09 (2021) 106

But for $y \ll R_p$, can compute DPDs perturbatively!



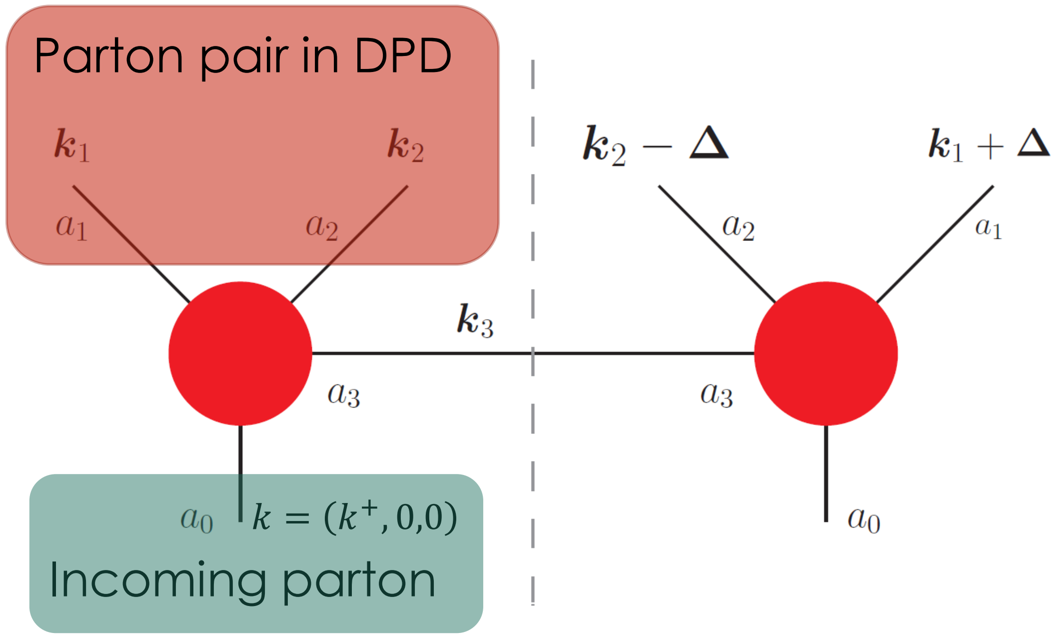

$$F_{a_1a_2}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2} \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} V_{a_1a_2,a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log\frac{\mu^2 y^2}{b_0^2}\right) f_{a_0}(z, \mu)$$

LO computation of $V$ performed in Diehl, Ostermeier, Schafer JHEP 03 (2012) 089

Here: NLO computation of $V$.

# CALCULATION

Strategy: Compute (bare) DPD in momentum space $F_B^{(2)}(\Delta)$ for a partonic initial state ($\Delta$ = Fourier conjugate to $y$)



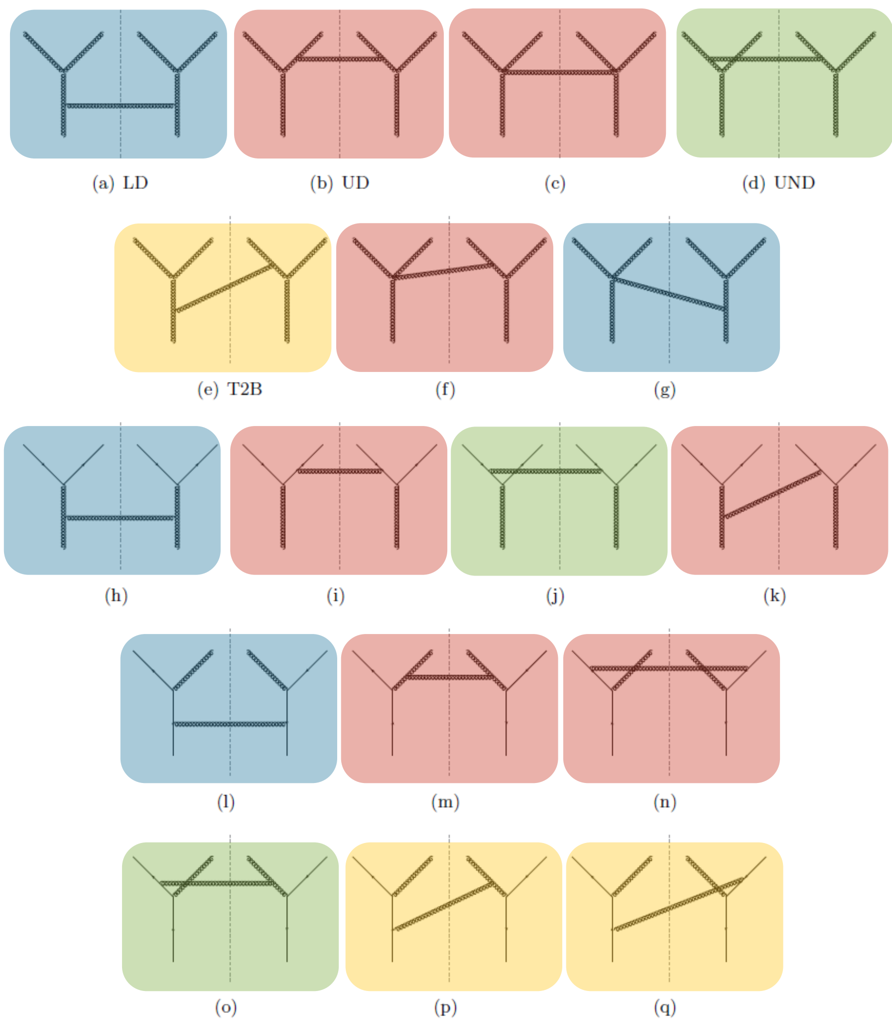

Plus components fixed:
$k_i^+ = z_i k^+, \sum z_i = 1$

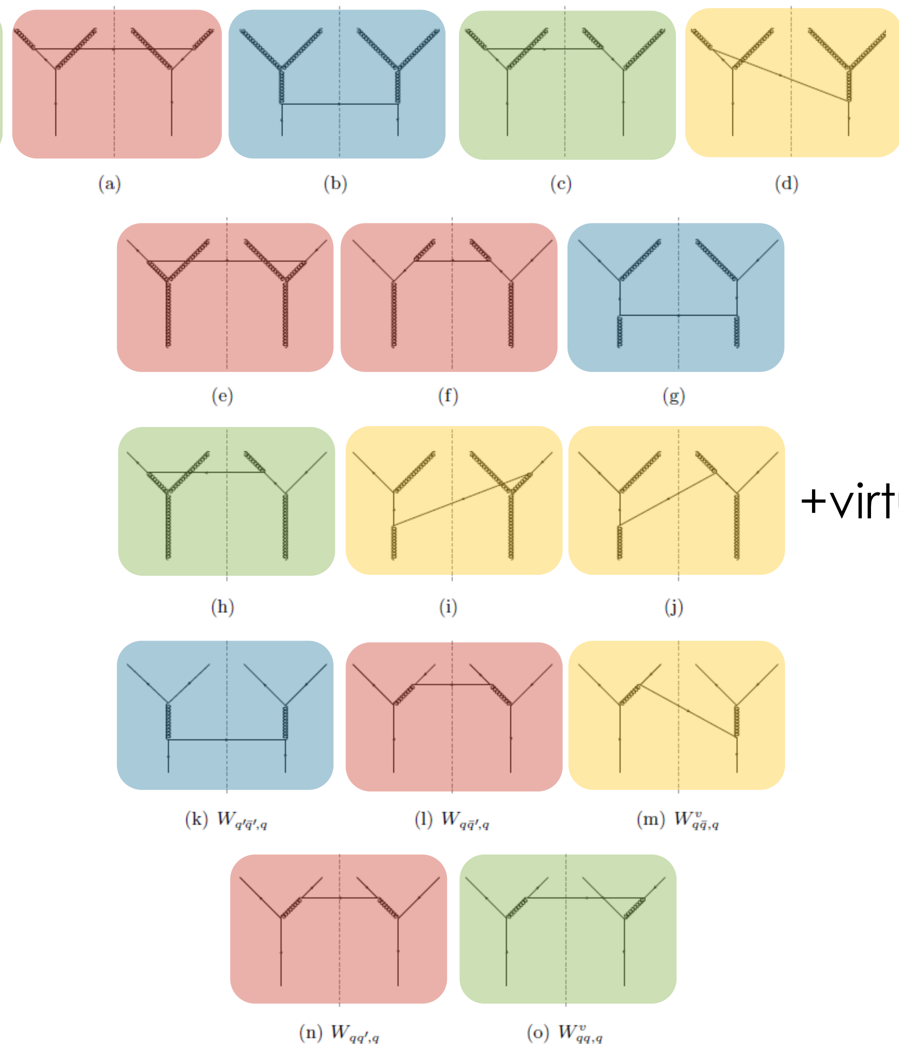
Must integrate over transverse components $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and minus components $k_1^-, k_2^-, \Delta^-$

Can extract $V^{(2)}(y)$ from $\epsilon^{-1}$ part of $F_B^{(2)}(\Delta)$.

# GRAPHS TO COMPUTE

Graphs to compute (in light-cone gauge):




+virtuals

**Four topologies.** Calculation done in both light-cone and Feynman gauge.

# METHOD: II

**Compute graph expressions** (`FORM, FeynCalc`). Integrate over minus components using contours.

[Kuipers, Ueda, Vermaseren, Vollinga, Comput. Phys. Commun. 184 (2013) 1453-1467]
[Shtabovenko, Mertig, Orellana, Comput. Phys. Commun. 207 (2016) 432-444]

$$\begin{bmatrix} \frac{\partial I_1(1,1,0,0)}{\partial x_1} \\ \frac{\partial I_1(0,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,0,1,1)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,1)}{\partial x_1} \\ \frac{\partial I_1(2,1,1,1)}{\partial x_1} \end{bmatrix} = \begin{bmatrix} \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacksquare & 0 & 0 & 0 \\ 0 & \blacklozenge & 0 & \blacksquare & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \end{bmatrix} \begin{bmatrix} I_1(1,1,0,0) \\ I_1(0,1,1,0) \\ I_1(1,1,1,0) \\ I_1(1,0,1,1) \\ I_1(1,1,1,1) \\ I_1(2,1,1,1) \end{bmatrix}$$

Construct **differential equations** in $z_1$ and solve (`Fuchsia`)

[Gituliar, Magerya, Comput. Phys. Commun. 219 (2017) 329-338]

Results for bare graphs!

$$D_1 = \frac{(\boldsymbol{k}_1+\boldsymbol{\Delta})^2}{z_1} + \frac{(\boldsymbol{k}_2-\boldsymbol{\Delta})^2}{z_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{z_3} \qquad D_2 = \frac{(\boldsymbol{k}_1)^2}{z_1} + \frac{(\boldsymbol{k}_2)^2}{z_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{z_3}$$

$$D_3 = (\boldsymbol{k}_1+\boldsymbol{\Delta})^2 \qquad D_4 = \boldsymbol{k}_2^2 \qquad \tilde{D}_4 = \boldsymbol{k}_1^2 \qquad \tilde{D}_5 = (\boldsymbol{k}_1+\boldsymbol{k}_2)^2$$

$$I_1(a_1,a_2,a_3,a_4) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1..4} D_i^{a_i}} \qquad I_2(a_1,a_2,a_3,a_4,a_5) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1\ldots3} D_i^{a_i} \prod_{i=4\ldots5} \tilde{D}_i^{a_i}}$$

$$I_1(0,1,1,0) \to \pi^{3-2\epsilon} {z_3}^{1-\epsilon} (z_1 z_2)^{\epsilon} \frac{\Gamma[-\epsilon]}{\sin[2\pi\epsilon]\Gamma[1-3\epsilon]}$$

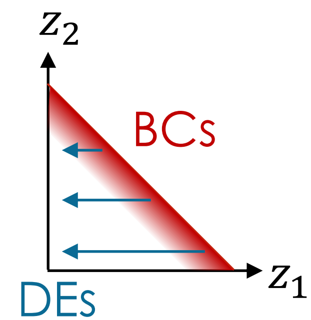
$I_1(1,1,0,0), I_1(0,1,1,0), I_1(1,1,1,0),$
$I_1(1,0,1,1), I_1(1,1,1,1), I_1(2,1,1,1)$

$I_2(0,1,1,0,1), I_2(1,1,1,1,0)$

**Integration-by-parts reduction to master integrals** (`LiteRed`)

[Lee, J. Phys. Conf. Ser. 523 (2014)]

Computation of $z_3 \to 0$ limit of master integrals using **method of regions** (boundary conditions)



# CROSS CHECKS

- Full computation of bare graphs done using **light-cone and covariant Feynman gauge** ✓
- Master integrals satisfy **differential equation in $x_2$** ✓
- Master integrals all checked **numerically** at 10 random points using FIESTA ✓ Smirnov, Smirnov, Tentyukov, Comput. Phys. Commun. 182, 790 (2011)
- Individual graphs have poles in $\epsilon$ up to $\epsilon^{-3}$. The $\epsilon^{-3}$ **pole cancels after summing over graphs, $\epsilon^{-2}$ pole is as predicted by renormalisation group equation** ✓
- Splitting kernels satisfy constraints related to **number and momentum sum rules**:

JG, Stirling, JHEP 1003 (2010) 005,
Blok, Dokshitzer, Frankfurt, Strikman, Eur.Phys.J. C74 (2014) 2926,
Ceccopieri, Phys.Lett. B734 (2014) 79-85,
Diehl, Plößl, Schäfer, Eur.Phys.J. C79 (2019) no.3, 253

$$\int_0^{1-x_1} dx_2 \left[P_{a_1 q, a_0}(x_1, x_2) - P_{a_1 \bar{q}, a_0}(x_1, x_2)\right] = \left(\delta_{a_1 \bar{q}} - \delta_{a_1 q} - \delta_{a_0 \bar{q}} + \delta_{a_0 q}\right) P_{a_1 a_0}(x_1),$$

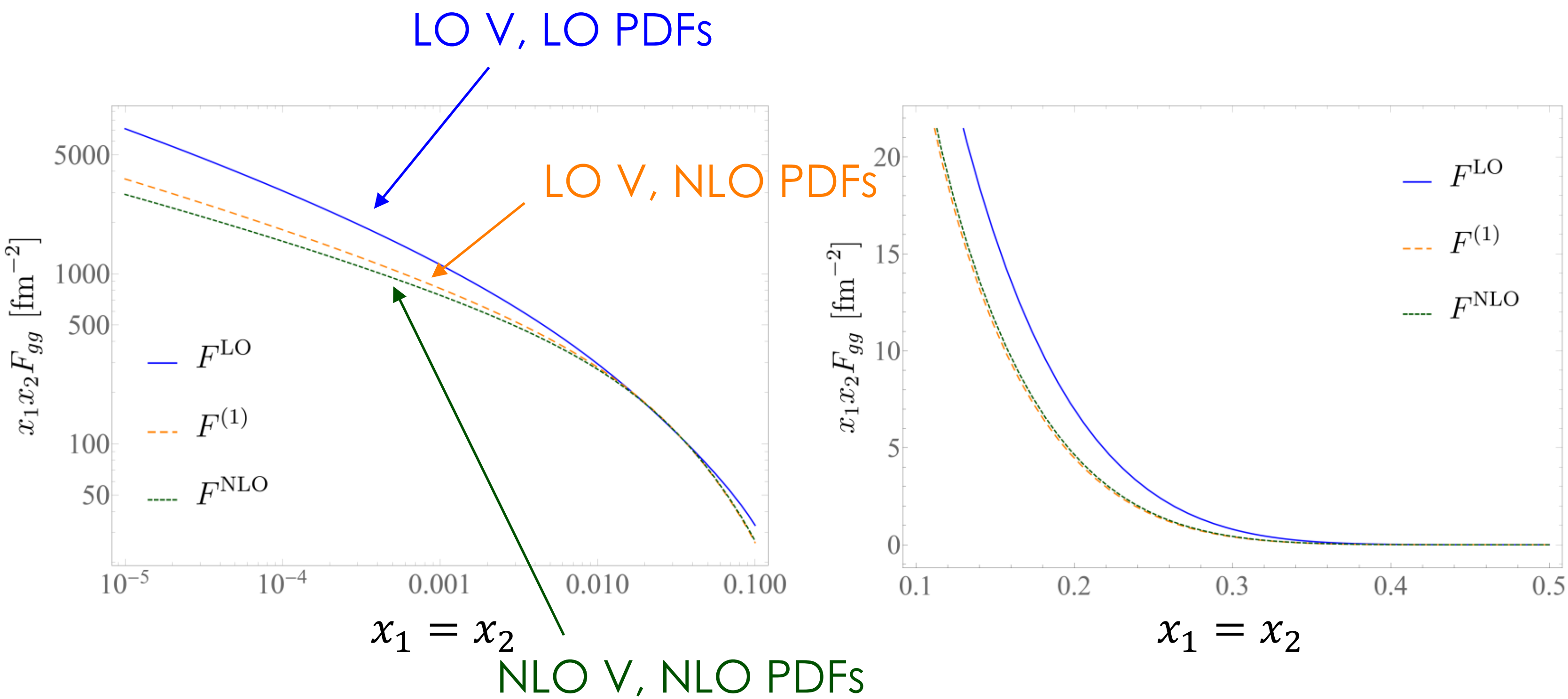
Closely linked to $V$ kernels

$$\sum_{a_2} \int_0^{1-x_1} dx_2 \, x_2 \, P_{a_1 a_2, a_0}(x_1, x_2) = (1 - x_1) \, P_{a_1 a_0}(x_1)$$

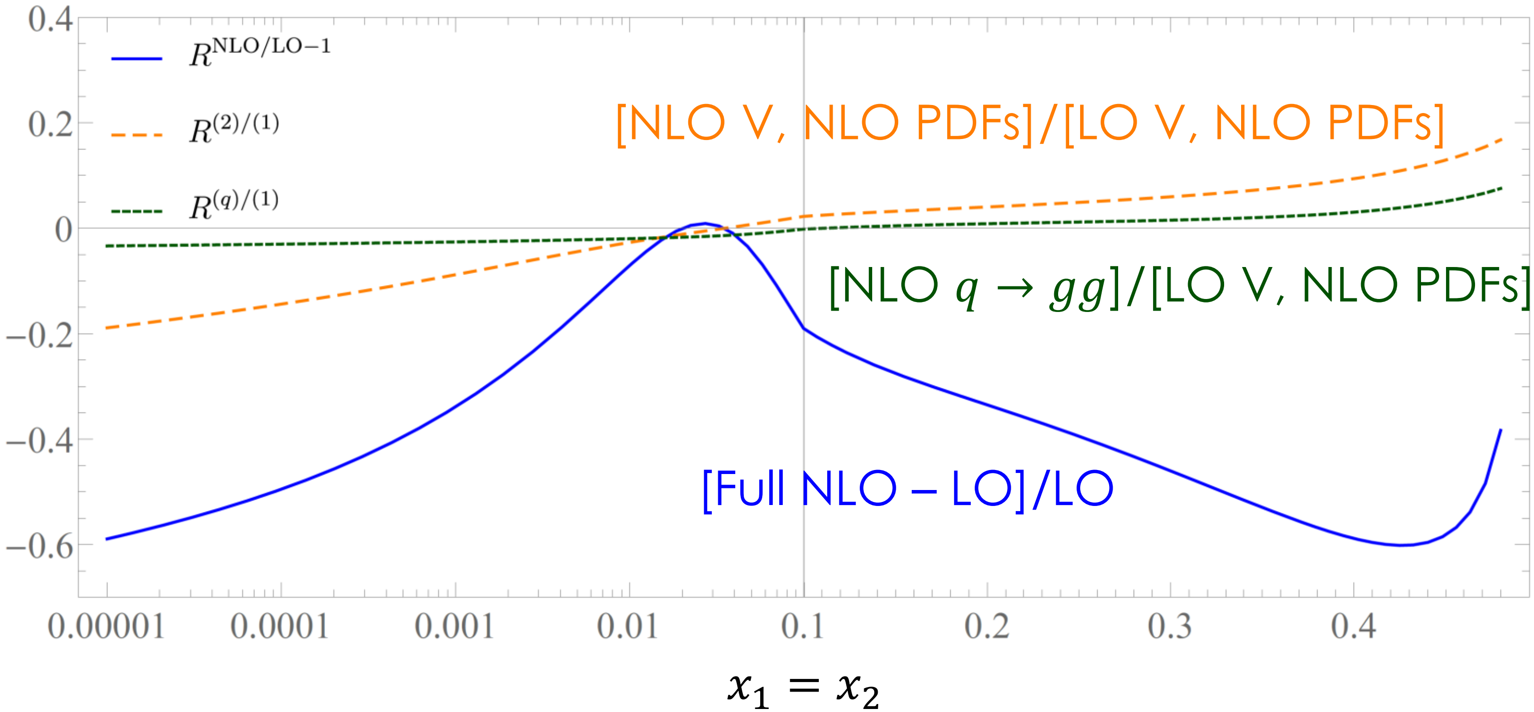
✓

# NLO: SOME NUMERICS

$y = 0.022$ fm, $\mu = b_0/y = 10$ GeV, splitting contribution only



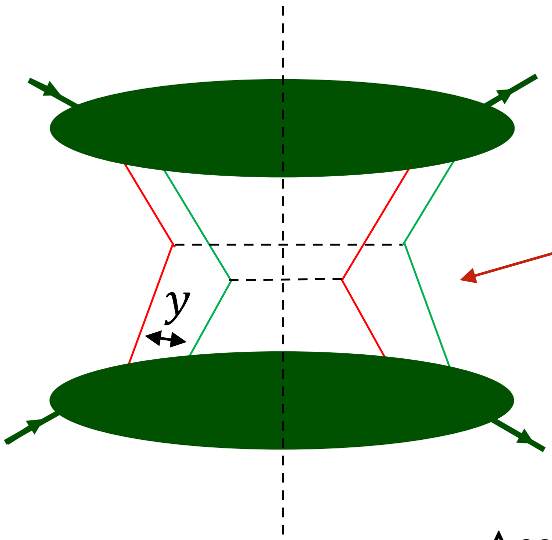
# NLO: SOME NUMERICS




- Large impact of NLO corrections, especially at low $x_i$ and $x_1 + x_2 = 1$.
- Much of this comes from switching LO PDF → NLO PDF, but **effect of $\alpha_s^2$ term in *V* ~5-10% of over wide kinematic range**.
- Effect of $q \rightarrow gg$ splitting is fairly small.

# COLOUR INTERFERENCE

Previous picture is not the full story…can also have interference contributions to DPS.




Example: colour interference

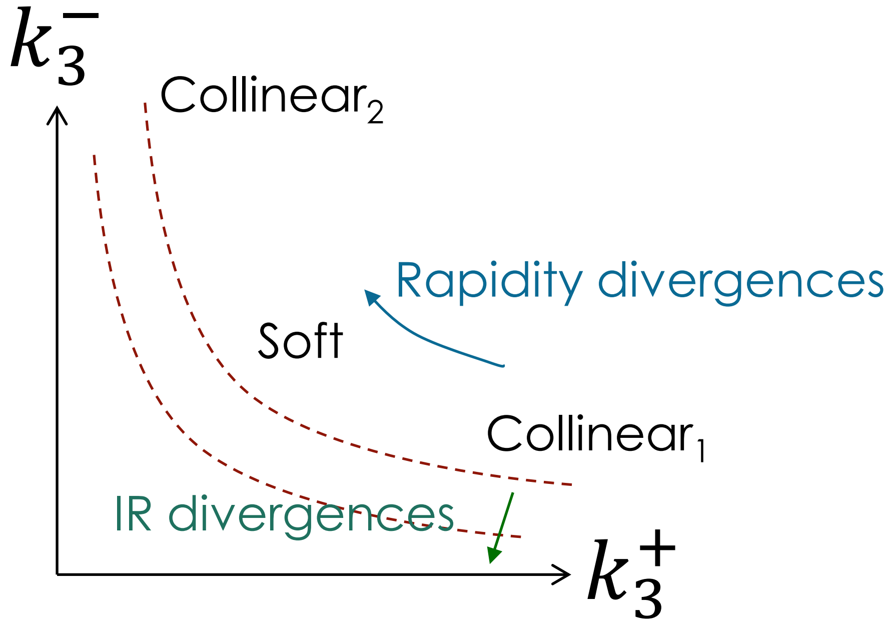
Mekhfi and Artru, Phys.Rev. D37 (1988) 2618–2622
Diehl, Ostermeier and Schafer (JHEP 1203 (2012) 089)
Manohar and Waalewijn, Phys.Rev. D85 (2012) 114009

Associated with modifications to DPS cross section due to colour correlations between partons

Can compute perturbative part of these colour correlations as before

# RAPIDITY DIVERGENCES

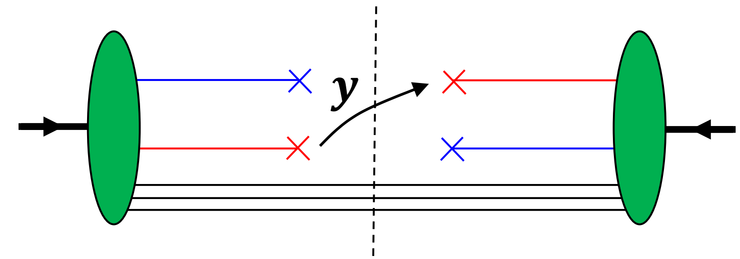
Subtlety here: naïve computation of DPDs for colour interference yields rapidity divergences!



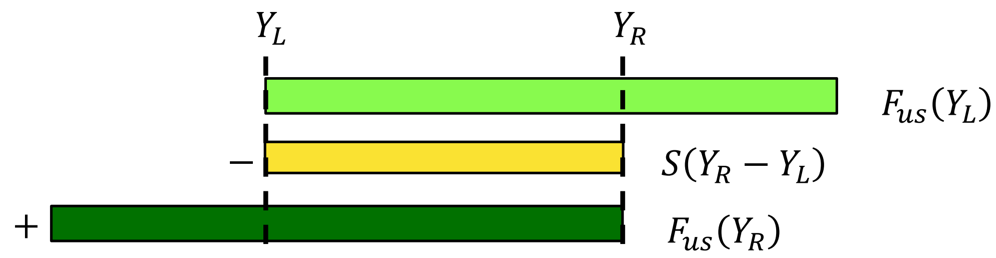

Rapidity divergences can't be regulated via dim reg. Need additional regulator!

# SOFT INTERACTIONS & COLOUR INTERFERENCE

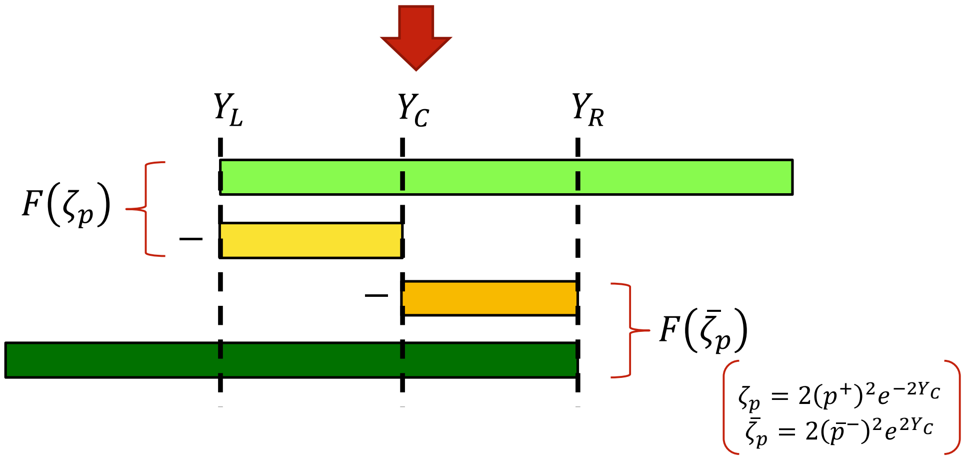
Sensitivity of this contribution to soft interactions, linked to shift of colour between amplitude & conjugate:




Initial factorisation formula needs soft function



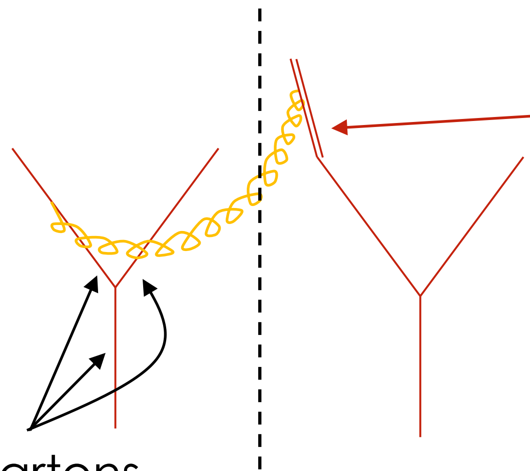

Soft function can be "divided up" between DPDs at central rapidity




Vladimirov, JHEP 1804 (2018) 045
Buffing, Diehl, Kasemets, JHEP 1801 (2018) 044

# RAPIDITY REGULATORS

Real diagrams with rapidity divergences:



$$\frac{1}{k_3^+ + i0}$$ from Wilson line

Any partons

See rapidity divergences as $z_3 \to 0$ (but $\boldsymbol{k}_3$ finite)

We use two different regulators:

(1) **$\boldsymbol{\delta}$ regulator**: $\frac{1}{k_3^+ + i0} \to \frac{1}{k_3^+ + i\delta^+}$

Echevarria, Scimemi, Vladimirov, Phys. Rev. D 93 (2016) 054004, JHEP 09 (2016) 004

(2) **Collins regulator**. Tilt Wilson line off the light-cone: $\frac{1}{k_3 \cdot n + i0} \to \frac{1}{k_3 \cdot v + i0}$, $v^2 < 0$

Collins, Foundations of perturbative QCD. Cambridge University Press, 2013.

# $\delta$ REGULATOR

Procedure with $\delta$ regulator:

$$\frac{1}{k_3^+ + i\delta^+} + \mathrm{c.c.} = \frac{2}{k^+} \frac{z_3}{z_3^2 + z_1 z_2/\rho} \Rightarrow \lim_{\rho\to\infty} \frac{z_3}{z_3^2 + z_1 z_2/\rho} = \frac{1}{[z_3]_+} + \frac{1}{2}\delta(z_3)[\log\rho - \log(z_1 z_2)]$$

Adding conjugate graph

$\rho = k_1^+ k_2^+/(\delta^+)^2$

IBPs, differential equations, method of region

Use elementary methods

$$\mathcal{L}_0(z_3) z_3^{-\epsilon} \to z_3^{-1-\epsilon} \to -\frac{1}{\epsilon}\delta(z_3) + \frac{1}{[z_3]_+} + \cdots, \quad \delta(z_3) z_3^{-\epsilon} \to 0$$

**Soft** divergence, regulated by **dim reg**

$\delta$ regulator easy to use here since it **doesn't interfere with transverse momentum integrations**. In fact one can just naively do IBP, DE method (ignoring rapidity regulator), perform transverse momentum integrations, then identify places where one has a $1/z_3$ factor (with no $z_3^{-\epsilon}$), then insert regulator and perform distributional expansion.

# COLLINS REGULATOR

Procedure with Collins regulator:

$$\lim_{\varepsilon\to 0} \frac{1}{v^- k_3^+ + v^+ k_3^- + i\varepsilon} + \mathrm{c.c.} = \frac{2}{v^- k^+} \mathrm{PV} \frac{z_3}{z_3^2 - \boldsymbol{k}_3^2 z_1 z_2/\rho}$$

$$\rho = k_1^+ k_2^+ v^- / |v^+|$$

Note transverse momentum dependence. Could consider following IBP, DE,.. procedure including also this denominator – however this is **very messy** (a lot of additional master integrals, with more severe rapidity divergences than final result)

# COLLINS REGULATOR

**Benefit from simplifications when $\rho \to \infty$:**

$$\lim_{\varepsilon\to 0}\frac{1}{v^-k_3^+ + v^+k_3^- + i\varepsilon} + \text{c.c.} = \frac{2}{v^-k^+}\,\text{PV}\frac{z_3}{z_3^2 - \boldsymbol{k}_3^2 z_1 z_2/\rho}$$

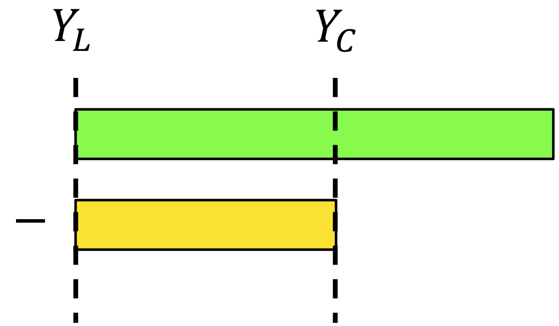
$$\lim_{\rho\to\infty}\text{PV}\frac{z_3}{z_3^2 - \boldsymbol{k}_3^2 z_1 z_2/\rho} = \frac{1}{[z_3]_+} + \frac{1}{2}\delta(z_3)\left[\log\frac{\rho}{\Delta^2} - \log(z_1 z_2) - \log\frac{\boldsymbol{k}_3^2}{\Delta^2}\right]$$

One extra term here, which can be evaluated with elementary methods after using: $\log\left(\boldsymbol{k}_3^2/\Delta^2\right) = \left[\frac{\partial}{\partial\alpha}\left(\boldsymbol{k}_3^2/\Delta^2\right)^\alpha\right]_{\alpha=0}$

# THE SUBTRACTED KERNEL

Construction of subtracted two-loop kernel:

$$ {}^{R_1R_2}V_B^{(2)}(\zeta) = \lim_{\rho\to\infty}\left\{ {}^{R_1R_2}V_{B,us}^{(2)}(\rho) - \frac{1}{2}\,{}^{R_1}S_B^{(1)}(\rho,\zeta)\,{}^{R_1R_2}V_{B,us}^{(1)} \right\} $$



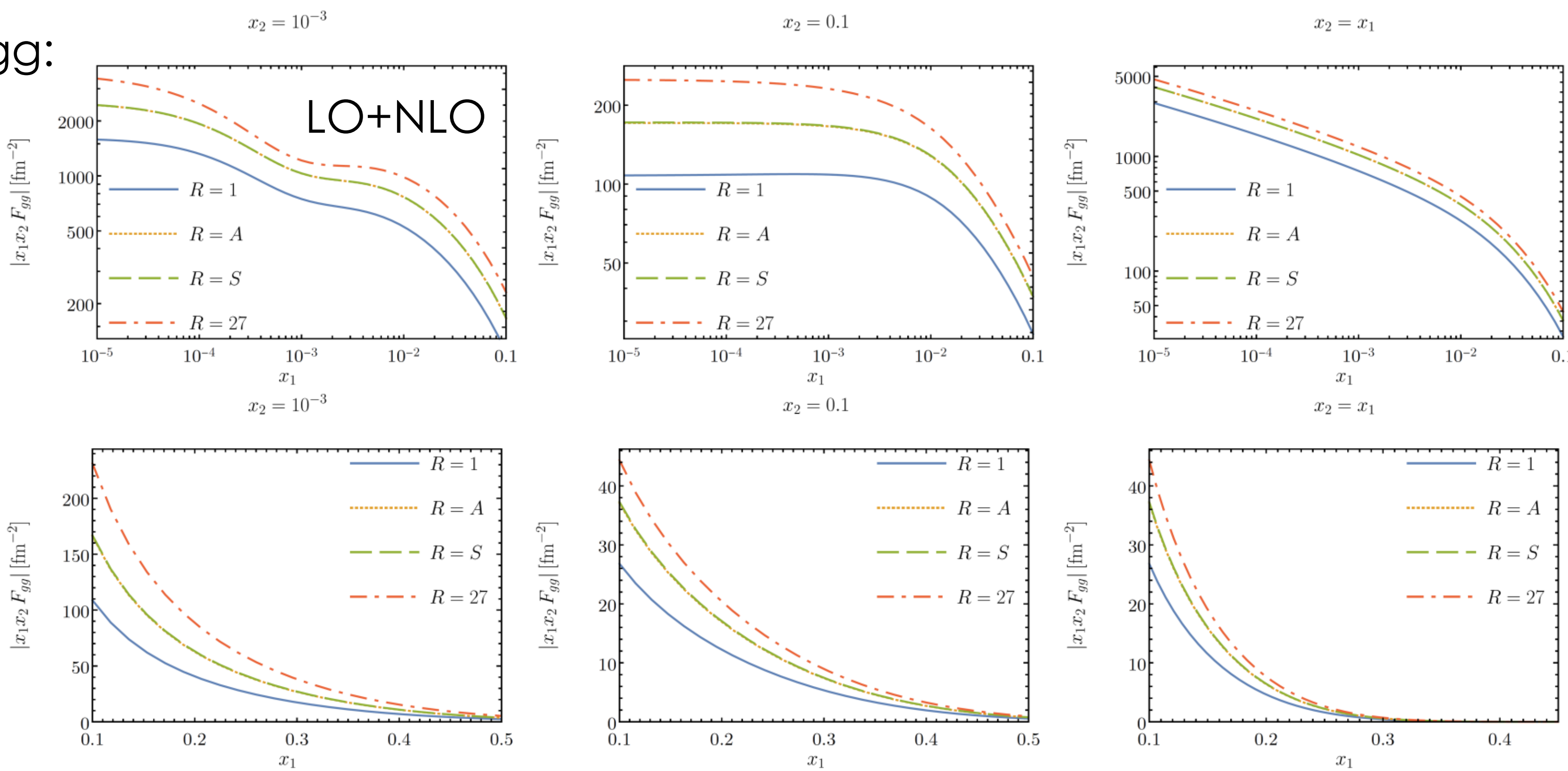

Dependence on rapidity regulator **cancels** between two terms.

Final result the **same** whether using Collins or $\delta$ regulator.

# NUMERICS: WITH COLOUR INTERFERENCE

$y = 0.022\ \mathrm{fm}, \mu = \sqrt{x_1 x_2 \zeta_p} = 10\ \mathrm{GeV}$, splitting contribution only

gg:

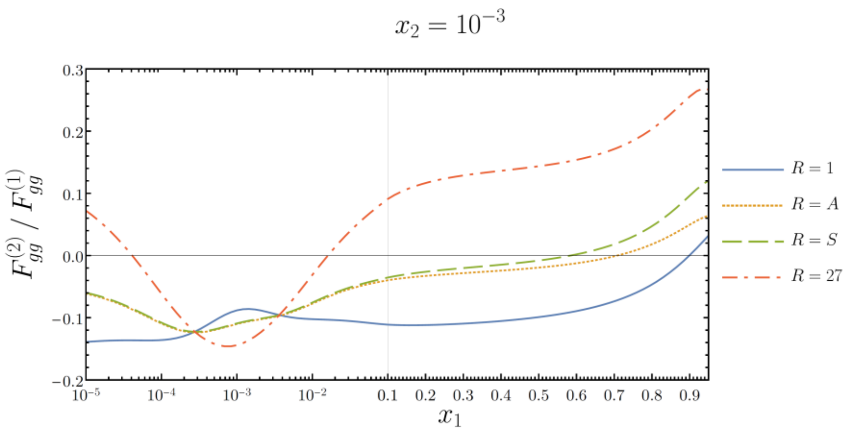


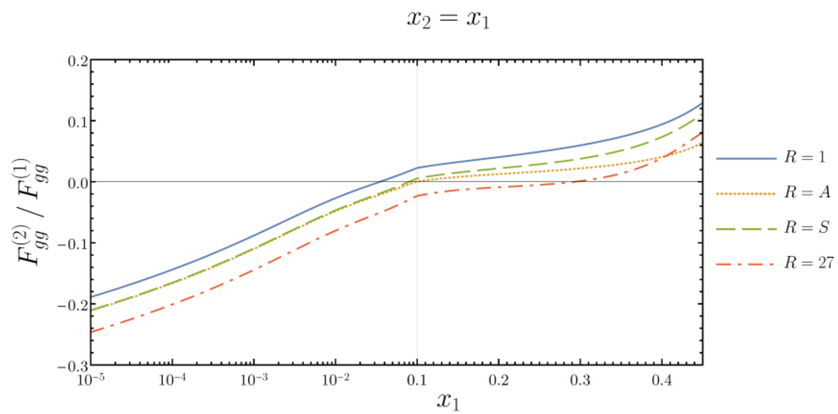

**Note: strong colour correlations generated by splitting!**

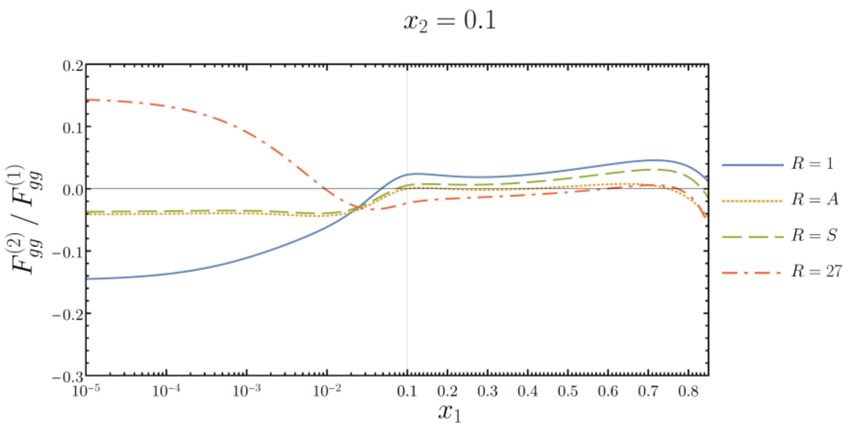
Colour interference distributions ~ colour singlet distribution

# NUMERICS: WITH COLOUR INTERFERENCE

Ratio NLO/LO:



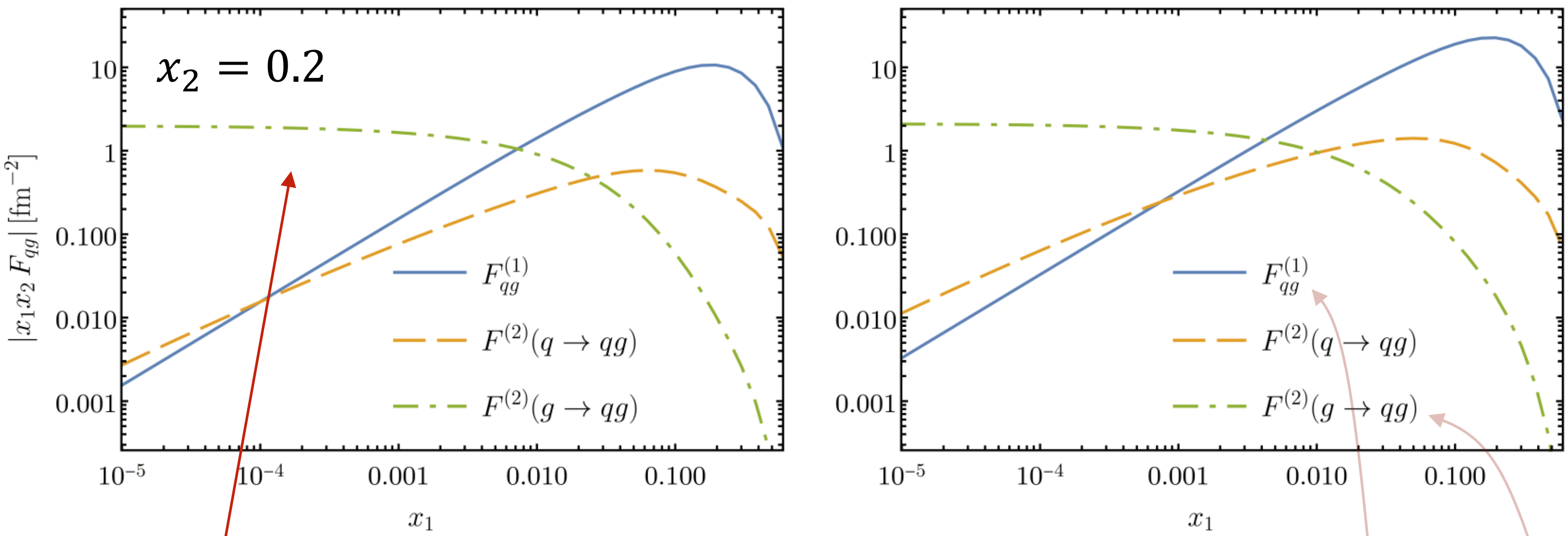




**$\mathcal{O}(10\%)$ NLO corrections**

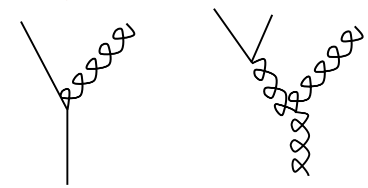
Varied structure as a function of $x_1, x_2$

# NUMERICS: WITH COLOUR INTERFERENCE

qg:




(a) $R_1 R_2 = 11$ (b) $R_1 R_2 = 8A$



Note at small $x_1$, NLO $g \to qg$ channel overtakes LO

# SUMMARY

- NLO matching of DPDs onto PDFs computed for all colour configurations of observed partons ✓

$$ {}^{R_1R_2}F(x_1, x_2, y; \mu) = \frac{1}{\pi y^2} \int \frac{dz}{z^2} \, {}^{R_1R_2}V\left(\frac{x_1}{z}, \frac{x_2}{z}, \alpha_s(\mu), \log\frac{y^2\mu^2}{b_0^2}, \log\frac{\mu^2}{x_1 x_2 \zeta_p}\right) f(z, \mu) $$

- Rapidity divergences at intermediate stages of the calculation. Used two regulators - $\delta$ regulator and Collins regulator.
  - $\delta$ regulator is straightforward to use (no $\boldsymbol{p}_T$ dependence in regulator)
  - Collins regulator calculation is only slightly more involved, after $\rho \to \infty$ limit.

- NLO corrections to splitting are of $\mathcal{O}(10\%)$ when $\mu = \sqrt{x_1 x_2 \zeta_p} = \mu_y$.

# STRUCTURE OF RESULTS

**General structure of results.**

Colour non-singlet kernels:

$$
{}^{R_1R_2}V^{(2)}_{a_1a_2,a_0}(z,u,y,\mu,\zeta) = {}^{R_1R_2}V^{[2,0]}_{a_1a_2,a_0}(z,u) + L\ {}^{R_1R_2}V^{[2,1]}_{a_1a_2,a_0}(z,u)
+ \left( L \log\frac{\mu^2}{\zeta} - \frac{L^2}{2} + c_{\overline{\text{MS}}} \right) \frac{{}^{R_1}\gamma_J^{(0)}}{2}\ {}^{R_1R_2}V^{(1)}_{a_1a_2,a_0}(z,u)
$$

where $L = \log\frac{y^2\mu^2}{b_0^2}$ and $b_0 = 2e^{-\gamma}$ and

$$
V^{[2,0]}(z,u) = V^{[2,0]}_{\text{regular}}(z,u) + \delta(1-z)\, V^{[2,0]}_{\delta}(u)\,,
$$

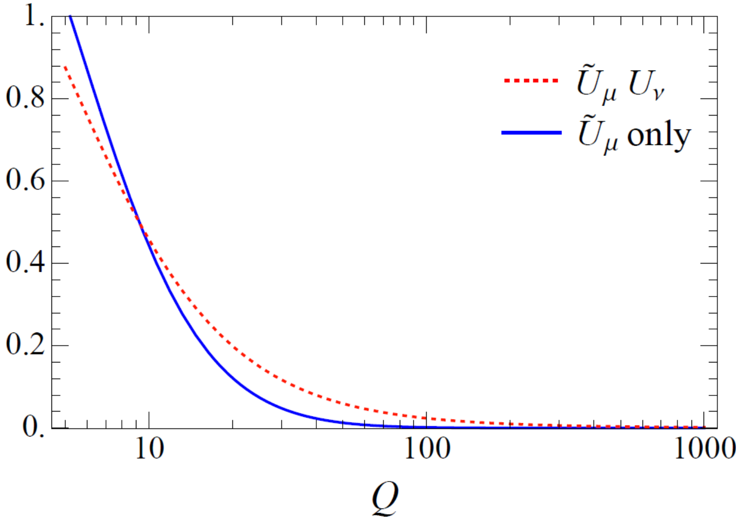
$$
V^{[2,1]}(z,u) = V^{[2,1]}_{\text{regular}}(z,u) + \frac{1}{[1-z]_+}\, V^{[2,1]}_{+}(u) + \delta(1-z)\, V^{[2,1]}_{\delta}(u)
$$

[Slide from Peter Plößl, talk at QCD evolution 2021]

# COLOUR CORRELATIONS

Colour correlations are strongly suppressed at high scales

[Technically: Sudakov suppression due to movement of colour between amplitude & conjugate by distance $\boldsymbol{y}$.]




Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

First estimate: negligible at 100 GeV, but could be relevant at moderate scales ~10 GeV.

(Enhanced by splitting?)

# SMALL $x_1, x_2$ LIMIT



Interesting processes/regions for studying DPS typically involve small $x$ values (higher density of partons→greater chance of DPS, plus smaller $Q$ such that power suppression is reduced).

→ Interesting to study matching coefficients and splitting functions in limits of small $x_i$. For example, small $x_1, x_2$ limit of $P^{(1)}_{gg,g}(x_1, x_2)$:

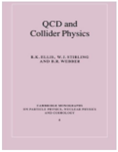
$$P^{(1)}_{gg,g}(x_1, x_2) \longrightarrow \frac{C_A^2\left(\left(1 - 6u + 6u^2\right) + \left(8 - \frac{2}{u} - 4u + 4u^2\right)\log[1-u] + \{u \leftrightarrow 1-u\}\right)}{x^2}$$

$x \equiv x_1 + x_2$
$u \equiv x_1/(x_1 + x_2)$

Same $1/x^2$ behaviour for other splitting functions, and $V$ kernels

$V^{(1)}(x_1, x_2) \sim 1/x^2 \Rightarrow F(x_1, x_2, \boldsymbol{y}) \sim \alpha_s^{n+2} \log^{n+1}(x)/x$ (for NLO splitting)
i.e. NLL in small $x$ logarithms!

[$V^{(1)}(x_1, x_2) \sim \log(x)/x^2 \Rightarrow F(x_1, x_2, \boldsymbol{y}) \sim \alpha_s^{n+2} \log^{n+2}(x)/x$, i.e. LL]




Similar cf usual splitting functions, where $P^{(1)}(x) \sim 1/x$ and not $\log(x)/x$ .

# REGIONS FOR $x_3 \to 0$

The leading behaviour of the integrals for $x_3 \to 0$ is computed using the method of regions [86]. Let us introduce the scaling parameter $\lambda \ll 1$, and say that $x_3 \sim \lambda$. Then for all master integrals the following region gives a leading contribution in $\lambda$:

$$R_1: \quad x_3 \sim \lambda\,, \quad x_1, x_2 \sim 1\,, \quad \boldsymbol{k}_1^2, \boldsymbol{k}_2^2, (\boldsymbol{k}_1 + \boldsymbol{\Delta})^2 \sim \Delta^2\,, \quad (\boldsymbol{k}_1 + \boldsymbol{k}_2)^2 \sim \lambda \Delta^2\,. \tag{106}$$

For $I_1(1,0,1,1)$ only, we identify a further leading region, namely

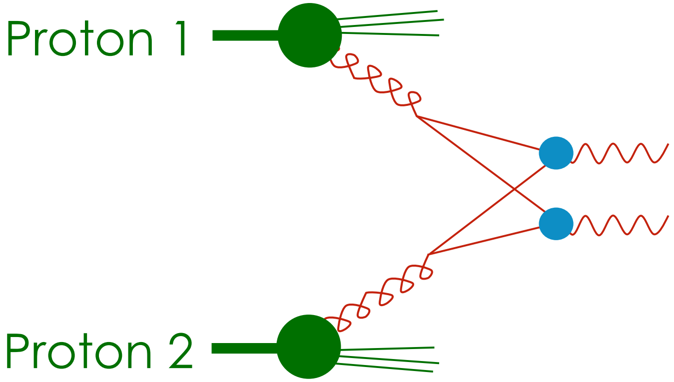
$$R_2: \quad x_3 \sim \lambda\,, \quad x_1, x_2 \sim 1\,, \quad \boldsymbol{k}_1^2, \boldsymbol{k}_2^2, (\boldsymbol{k}_1 + \boldsymbol{\Delta})^2, (\boldsymbol{k}_1 + \boldsymbol{k}_2)^2 \sim \Delta^2\,. \tag{107}$$

For each region, we use the appropriate scaling and drop terms in the denominator that are subleading in $\lambda$ (so that the result is homogeneous in $\lambda$). Following this approximation, every master integral has a sufficiently simple form to be solved to all orders in $\epsilon$ by the method of Feynman parameters. Then one adds together the contributions from the leading regions to obtain the leading behaviour in the limit $x_3 \to 0$. For any master integral, the region $R_1$ gives a non-integer power of $x_3$ for $\epsilon \neq 0$, namely $x_3^{1-\epsilon}$. This is because all denominators behave like $\lambda^0$, whilst the phase space contributes $\lambda^{1-\epsilon}$. For $I_1(1,0,1,1)$ the region $R_2$ gives an integer power of $x_3$, namely $x_3^1$, because $D_1$ behaves like $\lambda^{-1}$, whilst all other denominators and the phase space behave like $\lambda^0$.

# DOUBLE COUNTING PROBLEMS

Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

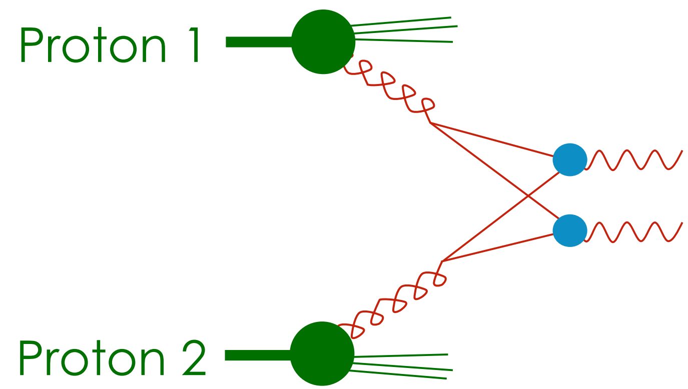
$$\int \frac{d^2y}{y^4} = ?$$



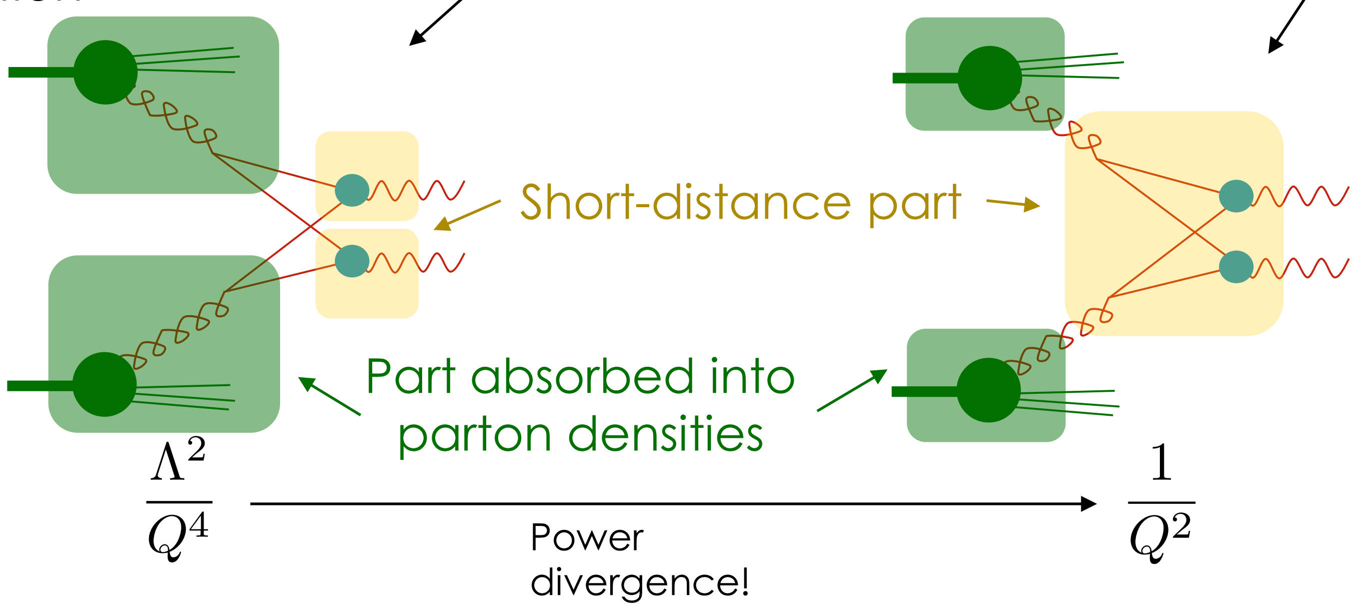
# DOUBLE COUNTING PROBLEMS

Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

$$\int \frac{d^2y}{y^4} = ?$$




This is related to the fact that this graph can also be regarded as an SPS loop correction

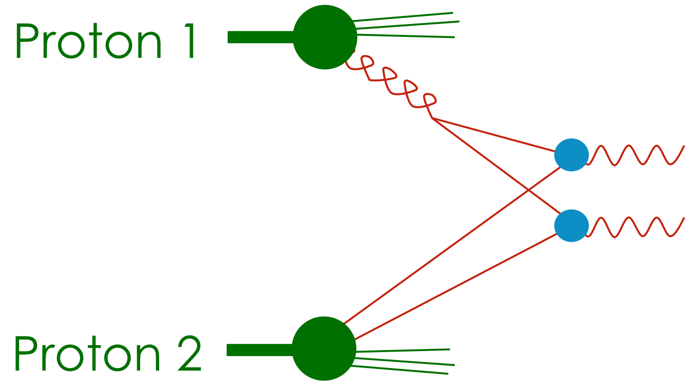



Diehl, Ostermeier and Schafer (JHEP 1203 (2012)), Manohar, Waalewijn Phys.Lett. 713 (2012) 196, **JG and Stirling, JHEP 1106 048 (2011)**, Blok et al. Eur.Phys.J. C72 (2012) 1963 Ryskin, Snigirev, Phys.Rev.D83:114047 ,2011, Cacciari, Salam, Sapeta JHEP 1004 (2010) 065

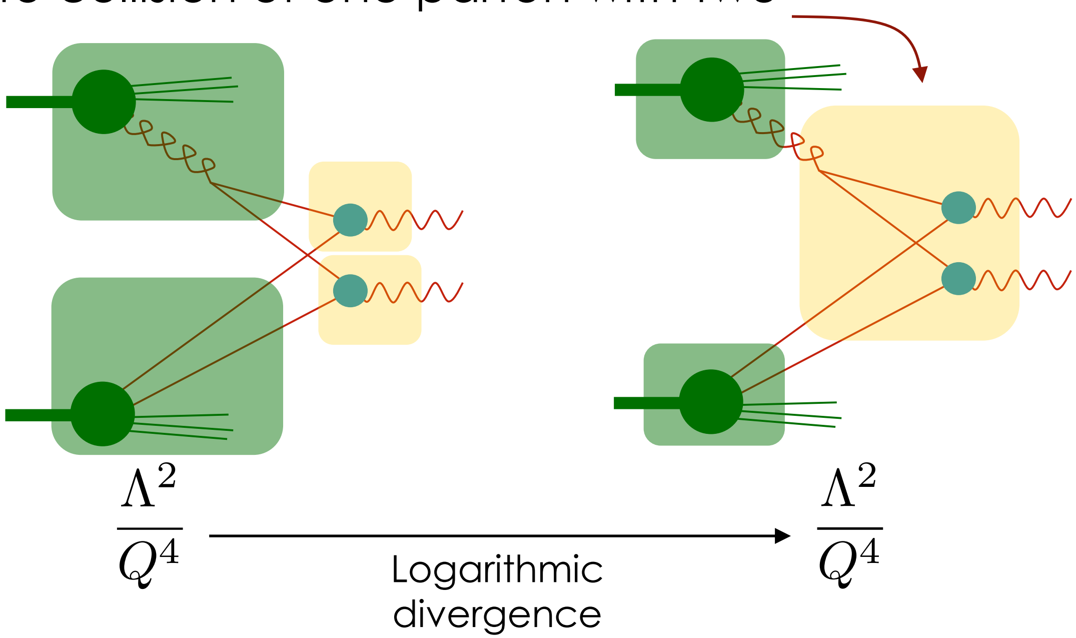
# DOUBLE COUNTING PROBLEMS

Also have graphs with perturbative 1→2 splitting in one proton only (2v1 graph).

This has a log divergence: $\int d^2y/y^2 F_{\text{non-split}}(x_1, x_2; y)$




Related to the fact that this graph can also be thought of as an NLO correction to collision of one parton with two




Blok et al., Eur. Phys. J. C72 (2012) 1963
Ryskin, Snigirev, Phys. Rev. D83:114047,2011,
**JG, JHEP 1301 (2013) 042**

# DOUBLE COUNTING PROBLEMS

Desired features of a solution to these issues:

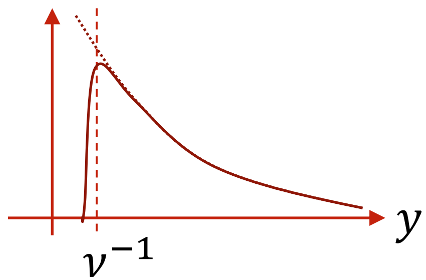
- DPS contribution finite + no double counting between DPS and SPS.
- Retain concept of the DPD for an individual hadron, with rigorous definition beyond perturbation theory.
- Should resum DGLAP logarithms in all types of diagram (1v1, 2v1, 2v2) where appropriate.
- All-order formulation, with corrections that are practicably computable.
- Re-use as many SPS results as possible.

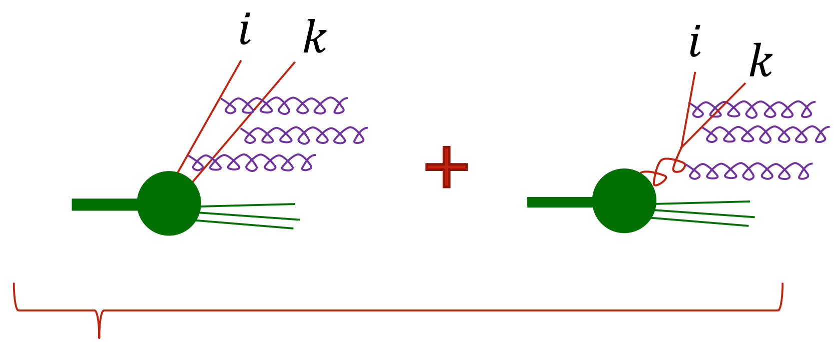
Solution with these features achieved in 'DGS framework' Diehl, JG, Schönwald JHEP 1706 (2017) 083.

# DPS WITHOUT DOUBLE COUNTING

I focus on SPS & 1v1 DPS overlap. Removal of overlap between 2v1 DPS & 3 particle collision is similar.

<u>Step 1:</u> insert cut-off function into DPS cross section formula




$$\sigma_{DPS}^{(A,B)} = \int dx_i dx'_i d^2\boldsymbol{y}\, \Phi^2(y\nu)\, F_{ik}(x_1, x_2, \boldsymbol{y}, \mu_A, \mu_B)\, F_{jl}(x'_1, x'_2, \boldsymbol{y}, \mu_A, \mu_B) \times \hat{\sigma}_{ij}^A\, \hat{\sigma}_{kl}^B$$

Choose $\nu \sim Q$ in practice.

Removed divergence. Double counting up to scale $\nu$.

# DPS WITHOUT DOUBLE COUNTING

<u>Step 2:</u> For total cross section for production of *AB*, include a subtraction term to remove double counting.

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

$\sigma_{sub}$: DPS cross section with DPDs replaced by fixed order splitting expression – i.e. combining the approximations used to compute double splitting piece in two approaches.

$$F_{ij}(x_1, x_2, y, \mu^2) \to \frac{1}{\pi y^2} \frac{f_k(x_1 + x_2, \mu^2)}{x_1 + x_2} \frac{\alpha_s(\mu^2)}{2\pi} P_{k\to ij}\left(\frac{x_1}{x_1 + x_2}\right)$$

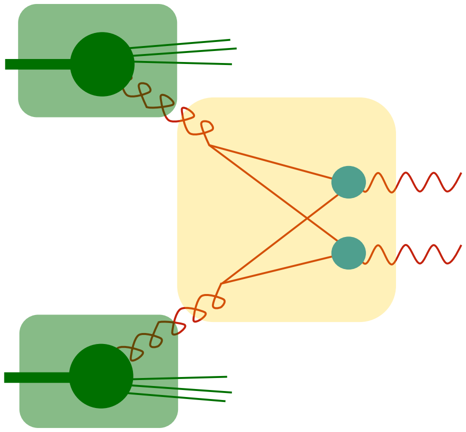
Similar philosophy used in subtraction terms in QCD factorisation, SCET zero bin subtractions, combination of NLO and parton shower…

# HOW THE SUBTRACTION WORKS

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

For small $\boldsymbol{y}$ (of order $1/Q$) the dominant contribution to $\sigma_{DPS}$ comes from the (fixed order) perturbative expression $\Rightarrow \sigma_{DPS} \approx \sigma_{sub}$
& $\sigma_{tot} \approx \sigma_{SPS}$ ✓

Dependence on $\nu$ cancels order-by-order between $\sigma_{DPS}$ & $\sigma_{sub}$



For large $\boldsymbol{y}$ (much larger than $1/Q$) the dominant contribution to $\sigma_{SPS}$ is the region of the 'double splitting' loop where DPS approximations are valid

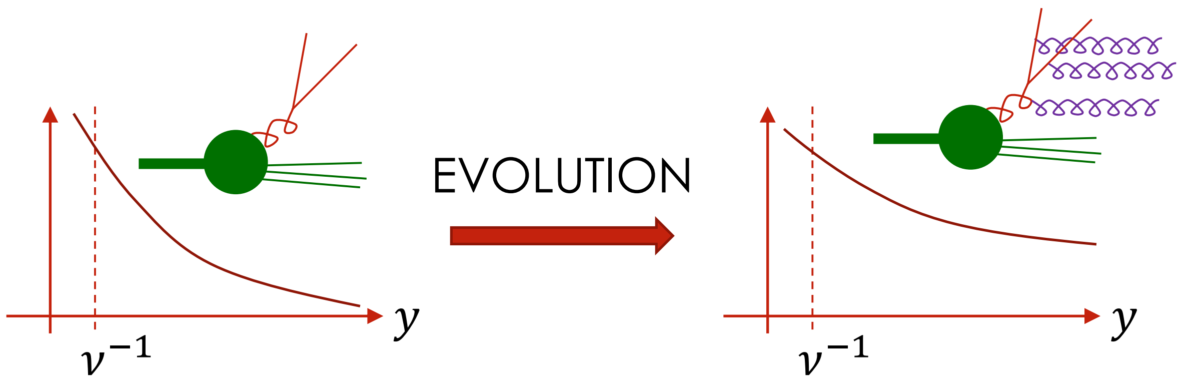
$\Rightarrow \sigma_{SPS} \approx \sigma_{sub}$
& $\sigma_{tot} \approx \sigma_{DPS}$ ✓

# CUTOFF DEPENDENCE

Important: $\sigma_{DPS}$ is not really 'meaningful' on its own. Can only measure $\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$

Generically $\propto \nu^2$

IN CERTAIN CASES:




Bulk of $\sigma_{DPS}$ shifts to large $\boldsymbol{y}$ where DPS approximations are valid. Small $\boldsymbol{y}$ is less important → reduced $\nu$ dependence, $\sigma_{sub}$ and two-loop $\sigma_{SPS}$ less important.