



MAX-PLANCK-INSTITUT
FÜR PHYSIK

Heaviside functions and the zero-jettiness soft function at N3LO QCD

Based on [2111.13594](#) & [2204.09459](#) and work in preparation with Daniel Baranowski, Maximilian Delto, Andrey Pikelner and Kirill Melnikov.

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Outline

1. Motivation
2. Integral reduction
3. Integral evaluation
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Motivation
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Integral reduction
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Motivation

- The ever-increasing experimental precision at the LHC and the HL-LHC in the future demands percent level precision from the theoretical side. *ATLAS 2019; CMS 2021*
- On the theoretical side many N3LO calculations and phenomenology results are available.
- Computing differential cross-section requires subtracting infrared divergences in the phase space:
 - Slicing:
 - q_T subtraction scheme *Catani and Grazzini 2007*
 - N-jettiness subtraction scheme *Boughezal, Focke, et al. 2015; Gaunt et al. 2015*
 - Subtraction:
 - CoLoRFull *Somogyi et al. 2005*
 - Antenna *Gehrmann-De Ridder et al. 2005* see Matteo's talk
 - STRIPPER *Czakon 2010*
 - Nested soft-collinear subtraction *Caola et al. 2017* see Chiara's talk
 - Local analytic sector subtraction *Magnea et al. 2018* see Gloria's talk
 - Project-to-Born *Cacciari et al. 2015*
 - ...

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Motivation

- To obtain differential cross sections, one can use slicing to extract and cancel infrared divergences properly:

$$\sigma(O) = \int_0 d\tau \frac{d\sigma(O)}{d\tau} = \int_0^{\tau_0} d\tau \frac{d\sigma(O)}{d\tau} + \int_{\tau_0} d\tau \frac{d\sigma(O)}{d\tau}.$$

- q_T subtraction scheme *Catani and Grazzini 2007*
- N-jettiness subtraction scheme *Boughezal, Focke, et al. 2015; Gaunt et al. 2015*
- q_T subtraction scheme is available up to N3LO *Li and Zhu 2017; Ebert et al. 2020b; Luo et al. 2020*
- N-jettiness factorization theorem derived in SCET *Stewart et al. 2010a,b*

$$\lim_{\tau \rightarrow 0} d\sigma(O) = B \otimes B \otimes \sum_i^N J_n \otimes S_N \otimes H.$$

- Beam function B @ N3LO *Ebert et al. 2020a; Baranowski et al. 2023* see Gherardo's talk
- Jet function J @ N3LO *Banerjee et al. 2018; Brüser et al. 2018*
- Soft function S_N @ N2LO *Hornig et al. 2011; Kelley et al. 2011; Monni et al. 2011; Boughezal, Liu, et al. 2015; Bell et al. 2018; Campbell et al. 2018; Jin and Liu 2019*

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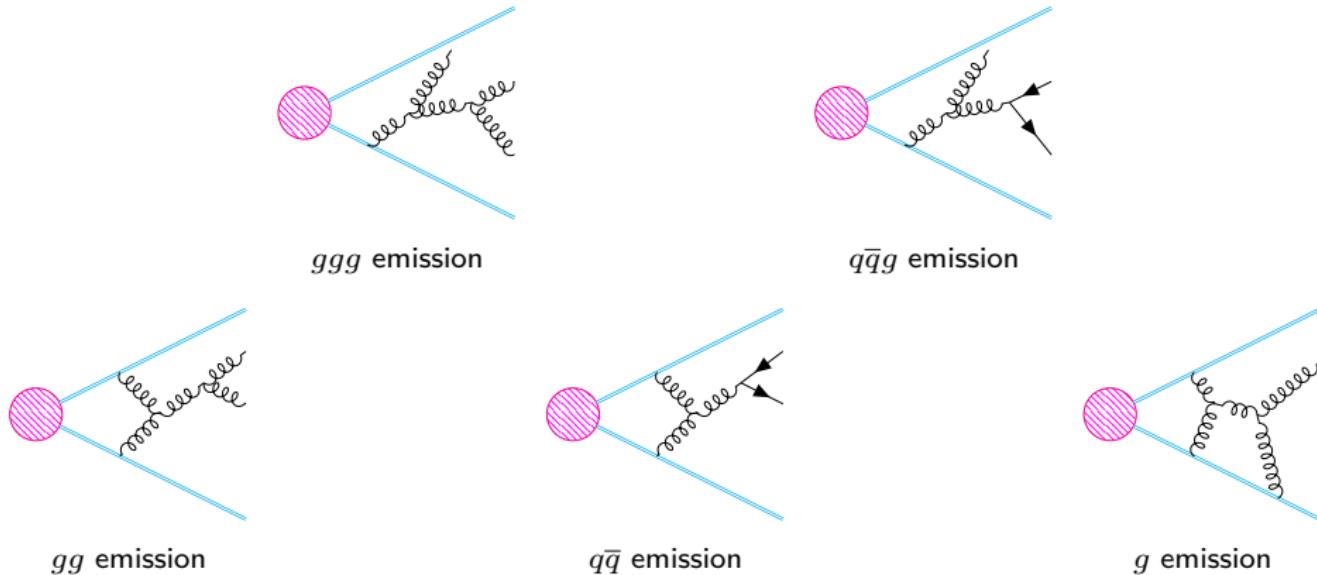
Integral reduction
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Motivation: zero-jettiness soft function at N3LO



- RVV and RRV are known in the literature. *Duhr and Gehrmann 2013; Li and Zhu 2013; Chen et al. 2022*

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Definition

- Zero-jettiness is defined as

$$\tau = \sum_{i=1}^m \min_{j \in 1,2} \left[\frac{2q_j \cdot k_i}{Q_j} \right] = \sum_{i=1}^m \min\{k_i \cdot n, k_i \cdot \bar{n}\}.$$

- By symmetry there are two independent configurations: $nnn / nn\bar{n}$.

$$\begin{aligned} d\Phi^{nnn} &= d\Phi_3 \delta(\tau - k_1 \cdot n - k_2 \cdot n - \cancel{k}_3 \cdot \cancel{n}) \\ &\quad \times \theta(k_1 \cdot \bar{n} - k_1 \cdot n) \theta(k_2 \cdot \bar{n} - k_2 \cdot n) \theta(\cancel{k}_3 \cdot \bar{n} - \cancel{k}_3 \cdot n), \end{aligned}$$

$$\begin{aligned} d\Phi^{nn\bar{n}} &= d\Phi_3 \delta(\tau - k_1 \cdot n - k_2 \cdot n - \cancel{k}_3 \cdot \bar{n}) \\ &\quad \times \theta(k_1 \cdot \bar{n} - k_1 \cdot n) \theta(k_2 \cdot \bar{n} - k_2 \cdot n) \theta(\cancel{k}_3 \cdot n - \cancel{k}_3 \cdot \bar{n}), \end{aligned}$$

where $d\Phi_3$ is the 3-body massless phase space measure.

- The amplitude needs to be integrated over these two configurations

$$S_{ggg} = \left(2 \int d\Phi^{nnn} + 6 \int d\Phi^{nn\bar{n}} \right) |J(k_1, k_2, k_3)|^2$$

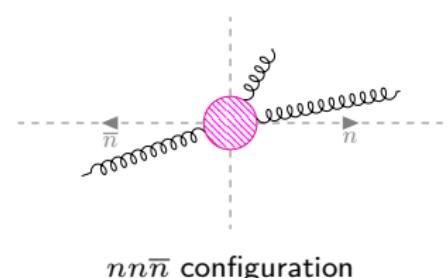
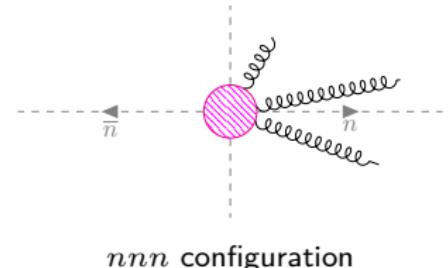
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Procedures

- Amplitude: expression available in the literature *Catani, Colferai, et al. 2020; Del Duca et al. 2023*

$$S = \sum_i C_i I_i.$$

- IBP reduction: *Chetyrkin and Tkachov 1981*

$$\int d^d k \frac{\partial}{\partial k_\mu} \left[p_\mu \frac{1}{\prod_i D_i} \right] = 0.$$

- Reverse unitarity: transform δ functions to denominators *Anastasiou and Melnikov 2002*

$$\delta(p^2 - m^2) = \frac{1}{2\pi} \left[\frac{i}{p^2 - m^2 + i\varepsilon} - \frac{i}{p^2 - m^2 - i\varepsilon} \right].$$

- How to deal with θ functions?
- Evaluate master integrals: **master integrals could still be hard to work out.**

$$S = \sum_i C'_i I'_i.$$

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Integral reduction

Rewrite Heaviside functions as δ functions

$$\theta(k_i \cdot \bar{n} - k_i \cdot n) = \int_0^1 dz_i \delta(z_i k_i \cdot \bar{n} - k_i \cdot n) k_i \cdot \bar{n}$$

- Standard IBP programs can be used.
- Needs to integrate over 3 auxillary parameters:

$$S = \int_0^1 dz_1 dz_2 dz_3 \sum_i C'_i(z_j) I'_i(z_j).$$

- Tested at N2LO. *Baranowski 2020*

Implement IBP for Heaviside functions

$$\frac{\partial}{\partial k_i \cdot \bar{n}} \theta(k_i \cdot \bar{n} - k_i \cdot n) = \delta(k_i \cdot \bar{n} - k_i \cdot n)$$

- Generate IBP identities manually.
- Needs auxillary families to perform the reduction.
- Simpler master integrals:

$$S = \sum_i C'_i I'_i.$$

- Parametric representation based approach *Chen 2020a,b, 2021*

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Integral reduction

IBP identity

$$\int d^d k \frac{\partial}{\partial k_\mu} [p_\mu f(k)] = 0 \implies \int d^d k \left[\left(\frac{\partial}{\partial k_\mu} p_\mu \right) + p \cdot k \frac{\partial}{\partial k^2} + p \cdot n \frac{\partial}{\partial k \cdot n} + p \cdot \bar{n} \frac{\partial}{\partial k \cdot \bar{n}} \right] f(k) = 0$$

- Example: $f(k) = \frac{\theta(k \cdot \bar{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \bar{n})^{a_3}}$ where $[...]_c$ denotes a δ function.
- The partial derivative generates two contributions:

$$\frac{\partial}{\partial k \cdot \bar{n}} f(k) = -a_3 \frac{\theta(k \cdot \bar{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \bar{n})^{a_3+1}} + \frac{\delta(k \cdot \bar{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \bar{n})^{a_3}}$$

- The **homogenous** term keeps the Heaviside function intact, while the **inhomogenous** term changes it to a δ function.

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Integral reduction

$$\frac{\partial}{\partial k \cdot \bar{n}} \frac{\theta(k \cdot \bar{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \bar{n})^{a_3}} = -a_3 \frac{\theta(k \cdot \bar{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \bar{n})^{a_3+1}} + \frac{\delta(k \cdot \bar{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \bar{n})^{a_3}}$$

- The **homogenous** term corresponds to the normal IBP identities without θ functions.
- The **inhomogenous** term introduces new families to the reduction and requires **partial fraction decomposition**.
- Auxillary families with δ functions in place of θ functions are required.
- Applicable to other observables involving θ functions.



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Integral reduction
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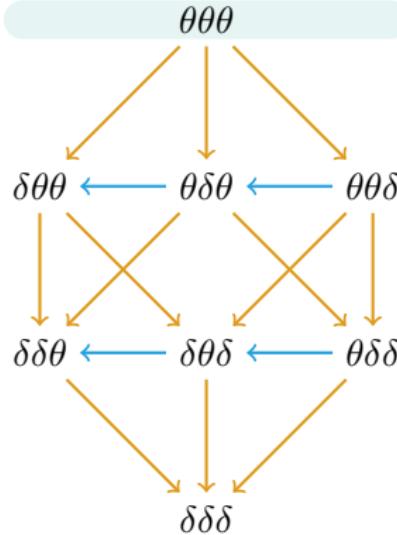
Conclusion
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$$d\Phi_{f_1 f_2 f_3}^{nnn} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - \mathbf{k}_3 \cdot \mathbf{n}) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot \bar{n} - k_3 \cdot n)$$

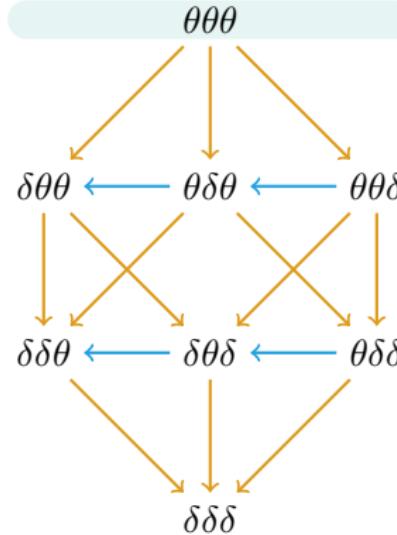
$$d\Phi_{f_1 f_2 f_3}^{nn\bar{n}} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - \mathbf{k}_3 \cdot \bar{n}) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot n - k_3 \cdot \bar{n})$$

Integral reduction

nnn configuration



nnn̄ configuration



- $f_i = \theta$ or δ .

- Starting from the measure $\theta\theta\theta$,

- **IBP identities** connect measures with fewer θ functions.
- **symmetry relations** connect measures with different permutations.

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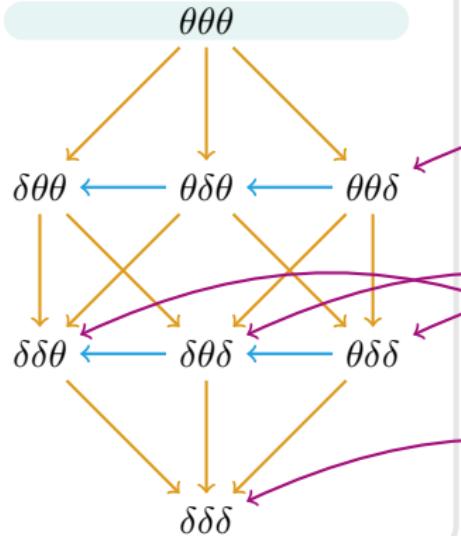
Conclusion
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$$d\Phi_{f_1 f_2 f_3}^{nnn} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - k_3 \cdot n) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot \bar{n} - k_3 \cdot n)$$

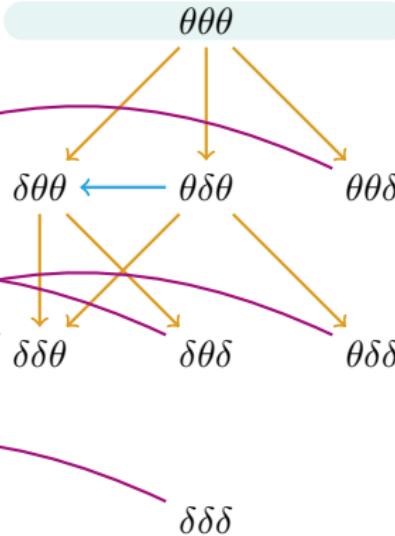
$$d\Phi_{f_1 f_2 f_3}^{nn\bar{n}} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - k_3 \cdot \bar{n}) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot n - k_3 \cdot \bar{n})$$

Integral reduction

nnn configuration



nn̄n configuration



- $f_i = \theta$ or δ .
- Starting from the measure $\theta\theta\theta$,
 - **IBP identities** connect measures with fewer θ functions.
 - **symmetry relations** connect measures with different permutations.
- More **symmetry relations** between configurations.

Motivation
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Integral reduction
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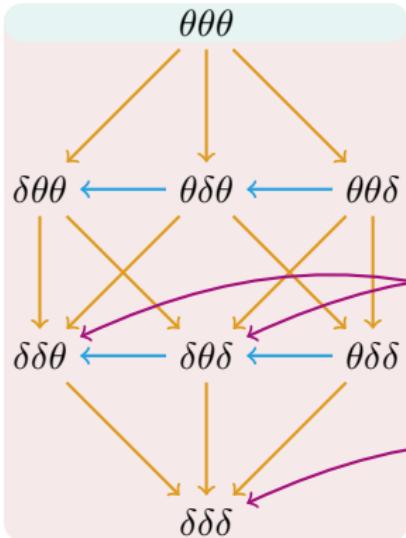
Conclusion
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$$d\Phi_{f_1 f_2 f_3}^{nnn} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - \mathbf{k}_3 \cdot \mathbf{n}) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot \bar{n} - k_3 \cdot n)$$

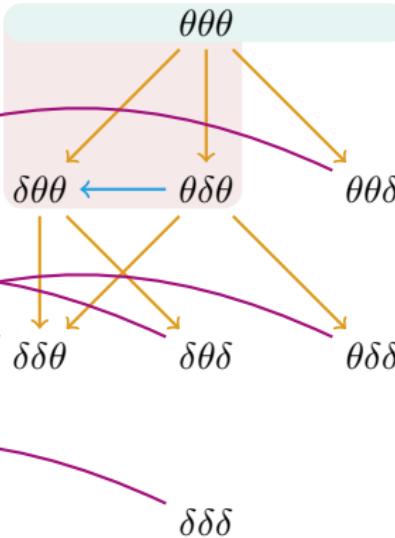
$$d\Phi_{f_1 f_2 f_3}^{nn\bar{n}} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - \mathbf{k}_3 \cdot \bar{n}) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot n - k_3 \cdot \bar{n})$$

Integral reduction

nnn configuration



nnn̄ configuration



- $f_i = \theta$ or δ .

- Starting from the measure $\theta\theta\theta$,

- **IBP identities** connect measures with fewer θ functions.
- **symmetry relations** connect measures with different permutations.

- More **symmetry relations** between configurations.

- Solve the whole system with Kira and FireFly (reduce_user_defined_system).

Maierhöfer et al. 2018; Klappert and Lange 2020;

Klappert, Lange, et al. 2020

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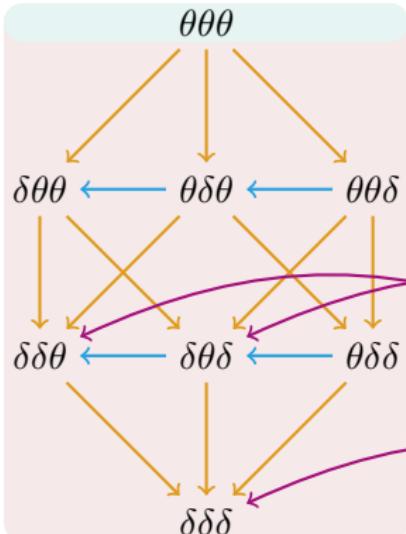
Conclusion
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$$d\Phi_{f_1 f_2 f_3}^{nnn} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - \mathbf{k}_3 \cdot \mathbf{n}) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot \bar{n} - k_3 \cdot n)$$

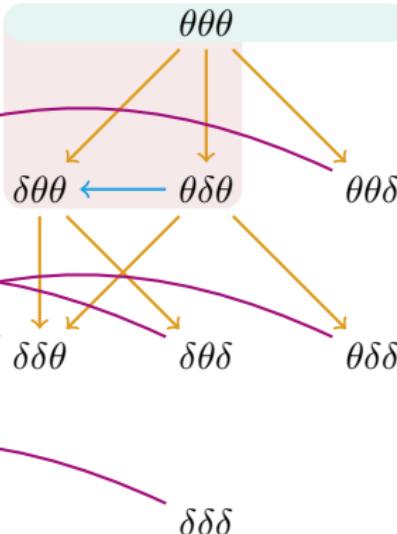
$$d\Phi_{f_1 f_2 f_3}^{nn\bar{n}} = d\Phi_3 \delta(1 - k_1 \cdot n - k_2 \cdot n - \mathbf{k}_3 \cdot \bar{n}) f_1(k_1 \cdot \bar{n} - k_1 \cdot n) f_2(k_2 \cdot \bar{n} - k_2 \cdot n) f_3(k_3 \cdot n - k_3 \cdot \bar{n})$$

Integral reduction

nnn configuration



nnn̄ configuration



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- Master integrals without $1/(k_1 + k_2 + k_3)^2$ denominator:
Calculate analytically using HypExp and HyperInt after subtracting ε poles. *Huber and Maitre 2006; Panzer 2015* (~ 150 integrals)
- Master integrals with $1/(k_1 + k_2 + k_3)^2$ denominator: Hard to calculate directly \Rightarrow Construct differential equation (~ 650 integrals)

Integral evaluation

- The problematic denominator $1/k_{123}^2$

$$\frac{1}{(k_1 + k_2 + k_3)^2} \sim \frac{1}{2k_1 \cdot k_2 + 2k_2 \cdot k_3 + 2k_3 \cdot k_1}$$

involves 3 dot products \Rightarrow **it would be great if this denominator can be “removed”**

- Naively if we add an auxillary mass-like parameter m to the denominator and in the limit $m \rightarrow \infty$ the dot products drop out:

$$(k_1 + k_2 + k_3)^2 \ll m^2: \frac{1}{(k_1 + k_2 + k_3)^2 + m^2} \sim \frac{1}{m^2}$$

- The integrals are relatively simple at $m \rightarrow \infty$ and differential equation w.r.t. m^2 helps us to recover the original integral at $m \rightarrow 0$. *Liu et al. 2018*
- Similar ideas of introducing auxillary parameters can be found in the literature. *Henn et al. 2014; Papadopoulos 2014; Zhu 2015; Chen et al. 2022; Lee et al. 2022*

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Integral evaluation

- Modify the master integrals by adding a m^2 to $1/k_{123}^2$:

$$I_i(\epsilon) = \lim_{m \rightarrow 0} J_i(\epsilon, m) \quad J_i(\epsilon, m) = \int d\Phi_{f_1 f_2 f_3} \frac{1}{k_{123}^2 + \textcolor{teal}{m^2}} \frac{\dots}{(k_1 \cdot k_2)(k_1 \cdot n) \dots}$$

- Construct a system of differential equations w.r.t. m^2 :

$$\frac{\partial}{\partial m^2} J = M J.$$

- Solve the differential equations **numerically** from $m \rightarrow \infty$ to $m \rightarrow 0$.
- Reconstruct **analytical** expression from numerical solutions.
- For $m^2 \geq 0$, $J_i(\epsilon, m)$ is real \Rightarrow consistency check to the solution.

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Integral evaluation

$$J = \int d\Phi_{\delta\theta\theta}^{nnn} \frac{1}{(k_{123}^2 + m^2)(k_2 \cdot \bar{n})}$$

m^2 plane

- Expand J around **boundary** in variable $w^2 = m^{-2} = 0$:

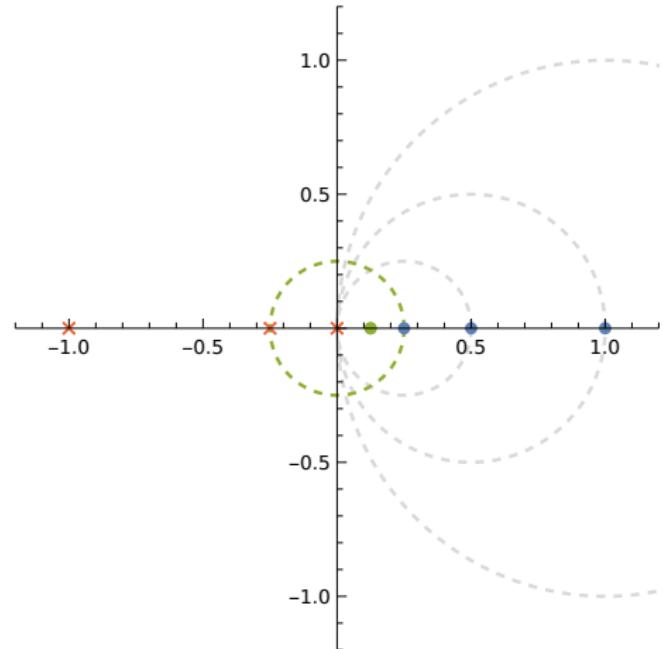
$$J = \sum_{i,j,k} c_{ijk}(\epsilon) w^{i+j\epsilon} \ln^k w$$

- Boundary conditions at $m \rightarrow \infty$ involves several regions as the Heaviside functions allow $k_i \cdot \bar{n}$ to be large:

$$J|_{m \rightarrow \infty} = \begin{cases} m^0 & \text{with } \alpha_1, \alpha_2, \alpha_3 \ll m^2 \\ m^{-2\epsilon} & \text{with } \alpha_1, \alpha_i \ll m^2, \text{ while } \alpha_j \sim m^2 \\ m^{-4\epsilon} & \text{with } \alpha_1 \ll m^2, \text{ while } \alpha_2, \alpha_3 \sim m^2 \end{cases}$$

where $\alpha_i = k_i \cdot \bar{n}$.

- Evaluate at a **regular point** $J(m = 1/w_0) = J|_{w=w_0}$.



Integral evaluation

$$J = \int d\Phi_{\delta\theta\theta}^{nnn} \frac{1}{(k_{123}^2 + m^2)(k_2 \cdot \bar{n})}$$

m^2 plane

- Expand and evaluate around **regular points**:

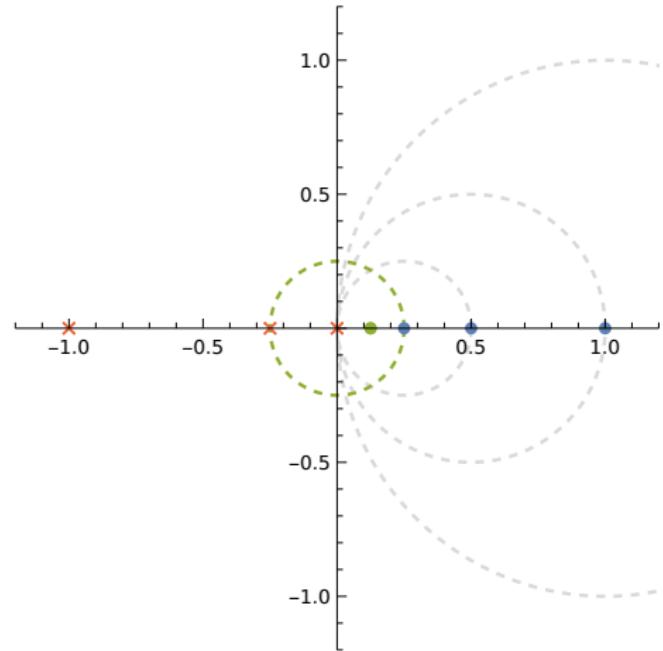
$$J = \sum_i c_i(\epsilon) m'^i$$

Repeat this procedure until we move into the radius of convergence around the **physical point** $m = 0$.

- Matching at the **physical point** $m = 0$:

$$J = \sum_{i,j,k} c_{ijk}(\epsilon) m^{i+j\epsilon} \ln^k m$$

- I corresponds to $\lim_{m \rightarrow 0} J(\epsilon, m) = c_{000}(\epsilon)$.
- Finally we reconstruct the analytical expression.



Result

Now we present the same-hemisphere *triple-gluon-emission* contribution to the N3LO zero-jettiness soft function:

$$S_{ggg}^{nnn} = \tau^{-1-6\epsilon} \frac{N_\epsilon^3}{3!} [C_a^3 S_{1+1+1}^{nnn} + C_a^2 C_A S_{1+2}^{nnn} + C_a C_A^2 S_3^{nnn}],$$

where $N_\epsilon = (4\pi)^{-2+\epsilon}/\Gamma(1-\epsilon)$, and $C_a = C_{F,A}$ for the quark (gluon) soft function. *Catani, Colferai, et al. 2020*

$$\begin{aligned} S_{1+1+1}^{nnn} &= \frac{48 \Gamma^3(1-2\epsilon)}{\epsilon^5 \Gamma(1-6\epsilon)}, \\ S_{1+2}^{nnn} &= -\frac{9 \Gamma(1-4\epsilon) \Gamma(1-2\epsilon)}{\epsilon^2 \Gamma(1-6\epsilon)} \times \left[\frac{8}{\epsilon^3} + \frac{44}{3\epsilon^2} + \frac{1}{\epsilon} \left(\frac{268}{9} - 8\zeta_2 \right) + \left(\frac{1544}{27} + \frac{88}{3}\zeta_2 - 72\zeta_3 \right) \right. \\ &\quad + \epsilon \left(\frac{9568}{81} + \frac{536\zeta_2}{9} + \frac{352}{3}\zeta_3 - 300\zeta_4 \right) + \epsilon^2 \left(\frac{55424}{243} + \frac{3520\zeta_2}{27} + \frac{2144\zeta_3}{9} + 352\zeta_4 + 96\zeta_2\zeta_3 - 1208\zeta_5 \right) \\ &\quad \left. + \epsilon^3 \left(\frac{297472}{729} + \frac{22592\zeta_2}{81} + \frac{14080\zeta_3}{27} + \frac{2144}{3}\zeta_4 - \frac{4576}{3}\zeta_2\zeta_3 + 3696\zeta_5 + 424\zeta_3^2 - 3596\zeta_6 \right) + \mathcal{O}(\epsilon^4) \right]. \end{aligned}$$

- Note that only zeta values are involved.

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Result

- Reconstructed from the numerical result and verified numerically with more than 2000 digits.
- Singular terms only contain zeta values.
- Regular terms contain multiple polylogarithm G with sixth root of unity $\exp(ik\pi/3)$ letters.

$$\begin{aligned} S_3^{nnn} = & \frac{24}{\epsilon^5} + \frac{308}{3\epsilon^4} + \frac{1}{\epsilon^3} \left(-12\pi^2 + \frac{3380}{9} \right) + \frac{1}{\epsilon^2} \left(-1000\zeta_3 + \frac{440\pi^2}{9} + \frac{10048}{9} \right) \\ & + \frac{1}{\epsilon} \left(-\frac{2377\pi^4}{45} + \frac{440\zeta_3}{3} + \frac{7192\pi^2}{27} + \frac{253252}{81} \right) \\ & + \left(-28064\zeta_5 + \frac{1972\zeta_3\pi^2}{3} - \frac{638\pi^4}{15} + 4224\text{Li}_4\left(\frac{1}{2}\right) + 3696\zeta_3 \ln(2) - 176\pi^2 \ln^2(2) + 176 \ln^4(2) \right. \\ & \left. + \frac{13208\zeta_3}{3} + \frac{78848\pi^2}{81} + 96 \ln(2) + \frac{1925074}{243} \right) \\ & + \epsilon \left(2304 \zeta_{-5,-1} - 4464\zeta_5 \ln(2) + 25784\zeta_3^2 - \frac{67351\pi^6}{567} - 6336G_R(0,0,r_2,1,-1) \right. \\ & \left. - 6336G_R(0,0,1,r_2,-1) - 3168G_R(0,0,1,r_2,r_4) - 6336G_R(0,0,r_2,-1) \ln(2) + \frac{268895\zeta_5}{3} \right. \\ & \left. - 45056\text{Li}_5\left(\frac{1}{2}\right) - 45056\text{Li}_4\left(\frac{1}{2}\right) \ln(2) + 176\text{Cl}_4\left(\frac{\pi}{3}\right)\pi - 1056\zeta_3 \text{Li}_2\left(\frac{1}{4}\right) - 3982\zeta_3\pi^2 \right. \\ & \left. - 21824\zeta_3 \ln^2(2) + 2112\zeta_3 \ln(2) \ln(3) - 1584\text{Cl}_2^2\left(\frac{\pi}{3}\right) \ln(3) - \frac{4400\text{Cl}_2\left(\frac{\pi}{3}\right)\pi^3}{27} + \frac{88\pi^4 \ln(2)}{45} \right. \\ & \left. - \frac{616\pi^4 \ln(3)}{27} + \frac{11264\pi^2 \ln^3(2)}{9} - \frac{22528\ln^5(2)}{15} + 8576\text{Li}_4\left(\frac{1}{2}\right) + 7504\zeta_3 \ln(2) + \frac{4174\pi^4}{27} \right. \\ & \left. - \frac{1072\pi^2 \ln^2(2)}{3} + \frac{1072\ln^4(2)}{3} + \frac{554032\zeta_3}{27} - 32\pi^2 \ln(2) + \frac{730378\pi^2}{243} - 384\ln^2(2) + 832\ln(2) \right. \\ & \left. + \frac{1408681}{81} + \sqrt{3} \left(192\Im\left\{\text{Li}_3\left(\frac{\exp(i\pi/3)}{2}\right)\right\} + 160\text{Cl}_2\left(\frac{\pi}{3}\right) \ln(2) - 16\pi \ln^2(2) - \frac{560\pi^3}{81} \right) \right) + \mathcal{O}(\epsilon^2). \end{aligned}$$

Motivation
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Integral reduction
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Integral evaluation
○○○

Result
○●

Conclusion
○

Conclusion

$nn\bar{n}$ configuration

- All master integrals have been computed, checking in progress.
- N3LO QCD corrections are crucial to the percent level phenomenology at LHC and HL-LHC.
- We compute the same-hemisphere triple-gluon zero-jettiness soft function at N3LO.
- **Custom IBP relations** enables reduction for integrals containing Heaviside functions.
 - Applicable to other observables involving θ functions.
- **Adding an auxiliary mass parameter** overcomes the technical difficulty of computing master integrals.
- Thank you!

Motivation
ooooo

Integral reduction
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Integral evaluation
ooo

Result
oo

Conclusion
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Analytic regulator

- Although the soft function itself is regularized dimensionally, we found that an additional regulator is required to obtain a correct result

$$d\Phi_{f_1 f_2 f_3}^{nnn} \rightarrow d\Phi_{f_1 f_2 f_3}^{nnn} (k_1 \cdot n)^\nu (k_2 \cdot n)^\nu (k_3 \cdot n)^\nu.$$

- The amplitude reduces to

$$S_{ggg}^{nnn} = \sum_{\alpha} c_{\alpha}(\nu) I_{\alpha}^{\nu} + \nu \sum_{\alpha} \tilde{c}_{\alpha}(\nu) \bar{I}_{\alpha}^{\nu},$$

where two of the \bar{I}_{α}^{ν} are $1/\nu$ -divergent.

- For integrals without $1/k_{123}^2$ denominator, we can proceed as before and obtain analytical results.
- For integrals with $1/k_{123}^2$ denominator, we now have two limits to take:

$$I(\epsilon) = \lim_{\nu \rightarrow 0} \lim_{m \rightarrow 0} J(\epsilon, \nu, m).$$

We find that these two limits do commute, thus we can set $\nu = 0$ beforehand and solve the equation.

Backup