Quantum simulation of colour in perturbative QCD

Herschel A. Chawdhry
University of Oxford

RADCOR, Crieff (Scotland), 1st June 2023

Based on arXiv:2303.04818
In collaboration with Mathieu Pellen

Outline

1. Introduction/motivation
2. Basics of quantum computing
3. Quantum circuits for colour
    - Overview
    - Details
4. Results/validation
5. Outlook and summary

Outline

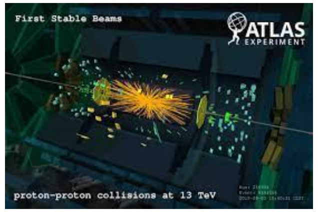
1. Introduction/motivation
    - Why perturbative QCD?
    - Why quantum computers?
    - Why now?
    - Proposed applications of quantum computing in high-energy physics
2. Basics of quantum computing
3. Quantum circuits for colour
    - Overview
    - Details
4. Results/validation
5. Outlook and summary

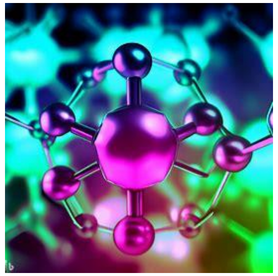
Why perturbative QCD?

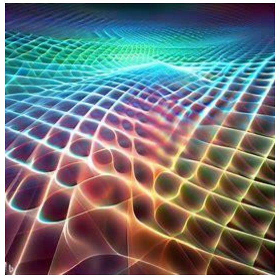
- High-precision predictions for colliders like the LHC
  - Stringent tests of the standard model
    - Could give first hints of new physics
    - High precision is worthwhile in its own right!
- Computationally intense
  - e.g. multi-loop amplitude calculations
  - e.g. Monte-Carlo integration of cross sections

What can quantum computers do?

- Prime factorisation
- Unstructured search
  - e.g. searching abstract spaces
  - e.g. Monte-Carlo integration
- Simulating quantum systems
  - Computational chemistry
  - Condensed matter systems
  - Lattice QFT/QCD
- Machine learning





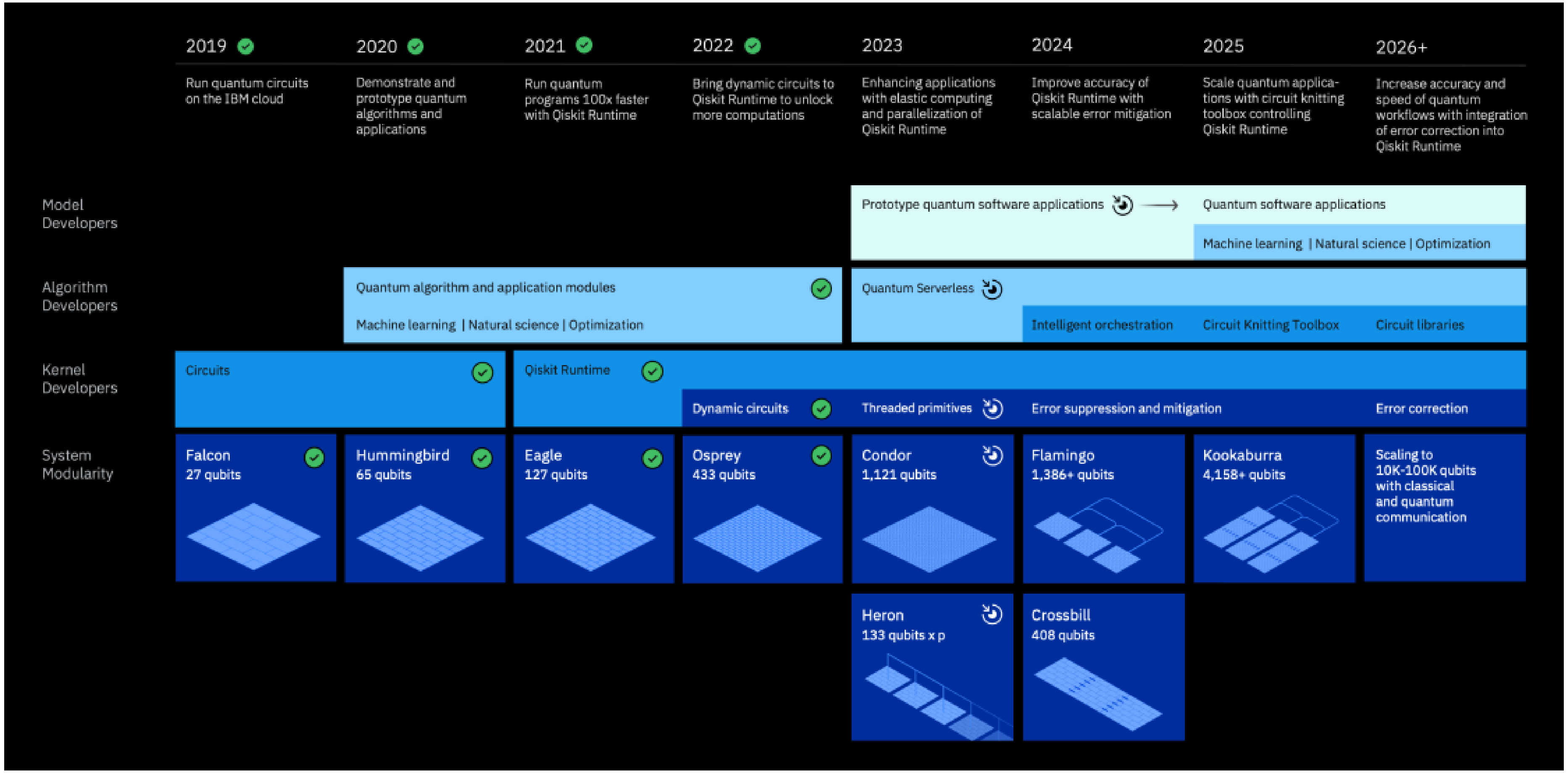


Why now?

- Hardware progress
  - Trapped ions
  - Neutral atoms
  - Photonic systems
  - Superconducting systems
  - ...
- Software progress
  - e.g. Error-correcting codes (e.g. "surface codes")
- Commercial interest

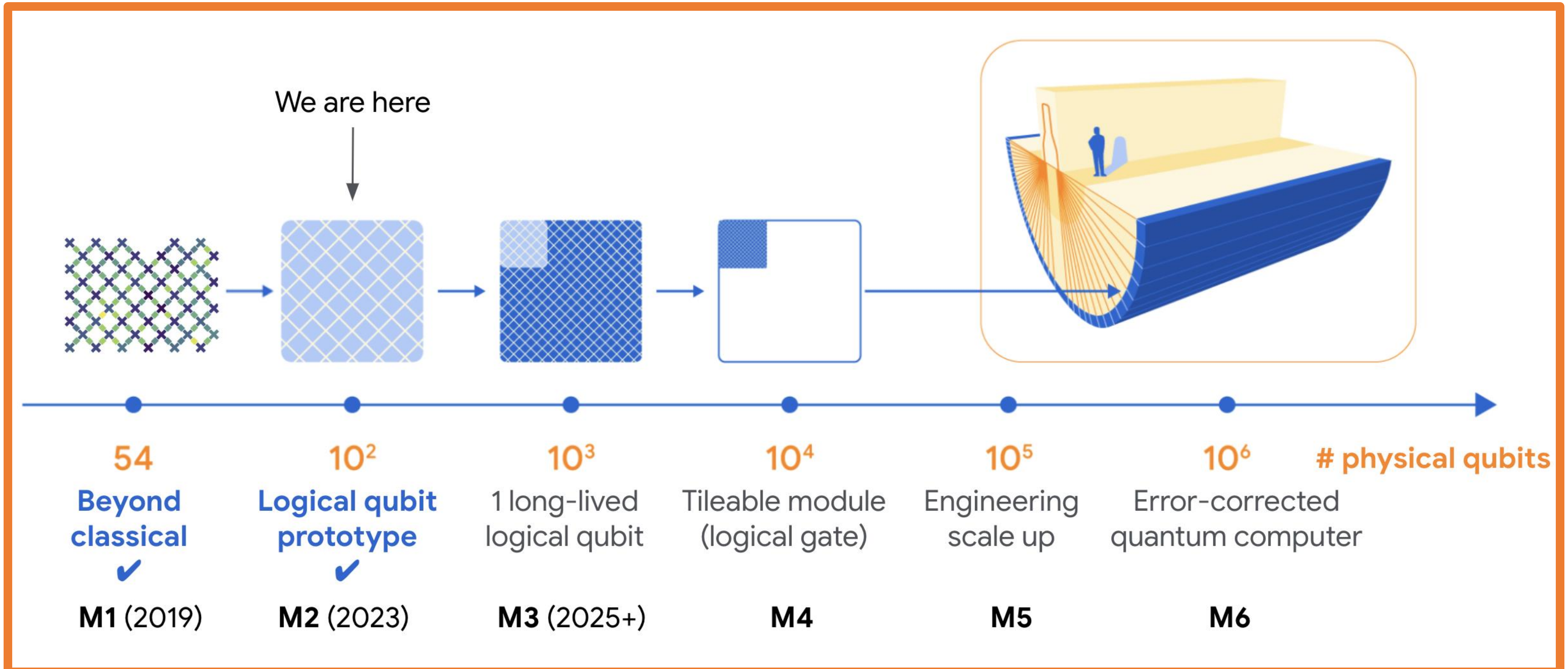
Why now?

IBM Quantum Development Roadmap



Why now?

Google's quantum roadmap



Proposed applications in high-energy physics

- Experiments / data analysis
- PDFs [Pérez-Salinas, Cruz-Martinez, Alhajri, Carrazza, '20], [QuNu Collaboration, '21]
- EFTs [Bauer, Freytsis, Nachman, '21]
- Monte Carlo for cross-sections [Agliardi, Grossi, Pellen, Prati, '22]
- Parton showers [Bauer, de Jong, Nachman, Provasoli, '19], [Bepari, Malik, Spannowsky, Williams, '20], [Gustafson, Prestel, Spannowsky, Williams, '22]
- Event generation [Gustafson, Prestel, Spannowsky, Williams, '22], [Bravo-Prieto, Baglio, Cè, Francis, Grabowska, Carrazza, '21], [Kiss, Grossi, Kajomovitz, Vallecorsa, '22]
- Lattice QCD (See reviews [Klco, Roggero, Savage, '21] and [Bauer et al., '22] and references therein)
- More [Cervera-Lierta, Latorre, Rojo, Rottoli, '17], [Ramírez-Uribe, Rentería-Olivo, Rodrigo, Sborlini, Vale Silva, '21], [Fedida, Serafini, '22], [Clemente, Crippa, Jansen, Ramírez-Uribe, Rentería-Olivo, Rodrigo, Sborlini, Vale Silva, '22]
- …

Spotlight: quantum simulation

- Quantum simulation: a flagship application of quantum computers
- Recent years: proposals for quantum simulation of lattice QFTs (e.g. lattice QCD)
- Quantum simulation of perturbative QCD remains largely unexplored
  - Notable exception: several papers on parton showers
- This talk: first steps towards generic perturbative QCD processes
  - Quantum simulation of colour in perturbative QCD

Motivation for quantum simulation of pQCD

1. Perturbative QCD requires quantum-coherent combination of contributions from many unobservable intermediate states
   - natural candidate to exploit superpositions of quantum states in quantum computers
2. Processes with high-multiplicity final states, with full interference effects
3. Improve speed/precision of perturbative QCD predictions by exploiting speed-ups of known quantum algorithms
   - e.g. quantum amplitude estimation; quantum Monte Carlo

Outline

1. Introduction
2. Basics of quantum computing
3. Quantum circuits for colour
   - Overview
   - Details
4. Results/validation
5. Outlook and summary

What quantum computers can and cannot do

- Formally, anything that can be computed on a quantum computer can also be computed on a classical Turing machine

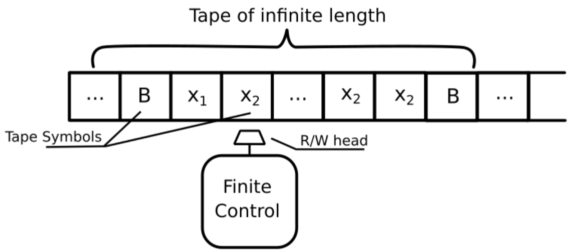
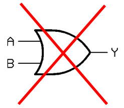


Figure from: opengenus.org

- But quantum computers are potentially (much) faster than classical computers for certain problems

Quantum circuit model

- Qubits
- Gates
  - Unitary, reversable
  - Can be controlled by other qubits



Quantum Mechanical Computers 511

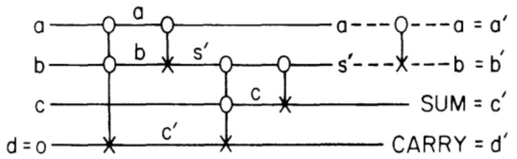
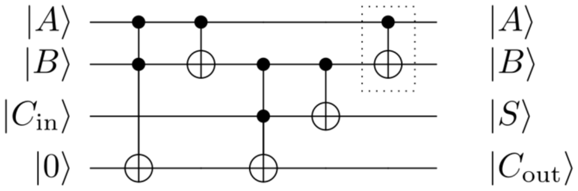


Fig. 5. Full adder.

Figure from: Feynman, R.P. Quantum mechanical computers. Found Phys ***16****, 507–531 (1986)*



Operator	Gate(s)	Matrix
Pauli-X (X)	$\boxed{\mathbf{X}}$	$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
Pauli-Y (Y)	$\boxed{\mathbf{Y}}$	$\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$
Pauli-Z (Z)	$\boxed{\mathbf{Z}}$	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$
Hadamard (H)	$\boxed{\mathbf{H}}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
Phase (S, P)	$\boxed{\mathbf{S}}$	$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$
$\pi/8$ (T)	$\boxed{\mathbf{T}}$	$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$
Controlled Z (CZ)	$\boxed{\mathbf{Z}}$	$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$
SWAP		$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$

Example: the increment circuit

$$|k\rangle \to \left|k+1 \pmod{2^N}\right\rangle$$

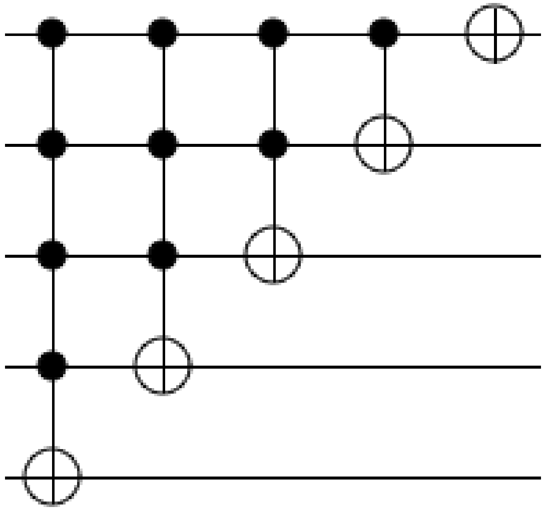


Figure adapted from: algassert.com/circuits/2015/06/12/Constructing-Large-Increment-Gates.html

- Examples:
  - $|00000\rangle \to |00001\rangle$
  - $|01011\rangle \to |01100\rangle$
  - $|11111\rangle \to |00000\rangle \quad \text{(overflow)}$
  - $\frac{\alpha|00000\rangle+\beta|01011\rangle}{|\alpha|^2+|\beta|^2} \to \frac{\alpha|00001\rangle+\beta|01100\rangle}{|\alpha|^2+|\beta|^2}$

Example: the increment circuit

$$|k\rangle \to \big|k+1 \pmod{2^N}\big\rangle$$

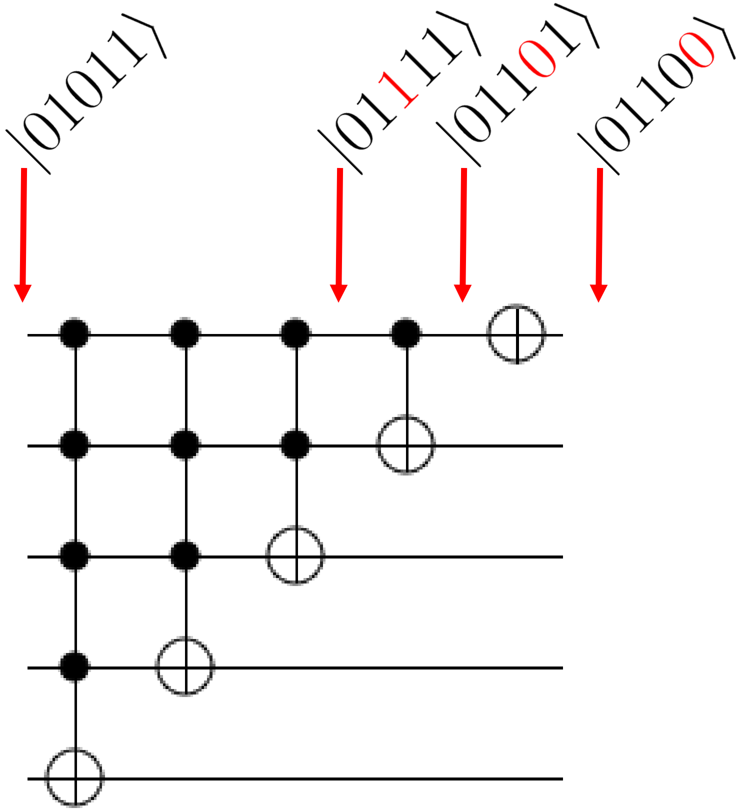


Figure adapted from: algassert.com/circuits/2015/06/12/Constructing-Large-Increment-Gates.html

- Examples:
  - $|00000\rangle \to |00001\rangle$
  - $|01011\rangle \to |01100\rangle$
  - $|11111\rangle \to |00000\rangle \quad \text{(overflow)}$
  - $\frac{\alpha|00000\rangle+\beta|01011\rangle}{|\alpha|^2+|\beta|^2} \to \frac{\alpha|00001\rangle+\beta|01100\rangle}{|\alpha|^2+|\beta|^2}$

Example: the increment circuit

$$|k\rangle \to \left|k+1 \pmod{2^N}\right\rangle$$

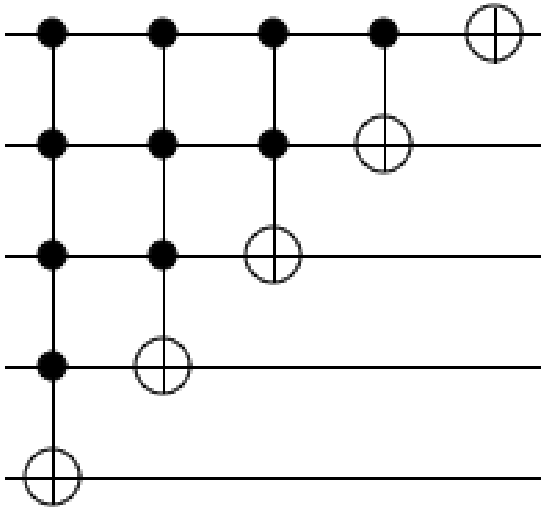


Figure adapted from: algassert.com/circuits/2015/06/12/Constructing-Large-Increment-Gates.html

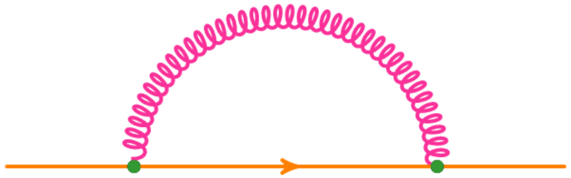
- Examples:
  - $|00000\rangle \to |00001\rangle$
  - $|01011\rangle \to |01100\rangle$
  - $|11111\rangle \to |00000\rangle \quad \text{(overflow)}$
  - $\frac{\alpha|00000\rangle+\beta|01011\rangle}{|\alpha|^2+|\beta|^2} \to \frac{\alpha|00001\rangle+\beta|01100\rangle}{|\alpha|^2+|\beta|^2}$

Outline

1. Introduction
2. Basics of quantum computing
3. Quantum circuits for colour
   - Overview
   - Details
4. Results/validation
5. Outlook and summary

Rapid reminder of colour in QCD calculations

- SU(3) structure function f^{abc} at each triple-gluon vertex
  - (4-gluon vertex can be written as linear combination of 3-gluon vertices)
- SU(3) generator T^a_{ij} at each quark-gluon vertex
- Trace over unmeasured (unmeasurable) colours
- e.g.



$$\sum_{\substack{a\in\{1,\ldots,8\} \\ j\in\{1,2,3\}}} T^a_{ij} T^a_{jk}$$

- Note: the large-N_c expansion is not used in this work

Idea: can Gell-Mann matrices become gates?

$$T^a_{ij} = \frac{1}{2}\lambda^a_{ij}$$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

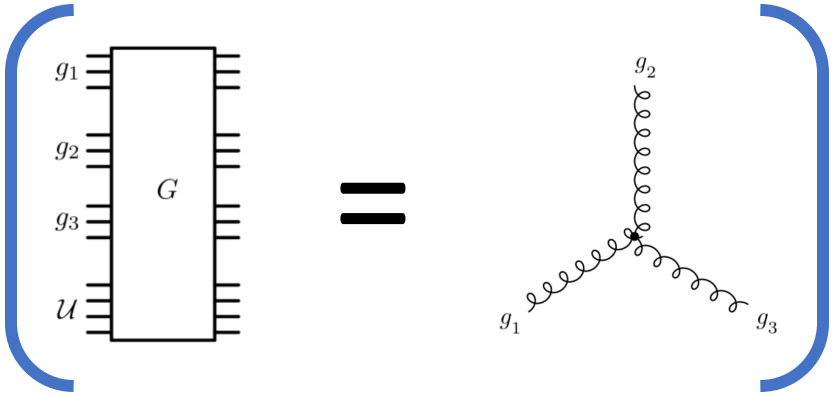
$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

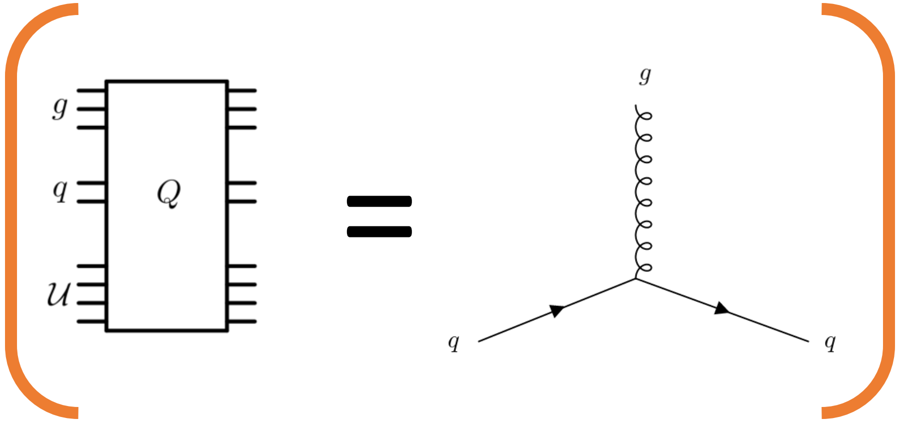
$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

- Short answer: yes, but there are complications:
  - Not 2^n x 2^n
  - Not unitary

Key results of this work

- Two quantum gates (**G** and **Q**) to simulate colour parts of the interactions of quarks and gluons

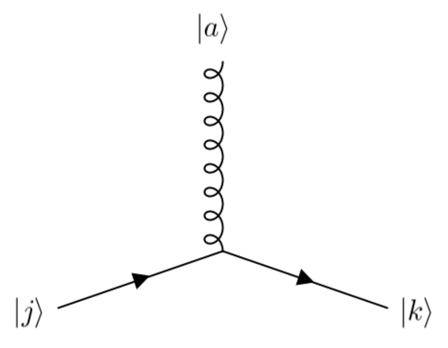




- Explicit construction of these gates: see later

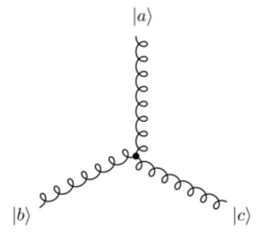
Methods

- Quark colours: represented by 2 qubits ($2^2 = 4$ basis states, of which 1 is unused)
- Gluon colours: represented by 3 qubits ($2^3 = 8$ basis states)
- Quark-gluon interaction gate is designed such that



$$Q\,|a\rangle_g\,|k\rangle_q\,|\Omega\rangle_{\mathcal{U}} = \sum_{j=1}^{3} T^a_{jk}\,|a\rangle_g\,|j\rangle_q\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$

- Triple-gluon interaction gate is designed such that

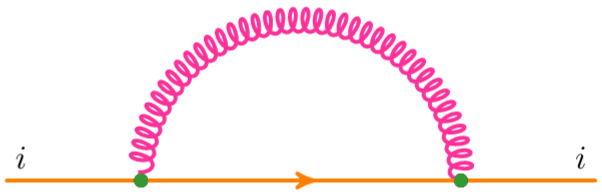


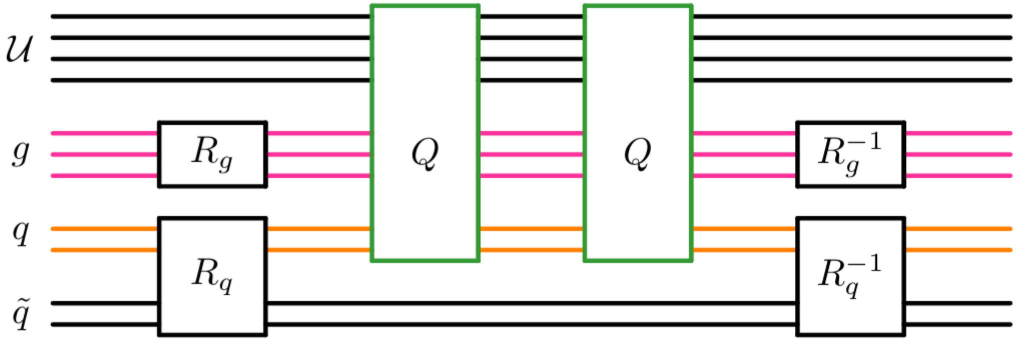
$$G\,|a\rangle_{g_1}\,|b\rangle_{g_2}\,|c\rangle_{g_3}\,|\Omega\rangle_{\mathcal{U}} = f^{abc}\,|a\rangle_{g_1}\,|b\rangle_{g_2}\,|c\rangle_{g_3}\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$

- Note: $|\Omega\rangle_{\mathcal{U}}$ is a reference state of a "Unitarisation register", which we introduce because in SU(3), T^a_{jk} and f^{abc} are non-unitary.

Calculating colour factors: illustrative example

(See later slides for more complicated examples)

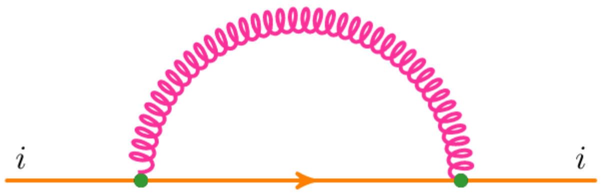


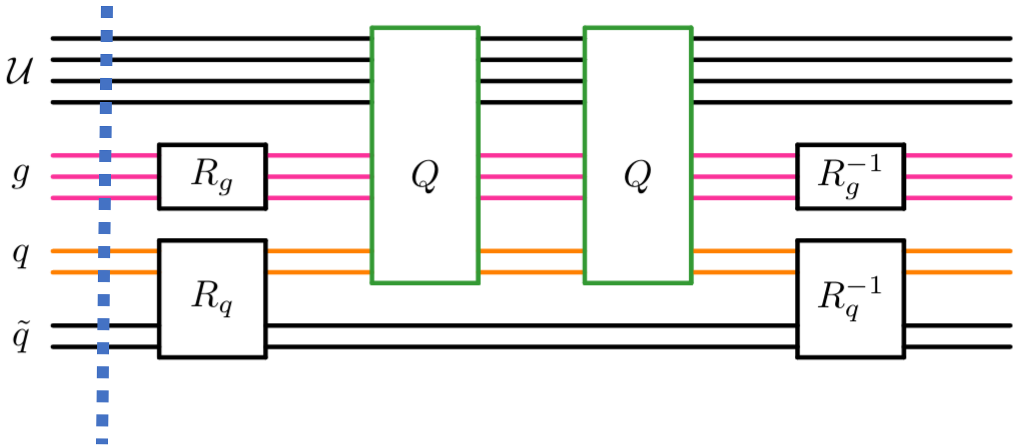


Calculating colour factors: illustrative example

(See later slides for more complicated examples)

$$|\Omega\rangle_g \, |\Omega\rangle_q \, |\Omega\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}$$

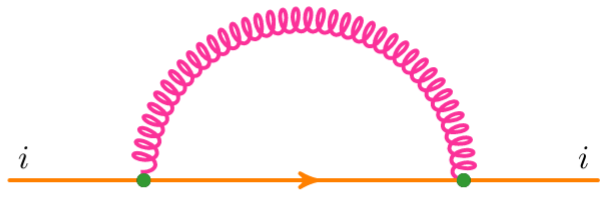


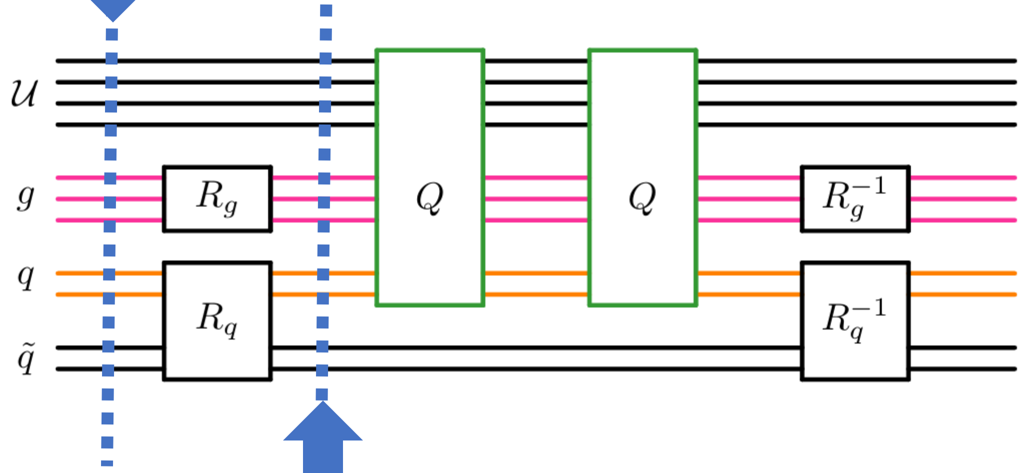


Calculating colour factors: illustrative example

(See later slides for more complicated examples)

$$|\Omega\rangle_g \, |\Omega\rangle_q \, |\Omega\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}$$



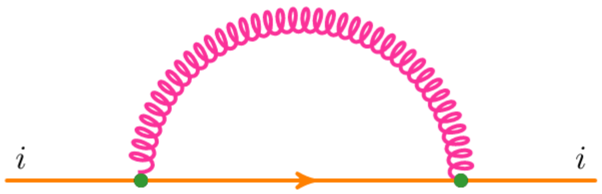


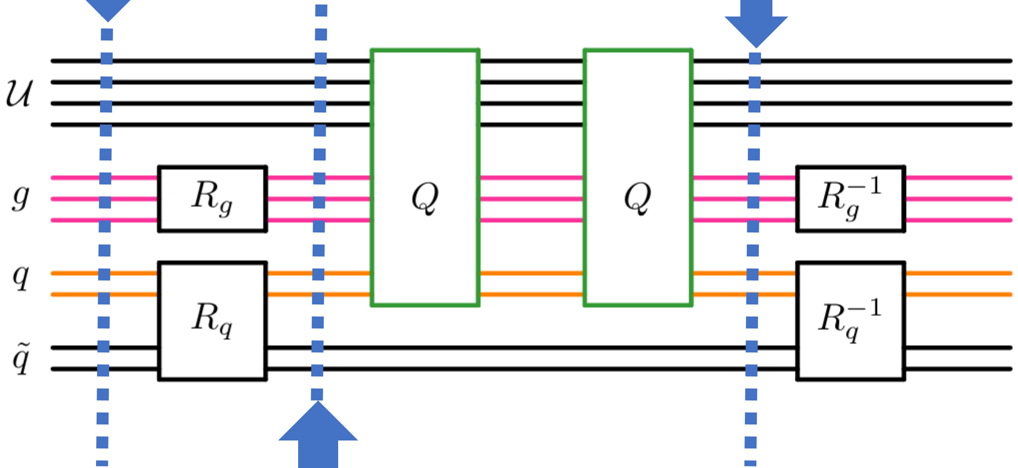
$$\frac{1}{\sqrt{24}} \sum_{a=1}^{8} \sum_{k=1}^{3} |a\rangle_g \, |k\rangle_q \, |k\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}$$

Calculating colour factors: illustrative example

$$|\Omega\rangle_g\,|\Omega\rangle_q\,|\Omega\rangle_{\tilde{q}}\,|\Omega\rangle_{\mathcal{U}}$$

$$\frac{1}{\sqrt{24}}\sum_{\substack{a\in\{1,\ldots,8\}\\ i,j,k\in\{1,2,3\}}} T^a_{ij}T^a_{jk}\,|a\rangle_g\,|i\rangle_q\,|k\rangle_{\tilde{q}}\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$



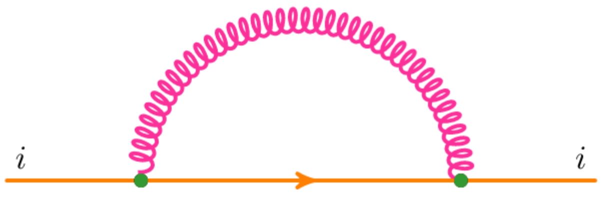


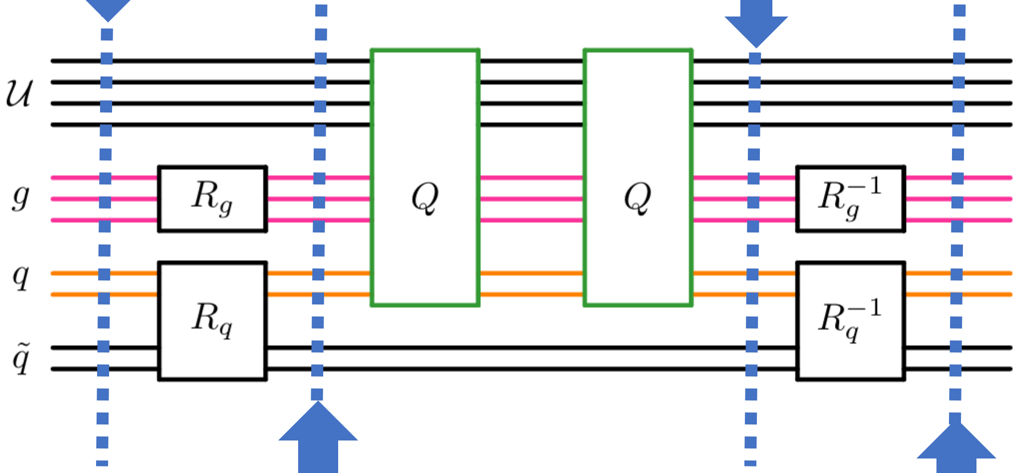
$$\frac{1}{\sqrt{24}}\sum_{a=1}^{8}\sum_{k=1}^{3}|a\rangle_g\,|k\rangle_q\,|k\rangle_{\tilde{q}}\,|\Omega\rangle_{\mathcal{U}}$$

Calculating colour factors: illustrative example

$$|\Omega\rangle_g\,|\Omega\rangle_q\,|\Omega\rangle_{\tilde q}\,|\Omega\rangle_{\mathcal U}$$

$$\frac{1}{\sqrt{24}}\sum_{\substack{a\in\{1,\dots,8\}\\ i,j,k\in\{1,2,3\}}} T^a_{ij}T^a_{jk}\,|a\rangle_g\,|i\rangle_q\,|k\rangle_{\tilde q}\,|\Omega\rangle_{\mathcal U} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal U})$$





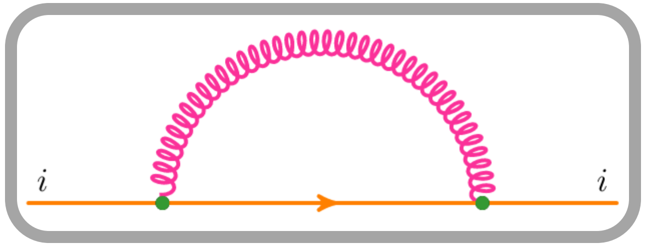
$$\frac{1}{\sqrt{24}}\sum_{a=1}^{8}\sum_{k=1}^{3}|a\rangle_g\,|k\rangle_q\,|k\rangle_{\tilde q}\,|\Omega\rangle_{\mathcal U}$$

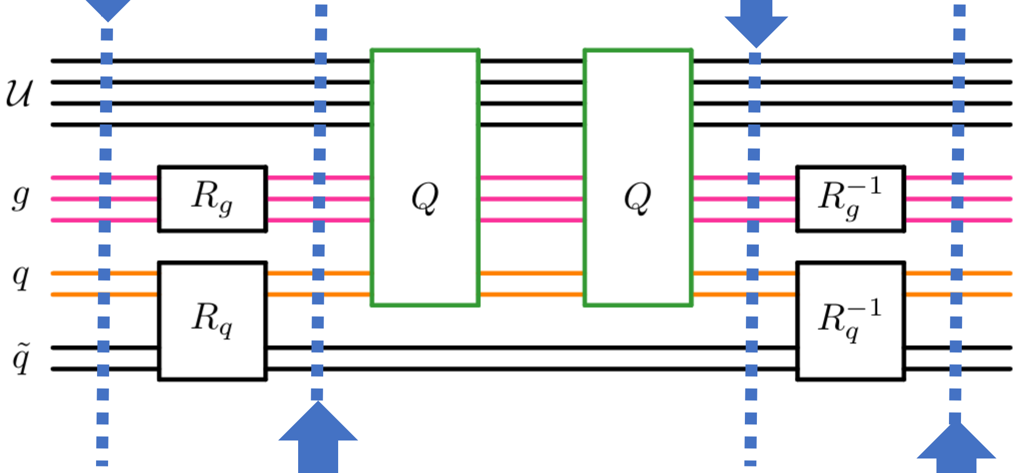
$$\frac{1}{24}\left(\sum_{\substack{a\in\{1,\dots,8\}\\ i,j\in\{1,2,3\}}} T^a_{ij}T^a_{ji}\right)|\Omega\rangle_g\,|\Omega\rangle_q\,|\Omega\rangle_{\tilde q}\,|\Omega\rangle_{\mathcal U} + \left(\text{terms orthogonal to } |\Omega\rangle_g\,|\Omega\rangle_q\,|\Omega\rangle_{\tilde q}\,|\Omega\rangle_{\mathcal U}\right)$$

Calculating colour factors: illustrative example

$$|\Omega\rangle_g \, |\Omega\rangle_q \, |\Omega\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}$$

$$\frac{1}{\sqrt{24}} \sum_{\substack{a\in\{1,\ldots,8\} \\ i,j,k\in\{1,2,3\}}} T^a_{ij} T^a_{jk} \, |a\rangle_g \, |i\rangle_q \, |k\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$





$$\frac{1}{\sqrt{24}} \sum_{a=1}^{8} \sum_{k=1}^{3} |a\rangle_g \, |k\rangle_q \, |k\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}$$

$$\frac{1}{24} \left(\sum_{\substack{a\in\{1,\ldots,8\} \\ i,j\in\{1,2,3\}}} T^a_{ij} T^a_{ji} \right) |\Omega\rangle_g \, |\Omega\rangle_q \, |\Omega\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}} + \left(\text{terms orthogonal to } |\Omega\rangle_g \, |\Omega\rangle_q \, |\Omega\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}\right)$$

Outline

1. Introduction
2. Basics of quantum computing
3. Quantum circuits for colour
    - Overview
    - Details
        - Non-unitary matrices
        - Constructing the Q and G gates
        - General algorithm for calculating colour factors for arbitrary Feynman diagrams
4. Results/validation
5. Outlook and summary

Non-unitary operators in perturbative QCD

- Would like quantum gates for the 8 linear operators

$$|j\rangle_q \to \sum_i T^a_{ij} |i\rangle_q$$

  and also for the (diagonal) operator

$$|a\rangle_{g_1} |b\rangle_{g_2} |c\rangle_{g_3} \to f^{abc} |a\rangle_{g_1} |b\rangle_{g_2} |c\rangle_{g_3}$$

- An operator is unitary iff the rows of its matrix representation are orthonormal
  - In matrices T^a_{ij} and f^{abc}, rows are orthogonal
    - But not necessarily of unit norm
- Need a unitary way to alter a state's norm

Recall:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Unitarisation register: expanding the space

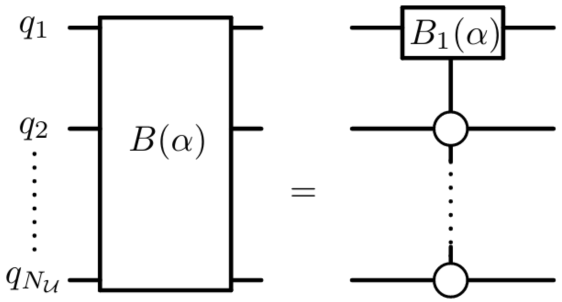
- Let L be an operator acting on a Hilbert space $\mathcal{H}_1$
- If L is non-unitary, it cannot be directly implemented as a circuit
- But it may be possible to define a new unitary operator $\hat{L}$ acting on a larger space $\mathcal{H}_1 \otimes \mathcal{H}_\mathcal{U}$ such that

$$\langle\Omega|_\mathcal{U} \langle\chi_2|\hat{L}|\chi_1\rangle |\Omega\rangle_\mathcal{U} = \langle\chi_2|L|\chi_1\rangle \qquad \begin{array}{l}\text{for some state } |\Omega_\mathcal{U}\rangle \in \mathcal{H}_\mathcal{U} \\ \text{for all states } |\chi_1\rangle, |\chi_2\rangle \in \mathcal{H}_1\end{array}$$

- In this work, we introduce a single additional register $\mathcal{U}$, whose size is small: $N_\mathcal{U} = \lceil \log_2(N_V + 1) \rceil$

Unitarisation register: gates A and B

- Let A denote the increment circuit described earlier
- Define a gate B(α):



where:

$$B_1(\alpha) = \begin{pmatrix} \sqrt{1-|\alpha|^2} & \alpha \\ -\alpha & \sqrt{1-|\alpha|^2} \end{pmatrix}$$

Unitarisation register: key properties

- Together, gates A and B(α) act on $\mathcal{U}$ in the following way:

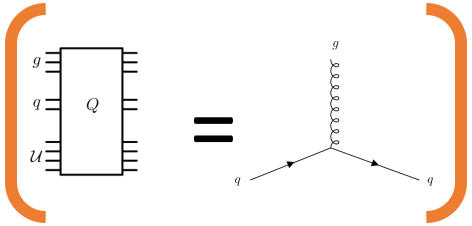
$$B(\alpha)A\,|k\rangle = \begin{cases} \alpha\,|0\rangle + \sqrt{1-|\alpha|^2}\,|1\rangle & \text{if } k=0 \\ |k+1\rangle & \text{if } 0<k<2^{N_{\mathcal{U}}}-1 \\ \sqrt{1-|\alpha|^2}\,|0\rangle - \alpha\,|1\rangle & \text{if } k=2^{N_{\mathcal{U}}}-1. \end{cases}$$

$$|0\rangle_{\mathcal{U}} \equiv |\Omega\rangle_{\mathcal{U}}$$

which means we can apply B(α)A repeatedly up to $2^{N_{\mathcal{U}}}-1$ times and satisfy

$$\langle\Omega|_{\mathcal{U}} \prod_{i=1} \{B(\alpha_i)A\}\,|\Omega\rangle_{\mathcal{U}} = \prod_{i=1} \alpha_i$$

Construction of the Q gate



$$\boxed{Q\,|a\rangle_g\,|k\rangle_q\,|\Omega\rangle_{\mathcal{U}} = \sum_{j=1}^{3} T^a_{jk}\,|a\rangle_g\,|j\rangle_q\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})}$$

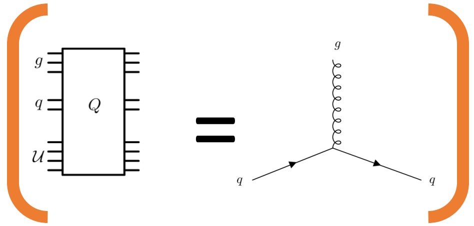
- Start by defining matrices $\overline{\lambda}_a$

$$\overline{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \overline{\lambda}_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \overline{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\overline{\lambda}_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \overline{\lambda}_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 1 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \overline{\lambda}_6 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

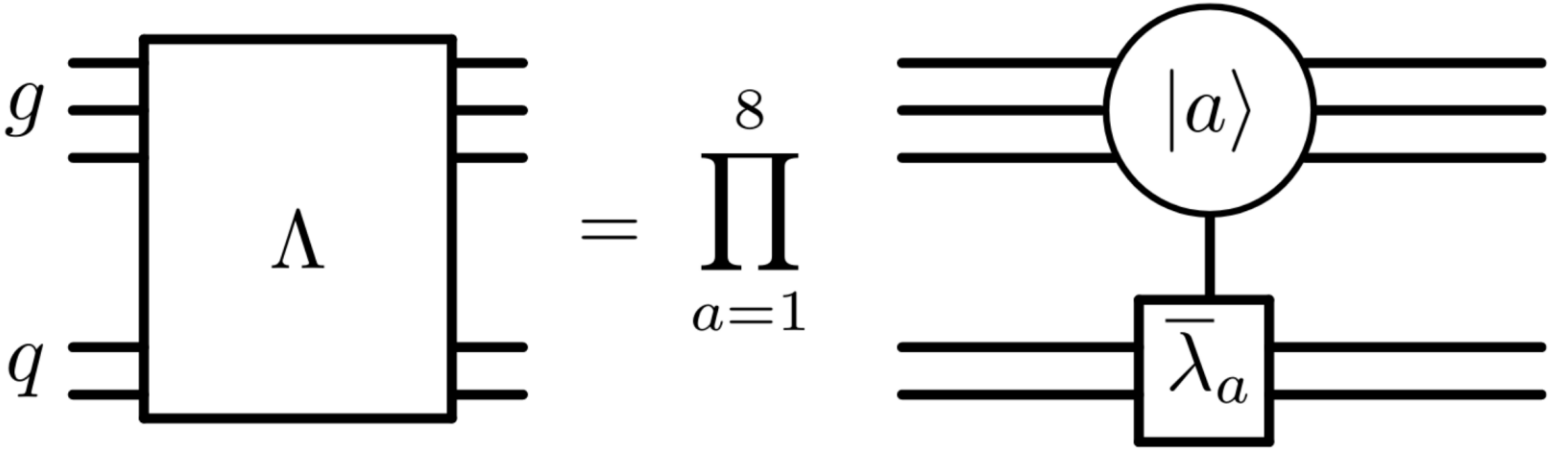
$$\overline{\lambda}_7 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \overline{\lambda}_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Construction of the Q gate

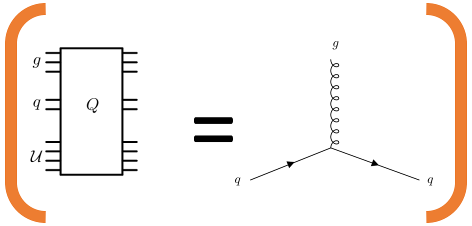


$$Q\,|a\rangle_g\,|k\rangle_q\,|\Omega\rangle_{\mathcal{U}} = \sum_{j=1}^{3} T^a_{jk}\,|a\rangle_g\,|j\rangle_q\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$

- Next, define a gate Λ

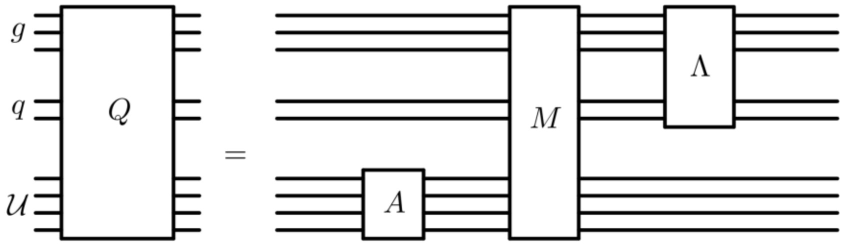


Construction of the Q gate



$$Q\left|a\right\rangle_g\left|k\right\rangle_q\left|\Omega\right\rangle_{\mathcal{U}} = \sum_{j=1}^{3} T^a_{jk}\left|a\right\rangle_g\left|j\right\rangle_q\left|\Omega\right\rangle_{\mathcal{U}} + (\text{terms orthogonal to } \left|\Omega\right\rangle_{\mathcal{U}})$$

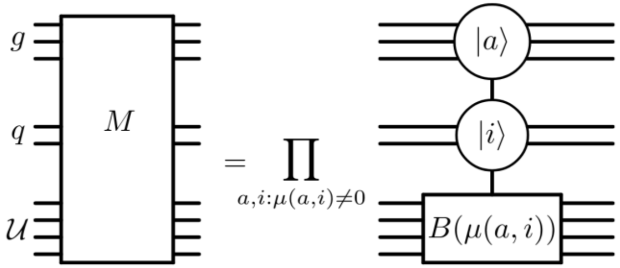
- Finally, define the gate Q



Recall:

$$\langle\Omega|_{\mathcal{U}} B(\alpha) A |\Omega\rangle_{\mathcal{U}} = \alpha$$

with



Explicitly:

$$\mu(a,i) = \begin{cases} \frac{1}{2} & \text{if } \left(\overline{\lambda}_a\right)_{ij} - \left(\lambda_a\right)_{ij} = 0 \quad \forall j \\ \frac{1}{2\sqrt{3}} & \text{if } a = 8 \text{ and } i \in \{1,2\} \\ \frac{-1}{\sqrt{3}} & \text{if } a = 8 \text{ and } i = 3 \\ 0 & \text{otherwise.} \end{cases}$$

where μ is defined such that $\mu(a,i)\overline{\lambda}_a\left|i\right\rangle = \frac{1}{2}\lambda_a\left|i\right\rangle$

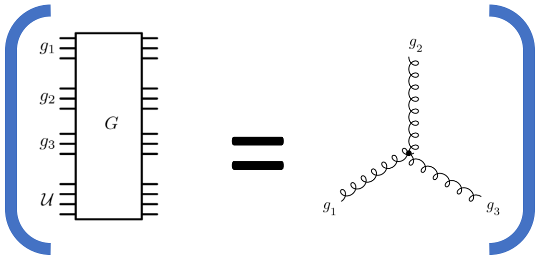
Recall:

$$\overline{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \overline{\lambda}_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \overline{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\overline{\lambda}_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \overline{\lambda}_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 1 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \overline{\lambda}_6 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

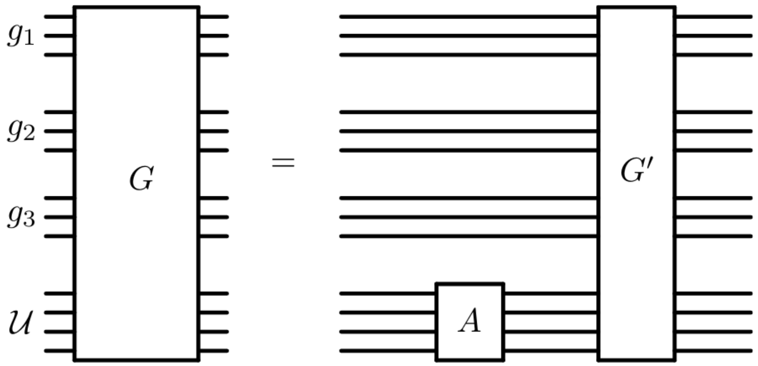
$$\overline{\lambda}_7 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \overline{\lambda}_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Construction of the G gate



$$\boxed{G\,|a\rangle_{g_1}\,|b\rangle_{g_2}\,|c\rangle_{g_3}\,|\Omega\rangle_{\mathcal{U}} = f^{abc}\,|a\rangle_{g_1}\,|b\rangle_{g_2}\,|c\rangle_{g_3}\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})}$$

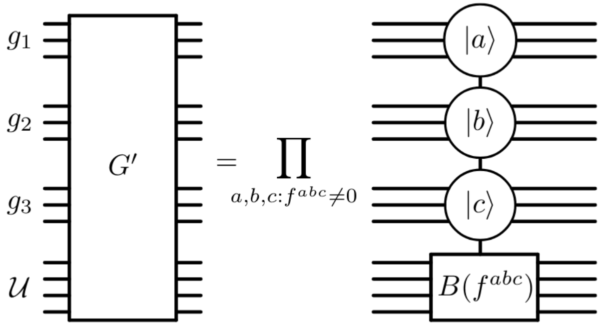
- Define G gate:



Recall:

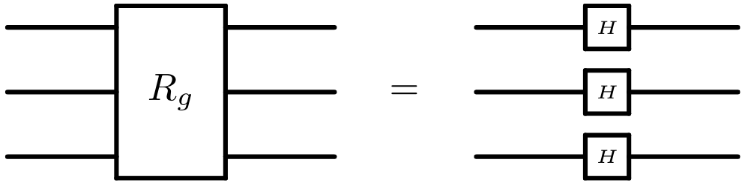
$$\langle\Omega|_{\mathcal{U}} B(\alpha) A |\Omega\rangle_{\mathcal{U}} = \alpha$$

where:



R_g and R_q gates for tracing

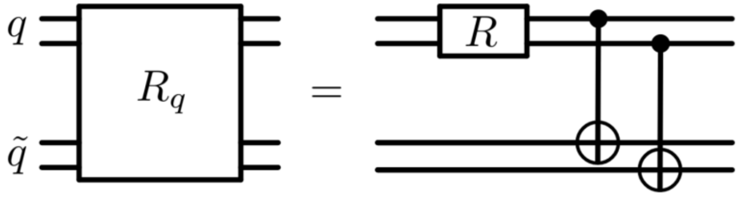
$$R_g \left|\Omega\right\rangle_g = \sum_{a=1}^{8} \frac{1}{\sqrt{8}} \left|a\right\rangle_g$$



$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$R_g^{-1} \sum_{a=1}^{8} c_a \left|a\right\rangle_g = \left(\frac{1}{\sqrt{8}} \sum_{a=1}^{8} c_a \right) \left|\Omega\right\rangle_g + \left(\text{terms orthogonal to } \left|\Omega\right\rangle_g \right)$$

$$R_q \left|\Omega\right\rangle_q \left|\Omega\right\rangle_{\tilde{q}} = \sum_{k=1}^{3} \frac{1}{\sqrt{3}} \left|k\right\rangle_q \left|k\right\rangle_{\tilde{q}}$$



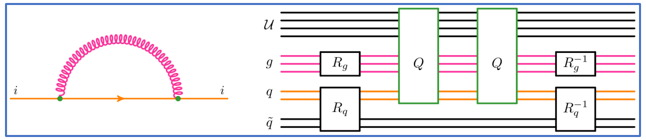
$$R = \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{6}} & 0 \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{6}} & 0 \\ \sqrt{\frac{1}{3}} & 0 & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_q^{-1} \sum_{i,k \in \{1,2,3\}} c_{ik} \left|i\right\rangle_q \left|k\right\rangle_{\tilde{q}} = \left(\frac{1}{\sqrt{3}} \sum_{i=1}^{3} c_{ii} \right) \left|\Omega\right\rangle_q \left|\Omega\right\rangle_{\tilde{q}} + \left(\text{terms orthogonal to } \left|\Omega\right\rangle_q \left|\Omega\right\rangle_{\tilde{q}} \right)$$

Calculating the colour factor of arbitrary Feynman diagrams

- Build a quantum circuit with:
  - For each gluon, 1 gluon register, with 3 qubits per register
  - For each quark line, a pair of quark registers: q and $\tilde{q}$, with 2 qubits per register
  - A unitarisation register with $N_{\mathcal{U}} = \lceil \log_2(N_V + 1) \rceil$ qubits
- Initialise each register r into the state $|\Omega\rangle_r$
- For each gluon, apply R_g
- For each quark, apply R_q
- For each quark-gluon vertex, apply Q gate to the corresponding g and q registers (not $\tilde{q}$)
- For each triple-gluon vertex, apply G gate to the corresponding g registers
- For each gluon, apply $(R_g)^{-1}$
- For each quark, apply $(R_q)^{-1}$
- Colour factor $\mathcal{C}$ is found encoded in the final state of the quantum computer, which is:

Recall the illustrative example:



$$\frac{1}{\mathcal{N}}\mathcal{C}\,|\Omega\rangle_{all} + (\text{terms orthogonal to } |\Omega\rangle_{all}) \qquad \text{where } \mathcal{N} = N_c^{n_q}\left(N_c^2 - 1\right)^{n_g}$$

Outline

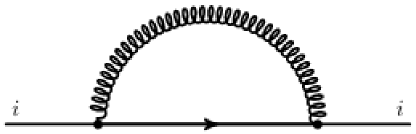
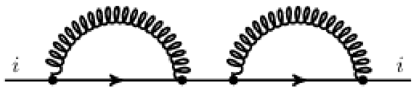
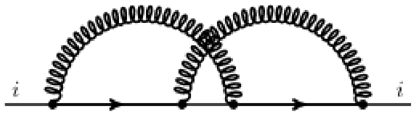
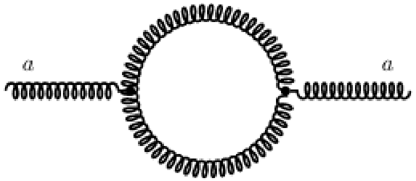
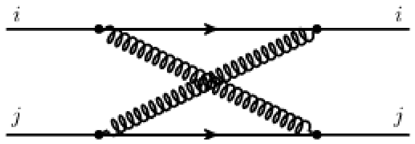
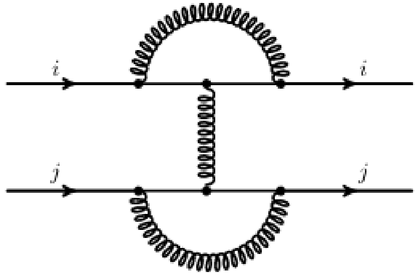
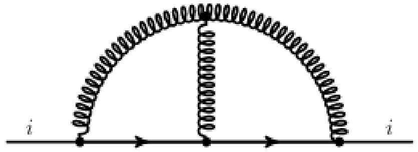
1. Introduction
2. Basics of quantum computing
3. Quantum circuits for colour
    - Overview
    - Details
4. Results/validation
5. Outlook and summary

Validation

- Implemented using Qiskit (IBM)
- Simulated various diagrams
  - Simulated noiseless quantum computer
    - These examples use up to 30 qubits
  - Ran each diagram 10^8 times
  - Measured output to infer colour factor

$$\frac{1}{N}\mathcal{C}\,|\Omega\rangle_{all} + (\text{terms orthogonal to } |\Omega\rangle_{all})$$

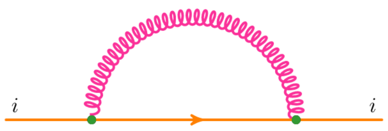
  - Full agreement with analytic expectation

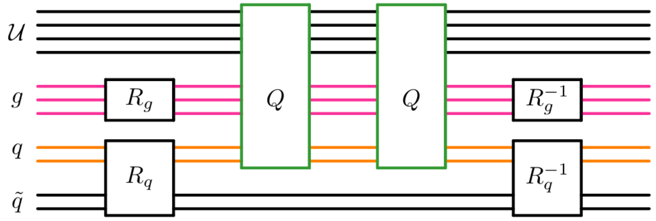
Diagram	Analytical	Numerical
	$C_F N = 4$	3.9988 ± 0.0012
	${C_F}^2 N = \frac{16}{3}$	5.331 ± 0.010
	$\frac{C_F}{2} = \frac{2}{3}$	0.673 ± 0.010
	$N(N^2-1) = 24$	23.95 ± 0.03
	$\frac{(N^2-1)}{4} = 2$	2.00 ± 0.03
	0	$0.0^{+0.5}_{-0.0}$
	$\frac{C_F N^2}{2} = 6$	5.92 ± 0.08

Directions for future work

- Interference of multiple diagrams
  - Natural application for a quantum computer
  - Can try with/without quantum simulation of kinematic parts
- Kinematic parts
  - Unitarisation register could be useful here too
  - Much larger Hilbert space since kinematic variables are continuous
- High-multiplicity processes
- Monte-Carlo integration of cross-sections
  - quadratic speed-up

Summary and outlook





- Designed quantum circuits to simulate colour part of perturbative QCD
  - Example application: colour factors for arbitrary Feynman diagrams
  - First step towards a full quantum simulation of generic perturbative QCD processes
- Natural avenues for follow-up work:
  - Interference of multiple Feynman diagrams
  - Kinematic parts of Feynman diagrams
  - Use in a quantum Monte Carlo calculation of cross-sections
    - Quadratic speed-up over classical Monte Carlo