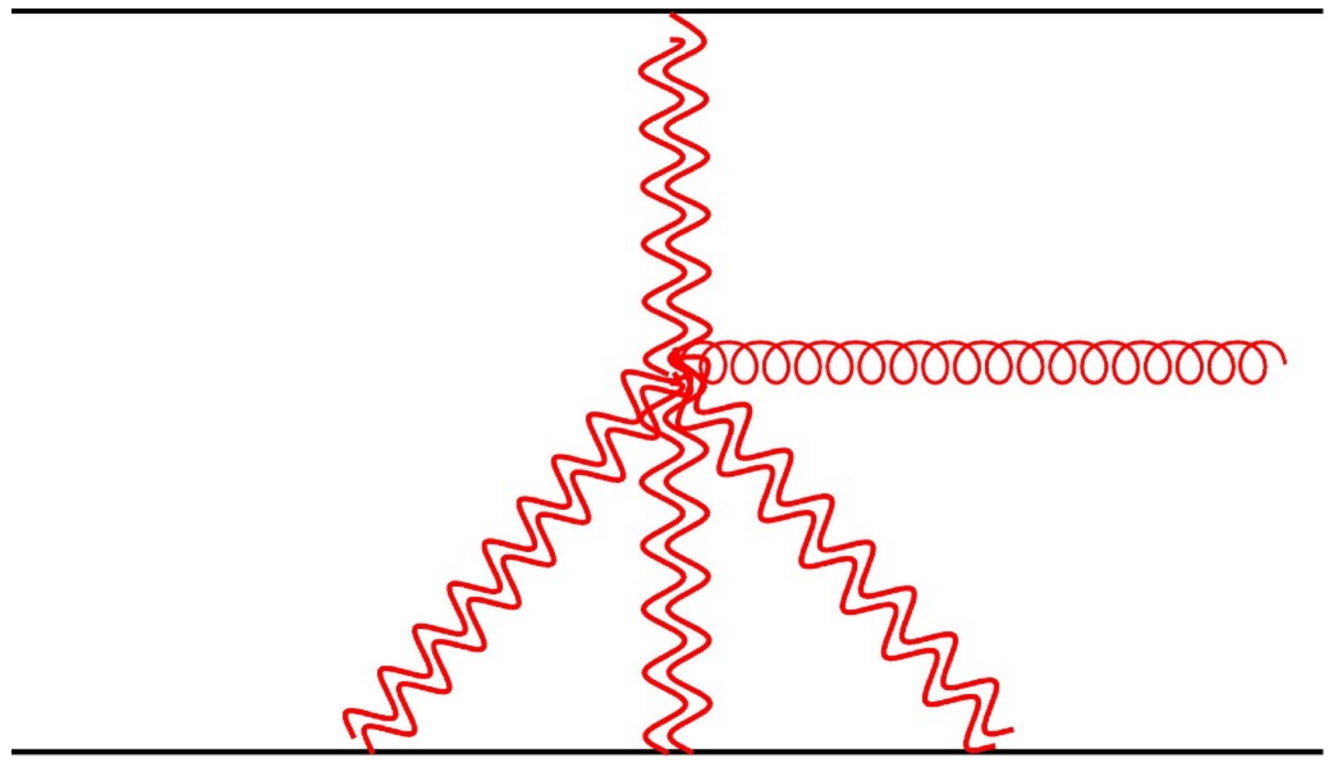
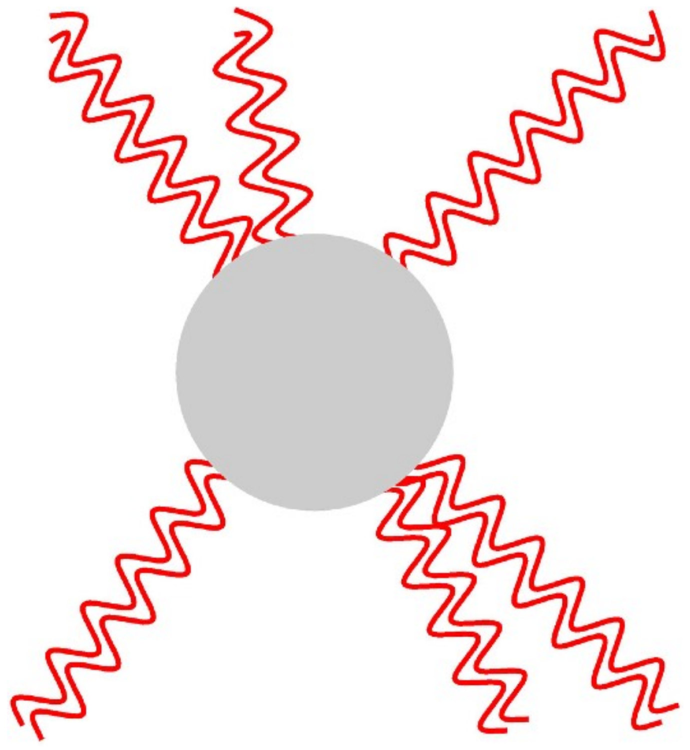
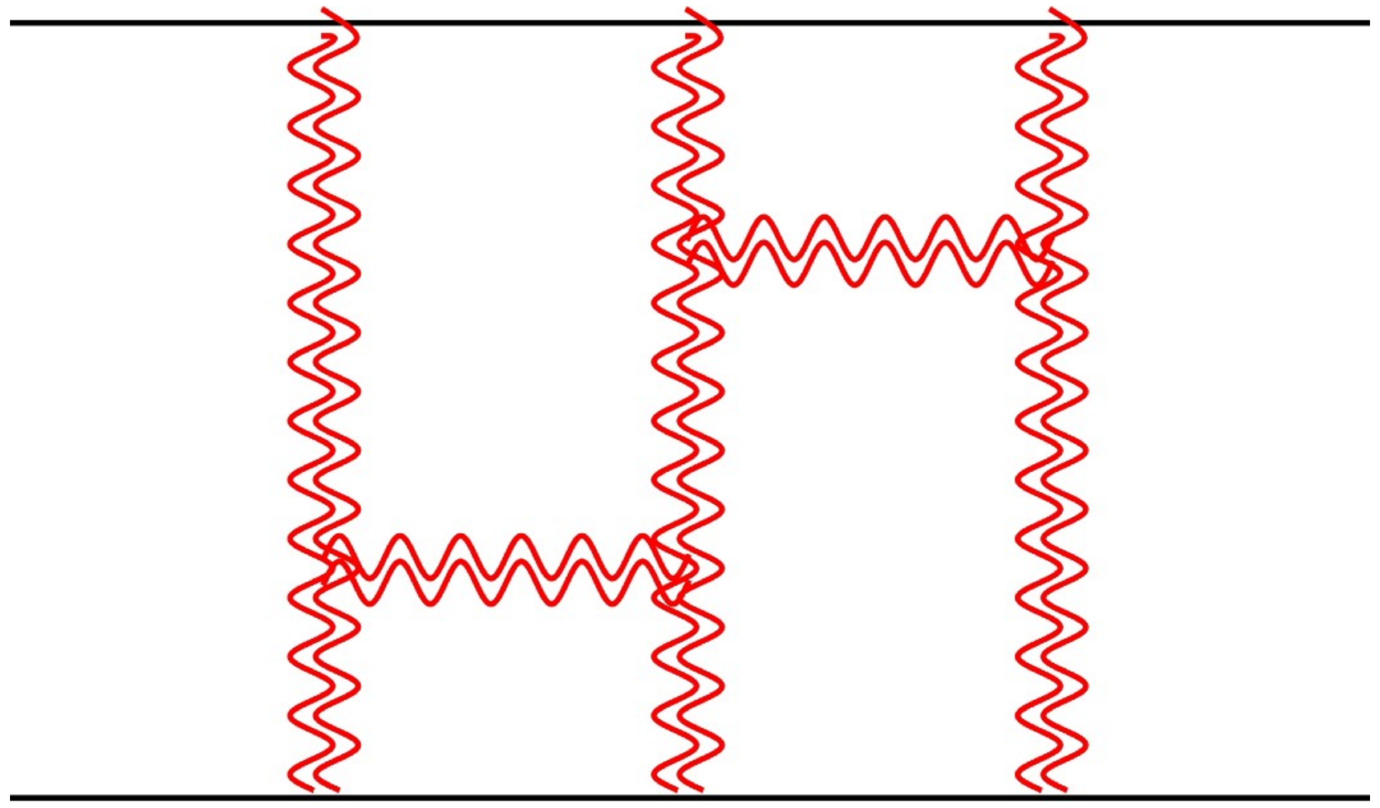


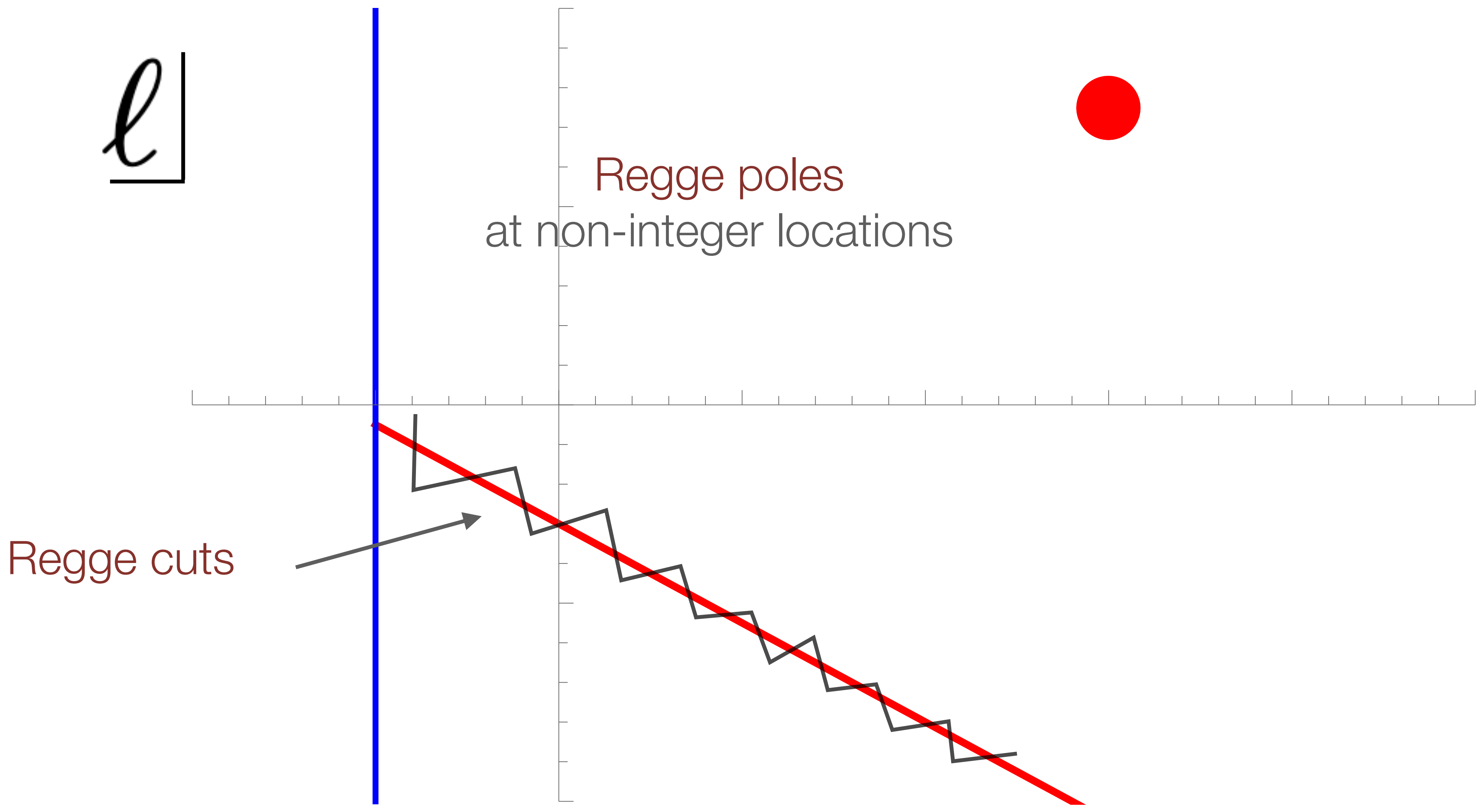




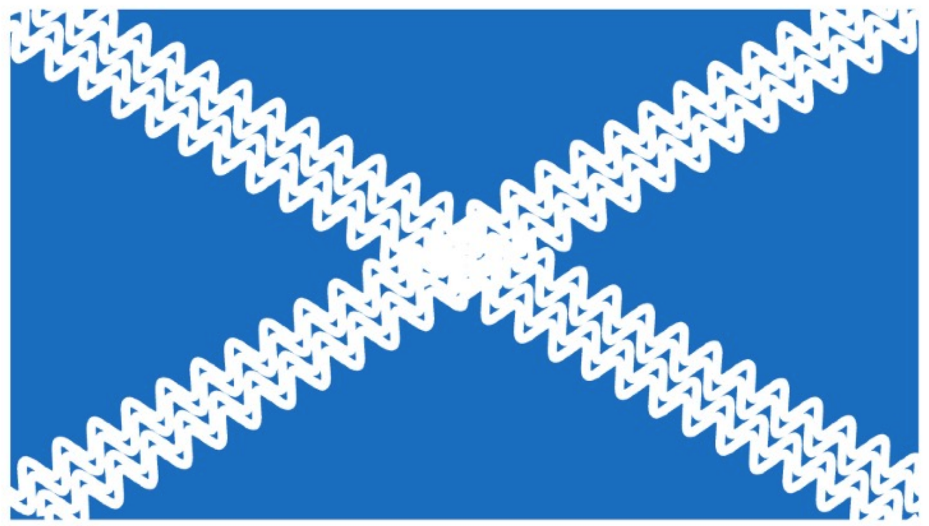
Regge poles and Regge cuts in multi-leg QCD amplitudes

Calum Milloy, University of Turin

with thanks to collaborators
Samuel Abreu, Giulio Falcioni, Einan Gardi,
Niamh Maher, Leonardo Vernazza



RADCOR, Crieff, Scotland

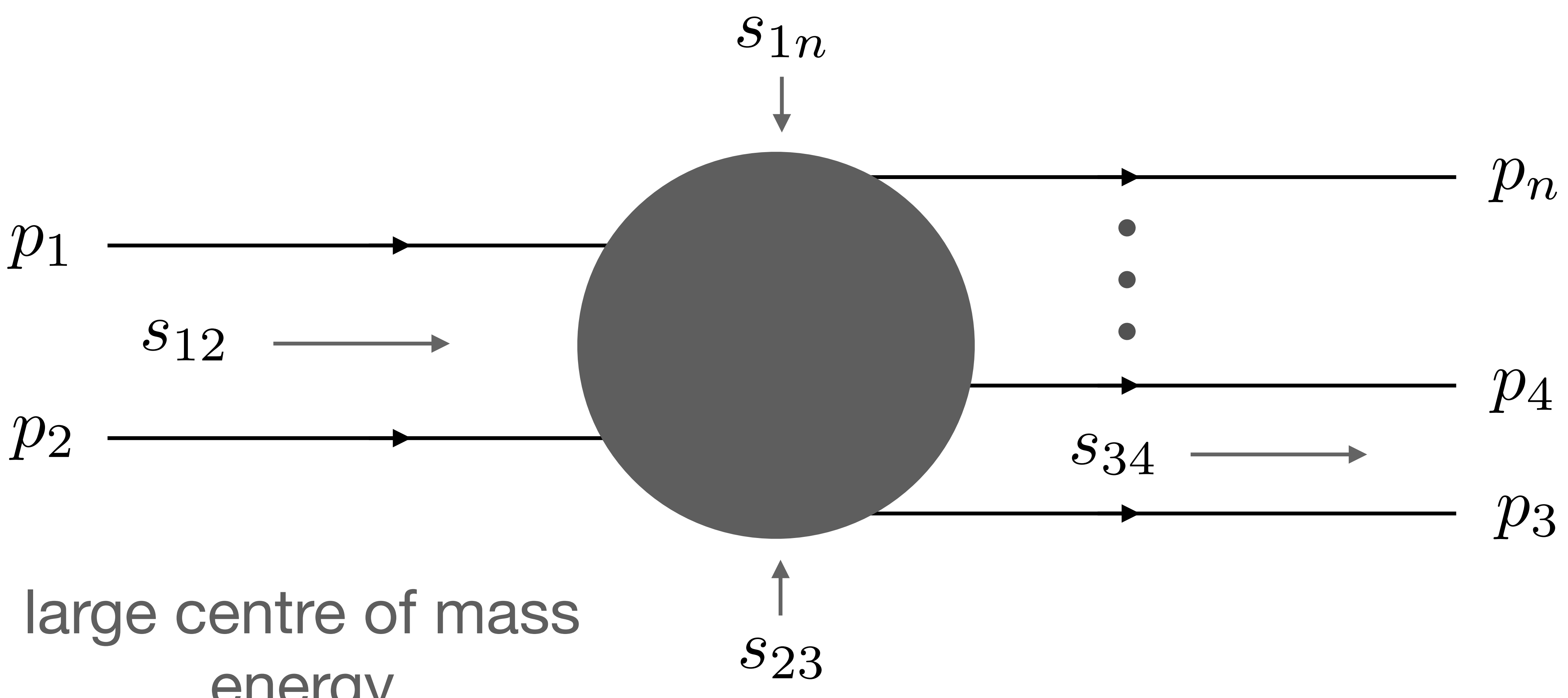


Contents

- The multi-Regge limit
- Motivation
- Regge poles and Regge cuts in the complex angular momentum plane
- 2 → 2 amplitudes
    - Impact factors and Regge trajectory
- 2 → 3 amplitudes
    - Lipatov vertex

The multi-Regge limit of n-point amplitudes

Multi-Regge kinematics (MRK)



large centre of mass energy

parameterise in the (12) scattering plane

$$p_1 = (0, p_1^-, \mathbf{0}) \qquad p_2 = (p_2^+, 0, \mathbf{0})$$

for all the outgoing partons non-zero transverse momenta

$$p_i = (p_i^+, p_i^-, \mathbf{p}_i)$$

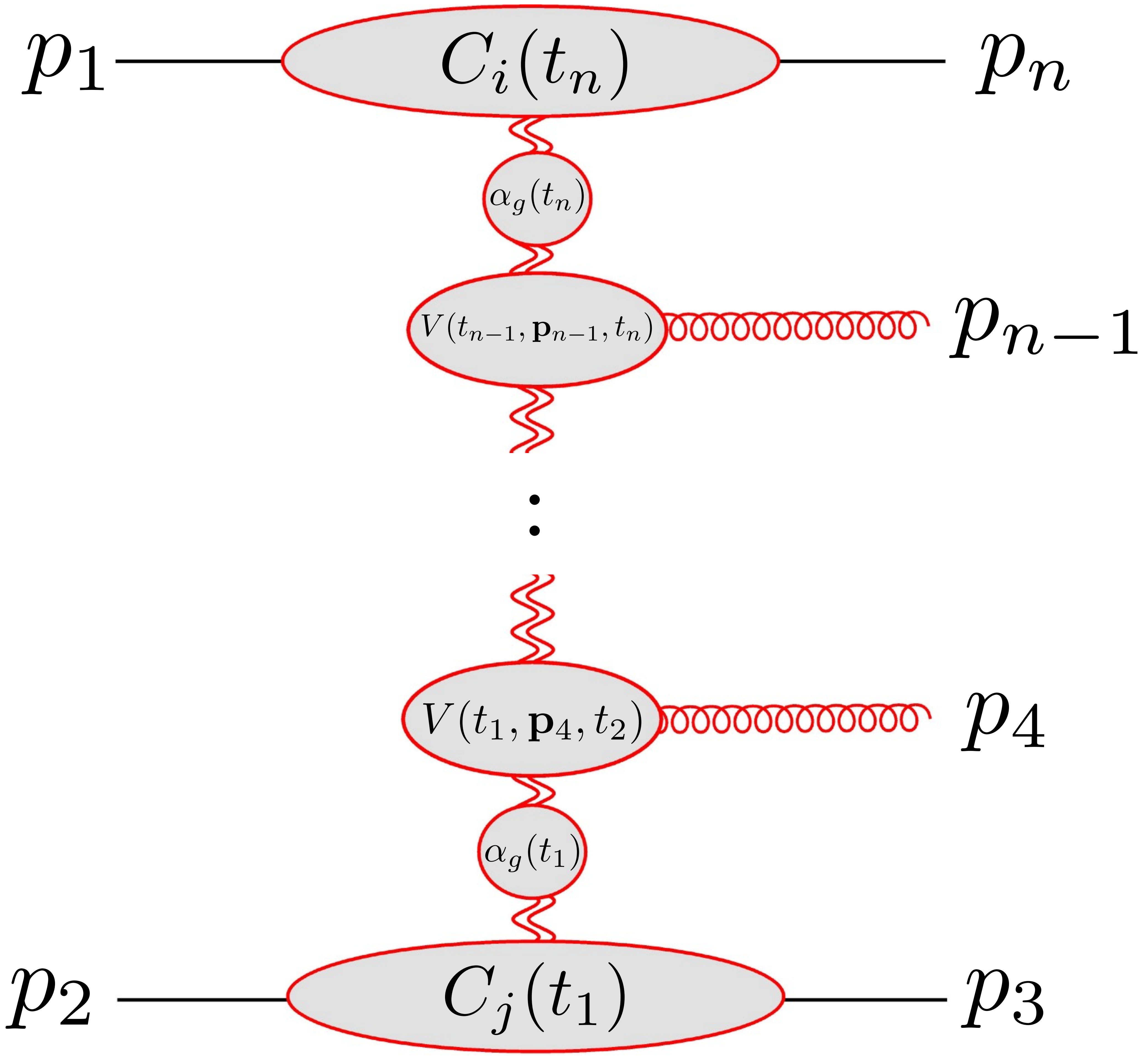
define rapidity variables through $p_i^\pm = |\mathbf{p}_i| e^{\pm y_i}$

- MRK can be defined as the strong ordering of the rapidity variables $y_3 \gg y_4 \gg \cdots \gg y_n$
- The separation of scales leads to simplification in the dynamics in the (+) and (-) directions
- Leaving non-trivial kinematics in the two-dimensional transverse plane

Why study the MRK limit?

Motivation

- Simplification gives opportunity to study high-loop orders and to resum perturbative amplitudes
  [Del Duca, Druc et. al. ’19]
- Explore universality of massless gauge theories
- Constraints on infrared structure
  [Del Duca, Duhr, Gardi, Magnea, White ’11; Del Duca, Falcioni, Magnea, Vernazza ’13; Falcioni, Gardi, Maher, **CM**, Vernazza ’21]
- Perturbative meaning to Regge poles and Regge cuts



Factorisation of 2 ➜ n amplitudes in MRK

[phenomenological talks: Michael Fucilla, Jeppe Andersen, Andreas Maier, Francesco Giovanni Celiberto, Sebastian Jaskiewicz …]

Complex Angular Momentum Plane

Let us travel back to the 1960s, prior to QCD

[Regge ’59, ’60; Eden, Landshoff, Olive, Polkinghorne ’66; Collins ‘77]

Easier to explain using the 2 ➡ 2 amplitude

Start with partial wave expansion of the scattering amplitude

$$\mathcal{M} \sim \sum_{\ell=0}^{\infty} (2\ell + 1) A_\ell(t) P_\ell(s)$$

angular momentum in t-channel

dependence of a state of angular momentum ℓ are the Legendre polynomials

Complex Angular Momentum Plane

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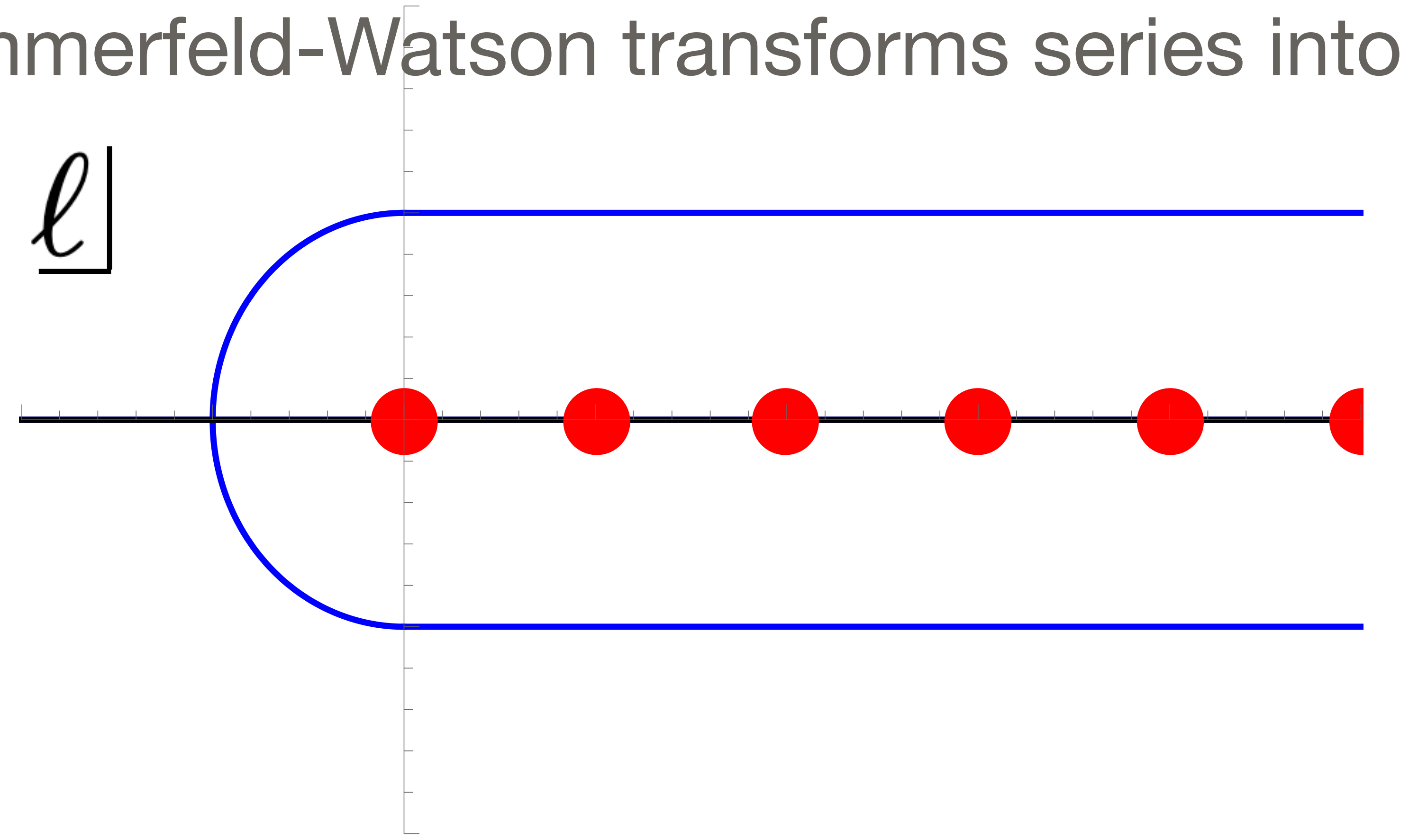
dependence of a state of angular momentum ℓ are the Legendre polynomials

Sommerfeld-Watson transforms series into a counter integral, picking up poles in the complex angular momentum plane



$$\mathcal{M} \sim \oint d\ell \, (2\ell+1) A_\ell(t) P_\ell(s) \frac{\pi}{\sin \pi \ell}$$

Legendre polynomials have the asymptotic behaviour

$$\lim_{s\to\infty} P_\ell(s) \sim s^\ell$$

Complex Angular Momentum Plane

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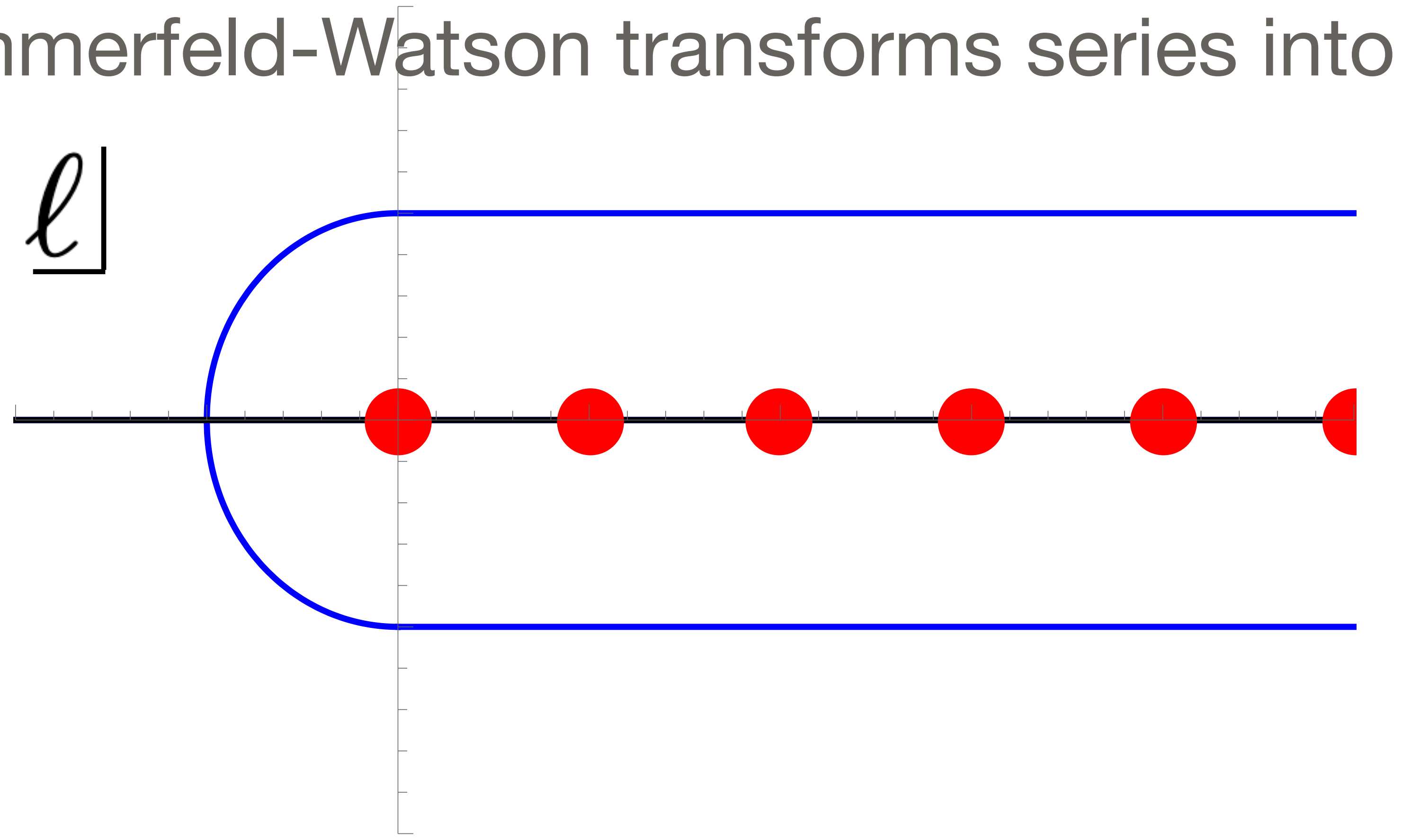

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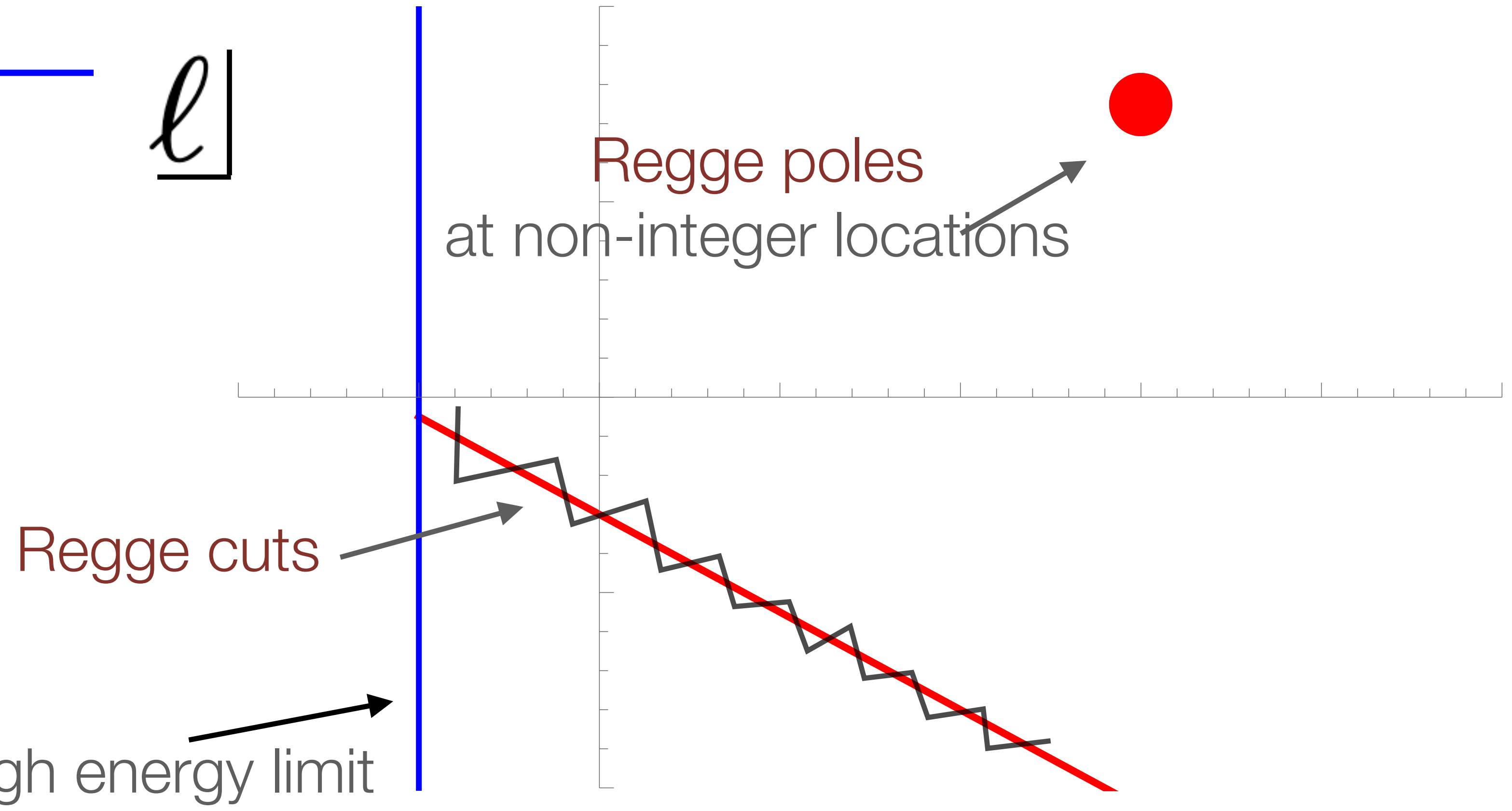
Legendre polynomials have the asymptotic behaviour

$$\lim_{s\to\infty} P_\ell(s) \sim s^\ell$$

Now we open up the contour

Doing so we will pick up other analytic behaviour



This contour is subleading in high energy limit

Evidence that Regge cuts are entirely nonplanar from diagram analysis [Mandelstam ’63]

Regge pole factorisation and universality

TOTAL CROSS SECTIONS

A Donnachie
Department of Physics, University of Manchester

P V Landshoff
CERN, Geneva*

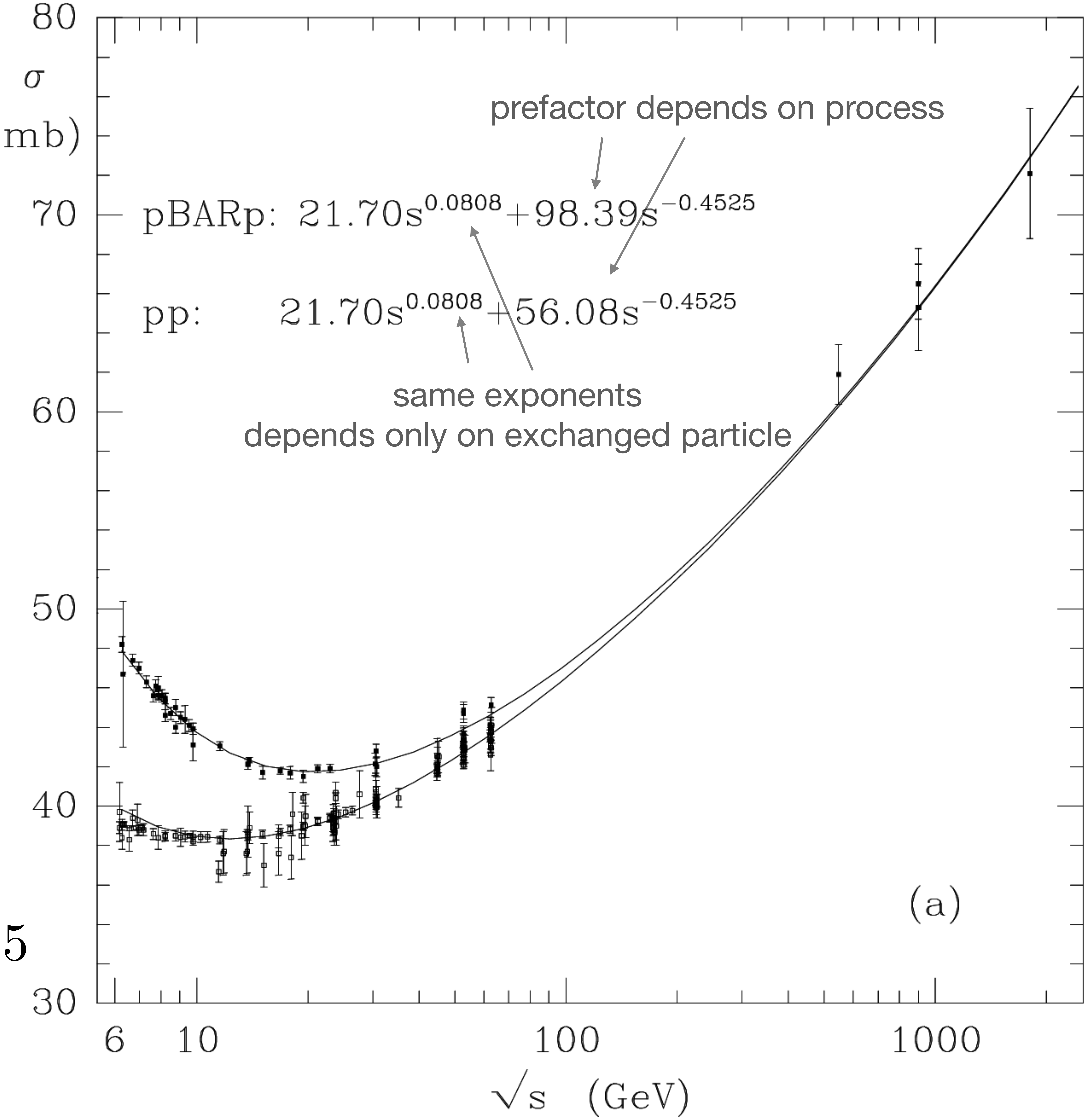
Fit against Regge pole behaviour

$$\sigma_{ab} = X_{ab}\, s^{\ell_1 - 1} + Y_{ab}\, s^{\ell_2 - 1}$$

angular momentum of exchanged particle

$\ell_1 - 1 = 0.0808$

$\ell_2 - 1 = -0.4525$



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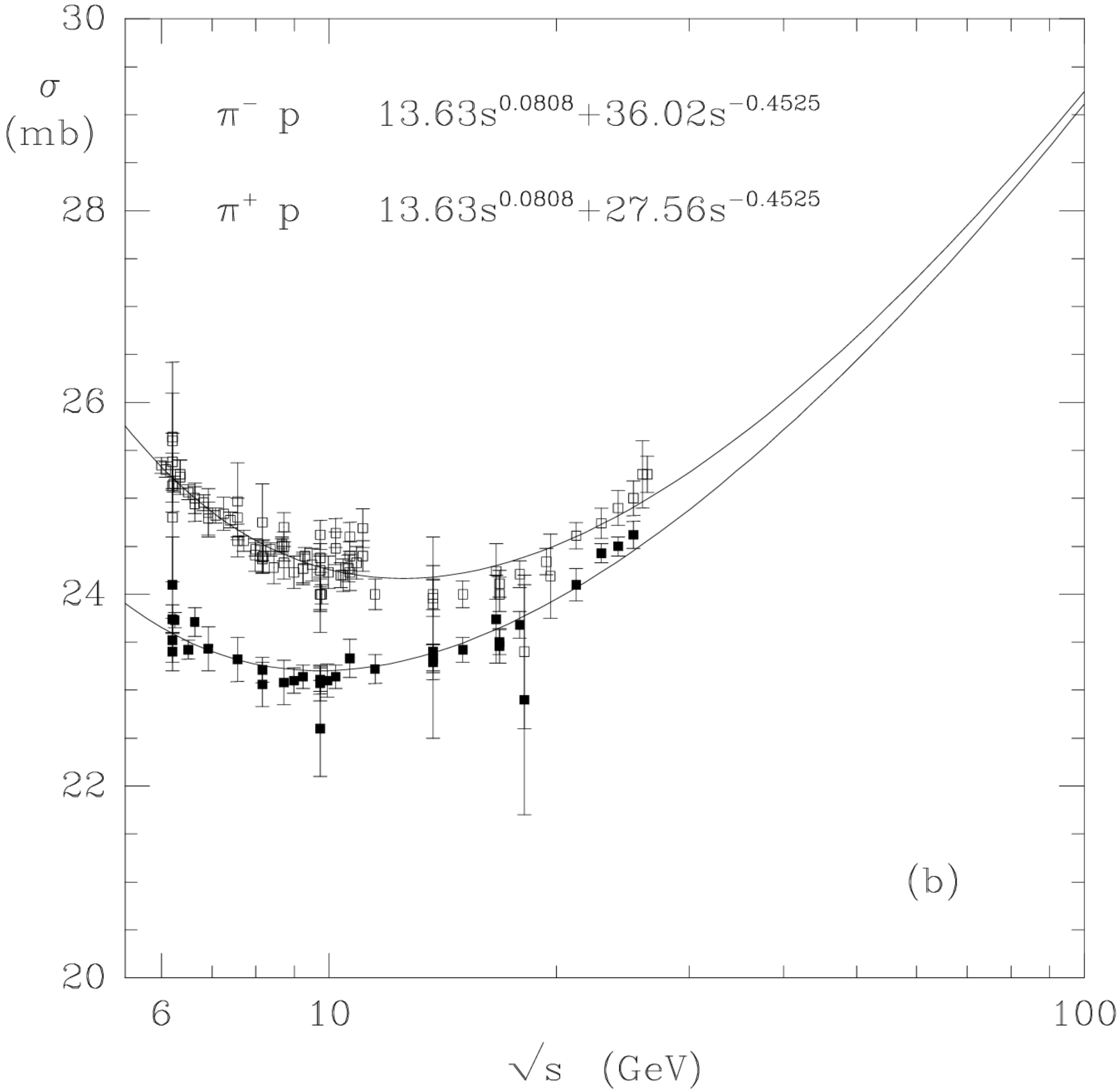

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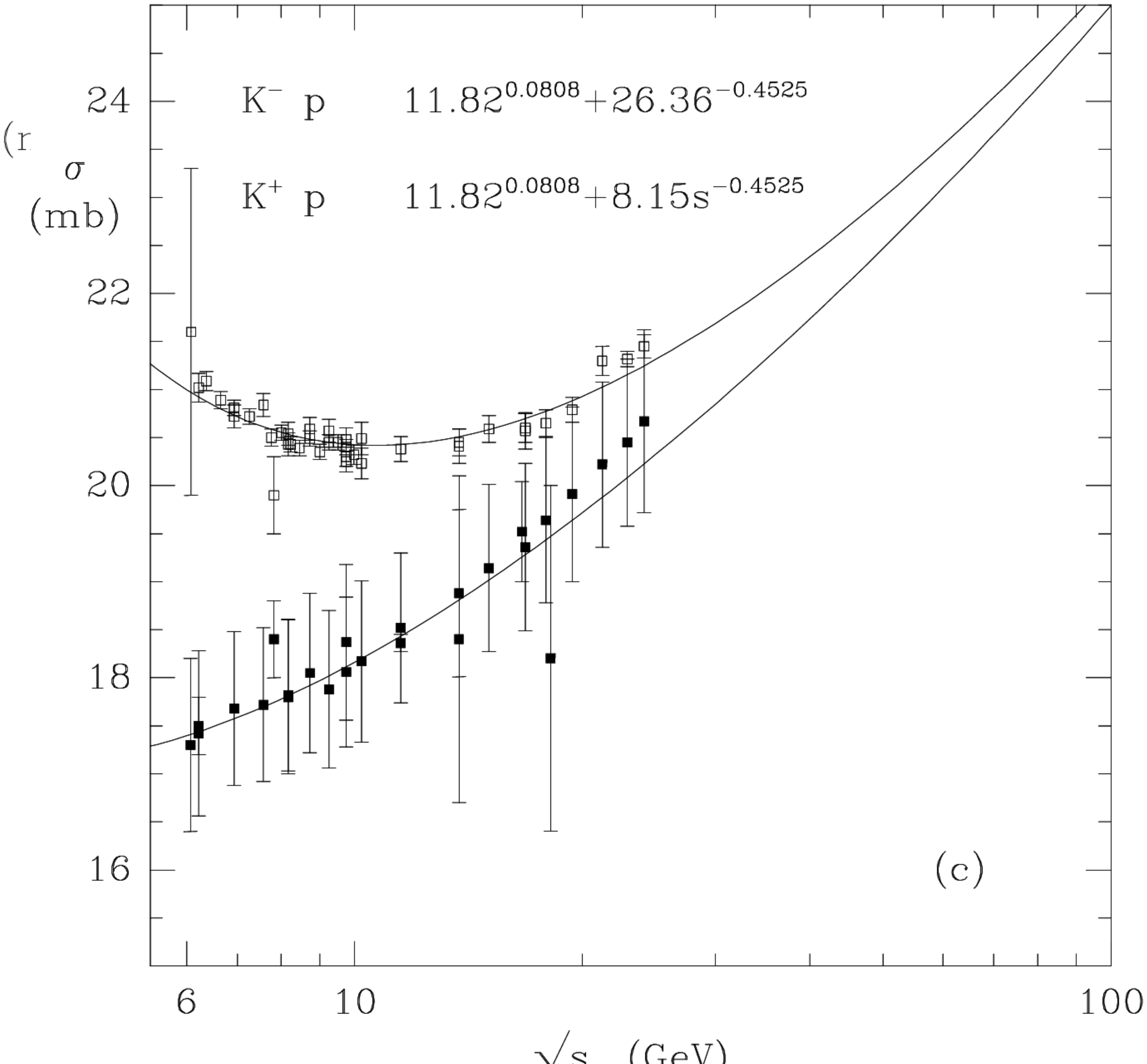

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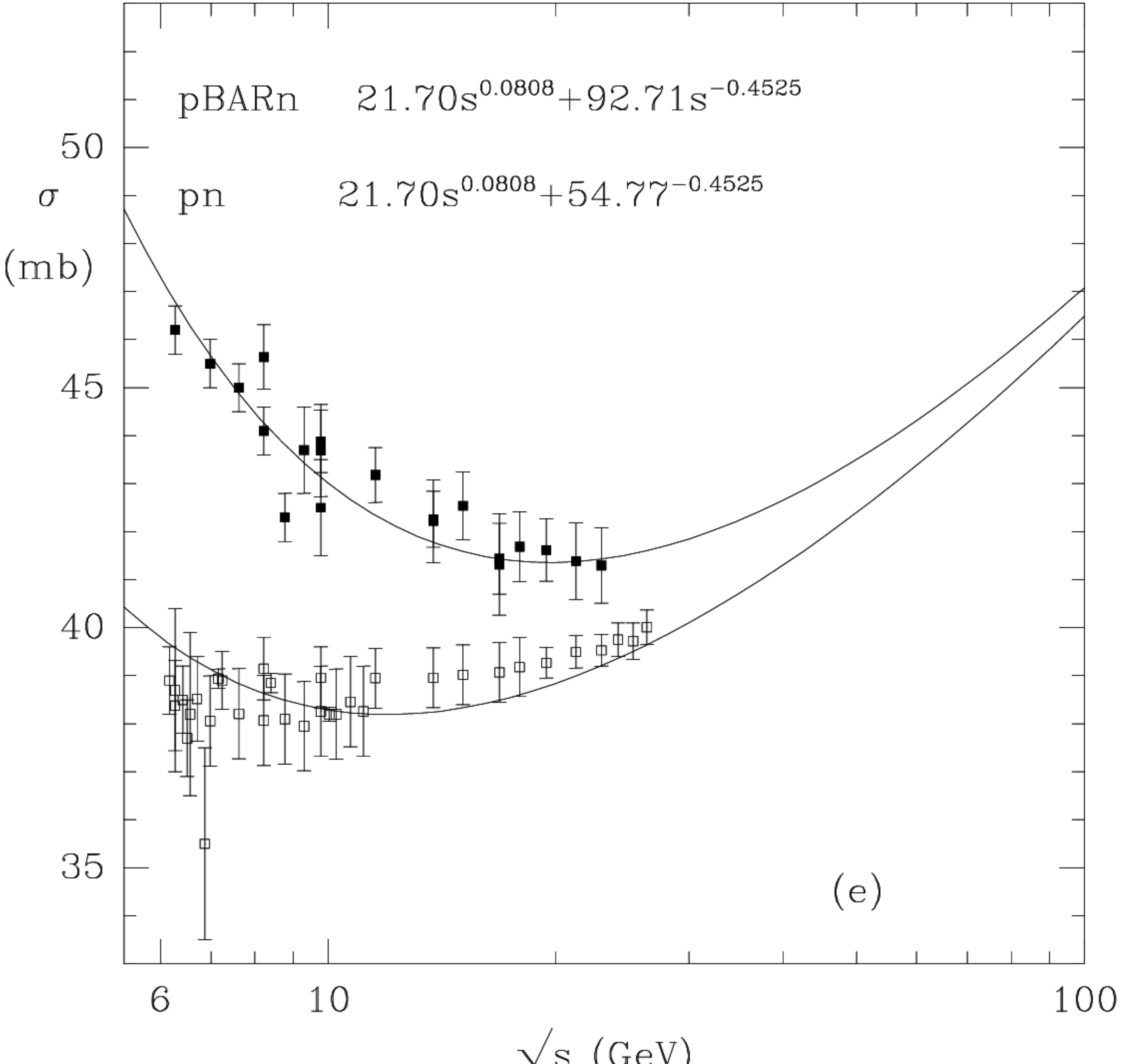
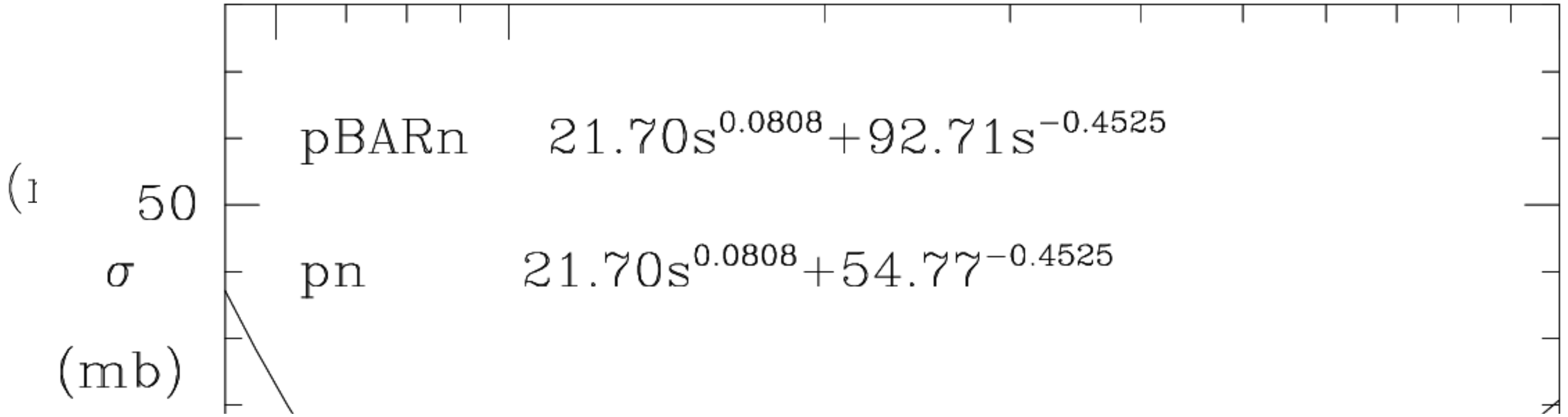


Figure 1

TOTAL CROSS SECTIONS

A Donnachie
Department of Physics, University of Manchester



Abstract

Regge theory provides a very simple and economical description of all total cross sections

$$\ell_1 - 1 = 0.0808 \quad \ell_2 - 1 = -0.45$$

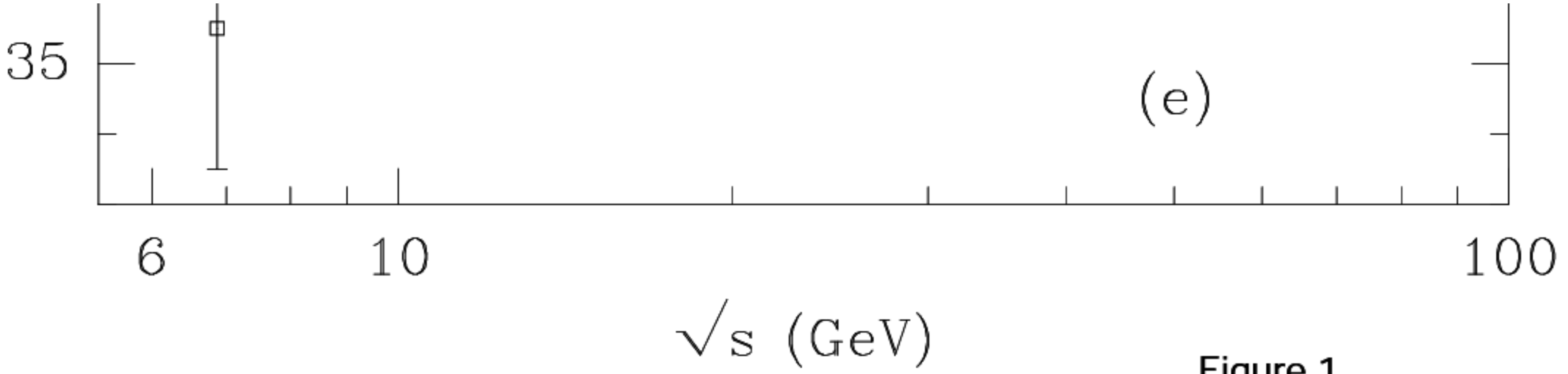
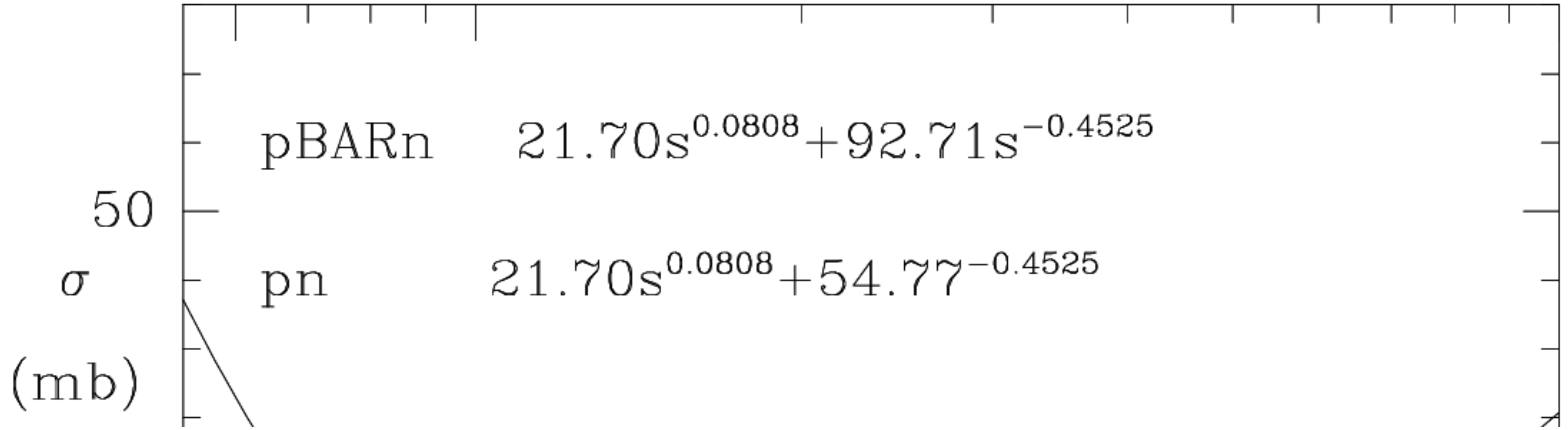


Figure 1

TOTAL CROSS SECTIONS

A Donnachie
Department of Physics, University of Manchester



Abstract

Regge theory provides a very simple and economical description of all total cross sections

Can we provide a very simple and economical description of amplitudes in MRK?

Does this universality also exist in perturbative QCD?

$$\ell_1 - 1 = 0.0808$$

$$\ell_2 - 1 = -0.45$$

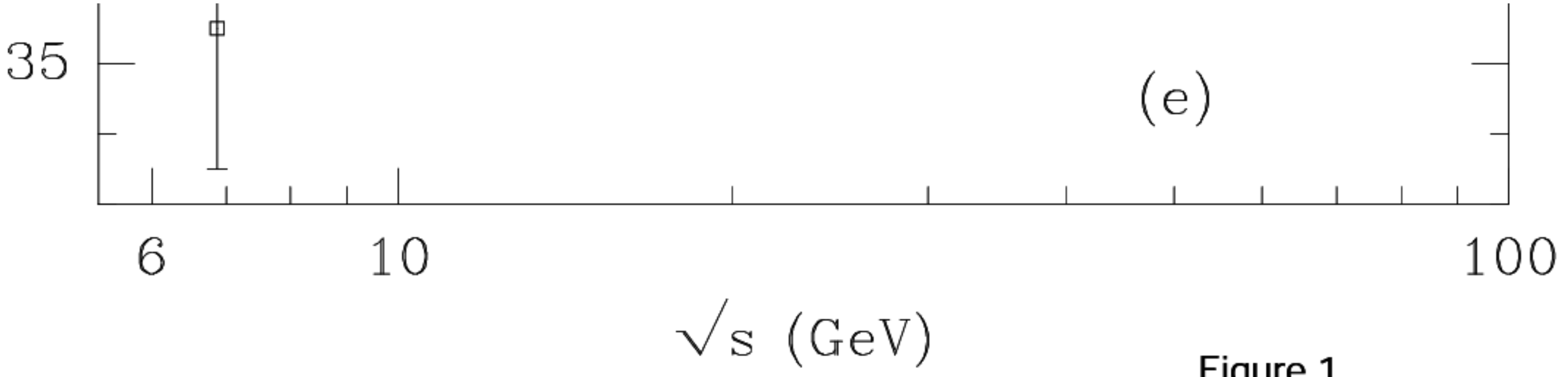
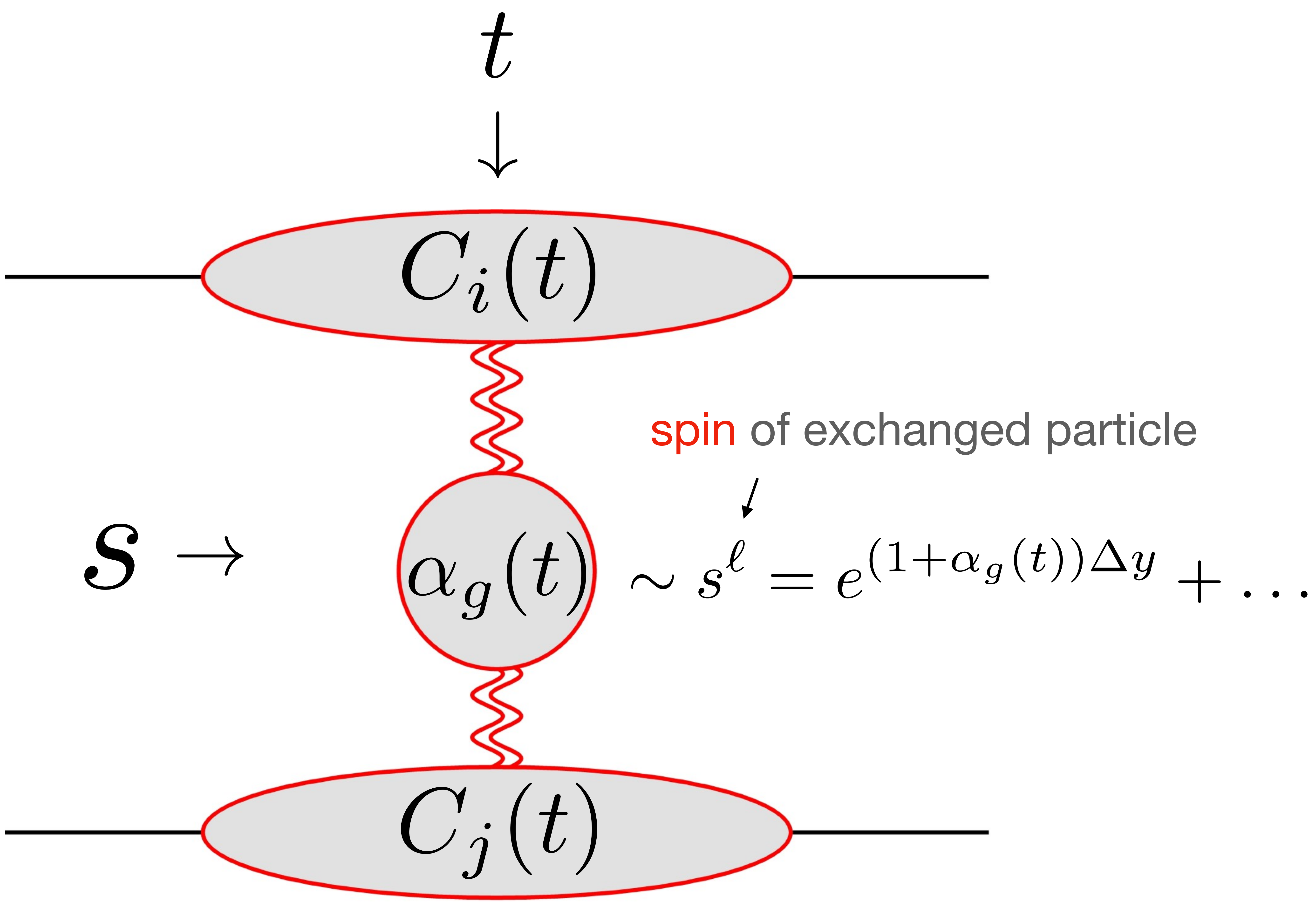


Figure 1

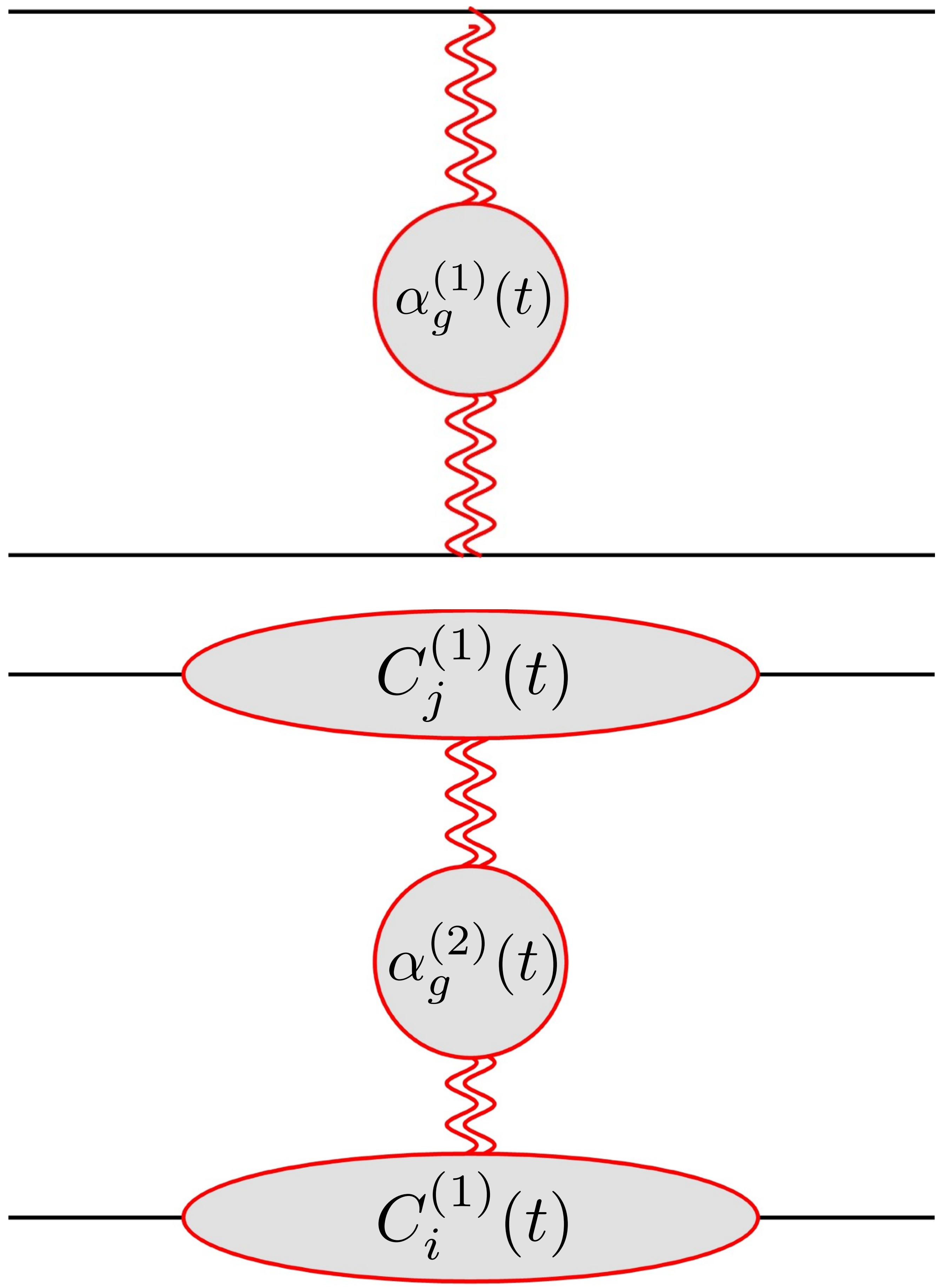
Factorisation of the Regge pole in 2 ➞ 2…



at leading power in QCD
gluon exchanges are dominant

2➔2 Amplitudes

The impact factors and Regge trajectory



[Kuraev, Fadin and Lipatov '76] [Fadin, Fiore, Kozlov, Reznichenko '06; Ioffe, Fadin, Lipatov '10; Fadin, Kozlov, Reznichenko '15]

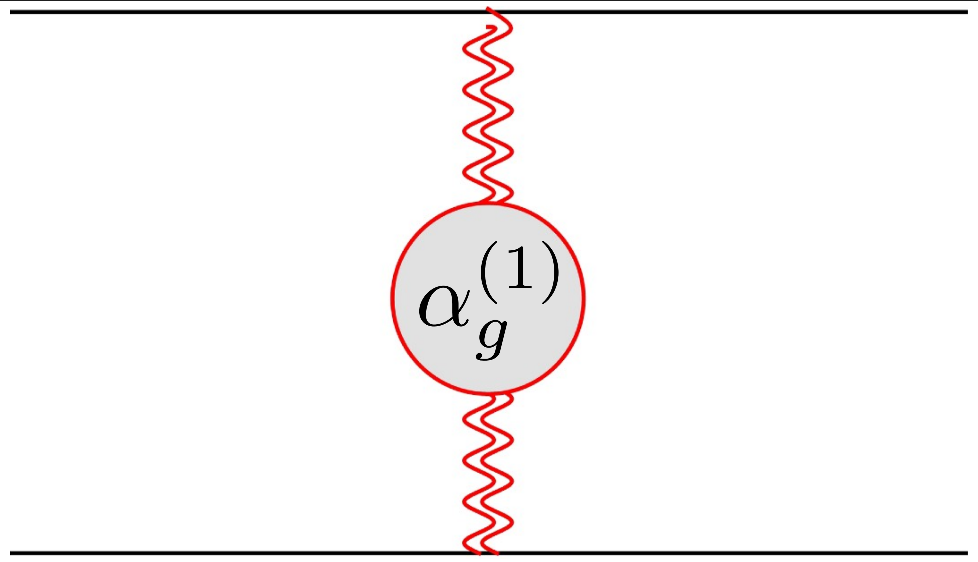
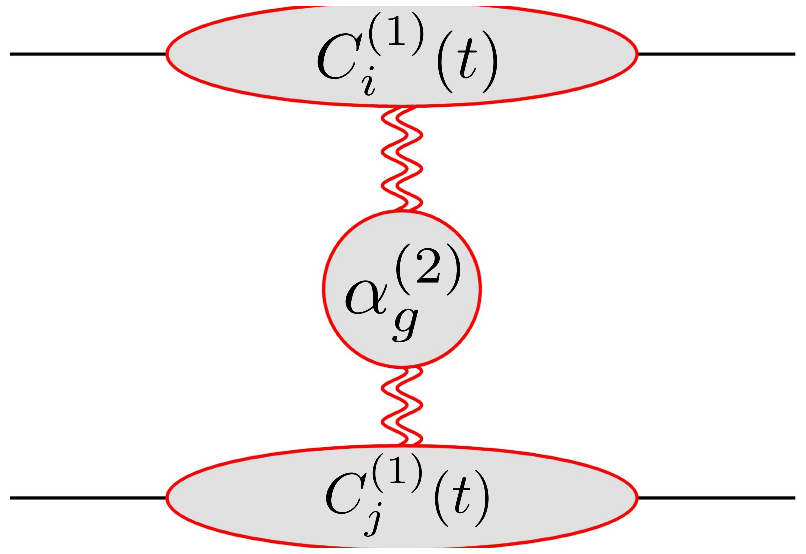
Describes the real part of the amplitude at LL and NLL

$$\mathcal{M} \sim C_i(t) C_j(t) e^{\alpha_g(t)\Delta y} \mathcal{M}_{\text{tree}}$$

- Universality as $\alpha_g(t)$ does not depend on external partons
- Parameters can be seen as Wilson coefficients
- Extracted from fixed order computations
- Infrared divergences of $\alpha_g(t)$ given by cusp anomalous dimension γ_K to two loops [Korchemskaya, Korchemsky '94, '96]

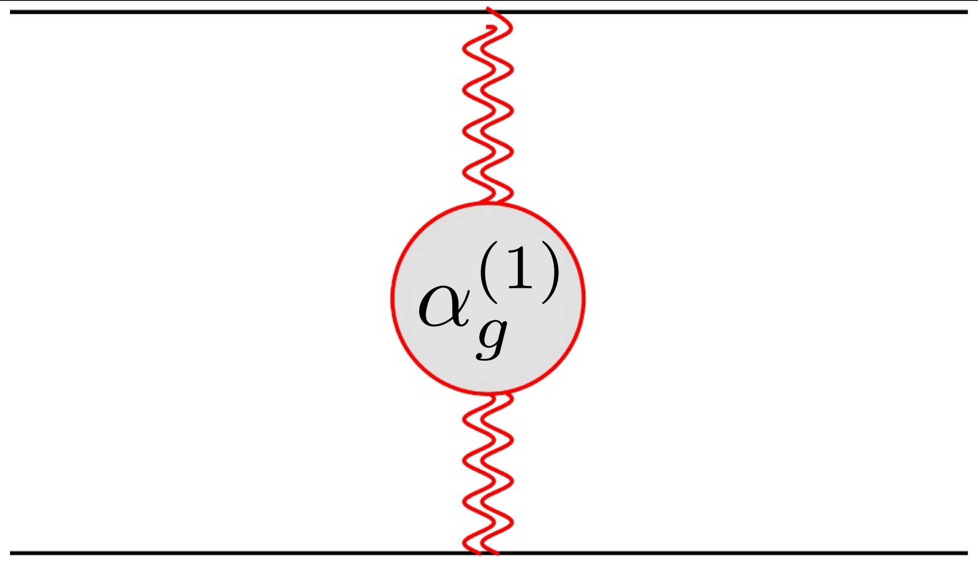
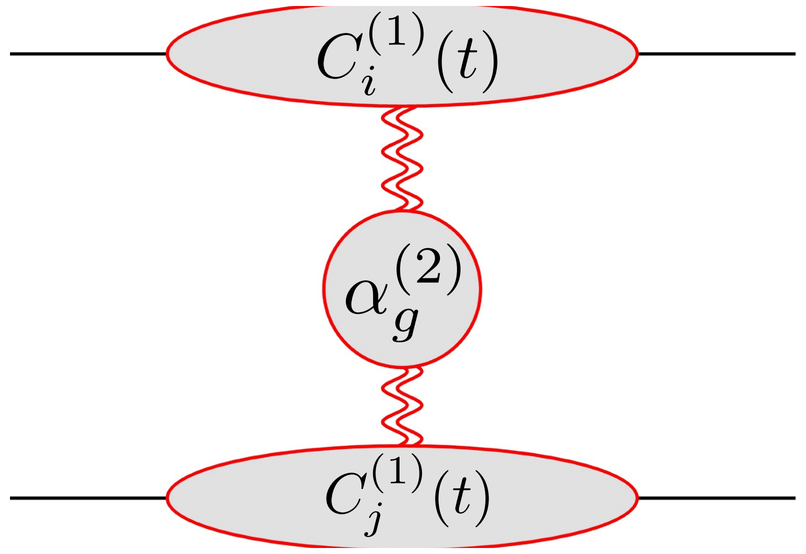
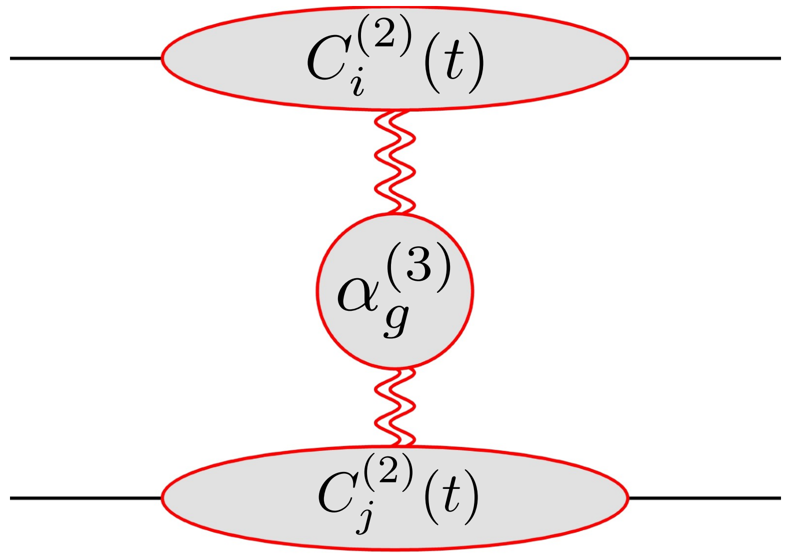
2 ➡ 2 Amplitudes

$$\mathcal{M} = C_i(t) C_j(t) e^{\alpha_g(t)\Delta y} \mathcal{M}_{\text{tree}}$$

LL $\alpha_s^n \log^n \left(\frac{s}{-t}\right)$	 one-loop Regge trajectory
NLL $\alpha_s^n \log^{n-1} \left(\frac{s}{-t}\right)$	 two-loop Regge trajectory one-loop impact factors
NNLL $\alpha_s^n \log^{n-2} \left(\frac{s}{-t}\right)$	

2 ➜ 2 Amplitudes

$$\mathcal{M} = C_i(t)C_j(t)e^{\alpha_g(t)\Delta y}\mathcal{M}_{\text{tree}}$$

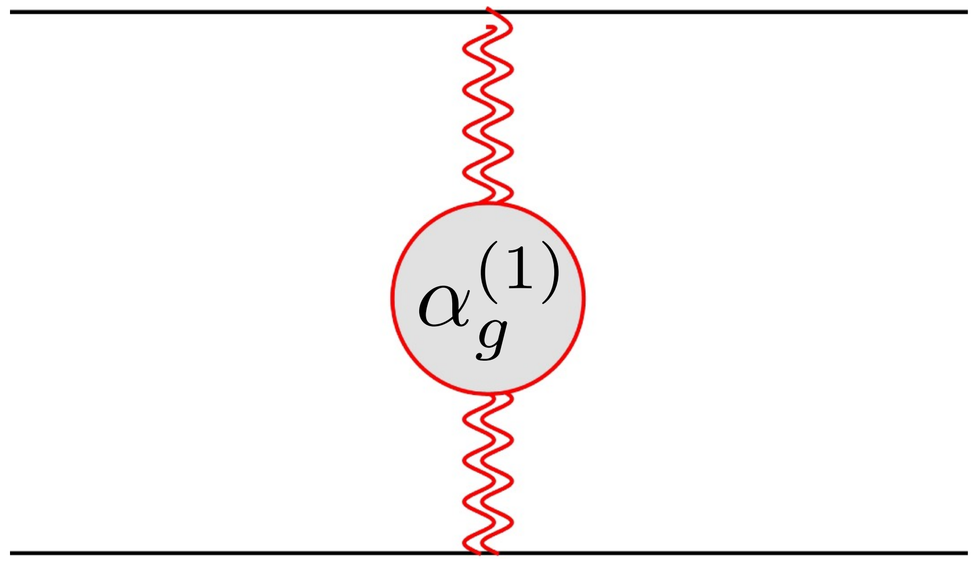
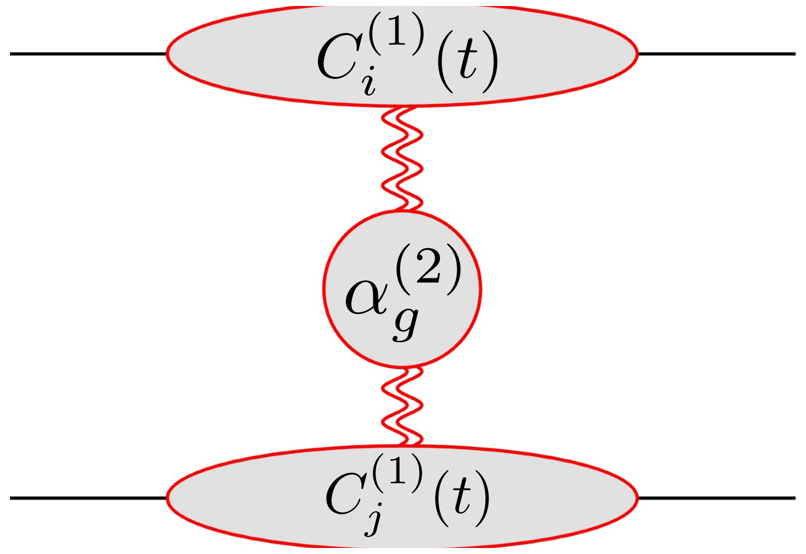
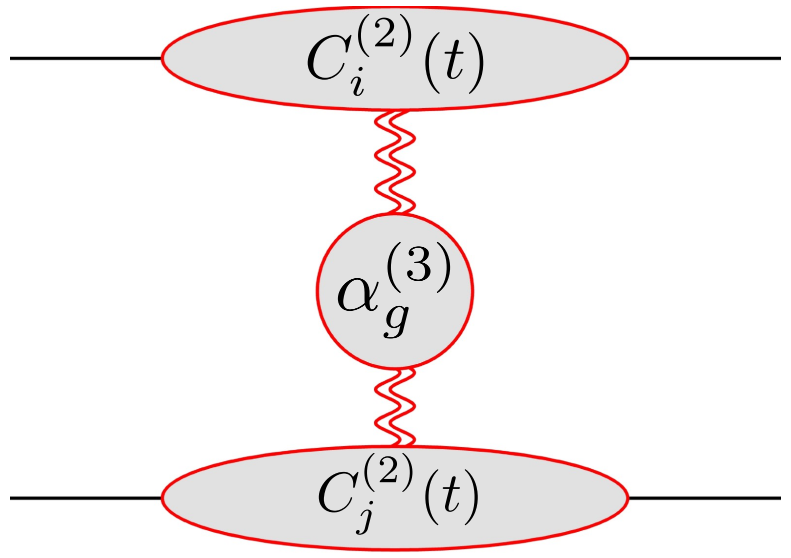
LL $\alpha_s^n \log^n\left(\frac{s}{-t}\right)$		one-loop Regge trajectory
NLL $\alpha_s^n \log^{n-1}\left(\frac{s}{-t}\right)$		two-loop Regge trajectory one-loop impact factors
NNLL $\alpha_s^n \log^{n-2}\left(\frac{s}{-t}\right)$		three-loop Regge trajectory two-loop impact factors

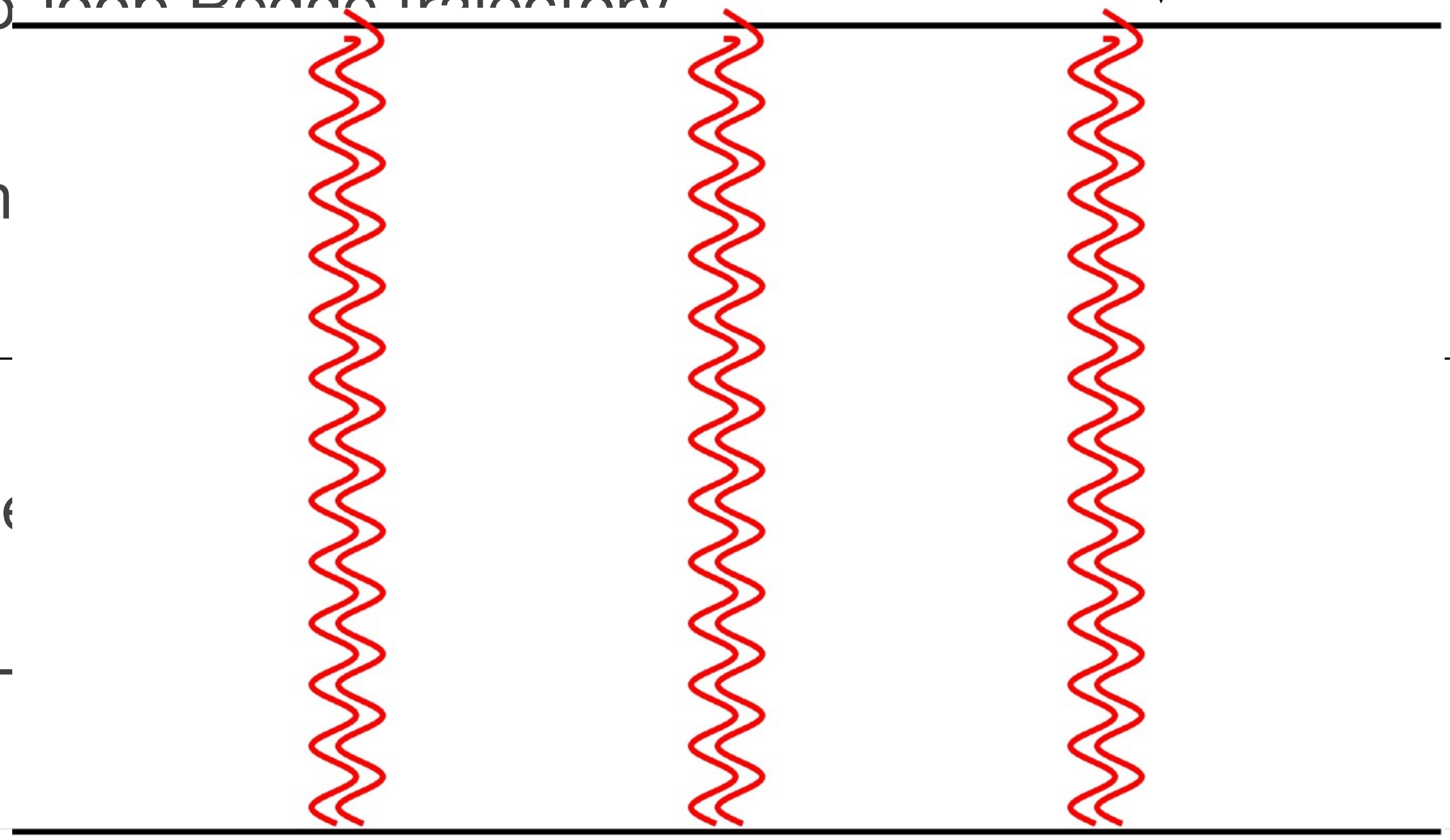
2 ➞ 2 Amplitudes

term from multiple *Reggeons*

$$\mathcal{M} = C_i(t)C_j(t)e^{\alpha_g(t)\Delta y}\mathcal{M}_{\text{tree}} + \text{MR}$$

Regge factorisation breaks
Long known
[Del Duca, Glover '01]
But until recently unknown
how to account for it

LL $\alpha_s^n \log^n\left(\frac{s}{-t}\right)$			
NLL $\alpha_s^n \log^{n-1}\left(\frac{s}{-t}\right)$		two loop Regge trajectory on	
NNLL $\alpha_s^n \log^{n-2}\left(\frac{s}{-t}\right)$		three two-	



2 ➞ 2 Amplitudes

term from multiple *Reggeons*

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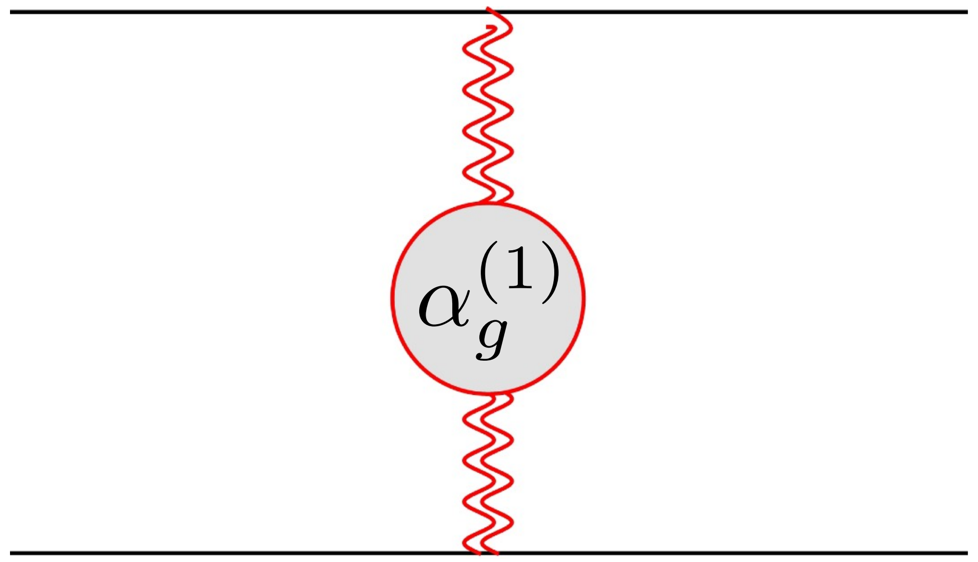
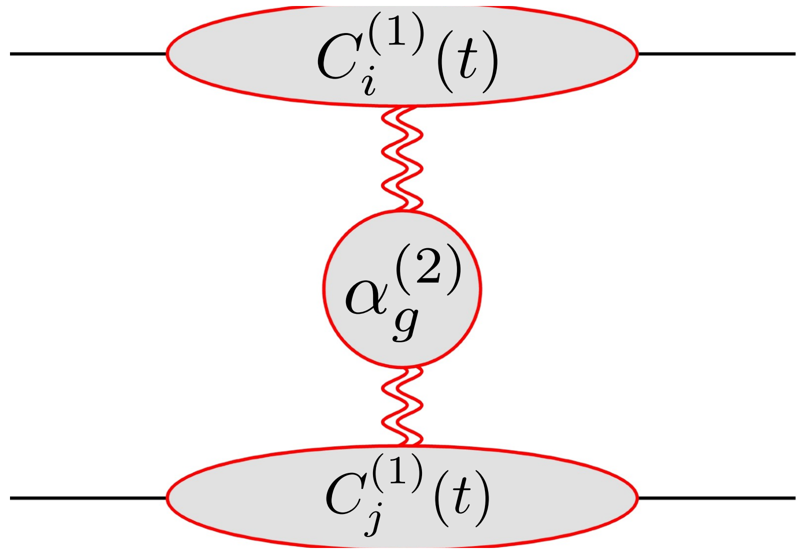
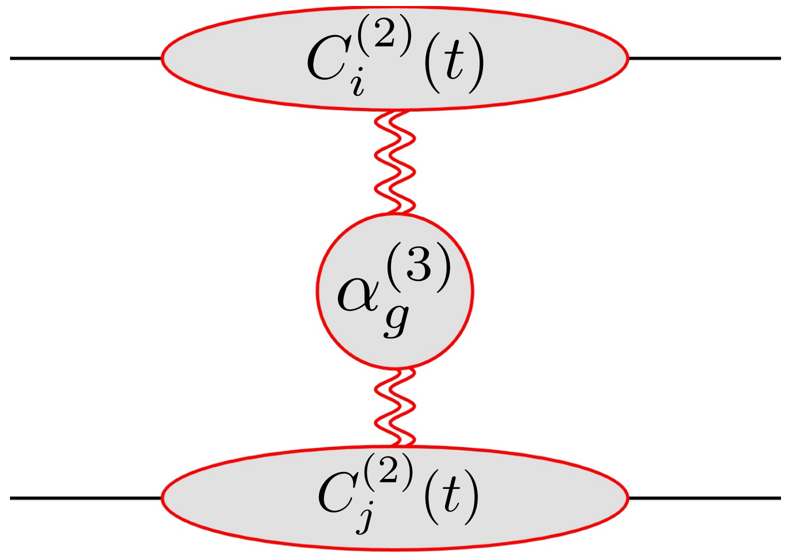
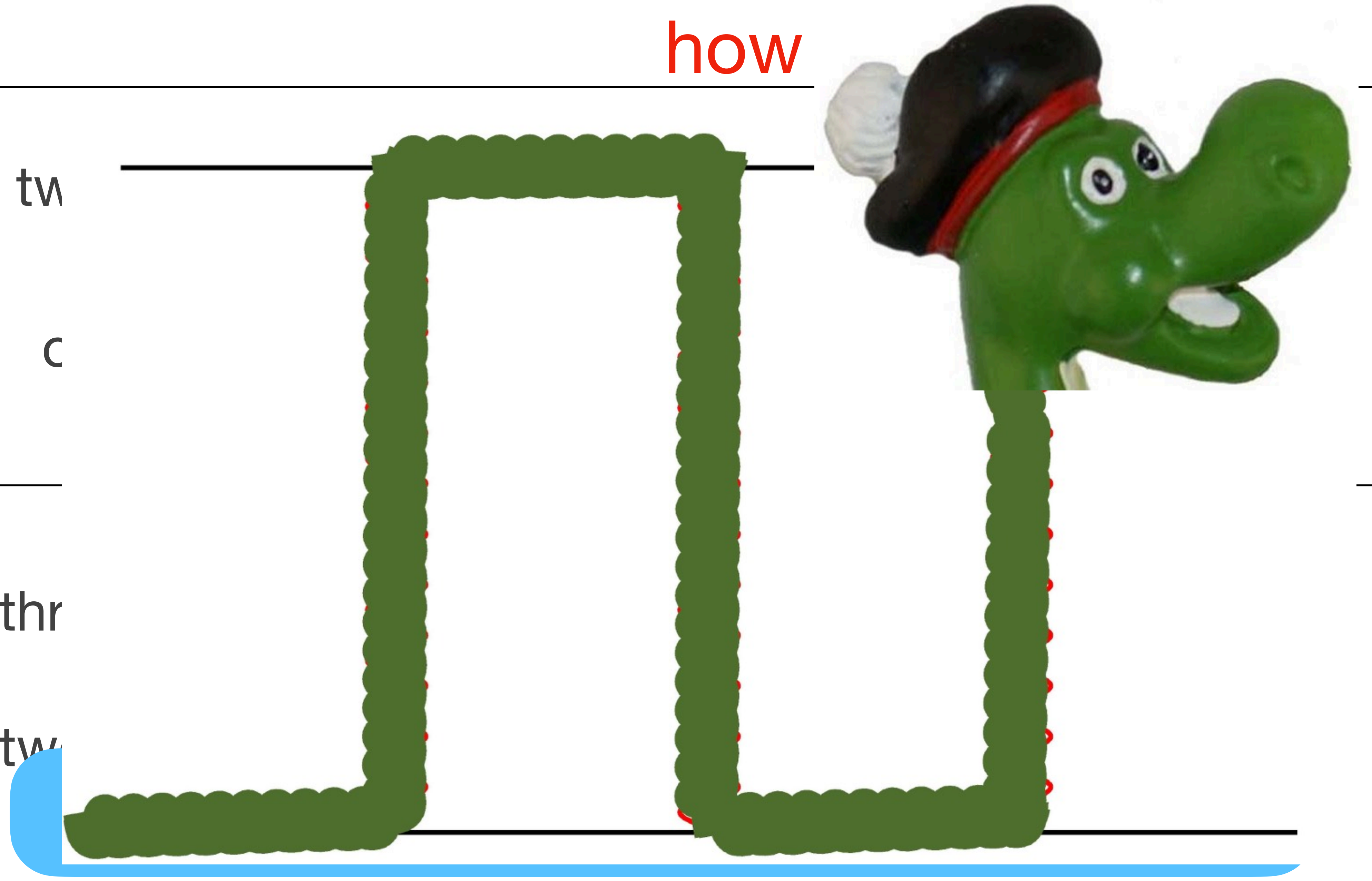
LL	$\alpha_s^n \log^n\left(\frac{s}{-t}\right)$		
NLL	$\alpha_s^n \log^{n-1}\left(\frac{s}{-t}\right)$		tw c
NNLL	$\alpha_s^n \log^{n-2}\left(\frac{s}{-t}\right)$		thr tw



2 ➔ 2 Amplitudes

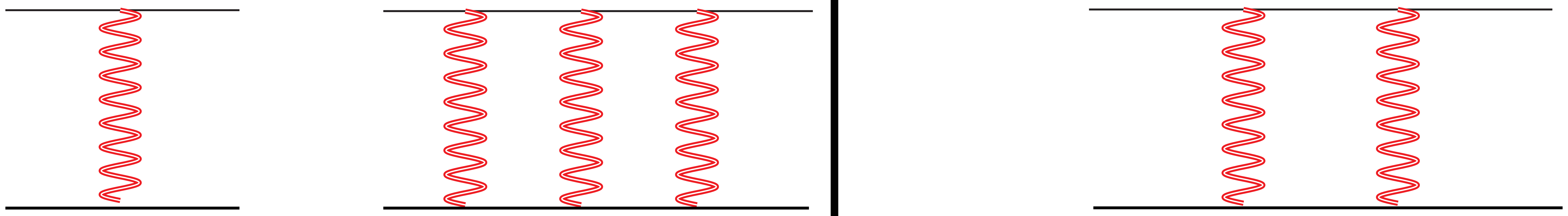
Colour and the number of Reggeons

SU(N) matrices in a generic representation

Use a basis that is orthogonal and one of the elements is the tree-level antisymmetric octet

$$\mathbf{T}_i^a \, \mathbf{T}_j^a$$

$$(\mathbf{8} \otimes \mathbf{8})_{gg} \qquad \mathbf{8}_a \oplus (\mathbf{10} \oplus \overline{\mathbf{10}}) \qquad \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_s \oplus \mathbf{27}$$



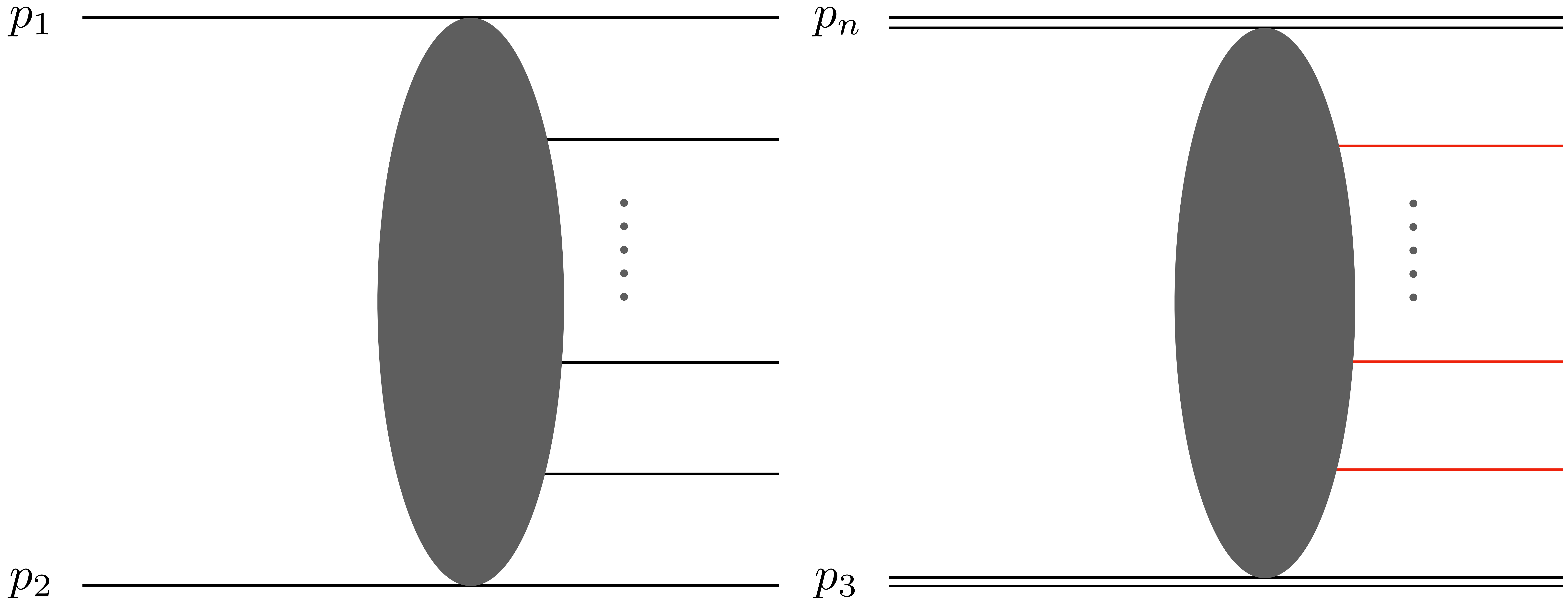
Odd number

Even number

Contributions to the octet channel from multiple Reggeons

How to compute amplitudes in MRK?

Formulating highly energetic particles as Wilson lines



$$a_{i,\lambda_3,a_3}(p_3)a^\dagger_{i,\lambda_2,a_2}(p_2) \sim \delta_{\lambda_2\lambda_3}U_i(p)_{a_2a_3}$$

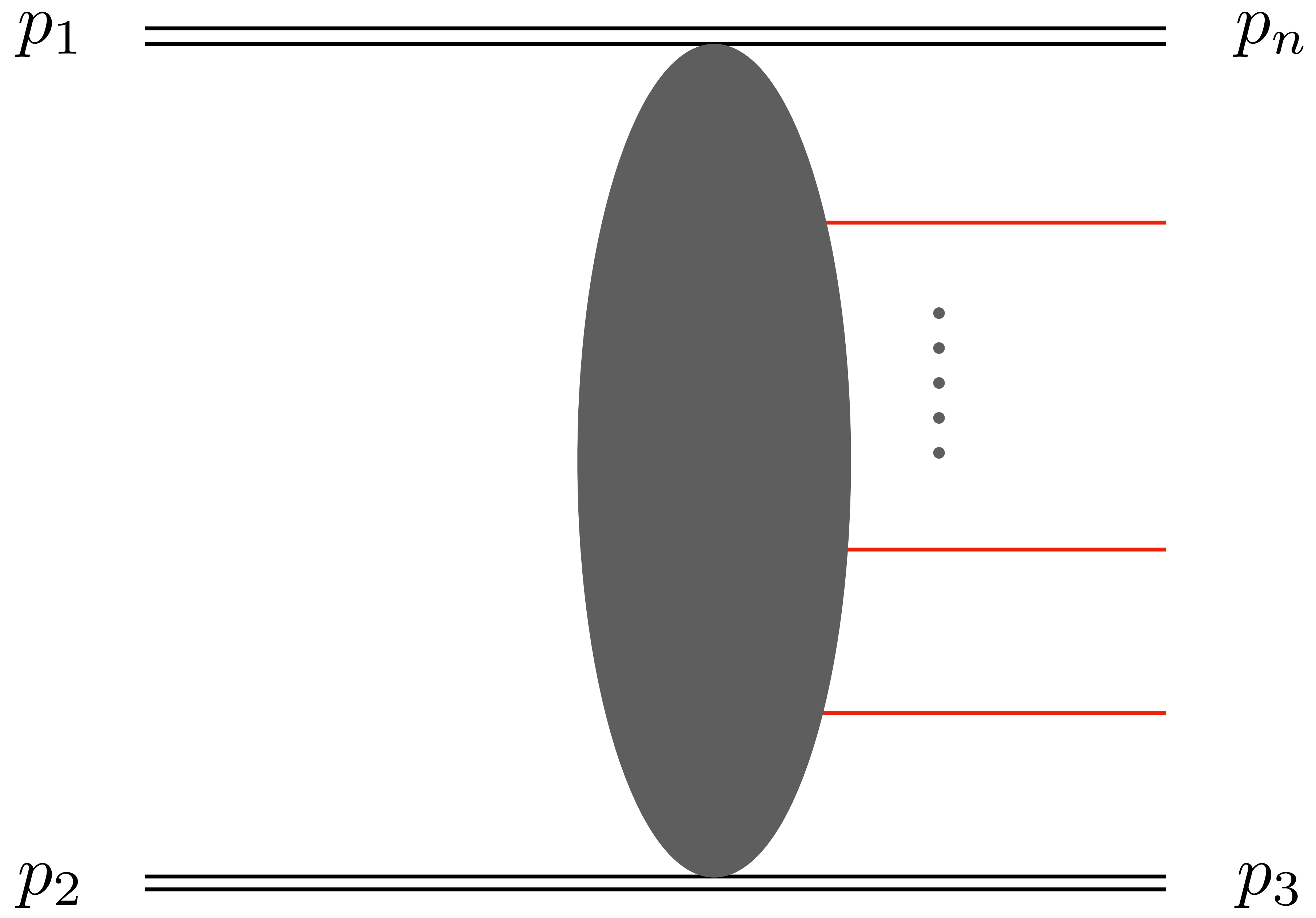
creation/annihilation operators

helicity is conserved

colour indices

How to compute amplitudes in MRK?

Formulating highly energetic particles as Wilson lines



$$\mathcal{M} \sim U_j(p_n)a(p_{n-1})\ldots a(p_4)U_i(p_3)$$

exist at different rapidities

evolve using Balitsky-JIMLWK equation

$$\frac{d}{dy}U(p_i) = H\,U(p_i)$$

large rapidity gaps

$$\mathcal{M} \sim U_j(\mathbf{p}_n)\ldots e^{H\Delta y_4}a(\mathbf{p}_4)e^{H\Delta y_3}U_i(\mathbf{p}_3)$$

"integrating out the rapidity degrees of freedom"

How to compute amplitudes in MRK?

The expansion in Reggeons

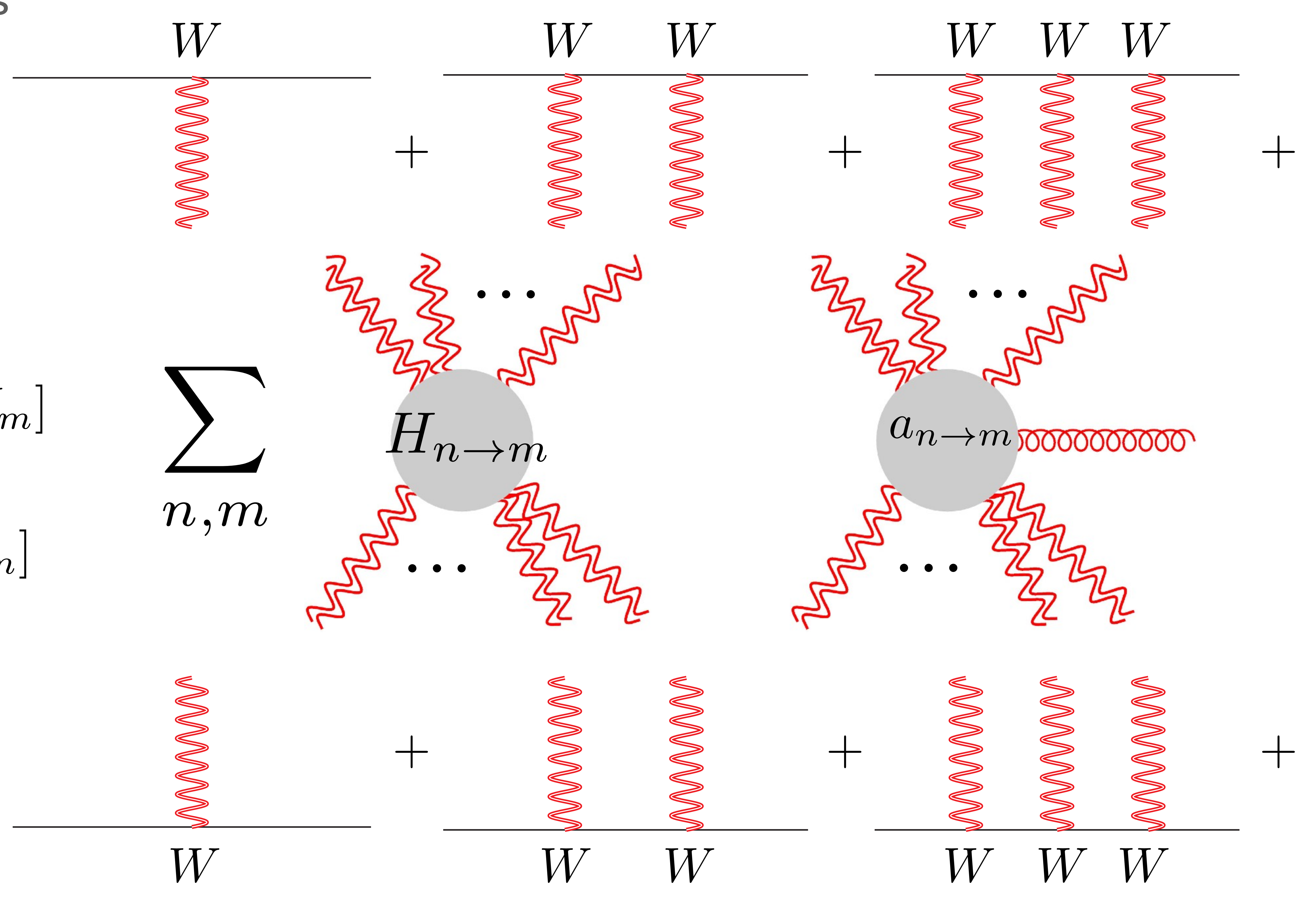
$$\mathcal{M} \sim U_j(\mathbf{p}_n) \dots e^{H\Delta y_4} a(\mathbf{p}_4) e^{H\Delta y_3} U_i(\mathbf{p}_3)$$

colour matrices

interested in weak coupling $\bar{U}(\mathbf{p}) = e^{ig_s W^a(\mathbf{p}) \mathbf{T}^a}$

[Caron-Huot '13]

"Reggeon field"



two sources of non-linearities

Hamiltonian $H\left[W_1 \dots W_n\right] \sim \sum_{m=1}^{\infty} H_{n\to m}\left[W_1 \dots W_m\right]$

OPE $\left[W_1 \dots W_n\right] a \sim \sum_{m=1}^{\infty} a_{n\to m}\left[W_1 \dots W_m\right]$

Expansion becomes a mess
but a *controllable* mess

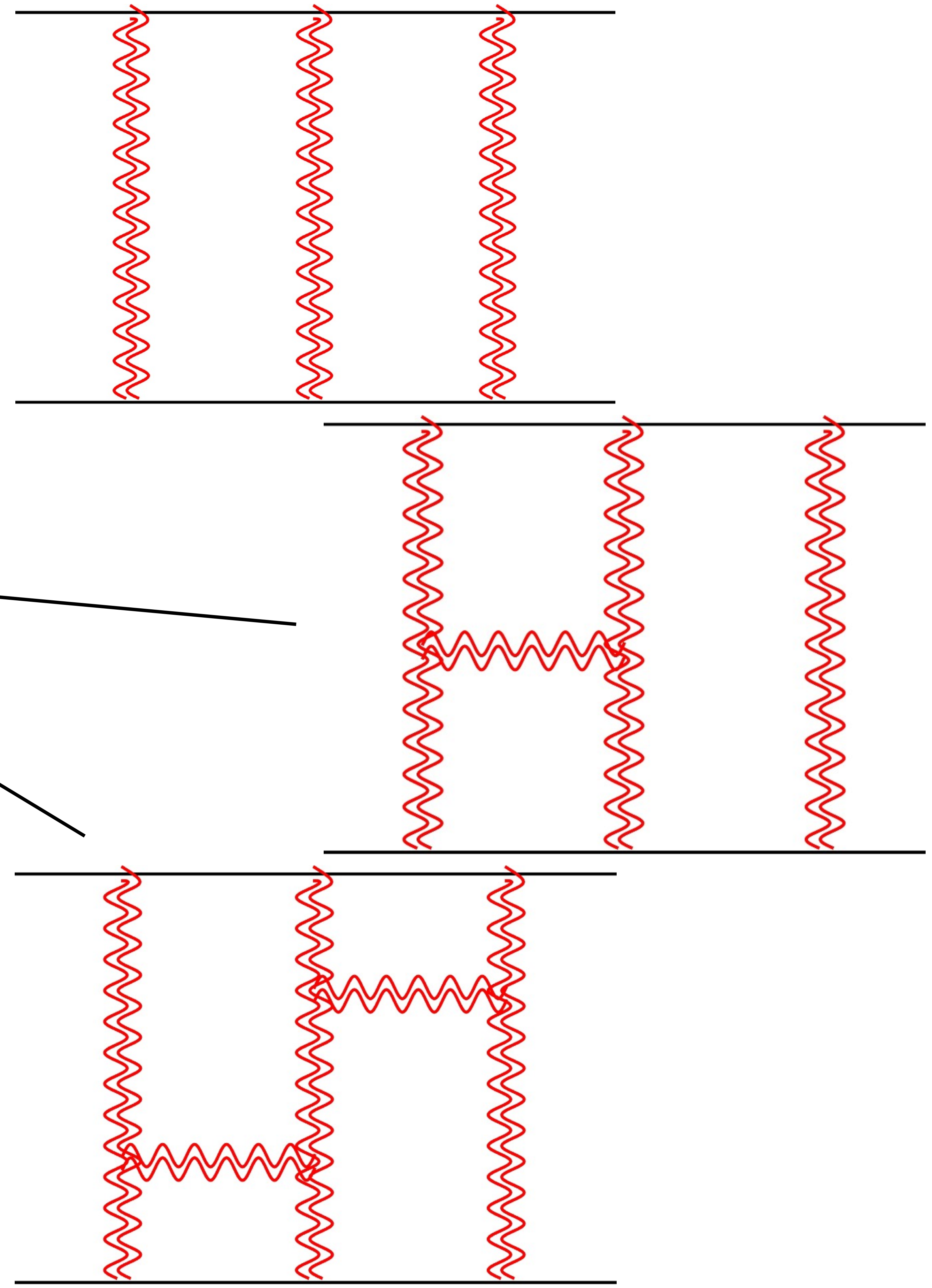
Multiple Reggeon Computations

For 2 → 2 amplitudes

- Integrals are single-scale as there is only one transverse momenta
- Computed up to four loops

[Caron-Huot, Gardi, Vernazza ’17; Falcioni, Gardi, **CM**, Vernazza ’20]

$$\mathcal{M} = C_i(t)C_j(t)e^{\alpha_g(t)\Delta y}\mathcal{M}_{\text{tree}} + \text{MR}$$



- MR terms proportional to the tree-level depend on the external partons
- If we put MR into parameters then this breaks Regge factorisation
- Ambiguous how to define NNLL impact factors and Regge trajectory

2 ➡ 2 Amplitudes

- In the planar limit all multiple Reggeon (MR) terms are independent on the scattered partons
  [Caron-Huot, Gardi, Vernazza ’17; Falcioni, Gardi, **CM**, Vernazza ’20]
- Shifting these into new parameters [Falcioni, Gardi, Maher, **CM**, Vernazza ’21]

$$\mathcal{M} = \left(C_i(t)C_j(t)e^{\alpha_g(t)\Delta y} + \mathrm{MR}|_{\text{planar}}\right)\mathcal{M}_{\text{tree}} + \mathrm{MR}|_{\text{non-planar}}$$
$$= \tilde{C}_i(t)\tilde{C}_j(t)e^{\tilde{\alpha}_g(t)\Delta y}\mathcal{M}_{\text{tree}} + \mathrm{MR}|_{\text{non-planar}}$$

2 ➔ 2 Amplitudes

- In the planar limit all multiple Reggeon (MR) terms are independent on the scattered partons
  [Caron-Huot, Gardi, Vernazza ’17; Falcioni, Gardi, **CM**, Vernazza ’20]
- Shifting these into new parameters [Falcioni, Gardi, Maher, **CM**, Vernazza ’21]

$$\mathcal{M} = \left(C_i(t)C_j(t)e^{\alpha_g(t)\Delta y} + \mathrm{MR}|_{\mathrm{planar}}\right)\mathcal{M}_{\mathrm{tree}} + \mathrm{MR}|_{\text{non-planar}}$$

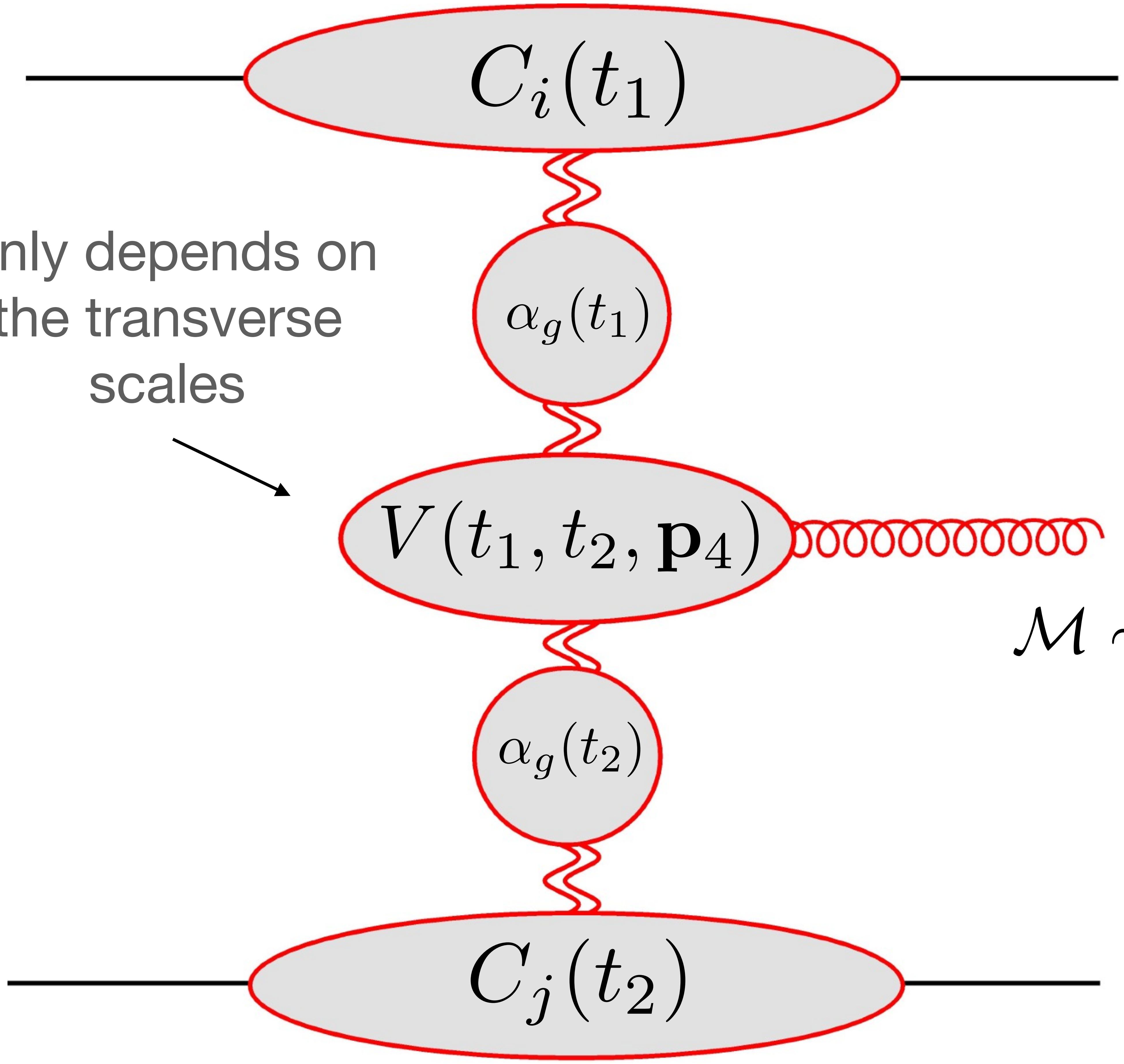
$$= \underbrace{\tilde{C}_i(t)\tilde{C}_j(t)e^{\tilde{\alpha}_g(t)\Delta y}\mathcal{M}_{\mathrm{tree}}}_{\text{Regge pole}} + \underbrace{\mathrm{MR}|_{\text{non-planar}}}_{\text{Regge cut}}$$

- Full four loop MR is non-planar as it must be: nae free parameters
- Gives a perturbative description of the Regge pole and Regge cut separation
- From results of amplitudes we can find explicit two-loop impact factors and three-loop trajectories
  [Falcioni, Gardi, Maher, **CM**, Vernazza ’21; Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi ’21]
- Infrared poles of three-loop $\tilde{\alpha}_g(t)$ given by γ_K despite the presence of the cut!

A similar story for 2 ➡ 3 amplitudes…

2 ➜ 3 Amplitudes

The Lipatov vertex



Only depends on the transverse scales

$$\mathcal{M} \sim U_j(\mathbf{p}_5) e^{H \Delta y_5} a(\mathbf{p}_4) e^{H \Delta y_4} U_i(\mathbf{p}_3)$$

Along with the same parameters extracted in 2 ➜ 2 we have a new parameter:
the Lipatov production vertex

[Lipatov '76; Fadin, Lipatov '93; Fadin, Fiore, Kotsky '96; Del Duca, Schmidt '99]

$$\mathcal{M} \sim C_i(t_1) e^{\alpha_g(t_1)\Delta y_5} V(t_1, t_2, \mathbf{p}_4) e^{\alpha_g(t_2)\Delta y_4} C_j(t_2) \mathcal{M}_{\text{tree}}$$

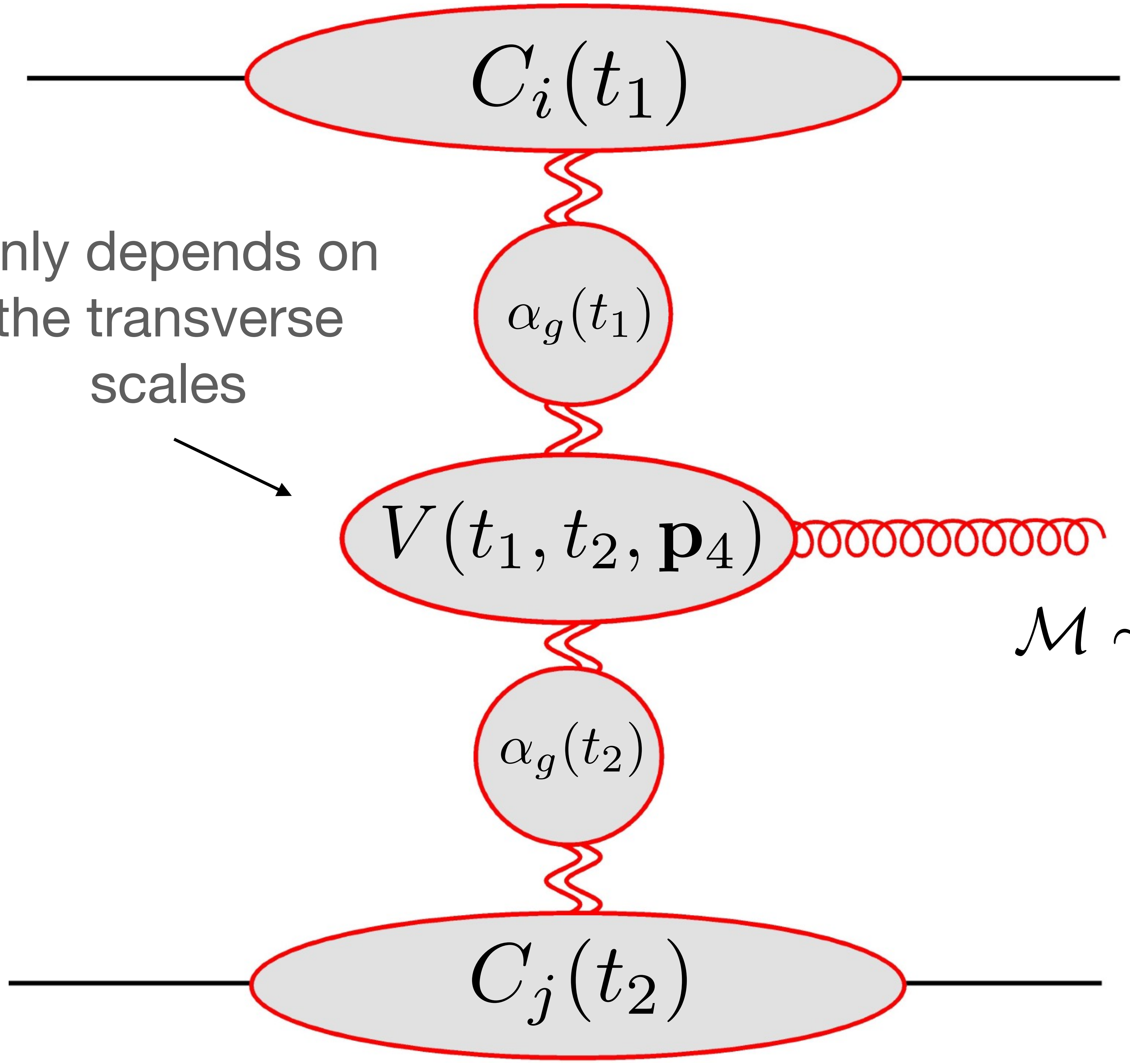
recent one-loop calculation of vertex to $\mathcal{O}(\epsilon^2)$

[Fadin, Fucilla, Papa '23]

2 ➞ 3 Amplitudes

The Lipatov vertex

Only depends on the transverse scales



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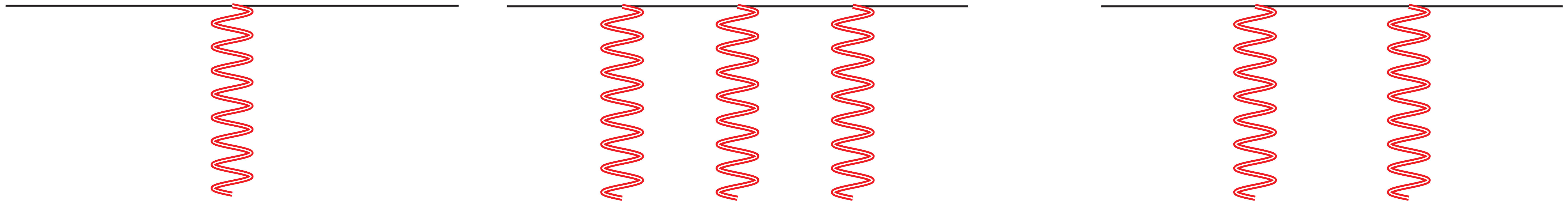
$$+ \text{multiple Reggeons at NNLL}$$

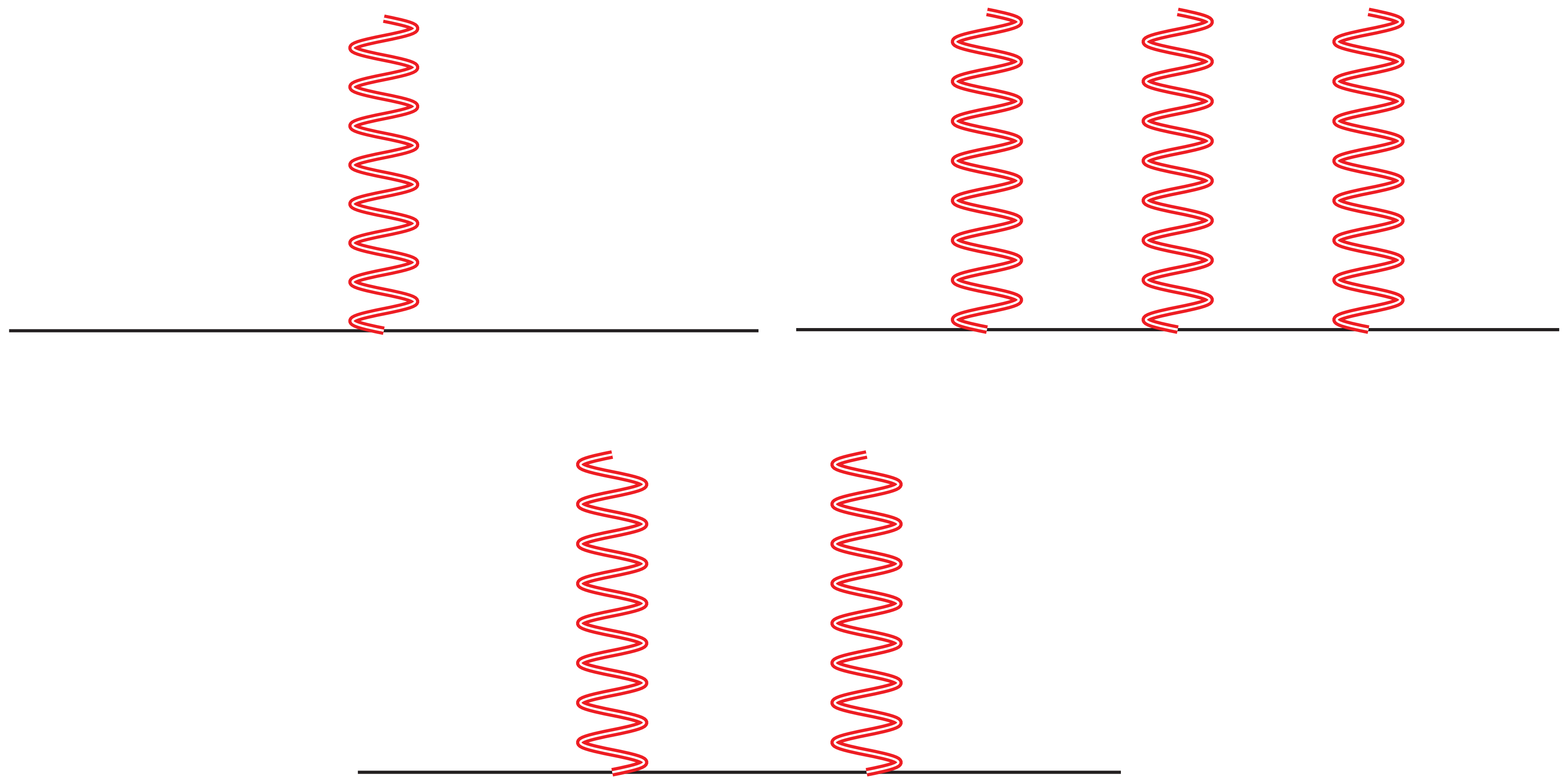
2 ➞ 3 Amplitudes

Colour and the number of Reggeons

- Due to the production vertex need to separate number of Reggeons between top and bottom leg
- Extra colour index for produced gluon leads to more representations

$$(\mathbf{8}\otimes\mathbf{8})_{gg}$$





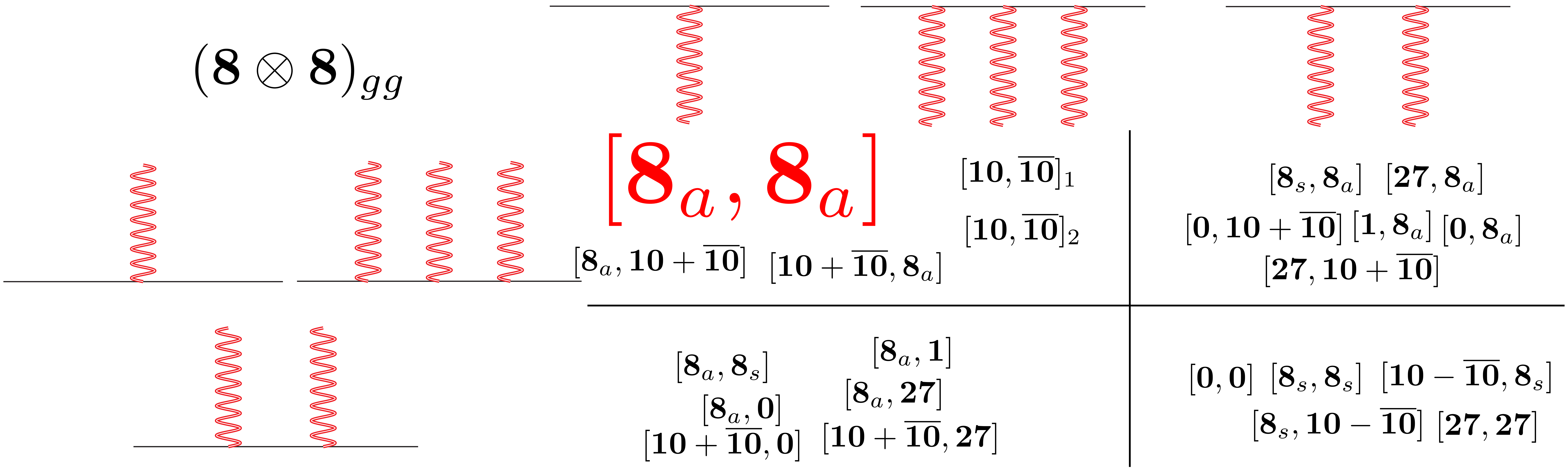
$[\mathbf{8}_a,\mathbf{8}_a]$ $[\mathbf{10},\overline{\mathbf{10}}]_1$ $[\mathbf{8}_a,\mathbf{10}+\overline{\mathbf{10}}]$ $[\mathbf{10},\overline{\mathbf{10}}]_2$ $[\mathbf{10}+\overline{\mathbf{10}},\mathbf{8}_a]$	$[\mathbf{8}_s,\mathbf{8}_a]$ $[\mathbf{27},\mathbf{8}_a]$ $[\mathbf{0},\mathbf{10}+\overline{\mathbf{10}}]$ $[\mathbf{1},\mathbf{8}_a]$ $[\mathbf{0},\mathbf{8}_a]$ $[\mathbf{27},\mathbf{10}+\overline{\mathbf{10}}]$
$[\mathbf{8}_a,\mathbf{8}_s]$ $[\mathbf{8}_a,\mathbf{1}]$ $[\mathbf{8}_a,\mathbf{0}]$ $[\mathbf{8}_a,\mathbf{27}]$ $[\mathbf{10}+\overline{\mathbf{10}},\mathbf{0}]$ $[\mathbf{10}+\overline{\mathbf{10}},\mathbf{27}]$	$[\mathbf{0},\mathbf{0}]$ $[\mathbf{8}_s,\mathbf{8}_s]$ $[\mathbf{10}-\overline{\mathbf{10}},\mathbf{8}_s]$ $[\mathbf{8}_s,\mathbf{10}-\overline{\mathbf{10}}]$ $[\mathbf{27},\mathbf{27}]$

Again similar for quark-quark and quark-gluon scattering

2→3 Amplitudes

Colour and the number of Reggeons

- Due to the production vertex need to separate number of Reggeons between top and bottom leg
- Extra colour index for produced gluon leads to more representations



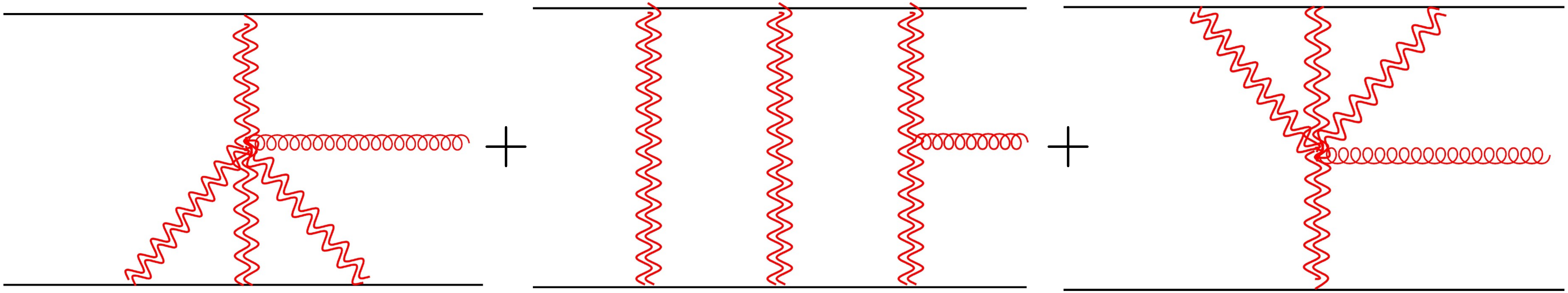
Contributions to multiple Reggeons in the (octet-octet) channel

$2 \rightarrow 3$ Amplitudes

Results of MR

[Abreu, Gardi, Falcioni, **CM**, Vernazza, in preparation]

Integrals are three-mass triangles $|\mathbf{p}_3|^2 = z\,\bar{z}\,|\mathbf{p}_4|^2 \quad |\mathbf{p}_5|^2 = (1-z)\,(1-\bar{z})\,|\mathbf{p}_4|^2$



[Caron-Huot, Chicherin, Henn, Zhang, Zoia '20]

$$D_2(z,\bar{z}) = \mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}\log\frac{1-z}{1-\bar{z}}\log z\bar{z}$$

Contribution to the octet-octet

$$C_{ij}\,\pi^2\left(\frac{1}{\epsilon^2} - \frac{1}{2\epsilon}\log|z|^2|1-z|^2 + 3\,D_2(z,\bar{z}) - \zeta_2 + \frac{5}{4}\log^2|z|^2 + \frac{5}{4}\log^2|1-z|^2 - \frac{1}{2}\log|z|^2\log|1-z|^2\right)$$

Colour factor depends on external partons

$2 \rightarrow 3$ Amplitudes

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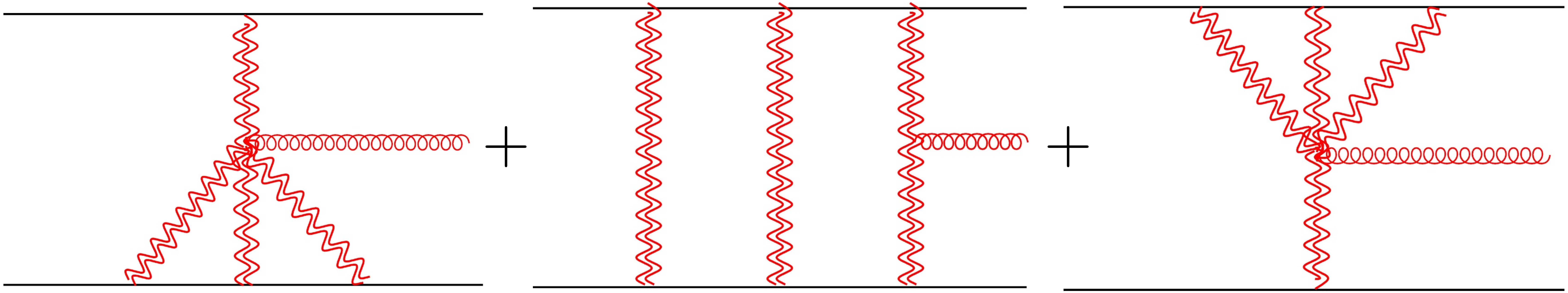

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Colour factor depends on external partons

However, in the planar limit it is independent $\quad C_{ij} \to \frac{N_c^2}{72}$

2→3 Amplitudes

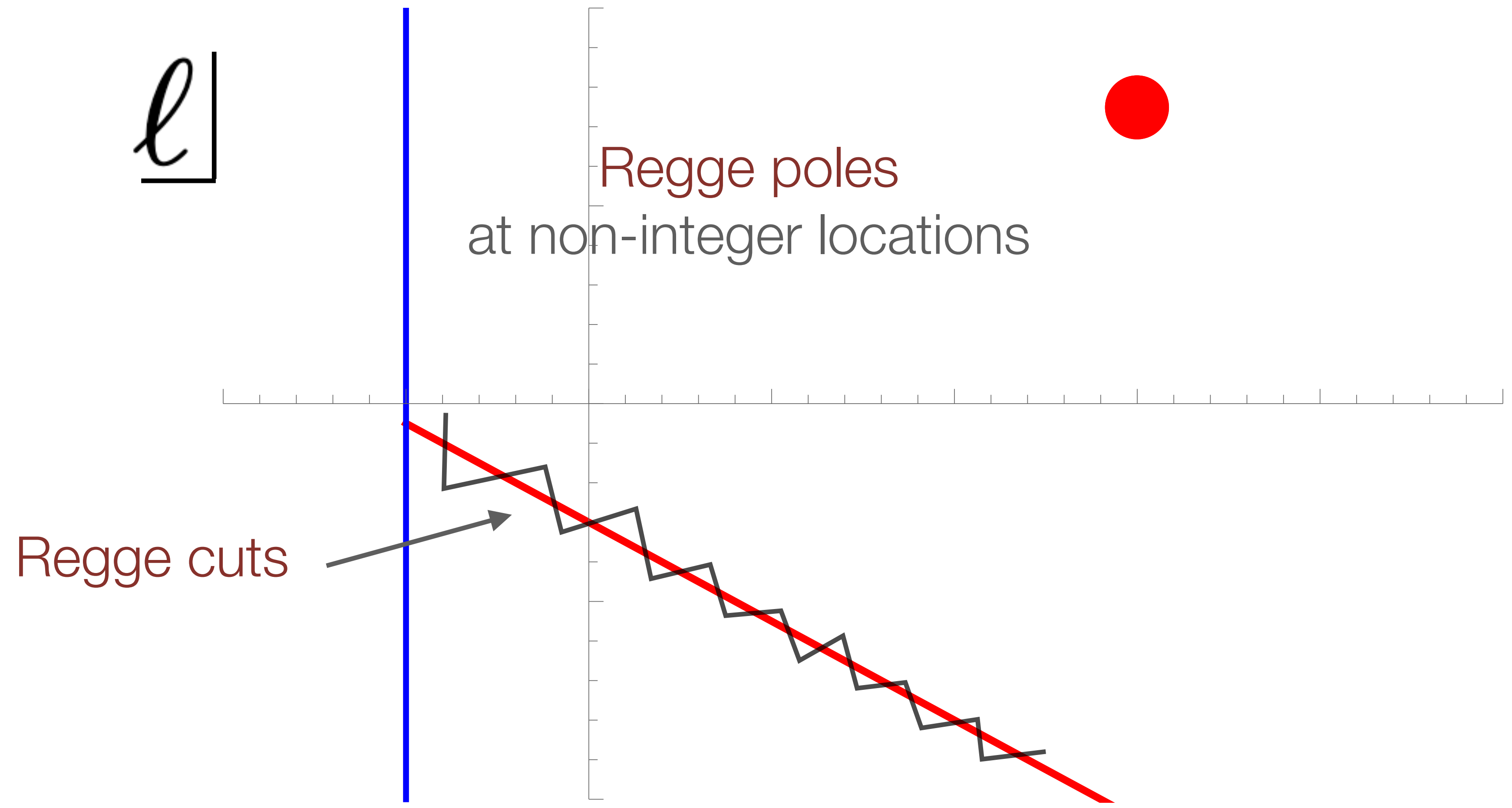
The two-loop Lipatov vertex

- As before we can define a new Lipatov vertex to absorb these planar terms

$$\mathcal{M} \sim \tilde{C}_i(t_1)e^{\tilde{\alpha}_g(t_1)\Delta y_5}\tilde{V}(t_1,t_2,\mathbf{p}_4)e^{\tilde{\alpha}_g(t_2)\Delta y_4}\tilde{C}_j(t_2)\mathcal{M}_{\text{tree}} + \mathcal{M}_{\text{MR}}\big|_{\text{non-planar}}$$

$$\mathcal{M} = \mathcal{M}_{\text{pole}} + \mathcal{M}_{\text{cut}}$$

[Caron-Huot, Chicherin, Henn, Zhang, Zoia '20]

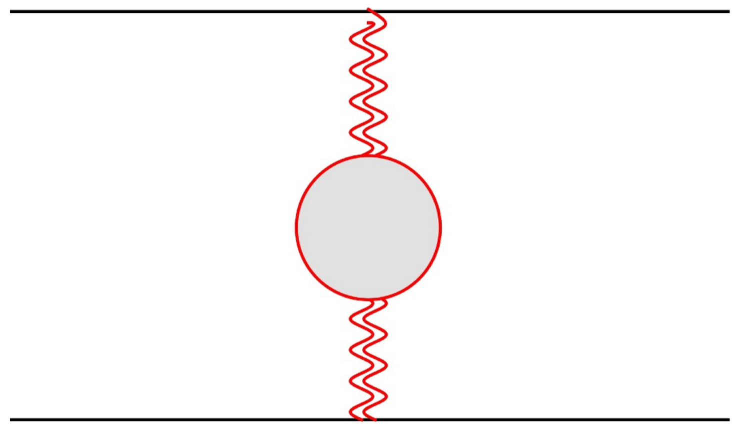
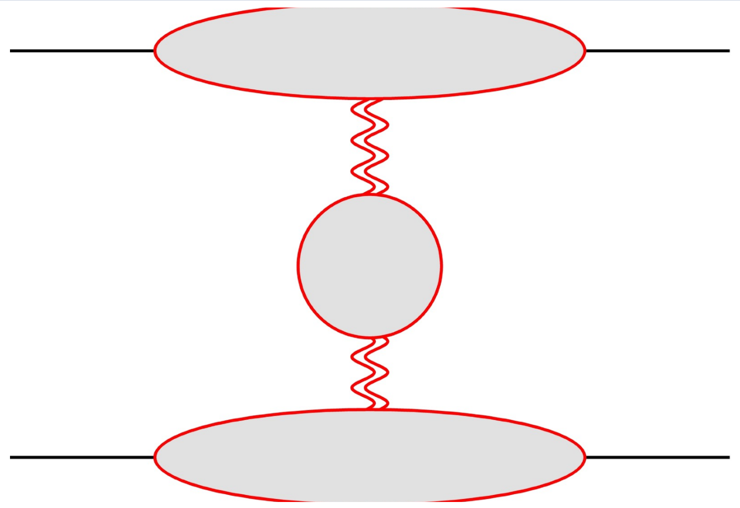
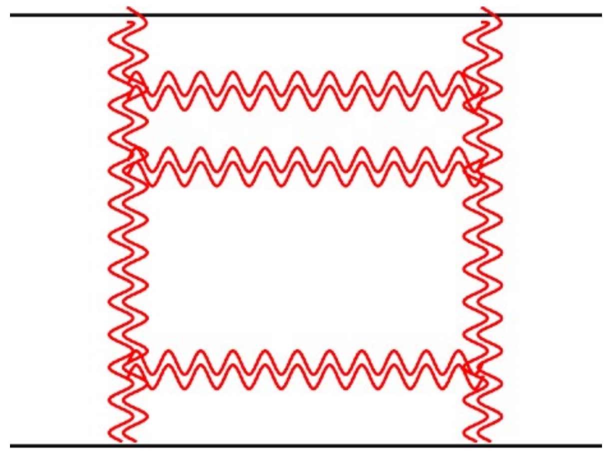
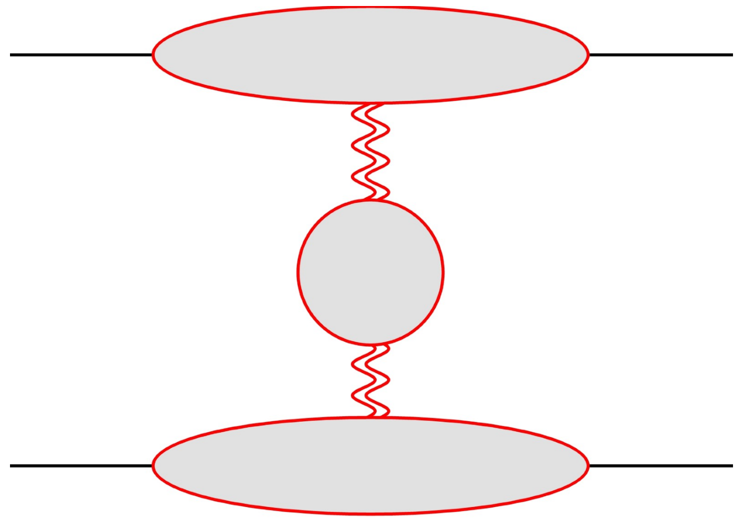
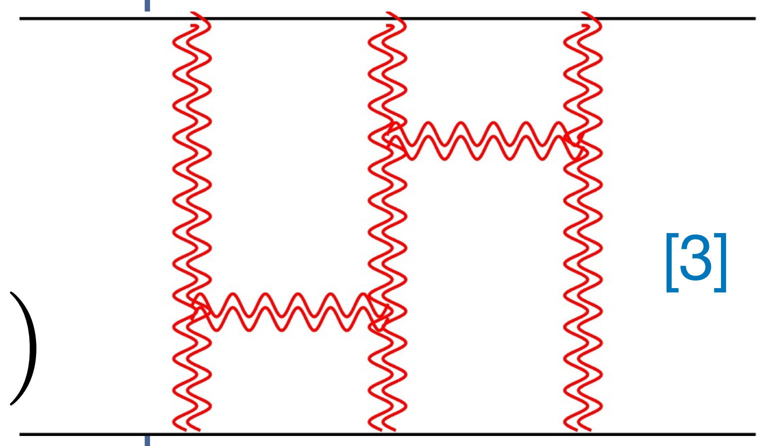


- Using known $\mathcal{N}=4$ super Yang-Mills amplitudes we can determine the two-loop vertex

[Abreu, Gardi, Falcioni, **CM**, Vernazza, in preparation]

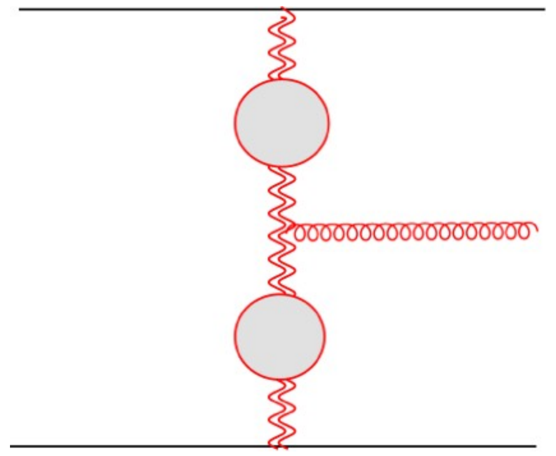
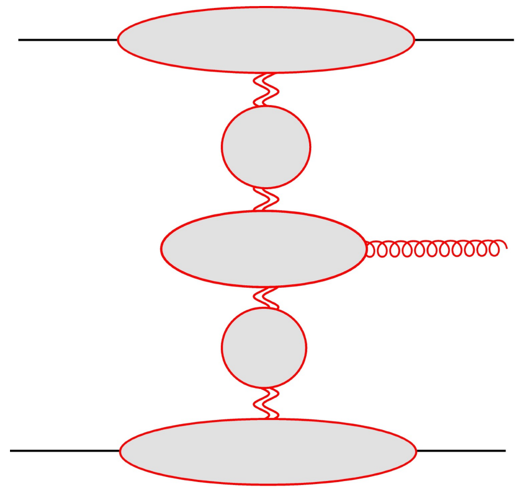
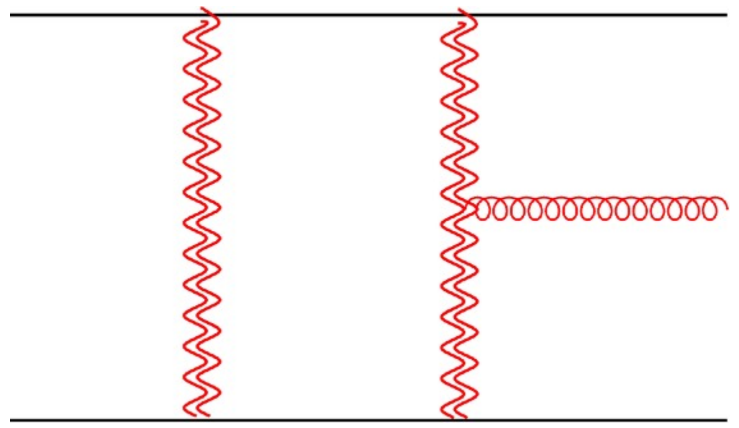
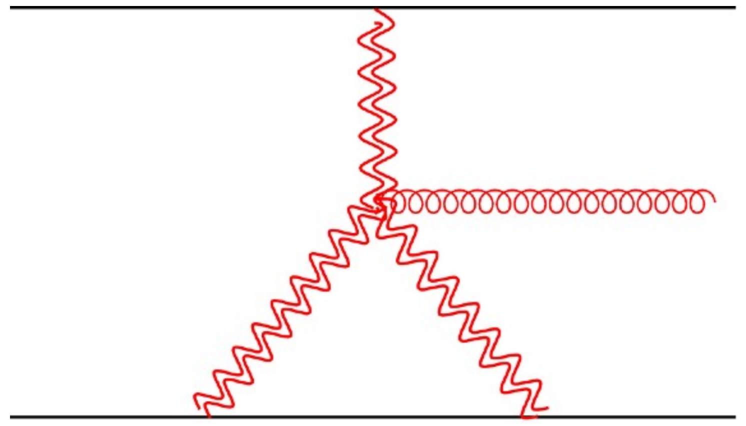
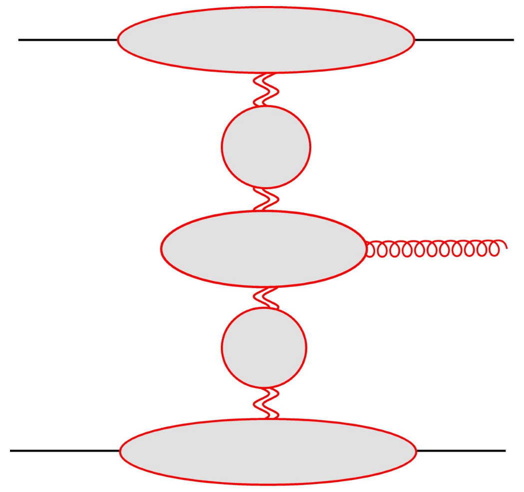
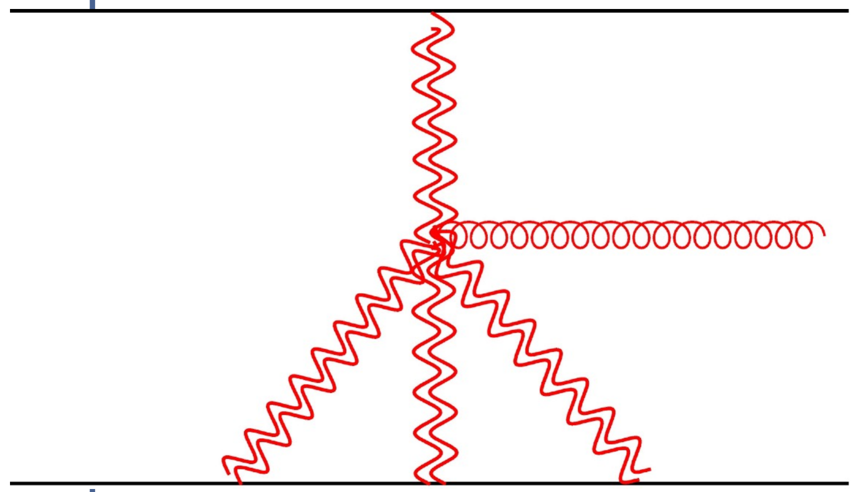
Perhaps soon in QCD! [see Giulio's talk]

$2 \rightarrow 2$ Amplitudes in MRK

		Real			Imaginary
		8_a		$10 \oplus \overline{10}$	Even $0 \oplus 1 \oplus 8_s \oplus 27$
LL	$\alpha_s^n \log^n\left(\frac{s}{-t}\right)$		$\alpha_g^{(1)}$	✗	✗
NLL	$\alpha_s^n \log^{n-1}\left(\frac{s}{-t}\right)$		$\alpha_g^{(2)}$ $C_i^{(1)}(t)$	✗	 infrared divergences resummed [1] finite terms to arbitrary order [2] resummation unknown
NNLL	$\alpha_s^n \log^{n-2}\left(\frac{s}{-t}\right)$		$\alpha_g^{(3)}$ $C_i^{(2)}(t)$	 [3]	?

[1] [Caron-Huot, Gardi, Reichel, Vernazza '17] [2] [Caron-Huot, Gardi, Reichel, Vernazza '20] [3] [Caron-Huot, Gardi, Vernazza '17; Falcioni, Gardi, **CM**, Vernazza '20]

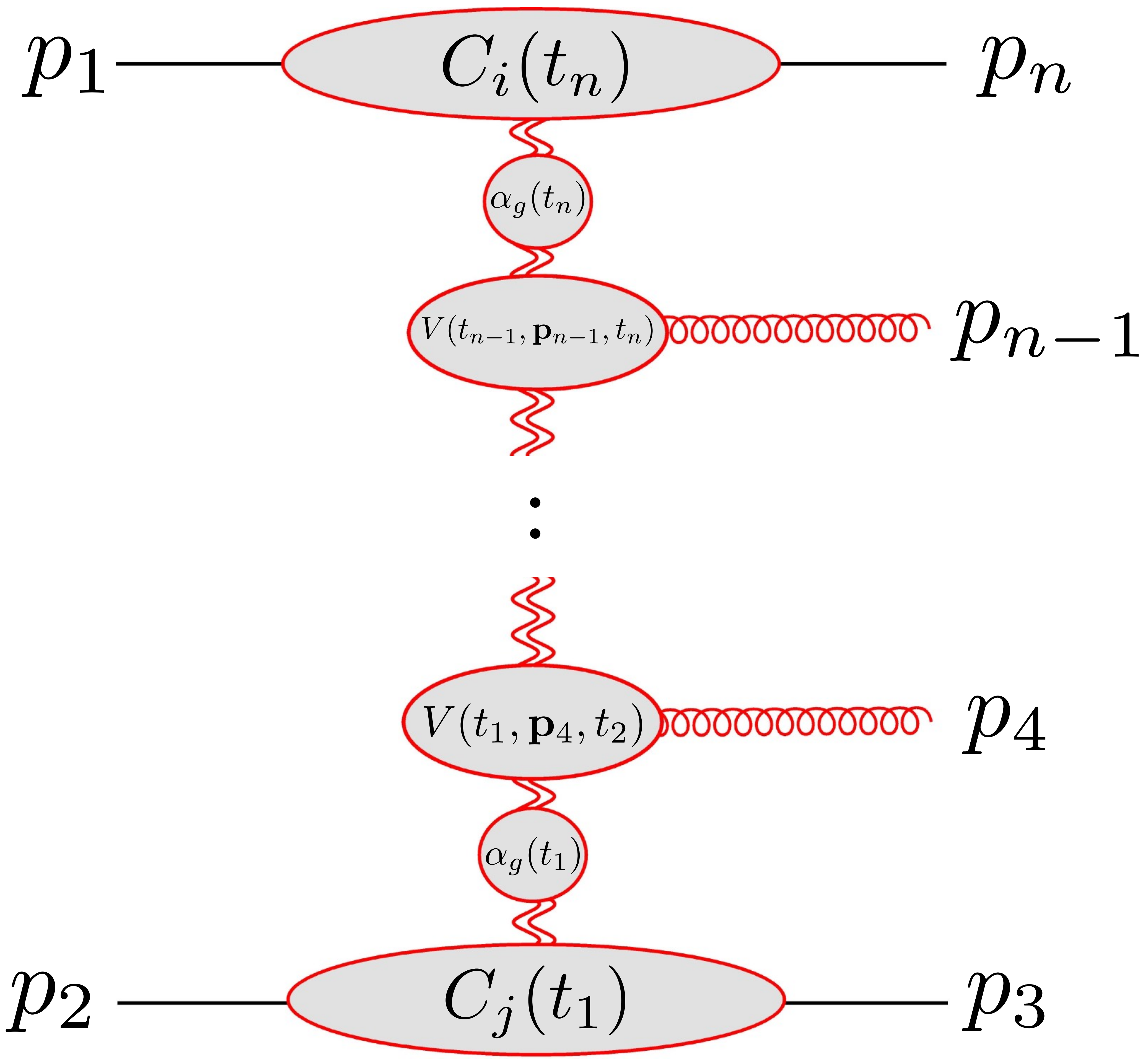
2 → 3 Amplitudes in MRK

	$[8_a, 8_a]$	other odd-odd	even-even	odd-even
LL	 $\alpha_g^{(1)}$	✗	✗	✗
NLL	 $\alpha_g^{(2)}$ $C_i^{(1)}$ $V^{(1)}$	✗	 one-loop [1] beyond in progress	 one-loop [2] beyond in progress
NNLL	 $\alpha_g^{(3)}$ $C_i^{(2)}$ $V^{(2)}$	 [2]	?	?

[1] [Caron-Huot, Chicherin, Henn, Zhang, Zoia '20] [2] [Abreu, Gardi, Falcioni, **CM**, Vernazza, in preparation]

Conclusion and Outlook

- Necessary ingredients fae n-leg amplitudes in MRK up to NLL can be found from only $2 \to 2$ and $2 \to 3$
- Define also NNLL parameters, three-loop Regge trajectory, two-loop impact factors and Lipatov vertex
- Need tae take into account multiple Reggeons
- Regge poles and Regge cuts in perturbative QCD
- Framework allows for calculations in different colour channels
- Different avenues to explore:
    - Higher loop orders, higher leg orders
    - Can you achieve the same in Next-to-MRK?
    - Connect to integrability, resum amplitudes



Conclusion and Outlook

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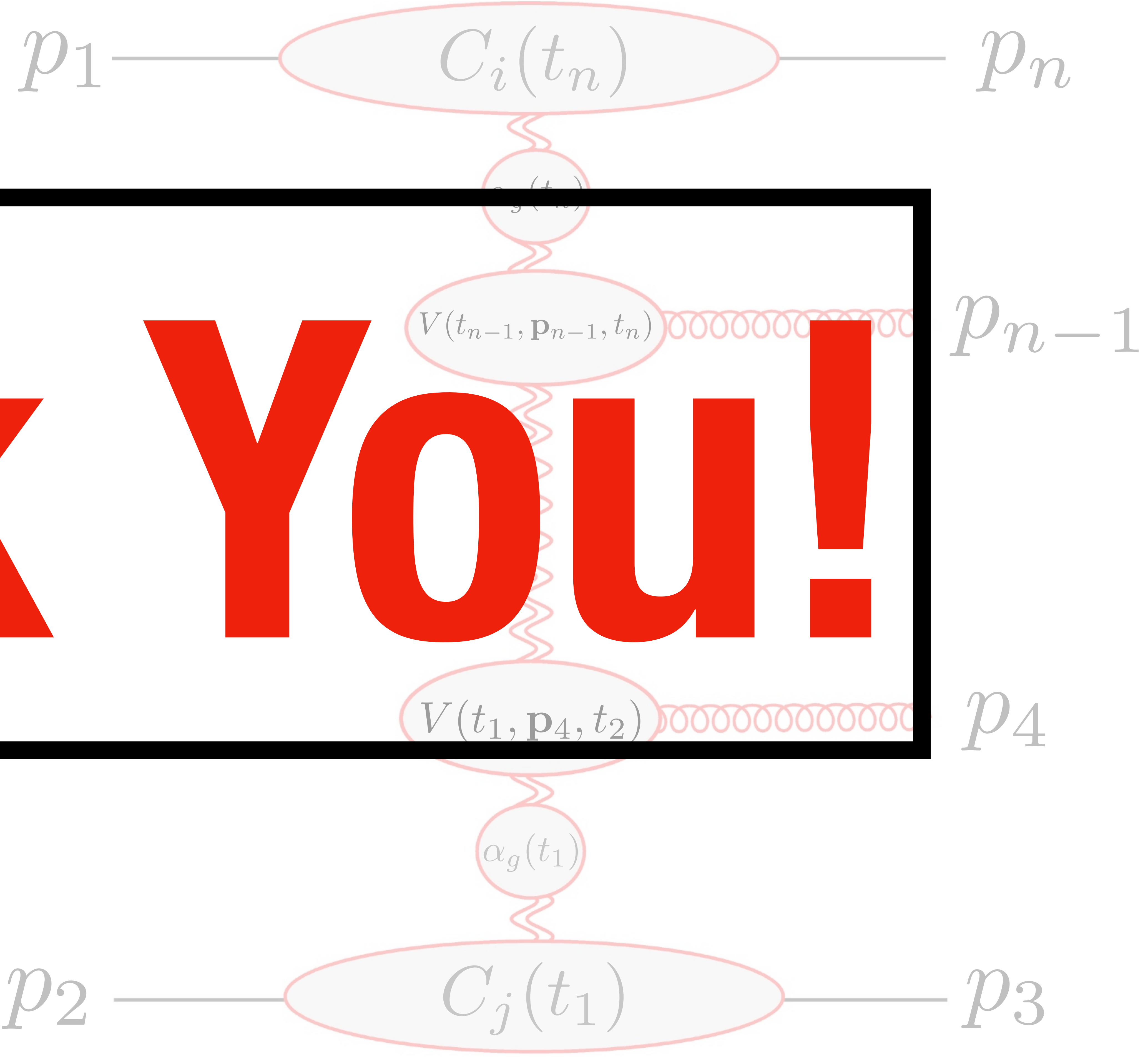


Thank You!

Backup Slides

Explicit results

Two-loop and three-loop Regge trajectories

$$\hat{\hat{\alpha}}_g^{(2)} = C_A\left(\frac{101}{108} - \frac{\zeta_3}{8}\right) - \frac{7n_f}{54} + \epsilon\left[C_A\left(\frac{607}{324} - \frac{67\zeta_2}{144} - \frac{33\zeta_3}{8} - \frac{3\zeta_4}{16}\right) + n_f\left(-\frac{41}{162} + \frac{5\zeta_2}{72} + \frac{3\zeta_3}{4}\right)\right]$$
$$+ \epsilon^2\left[C_A\left(\frac{911}{243} - \frac{101\zeta_2}{108} - \frac{1139\zeta_3}{108} - \frac{2321\zeta_4}{384} + \frac{41\zeta_5}{8} + \frac{71\zeta_2\zeta_3}{24}\right) + n_f\left(\frac{7\zeta_2}{54} + \frac{85\zeta_3}{54} + \frac{211\zeta_4}{192} - \frac{122}{243}\right)\right] + \mathcal{O}(\epsilon^3)$$

$$\hat{\hat{\alpha}}_g^{(3)} = C_A^2\left(\frac{297029}{93312} - \frac{799\zeta_2}{1296} - \frac{833\zeta_3}{216} - \frac{77\zeta_4}{192} + \frac{5}{24}\zeta_2\zeta_3 + \frac{\zeta_5}{4}\right) + C_A n_f\left(\frac{103\zeta_2}{1296} + \frac{139\zeta_3}{144} - \frac{5\zeta_4}{96} - \frac{31313}{46656}\right)$$
$$+ C_F n_f\left(\frac{19\zeta_3}{72} + \frac{\zeta_4}{8} - \frac{1711}{3456}\right) + n_f^2\left(\frac{29}{1458} - \frac{2\zeta_3}{27}\right) + \mathcal{O}(\epsilon),$$

One-loop impact factors

$$C_q^{(1)} = -\frac{C_F}{2\epsilon^2} - \frac{3C_F}{4\epsilon} + C_A\left(\frac{85}{72} + \frac{3\zeta_2}{4}\right) + C_F\left(\frac{\zeta_2}{4} - 2\right) - \frac{5n_f}{36} + \mathcal{O}(\epsilon)$$

$$C_g^{(1)} = -\frac{C_A}{2\epsilon^2} - \frac{b_0}{4\epsilon} + C_A\left(\zeta_2 - \frac{67}{72}\right) + \frac{5n_f}{36} + \mathcal{O}(\epsilon)$$

Formulating highly energetic partons as Wilson lines

[Caron-Huot '13; Caron-Huot, Gardi, Vernazza '17]

"eikonal approximation"

Fourier conjugate of t

$$U(z_\perp) = \mathbf{P}\exp\left[ig_s \mathbf{T}^a \int_{-\infty}^{\infty} dx^+ A_+^a(x^+, x^- = 0, z_\perp)\right]$$

regulate rapidity divergences by tilting Wilson-line off the light cone

$$\eta = \frac{1}{2}\log\frac{p_+}{p_-}$$

$$T^a_{i,L} = [\mathbf{T}^a U(z_i)]\frac{\delta}{\delta U(z_i)}$$

$$T^a_{i,R} = [U(z_i)\mathbf{T}^a]\frac{\delta}{\delta U(z_i)}$$

our parton is a collection of such Wilson lines

evolves according to Balitsky-JIMWLK

$$\frac{d}{d\eta}|\psi_i\rangle = -H|\psi_i\rangle \qquad H = \frac{\alpha_s}{2\pi^2}\int d^d z_i d^d z_j d^d z_0 \frac{z_{0i}\cdot z_{0j}}{(z_{0i}^2 z_{0j}^2)^{1-\epsilon}}\left\{T^a_{i,L}T^a_{j,L} + T^a_{i,R}T^a_{j,R} - U^{ab}_{\text{ad}}(z_0)\left(T^a_{i,L}T^b_{j,R} + T^a_{j,L}T^b_{i,R}\right)\right\}$$

The amplitude is then written as $\mathcal{M}_{ij\to ij} \sim \langle\psi_j|e^{-HL}|\psi_i\rangle$

Target — Projectile

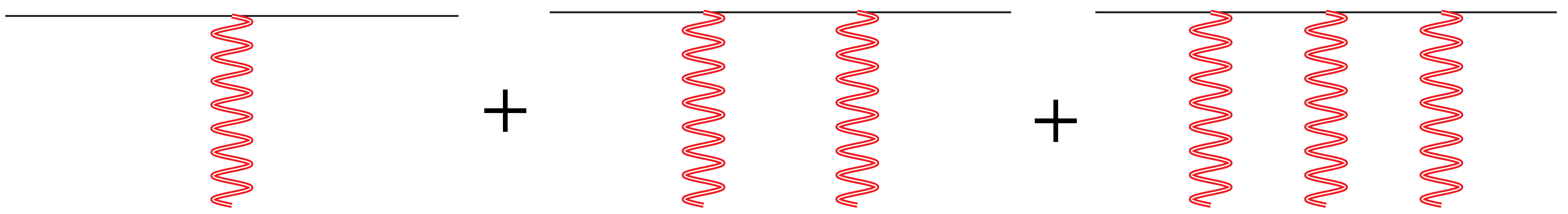
Evolve so that they are at equal rapidity

Target and projectile separated by some $z_\perp^2 \leftrightarrow t$

Expand Wilson line in *Reggeons*

Reggeon field

$$U = \exp\left[ig_s\mathbf{T}^a W^a\right] \sim$$



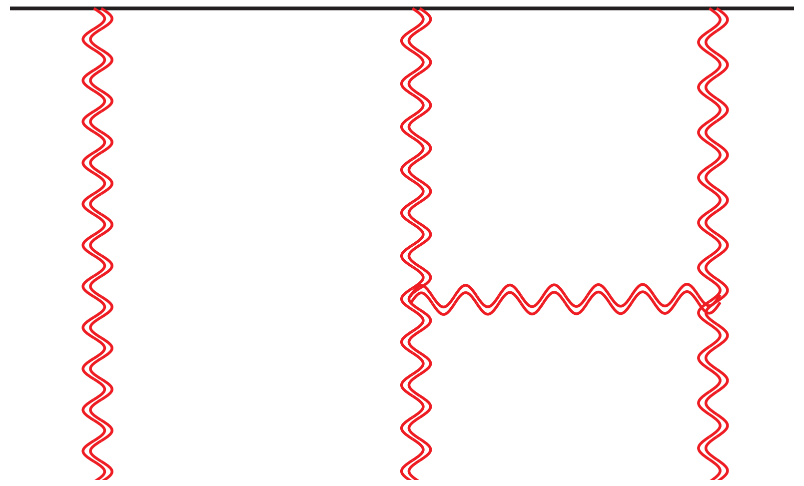
Only odd/even number of Reggeons contribute to the odd/even amplitude

Explicit momentum-space Hamiltonians

The diagonal transitions are

dresses one Reggeon with the trajectory

adds a *rung* between two Reggeons



$$H_{k\to k} = -\int d^d p\, C_A \alpha_g(p) W^a(p) \frac{\delta}{\delta W^a(p)}$$
$$+\alpha_s \int d^d q d^d p_1 d^d p_2\, H_{22}(q;p_1,p_2) W^x(p_1+q) W^y(p_2-q)(F^x F^y)^{ab} \frac{\delta}{\delta W^a(p_1)} \frac{\delta}{\delta W^b(p_1)}$$

with kernel $H_{22}(q;p_1,p_2) = \frac{(p_1+p_2)^2}{p_1^2 p_2^2} - \frac{(p_1+q)^2}{p_1^2 q^2} - \frac{(p_2-q)^2}{q^2 p_2^2}$

Source of the difficulty at NNLL. Three Reggeons spoil the symmetry between colour and kinematics, which is there for two Reggeons (NLL).

The off-diagonal transitions are

$$H_{1\to 3} = \alpha_s^2 \int d^d p_1 d^d p_2 d^d p \;\mathrm{tr}\left[F^a F^b F^c F^d\right] W^b(p_1) W^c(p_2) W^d(p_3) H_{13}(p_1,p_2,p_3) \frac{\delta}{\delta W^a(p)}$$

with kernel $H_{13}(p_1,p_2,p_3) = \frac{r_\Gamma}{3\epsilon}\left[\left(\frac{\mu^2}{(p_1+p_2+p_3)^2}\right)^\epsilon + \left(\frac{\mu^2}{p_2^2}\right)^\epsilon - \left(\frac{\mu^2}{(p_1+p_2)^2}\right)^\epsilon - \left(\frac{\mu^2}{(p_2+p_3)^2}\right)^\epsilon\right]$