Progress towards full Next-to-Leading Logarithmic Accuracy to Scattering at High Energy

including their relevance in describing e.g. $R_{3/2}$ and related observables

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with E. Byrne, A. Maier, J. Smillie and the rest of HEJ

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Introduction

Outline of talk:

1. Amplitudes in the High Energy Limit
2. Progress towards full next-to-leading logarithmic accuracy
3. Recent results including Pure jets, H+jets, W+jets,. . .

High Energy Jets:

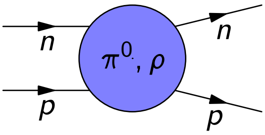
- **Factorisation of matrix elements** using **currents** retains analytic properties such as **crossing symmetries**
- systematic **power expansion of QCD amplitudes**
- all-order **leading and sub-leading logarithmic corrections**
- **matching**, **results**. . .

Regge theory

Regge theory describes scattering from a **central potential** in terms of the projections on Legendre polynomial and states of **definite orbital angular momentum** (partial wave analysis)

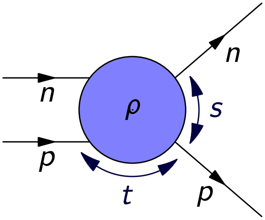
The analysis of **analytic scattering amplitudes** in terms of Regge Theory: Regge (1959)

$$\mathcal{M} = \sum_i \Gamma_i(t)\,(s)^{j_i}$$



At **large energies** s, the contribution from particle of **highest spin j dominates**

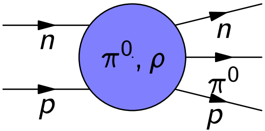
$$\mathcal{M} \to \Gamma(t)\,(s)^j$$



Regge limit: $s \gg -t$ or $s \gg p_t^2$

Multi-Regge theory

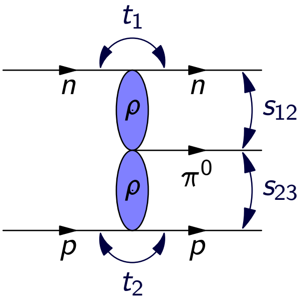
Large *s* of course leads to the possibility of **multi-particle production**



Multi-Regge limit: Brower, DeTAR, Weis (1974)

$s_{12}, s_{23} \gg p_{t_i}^2, |t_i|, \quad |t_i| \sim |t_j|, \quad |p_{t_i}| \sim |p_{t_j}|$

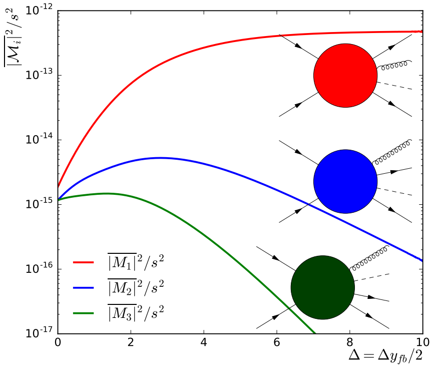
$$\mathcal{M} = \ s_{12}^j s_{23}^j \ \Gamma\left(t_1, t_2, s/(s_{12}s_{23})\right)$$

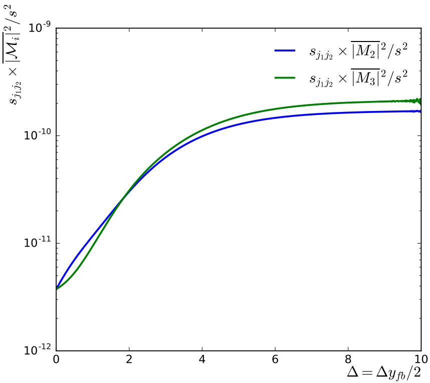


No underlying theory for strong interactions; derives constraints on the high energy behaviour based on the constraints from an **analytic scattering amplitude**.

Scaling of QCD Amplitudes

The scaling extends to all QCD processes involving also Higgs bosons, W, Z and photon production.

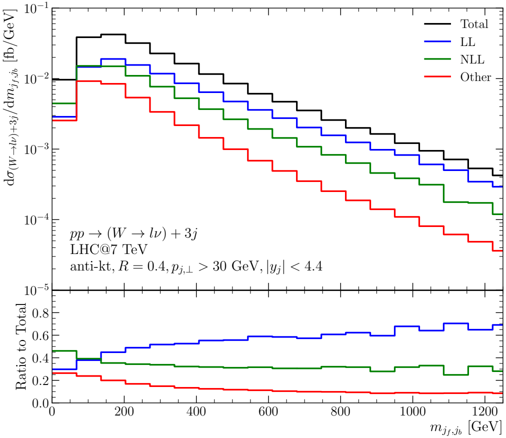


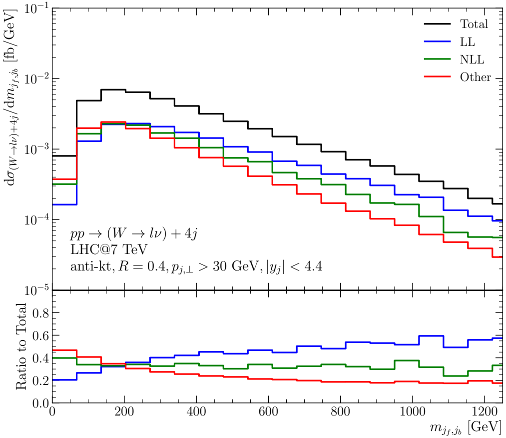


The **scaling** for different kinematic evaluations of the same amplitude is exactly as predicted by Regge theory applied to the **planar graph** connecting the rapidity-ordered configuration.

M. Heil, A. Maier, J.M. Smillie, JRA, arXiv:1706.01002

Cross sections vs logarithmic ordering

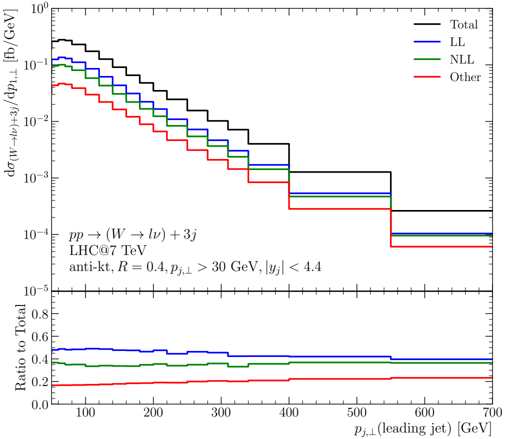


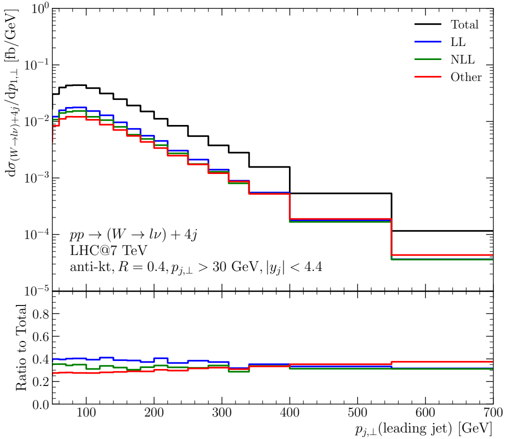


$pp \to W3j$ J. Black, H. Brooks, A. Maier, J.M. Smillie, JRA, arXiv:2012.10310 $pp \to W4j$

The cross sections really do follow the logarithmic ordering.

Logartihmic split for other observables





pp $\to$ *W3j* J. Black, H. Brooks, A. Maier, J.M. Smillie, JRA, arXiv:2012.10310 *pp* $\to$ *W4j*

The logarithmic ordering is less good for p_t-based observables – expect NLL to be as important as LL, and therefore corrections necessary.

Perurbative Corrections in the High Energy Limit

Since the real emision perturbative corrections have $|M|^2/s^2 \to \text{constant}$ for large $\Delta y \sim \log(s/p_t^2)$, it will contribute a correction $\alpha_s \log(s/p_t^2)$ after integration.
The other orderings of momenta (and other processes) contribute sub-leading corrections which can be included at next-to-leading order (see later).
Also virtual corrections for $gg \to gg$ at one loop have logarithmic piece:

$$
\begin{aligned}
m_{4:1}(-,-,+,+) = {} & m_4(-,-,+,+)\ c_\Gamma \\
& \times \left\{ \left(-\frac{\mu^2}{s_{14}}\right)^\epsilon \left[N_c \left(-\frac{4}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{2}{\epsilon} \ln \frac{s_{12}}{s_{14}} - \frac{64}{9} - \frac{1}{3} + \pi^2 \right) \right.\right. \\
& \left.\left. + N_f \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right] - \frac{\beta_0}{\epsilon} \right\}
\end{aligned}
$$

Logarithmic structure predicted to all orders (BFKL, Regge, VDD,...).
Control perturbative corrections of $\alpha_s^n \log^n(s/p_t^2)$ (leading logarithm).

Regge Theory and QCD

While QCD allows for the calculation of the scattering amplitudes, the amplitudes are still **analytic**, and a Regge analysis can be applied. The amplitude can be reconstructed (to ensure logarithmic accuracy of the cross section) by effective vertices. These **building blocks** can be **calculated in QCD**.

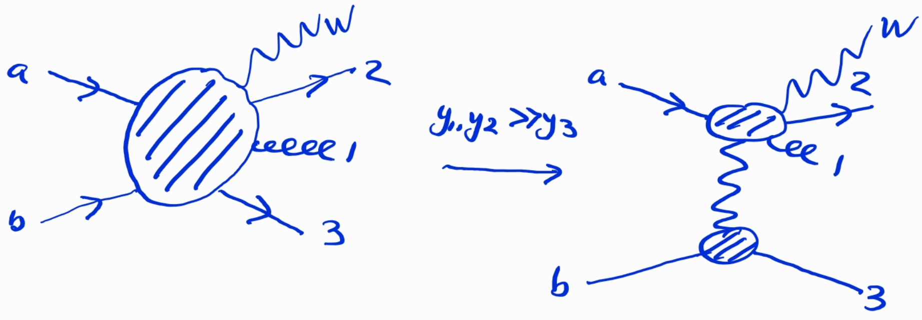
These are the impact factors and kernels in the BFKL language. So what is different with High Energy Jets?

The Regge analysis relies on analyticity, e.g. crossing symmetry. However, even the building blocks of BFKL and the standard analysis of the high energy limits do not respect such crossing symmetries. The components in HEJ respects **crossing symmetry**.

The components also respect **gauge invariance** - everywhere in phase space, not just in asymptotic limits.

NLL components for Reggeisation

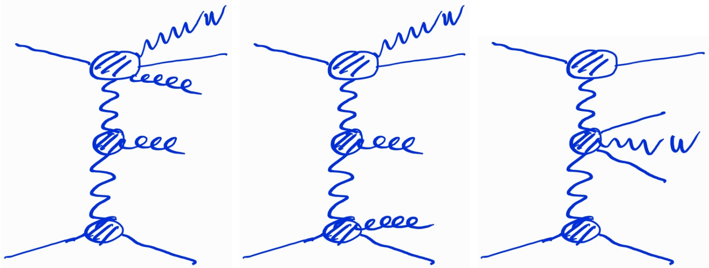
Consider again $pp \to W3j$. Next-to-leading logarithmic corrections arise e.g. from the amplitudes in the quasi-multi-Regge-kinematic limit, where the invariant mass between one pair of partons is not large.



Amplitude expressed as $\mathcal{M} = I^\mu(a, w, 1, 2)\ J_\mu(b, 3)$. Full crossing symmetry, gauge invariance etc. in each component. $I^\mu(a, w, 1, 2)$ calculated by projection onto colour octet exchange in the t-channel.

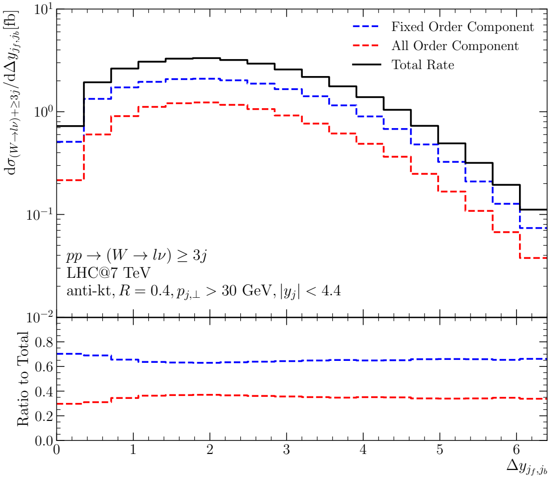
Higher Order Corrections

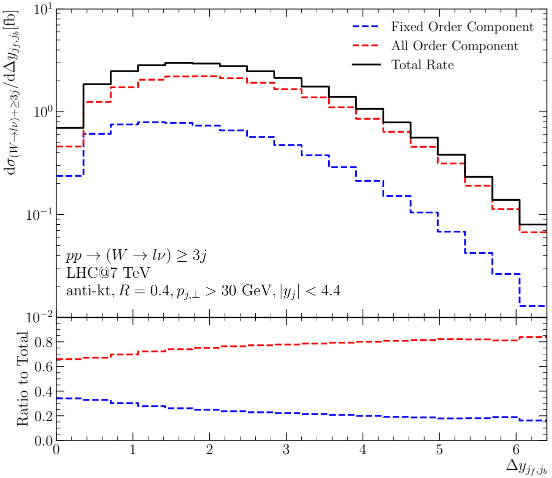
Can calculate higher order corrections with NLL components by explicit MC integration over the regulated amplitudes, represented by a Reggeised graph



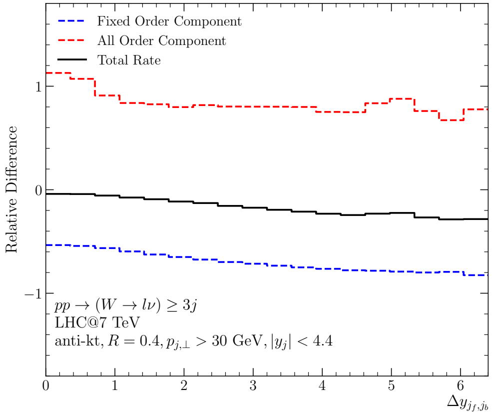
Virtual corrections encoded in the t-channel propagators.
Sub-processes and phase space points not reached with LL or NLL Reggeised description are treated at fixed order.

Impact of NLL corrections for W3J



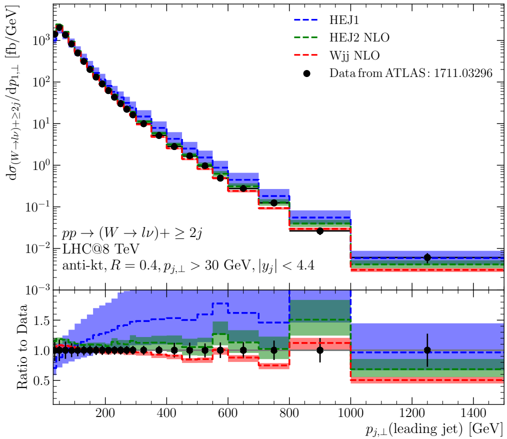


Impact of NLL corrections for W3J



Much less fixed order matching, much bigger resummation component. Final result of the inclusive distribution changes by up to 25%.

Comparison to Data (WJJ)



The NLL terms included and improvement in matching are sufficient to ensure the predictions agree well with data even in the most difficult regions of phase space.

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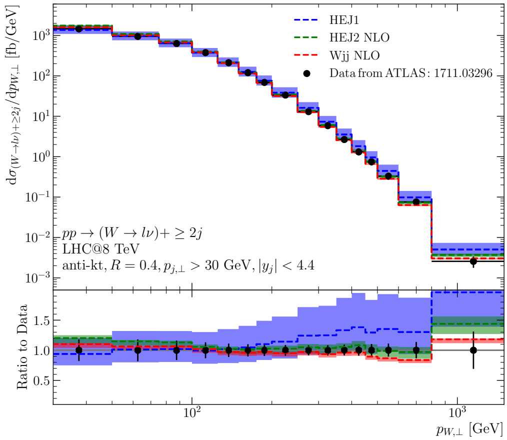

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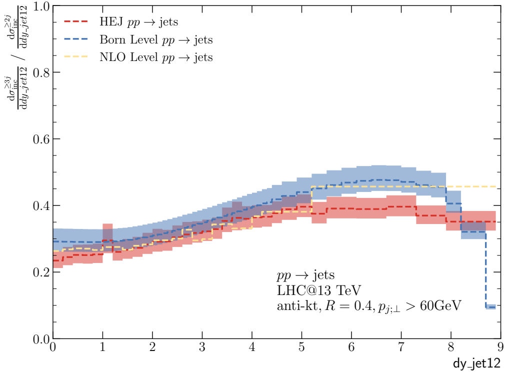
R_{32} (jets)

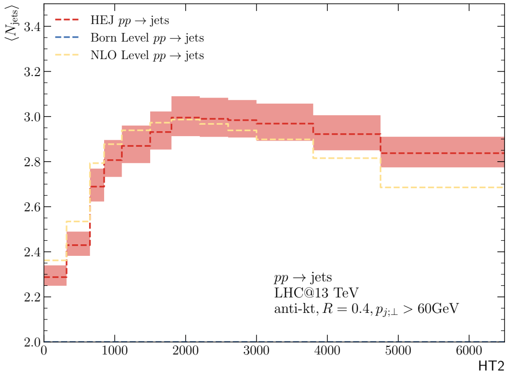
R_{32} with cuts relevant for upcoming ATLAS study.



C. Elrick

Relatively small differences between LO, NLO and HEJ for R_{32} vs. rapidity difference between hardest jets.

Average number of jets (asking for at least two)



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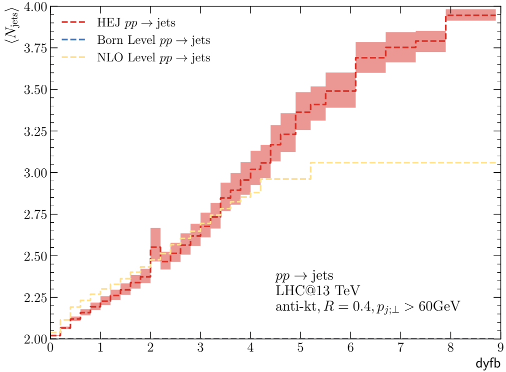
Similar results between NLO and HEJ for pt-based observables.

Average number of jets (asking for at least two)



C. Elrick

Similar results at also for $\langle$jets$\rangle$ vs. Δy_{fb} until reaching the limitations of the NLO calculation.

Ongoing work

The **work to include full NLL accuracy** in the resummation of High Energy Jets is ongoing - need one-loop corrections to the one-particle production components at LL, two-loop component of the reggeised t-channel propagator.

The calculation within High Energy Jets using the Reggeised amplitudes captures the logarithmically enhanced terms $\log(s/pt^2)$. A systematic treatment of hierarchies in transverse scales is achieved by matching to a **parton shower** (Pythia) [see talk by Sebastian Jaskiewicz]

The process of $pp \to HJ$ (one-jet process) was recently included in the family of processes within HEJ, **extending the applicability** of the framework [see talk by Andreas Maier].

Conclusions

Unsurprisingly, the inclusion of sub-leading logarithms leads to

- small changes in the leading regions of phase space
- a better description in sub-leading regions of phase space

Hall-marks of a well-behaved perturbative expansion.
Further improvements ongoing.