High-Energy and Soft-Collinear Resummation for Jet Production at the LHC: HEJ+Pythia

Sebastian Jaskiewicz

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In collaboration with Jeppe Andersen, Hitham Hassan, and Leif Lönnblad
Introduction

▶ Large high-energy logarithms are important to understand at the LHC: *High Energy Jets (HEJ)* framework

▶ Transverse momentum hierarchies, collinear splittings, MPI, and hadronization described by parton showers: *Pythia*.

▶ This talk: Merging High-Energy resummation of HEJ with Pythia parton shower → HEJ+Pythia: showered events from HEJ input.
High Energy Jets (HEJ) framework

High Energy Jets resums large logarithms in $s/t$ according to the BFKL equation.

\[ J. \text{ Andersen, J. Smillie, 0908.2786} \quad [J. \text{ Andersen, J. Smillie, 0910.5113}] \quad [J. \text{ Andersen, J. Smillie, 1101.5394}] \]

\[ J. \text{ Andersen, J. Black, H. Brooks, B. Ducloué, M. Heil, A. Maier, J. Smillie, 2110.15692} \quad [J. \text{ Andersen, B. Ducloué, H. Hassan, C. Elrick, A. Maier, G. Nail, J. Paltrinieri, A. Papaefstathiou, J. Smillie, 2303.15778}] \]

See also talks by J. Andersen and A. Maier.

High Energy limit for $2 \to n$ scattering

\[ \hat{s} \gg \hat{s}_{ij} \gg k_\perp \quad \hat{s}_{ij} = (p_i + p_j)^2 \]

\[ y_1 \ll y_2 \ll \ldots \ll y_{n-1} \ll y_n \quad p_{i\perp} \approx k_\perp \]

HEJ is built to describe effects from hard, wide angle emissions. Large $\ln(s/t)$ logs.
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HEJ is built to describe effects from hard, wide angle emissions. Large $\ln(s/t)$ logs.

LL + LO accurate
HEJ Motivations

HEJ describes data well in the High Energy region. Parton showers do not resum these logarithms.

Measurement of $W + \geq 2$ jets.

Average number of jets as rapidity between most forward ($j_F$) and backward ($j_B$) increases.

[D0 Collaboration, 1302.6508]

Corrections proportional to $\Delta y_{j_F, j_B}$

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HEJ Motivations: Higgs + DiJets

These logarithms are crucial in describing regions selected by VBF cuts

\[ \Delta y_{jj} > 2.8 \quad m_{jj} > 400\text{GeV} \]
Beyond the High-Energy limit

- HEJ describes high-energy logarithms well
- Transverse momentum hierarchies are not captured → Parton Showers

[ATLAS collaboration, 1407.5756]

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Parton Showers

Soft and collinear splittings described by parton showers.
Our method uses Pythia.


\[
\Delta_{ab}(t_n, t_m) = \exp \left[ - \int_{t_m}^{t_n} \frac{dt}{t} \left( \frac{\alpha_s(t)}{2\pi} \int dz P_{ab}(z) \right) \right]
\]

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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
The HEJ+Ariadne method uses subtracted Sudakov factors to generate parton shower emissions.

[J. Andersen, L. Lönnblad, J. Smillie, 1104.1316][ATLAS collaboration, 1407.5756]

The splitting function in HEJ is

$$P_{\text{HEJ}} = \frac{|M_{n+1}^{\text{HEJ}}|^2}{|M_n^{\text{HEJ}}|^2}$$

Procedure starts with HEJ.

Then, parton shower emissions are generated according to $P_{\text{Ariadne}}$ - parton shower splitting function.

If these could not have been produced by HEJ they are accepted. Otherwise veto with probability $P_{\text{veto}} = P_{\text{HEJ}} / P_{\text{Ariadne}}$. 
The HEJ+Ariadne method uses subtracted Sudakov factors to generate parton shower emissions. [J. Andersen, L. Lönnblad, J. Smillie, 1104.1316][ATLAS collaboration, 1407.5756]

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If these could not have been produced by HEJ they are accepted. Otherwise veto with probability \( P_{\text{veto}} = P_{\text{HEJ}} / P_{\text{Ariadne}} \).

No parton shower emissions at earlier stages
CKKW-L method

CKKW-L are method for merging Fixed-Order matrix elements with Parton Showers.


Aims:

▶ generate $N$ hardest emissions using matrix elements
▶ add Parton Shower

Phase space is divided at some merging scale $t_{ms}$ if:

▶ $t > t_{ms}$ – matrix element region
▶ $t < t_{ms}$ – parton shower region

[A. Buckley et al, 1101.2599]

\[
d\sigma^{\text{CKKW-L}} = d\Phi_0 B(\Phi_0) + \sum_{n=1}^{N_{\text{max}}} d\Phi_n R(t_n) \theta(t_n - t_{ms}) \Delta(t_n, t_{ms}) \\
\times \prod_{m=1}^{n} P(t_m, z_m) \theta(t_{ms} - t_m) \Delta(t_{m-1}, t_m)
\]

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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
Previous method: HEJ+Pythia (2017)

The first HEJ+Pythia merging was based on the CKKW-L.

This method uses HEJ as a matrix element generator.

History of states is generated by Pythia, which then performs trial emissions.

If trial emission is a HEJ like emission, veto with probability $P^{HEJ}/P^{Pythia}$ and move on.

If trial emission is not a HEJ state, accept it and shower freely from here.

First collinear (PS) emission is accepted.

Data from: [ATLAS Collaboration, 1407.5756]
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Data from: [ATLAS Collaboration, 1407.5756]
HEJ States Classification

In the Regge limit the amplitudes have the following scaling

\[ M \propto s_{12}^{\alpha_{12}} s_{23}^{\alpha_{23}} \cdots s_{n-1,n}^{\alpha_{n-1,n}} f(\{pT_i\}) \]

\[ s_{ij} = (p_i + p_j)^2 \]

[R. Brower, C. DeTar, J. Weis, 1974]
HEJ States Classification

In the Regge limit the amplitudes have the following scaling

$$M \propto s_{12}^{\alpha_{12}} s_{23}^{\alpha_{23}} \cdots s_{n-1,n}^{\alpha_{n-1,n}} f(\{p_{Ti}\})$$

$\alpha_{ij}$ is the spin in the effective t-channel.
HEJ States Classification

In the Regge limit the amplitudes have the following scaling

\[ M \propto s_{12}^{\alpha_{12}} s_{23}^{\alpha_{23}} \cdots s_{n-1,n}^{\alpha_{n-1,n}} f(\{p_T\}) \]

\( \alpha_{ij} \) is the spin in the effective t-channel.

The leading order input into HEJ is classified as either “resummable” or “non-resummable”
HEJ+Pythia: Merging procedure

We express HEJ in the parton shower language and feed in HEJ events.

The HEJ+Pythia method constructs histories for resummable HEJ events. → this is to ensure that the full phase space of HEJ is explored by the shower.

A history is a sequence of states \( \mathcal{H} = \{S_0, \ldots, S_l\} \), which charts the evolution in a parton shower from the most clustered state \( S_0 \) to the input event \( S_l \).

Then, we let parton shower add subtracted shower emissions in between the states of the history, and continue the Pythia shower with a subtracted splitting kernel until hadronisation

\[
d\sigma^\text{HEJ+Pythia}_{m,n} = d\sigma^*_2 \prod_{i=1}^{m-2} P_i^H \Delta^{H}_{i-1,i} \left[ \prod_{\lambda=1}^{\lambda_i} P_{i,\lambda} \Delta^S_{i,\lambda-1,i,\lambda} \right] \prod_{j=m-2+N}^{n} P_j^S \Delta^S_{j-1,j}
\]

\[
\Delta^S(t_{i-1}, t_i) = \exp \left\{ - \int_{t_i}^{t_{i-1}} dt \int dz \, \Theta(P^P(t, z) - P^H(t, z)) \left[ P^P(t, z) - P^H(t, z) \right] \right\}
\]

Subtracted splitting probability: \( P^S(t, z) \)
HEJ+PYTHIA: Merging

Hej Input

Cluster back

Trial 1 - keep:
Input for next trial

Trial 2 - veto

Trial 3 - keep:
Input for next trial

Trial 4: $t_4^P < t_2^H$
Move to next state

Input for trial 5

...
HEJ+Pythia: Merging

HEJ Input

Cluster back

Trial 1 - keep:
Input for next trial

Trial 2 - veto

Trial 3 - keep:
Input for next trial

Trial 4: $t_4^P < t_2^H$
Move to next state

Input for trial 5

...
HEJ+PYTHIA: Merging

- HEJ Input
- Cluster back
- Trial 1 - keep:
  Input for next trial
- Trial 2 - veto
- Trial 3 - keep:
  Input for next trial
- Trial 4: $t_4^P < t_2^H$
  Move to next state
- Input for trial 5

...
HEJ+PYTHIA: Merging

**HEJ Input**

$t^H_1$ $t^H_2$ $t^H_3$ $t$

**Cluster back**

$t^H_1$ $t$

**Trial 1 - keep:**

Input for next trial

$t^H_1$ $t^P_1$ $t$

**Trial 2 - veto**

$t^H_1$ $t^P_1$ $t^P_2$ $t$

**Trial 3 - keep:**

Input for next trial

$t^H_1$ $t^P_1$ $t^P_3$ $t$

**Trial 4: $t^P_4 < t^H_2$**

Move to next state

$t^H_1$ $t^P_1$ $t^P_3$ $t^P_4$ $t$

**Input for trial 5**

$t^H_1$ $t^P_1$ $t^P_3$ $t^H_2$ $t$

...
HEJ+PYTHIA: Merging

HEJ Input

Cluster back

Trial 1 - keep:
Input for next trial

Trial 2 - veto

Trial 3 - keep:
Input for next trial

Trial 4: $t_4^P < t_2^H$
Move to next state

Input for trial 5

::

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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
HEJ+Pythia: Merging

**HEJ Input**

**Cluster back**

**Trial 1 - keep:**
Input for next trial

**Trial 2 - veto**

**Trial 3 - keep:**
Input for next trial

**Trial 4:** $t_4^H < t_2^H$
Move to next state

Input for trial 5

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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
HEJ+PYTHIA: Merging

HEJ Input

Cluster back

Trial 1 - keep:
  Input for next trial

Trial 2 - veto

Trial 3 - keep:
  Input for next trial

Trial 4: $t_4^P < t_2^H$
  Move to next state

Input for trial 5

...
HEJ+PYTHIA: Merging

HEJ Input

Cluster back

Trial 1 - keep:
Input for next trial

Trial 2 - veto

Trial 3 - keep:
Input for next trial

Trial 4: $t_4^P < t_2^H$
Move to next state

Input for trial 5

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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
Classification of events
Consider the following evolution history:
Classification of events

Consider the following evolution history:
**HEJ+Pythia: subtracted shower**

Once the original HEJ input state is recovered and dressed, the subtracted shower continues until hadronization.

\[
d\sigma_{m,n}^{\text{HEJ+Pythia}} = d\sigma_2^* \prod_{i=1}^{m-2} P_i^H \Delta_{i-1,i}^H \left[ \prod_{\lambda=1}^{\lambda_i} P_{i\lambda}^S \Delta_{i\lambda-1,i\lambda}^S \right] \prod_{j=m-2+N}^{n} P_j^S \Delta_{j-1,j}^S
\]

Showered low multiplicity HEJ events can resemble higher multiplicity HEJ events \(
\rightarrow\)
subtraction needed.
Results
Results: Jet Shapes

$pp \rightarrow 2j$, LHC at $\sqrt{s} = 7$ TeV
anti-$k_{T}$ jets, $R = 0.6$, $|y_{j}| < 2.8$
ATLAS: arXiv:1101.0070
HEJ+Pythia (LL+subl.)
CKKW-L
Pythia

$30 \leq p_{T,j} < 40 \text{ GeV} \times 100 \times 1.0$
$40 \leq p_{T,j} < 60 \text{ GeV} \times 100 \times 1.0$
$60 \leq p_{T,j} < 80 \text{ GeV} \times 100 \times 1.0$
$80 \leq p_{T,j} < 110 \text{ GeV} \times 100 \times 1.0$
$110 \leq p_{T,j} < 160 \text{ GeV} \times 100 \times 1.0$
$160 \leq p_{T,j} < 210 \text{ GeV} \times 100 \times 1.0$

$pp \rightarrow \geq 2j$, LHC at $\sqrt{s} = 7$ TeV
anti-$k_{T}$ jets, $R = 0.6$, $|y_{j}| < 2.8$
ATLAS: arXiv:1101.0070
HEJ+Pythia (LL+subl.)
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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
Results

\[
\langle N_{\text{jets}} \rangle \text{ in rapidity interval}
\]

\[
pp \rightarrow \geq 2j, \text{ LHC at } \sqrt{s} = 7 \text{ TeV, } \left| y_j \right| < 4.4
\]

\[
\Delta y_{j_1, j_2}
\]

ATLAS

\[
\sqrt{s} = 7 \text{ TeV, } \int \mathcal{L} dt = 38 \text{ pb}^{-1}
\]

Theory/Data

\[
Q_0 = 20 \text{ GeV}
\]

Data 2010
POWHEG+PYTHIA 8
POWHEG+HERWIG
HEJ (partonic)
HEJ+ARIADNE

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<td>6</td>
</tr>
</tbody>
</table>

\[
\Delta y_{j_1, j_2}
\]

\[
\text{Ratio to Data}
\]

\[
\text{Theory/Data}
\]

\[
\text{HEJ (LL+subl.)}
\]

\[
\text{HEJ+Pythia (LL+subl.)}
\]

\[
\text{CKKW-L}
\]

\[
\text{ATLAS: arXiv:1407.5756}
\]

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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
Results: Multi-jet Cross-sections

$pp \rightarrow \geq 2j$, LHC at $\sqrt{s} = 7$ TeV

anti-$k_{T}$ jets, $R = 0.4$, $p_{T,j} > 60$ GeV, $|y_j| < 2.8$

- HEJ (LL+subl.)
- HEJ+Pythia (LL+subl.)
- CKKW-L
- ATLAS: arXiv:1107.2092

$pp \rightarrow \geq 3j$, LHC at $\sqrt{s} = 7$ TeV

anti-$k_{T}$ jets, $R = 0.4$, $p_{T,j} > 60$ GeV, $|y_j| < 2.8$

- HEJ (LL+subl.)
- HEJ+Pythia (LL+subl.)
- CKKW-L
- ATLAS: arXiv:1107.2092

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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia
Summary

- Introduced improved method to combine high-energy resummation implemented in HEJ with soft-collinear logarithms of Pythia
- Implemented in HEJ+Pythia software
- We cover the full phase space of both HEJ and Pythia and subtract overlapping contributions, keeping accuracy of both and retaining LO accuracy
- Demonstrated correct implementation using jet shapes and multi-jet observables

Outlook

- Implementation of this procedure to other processes, for example $W +$ jets
Thank you!
Auxiliary slides
Phase space

$p_{\perp}$

$p_{\perp j}$

$y$

HEJ-Inclusive

HEJ-Exclusive

HEJ-Inclusive

ext jet

jet

soft jet

sebastian jaskiewicz

merging high-energy and soft-collinear resummation: HEJ+Pythia
Recoil Strategy

When we add back an emission from a trial event to an arbitrary later stage in the history, we recoil the additional momentum according to the following global strategy:

1. Reshuffle the excess transverse momentum across the final state partons, conserving the mass and rapidity of each.

2. Rescale all transverse momenta by a constant factor $\lambda$ such that the invariant mass of the initial state $\sqrt{\hat{s}}$ is conserved and again reassign the $E$ and $p_z$ components of each final state particle such that the rapidities and masses of each are conserved.

3. Sum over positive and negative lightcone components of the final state momenta to find physical analogues for the momenta of the initial state partons.

4. Boost along the $z$-axis such that that the initial state momenta are the same as they were in the original state, using the momenta in step 3. to derive the rapidity $\psi$. This ensures that beam energies are not exceeded if many emissions are added.
Thank you