

High-Energy and Soft-Collinear Resummation for Jet Production at the LHC: HEJ+Pythia

Sebastian Jaskiewicz

RADCOR 2023

Crieff, Scotland

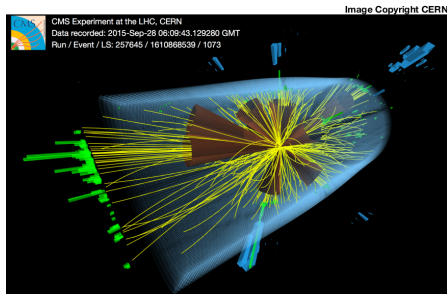
June 1st, 2023

In collaboration with Jeppe Andersen, Hitham Hassan, and Leif Lönnblad



Introduction

- ▶ Large high-energy logarithms are important to understand at the LHC: *High Energy Jets* (HEJ) framework
- ▶ Transverse momentum hierarchies, collinear splittings, MPI, and hadronization described by parton showers: *Pythia*.
- ▶ This talk: Merging High-Energy resummation of HEJ with *Pythia* parton shower → HEJ+*Pythia*: showered events from HEJ input.



High Energy Jets (HEJ) framework

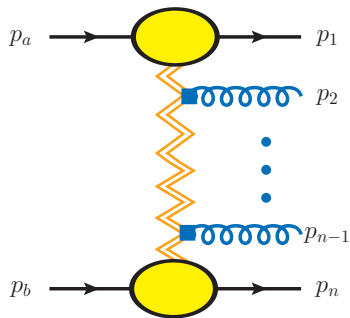
High Energy Jets resums large logarithms in s/t according to the BFKL equation.

[J. Andersen, J. Smillie, 0908.2786] [J. Andersen, J. Smillie, 0910.5113] [J. Andersen, J. Smillie, 1101.5394]

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See also talks by [J. Andersen](#) and [A. Maier](#).



High Energy limit for $2 \rightarrow n$ scattering

$$\hat{s} \gg \hat{s}_{ij} \gg k_{\perp} \quad \hat{s}_{ij} = (p_i + p_j)^2$$

$$y_1 \ll y_2 \ll \dots \ll y_{n-1} \ll y_n \quad p_{i\perp} \approx k_{\perp}$$

HEJ is built to describe effects from hard, wide angle emissions. Large $\ln(s/t)$ logs.

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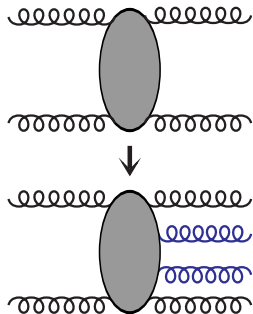
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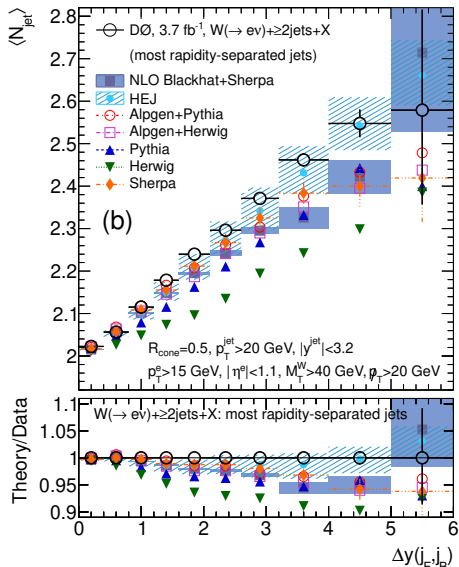
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HEJ is built to describe effects from hard, wide angle emissions. Large $\ln(s/t)$ logs.

LL + LO accurate

HEJ Motivations



HEJ describes data well in the High Energy region. Parton showers do not resum these logarithms.

Measurement of $W + \geq 2\text{jets}$.

Average number of jets as rapidity between most forward (j_F) and backward (j_B) increases.

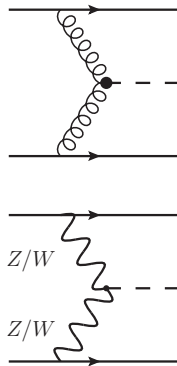
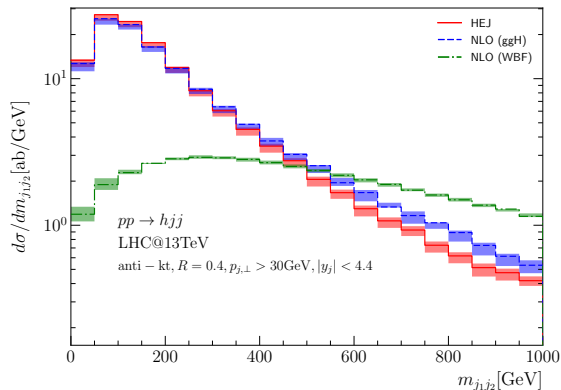
[DØ Collaboration, 1302.6508]

Corrections proportional to $\Delta y_{j_F, j_B}$

HEJ Motivations: Higgs + DiJets

These logarithms are crucial in describing regions selected by VBF cuts

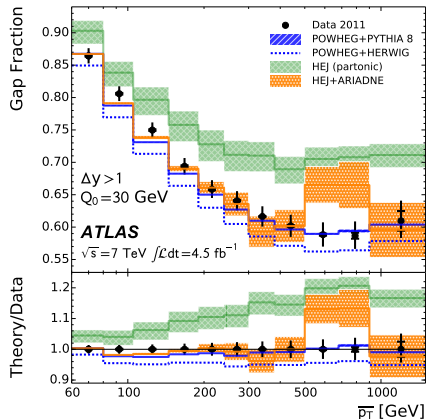
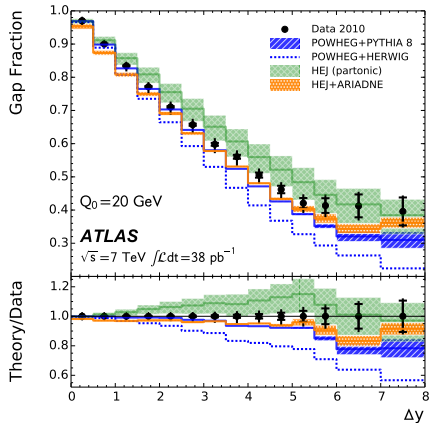
$$\Delta y_{jj} > 2.8 \quad m_{jj} > 400 \text{ GeV}$$



[J. Andersen, M. Heil, A. Maier, J. Smillie, in 1803.07977]

Beyond the High-Energy limit

- ▶ HEJ describes high-energy logarithms well
- ▶ Transverse momentum hierarchies are not captured → [Parton Showers](#)



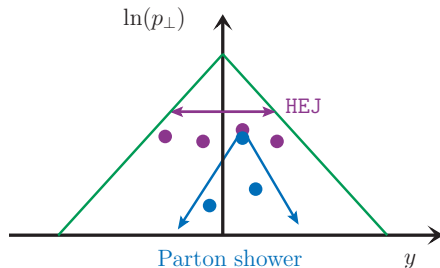
[ATLAS collaboration, 1407.5756]

Parton Showers

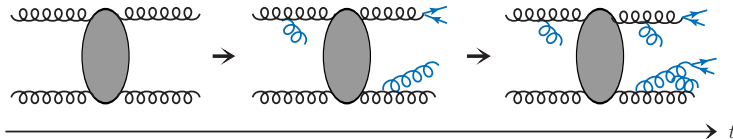
Soft and collinear splittings described by parton showers.

Our method uses **Pythia**.

[T. Sjöstrand, S. Mrenna, P. Skands, hep-ph/0603175][T. Sjöstrand, S. Ask, J. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. Rasmussen, P. Skands, 1410.3012]

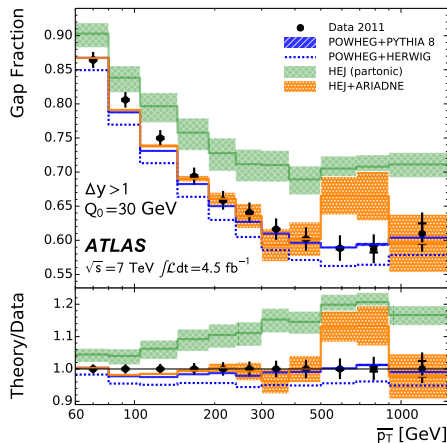


$$\Delta_{ab}(t_n, t_m) = \exp \left[- \int_{t_m}^{t_n} \frac{dt}{t} \left(\frac{\alpha_s(t)}{2\pi} \int dz P_{ab}(z) \right) \right]$$



The HEJ+Ariadne method uses subtracted Sudakov factors to generate parton shower emissions.

[J. Andersen, L. Lönnblad, J. Smillie, 1104.1316][ATLAS collaboration, 1407.5756]



The splitting function in HEJ is

$$P^{\text{HEJ}} = \frac{|\mathcal{M}_{n+1}^{\text{HEJ}}|^2}{|\mathcal{M}_n^{\text{HEJ}}|^2}$$

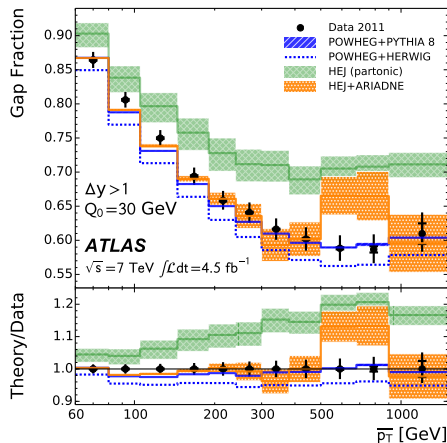
Procedure starts with HEJ.

Then, parton shower emissions are generated according to P^{Ariadne} - parton shower splitting function.

If these could not have been produced by HEJ they are accepted. Otherwise veto with probability $\mathcal{P}^{\text{veto}} = P^{\text{HEJ}}/P^{\text{Ariadne}}$.

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No parton shower emissions at earlier stages

CKKW-L method

CKKW-L are method for merging Fixed-Order matrix elements with Parton Showers.

[S. Catani, F. Krauss, R. Kuhn, B. Webber, hep-ph/0109231] [L. Lönnblad, hep-ph/0112284]

Aims:

- ▶ generate N hardest emissions using matrix elements
- ▶ add Parton Shower

Phase space is divided at some *merging scale* t_{ms} if:

- ▶ $t > t_{ms}$ – matrix element region
- ▶ $t < t_{ms}$ – parton shower region

[A. Buckley et al, 1101.2599]

$$\begin{aligned} d\sigma^{\text{CKKW-L}} &= d\Phi_0 B(\Phi_0) + \sum_{n=1}^{N_{\text{max}}} d\Phi_n R(t_n) \theta(t_n - t_{ms}) \Delta(t_n, t_{ms}) \\ &\quad \times \prod_{m=1}^n P(t_m, z_m) \theta(t_{ms} - t_m) \Delta(t_{m-1}, t_m) \end{aligned}$$

Previous method: HEJ+Pythia (2017)

The first HEJ+Pythia merging was based on the CKKW-L.

[J. Andersen, H. Brooks, L. Lönnblad, 1712.00178]

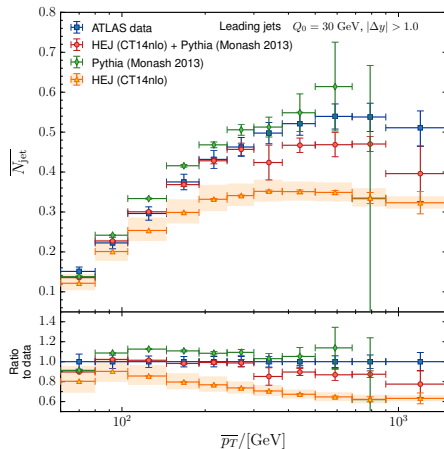
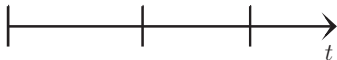
This method uses HEJ as a matrix element generator.

History of states is generated by Pythia, which then preforms trial emissions.

If trial emission is a HEJ like emission, veto with probability $P^{\text{HEJ}}/P^{\text{Pythia}}$ and move on.

If trial emission is not a HEJ state, accept it and shower freely from here.

First collinear (PS) emission is accepted.



Data from: [ATLAS Collaboration, 1407.5756]

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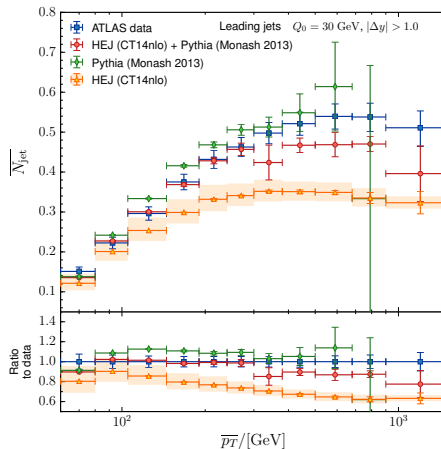
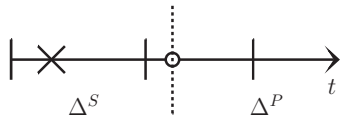
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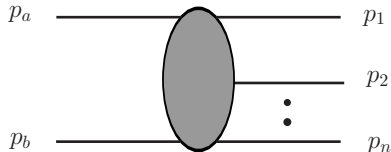
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Merging High-Energy and Soft-Collinear Resummation: HEJ+Pythia

HEJ States Classification

In the Regge limit the amplitudes have the following scaling

[R. Brower, C. DeTar, J. Weis, 1974]



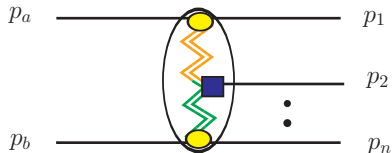
$$\mathcal{M} \propto s_{12}^{\alpha_{12}} s_{23}^{\alpha_{23}} \cdots s_{n-1,n}^{\alpha_{n-1,n}} f(\{p_{Ti}\})$$

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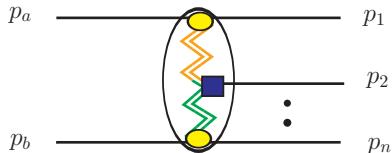
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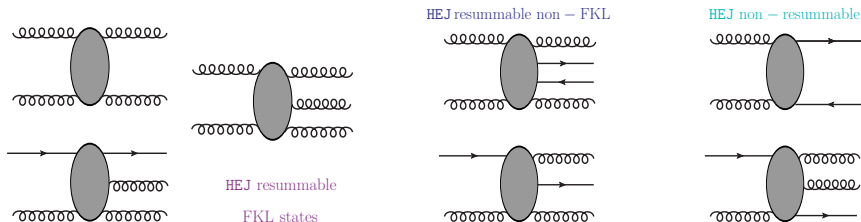
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α_{ij} is the spin in the effective t-channel.

The leading order input into HEJ is classified as either “resummable” or “non-resummable”



HEJ+Pythia: Merging procedure

We express HEJ in the parton shower language and feed in HEJ events.

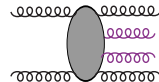
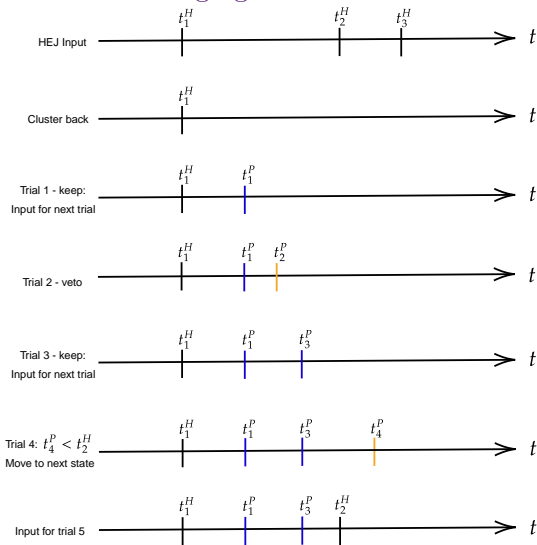
The HEJ+Pythia method constructs **histories** for resumable HEJ events. → this is to ensure that the full phase space of HEJ is explored by the shower.

A *history* is a sequence of states $\mathcal{H} = \{S_0, \dots, S_l\}$, which charts the evolution in a parton shower from the **most clustered** state S_0 to the input event S_l .

Then, we let parton shower add **subtracted shower emissions** in between the states of the history, and **continue the Pythia shower** with a subtracted splitting kernel until hadronisation

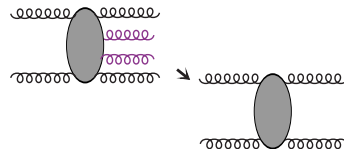
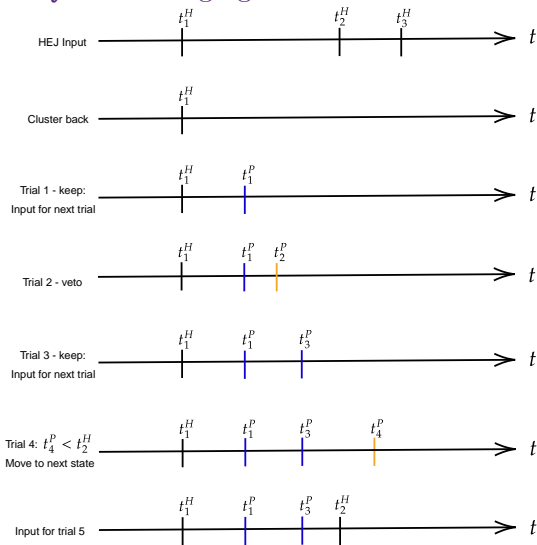
$$d\sigma_{m,n}^{\text{HEJ+Pythia}} = d\sigma_2^* \prod_{i=1}^{m-2} P_i^H \Delta_{i-1,i}^H \left[\prod_{\lambda=1}^{\lambda_i} P_{i_\lambda}^S \Delta_{i_{\lambda-1},i_\lambda}^S \right] \prod_{j=m-2+\mathcal{N}}^n P_j^S \Delta_{j-1,j}^S$$
$$\Delta^S(t_{i-1}, t_i) = \exp \left\{ - \int_{t_i}^{t_{i-1}} dt \int dz \underbrace{\Theta(P^P(t, z) - P^H(t, z)) [P^P(t, z) - P^H(t, z)]}_{\text{Subtracted splitting probability: } P^S(t, z)} \right\}$$

HEJ+PYTHIA: Merging



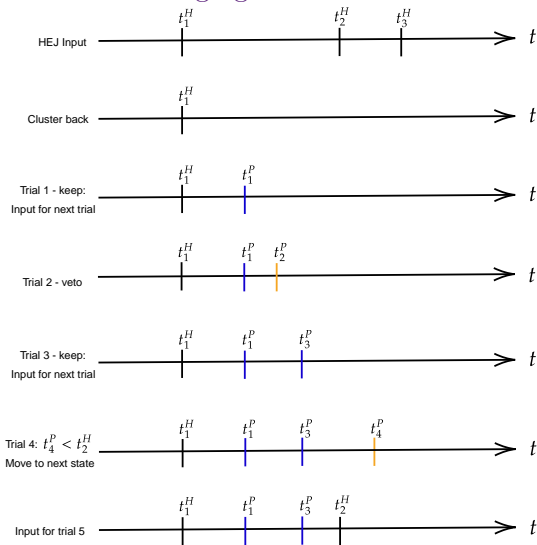
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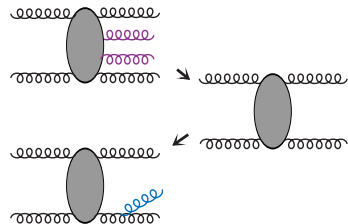


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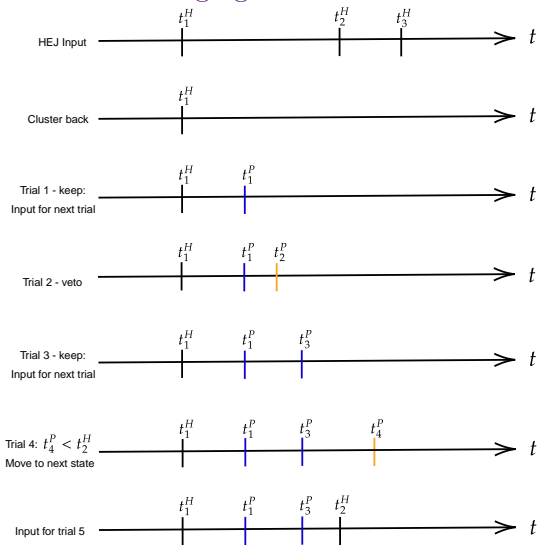
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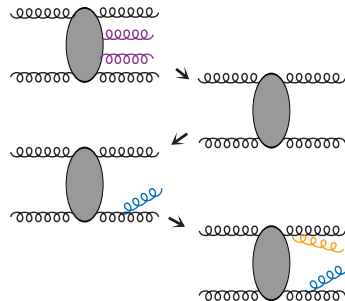
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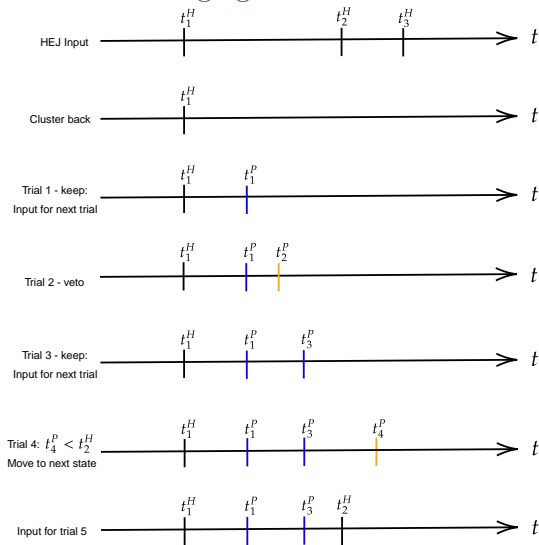
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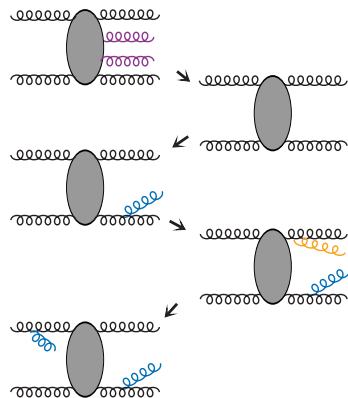
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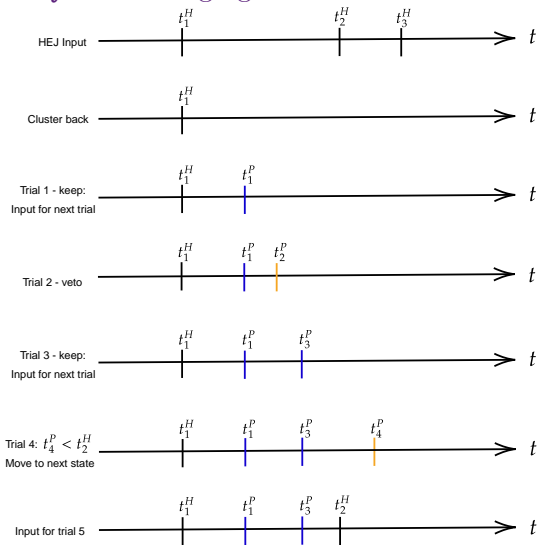
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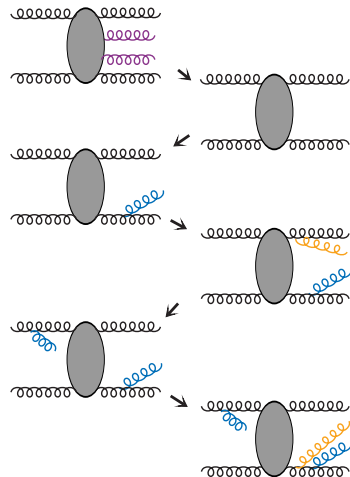
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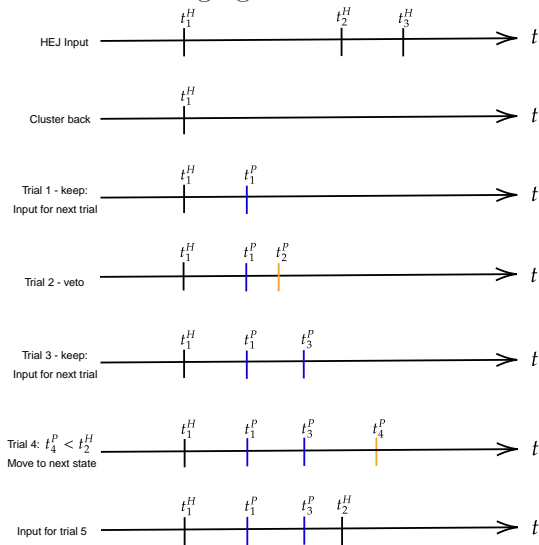
HEJ+Pythia: Merging



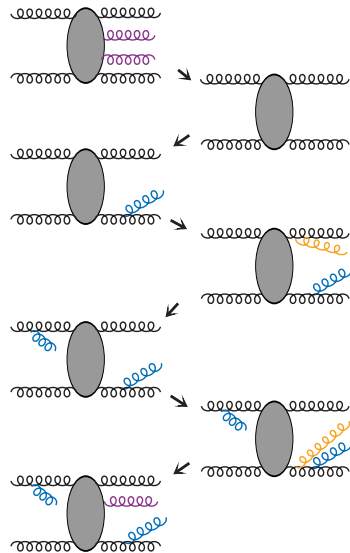
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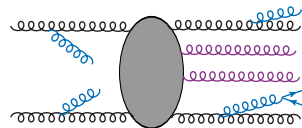
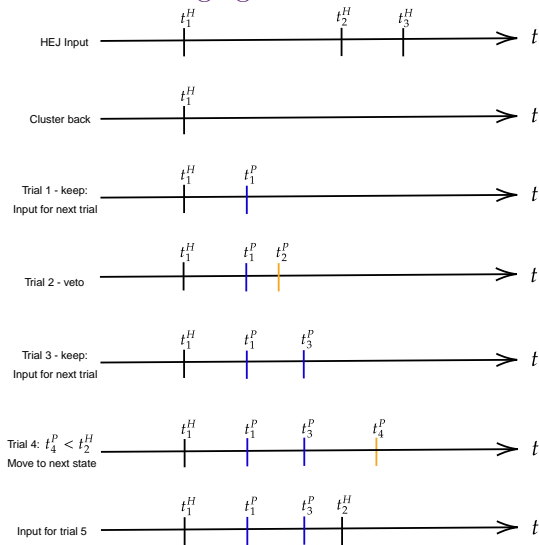
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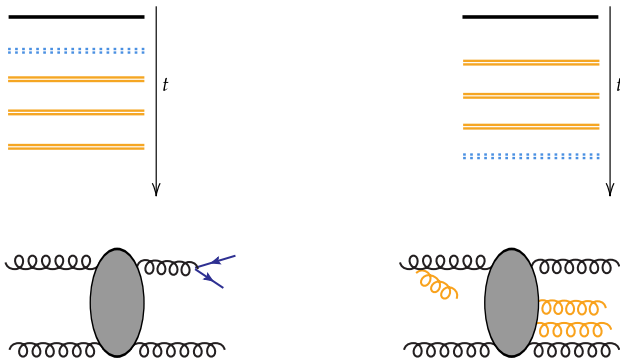
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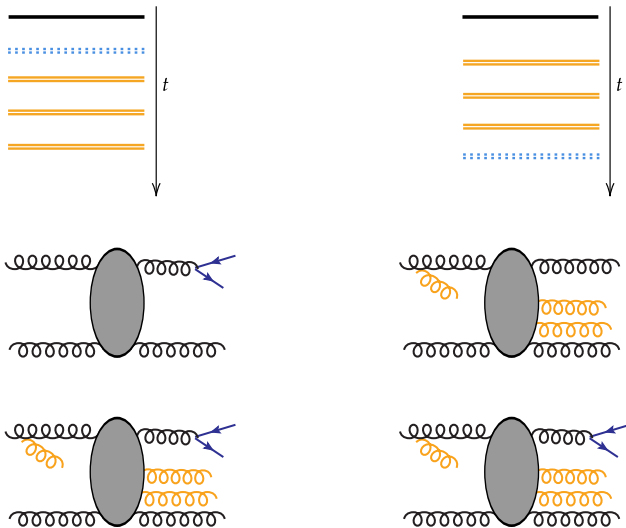
Classification of events

Consider the following evolution history:



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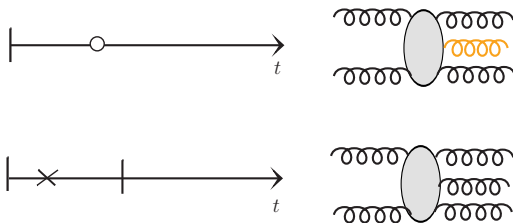


HEJ+Pythia: subtracted shower

Once the original HEJ input state is recovered and dressed, the subtracted shower continues until hadronization.

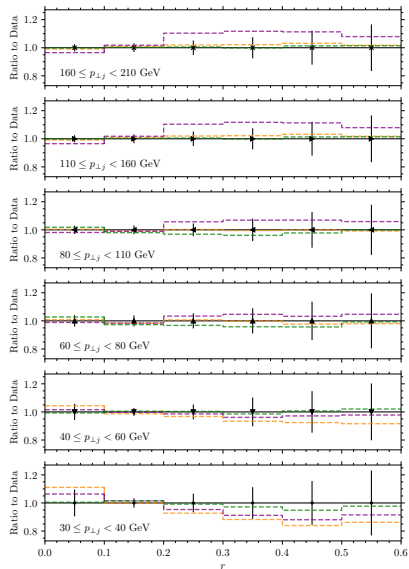
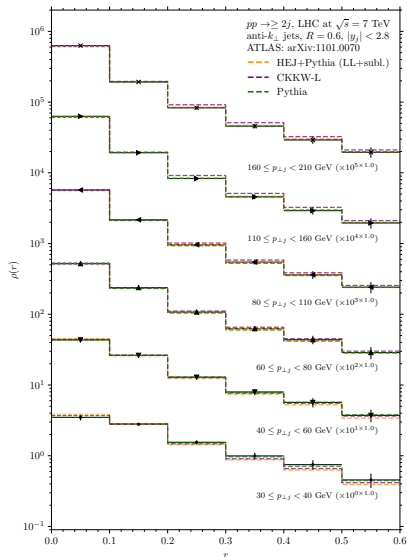
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Showered low multiplicity HEJ events can resemble higher multiplicity HEJ events \rightarrow subtraction needed.

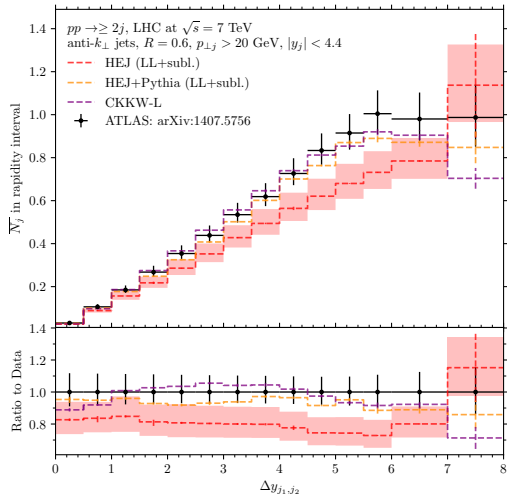
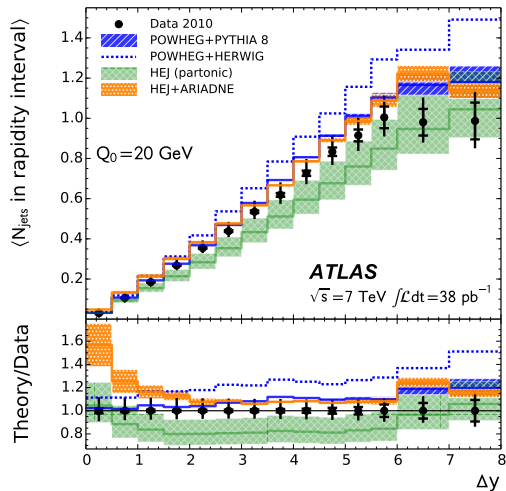


Results

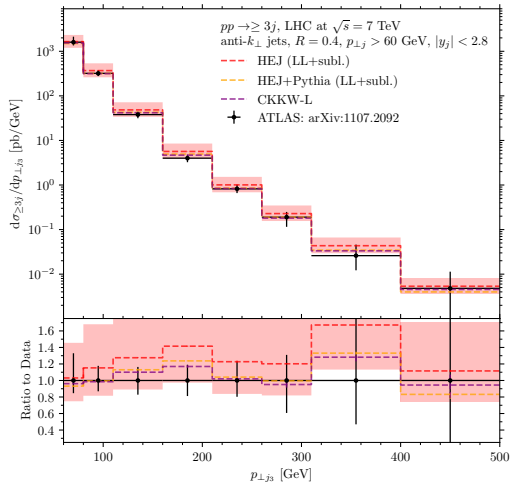
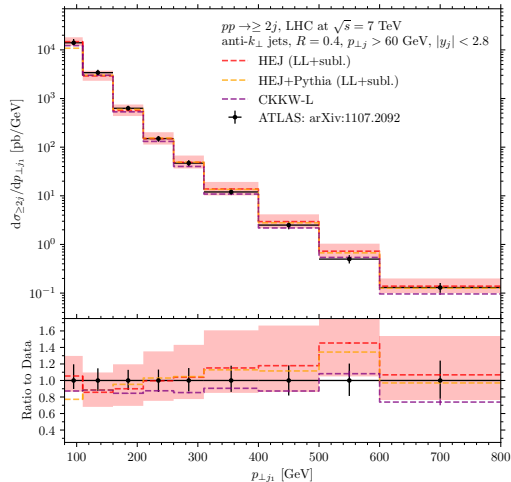
Results: Jet Shapes



Results



Results: Multi-jet Cross-sections



Summary

- ▶ Introduced improved method to combine high-energy resummation implemented in **HEJ** with soft-collinear logarithms of **Pythia**
- ▶ Implemented in **HEJ+Pythia** software
- ▶ We cover the full phase space of both **HEJ** and **Pythia** and subtract overlapping contributions, keeping accuracy of both and retaining LO accuracy
- ▶ Demonstrated correct implementation using jet shapes and multi-jet observables

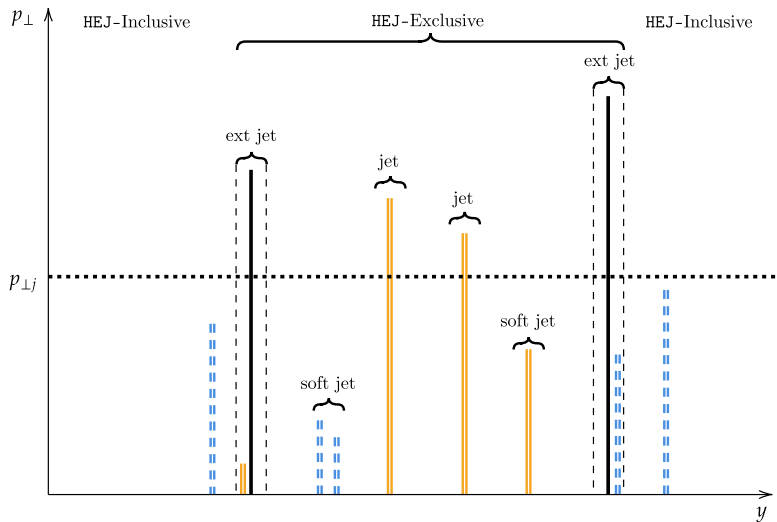
Outlook

- ▶ Implementation of this procedure to other processes, for example $W + \text{jets}$

Thank you!

Auxiliary slides

Phase space



When we add back an emission from a trial event to an arbitrary later stage in the history, we recoil the additional momentum according to the following global strategy:

1. Reshuffle the excess transverse momentum across the final state partons, conserving the mass and rapidity of each.
2. Rescale all transverse momenta by a constant factor λ such that the invariant mass of the initial state $\sqrt{\hat{s}}$ is conserved and again reassign the E and p_z components of each final state particle such that the rapidities and masses of each are conserved.
3. Sum over positive and negative lightcone components of the final state momenta to find physical analogues for the momenta of the initial state partons.
4. Boost along the z -axis such that the initial state momenta are the same as they were in the original state, using the momenta in step 3. to derive the rapidity ψ . This ensures that beam energies are not exceeded if many emissions are added.

Thank you