

# Absolute-mass threshold resummation for the production of four top quarks



Science & Technology  
Facilities Council



Melissa van Beekveld (RADCOR 2023)

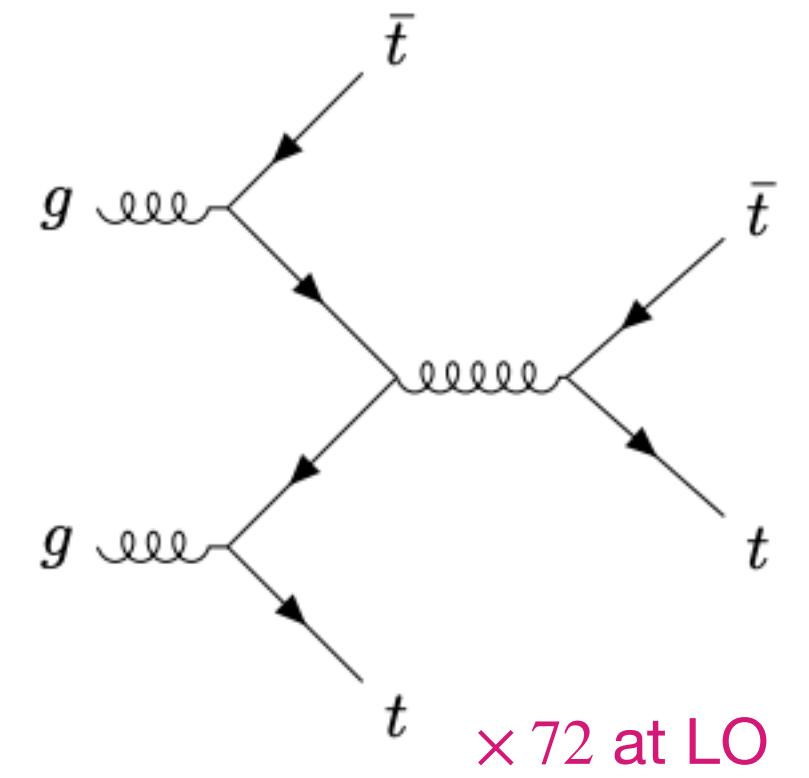
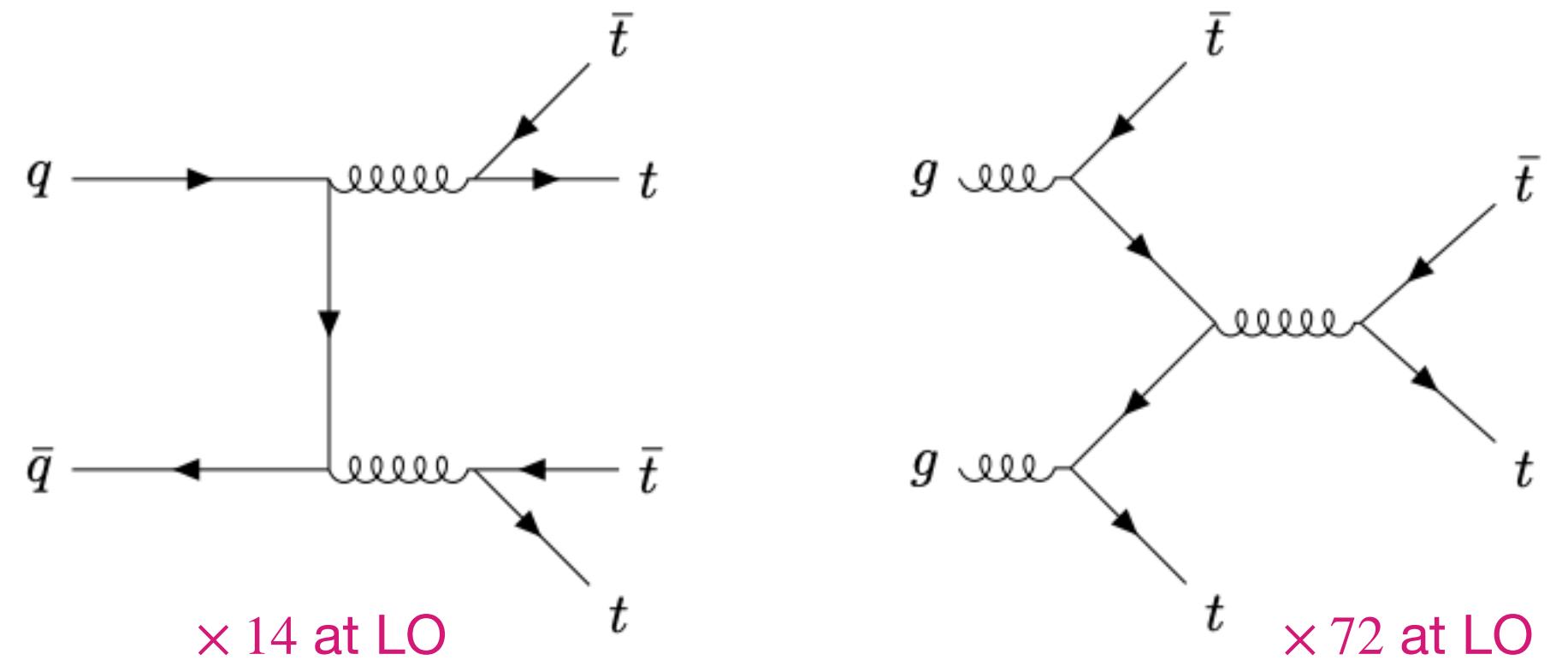
[2212.03259]

with Anna Kulesza, Laura Moreno Valero (Universität Münster)



UNIVERSITY OF  
**OXFORD**

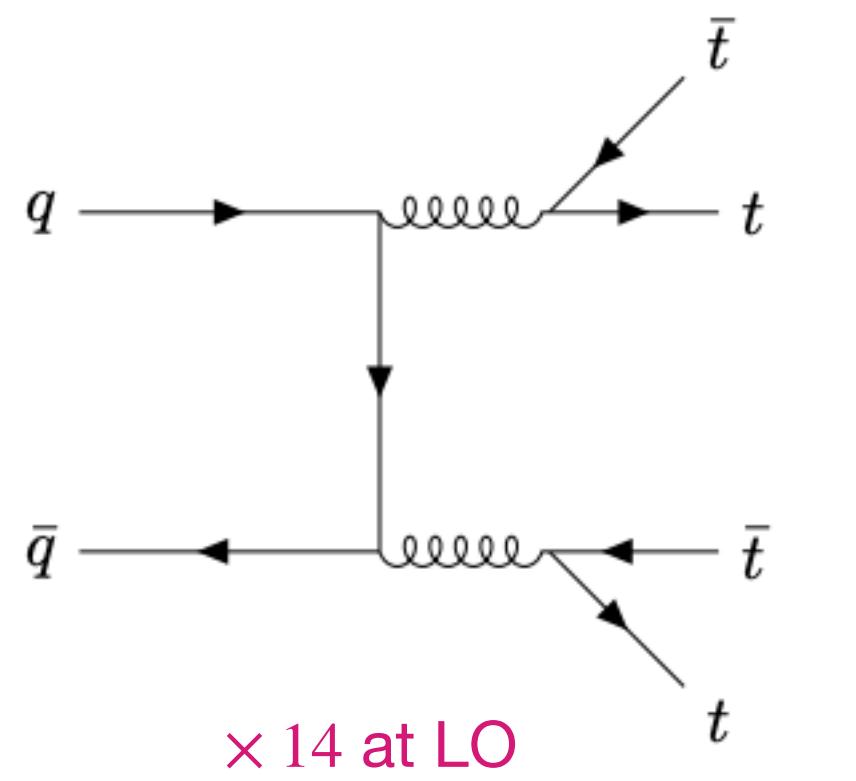
# Status of 4top - theory



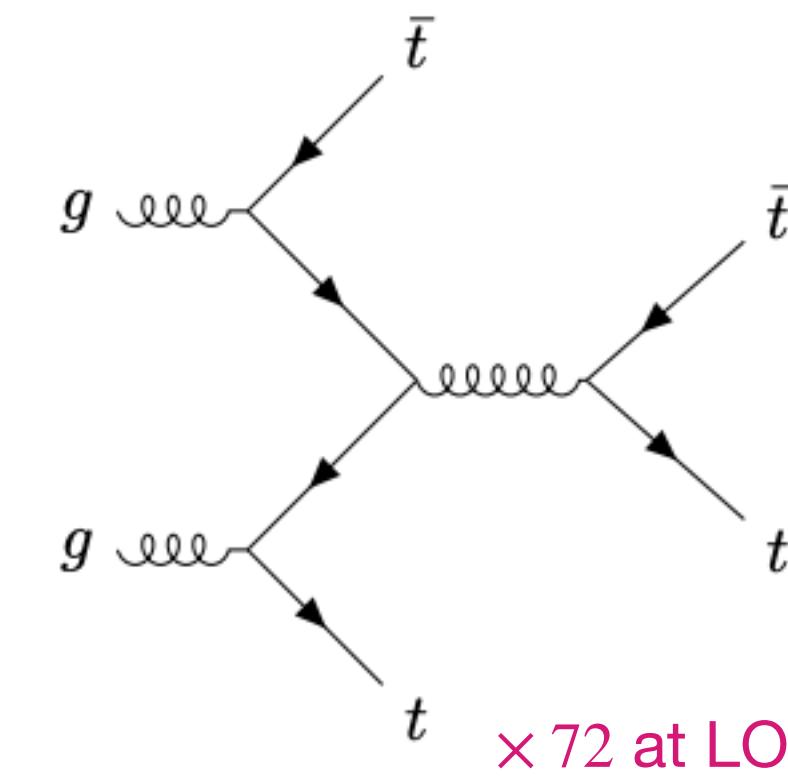
Pure QCD ( $\mathcal{O}(\alpha_s^4)$ )

- NLO calculated some time ago [1206.3064]
- MadGraph@NLO matched with parton showers [1405.0301, 1507.05640]

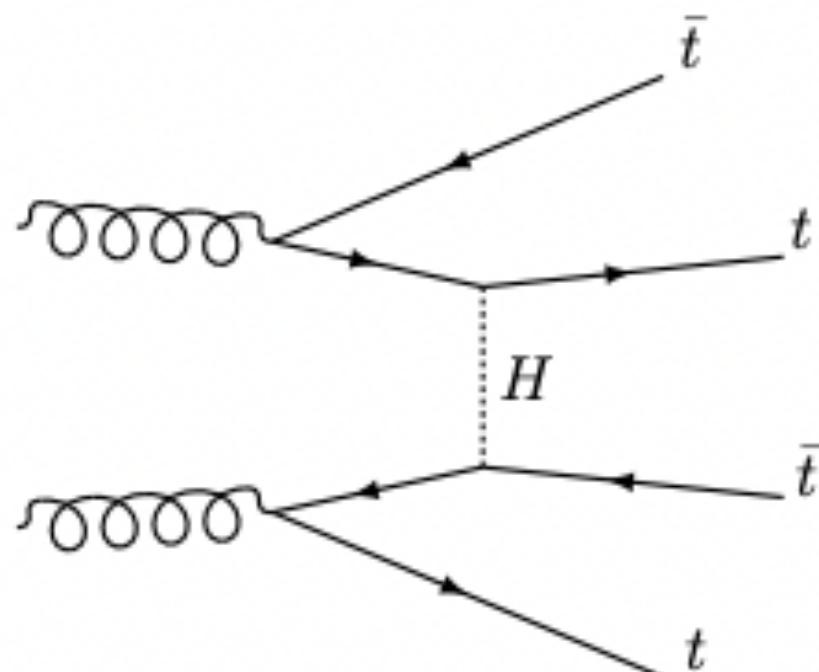
# Status of 4top - theory



Pure QCD ( $\mathcal{O}(\alpha_s^4)$ )

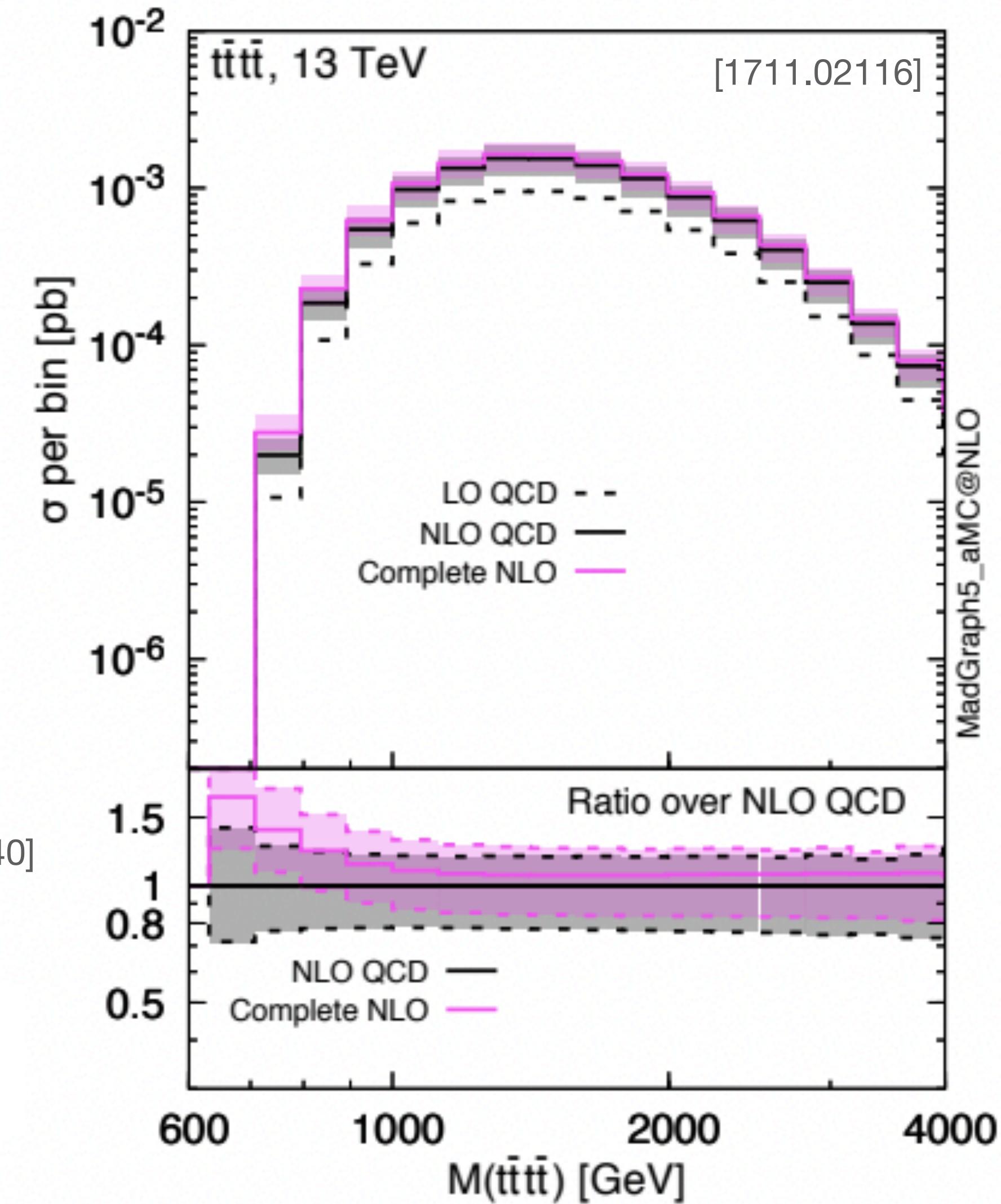


$\times 72$  at LO

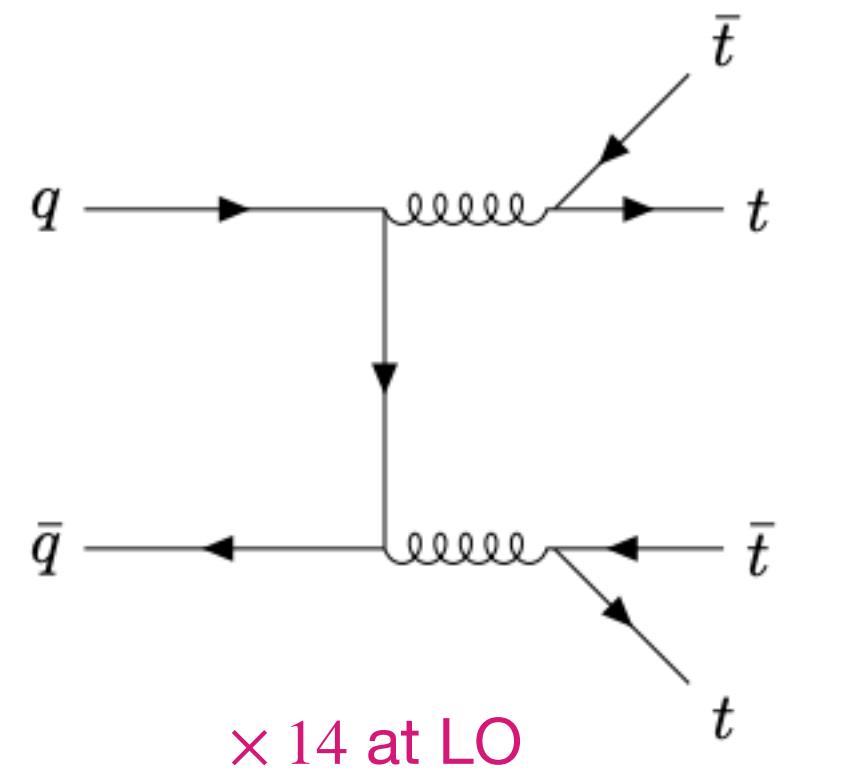


EW + QCD (i.e.  $\mathcal{O}(\alpha_s^2 \alpha^2)$ )

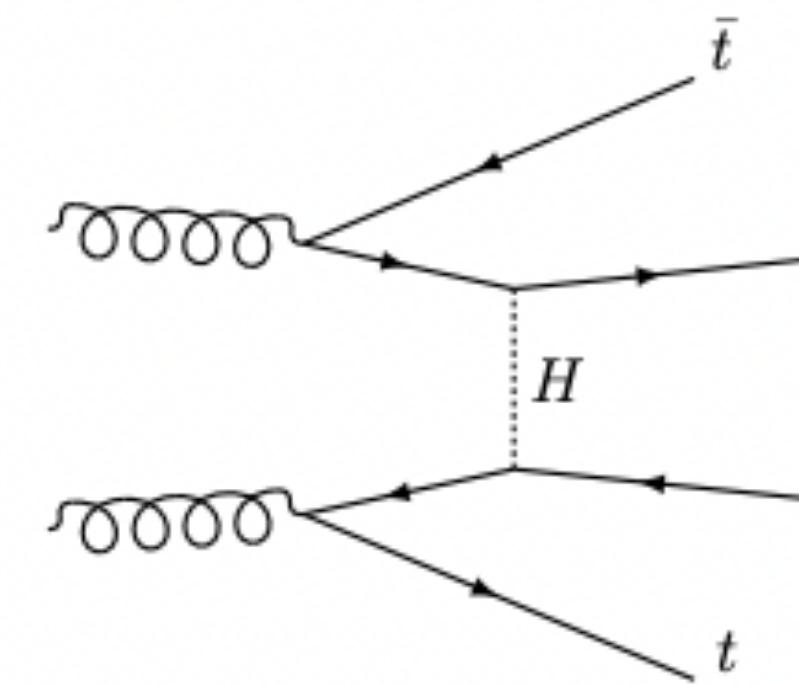
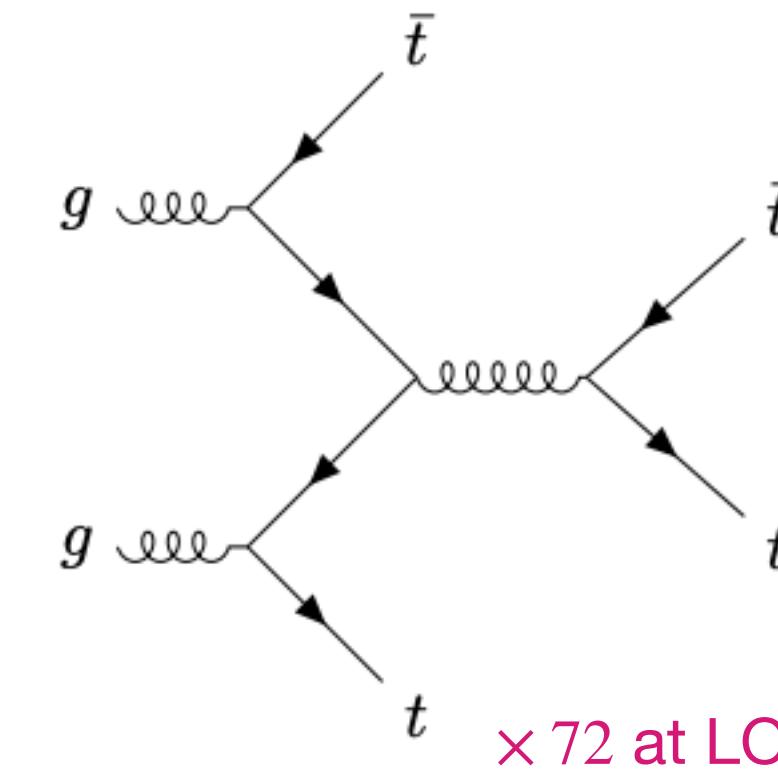
- NLO calculated some time ago [1206.3064]
- MadGraph@NLO matched with parton showers [1405.0301, 1507.05640]
- Including EW corrections [1711.02116]



# Status of 4top - theory

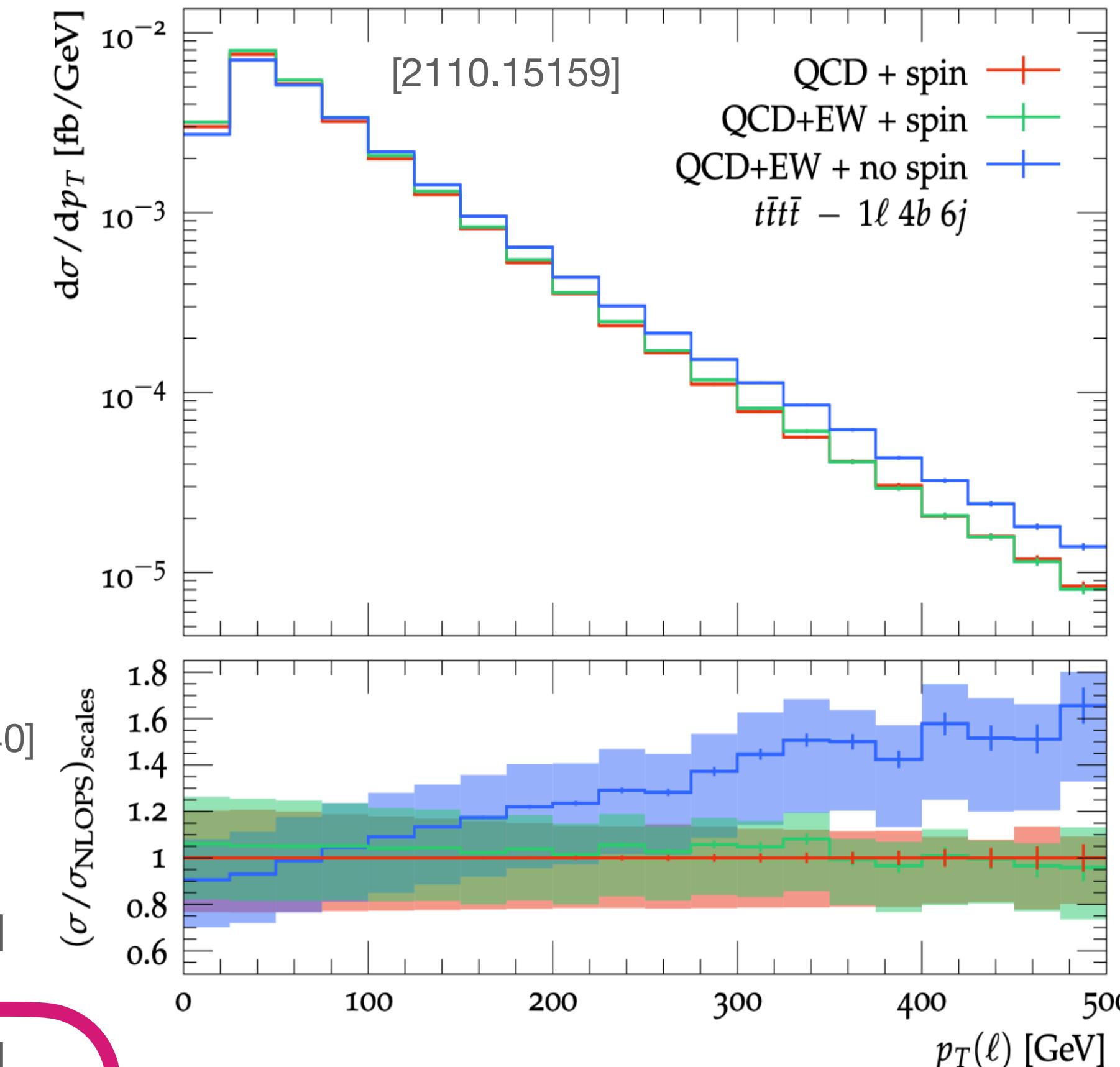


Pure QCD ( $\mathcal{O}(\alpha_s^4)$ )



EW + QCD (i.e.  $\mathcal{O}(\alpha_s^2 \alpha^2)$ )

- NLO calculated some time ago [1206.3064]
- MadGraph@NLO matched with parton showers [1405.0301, 1507.05640]
- Including EW corrections [1711.02116]
- POWHEG-BOX implementation with spin correlations [2110.15159]

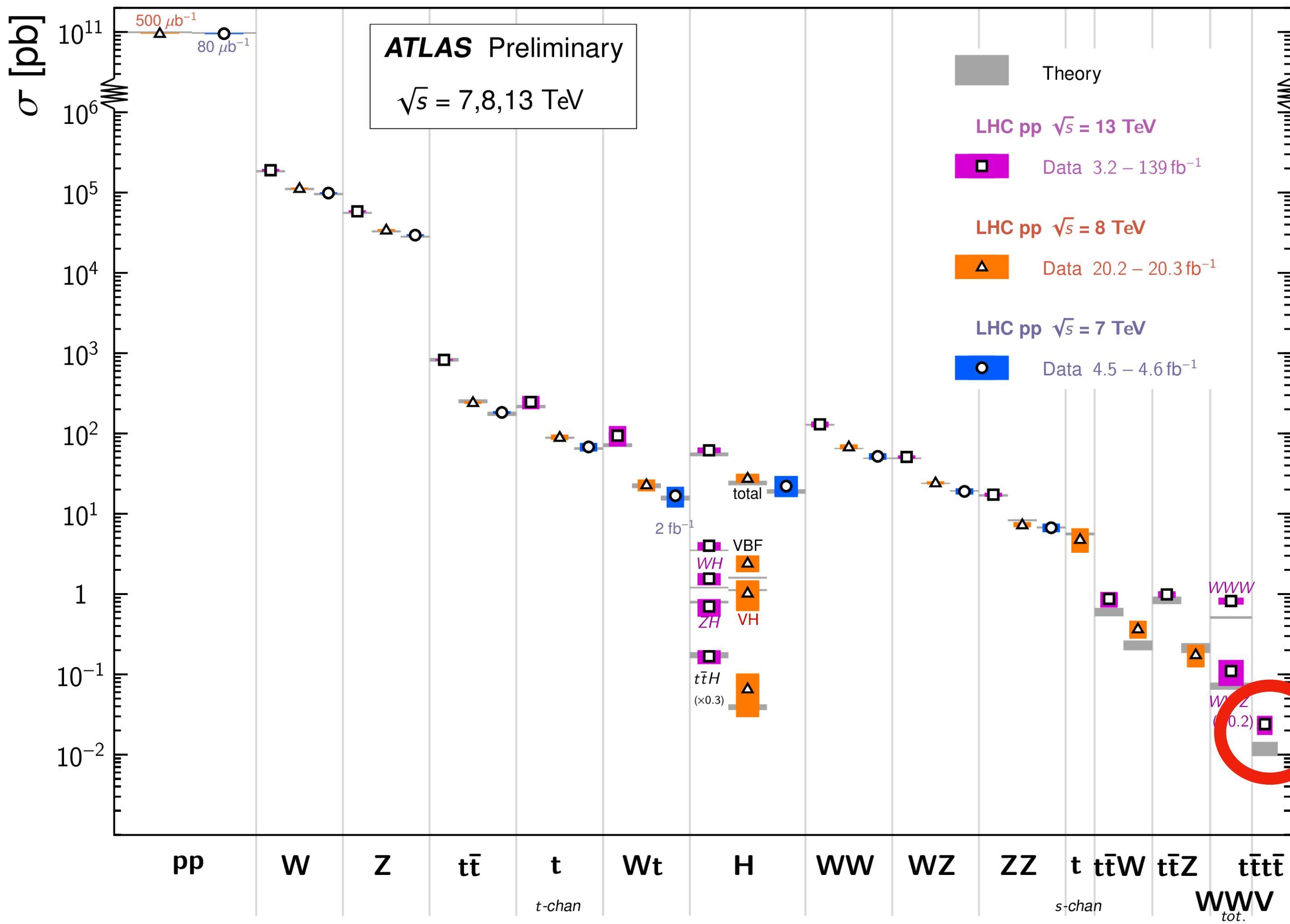


PDF	$\sigma^{\text{NLO}} \text{ [fb]}$	$\delta_{\text{scale}}$	$\delta_{\text{PDF}}$	$\mathcal{K} = \frac{\text{NLO}}{\text{LO}}$	[2110.15159]
NNPDF3.1	11.65	+1.98 (17%) -2.57 (22%)	+0.28 (2%) -0.28 (2%)	1.33	$\mu_0 = H_T/4$
MMHT	11.62	+1.95 (17%) -2.54 (22%)	+0.63 (5%) -0.53 (5%)	1.04	$m_t = 172.5 \text{ GeV}$
CT18	11.74	+1.97 (17%) -2.56 (22%)	+0.46 (4%) -0.36 (3%)	1.06	Including EW corrections

# Status of 4top - experiment

## Standard Model Total Production Cross Section Measurements

Status: February 2022



Significance of  $> 5\sigma$  obtained!

In two same-sign /  $> 3$  lepton final-state channel

$$\sigma_{t\bar{t}t\bar{t}}^{\text{ATLAS}} = 22.5^{+4.7}_{-3.4}(\text{stat})^{+4.6}_{-3.4}(\text{sys}) \text{ fb} \quad [2303.15061]$$

$$\sigma_{t\bar{t}t\bar{t}}^{\text{CMS}} = 17.7^{+3.7}_{-3.5}(\text{stat})^{+2.3}_{-1.9}(\text{sys}) \text{ fb} \quad [2305.13439]$$

Both relatively high in comparison to

$$\sigma_{t\bar{t}t\bar{t}}^{\text{theory}} = 12.0 \text{ fb} \quad [1711.02116]$$

Evidence also in other final states

In one/two opposite-sign lepton final-state channel

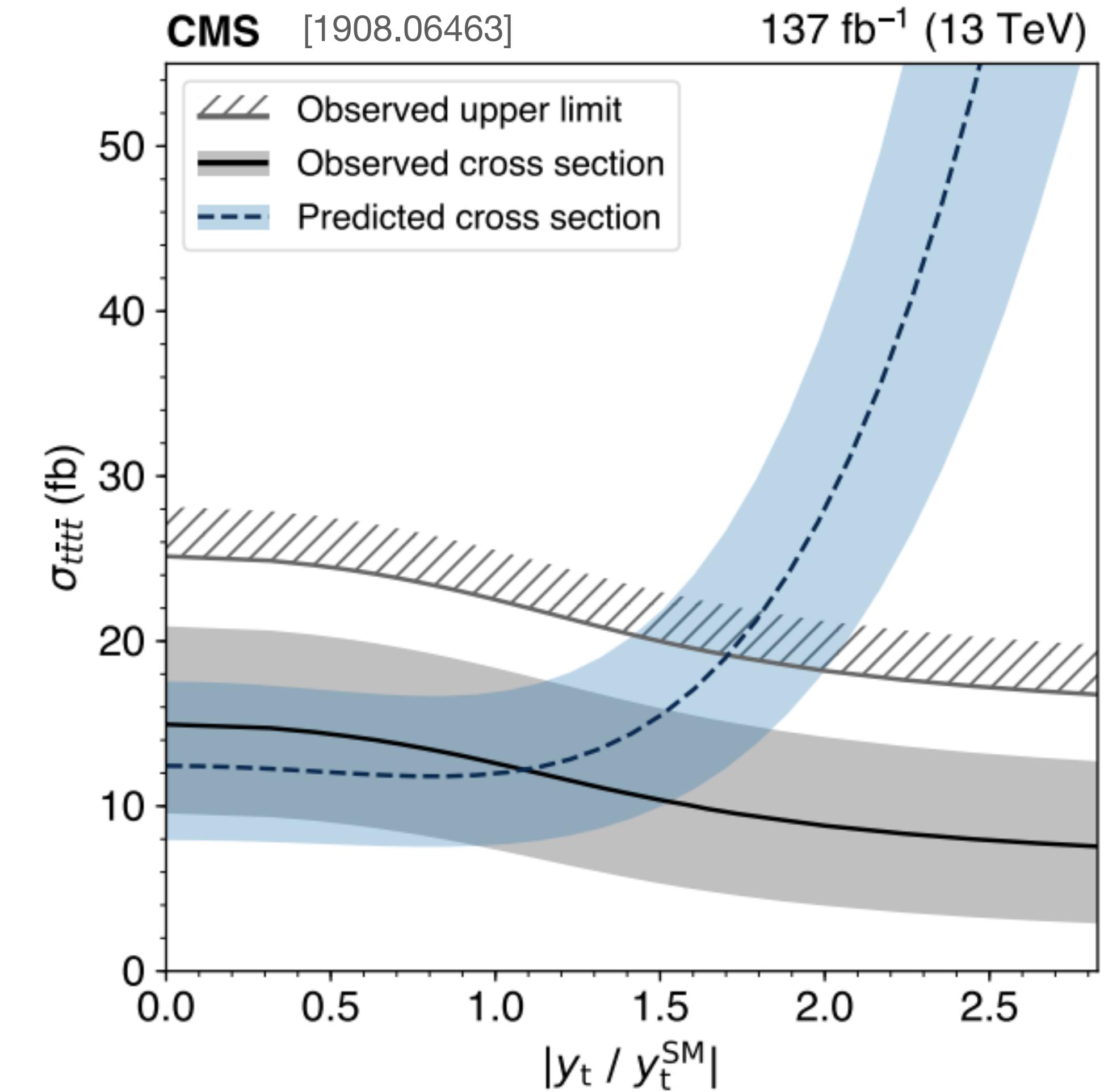
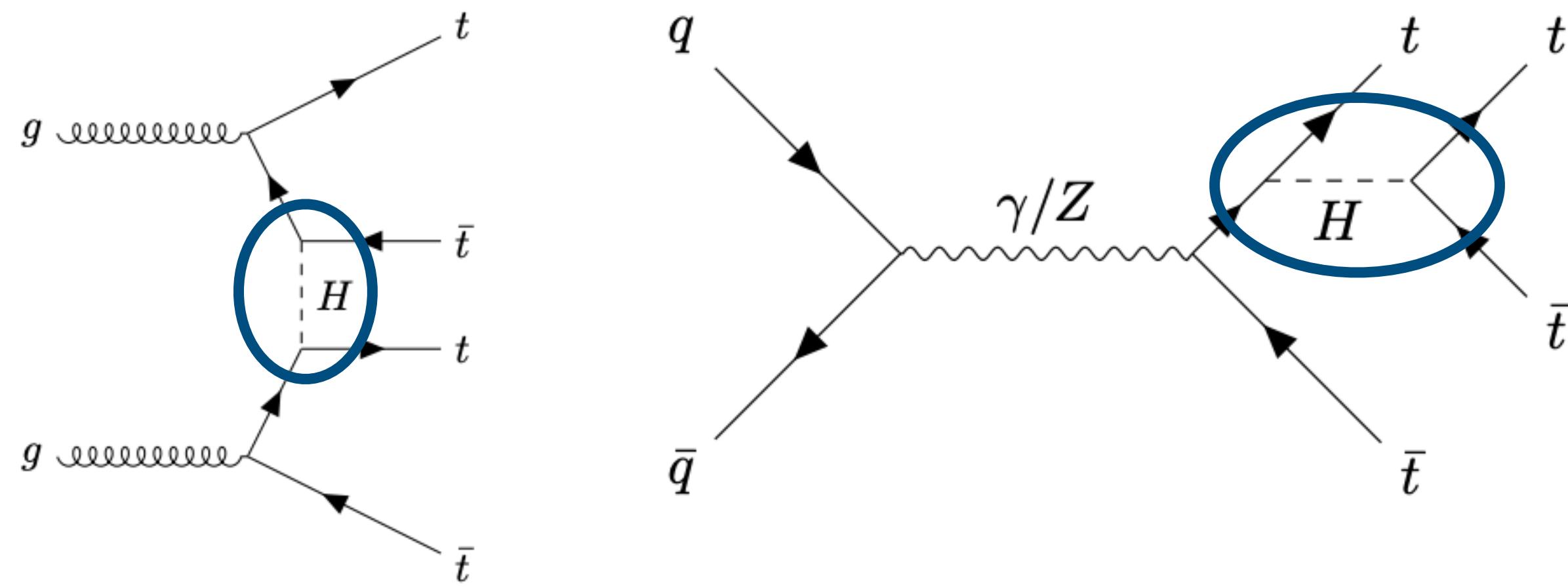
+ 10/8 jets...

$$\sigma_{t\bar{t}t\bar{t}}^{\text{ATLAS}} = 26 \pm 8(\text{stat.})^{+15}_{-13}(\text{syst.}) \text{ fb} \quad [2106.11683]$$

$$\sigma_{t\bar{t}t\bar{t}}^{\text{CMS}} = 36^{+12}_{-11} \text{ fb} \quad \text{includes 0 lepton channel} \quad [2303.03864]$$

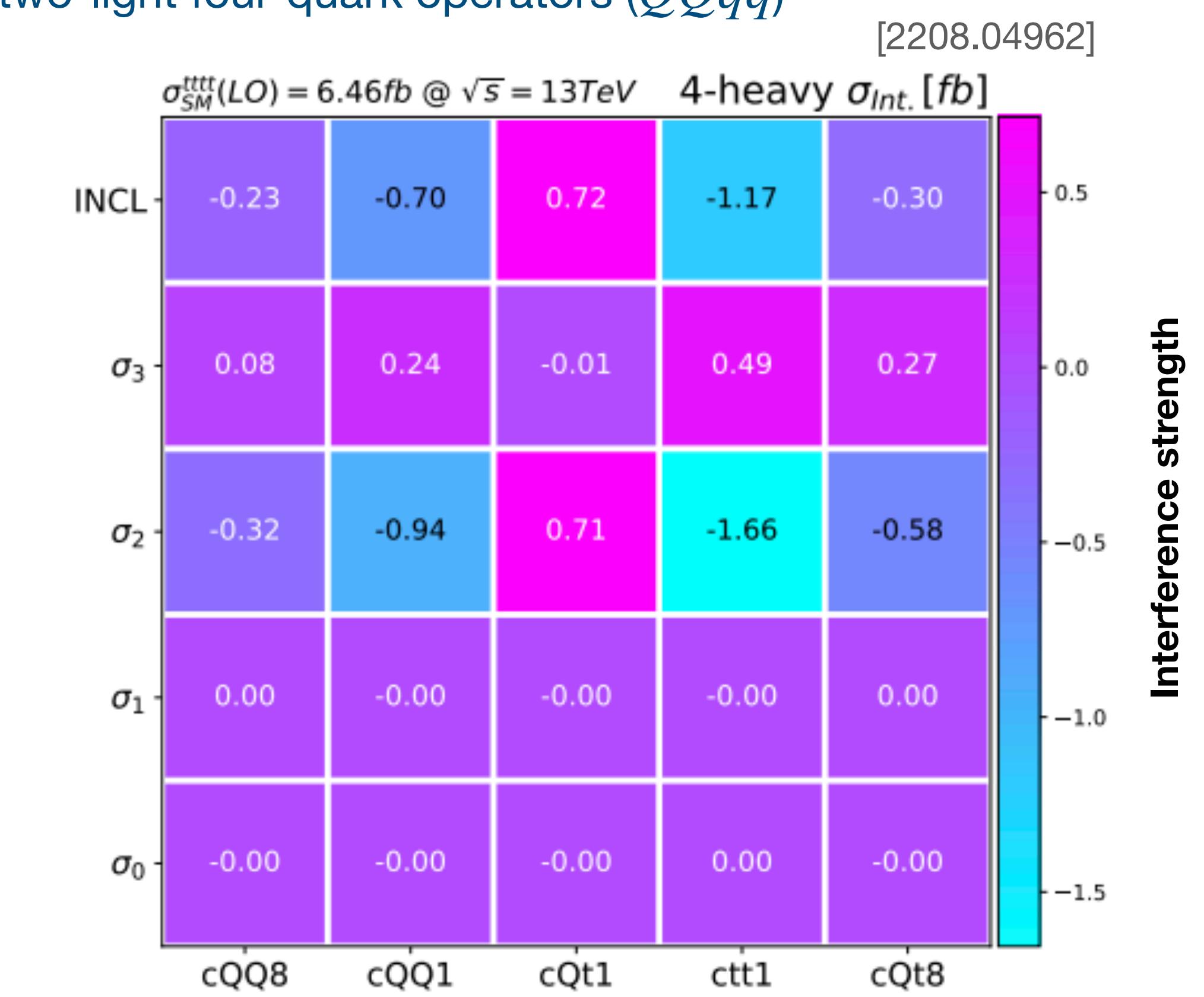
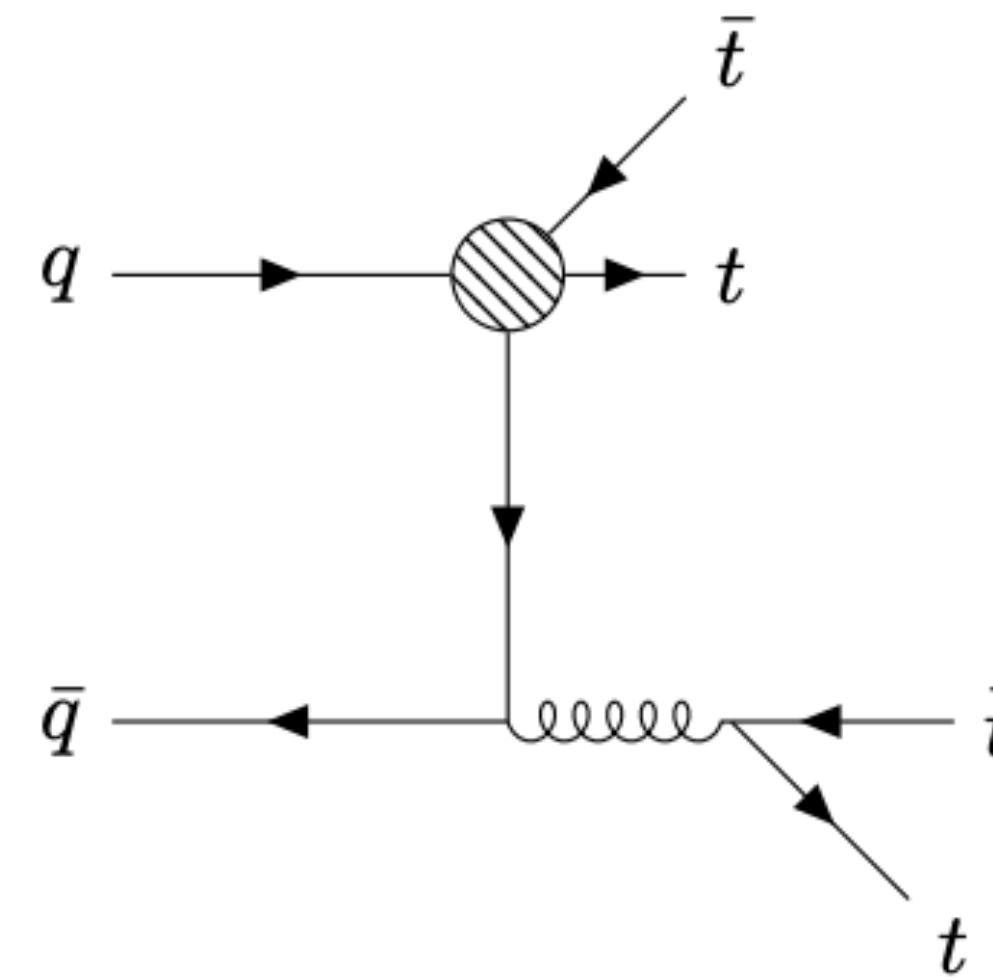
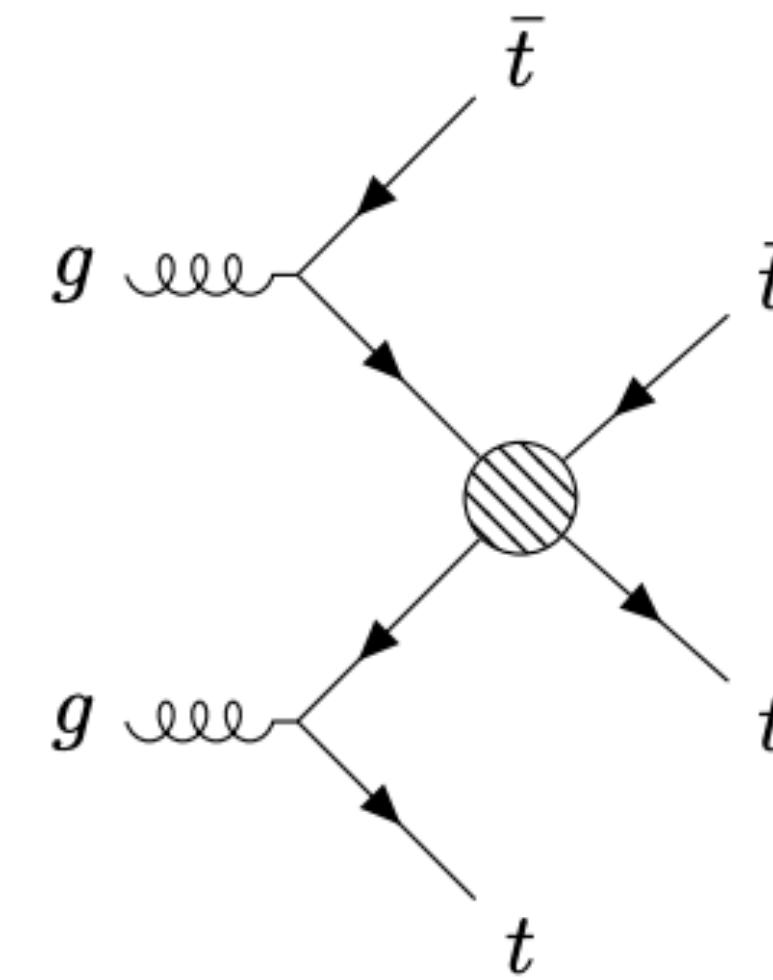
# Why 4top?

- Sensitive to the Yukawa coupling



# Why 4top?

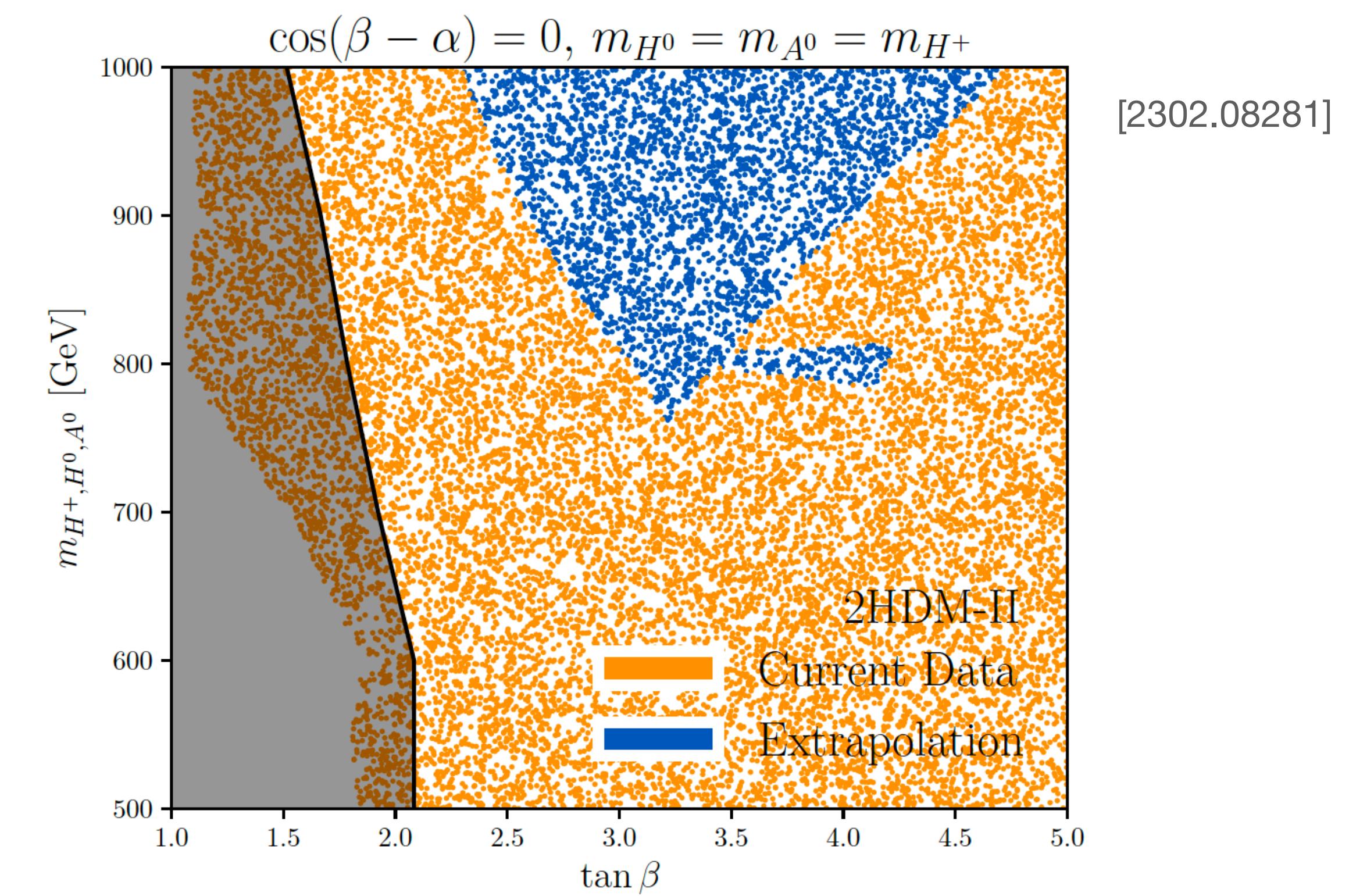
- Sensitive to the Yukawa coupling
- Constrain EFT coefficients
  - Effective **four-heavy-quark operators** ( $QQQQ$ ) and two-heavy-two-light four-quark operators ( $QQqq$ )
  - Dimension-6 operator  $\hat{H}$  that modifies Higgs propagator



# Why 4top?

- Sensitive to the Yukawa coupling
- Constrain EFT coefficients
  - Effective four-heavy-quark operators ( $QQQQ$ ) and two-heavy-two-light four-quark operators ( $QQqq$ )
  - Dimension-6 operator  $\hat{H}$  that modifies Higgs propagator
- Probe the presence of new particles
  - Simplified DM models
  - Type II **two Higgs doublet models**
  - SUSY (both minimal and non-minimal models)

$$\mathcal{L}_{\text{2HDM}} \supset -\frac{m_t}{v} (\xi_h \bar{t} t h + \xi_H \bar{t} t H - i \xi_A \bar{t} \gamma^5 t A)$$



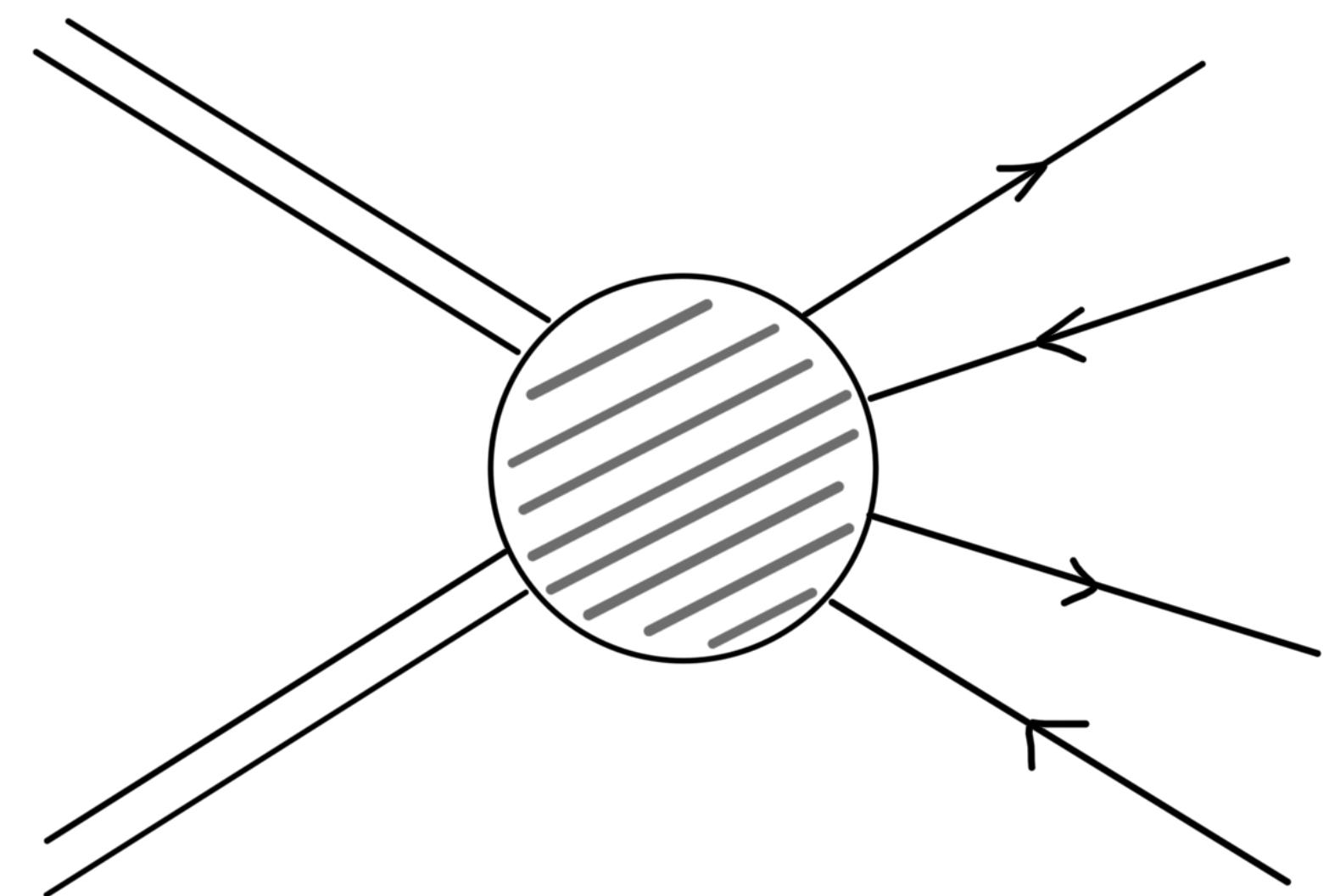
# Improving perturbation theory

Fixed-order description of the total cross section

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \dots$$

( $c_0$  normalised to  $\alpha_s^4$ )

Far out of reach for 4top!



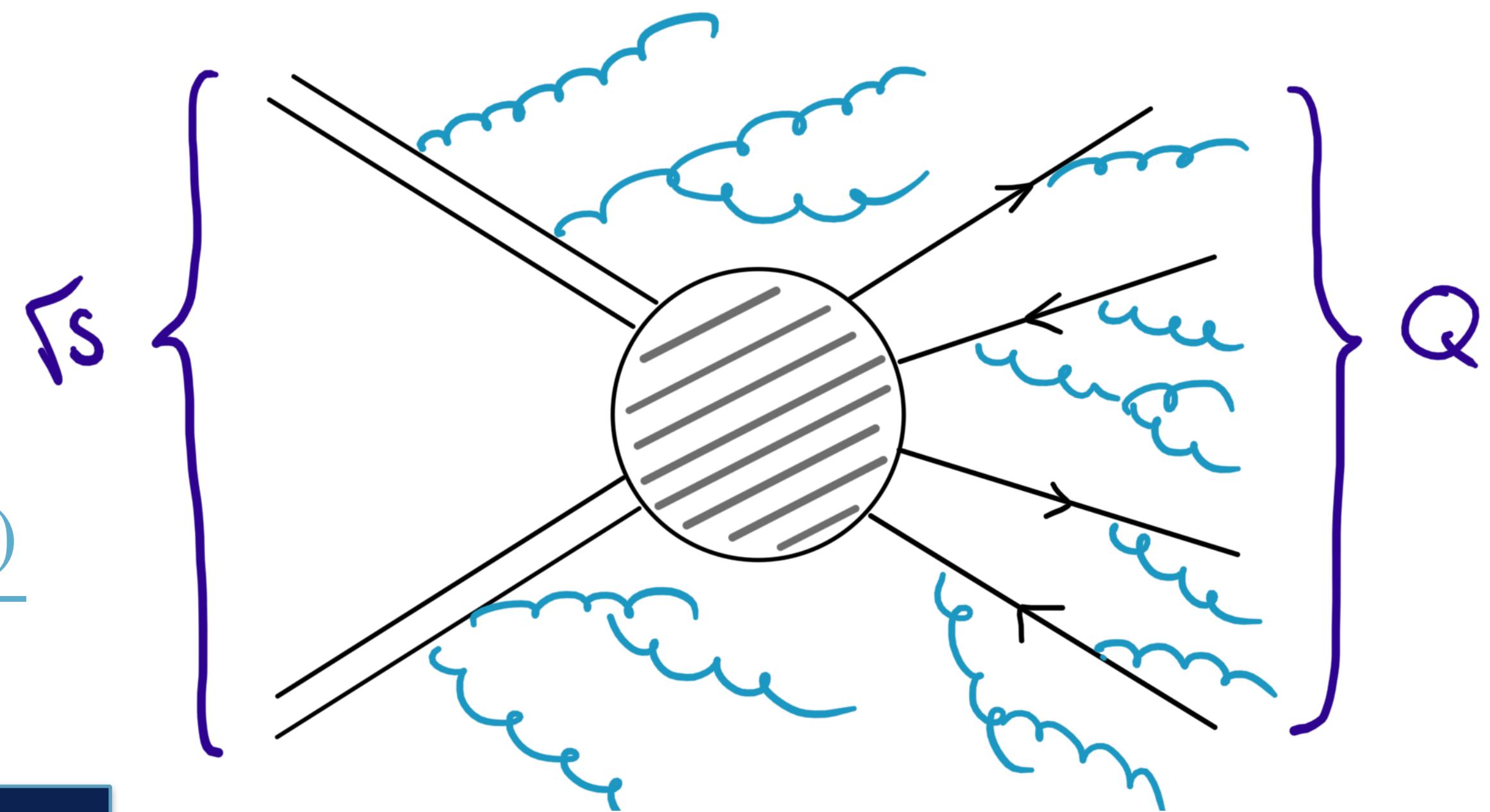
# Improving perturbation theory

Fixed-order description of the total cross section

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \dots$$

$$c_n = \sum_{k=0}^{k_{\max,n}} d_{nk} L^k + f_n,$$

$$L \text{ depends on observable, e.g. } L^k = \frac{\ln^{2k-1}(1 - Q^2/s)}{1 - Q^2/s}$$



Corrections can be predicted accurately in the limit that  
 $L^k \rightarrow \infty$  for  $Q^2 \simeq s$  (soft/collinear gluon emissions)

# Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
$N^n$ LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	...

$$\sigma_{\text{resum}} = h(\alpha_s) e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \dots$$

i.e. [hep-ph/0306211]

# Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
N <sup>n</sup> LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	...

$$\sigma_{\text{resum}} = h(\alpha_s) e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \dots$$

Leading-Log (LL)

i.e. [hep-ph/0306211]

# Resummation: A new series

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i.e. [hep-ph/0306211]

Next-to-Leading-Log (NLL)

# Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
N <sup>n</sup> LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	...

$$\sigma_{\text{resum}} = h(\alpha_s) e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \dots$$

i.e. [hep-ph/0306211]

obeys perturbative expansion;  
including  $\mathcal{O}(\alpha_s)$  leads to NLL' accuracy

# Resummation for 4top

Logarithmic corrections grow large when  $\sqrt{s} \rightarrow M \equiv 4m_t$  (*the absolute-threshold limit*)

$$\sigma_{t\bar{t}t\bar{t}}(N) = \int_0^1 d\tau \tau^{N-1} \sigma_{t\bar{t}t\bar{t}}(\tau) \quad \tau = \frac{M^2}{S} = x_1 x_2 \rho$$

*Transformation to Mellin space helps to obtain closed forms for the phase-space integrals*

Usual factorisation for the hadronic total production cross section:

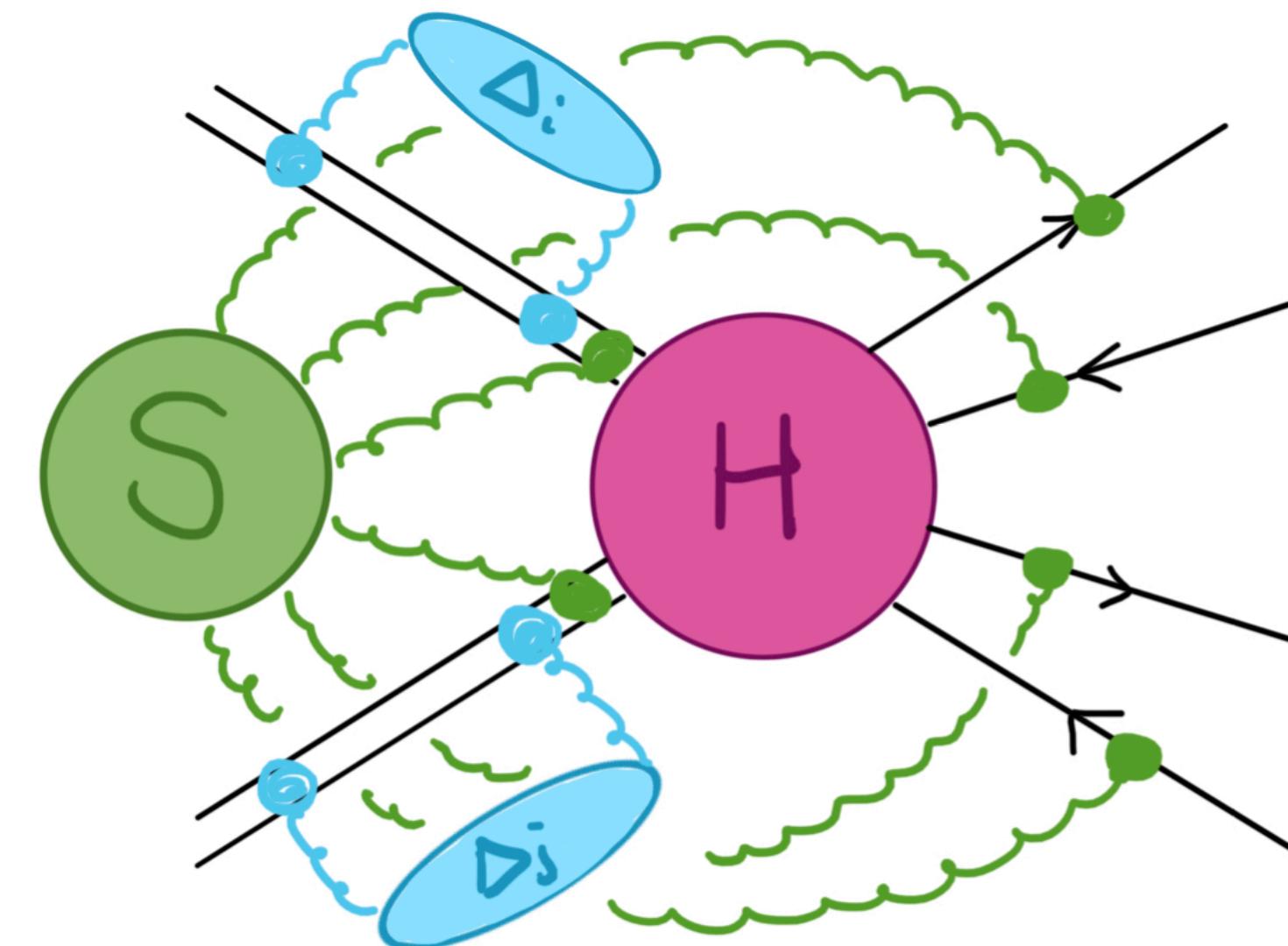
$$\sigma_{t\bar{t}t\bar{t}}(\rho) = \int_0^1 dx_1 f_i(x_1, \mu_F^2) \int_0^1 dx_2 f_j(x_2, \mu_F^2) \int_0^1 d\rho \delta\left(\rho - \frac{\tau}{x_1 x_2}\right) \hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}(\rho)$$

*partonic cross section obeys refactorisation  
when all radiation is soft and/or collinear*

# Resummation for 4top

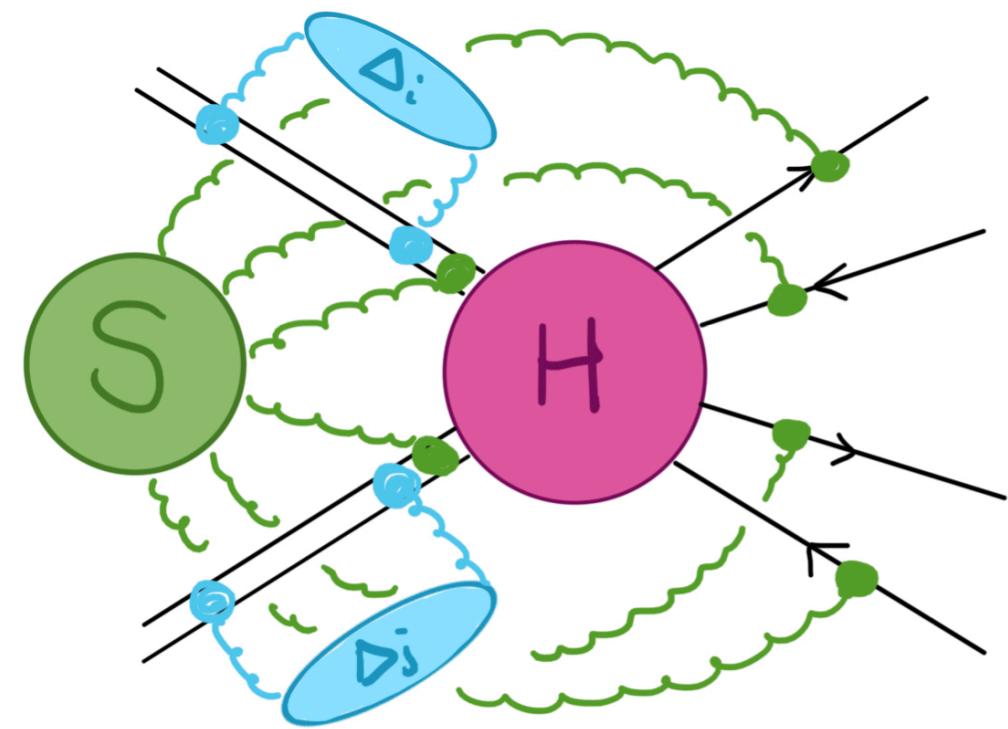
Mellin-space resummed cross section

$$\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N) = \text{Tr} \left[ \begin{array}{c} \mathbf{H}_{ij \rightarrow t\bar{t}t\bar{t}} \quad \bar{\mathbf{U}}_{ij \rightarrow t\bar{t}t\bar{t}} \quad \tilde{\mathbf{S}}_{ij \rightarrow t\bar{t}t\bar{t}} \quad \mathbf{U}_{ij \rightarrow t\bar{t}t\bar{t}} \end{array} \right] \Delta_i \Delta_j$$



# Resummation for 4top

Mellin-space resummed cross section



$$\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N) = \text{Tr} \left[ \begin{array}{c} \mathbf{H}_{ij \rightarrow t\bar{t}t\bar{t}} \quad \bar{\mathbf{U}}_{ij \rightarrow t\bar{t}t\bar{t}} \quad \tilde{\mathbf{S}}_{ij \rightarrow t\bar{t}t\bar{t}} \quad \mathbf{U}_{ij \rightarrow t\bar{t}t\bar{t}} \end{array} \right] \Delta_i \Delta_j$$

Incoming jet functions  
capture soft-collinear enhancements

$$\Delta_i = \exp \left[ \frac{1}{\alpha_s} g_1(\lambda) + g_2(\lambda, \mu_R/M, \mu_F/M) \right]$$

Needed at LL

Needed at NLL

$$\lambda = \alpha_s \ln (\bar{N}) = \alpha_s (\ln N + \gamma_E)$$

# Resummation for 4top

Mellin-space resummed cross section

Soft function  
*captures wide-angle-soft enhancements*

$$\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N) = \text{Tr} \left[ H_{ij \rightarrow t\bar{t}t\bar{t}} \begin{array}{|c|} \hline \bar{U}_{ij \rightarrow t\bar{t}t\bar{t}} \tilde{S}_{ij \rightarrow t\bar{t}t\bar{t}} U_{ij \rightarrow t\bar{t}t\bar{t}} \\ \hline \end{array} \right] \Delta_i \Delta_j$$

Incoming jet functions  
*capture soft-collinear enhancements*

Result of RGE equation with evolution matrix

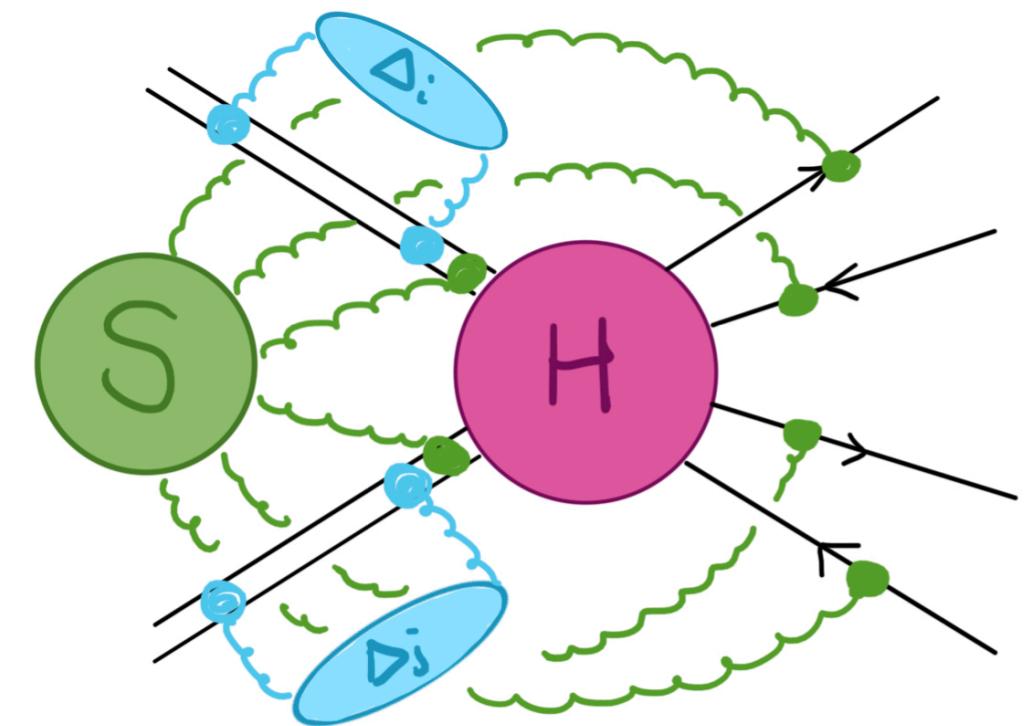
$$U_{ij \rightarrow t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathcal{P} \exp \left[ \int_{\mu_R}^{M/\bar{N}} \frac{dq}{q} \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right]$$

And boundary condition at  $\bar{N} = M/\mu_R$

$$\tilde{S} = \tilde{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi} \tilde{S}^{(1)} + \dots$$

Needed  
at NLL

Needed  
at NLL'



# Colour structure 4top

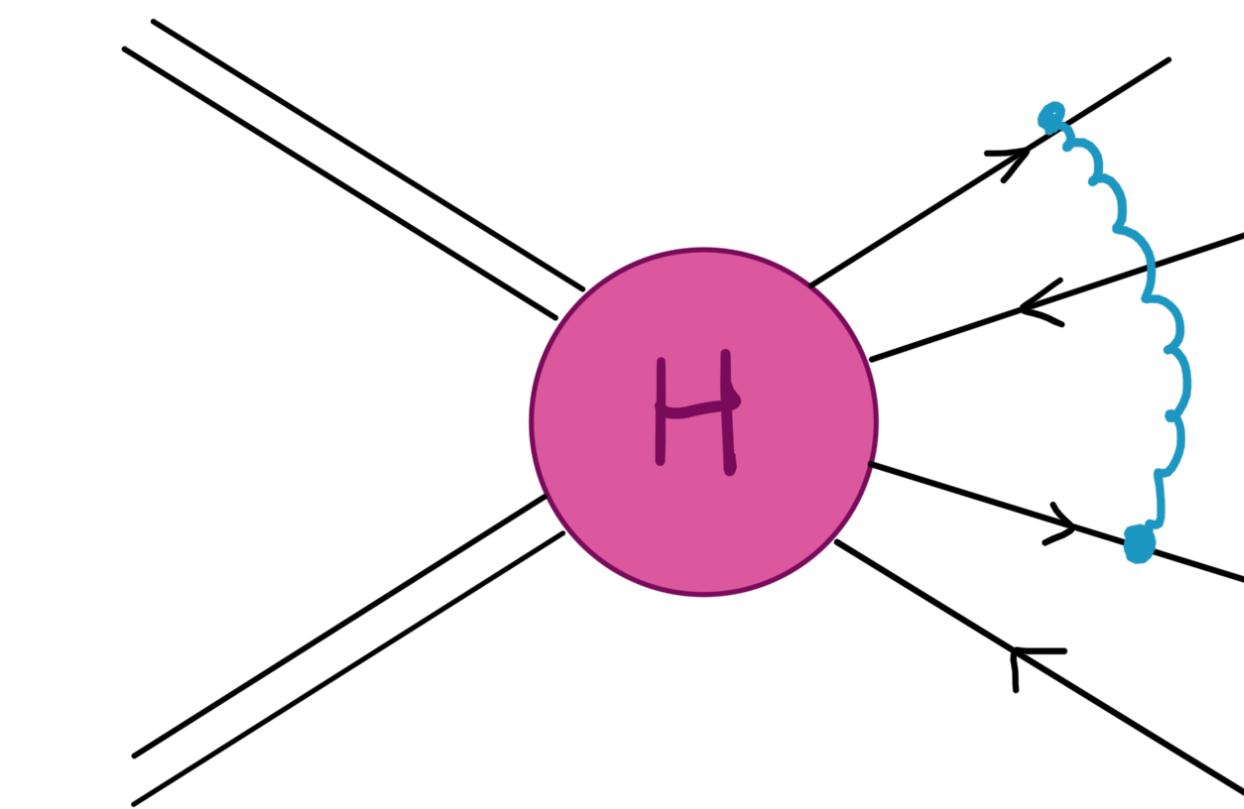
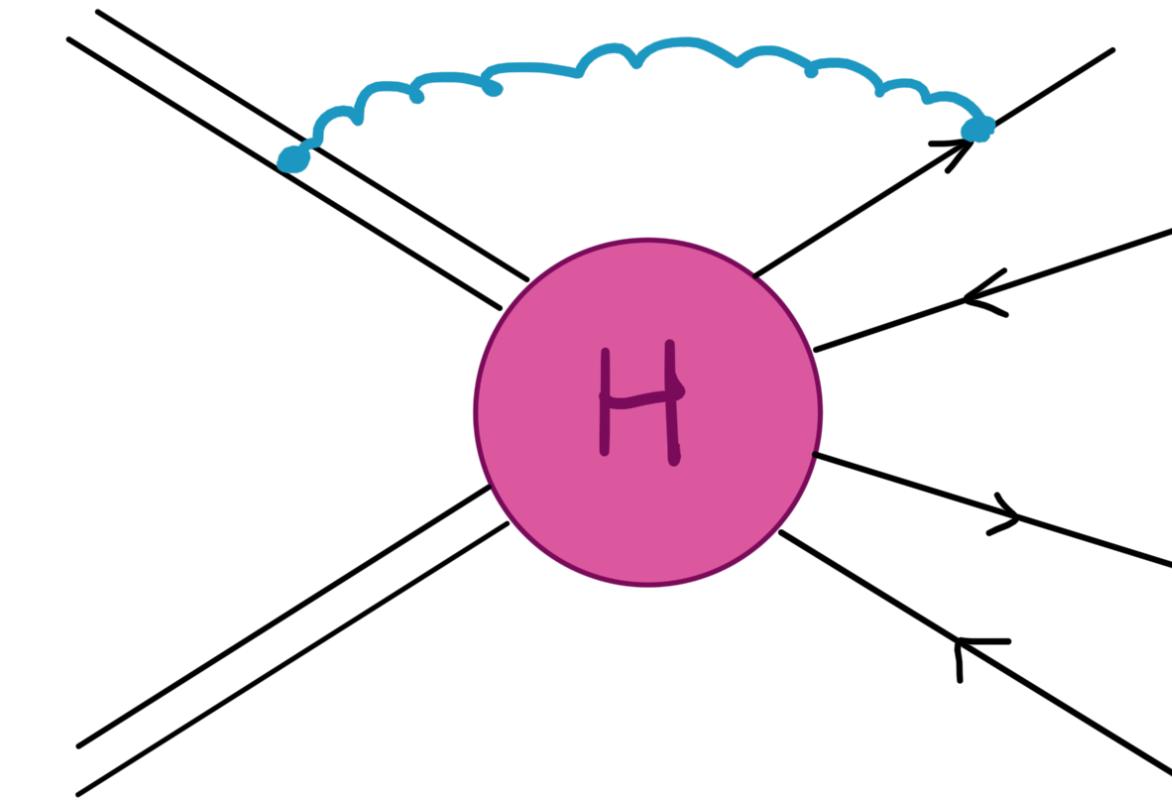
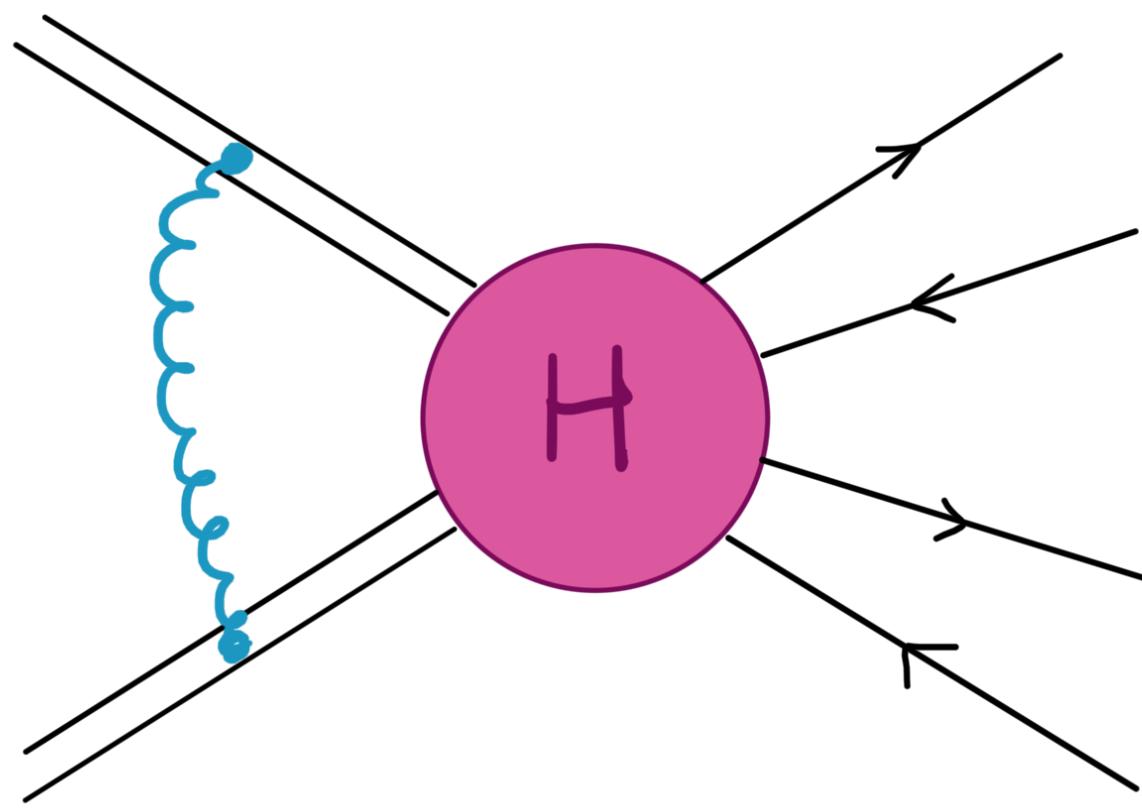
$$U_{ij \rightarrow t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathcal{P} \exp \left[ \int_{\mu_R}^{M/\bar{N}} \frac{dq}{q} \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right]$$

Soft-anomalous dimension (SAD) matrix

$$\Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s) = \left( \frac{\alpha_s}{\pi} \right) \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(2)} + \dots$$

Needed  
at NLL

Needed  
at NNLL



*Mixes colour structure → in general not diagonal in colour space*

# Colour structure 4top

$$U_{ij \rightarrow t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathcal{P} \exp \left[ \int_{\mu_R}^{M/\bar{N}} \frac{dq}{q} \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right]$$

$$\Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s) = \left( \frac{\alpha_s}{\pi} \right) \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(2)} + \dots$$

Needed  
at NLL

Needed  
at NNLL

$$q\bar{q} \rightarrow t\bar{t}t\bar{t}$$

$$3 \otimes \bar{3} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \longrightarrow 1 \oplus 8 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27$$

leads to a 6-dimensional SAD

$$gg \rightarrow t\bar{t}t\bar{t}$$

$$8 \otimes 8 = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \longrightarrow 0 \oplus 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27$$

leads to a 14-dimensional SAD

## SAD for $q\bar{q}$ (Full kinematics)

In colour basis of [1207.0609]

$$\Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathcal{P} \exp \left[ \sum_{i=1}^{16} \Gamma_i \right] \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s) = \left( \frac{\alpha_s}{\pi} \right) \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(2)} + \dots$$

$$\Gamma_{11} = \frac{(L34 + L56 + 2)(Nc^2 - 1)}{2 Nc}$$

$$\Gamma_{12} = \frac{(-L35 + L36 + L45 - L46)\sqrt{Nc^2 - 1}}{2 Nc}$$

$$\Gamma_{13} = \frac{\sqrt{Nc^2 - 1} (-T15 + T16 + T25 - T26)}{2 Nc}$$

$$\Gamma_{14} = \frac{\sqrt{Nc^2 - 1} (-T13 + T14 + T23 - T24)}{2 Nc}$$

$$\Gamma_{15=0} = \frac{2 Nc}{U_{ij \rightarrow t\bar{t}t\bar{t}}(N, M^2/\mu_R^2)} = \mathcal{P} \exp \left[ \sum_{i=1}^{16} \Gamma_i \right]$$

$$\Gamma_{16=0}$$

$$\Gamma_{21} = \frac{(-L35 + L36 + L45 - L46)\sqrt{Nc^2 - 1}}{2 Nc}$$

$$\Gamma_{22} = -\frac{-L36 Nc^2 - L45 Nc^2 - 2 Nc^2 + L34 - 2 L35 + 2 L36 + 2 L45 - 2 L46 + L56 + 2}{2 Nc}$$

$$\Gamma_{23} = \frac{-T13 + T14 + T23 - T24}{2 Nc}$$

$$\Gamma_{24} = \frac{-T15 + T16 + T25 - T26}{2 Nc}$$

$$\Gamma_{25} = \frac{\sqrt{Nc^2 - 4} (-T13 + T14 - T15 + T16 + T23 - T24 + T25 - T26)}{2 \sqrt{2} Nc}$$

$$\Gamma_{26} = \frac{T13 + T14 - T15 - T16 - T23 - T24 + T25 + T26}{2 \sqrt{2}}$$

$$\Gamma_{31} = \frac{\sqrt{Nc^2 - 1} (-T15 + T16 + T25 - T26)}{2 Nc}$$

$$\Gamma_{32} = \frac{-T13 + T14 + T23 - T24}{2 Nc}$$

$$\Gamma_{33} = -\frac{L34 Nc^2 + T15 Nc^2 + T26 Nc^2 - Nc^2 + L34 + L56 - 2 T15 + 2 T16 + 2 T25 - 2 T26 + 2}{2 Nc}$$

$$\Gamma_{34} = \frac{-L35 + L36 + L45 - L46}{2 Nc}$$

$$\Gamma_{35} = \frac{\sqrt{Nc^2 - 4} (-L35 + L36 + L45 - L46 - T13 + T14 + T23 - T24)}{2 \sqrt{2} Nc}$$

$$\Gamma_{36} = \frac{L35 + L36 - L45 - L46 + T13 - T14 + T23 - T24}{2 \sqrt{2}}$$

$$\Gamma_{41} = \frac{\sqrt{Nc^2 - 1} (-T13 + T14 + T23 - T24)}{2 Nc}$$

$$\Gamma_{42} = \frac{-T15 + T16 + T25 - T26}{2 Nc}$$

$$\Gamma_{43} = \frac{-L35 + L36 + L45 - L46}{2 Nc}$$

$$\Gamma_{44} = \frac{-L56 Nc^2 + T13 Nc^2 + T24 Nc^2 - Nc^2 + L34 + L56 - 2 T13 + 2 T14 + 2 T23 - 2 T24 + 2}{2 Nc}$$

$$\Gamma_{45} = \frac{\sqrt{Nc^2 - 4} (-L35 + L36 + L45 - L46 - T15 + T16 + T25 - T26)}{2 \sqrt{2} Nc}$$

$$\Gamma_{46} = \frac{-L35 + L36 - L45 + L46 - T15 + T16 - T25 + T26}{2 \sqrt{2}}$$

$$\Gamma_{51=0}$$

$$\Gamma_{52} = \frac{\sqrt{Nc^2 - 4} (-T13 + T14 - T15 + T16 + T23 - T24 + T25 - T26)}{2 \sqrt{2} Nc}$$

$$\Gamma_{53} = \frac{\sqrt{Nc^2 - 4} (-L35 + L36 + L45 - L46 - T13 + T14 + T23 - T24)}{2 \sqrt{2} Nc}$$

$$\Gamma_{54} = \frac{\sqrt{Nc^2 - 4} (-L35 + L36 + L45 - L46 - T15 + T16 + T25 - T26)}{2 \sqrt{2} Nc}$$

$$\Gamma_{55} = -\frac{-L36 Nc^2 - L45 Nc^2 + T13 Nc^2 + T15 Nc^2 + T24 Nc^2 + T26 Nc^2 - 2 Nc^2 + 2 L34 - 6 L35 + 6 L36 + 6 L45 - 6 L46 + 2 L56 - 6 T13 + 6 T14 - 6 T15 + 6 T16 + 6 T23 - 6 T24 + 6 T25 - 6 T26 + 4}{4 Nc}$$

$$\Gamma_{56} = \frac{1}{4} \sqrt{Nc^2 - 4} (L36 - L45 + T13 - T15 - T24 + T26)$$

$$\Gamma_{61=0}$$

$$\Gamma_{62} = \frac{T13 + T14 - T15 - T16 - T23 - T24 + T25 + T26}{2 \sqrt{2}}$$

$$\Gamma_{63} = \frac{L35 + L36 - L45 - L46 + T13 - T14 + T23 - T24}{2 \sqrt{2}}$$

$$\Gamma_{64} = \frac{-L35 + L36 - L45 + L46 - T15 + T16 - T25 + T26}{2 \sqrt{2}}$$

$$\Gamma_{65} = \frac{1}{4} \sqrt{Nc^2 - 4} (L36 - L45 + T13 - T15 - T24 + T26)$$

$$\Gamma_{66} = -\frac{-L36 Nc^2 - L45 Nc^2 + T13 Nc^2 + T15 Nc^2 + T24 Nc^2 + T26 Nc^2 - 2 Nc^2 + 2 L34 - 2 L35 + 2 L36 + 2 L45 - 2 L46 + 2 L56 - 2 T13 + 2 T14 - 2 T15 + 2 T16 + 2 T23 - 2 T24 + 2 T25 - 2 T26 + 4}{4 Nc}$$

leads to a 14-dimensional SAD

# SAD for $gg$ (Full kinematics)

# SAD for gg (Full kinematics)

# SAD for $gg$ (Full kinematics)

$$J, M^2/\mu_R^2) = \mathcal{P} \exp \left[ \int_{\mu_R}^{M/\bar{N}} \frac{dq}{q} \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right]$$

solve this: go to colour space (R) where SAD is diagonal

$$J \tilde{S} U = \tilde{S}_R \exp \left[ \frac{2\text{Re}(\Gamma_R)}{2\pi b_0} \ln(1 - 2\lambda) \right]$$

$\{0, ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Nc), (-2*L45 + 2*L46 - L35 - L36 + L45 - L46 + L36*(2 + Nc)^2 + L45 + Nc^2 + L46 + Nc^2 + L36*(-4 + Nc^2) - 4*Nc*T15 + 2*Nc^2*T15 + 4*Nc*T16 + 2*Nc^2*T16 + 4*Nc*T23 + 2*Nc^2*T23 - 4*Nc*T24 + 2*Nc^2*T24)/(8*Nc*Sqrt[-4 + Nc^2]),$   
 $(2*L45 - 2*L46 - L45 + Nc - L35*(2 + Nc) + L36*(2 + Nc) - 2*Nc*T15 + 2*Nc*T16)/(8*Nc), ((L35 - L36 - L45 + L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (2*L45 - 2*L46 + L35*(-2 + Nc) - L45 + Nc + L36*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(8*Nc),$   
 $((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(8*Nc), -(4 + 2*L34 - 4*L35 + 4*L36 + 4*L45 - 4*L46 + 2*L56 - L36 + Nc + L45 - 2*Nc^2 - L36 + Nc^2 - L45 + Nc^2 - 2*Nc*T13 + Nc*T14 + Nc^2*T15 + Nc*T16 + Nc^2*T23 - Nc*T24 + Nc^2*T25 - 2*Nc*T26)/(4*Nc),$   
 $0, ((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]), ((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]),$   
 $\{0, ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Nc), (-2*L45 + 2*L46 + L36*(-2 + Nc) - L45 + Nc + L35*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(4*Sqrt[2]*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),$   
 $(-2*L45 + 2*L46 - L35*(-2 + Nc) + L36*(-2 + Nc) - L45 + Nc + L46 + Nc - 2*Nc*T15 + 2*Nc*T16)/(4*Sqrt[2]*Nc),$   
 $(4*L45 - 4*L46 + L36*(-2 + Nc)^2 + 4*L45 + Nc + 4*Nc^2 + L45 + Nc^2 + L46 + Nc^2 + L35*(-4 + Nc^2) + 4*Nc*T15 + 2*Nc^2*T15 - 4*Nc*T16 + 2*Nc^2*T16 - 4*Nc*T23 + 2*Nc^2*T23 + 4*Nc*T24 + 2*Nc^2*T24)/(8*Nc*Sqrt[-4 + Nc^2]),$   
 $(-2*L45 + 2*L46 - L35*(-2 + Nc) + L36*(-2 + Nc) - L45 + Nc + L46 + Nc - 2*Nc*T15 + 2*Nc*T16)/(8*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (-2*L45 + 2*L46 + L36*(-2 + Nc) - L45 + Nc + L35*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(8*Nc),$   
 $((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(8*Nc), 0, -(4 + 2*L34 - 4*L35 + 4*L36 + 4*L45 - 4*L46 + 2*L56 + L36 + Nc - L45 - 2*Nc^2 - L36 + Nc^2 + 2*Nc*T13 - 2*Nc*T14 + Nc^2*T15 - Nc*T16 + Nc^2*T23 + Nc*T24 + Nc^2*T25 - 2*Nc*T26)/(4*Nc),$   
 $((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]), ((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]),$   
 $\{0, ((-L35 + L36 + L45 - L46)*Sqrt[(3 + Nc)/(1 + Nc)])/4, ((-L35 + L36 + L45 - L46)*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 + 3*Nc + Nc^2)), (\text{Sqrt}[3 + Nc]/(2 + 2*Nc))*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/4,$   
 $((-L35 + L36 + L45 - L46)*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 + 3*Nc + Nc^2)), -((L35 - L36 - L45 + L46)*(-6 + Nc + Nc^2))/(8*(2 + Nc)*Sqrt[3 + 4*Nc + Nc^2]), (\text{Sqrt}[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(8*(2 + 3*Nc + Nc^2)),$   
 $(\text{Sqrt}[3 + Nc]/(2 + 2*Nc))*(L35 + L36 - L45 + L46 - 2*T15 + 2*T16))/4, (\text{Sqrt}[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]*(-L35 + L36 - L45 + L46 - 2*T15 + 2*T16))/(8*(2 + 3*Nc + Nc^2)), (\text{Sqrt}[3 + Nc]/(1 + Nc))*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/8,$   
 $((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]), ((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]),$   
 $-8 + 4*L36 + 4*L45 - 4*L46 + 4*L56 + 2*L36 + Nc + 2*L45 + Nc + 2*L46 + Nc - 6*Nc^2 - 3*L36 + Nc^2 - L36 + Nc^3 - L45 + Nc^3 - 4*L35*(1 + Nc) + 2*L34*(2 + Nc) - 4*Nc*T13 - 2*Nc^2*T13 - 4*Nc*T14 - 2*Nc^2*T14 + 2*Nc*T15 + 3*Nc^2*T15 + Nc^3*T15 + 2*Nc*T16 + 3*Nc^2*T16 + Nc^3*T16 + 2*Nc*T23 + 3*Nc^2*T23 + Nc^3*T23 + 2*Nc^2*T24 + Nc^3*T24 - 4*Nc*T25 - 2*Nc^2*T25 - 4*Nc*T26 - 2*Nc^2*T26)/(4*Nc*(2 + Nc)), 0\},$   
 $\{0, ((-L35 + L36 + L45 - L46)*Sqrt[(-3 + Nc)/(-1 + Nc)])/4, ((L35 - L36 - L45 + L46)*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 - 3*Nc + Nc^2)), (\text{Sqrt}[-3 + Nc]/(-1 + Nc))*(L35 - L36 + L45 + L46 - 2*T23 + 2*T24))/(4*Sqrt[2]),$   
 $((L35 - L36 - L45 + L46)*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 - 3*Nc + Nc^2)), -((L35 - L36 - L45 + L46)*(-6 - Nc + Nc^2))/(8*(-2 + Nc)*Sqrt[3 - 4*Nc + Nc^2]), ((-6 - Nc + Nc^2)*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(8*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]),$   
 $(\text{Sqrt}[-3 + Nc]/(-1 + Nc))*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(4*Sqrt[2]), -((-6 - Nc + Nc^2)*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(8*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]), (\text{Sqrt}[-3 + Nc]/(-1 + Nc))*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/8,$   
 $((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]), ((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]), 0,$   
 $-8 - 4*L36 - 4*L45 + 4*L46 - 4*L56 + 2*L36 + Nc + 2*L45 + Nc + 2*L46 + Nc - 6*Nc^2 + 3*L36 + Nc^2 - L45 + Nc^2 - 4*Nc*T13 + 2*Nc^2*T13 - 4*Nc*T14 + 2*Nc^2*T14 + 2*Nc*T15 - 3*Nc^2*T15 + Nc^3*T15 + 2*Nc^2*T16 + Nc^3*T16 + 2*Nc*T23 - 3*Nc^2*T23 + Nc^3*T24 - 4*Nc*T25 + 2*Nc^2*T25 + 2*Nc^2*T26 + 2*Nc^2*T26 + 2*Nc^2*T26)/(4*(-2 + Nc)*Nc)\}$

# Colour structure 4top

$$\bar{\mathbf{U}} \tilde{\mathbf{S}} \mathbf{U} = \tilde{\mathbf{S}}_R \exp \left[ \frac{2\text{Re}(\Gamma_R)}{2\pi b_0} \ln(1 - 2\lambda) \right]$$

$$C_2(\mathbf{1}) = 0$$

$$C_2(\mathbf{8}_{(\text{A/S})}) = N_c = 3$$

$$C_2(\mathbf{10}, \bar{\mathbf{10}}) = 2N_c = 6$$

$$C_2(\mathbf{27}) = 2(N_c + 1) = 8$$

$$C_2(N_c^2(N_c - 3)(N_c + 1)/4 = \mathbf{0}) = 2(N_c - 1) = 4$$

$$q\bar{q} \rightarrow t\bar{t}t\bar{t}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{1} \oplus \mathbf{8} = \mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \bar{\mathbf{10}} \oplus \mathbf{27}$$

Absolute threshold limit:  $2\text{Re}[\Gamma_{R,q\bar{q}}^{(1)}] = \text{diag}\left(0, 0, -3, -3, -3, -3\right)$

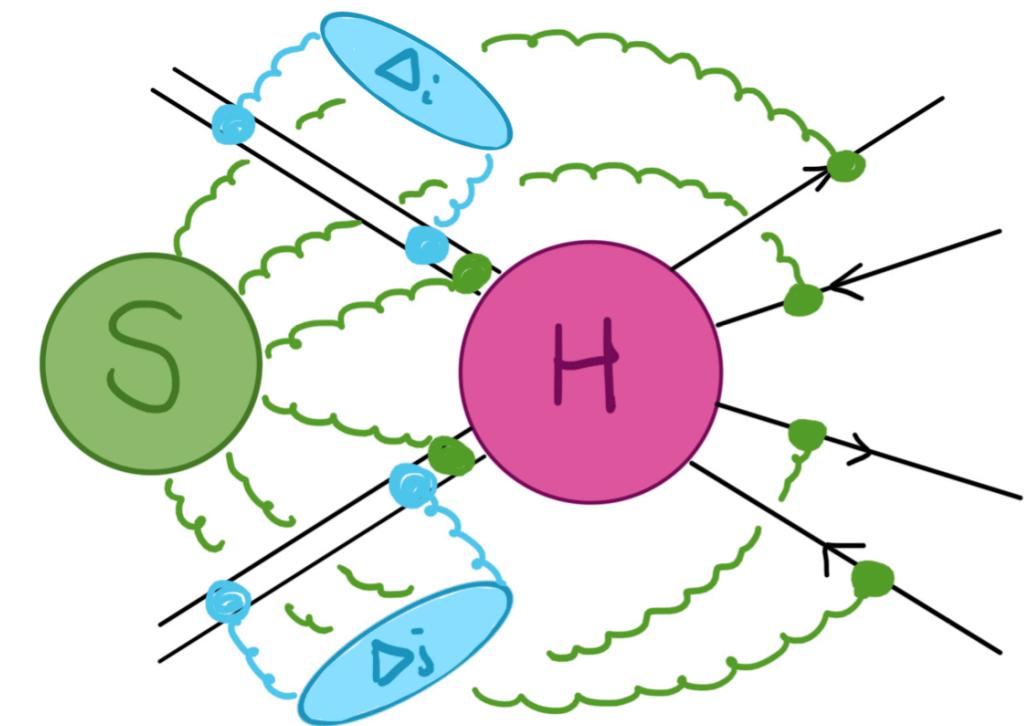
$$gg \rightarrow t\bar{t}t\bar{t}$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \bar{\mathbf{10}} \oplus \mathbf{27} = \mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \bar{\mathbf{10}} \oplus \mathbf{27}$$

Absolute threshold limit:  $2\text{Re}[\Gamma_{R,gg}^{(1)}] = \text{diag}\left(-8, -6, -6, -4, -3, -3, -3, -3, -3, -3, 0, 0\right)$

# Resummation for 4top

Mellin-space resummed cross section



Soft function  
*captures wide-angle-soft enhancements*

$$\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N) = \text{Tr} \left[ \begin{array}{c|c|c} \text{H}_{ij \rightarrow t\bar{t}t\bar{t}} & \bar{\mathbf{U}}_{ij \rightarrow t\bar{t}t\bar{t}} \tilde{\mathbf{S}}_{ij \rightarrow t\bar{t}t\bar{t}} \mathbf{U}_{ij \rightarrow t\bar{t}t\bar{t}} & \Delta_i \Delta_j \\ \hline & & \end{array} \right]$$

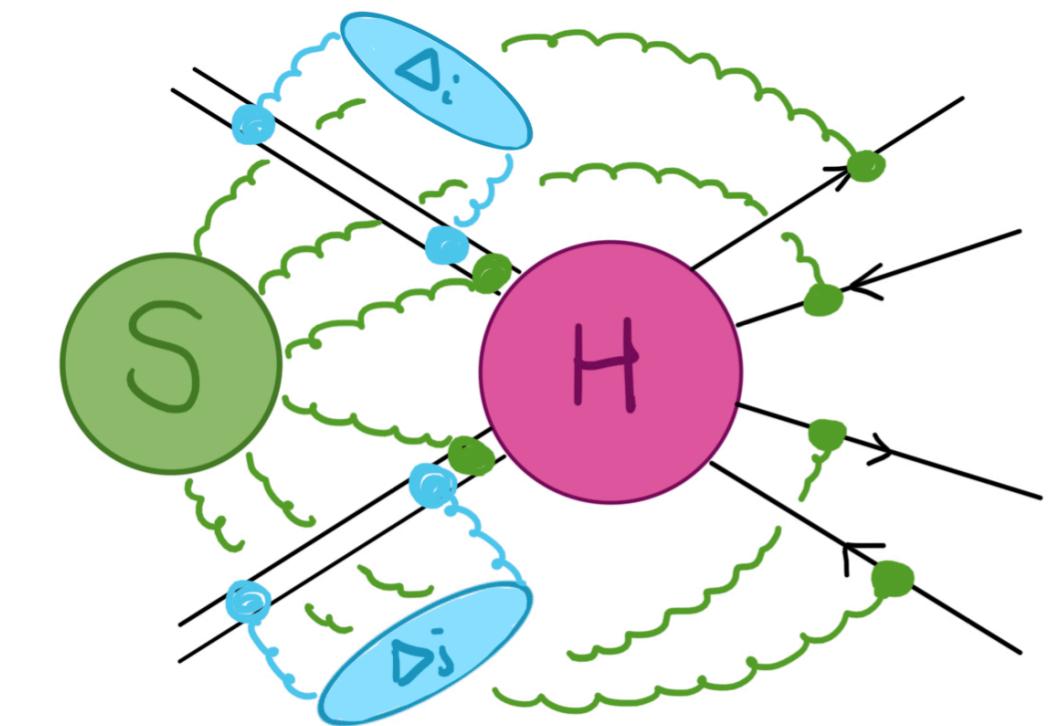
Hard function  
*captures constant contributions as  $N \rightarrow \infty$*

Incoming jet functions  
*capture soft-collinear enhancements*

# Resummation for 4top

Mellin-space resummed cross section

Soft function  
*captures wide-angle-soft enhancements*



$$\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N) = \text{Tr} \left[ \begin{array}{c|c|c} \text{H}_{ij \rightarrow t\bar{t}t\bar{t}} & \bar{\mathbf{U}}_{ij \rightarrow t\bar{t}t\bar{t}} \tilde{\mathbf{S}}_{ij \rightarrow t\bar{t}t\bar{t}} \mathbf{U}_{ij \rightarrow t\bar{t}t\bar{t}} & \Delta_i \Delta_j \\ \hline \end{array} \right]$$

Hard function  
*captures constant contributions as  $N \rightarrow \infty$*

Incoming jet functions  
*capture soft-collinear enhancements*

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi} \mathbf{H}^{(1)} + \dots,$$

Needed at NLL

Projection on  
colour space R

$$H_{IJ}^{(0)} = \frac{1}{2s} \int_0^1 d\rho \rho^{N-1} \int d\Phi^B \sum_{\text{colour(K,L),spin}} \mathcal{A}_K^{(0)} \mathcal{A}_L^{\dagger(0)} \langle c_L | c_J \rangle \langle c_I | c_K \rangle$$

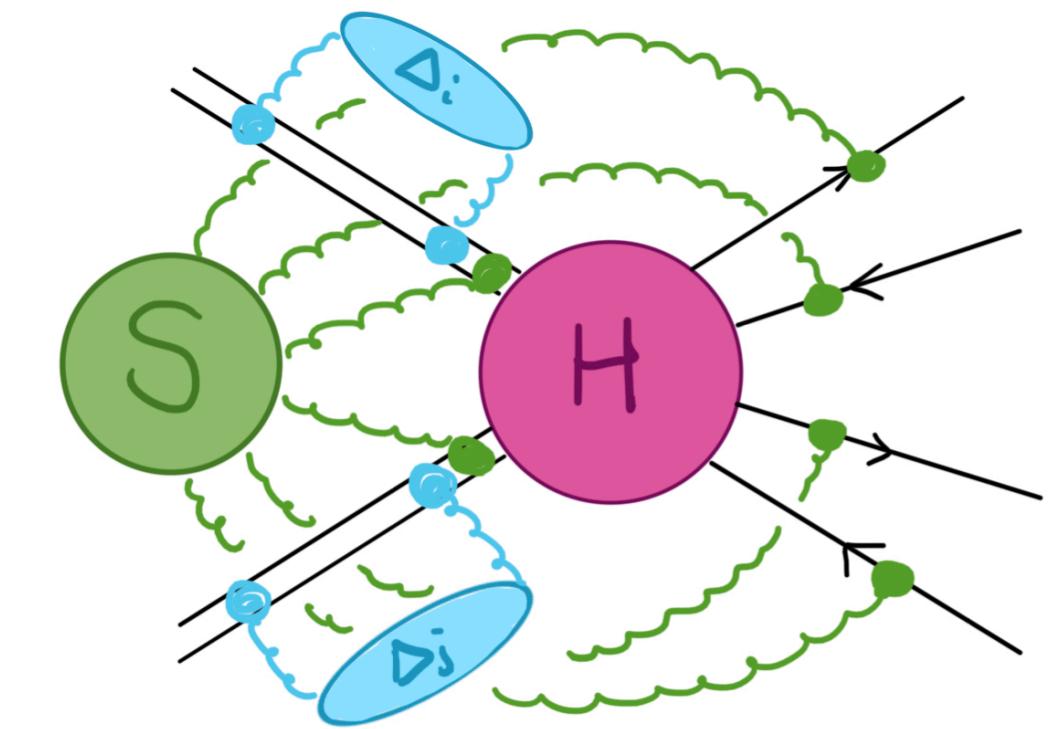
Born phase-space integration

Colour-stripped  
amplitude

# Resummation for 4top

Mellin-space resummed cross section

Soft function  
*captures wide-angle-soft enhancements*



$$\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N) = \text{Tr} \left[ \begin{array}{c|c|c} \text{H}_{ij \rightarrow t\bar{t}t\bar{t}} & \bar{\mathbf{U}}_{ij \rightarrow t\bar{t}t\bar{t}} \tilde{\mathbf{S}}_{ij \rightarrow t\bar{t}t\bar{t}} \mathbf{U}_{ij \rightarrow t\bar{t}t\bar{t}} & \Delta_i \Delta_j \\ \hline & & \end{array} \right]$$

Hard function  
*captures constant contributions as  $N \rightarrow \infty$*

Incoming jet functions  
*capture soft-collinear enhancements*

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi} \mathbf{H}^{(1)} + \dots,$$

Needed at NLL'

$$\mathbf{H}^{(1)} = \mathbf{V}^{(1)} + \mathbf{C}^{(1)}$$

Virtual  
one-loop  
contributions  
(Extracted from  
MC@NLO)

Collinear  
end-point  
contributions

# Matching to fixed order

Go back to physical space by an inverse Mellin transform

$$\sigma_{t\bar{t}t\bar{t}}^{\text{NLL}(\prime)}(\tau) = \int_{\mathcal{C}} \frac{dN}{2\pi i} \tau^{-N} f_i(N+1, \mu_F^2) f_j(N+1, \mu_F^2) \times \hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N)$$

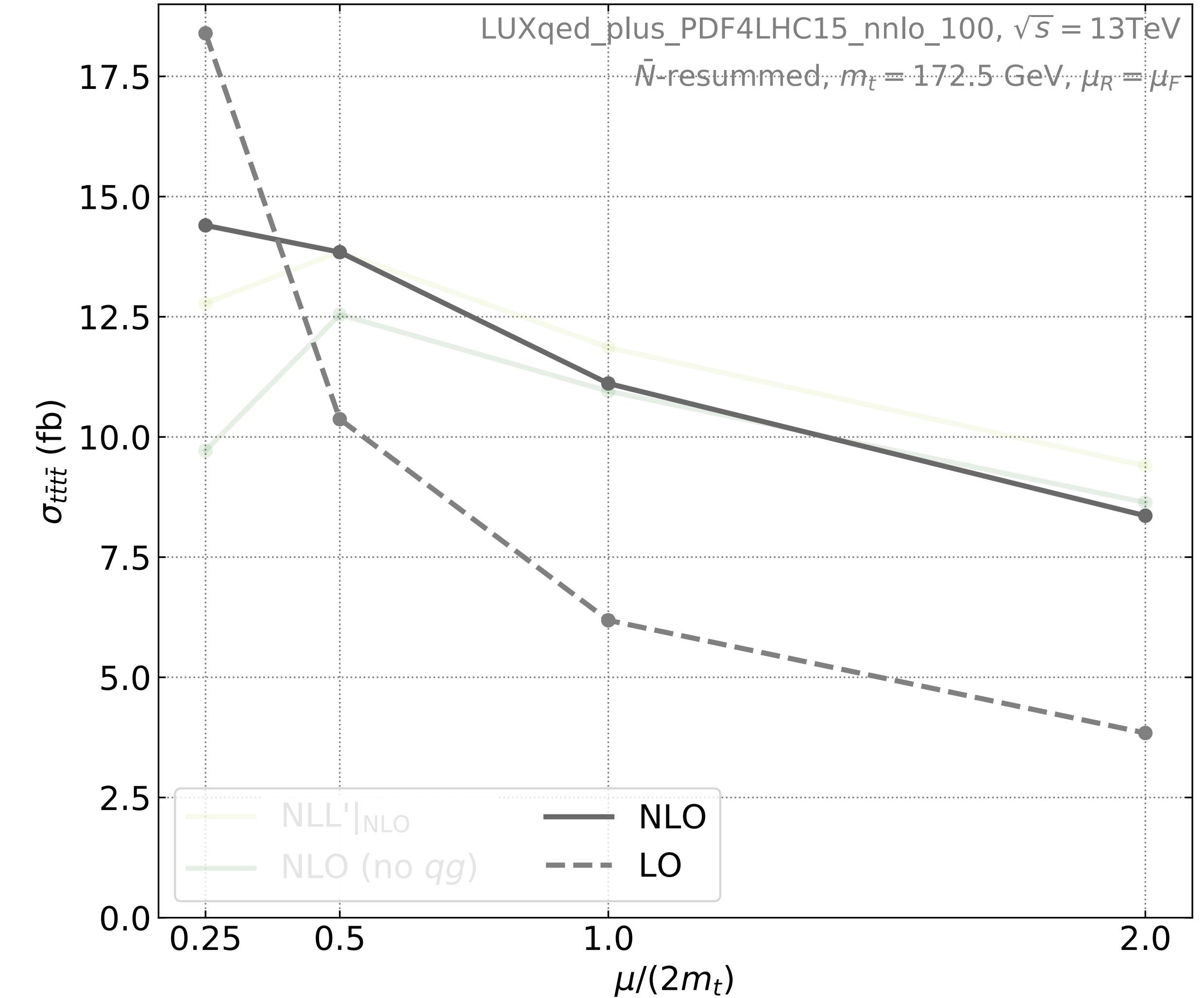
Match to fixed-order predictions through

$$\sigma_{t\bar{t}t\bar{t}}^{\text{NLO+NLL}(\prime)}(\tau) = \sigma_{t\bar{t}t\bar{t}}^{\text{NLO}}(\tau) + \int_{\mathcal{C}} \frac{dN}{2\pi i} \tau^{-N} f_i(N+1, \mu_F^2) f_j(N+1, \mu_F^2) \times \left[ \hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N) - \hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}}(N)|_{\text{NLO}} \right]$$

Consider both QCD-only NLO corrections  
and QCD + EW (LO only)

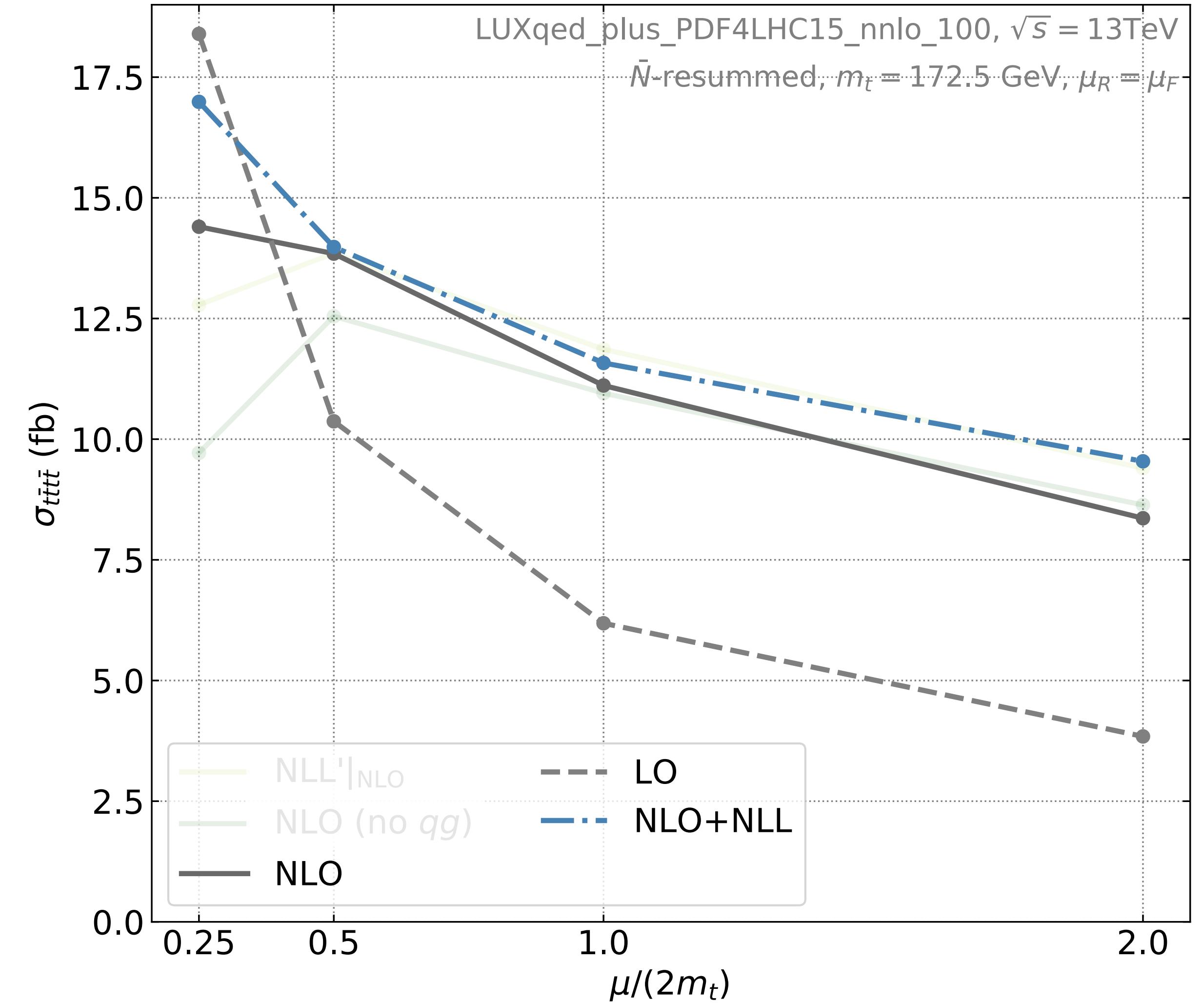
# Results

- LHC at 13 TeV
- $\mu_R = \mu_F = 2m_t$
- $m_t = 172.5$  GeV
- NNLO PDF4LHC with LUXqed
  - (includes photon PDFs)



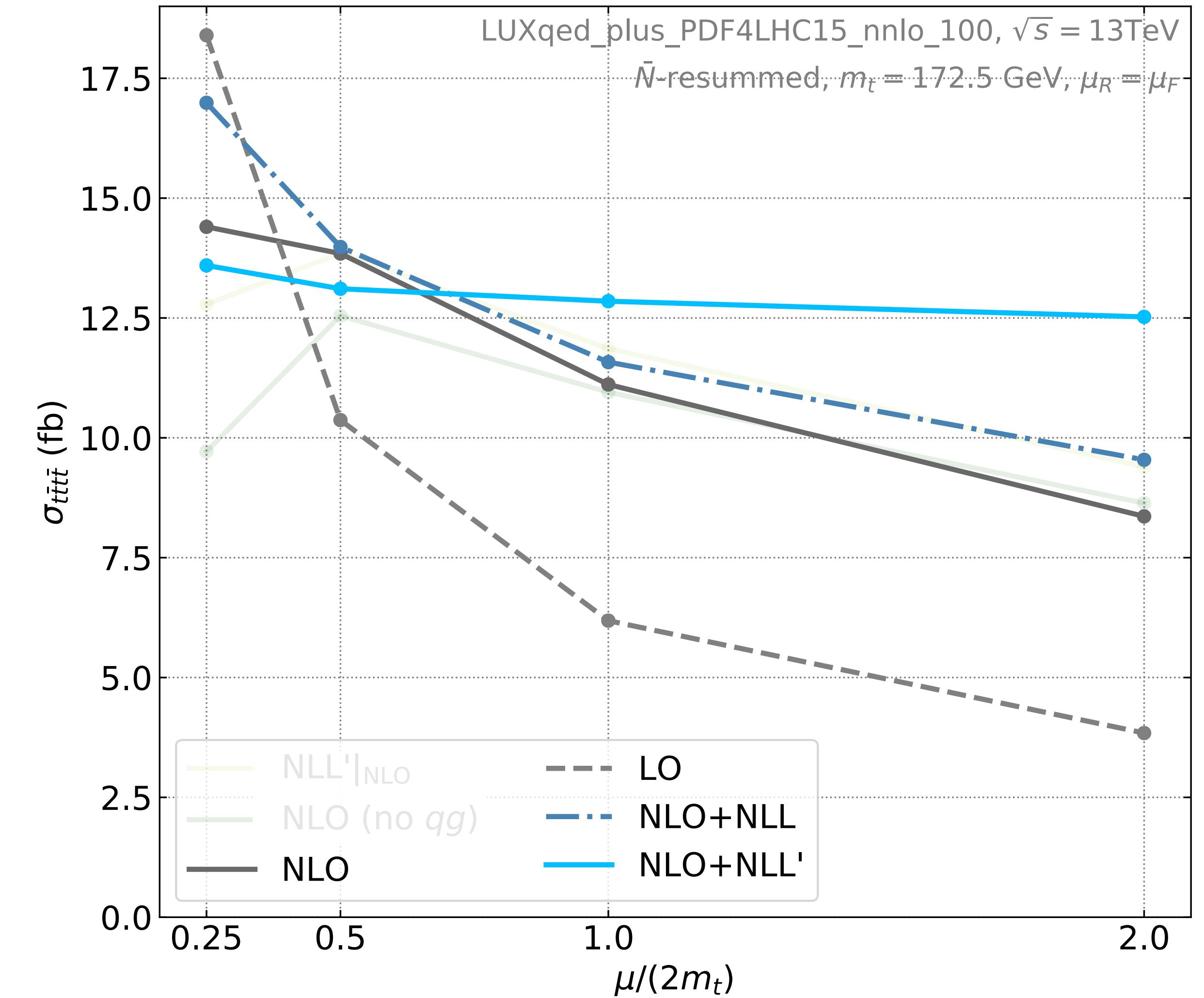
# Results

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# Results

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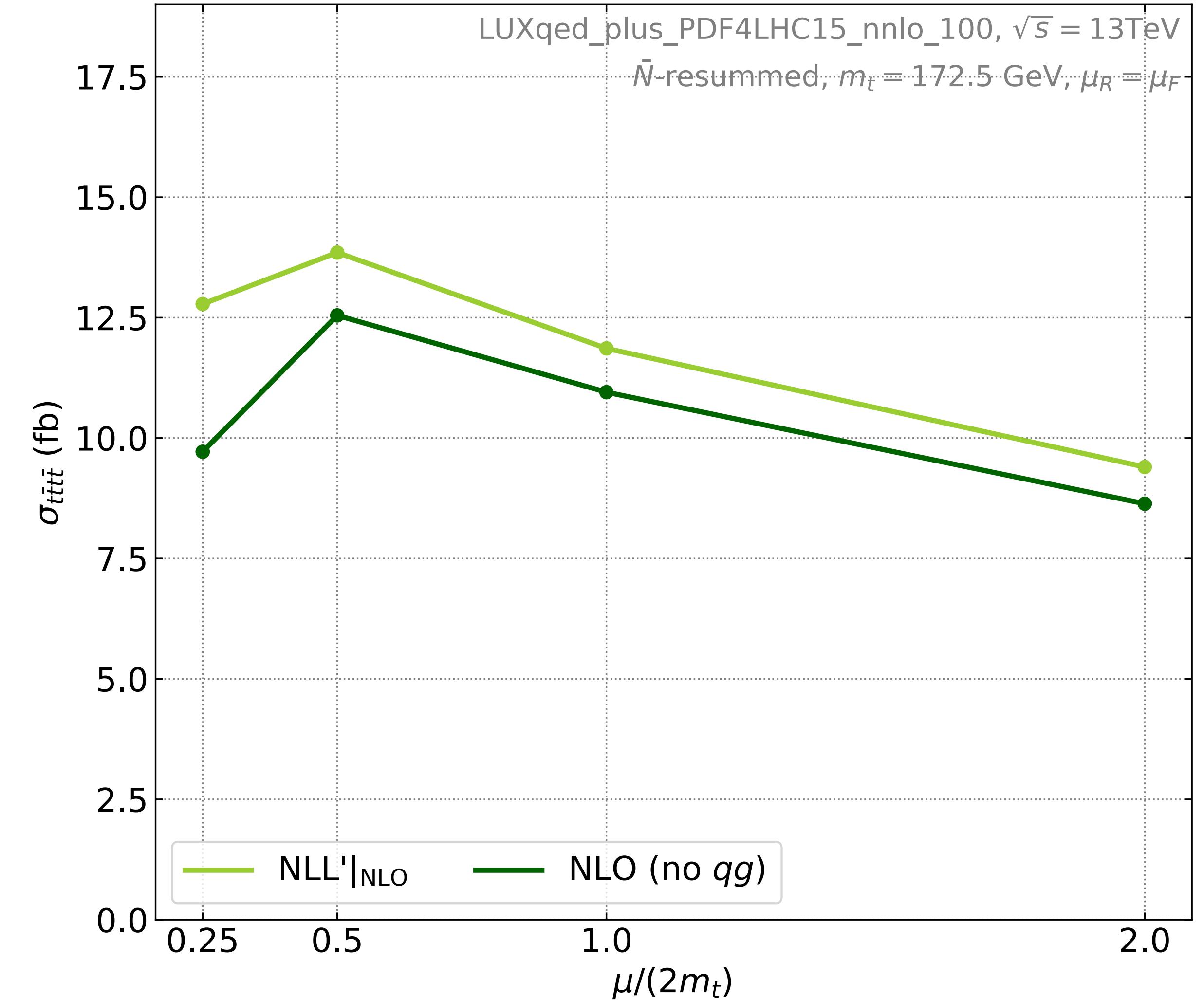


# Results

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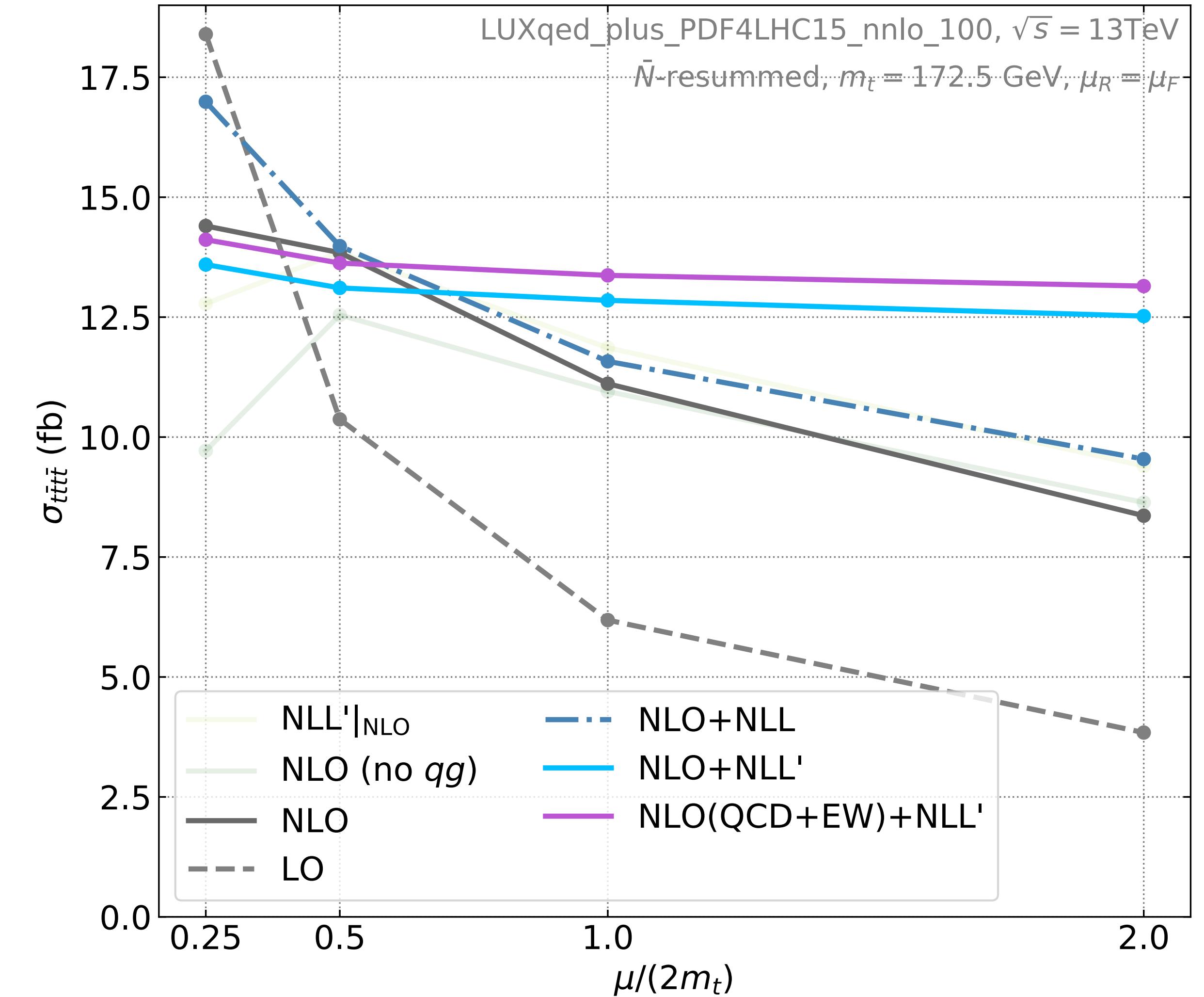
(includes photon PDFs)

Expanded NLL' result up to NLO shows that  
the dominant part of the NLO corrections are  
captured by soft-gluon radiation



# Results

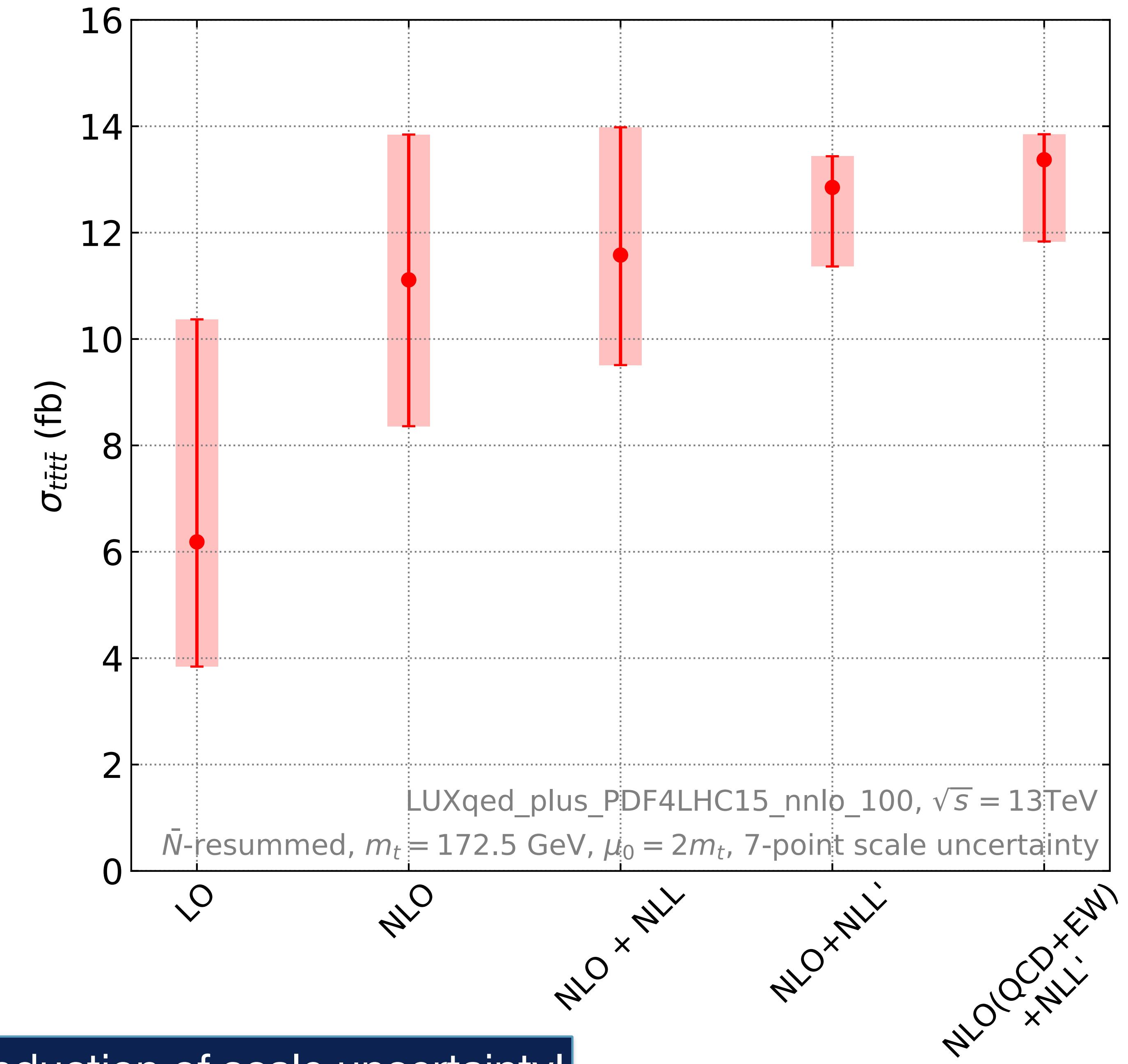
- LHC at 13 TeV
- $\mu_R = \mu_F = 2m_t$
- $m_t = 172.5$  GeV
- NNLO PDF4LHC with LUXqed  
(includes photon PDFs)



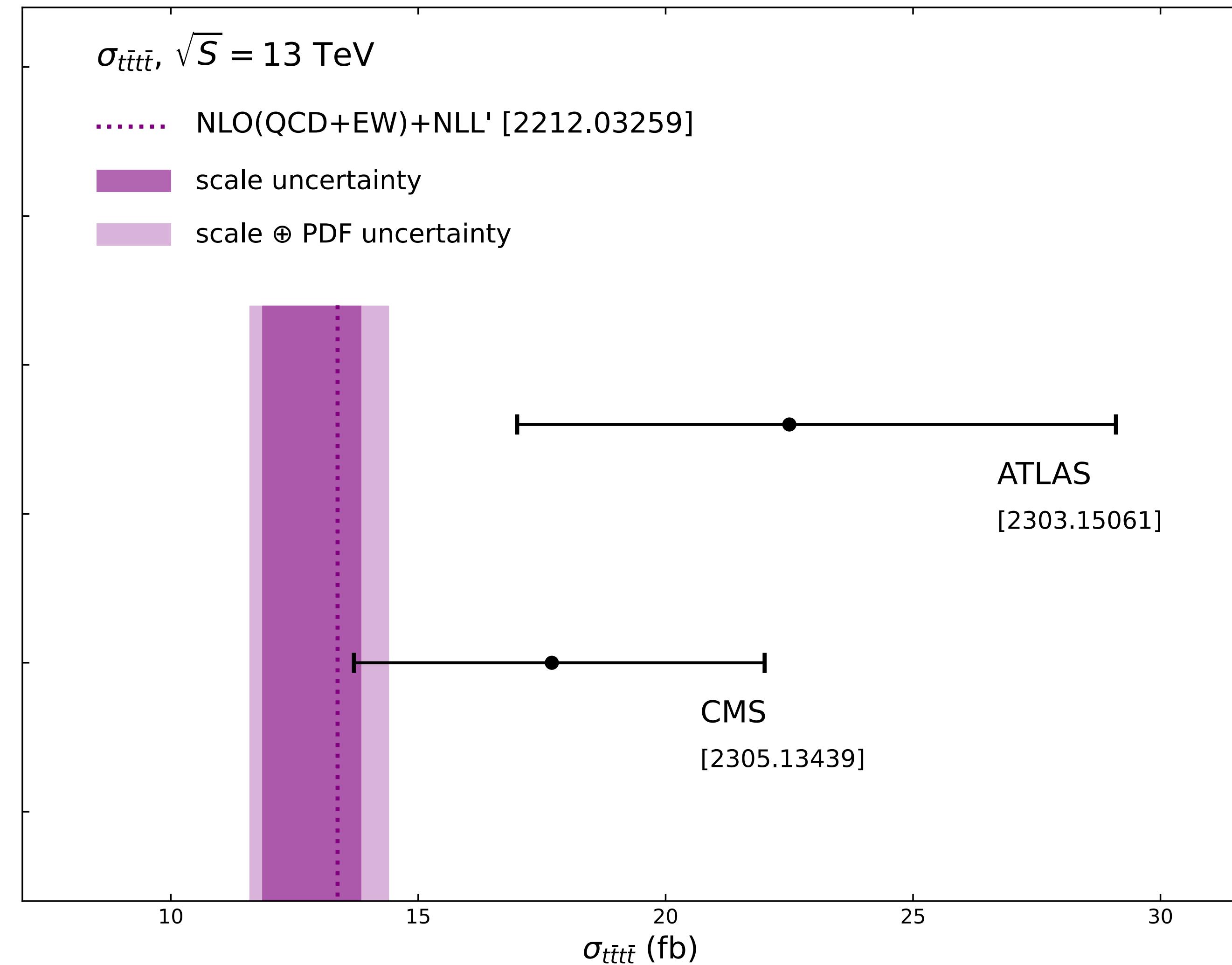
# Results

	$\sigma_{t\bar{t}t\bar{t}}$ (fb)	$K$ -factor
NLO	$11.00(2)^{+25.2\%}_{-24.5\%}$	
NLO+NLL	$11.46(2)^{+21.3\%}_{-17.7\%}$	1.04
NLO+NLL'	$12.73(2)^{+4.1\%}_{-11.8\%}$	1.16
NLO(QCD+EW)	$11.64(2)^{+23.2\%}_{-22.8\%}$	
NLO(QCD+EW)+NLL	$12.10(2)^{+19.5\%}_{-16.3\%}$	1.04
NLO(QCD+EW)+NLL'	$13.37(2)^{+3.6\%}_{-11.4\%}$	1.15

PDF error is 6.9 %



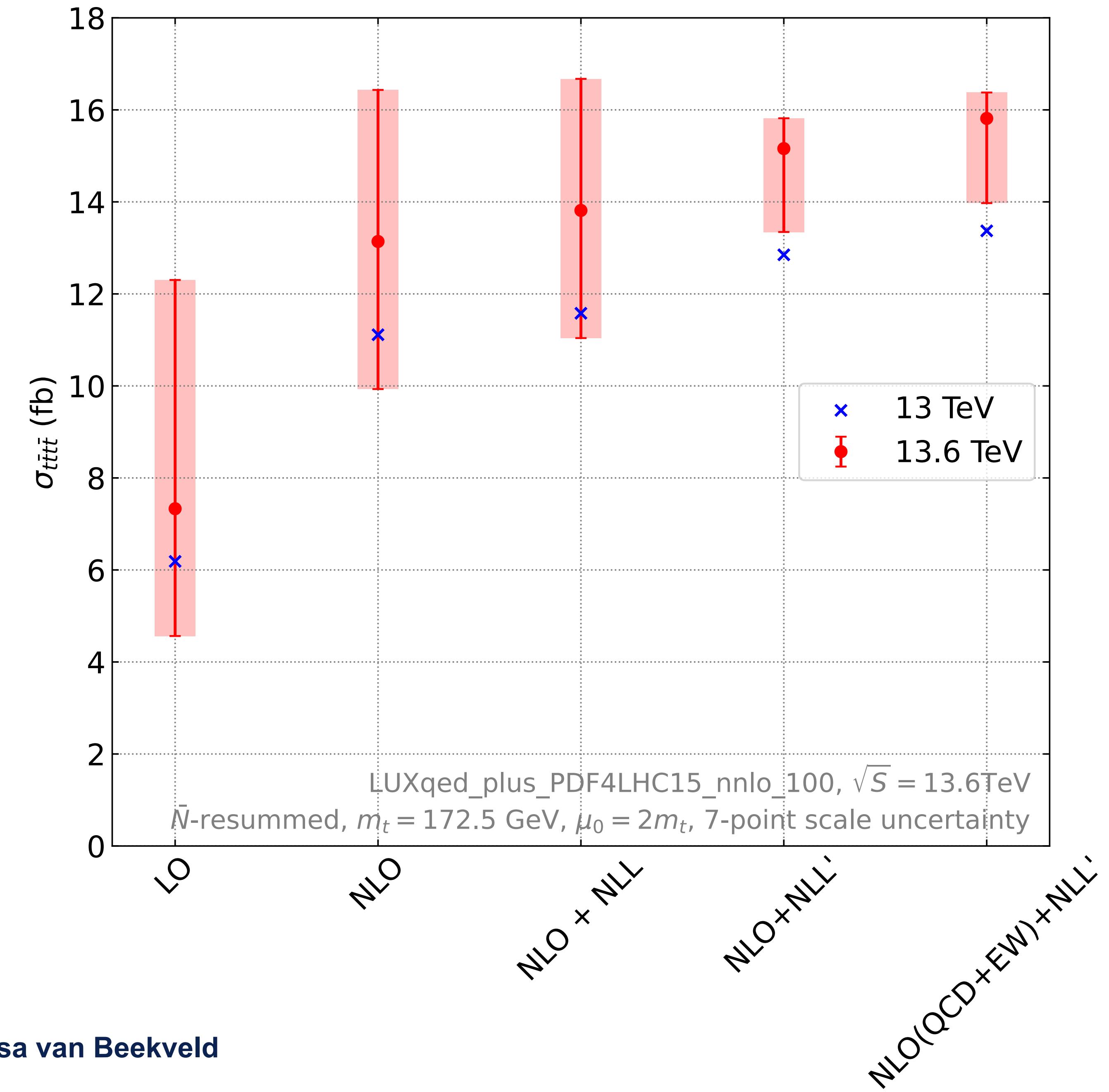
# Comparison with experiment



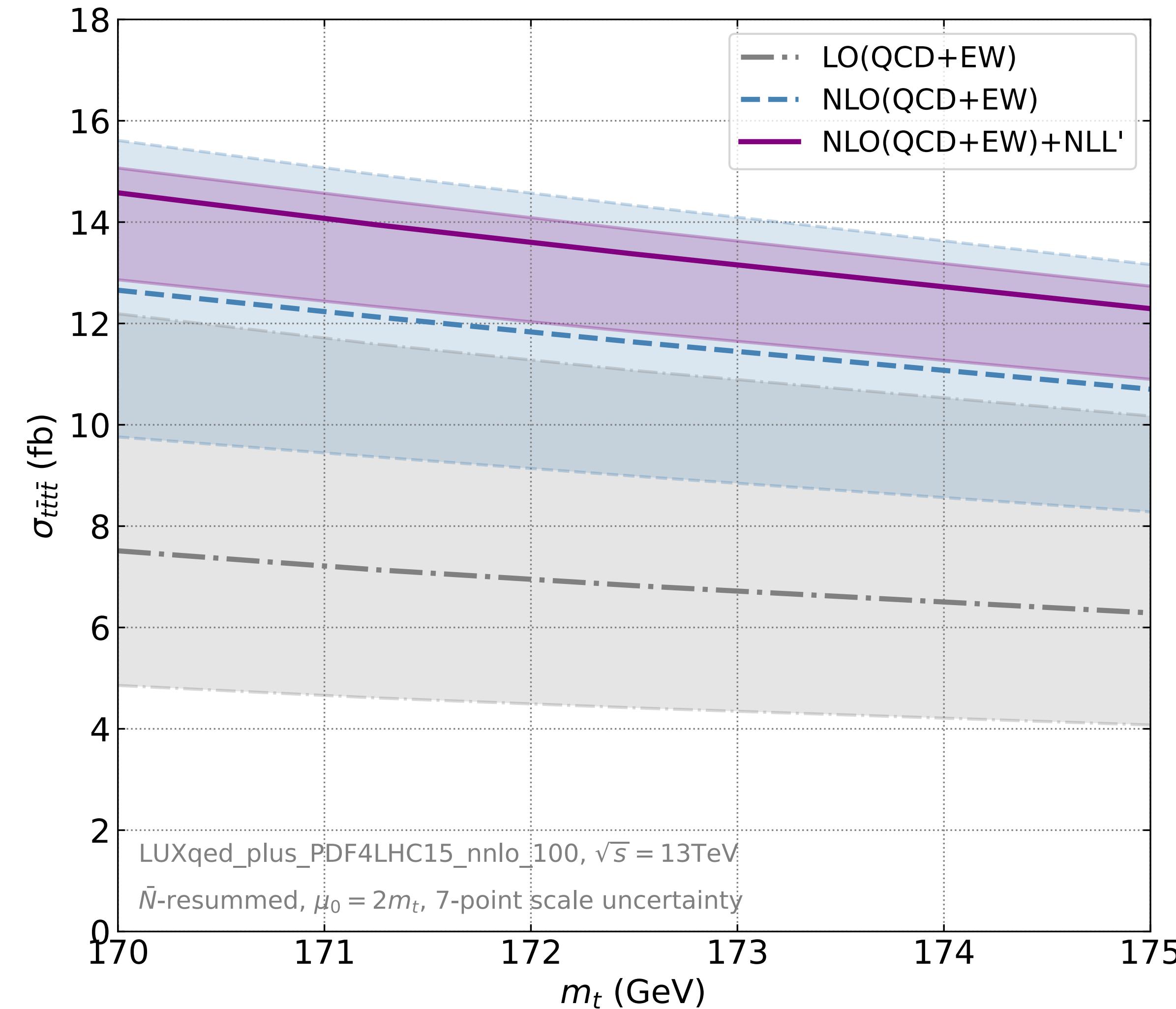
# Results for 13.6 TeV

	$\sigma_{t\bar{t}t\bar{t}}$ (fb)	K-factor
NLO	$13.14(2)^{+25.1\%}_{-24.4\%}$	
NLO+NLL	$13.81(2)^{+20.7\%}_{-20.1\%}$	1.05
NLO+NLL'	$15.16(2)^{+4.3\%}_{-11.9\%}$	1.15
NLO(QCD+EW)	$13.80(2)^{+22.9\%}_{-22.6\%}$	
NLO(QCD+EW)+NLL	$14.47(2)^{+18.4\%}_{-18.5\%}$	1.05
NLO(QCD+EW)+NLL'	$15.81(2)^{+3.6\%}_{-11.6\%}$	1.14

PDF error is 6.7 %

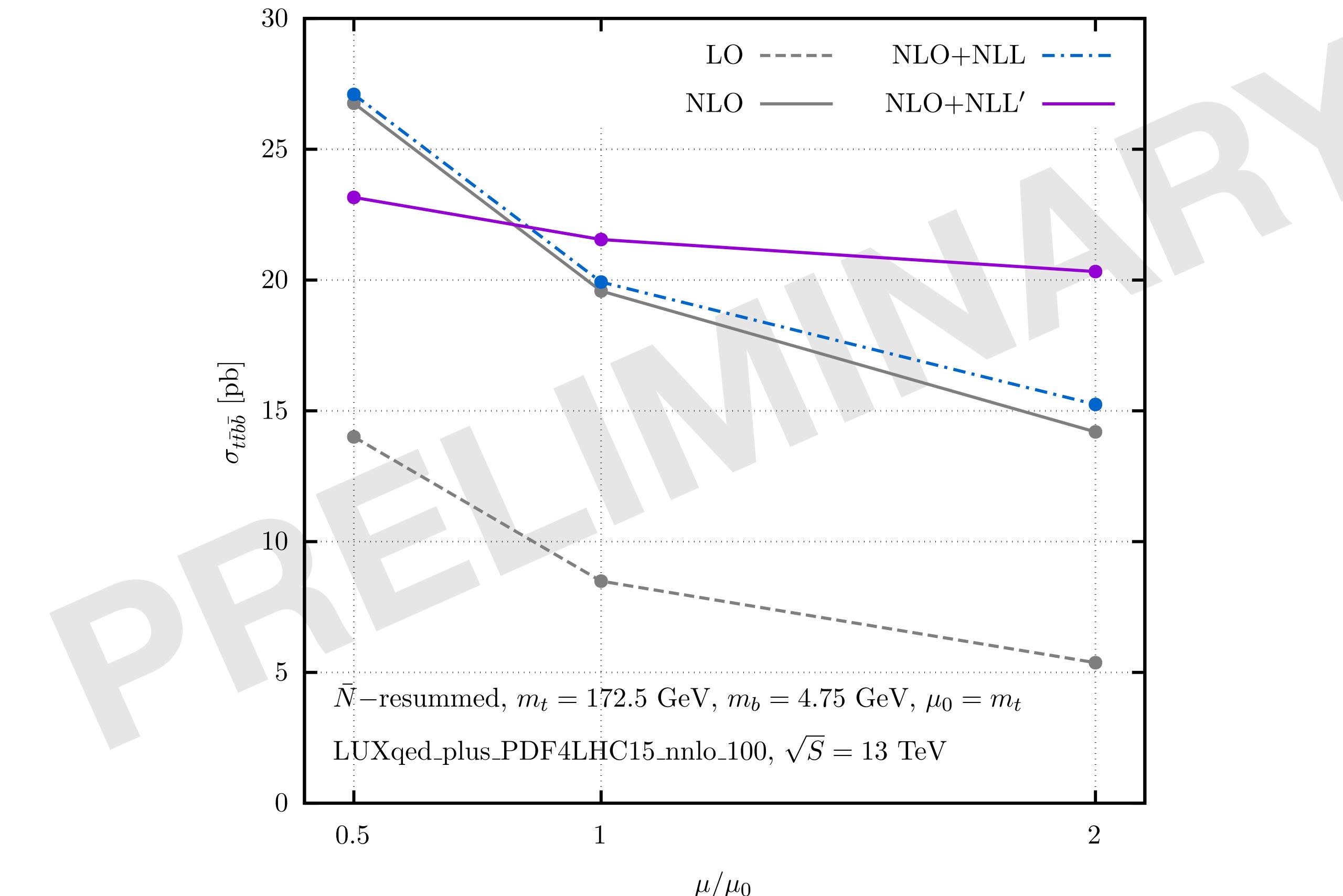


# Variation of the top mass



# Conclusions

- Performed first resummation of a process with 6 coloured particles at Born level
- Find a significant reduction of the total scale uncertainty
- Next steps:  $t\bar{t}b\bar{b}$ , invariant-mass resummation, extension to NNLL



# Back-up

# Building $H^{(1)}$

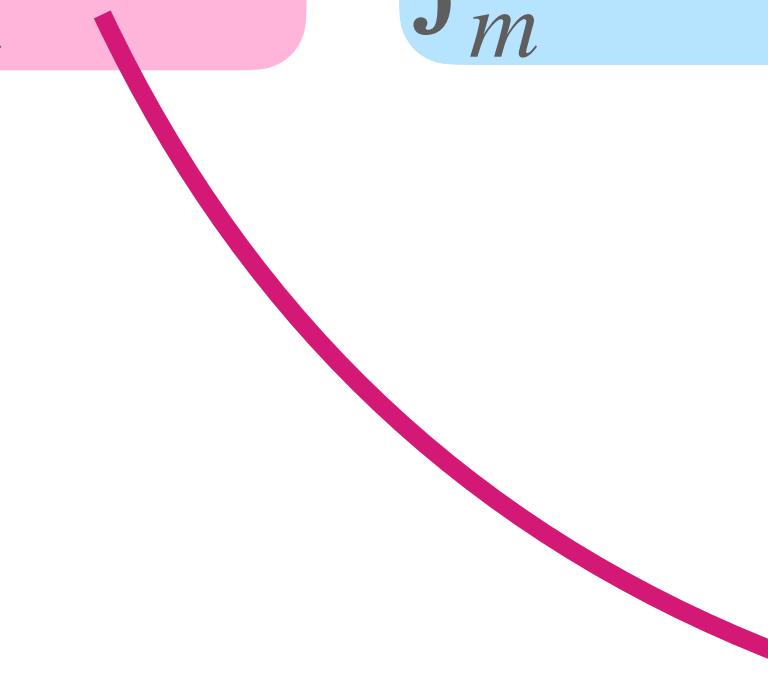
Using standard formula for construction of NLO cross section Catani-Seymour [9605323]

$$\sigma_{ab}^{\text{NLO}} = \int_{m+1} d\sigma_{ab}^{R,c} + \int_m d\sigma_{ab}^V + \int_m d\sigma_{ab}^C$$

Real gluon emission off the initial-state legs contributing to  $C^{(1)}$  after removing  $N$ -dependent terms (double pole and single poles)

Virtual correction (double pole and single pole obtained via FKS [0908.4272], finite part contributes to  $V^{(1)}$ )

Counterterm from PDF factorisation (constant terms contribute to  $C^{(1)}$ , single pole)



# Building $\tilde{S}^{(1)}$

Consider the soft eikonal diagrams

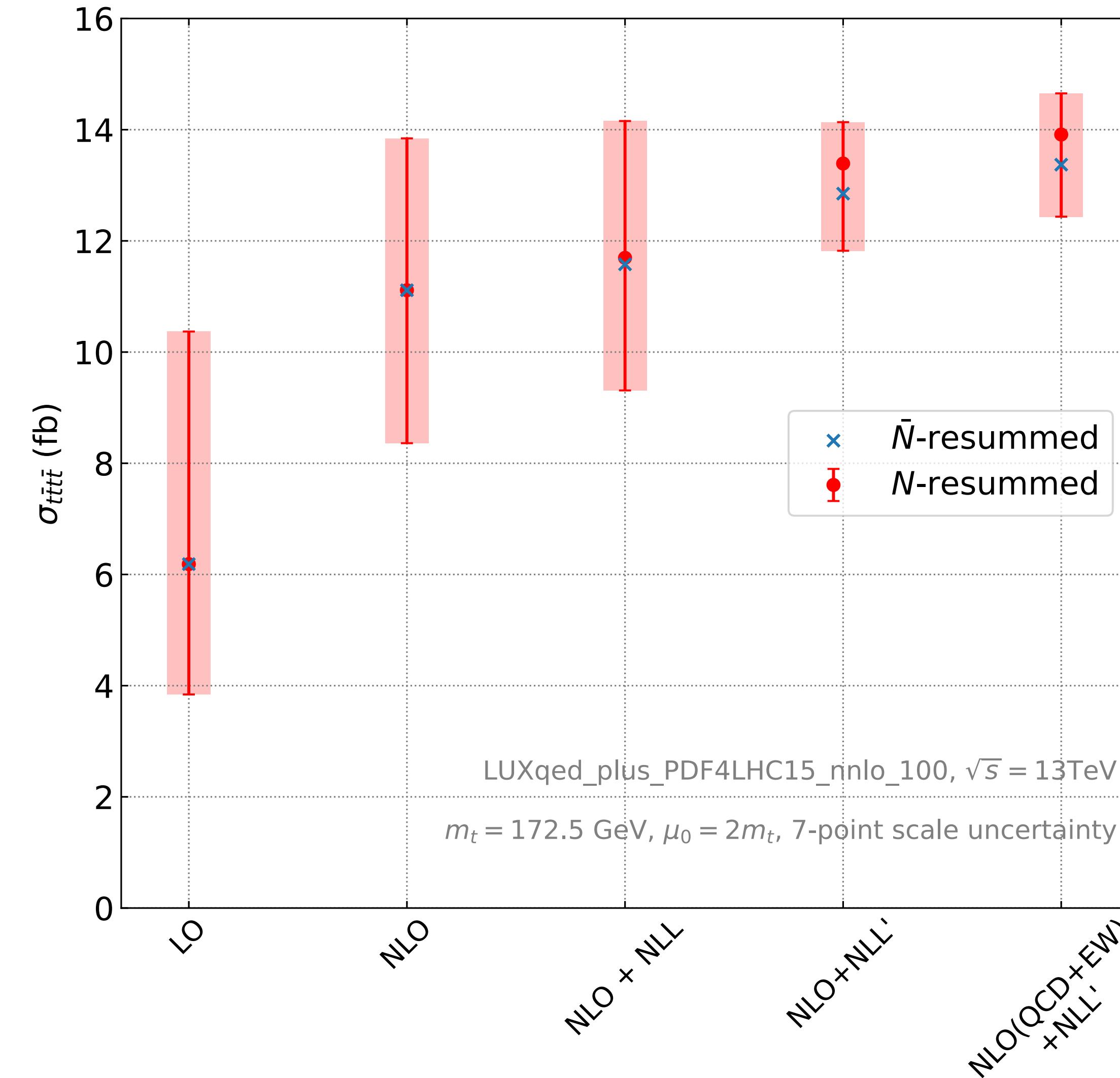
$$\mathcal{S} = -4\pi\mu^{2\epsilon}\alpha_s \sum_{i,j} \frac{p_i \cdot p_j}{p_i \cdot k \, p_j \cdot k} \langle \mathcal{A}^{(0)} | \mathbf{T}_i \cdot \mathbf{T}_j | \mathcal{A}^{(0),\dagger} \rangle$$

Integrating over the 1-gluon phase space gives the kinematic contribution

$$S_{ij} \propto \int dz z^{N-1} (1-z)^{-1-2\epsilon} \int d^{1-2\epsilon} \Omega_{ik} \frac{p_i \cdot p_j}{p_i \cdot k \, p_j \cdot k}$$

Needs to be computed for the II, IF, FI and FF dipoles  
(results consistent with [2102.08943])

# $N$ versus $\bar{N}$ resummation



# Colour basis $q\bar{q}$

$$c_1^{q\bar{q}} = \frac{1}{\sqrt{N_c^3}} \delta_{c_1 c_3} \delta_{c_2 c_4} \delta_{c_6 c_8}$$

$$c_2^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} \delta_{c_1 c_3} t_{c_2 c_4}^{a_1} t_{c_6 c_8}^{a_1}$$

$$c_3^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} t_{c_1 c_3}^{a_1} \delta_{c_2 c_4} t_{c_6 c_8}^{a_1}$$

$$c_4^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} t_{c_1 c_3}^{a_1} t_{c_2 c_4}^{a_1} \delta_{c_6 c_8}$$

$$c_5^{q\bar{q}} = \frac{\sqrt{N_c}}{T_R^2 \sqrt{2(N_c^4 - 5N_c^2 + 4)}} t_{c_1 c_3}^{a_1} d^{a_1 a_2 b_3} t_{c_2 c_4}^{a_2} t_{c_6 c_8}^{b_3}$$

$$c_6^{q\bar{q}} = \frac{1}{T_R^2 \sqrt{2N_c(N_c^2 - 1)}} t_{c_1 c_3}^{a_1} i f^{a_1 a_2 b_3} t_{c_2 c_4}^{a_2} t_{c_6 c_8}^{b_3}$$

# Colour basis $gg$

$$\begin{aligned}
\bar{c}_1^{gg} &= \frac{3\sqrt{3}}{8}c_1^{gg} + \frac{3}{10}\sqrt{\frac{3}{2}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{3}{2}}c_{10}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{11}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{12}^{gg} + \frac{7}{40}c_{13}^{gg}, \\
\bar{c}_2^{gg} &= -\frac{\sqrt{5}}{4}c_1^{gg} + \sqrt{\frac{2}{5}}c_6^{gg} - \frac{1}{2\sqrt{2}}c_{11}^{gg} - \frac{1}{2\sqrt{2}}c_{12}^{gg} + \frac{1}{4}\sqrt{\frac{3}{5}}c_{13}^{gg}, \\
\bar{c}_3^{gg} &= -\frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}, \\
\bar{c}_5^{gg} &= -\frac{1}{2\sqrt{2}}c_1^{gg} - \frac{1}{2}c_6^{gg} - \frac{1}{2}c_{10}^{gg} + \frac{1}{2}\sqrt{\frac{3}{2}}c_{13}^{gg}, \\
\bar{c}_6^{gg} &= -\frac{1}{2}\sqrt{\frac{5}{14}}c_1^{gg} + \frac{3}{2\sqrt{35}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{5}{7}}c_{10}^{gg} + \frac{2}{\sqrt{7}}c_{12}^{gg} - \frac{3}{2}\sqrt{\frac{3}{70}}c_{13}^{gg}, \\
\bar{c}_7^{gg} &= -\frac{1}{2\sqrt{7}}c_1^{gg} + \frac{3}{5\sqrt{14}}c_6^{gg} - \frac{1}{\sqrt{14}}c_{10}^{gg} + \sqrt{\frac{7}{10}}c_{11}^{gg} - \frac{3}{\sqrt{70}}c_{12}^{gg} - \frac{3}{10}\sqrt{\frac{3}{7}}c_{13}^{gg}, \\
\bar{c}_8^{gg} &= \frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}, \\
\bar{c}_{13}^{gg} &= \frac{1}{8}c_1^{gg} + \frac{1}{2\sqrt{2}}c_6^{gg} + \frac{1}{2\sqrt{2}}c_{10}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{11}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{12}^{gg} + \frac{3\sqrt{3}}{8}c_{13}^{gg}, \\
\bar{c}_4^{gg} &= c_{14}^{gg}, \quad \bar{c}_9^{gg} = c_8^{gg}, \quad \bar{c}_{10}^{gg} = c_5^{gg}, \quad \bar{c}_{11}^{gg} = c_4^{gg}, \quad \bar{c}_{12}^{gg} = c_3^{gg}, \quad \bar{c}_{14}^{gg} = c_2^{gg}.
\end{aligned}$$

# Colour basis $gg$

With

$$c_1^{gg} = \frac{1}{T_R} \frac{1}{N_c^2 - 1} t_{c_2 c_4}^{a_1} t_{c_6 c_8}^{a_2},$$

$$c_3^{gg} = \frac{1}{T_R} \frac{1}{\sqrt{2(N_c^4 - 5N_c^2 + 4)}} \delta_{c_2 c_4} d_{a_1 a_2 b_1} t_{c_6 c_8}^{b_1},$$

$$c_5^{gg} = \frac{1}{T_R} \frac{1}{\sqrt{2(N_c^4 - 5N_c^2 + 4)}} t_{c_2 c_4}^{b_1} d_{b_1 a_1 a_2} \delta_{c_6 c_8},$$

$$c_7^{gg} = \frac{1}{T_R^2} \frac{1}{2\sqrt{N_c^4 - 5N_c^2 + 4}} t_{c_2 c_4}^{b_1} d_{b_1 a_1 b_2} i f_{b_2 a_2 b_3} t_{c_6 c_8}^{b_3},$$

$$c_9^{gg} = \frac{1}{T_R^2} \frac{1}{2\sqrt{N_c^4 - 5N_c^2 + 4}} t_{c_2 c_4}^{b_1} i f_{b_1 a_1 b_2} d_{b_2 a_2 b_3} t_{c_6 c_8}^{b_3},$$

$$c_{11}^{gg} = \frac{1}{T_R} \frac{2}{\sqrt{N_c^4 - 5N_c^2 + 4}} t_{c_2 c_4}^{b_1} \mathbf{P}_{a_1 b_1 a_2 b_2}^{10} t_{c_6 c_8}^{b_2},$$

$$c_{13}^{gg} = \frac{1}{T_R} \frac{2}{N_c \sqrt{N_c^2 + 2N_c - 3}} t_{c_2 c_4}^{b_1} \mathbf{P}_{a_1 b_1 a_2 b_2}^{27} t_{c_6 c_8}^{b_2},$$

$$c_2^{gg} = \frac{1}{N_c \sqrt{N_c^2 - 1}} \delta_{a_1 a_2} \delta_{c_2 c_4} \delta_{c_6 c_8},$$

$$c_4^{gg} = \frac{1}{T_R} \frac{1}{N_c \sqrt{2(N_c^2 - 1)}} \delta_{c_2 c_4} i f_{a_1 a_2 b_1} t_{c_6 c_8}^{b_1},$$

$$c_6^{gg} = \frac{1}{T_R^2} \frac{N_c}{2(N_c^2 - 4) \sqrt{N_c^2 - 1}} t_{c_2 c_4}^{b_1} d_{b_1 a_1 b_2} d_{b_2 a_2 b_3} t_{c_6 c_8}^{b_3}$$

$$c_8^{gg} = \frac{1}{T_R} \frac{1}{N_c \sqrt{2(N_c^2 - 1)}} t_{c_2 c_4}^{b_1} i f_{b_1 a_1 a_2} \delta_{c_6 c_8},$$

$$c_{10}^{gg} = \frac{1}{T_R^2} \frac{1}{2N_c \sqrt{N_c^2 - 1}} t_{c_2 c_4}^{b_1} i f_{b_1 a_1 b_2} i f_{b_2 a_2 b_3} t_{c_6 c_8}^{b_3},$$

$$c_{12}^{gg} = \frac{1}{T_R} \frac{2}{\sqrt{N_c^4 - 5N_c^2 + 4}} t_{c_2 c_4}^{b_1} \mathbf{P}_{a_1 b_1 a_2 b_2}^{\overline{10}} t_{c_6 c_8}^{b_2},$$

$$c_{14}^{gg} = \frac{1}{T_R} \frac{2}{N_c \sqrt{N_c^2 - 2N_c - 3}} t_{c_2 c_4}^{b_1} \mathbf{P}_{a_1 b_1 a_2 b_2}^0 t_{c_6 c_8}^{b_2}.$$

# Colour basis $gg$

With

$$\begin{aligned}
 \mathbf{P}_{a_1 b_1 a_2 b_2}^{10(\overline{10})} &= \frac{1}{4} (\delta_{a_1 b_3} \delta_{b_1 b_4} - \delta_{a_1 b_4} \delta_{b_1 b_3}) \left( \delta_{b_3 a_2} \delta_{b_4 b_2} \pm \frac{1}{T_R^2} \text{Tr} [t^{b_3} t^{b_2} t^{b_4} t^{a_2}] \right) - \frac{1}{4N_c T_R} f_{a_1 b_1 b_5} f_{b_5 a_2 b_2}, \\
 \mathbf{P}_{a_1 b_1 a_2 b_2}^{27} &= \frac{1}{4} (\delta_{a_1 b_3} \delta_{b_1 b_4} + \delta_{a_1 b_4} \delta_{b_1 b_3}) \left( \delta_{b_3 a_2} \delta_{b_4 b_2} + \frac{1}{T_R^2} \text{Tr} [t^{b_3} t^{b_2} t^{b_4} t^{a_2}] \right) \\
 &\quad - \frac{1}{4T_R(2+N_c)} d_{a_1 b_1 b_3} d_{b_3 a_2 b_2} - \frac{1}{2N_c(1+N_c)} \delta_{a_1 b_1} \delta_{a_2 b_2}, \\
 \mathbf{P}_{a_1 b_1 a_2 b_2}^0 &= \frac{1}{4} (\delta_{a_1 b_3} \delta_{b_1 b_4} + \delta_{a_1 b_4} \delta_{b_1 b_3}) \left( \delta_{b_3 a_2} \delta_{b_4 b_2} - \frac{1}{T_R^2} \text{Tr} [t^{b_3} t^{b_2} t^{b_4} t^{a_2}] \right) \\
 &\quad + \frac{1}{4T_R(2-N_c)} d_{a_1 b_1 b_3} d_{b_3 a_2 b_2} + \frac{1}{2N_c(1-N_c)} \delta_{a_1 b_1} \delta_{a_2 b_2}
 \end{aligned}$$